

Computer algebra independent integration tests

6-Hyperbolic-functions/6.3-Hyperbolic-tangent/6.3.7-d-hyper-^m-a+b-c-tanh-ⁿ-^p

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3.154	$\int \coth^6(c + dx) (a + b \tanh^2(c + dx))^2 dx$	762
3.155	$\int \coth^7(c + dx) (a + b \tanh^2(c + dx))^2 dx$	765
3.156	$\int \tanh^4(c + dx) (a + b \tanh^2(c + dx))^3 dx$	770
3.157	$\int \tanh^3(c + dx) (a + b \tanh^2(c + dx))^3 dx$	774
3.158	$\int \tanh^2(c + dx) (a + b \tanh^2(c + dx))^3 dx$	781
3.159	$\int \tanh(c + dx) (a + b \tanh^2(c + dx))^3 dx$	785
3.160	$\int (a + b \tanh^2(c + dx))^3 dx$	790
3.161	$\int \coth(c + dx) (a + b \tanh^2(c + dx))^3 dx$	793
3.162	$\int \coth^2(c + dx) (a + b \tanh^2(c + dx))^3 dx$	797
3.163	$\int \coth^3(c + dx) (a + b \tanh^2(c + dx))^3 dx$	800
3.164	$\int \coth^4(c + dx) (a + b \tanh^2(c + dx))^3 dx$	804
3.165	$\int \coth^5(c + dx) (a + b \tanh^2(c + dx))^3 dx$	807
3.166	$\int \coth^6(c + dx) (a + b \tanh^2(c + dx))^3 dx$	811
3.167	$\int \coth^7(c + dx) (a + b \tanh^2(c + dx))^3 dx$	815
3.168	$\int (a + b \tanh^2(c + dx))^4 dx$	820
3.169	$\int (a + b \tanh^2(c + dx))^5 dx$	824
3.170	$\int \frac{\tanh^5(c+dx)}{a+b \tanh^2(c+dx)} dx$	829
3.171	$\int \frac{\tanh^4(c+dx)}{a+b \tanh^2(c+dx)} dx$	833
3.172	$\int \frac{\tanh^3(c+dx)}{a+b \tanh^2(c+dx)} dx$	837
3.173	$\int \frac{\tanh^2(c+dx)}{a+b \tanh^2(c+dx)} dx$	840
3.174	$\int \frac{\tanh(c+dx)}{a+b \tanh^2(c+dx)} dx$	844
3.175	$\int \frac{1}{a+b \tanh^2(c+dx)} dx$	847
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3.182	$\int \frac{\tanh^3(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$	875

3.183	$\int \frac{\tanh^2(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$	878
3.184	$\int \frac{\tanh(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$	882
3.185	$\int \frac{1}{(a+b \tanh^2(c+dx))^2} dx$	885
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3.200	$\int \frac{\coth^4(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$	984
3.201	$\int \frac{1}{(a+b \tanh^2(c+dx))^4} dx$	991
3.202	$\int \sqrt{1 - \tanh^2(x)} dx$	997
3.203	$\int \sqrt{-1 + \tanh^2(x)} dx$	999
3.204	$\int (1 - \tanh^2(x))^{3/2} dx$	1002
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3.206	$\int \frac{1}{\sqrt{1 - \tanh^2(x)}} dx$	1008
3.207	$\int \frac{1}{\sqrt{-1 + \tanh^2(x)}} dx$	1010
3.208	$\int \tanh^5(x) \sqrt{a + b \tanh^2(x)} dx$	1013
3.209	$\int \tanh^4(x) \sqrt{a + b \tanh^2(x)} dx$	1019
3.210	$\int \tanh^3(x) \sqrt{a + b \tanh^2(x)} dx$	1028
3.211	$\int \tanh^2(x) \sqrt{a + b \tanh^2(x)} dx$	1033
3.212	$\int \tanh(x) \sqrt{a + b \tanh^2(x)} dx$	1039

3.213	$\int \sqrt{a + b \tanh^2(x)} dx$	1043
3.214	$\int \coth(x) \sqrt{a + b \tanh^2(x)} dx$	1048
3.215	$\int \coth^2(x) \sqrt{a + b \tanh^2(x)} dx$	1053
3.216	$\int \coth^3(x) \sqrt{a + b \tanh^2(x)} dx$	1057
3.217	$\int \coth^4(x) \sqrt{a + b \tanh^2(x)} dx$	1063
3.218	$\int \coth^5(x) \sqrt{a + b \tanh^2(x)} dx$	1068
3.219	$\int \tanh^3(x) (a + b \tanh^2(x))^{3/2} dx$	1076
3.220	$\int \tanh^2(x) (a + b \tanh^2(x))^{3/2} dx$	1082
3.221	$\int \tanh(x) (a + b \tanh^2(x))^{3/2} dx$	1087
3.222	$\int (a + b \tanh^2(x))^{3/2} dx$	1092
3.223	$\int \coth(x) (a + b \tanh^2(x))^{3/2} dx$	1098
3.224	$\int \coth^2(x) (a + b \tanh^2(x))^{3/2} dx$	1103
3.225	$\int \sqrt{1 + \tanh^2(x)} dx$	1108
3.226	$\int \sqrt{-1 - \tanh^2(x)} dx$	1111
3.227	$\int (1 + \tanh^2(x))^{3/2} dx$	1114
3.228	$\int (-1 - \tanh^2(x))^{3/2} dx$	1118
3.229	$\int \frac{\tanh^5(x)}{\sqrt{a+b \tanh^2(x)}} dx$	1122
3.230	$\int \frac{\tanh^4(x)}{\sqrt{a+b \tanh^2(x)}} dx$	1127
3.231	$\int \frac{\tanh^3(x)}{\sqrt{a+b \tanh^2(x)}} dx$	1133
3.232	$\int \frac{\tanh^2(x)}{\sqrt{a+b \tanh^2(x)}} dx$	1137
3.233	$\int \frac{\tanh(x)}{\sqrt{a+b \tanh^2(x)}} dx$	1142
3.234	$\int \frac{1}{\sqrt{a+b \tanh^2(x)}} dx$	1146
3.235	$\int \frac{\coth(x)}{\sqrt{a+b \tanh^2(x)}} dx$	1149
3.236	$\int \frac{\coth^2(x)}{\sqrt{a+b \tanh^2(x)}} dx$	1154
3.237	$\int \frac{\coth^3(x)}{\sqrt{a+b \tanh^2(x)}} dx$	1158
3.238	$\int \frac{\tanh^5(x)}{(a+b \tanh^2(x))^{3/2}} dx$	1164
3.239	$\int \frac{\tanh^4(x)}{(a+b \tanh^2(x))^{3/2}} dx$	1169
3.240	$\int \frac{\tanh^3(x)}{(a+b \tanh^2(x))^{3/2}} dx$	1176
3.241	$\int \frac{\tanh^2(x)}{(a+b \tanh^2(x))^{3/2}} dx$	1181
3.242	$\int \frac{\tanh(x)}{(a+b \tanh^2(x))^{3/2}} dx$	1185
3.243	$\int \frac{1}{(a+b \tanh^2(x))^{3/2}} dx$	1190

3.244	$\int \frac{\coth(x)}{(a+b \tanh^2(x))^{3/2}} dx$	1194
3.245	$\int \frac{\coth^2(x)}{(a+b \tanh^2(x))^{3/2}} dx$	1200
3.246	$\int \frac{\tanh^6(x)}{(a+b \tanh^2(x))^{5/2}} dx$	1205
3.247	$\int \frac{\tanh^5(x)}{(a+b \tanh^2(x))^{5/2}} dx$	1209
3.248	$\int \frac{\tanh^4(x)}{(a+b \tanh^2(x))^{5/2}} dx$	1215
3.249	$\int \frac{\tanh^3(x)}{(a+b \tanh^2(x))^{5/2}} dx$	1221
3.250	$\int \frac{\tanh^2(x)}{(a+b \tanh^2(x))^{5/2}} dx$	1227
3.251	$\int \frac{\tanh(x)}{(a+b \tanh^2(x))^{5/2}} dx$	1234
3.252	$\int \frac{1}{(a+b \tanh^2(x))^{5/2}} dx$	1240
3.253	$\int \frac{\coth(x)}{(a+b \tanh^2(x))^{5/2}} dx$	1247
3.254	$\int \frac{\coth^2(x)}{(a+b \tanh^2(x))^{5/2}} dx$	1251
3.255	$\int \frac{1}{\sqrt{1+\tanh^2(x)}} dx$	1260
3.256	$\int \frac{1}{\sqrt{-1-\tanh^2(x)}} dx$	1263
3.257	$\int (a + b \tanh^3(c + dx))^2 dx$	1266
3.258	$\int \frac{1}{1+\tanh^3(x)} dx$	1270
3.259	$\int \tanh(x) (a + b \tanh^4(x))^{3/2} dx$	1273
3.260	$\int \tanh(x) \sqrt{a + b \tanh^4(x)} dx$	1277
3.261	$\int \frac{\tanh(x)}{\sqrt{a+b \tanh^4(x)}} dx$	1283
3.262	$\int \frac{\tanh(x)}{(a+b \tanh^4(x))^{3/2}} dx$	1287
3.263	$\int \frac{\tanh(x)}{(a+b \tanh^4(x))^{5/2}} dx$	1292

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Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [263]. This is test number [173].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric $2F1$ functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (263)	% 0.00 (0)
Mathematica	% 100.00 (263)	% 0.00 (0)
Maple	% 94.68 (249)	% 5.32 (14)
Maxima	% 67.30 (177)	% 32.70 (86)
Fricas	% 93.54 (246)	% 6.46 (17)
Sympy	% 15.21 (40)	% 84.79 (223)
Giac	% 88.59 (233)	% 11.41 (30)
Mupad	% 70.34 (185)	% 29.66 (78)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

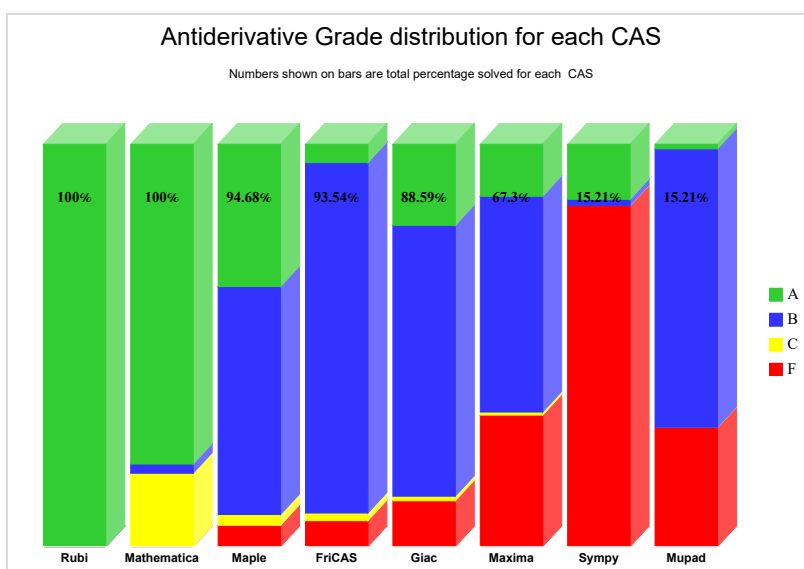
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

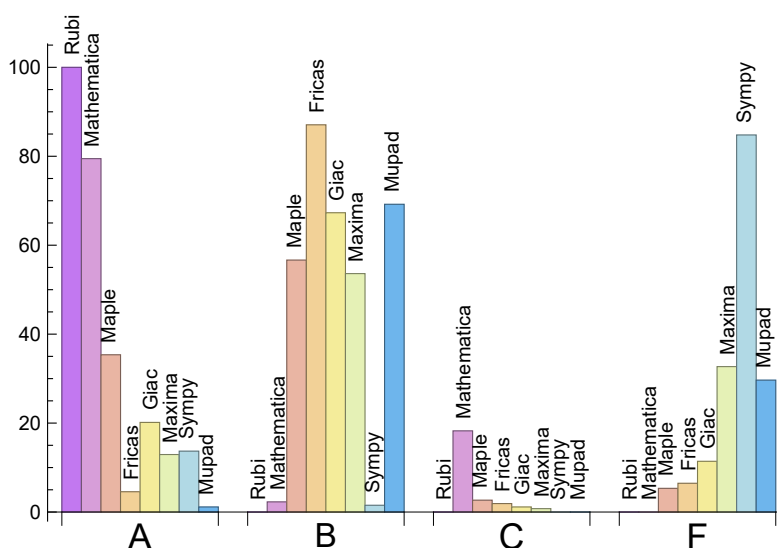
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	79.47	2.28	18.25	0.00
Maple	35.36	56.65	2.66	5.32
Maxima	12.93	53.61	0.76	32.70
Fricas	4.56	87.07	1.90	6.46
Sympy	13.69	1.52	0.00	84.79
Giac	20.15	67.30	1.14	11.41
Mupad	1.14	69.20	0.00	29.66

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure F .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned F(-1).

The third is due to an exception generated. Assigned F(-2). This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	14	100.00 %	0.00 %	0.00 %
Maxima	86	98.84 %	0.00 %	1.16 %
Fricas	17	0.00 %	100.00 %	0.00 %
Sympy	223	81.61 %	18.39 %	0.00 %
Giac	30	16.67 %	0.00 %	83.33 %
Mupad	78	98.72 %	1.28 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

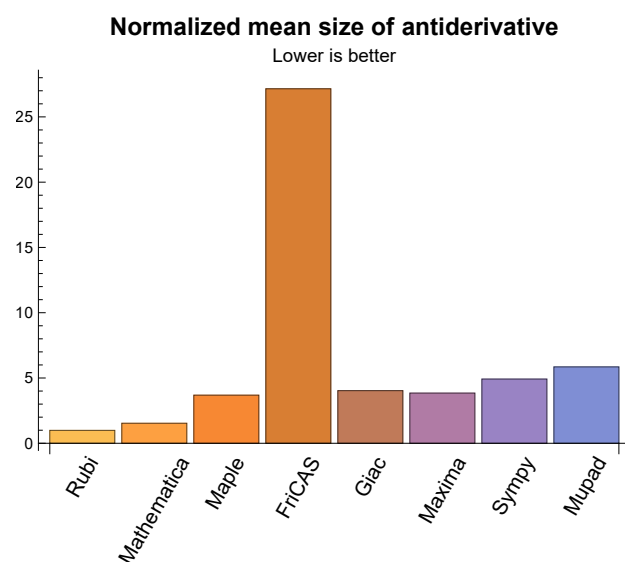
1.3 Performance

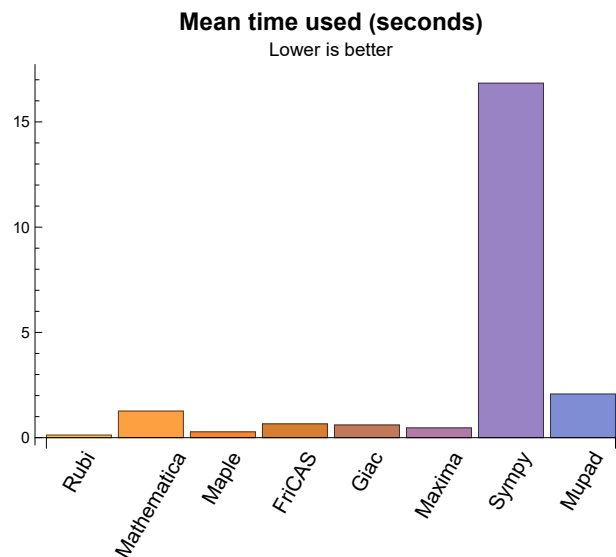
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.12	91.19	0.99	78.00	1.00
Mathematica	1.26	125.33	1.53	86.00	1.00
Maple	0.28	363.71	3.69	189.00	2.28
Maxima	0.47	410.79	3.84	231.00	2.82
Fricas	0.66	2758.11	27.15	1626.50	21.91
Sympy	16.84	406.25	4.92	124.00	1.96
Giac	0.60	374.85	4.03	241.00	2.60
Mupad	2.07	434.32	5.85	177.00	2.67

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{74,76,77,79}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {74,76,77,79}

Maple {74,76,77,79}

Maxima {}

Fricas {}

Sympy {}

Giac {74,76,77,79}

Mupad {76,77,79}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {93,95,98,101,217,236,243,250,252}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate if the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

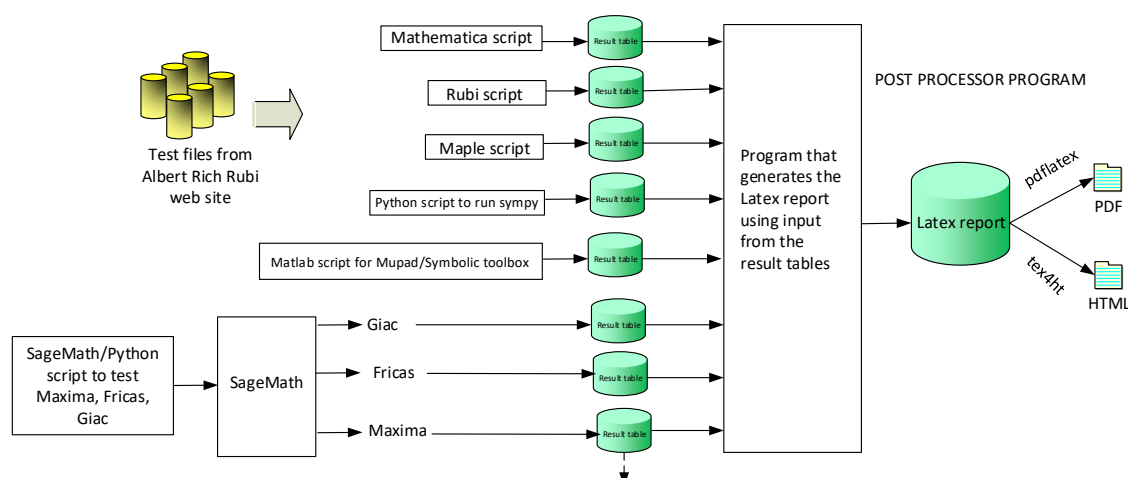
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives $\sin(x)^2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
- The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.
- The following fields present only in Rubi Tables*
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,..}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 30, 32, 33, 35, 38, 40, 41, 43, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 74, 76, 77, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 96, 97, 99, 100, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 141, 143, 145, 147, 148, 149, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 208, 210, 212, 214, 216, 218, 219, 221, 222, 223, 225, 226, 227, 228, 229, 231, 233, 234, 235, 237, 240, 248, 255, 256, 257, 258, 259, 260, 261, 262, 263 }

B grade: { 104, 144, 146, 202, 213, 241 }

C grade: { 26, 28, 29, 31, 34, 36, 37, 39, 42, 44, 45, 47, 73, 75, 78, 80, 93, 95, 98, 101, 133, 140, 142, 150, 154, 209, 211, 215, 217, 220, 224, 230, 232, 236, 238, 239, 242, 243, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 14, 15, 16, 17, 19, 21, 23, 24, 29, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 71, 72, 74, 76, 77, 79, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 139, 140, 141, 142, 143, 149, 150, 151, 152, 153, 154, 155, 161, 162, 163, 164, 165, 166, 167, 170, 171, 172, 173, 174, 180, 182, 184, 202, 203, 204, 205, 206, 207, 257, 258, 260, 261 }

B grade: { 10, 12, 18, 20, 22, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 62, 70, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 144, 145, 146, 147, 148, 156, 157, 158, 159, 160, 168, 169, 175, 176, 177, 178, 179, 181, 183, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 208, 209, 210, 211, 212, 213, 219, 220, 221, 222, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 238, 239, 240, 241, 242, 243, 246, 247, 248, 249, 250, 251, 252, 255, 256 }

C grade: { 73, 75, 78, 80, 259, 262, 263 }

F grade: { 214, 215, 216, 217, 218, 223, 224, 235, 236, 237, 244, 245, 253, 254 }

2.1.4 Maxima

A grade: { 5, 6, 30, 49, 50, 51, 52, 53, 54, 66, 68, 74, 76, 77, 79, 81, 86, 94, 102, 110, 112, 121, 138, 139, 140, 150, 172, 174, 175, 176, 178, 202, 204, 206 }

B grade: { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 32, 33, 35, 38, 40, 41, 43, 46, 48, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 69, 70, 71, 72, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 107, 114, 116, 119, 123, 125, 128, 130, 132, 134, 135, 136, 137, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 177, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 207, 257, 258 }

C grade: { 203, 205 }

F grade: { 26, 28, 29, 31, 34, 36, 37, 39, 42, 44, 45, 47, 73, 75, 78, 80, 106, 108, 109, 111, 113, 115, 117, 118, 120, 122, 124, 126, 127, 129, 131, 133, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 259, 260, 261, 262, 263 }

2.1.5 FriCAS

A grade: { 1, 3, 4, 81, 82, 83, 89, 138, 203, 205, 206, 207 }

B grade: { 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 78, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 202, 204, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 223, 224, 225, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 247, 248, 249, 250, 251, 252, 254, 255, 257, 258, 260, 261, 262 }

C grade: { 75, 80, 226, 228, 256 }

F grade: { 41, 42, 47, 48, 65, 73, 74, 76, 77, 79, 200, 201, 220, 246, 253, 259, 263 }

2.1.6 Sympy

A grade: { 77, 79, 134, 135, 136, 137, 138, 144, 145, 146, 147, 148, 156, 157, 158, 159, 160, 168, 169, 170, 171, 172, 173, 174, 175, 191, 193, 195, 206, 208, 210, 212, 233, 242, 251, 257 }

B grade: { 140, 219, 221, 258 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 139, 141, 142, 143, 149, 150, 151, 152, 153, 154, 155, 161, 162, 163, 164, 165, 166, 167, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 192, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 207, 209, 211, 213, 214, 215, 216, 217, 218, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 256, 259, 260, 261, 262, 263 }

2.1.7 Giac

A grade: { 5, 6, 8, 14, 23, 30, 49, 50, 51, 52, 53, 54, 58, 60, 61, 63, 66, 68, 73, 74, 75, 76, 77, 78, 79, 80, 81, 85, 93, 110, 112, 137, 138, 139, 140, 141, 150, 153, 171, 173, 174, 175, 176, 177, 178, 189, 202, 203, 205, 206, 207, 257, 258 }

B grade: { 1, 2, 3, 4, 7, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 55, 56, 57, 59, 62, 64, 65, 67, 69, 70, 71, 72, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 204, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 234, 255 }

C grade: { 226, 228, 256 }

F grade: { 229, 230, 231, 232, 233, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 259, 260, 261, 262, 263 }

2.1.8 Mupad

A grade: { 76, 77, 79 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 190, 191, 192, 193, 194, 195, 196, 201, 202, 203, 204, 205, 206, 207, 208, 210, 212, 219, 221, 225, 226, 227, 229, 231, 233, 234, 238, 240, 242, 247, 249, 251, 255, 256, 257, 258 }

C grade: { }

F grade: { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 74, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 186, 187, 188, 189, 197, 198, 199, 200, 209, 211, 213, 214, 215, 216, 217, 218, 220, 222, 223, 224, 228, 230, 232, 235, 236, 237, 239, 241, 243, 244, 245, 246, 248, 250, 252, 253, 254, 259, 260, 261, 262, 263 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	56	96	154	120	0	145	101
normalized size	1	1.00	0.77	1.32	2.11	1.64	0.00	1.99	1.38
time (sec)	N/A	0.076	0.369	0.267	1.130	0.512	0.000	0.202	0.245
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	73	75	136	91	0	120	99
normalized size	1	1.00	1.55	1.60	2.89	1.94	0.00	2.55	2.11
time (sec)	N/A	0.055	0.054	0.263	0.391	0.785	0.000	0.168	1.174
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	66	101	71	0	109	64
normalized size	1	1.00	0.93	1.50	2.30	1.61	0.00	2.48	1.45
time (sec)	N/A	0.051	0.210	0.176	1.384	0.651	0.000	0.176	0.145
Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	45	44	67	42	0	79	27
normalized size	1	1.00	1.80	1.76	2.68	1.68	0.00	3.16	1.08
time (sec)	N/A	0.032	0.056	0.171	0.367	0.563	0.000	0.150	0.118
Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	52	27	40	167	0	52	64
normalized size	1	1.00	2.00	1.04	1.54	6.42	0.00	2.00	2.46
time (sec)	N/A	0.033	0.029	0.249	0.348	0.745	0.000	0.155	0.118

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	23	39	88	0	45	43
normalized size	1	1.00	1.00	0.96	1.62	3.67	0.00	1.88	1.79
time (sec)	N/A	0.033	0.023	0.333	0.340	0.471	0.000	0.162	1.050
Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	87	50	152	924	0	113	156
normalized size	1	1.00	1.71	0.98	2.98	18.12	0.00	2.22	3.06
time (sec)	N/A	0.059	0.048	0.345	0.384	0.660	0.000	0.148	1.133
Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	61	55	113	244	0	80	173
normalized size	1	1.00	1.39	1.25	2.57	5.55	0.00	1.82	3.93
time (sec)	N/A	0.043	0.072	0.371	0.358	0.723	0.000	0.151	1.097
Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	94	166	295	394	0	295	293
normalized size	1	1.00	0.80	1.41	2.50	3.34	0.00	2.50	2.48
time (sec)	N/A	0.133	1.449	0.293	0.358	0.610	0.000	0.458	0.309
Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	71	148	265	259	0	290	215
normalized size	1	1.00	0.92	1.92	3.44	3.36	0.00	3.77	2.79
time (sec)	N/A	0.098	0.538	0.293	0.365	0.524	0.000	0.420	0.287
Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	70	118	217	291	0	215	248
normalized size	1	1.00	0.89	1.49	2.75	3.68	0.00	2.72	3.14
time (sec)	N/A	0.111	0.902	0.210	0.347	0.476	0.000	0.326	1.177

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	46	98	171	167	0	162	154
normalized size	1	1.00	0.94	2.00	3.49	3.41	0.00	3.31	3.14
time (sec)	N/A	0.053	0.341	0.210	0.354	0.545	0.000	0.216	0.191
Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	50	63	196	890	0	126	160
normalized size	1	1.00	0.98	1.24	3.84	17.45	0.00	2.47	3.14
time (sec)	N/A	0.065	0.169	0.243	0.349	0.897	0.000	0.232	0.154
Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	68	136	264	0	86	209
normalized size	1	1.00	0.93	1.48	2.96	5.74	0.00	1.87	4.54
time (sec)	N/A	0.057	0.492	0.431	0.339	0.516	0.000	0.265	1.157
Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	96	67	181	2462	0	168	261
normalized size	1	1.00	1.17	0.82	2.21	30.02	0.00	2.05	3.18
time (sec)	N/A	0.119	1.591	0.382	0.316	0.514	0.000	0.266	0.162
Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	59	81	210	393	0	143	143
normalized size	1	1.00	0.82	1.12	2.92	5.46	0.00	1.99	1.99
time (sec)	N/A	0.079	0.520	0.455	0.319	0.714	0.000	0.259	1.080
Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	125	246	480	879	0	507	730
normalized size	1	1.00	0.69	1.35	2.64	4.83	0.00	2.79	4.01
time (sec)	N/A	0.222	3.925	0.357	0.340	0.551	0.000	0.793	1.390

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	91	239	439	540	0	442	361
normalized size	1	1.00	0.87	2.28	4.18	5.14	0.00	4.21	3.44
time (sec)	N/A	0.123	0.303	0.319	0.350	0.588	0.000	0.660	0.412
Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	139	95	180	377	725	0	395	668
normalized size	1	1.14	0.78	1.48	3.09	5.94	0.00	3.24	5.48
time (sec)	N/A	0.185	2.198	0.234	0.329	0.875	0.000	0.643	0.315
Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	63	170	321	383	0	322	308
normalized size	1	1.00	0.90	2.43	4.59	5.47	0.00	4.60	4.40
time (sec)	N/A	0.068	0.880	0.230	0.326	0.636	0.000	0.391	1.245
Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	79	118	560	2277	0	262	317
normalized size	1	1.00	0.94	1.40	6.67	27.11	0.00	3.12	3.77
time (sec)	N/A	0.084	0.326	0.245	0.325	0.793	0.000	0.365	0.246
Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	70	141	348	572	0	202	590
normalized size	1	1.00	1.09	2.20	5.44	8.94	0.00	3.16	9.22
time (sec)	N/A	0.064	0.714	0.510	0.326	0.487	0.000	0.443	1.235
Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	127	103	403	5037	0	281	412
normalized size	1	1.00	0.84	0.68	2.65	33.14	0.00	1.85	2.71
time (sec)	N/A	0.225	6.197	0.391	0.332	0.967	0.000	0.495	1.256

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	87	136	493	925	0	257	622
normalized size	1	1.00	0.89	1.39	5.03	9.44	0.00	2.62	6.35
time (sec)	N/A	0.097	1.255	0.529	0.344	0.594	0.000	0.462	0.279
Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	93	865	514	2024	0	301	250
normalized size	1	1.00	0.79	7.33	4.36	17.15	0.00	2.55	2.12
time (sec)	N/A	0.172	0.273	0.344	0.510	0.802	0.000	1.093	1.678
Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	135	202	0	1367	0	810	955
normalized size	1	1.00	1.80	2.69	0.00	18.23	0.00	10.80	12.73
time (sec)	N/A	0.128	0.577	0.301	0.000	0.650	0.000	0.495	2.655
Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	67	605	316	916	0	176	198
normalized size	1	1.00	0.86	7.76	4.05	11.74	0.00	2.26	2.54
time (sec)	N/A	0.104	0.153	0.316	0.461	0.566	0.000	0.590	1.512
Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	107	104	0	666	0	1065	520
normalized size	1	1.00	2.02	1.96	0.00	12.57	0.00	20.09	9.81
time (sec)	N/A	0.063	0.273	0.217	0.000	0.915	0.000	0.250	2.002
Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	123	69	0	587	0	958	284
normalized size	1	1.00	2.24	1.25	0.00	10.67	0.00	17.42	5.16
time (sec)	N/A	0.078	0.211	0.394	0.000	0.551	0.000	0.215	1.921

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	413	62	618	0	69	136
normalized size	1	1.00	1.00	8.60	1.29	12.88	0.00	1.44	2.83
time (sec)	N/A	0.063	0.128	0.356	0.433	0.538	0.000	0.348	1.311
Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	170	181	0	1790	0	475	787
normalized size	1	1.00	2.00	2.13	0.00	21.06	0.00	5.59	9.26
time (sec)	N/A	0.119	0.697	0.391	0.000	1.272	0.000	0.377	1.802
Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	71	750	134	1628	0	132	254
normalized size	1	1.00	1.01	10.71	1.91	23.26	0.00	1.89	3.63
time (sec)	N/A	0.088	0.319	0.372	0.463	0.625	0.000	0.360	1.439
Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	132	1246	1690	7366	0	525	-1
normalized size	1	1.00	0.69	6.49	8.80	38.36	0.00	2.73	-0.01
time (sec)	N/A	0.252	1.039	0.350	0.718	1.078	0.000	1.906	0.000
Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	160	267	0	5025	0	1929	-1
normalized size	1	1.00	1.29	2.15	0.00	40.52	0.00	15.56	-0.01
time (sec)	N/A	0.218	1.371	0.321	0.000	0.755	0.000	0.943	0.000
Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	105	1128	840	3918	0	400	-1
normalized size	1	1.00	0.80	8.55	6.36	29.68	0.00	3.03	-0.01
time (sec)	N/A	0.165	0.737	0.335	0.586	0.812	0.000	1.132	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	133	167	0	2252	0	924	-1
normalized size	1	1.00	1.45	1.82	0.00	24.48	0.00	10.04	-0.01
time (sec)	N/A	0.075	0.775	0.318	0.000	0.613	0.000	0.408	0.000
Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	175	331	0	2614	0	1405	-1
normalized size	1	1.00	1.70	3.21	0.00	25.38	0.00	13.64	-0.01
time (sec)	N/A	0.144	0.746	0.472	0.000	1.094	0.000	0.373	0.000
Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	86	552	212	2562	0	227	-1
normalized size	1	1.00	1.05	6.73	2.59	31.24	0.00	2.77	-0.01
time (sec)	N/A	0.076	0.502	0.459	0.505	0.633	0.000	0.621	0.000
Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	203	367	0	6335	0	1074	-1
normalized size	1	1.00	1.44	2.60	0.00	44.93	0.00	7.62	-0.01
time (sec)	N/A	0.209	4.346	0.463	0.000	0.733	0.000	0.713	0.000
Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	114	1012	282	5062	0	221	-1
normalized size	1	1.00	1.01	8.96	2.50	44.80	0.00	1.96	-0.01
time (sec)	N/A	0.154	0.929	0.467	0.550	0.859	0.000	0.563	0.000
Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	184	2366	3392	0	0	923	-1
normalized size	1	1.00	0.77	9.86	14.13	0.00	0.00	3.85	-0.00
time (sec)	N/A	0.345	0.858	0.394	1.120	0.000	0.000	3.004	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-2)	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	227	341	0	0	0	1748	-1
normalized size	1	1.00	1.37	2.05	0.00	0.00	0.00	10.53	-0.01
time (sec)	N/A	0.286	2.004	0.336	0.000	0.000	0.000	1.639	0.000
Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	158	2110	1806	12965	0	584	-1
normalized size	1	1.00	0.85	11.41	9.76	70.08	0.00	3.16	-0.01
time (sec)	N/A	0.251	1.340	0.355	0.817	1.712	0.000	1.940	0.000
Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	157	252	0	7119	0	1718	-1
normalized size	1	1.00	1.25	2.00	0.00	56.50	0.00	13.63	-0.01
time (sec)	N/A	0.097	1.828	0.305	0.000	1.381	0.000	0.615	0.000
Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	236	1132	0	10716	0	2033	-1
normalized size	1	1.00	1.51	7.26	0.00	68.69	0.00	13.03	-0.01
time (sec)	N/A	0.254	1.513	0.447	0.000	1.775	0.000	0.502	0.000
Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	109	816	478	8312	0	351	-1
normalized size	1	1.00	0.97	7.29	4.27	74.21	0.00	3.13	-0.01
time (sec)	N/A	0.087	1.060	0.439	0.619	0.675	0.000	0.802	0.000
Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	269	1083	0	0	0	1877	-1
normalized size	1	1.00	1.37	5.53	0.00	0.00	0.00	9.58	-0.01
time (sec)	N/A	0.334	4.447	0.476	0.000	0.000	0.000	1.026	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	149	1416	615	0	0	407	-1
normalized size	1	1.00	0.99	9.38	4.07	0.00	0.00	2.70	-0.01
time (sec)	N/A	0.201	1.401	0.480	0.700	0.000	0.000	0.847	0.000
Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	92	122	194	1530	0	206	156
normalized size	1	1.00	0.70	0.92	1.47	11.59	0.00	1.56	1.18
time (sec)	N/A	0.172	0.200	0.299	0.418	0.571	0.000	0.251	0.269
Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	104	129	174	1070	0	142	171
normalized size	1	1.00	1.06	1.32	1.78	10.92	0.00	1.45	1.74
time (sec)	N/A	0.120	0.276	0.353	0.412	0.740	0.000	0.192	0.228
Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	69	79	141	924	0	142	115
normalized size	1	1.00	0.69	0.79	1.41	9.24	0.00	1.42	1.15
time (sec)	N/A	0.117	0.115	0.205	0.410	0.590	0.000	0.177	1.147
Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	72	85	105	528	0	95	128
normalized size	1	1.00	1.14	1.35	1.67	8.38	0.00	1.51	2.03
time (sec)	N/A	0.076	0.126	0.280	0.414	0.948	0.000	0.172	0.139
Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	75	65	83	522	0	74	233
normalized size	1	1.00	1.53	1.33	1.69	10.65	0.00	1.51	4.76
time (sec)	N/A	0.074	0.032	0.367	0.408	0.958	0.000	0.159	2.471

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	44	141	0	45	79
normalized size	1	1.00	1.00	0.90	1.52	4.86	0.00	1.55	2.72
time (sec)	N/A	0.034	0.027	0.356	0.302	1.299	0.000	0.189	0.165
Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	95	62	156	1188	0	143	173
normalized size	1	1.00	1.34	0.87	2.20	16.73	0.00	2.01	2.44
time (sec)	N/A	0.090	0.028	0.452	0.421	1.683	0.000	0.165	2.481
Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	74	60	184	1739	0	149	162
normalized size	1	1.00	1.32	1.07	3.29	31.05	0.00	2.66	2.89
time (sec)	N/A	0.058	0.223	0.385	0.401	0.739	0.000	0.178	1.235
Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	206	156	243	379	5034	0	376	359
normalized size	1	1.21	0.92	1.43	2.23	29.61	0.00	2.21	2.11
time (sec)	N/A	0.297	2.829	0.320	0.408	0.749	0.000	0.711	0.444
Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	121	250	348	3341	0	300	397
normalized size	1	1.00	0.66	1.37	1.91	18.36	0.00	1.65	2.18
time (sec)	N/A	0.225	0.780	0.378	0.407	0.660	0.000	0.485	1.436
Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	159	137	173	301	3649	0	298	306
normalized size	1	1.23	1.06	1.34	2.33	28.29	0.00	2.31	2.37
time (sec)	N/A	0.205	1.602	0.252	0.411	0.742	0.000	0.444	1.360

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	90	179	253	2230	0	214	338
normalized size	1	1.00	0.73	1.46	2.06	18.13	0.00	1.74	2.75
time (sec)	N/A	0.147	0.438	0.320	0.409	0.720	0.000	0.313	1.290
Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	106	134	447	2498	0	178	522
normalized size	1	1.00	1.08	1.37	4.56	25.49	0.00	1.82	5.33
time (sec)	N/A	0.136	0.171	0.392	0.407	0.775	0.000	0.285	3.283
Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	94	98	256	518	0	122	483
normalized size	1	1.00	2.00	2.09	5.45	11.02	0.00	2.60	10.28
time (sec)	N/A	0.058	0.252	0.457	0.315	0.566	0.000	0.322	1.264
Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	138	108	378	4642	0	192	561
normalized size	1	1.00	1.29	1.01	3.53	43.38	0.00	1.79	5.24
time (sec)	N/A	0.160	0.164	0.529	0.411	1.007	0.000	0.312	2.896
Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	147	146	468	4125	0	249	344
normalized size	1	1.00	1.52	1.51	4.82	42.53	0.00	2.57	3.55
time (sec)	N/A	0.098	0.218	0.493	0.410	0.617	0.000	0.331	0.261
Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	306	294	385	647	0	0	708	682
normalized size	1	1.11	1.07	1.40	2.35	0.00	0.00	2.57	2.48
time (sec)	N/A	0.494	6.271	0.409	0.432	0.000	0.000	1.633	1.703

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	291	506	604	8462	0	601	757
normalized size	1	1.00	0.83	1.44	1.72	24.11	0.00	1.71	2.16
time (sec)	N/A	0.350	6.614	0.486	0.429	0.603	0.000	1.248	1.598
Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	241	244	289	544	9862	0	592	617
normalized size	1	1.10	1.11	1.31	2.47	44.83	0.00	2.69	2.80
time (sec)	N/A	0.315	6.253	0.293	0.425	0.637	0.000	1.276	0.556
Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	233	410	484	6410	0	485	707
normalized size	1	1.00	0.87	1.52	1.80	23.83	0.00	1.80	2.63
time (sec)	N/A	0.249	6.329	0.428	0.424	0.525	0.000	0.678	1.487
Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	154	339	654	7127	0	422	671
normalized size	1	1.00	0.70	1.55	2.99	32.54	0.00	1.93	3.06
time (sec)	N/A	0.264	3.206	0.524	0.433	0.550	0.000	0.598	6.399
Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	113	171	679	1192	0	311	1515
normalized size	1	1.00	1.59	2.41	9.56	16.79	0.00	4.38	21.34
time (sec)	N/A	0.065	0.770	0.505	0.336	0.404	0.000	0.765	1.378
Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	243	286	586	10985	0	434	731
normalized size	1	1.00	1.05	1.23	2.53	47.35	0.00	1.87	3.15
time (sec)	N/A	0.316	6.365	0.614	0.427	0.606	0.000	0.841	8.048

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	213	219	997	9459	0	437	646
normalized size	1	1.00	1.54	1.59	7.22	68.54	0.00	3.17	4.68
time (sec)	N/A	0.115	0.197	0.556	0.426	0.582	0.000	0.825	0.540
Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	491	491	645	603	0	0	0	362	3313
normalized size	1	1.00	1.31	1.23	0.00	0.00	0.00	0.74	6.75
time (sec)	N/A	0.892	4.831	0.509	0.000	0.000	0.000	0.759	4.355
Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	F(-1)
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	826	346	0	0	0	344	-1
normalized size	1	0.00	25.03	10.48	0.00	0.00	0.00	10.42	-0.03
time (sec)	N/A	0.046	0.535	0.467	0.000	0.000	0.000	3.164	0.000
Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	423	356	0	10695	0	223	2100
normalized size	1	1.00	1.10	0.93	0.00	27.85	0.00	0.58	5.47
time (sec)	N/A	0.629	3.813	0.459	0.000	1.413	0.000	0.365	2.815
Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	F(-1)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	409	164	0	0	0	189	4474
normalized size	1	0.00	13.19	5.29	0.00	0.00	0.00	6.10	144.32
time (sec)	N/A	0.028	0.269	0.528	0.000	0.000	0.000	1.841	87.988
Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	319	98	0	0	0	147	3679
normalized size	1	0.00	10.29	3.16	0.00	0.00	0.00	4.74	118.68
time (sec)	N/A	0.041	0.179	0.619	0.000	0.000	0.000	1.398	16.503

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	190	121	0	640	0	21	669
normalized size	1	1.00	1.21	0.77	0.00	4.08	0.00	0.13	4.26
time (sec)	N/A	0.140	0.148	0.570	0.000	0.720	0.000	0.252	8.707
Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	201	144	0	0	0	68	3643
normalized size	1	0.00	6.09	4.36	0.00	0.00	0.00	2.06	110.39
time (sec)	N/A	0.047	0.386	0.595	0.000	0.000	0.000	0.788	26.921
Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	322	187	0	1954	0	180	4563
normalized size	1	1.00	1.50	0.87	0.00	9.09	0.00	0.84	21.22
time (sec)	N/A	0.239	3.618	0.600	0.000	1.869	0.000	0.285	3.218
Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	44	82	104	63	0	107	74
normalized size	1	1.00	0.70	1.30	1.65	1.00	0.00	1.70	1.17
time (sec)	N/A	0.050	0.169	0.325	0.349	0.412	0.000	0.177	0.212
Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	44	37	83	45	0	94	74
normalized size	1	1.00	1.47	1.23	2.77	1.50	0.00	3.13	2.47
time (sec)	N/A	0.036	0.018	0.318	0.344	0.392	0.000	0.159	0.210
Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	32	54	69	30	0	81	27
normalized size	1	1.00	0.97	1.64	2.09	0.91	0.00	2.45	0.82
time (sec)	N/A	0.039	0.060	0.208	0.322	0.394	0.000	0.140	0.146

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	47	37	55	102	0	56	66
normalized size	1	1.00	1.74	1.37	2.04	3.78	0.00	2.07	2.44
time (sec)	N/A	0.032	0.037	0.197	0.440	0.408	0.000	0.143	0.139
Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	48	65	80	323	0	63	125
normalized size	1	1.00	1.20	1.62	2.00	8.08	0.00	1.58	3.12
time (sec)	N/A	0.031	0.027	0.270	0.426	0.393	0.000	0.145	0.132
Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	53	34	159	0	59	59
normalized size	1	1.00	1.00	1.89	1.21	5.68	0.00	2.11	2.11
time (sec)	N/A	0.029	0.013	0.368	0.344	0.382	0.000	0.156	1.218
Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	93	103	181	1046	0	132	280
normalized size	1	1.00	1.41	1.56	2.74	15.85	0.00	2.00	4.24
time (sec)	N/A	0.046	0.034	0.386	0.438	0.401	0.000	0.154	1.251
Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	86	75	371	345	0	95	304
normalized size	1	1.00	1.79	1.56	7.73	7.19	0.00	1.98	6.33
time (sec)	N/A	0.040	0.053	0.361	0.347	0.393	0.000	0.159	0.165
Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	63	124	171	95	0	189	102
normalized size	1	1.00	0.74	1.46	2.01	1.12	0.00	2.22	1.20
time (sec)	N/A	0.087	0.327	0.380	0.361	0.412	0.000	0.388	0.266

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	71	98	161	519	0	164	130
normalized size	1	1.00	1.31	1.81	2.98	9.61	0.00	3.04	2.41
time (sec)	N/A	0.060	0.458	0.274	0.502	0.419	0.000	0.364	0.257
Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	54	96	140	105	0	170	77
normalized size	1	1.00	1.06	1.88	2.75	2.06	0.00	3.33	1.51
time (sec)	N/A	0.076	0.411	0.213	0.340	0.427	0.000	0.276	1.289
Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	122	152	774	0	137	182
normalized size	1	1.00	0.90	2.03	2.53	12.90	0.00	2.28	3.03
time (sec)	N/A	0.082	0.178	0.306	0.446	0.428	0.000	0.248	0.237
Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	427	173	199	1373	0	157	303
normalized size	1	1.00	4.69	1.90	2.19	15.09	0.00	1.73	3.33
time (sec)	N/A	0.085	7.277	0.370	0.461	0.418	0.000	0.203	0.162
Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	126	53	391	0	169	482
normalized size	1	1.00	1.00	2.57	1.08	7.98	0.00	3.45	9.84
time (sec)	N/A	0.053	0.213	0.458	0.329	0.402	0.000	0.229	1.240
Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	792	236	345	2824	0	291	572
normalized size	1	1.00	6.34	1.89	2.76	22.59	0.00	2.33	4.58
time (sec)	N/A	0.152	8.761	0.506	0.426	0.470	0.000	0.215	0.181

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	83	158	928	677	0	238	732
normalized size	1	1.00	1.09	2.08	12.21	8.91	0.00	3.13	9.63
time (sec)	N/A	0.070	0.577	0.466	0.342	0.403	0.000	0.244	1.220
Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	81	184	267	227	0	285	133
normalized size	1	1.00	0.89	2.02	2.93	2.49	0.00	3.13	1.46
time (sec)	N/A	0.130	0.703	0.380	0.329	0.423	0.000	0.755	1.407
Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	494	206	284	1840	0	279	232
normalized size	1	1.00	5.68	2.37	3.26	21.15	0.00	3.21	2.67
time (sec)	N/A	0.105	6.885	0.380	0.431	0.451	0.000	0.656	0.355
Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	69	148	256	369	0	267	243
normalized size	1	1.00	0.88	1.90	3.28	4.73	0.00	3.42	3.12
time (sec)	N/A	0.090	0.969	0.240	0.410	0.411	0.000	0.599	0.289
Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	89	257	295	2411	0	257	355
normalized size	1	1.00	0.90	2.60	2.98	24.35	0.00	2.60	3.59
time (sec)	N/A	0.116	0.422	0.397	0.698	0.452	0.000	0.384	0.309
Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	1341	334	362	3465	0	321	535
normalized size	1	1.00	9.00	2.24	2.43	23.26	0.00	2.15	3.59
time (sec)	N/A	0.154	16.676	0.421	0.432	0.446	0.000	0.309	1.394

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	227	71	786	0	347	1050
normalized size	1	1.00	1.00	3.39	1.06	11.73	0.00	5.18	15.67
time (sec)	N/A	0.061	0.179	0.547	0.325	0.396	0.000	0.392	1.333
Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	158	421	553	6114	0	517	951
normalized size	1	1.00	0.80	2.13	2.79	30.88	0.00	2.61	4.80
time (sec)	N/A	0.239	11.837	0.589	0.693	1.184	0.000	0.410	1.364
Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	218	269	1847	1185	0	447	1424
normalized size	1	1.00	2.14	2.64	18.11	11.62	0.00	4.38	13.96
time (sec)	N/A	0.089	0.910	0.563	0.350	0.401	0.000	0.412	1.333
Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	115	857	514	2180	0	326	967
normalized size	1	1.00	0.96	7.14	4.28	18.17	0.00	2.72	8.06
time (sec)	N/A	0.168	0.303	0.465	0.509	0.482	0.000	1.068	1.930
Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	79	468	0	1850	0	829	2194
normalized size	1	1.00	0.99	5.85	0.00	23.12	0.00	10.36	27.42
time (sec)	N/A	0.109	0.430	0.444	0.000	0.493	0.000	0.487	2.877
Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	77	608	316	948	0	172	880
normalized size	1	1.00	1.00	7.90	4.10	12.31	0.00	2.23	11.43
time (sec)	N/A	0.103	0.161	0.432	0.616	0.447	0.000	0.564	1.933

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	315	0	766	0	1117	154
normalized size	1	1.00	1.00	5.94	0.00	14.45	0.00	21.08	2.91
time (sec)	N/A	0.070	0.107	0.376	0.000	0.451	0.000	0.268	1.676
Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	235	0	511	0	480	147
normalized size	1	1.00	1.00	6.53	0.00	14.19	0.00	13.33	4.08
time (sec)	N/A	0.044	0.047	0.317	0.000	0.423	0.000	0.182	0.335
Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	363	36	455	0	44	81
normalized size	1	1.00	1.00	11.34	1.12	14.22	0.00	1.38	2.53
time (sec)	N/A	0.056	0.058	0.356	0.525	0.455	0.000	0.329	1.459
Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	494	0	540	0	563	449
normalized size	1	1.00	1.00	8.98	0.00	9.82	0.00	10.24	8.16
time (sec)	N/A	0.073	0.208	0.344	0.000	0.448	0.000	0.397	1.705
Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	648	63	649	0	84	176
normalized size	1	1.00	1.00	12.96	1.26	12.98	0.00	1.68	3.52
time (sec)	N/A	0.070	0.146	0.303	0.449	0.440	0.000	0.342	1.615
Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	79	836	0	1584	0	428	1012
normalized size	1	1.00	0.92	9.72	0.00	18.42	0.00	4.98	11.77
time (sec)	N/A	0.120	0.601	0.329	0.000	0.471	0.000	0.402	2.096

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	71	1077	140	2032	0	153	252
normalized size	1	1.00	0.95	14.36	1.87	27.09	0.00	2.04	3.36
time (sec)	N/A	0.092	0.401	0.349	0.456	0.482	0.000	0.361	1.654
Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	111	875	0	6934	0	2074	-1
normalized size	1	1.00	0.87	6.84	0.00	54.17	0.00	16.20	-0.01
time (sec)	N/A	0.182	1.186	0.523	0.000	0.563	0.000	0.937	0.000
Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	110	1146	840	4324	0	449	-1
normalized size	1	1.00	0.79	8.19	6.00	30.89	0.00	3.21	-0.01
time (sec)	N/A	0.194	0.836	0.472	0.704	0.526	0.000	1.096	0.000
Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	84	729	0	3502	0	1838	-1
normalized size	1	1.00	0.83	7.22	0.00	34.67	0.00	18.20	-0.01
time (sec)	N/A	0.139	0.762	0.415	0.000	0.543	0.000	0.453	0.000
Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	78	666	0	2041	0	1309	-1
normalized size	1	1.00	0.94	8.02	0.00	24.59	0.00	15.77	-0.01
time (sec)	N/A	0.071	0.306	0.386	0.000	0.463	0.000	0.324	0.000
Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	63	498	125	1515	0	138	-1
normalized size	1	1.00	0.95	7.55	1.89	22.95	0.00	2.09	-0.02
time (sec)	N/A	0.066	0.268	0.400	0.518	0.435	0.000	0.558	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	69	375	0	1555	0	978	-1
normalized size	1	1.00	0.96	5.21	0.00	21.60	0.00	13.58	-0.01
time (sec)	N/A	0.077	0.140	0.390	0.000	0.438	0.000	0.599	0.000
Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	83	746	127	1443	0	156	-1
normalized size	1	1.00	1.08	9.69	1.65	18.74	0.00	2.03	-0.01
time (sec)	N/A	0.079	0.327	0.367	0.538	0.444	0.000	0.554	0.000
Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	203	1007	0	2140	0	1035	-1
normalized size	1	1.00	1.99	9.87	0.00	20.98	0.00	10.15	-0.01
time (sec)	N/A	0.127	0.555	0.364	0.000	0.516	0.000	0.675	0.000
Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	102	1283	209	2869	0	248	-1
normalized size	1	1.00	1.05	13.23	2.15	29.58	0.00	2.56	-0.01
time (sec)	N/A	0.129	0.654	0.365	0.604	0.502	0.000	0.596	0.000
Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	265	1477	0	6396	0	870	-1
normalized size	1	1.00	1.71	9.53	0.00	41.26	0.00	5.61	-0.01
time (sec)	N/A	0.249	1.414	0.402	0.000	0.573	0.000	0.687	0.000
Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	164	2132	1806	13887	0	595	-1
normalized size	1	1.00	0.83	10.77	9.12	70.14	0.00	3.01	-0.01
time (sec)	N/A	0.310	1.424	0.514	1.048	0.714	0.000	1.913	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	136	1570	0	11392	0	2145	-1
normalized size	1	1.00	0.88	10.19	0.00	73.97	0.00	13.93	-0.01
time (sec)	N/A	0.223	2.137	0.492	0.000	0.652	0.000	0.684	0.000
Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	134	1676	0	7909	0	1137	-1
normalized size	1	1.00	0.93	11.64	0.00	54.92	0.00	7.90	-0.01
time (sec)	N/A	0.141	1.184	0.451	0.000	0.565	0.000	0.445	0.000
Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	77	764	366	5840	0	320	-1
normalized size	1	1.00	0.80	7.96	3.81	60.83	0.00	3.33	-0.01
time (sec)	N/A	0.075	0.828	0.483	0.632	0.544	0.000	0.750	0.000
Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	123	1226	0	6614	0	1495	-1
normalized size	1	1.00	0.95	9.50	0.00	51.27	0.00	11.59	-0.01
time (sec)	N/A	0.120	0.724	0.486	0.000	0.567	0.000	0.991	0.000
Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	115	1270	360	5659	0	332	-1
normalized size	1	1.00	1.00	11.04	3.13	49.21	0.00	2.89	-0.01
time (sec)	N/A	0.097	1.051	0.401	0.697	0.534	0.000	0.731	0.000
Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	88	634	0	5077	0	690	-1
normalized size	1	1.00	0.85	6.10	0.00	48.82	0.00	6.63	-0.01
time (sec)	N/A	0.091	0.311	0.401	0.000	0.516	0.000	0.823	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	128	1776	332	5233	0	338	-1
normalized size	1	1.00	0.98	13.56	2.53	39.95	0.00	2.58	-0.01
time (sec)	N/A	0.124	0.995	0.418	0.677	0.546	0.000	0.752	0.000
Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	317	1907	0	8070	0	1040	-1
normalized size	1	1.00	2.03	12.22	0.00	51.73	0.00	6.67	-0.01
time (sec)	N/A	0.228	3.382	0.402	0.000	0.598	0.000	0.929	0.000
Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	97	128	199	339	82	134	50
normalized size	1	1.00	1.80	2.37	3.69	6.28	1.52	2.48	0.93
time (sec)	N/A	0.052	0.039	0.015	0.328	0.412	0.553	0.210	1.218
Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	43	104	168	1205	88	92	53
normalized size	1	1.00	0.88	2.12	3.43	24.59	1.80	1.88	1.08
time (sec)	N/A	0.059	0.249	0.014	0.405	0.444	0.419	0.182	0.118
Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	65	100	105	160	54	86	34
normalized size	1	1.00	1.81	2.78	2.92	4.44	1.50	2.39	0.94
time (sec)	N/A	0.040	0.028	0.016	0.326	0.411	0.292	0.175	1.170
Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	41	76	76	399	60	57	37
normalized size	1	1.00	1.32	2.45	2.45	12.87	1.94	1.84	1.19
time (sec)	N/A	0.032	0.024	0.016	0.406	0.443	0.205	0.162	1.168

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	28	47	31	37	20	29	18
normalized size	1	1.00	1.47	2.47	1.63	1.95	1.05	1.53	0.95
time (sec)	N/A	0.013	0.007	0.016	0.301	0.403	0.155	0.134	0.073
Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	33	26	35	70	0	46	228
normalized size	1	1.00	1.32	1.04	1.40	2.80	0.00	1.84	9.12
time (sec)	N/A	0.041	0.042	0.267	0.312	0.421	0.000	0.159	1.312
Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	32	28	31	38	49	30	25
normalized size	1	1.00	1.78	1.56	1.72	2.11	2.72	1.67	1.39
time (sec)	N/A	0.029	0.027	0.211	0.323	0.404	7.559	0.170	1.267
Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	39	40	106	407	0	58	76
normalized size	1	1.00	1.26	1.29	3.42	13.13	0.00	1.87	2.45
time (sec)	N/A	0.042	0.122	0.268	0.320	0.426	0.000	0.196	1.264
Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	61	46	105	156	0	86	162
normalized size	1	1.00	1.69	1.28	2.92	4.33	0.00	2.39	4.50
time (sec)	N/A	0.039	0.040	0.235	0.329	0.425	0.000	0.188	1.168
Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	51	68	206	1216	0	93	177
normalized size	1	1.00	1.04	1.39	4.20	24.82	0.00	1.90	3.61
time (sec)	N/A	0.062	0.335	0.252	0.337	0.432	0.000	0.236	1.246

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	190	236	369	796	165	300	91
normalized size	1	1.00	2.29	2.84	4.45	9.59	1.99	3.61	1.10
time (sec)	N/A	0.079	0.082	0.016	0.344	0.416	1.110	0.318	0.180
Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	66	196	333	3441	170	191	100
normalized size	1	1.00	0.87	2.58	4.38	45.28	2.24	2.51	1.32
time (sec)	N/A	0.110	0.426	0.016	0.429	0.475	0.886	0.287	1.315
Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	137	189	231	483	117	218	67
normalized size	1	1.00	2.17	3.00	3.67	7.67	1.86	3.46	1.06
time (sec)	N/A	0.076	0.053	0.017	0.338	0.429	0.643	0.256	1.300
Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	50	149	186	1638	122	116	76
normalized size	1	1.00	0.88	2.61	3.26	28.74	2.14	2.04	1.33
time (sec)	N/A	0.073	0.323	0.016	0.420	0.431	0.474	0.222	1.206
Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	65	144	114	201	68	103	47
normalized size	1	1.00	1.51	3.35	2.65	4.67	1.58	2.40	1.09
time (sec)	N/A	0.031	0.750	0.019	0.327	0.396	0.322	0.144	1.243
Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	48	60	104	668	0	141	210
normalized size	1	1.00	0.98	1.22	2.12	13.63	0.00	2.88	4.29
time (sec)	N/A	0.077	0.135	0.264	0.422	0.417	0.000	0.208	1.319

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	64	49	64	97	0	71	59
normalized size	1	1.00	1.78	1.36	1.78	2.69	0.00	1.97	1.64
time (sec)	N/A	0.067	0.107	0.209	0.347	0.391	0.000	0.253	1.244
Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	50	60	134	677	0	141	211
normalized size	1	1.00	0.96	1.15	2.58	13.02	0.00	2.71	4.06
time (sec)	N/A	0.095	0.171	0.267	0.330	0.434	0.000	0.270	1.415
Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	65	59	114	197	0	103	175
normalized size	1	1.00	1.51	1.37	2.65	4.58	0.00	2.40	4.07
time (sec)	N/A	0.071	0.619	0.228	0.332	0.404	0.000	0.279	0.175
Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	58	91	236	1649	0	118	197
normalized size	1	1.00	0.81	1.26	3.28	22.90	0.00	1.64	2.74
time (sec)	N/A	0.099	0.429	0.288	0.335	0.465	0.000	0.335	1.272
Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	98	87	231	473	0	218	529
normalized size	1	1.00	1.56	1.38	3.67	7.51	0.00	3.46	8.40
time (sec)	N/A	0.077	0.094	0.238	0.351	0.410	0.000	0.360	0.204
Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	74	138	390	3454	0	192	362
normalized size	1	1.00	0.80	1.50	4.24	37.54	0.00	2.09	3.93
time (sec)	N/A	0.113	0.517	0.257	0.354	0.472	0.000	0.431	0.274

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	123	365	583	1563	260	534	138
normalized size	1	1.00	1.08	3.20	5.11	13.71	2.28	4.68	1.21
time (sec)	N/A	0.101	1.519	0.017	0.368	0.449	2.091	0.539	0.260
Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	98	307	540	7502	279	309	155
normalized size	1	1.00	0.92	2.87	5.05	70.11	2.61	2.89	1.45
time (sec)	N/A	0.152	0.313	0.017	0.454	0.558	1.683	0.466	1.241
Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	108	299	400	1036	192	418	106
normalized size	1	1.00	1.15	3.18	4.26	11.02	2.04	4.45	1.13
time (sec)	N/A	0.093	1.724	0.017	0.347	0.431	1.200	0.394	1.218
Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	76	241	351	4298	211	216	123
normalized size	1	1.00	0.92	2.90	4.23	51.78	2.54	2.60	1.48
time (sec)	N/A	0.101	0.249	0.019	0.441	0.488	0.969	0.352	1.253
Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	95	235	239	567	126	241	86
normalized size	1	1.00	1.28	3.18	3.23	7.66	1.70	3.26	1.16
time (sec)	N/A	0.046	0.615	0.017	0.348	0.410	0.685	0.177	1.282
Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	67	111	214	2381	0	267	380
normalized size	1	1.00	0.93	1.54	2.97	33.07	0.00	3.71	5.28
time (sec)	N/A	0.098	0.547	0.286	0.437	0.468	0.000	0.331	0.401

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	81	80	147	341	0	135	218
normalized size	1	1.00	1.37	1.36	2.49	5.78	0.00	2.29	3.69
time (sec)	N/A	0.080	2.312	0.246	0.341	0.428	0.000	0.413	1.308
Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	63	94	203	1686	0	274	327
normalized size	1	1.00	0.88	1.31	2.82	23.42	0.00	3.81	4.54
time (sec)	N/A	0.108	0.462	0.275	0.438	0.445	0.000	0.443	2.476
Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	82	80	147	341	0	135	219
normalized size	1	1.00	1.39	1.36	2.49	5.78	0.00	2.29	3.71
time (sec)	N/A	0.081	1.284	0.246	0.340	0.437	0.000	0.523	1.287
Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	67	111	264	2393	0	267	381
normalized size	1	1.00	0.81	1.34	3.18	28.83	0.00	3.22	4.59
time (sec)	N/A	0.117	0.583	0.270	0.350	0.461	0.000	0.577	0.395
Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	100	100	239	557	0	241	568
normalized size	1	1.00	1.35	1.35	3.23	7.53	0.00	3.26	7.68
time (sec)	N/A	0.089	1.781	0.283	0.349	0.441	0.000	0.557	1.299
Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	76	161	420	4305	0	217	380
normalized size	1	1.00	0.74	1.56	4.08	41.80	0.00	2.11	3.69
time (sec)	N/A	0.129	0.254	0.273	0.355	0.493	0.000	0.782	0.318

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	128	344	410	1176	209	447	133
normalized size	1	1.00	1.16	3.13	3.73	10.69	1.90	4.06	1.21
time (sec)	N/A	0.070	1.793	0.018	0.353	0.427	1.276	0.216	0.200
Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	170	472	624	2133	308	721	188
normalized size	1	1.00	1.06	2.95	3.90	13.33	1.92	4.51	1.18
time (sec)	N/A	0.094	2.338	0.020	0.357	0.437	2.316	0.308	1.321
Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	60	93	133	742	425	132	72
normalized size	1	1.00	0.91	1.41	2.02	11.24	6.44	2.00	1.09
time (sec)	N/A	0.115	0.180	0.089	0.428	0.519	13.457	0.260	0.269
Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	66	95	509	777	495	87	56
normalized size	1	1.00	1.12	1.61	8.63	13.17	8.39	1.47	0.95
time (sec)	N/A	0.108	0.199	0.097	0.603	0.465	11.281	0.226	1.230
Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	42	75	82	118	316	96	46
normalized size	1	1.00	0.91	1.63	1.78	2.57	6.87	2.09	1.00
time (sec)	N/A	0.104	0.039	0.107	0.409	0.462	9.647	0.229	1.236
Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	47	77	215	486	294	65	38
normalized size	1	1.00	1.02	1.67	4.67	10.57	6.39	1.41	0.83
time (sec)	N/A	0.083	0.031	0.072	0.451	0.489	8.966	0.184	0.106

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	35	71	58	82	156	61	43
normalized size	1	1.00	0.83	1.69	1.38	1.95	3.71	1.45	1.02
time (sec)	N/A	0.061	0.037	0.087	0.327	0.415	8.830	0.174	1.170
Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	65	76	57	484	280	63	37
normalized size	1	1.00	1.44	1.69	1.27	10.76	6.22	1.40	0.82
time (sec)	N/A	0.074	0.084	0.144	0.422	0.458	8.957	0.149	0.091
Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	121	101	118	0	97	194
normalized size	1	1.00	0.90	2.02	1.68	1.97	0.00	1.62	3.23
time (sec)	N/A	0.101	0.076	0.403	0.329	0.453	0.000	0.175	1.468
Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	67	494	329	784	0	89	402
normalized size	1	1.00	1.12	8.23	5.48	13.07	0.00	1.48	6.70
time (sec)	N/A	0.106	0.191	0.398	0.475	0.453	0.000	0.205	1.616
Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	60	180	159	747	0	133	313
normalized size	1	1.00	0.71	2.12	1.87	8.79	0.00	1.56	3.68
time (sec)	N/A	0.137	0.182	0.422	0.331	0.519	0.000	0.251	1.539
Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	91	580	1038	2368	0	147	519
normalized size	1	1.00	1.11	7.07	12.66	28.88	0.00	1.79	6.33
time (sec)	N/A	0.177	0.710	0.484	0.613	0.462	0.000	0.252	1.614

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	69	156	217	1141	0	194	170
normalized size	1	1.00	0.83	1.88	2.61	13.75	0.00	2.34	2.05
time (sec)	N/A	0.150	0.560	0.101	0.438	0.549	0.000	0.446	1.609
Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	90	172	1010	1950	0	195	1655
normalized size	1	1.00	1.01	1.93	11.35	21.91	0.00	2.19	18.60
time (sec)	N/A	0.117	0.599	0.098	0.792	0.477	0.000	0.336	1.692
Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	57	118	170	629	0	149	210
normalized size	1	1.00	0.79	1.64	2.36	8.74	0.00	2.07	2.92
time (sec)	N/A	0.118	0.530	0.098	0.341	0.466	0.000	0.318	0.427
Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	86	162	614	2025	0	177	106
normalized size	1	1.00	1.01	1.91	7.22	23.82	0.00	2.08	1.25
time (sec)	N/A	0.108	0.472	0.096	0.605	0.461	0.000	0.307	0.660
Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	55	113	170	623	0	149	129
normalized size	1	1.00	0.81	1.66	2.50	9.16	0.00	2.19	1.90
time (sec)	N/A	0.086	0.466	0.120	0.342	0.443	0.000	0.266	1.466
Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	97	172	206	1942	0	195	110
normalized size	1	1.00	1.09	1.93	2.31	21.82	0.00	2.19	1.24
time (sec)	N/A	0.088	0.578	0.153	0.461	0.473	0.000	0.184	1.541

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	83	325	235	1148	0	195	-1
normalized size	1	1.00	0.87	3.42	2.47	12.08	0.00	2.05	-0.01
time (sec)	N/A	0.149	2.164	0.464	0.335	0.563	0.000	0.314	0.000
Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	111	1061	976	3725	0	336	-1
normalized size	1	1.00	0.93	8.92	8.20	31.30	0.00	2.82	-0.01
time (sec)	N/A	0.194	1.863	0.422	0.637	0.502	0.000	0.361	0.000
Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	93	383	402	3468	0	323	-1
normalized size	1	1.00	0.75	3.09	3.24	27.97	0.00	2.60	-0.01
time (sec)	N/A	0.189	0.920	0.482	0.346	0.671	0.000	0.378	0.000
Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	139	1137	2345	8482	0	281	-1
normalized size	1	1.00	0.87	7.15	14.75	53.35	0.00	1.77	-0.01
time (sec)	N/A	0.283	1.565	0.469	1.084	0.593	0.000	0.442	0.000
Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	144	352	3354	7528	0	408	2669
normalized size	1	1.00	1.00	2.44	23.29	52.28	0.00	2.83	18.53
time (sec)	N/A	0.208	1.293	0.102	1.891	0.581	0.000	0.602	0.922
Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	91	234	376	2584	3937	245	416
normalized size	1	1.00	0.83	2.15	3.45	23.71	36.12	2.25	3.82
time (sec)	N/A	0.170	1.098	0.107	0.363	0.482	157.941	0.514	0.825

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	135	340	2432	7757	0	384	2574
normalized size	1	1.00	0.99	2.48	17.75	56.62	0.00	2.80	18.79
time (sec)	N/A	0.195	1.150	0.121	1.345	0.578	0.000	0.488	1.856
Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	80	196	384	2611	3434	245	397
normalized size	1	1.00	0.82	2.00	3.92	26.64	35.04	2.50	4.05
time (sec)	N/A	0.138	0.505	0.104	0.367	0.480	157.413	0.477	1.805
Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	137	340	1472	7791	0	388	255
normalized size	1	1.00	1.00	2.48	10.74	56.87	0.00	2.83	1.86
time (sec)	N/A	0.157	1.166	0.108	0.901	0.577	0.000	0.441	3.471
Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	77	193	378	2554	3442	245	235
normalized size	1	1.00	0.82	2.05	4.02	27.17	36.62	2.61	2.50
time (sec)	N/A	0.105	0.592	0.120	0.370	0.467	157.269	0.394	2.255
Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	147	352	507	7496	0	409	260
normalized size	1	1.00	1.04	2.48	3.57	52.79	0.00	2.88	1.83
time (sec)	N/A	0.163	0.321	0.153	0.566	0.560	0.000	0.211	0.897
Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	117	952	498	4800	0	295	-1
normalized size	1	1.00	0.85	6.90	3.61	34.78	0.00	2.14	-0.01
time (sec)	N/A	0.205	1.756	0.499	0.397	0.873	0.000	0.491	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	166	2045	1944	11865	0	437	-1
normalized size	1	1.00	0.93	11.49	10.92	66.66	0.00	2.46	-0.01
time (sec)	N/A	0.291	6.311	0.481	0.942	0.680	0.000	0.549	0.000
Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	138	1020	770	10720	0	474	-1
normalized size	1	1.00	0.81	5.96	4.50	62.69	0.00	2.77	-0.01
time (sec)	N/A	0.262	1.906	0.496	0.421	1.293	0.000	1.062	0.000
Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	194	2139	4285	0	0	493	-1
normalized size	1	1.00	0.85	9.38	18.79	0.00	0.00	2.16	-0.00
time (sec)	N/A	0.368	3.866	0.496	2.671	0.000	0.000	1.017	0.000
Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	203	608	925	0	0	1356	3685
normalized size	1	1.00	1.01	3.02	4.60	0.00	0.00	6.75	18.33
time (sec)	N/A	0.277	0.680	0.162	0.743	0.000	0.000	0.530	1.380
Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	19	4	5	8	0	5	3
normalized size	1	1.00	6.33	1.33	1.67	2.67	0.00	1.67	1.00
time (sec)	N/A	0.017	0.009	0.082	0.475	0.744	0.000	0.133	0.045
Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	21	15	5	1	0	1	14
normalized size	1	1.00	1.31	0.94	0.31	0.06	0.00	0.06	0.88
time (sec)	N/A	0.021	0.008	0.112	0.493	2.082	0.000	0.126	0.248

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	29	21	28	140	0	45	20
normalized size	1	1.00	1.32	0.95	1.27	6.36	0.00	2.05	0.91
time (sec)	N/A	0.020	0.021	0.066	0.478	0.451	0.000	0.125	0.100
Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	28	32	1	0	41	27
normalized size	1	1.00	0.80	0.80	0.91	0.03	0.00	1.17	0.77
time (sec)	N/A	0.024	0.023	0.119	0.461	0.613	0.000	0.118	1.173
Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	11	2	12	11	2
normalized size	1	1.00	1.00	1.27	1.00	0.18	1.09	1.00	0.18
time (sec)	N/A	0.021	0.008	0.051	0.417	0.552	0.405	0.141	0.145
Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	25	1	0	21	14
normalized size	1	1.00	1.00	0.92	1.92	0.08	0.00	1.62	1.08
time (sec)	N/A	0.021	0.007	0.089	0.421	0.461	0.000	0.130	0.095
Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	85	288	0	4529	97	980	119
normalized size	1	1.00	0.98	3.31	0.00	52.06	1.11	11.26	1.37
time (sec)	N/A	0.161	0.521	0.134	0.000	0.867	6.393	2.950	9.603
Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	580	337	0	9360	0	938	-1
normalized size	1	1.00	4.79	2.79	0.00	77.36	0.00	7.75	-0.01
time (sec)	N/A	0.200	6.217	0.104	0.000	0.951	0.000	3.036	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	253	0	2329	71	630	66
normalized size	1	1.00	0.95	4.02	0.00	36.97	1.13	10.00	1.05
time (sec)	N/A	0.119	0.176	0.091	0.000	0.584	4.277	2.177	3.465
Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	193	276	0	4825	0	554	-1
normalized size	1	1.00	2.27	3.25	0.00	56.76	0.00	6.52	-0.01
time (sec)	N/A	0.125	3.437	0.089	0.000	0.685	0.000	1.744	0.000
Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	238	0	1543	51	349	51
normalized size	1	1.00	1.00	5.41	0.00	35.07	1.16	7.93	1.16
time (sec)	N/A	0.077	0.028	0.096	0.000	0.505	2.257	0.989	1.694
Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	137	238	0	3443	0	253	-1
normalized size	1	1.00	2.28	3.97	0.00	57.38	0.00	4.22	-0.02
time (sec)	N/A	0.047	0.235	0.106	0.000	0.571	0.000	0.690	0.000
Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	3467	0	255	-1
normalized size	1	1.00	1.00	0.00	0.00	61.91	0.00	4.55	-0.02
time (sec)	N/A	0.112	0.031	0.384	0.000	0.602	0.000	0.765	0.000
Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	42	0	0	1539	0	348	-1
normalized size	1	1.00	0.88	0.00	0.00	32.06	0.00	7.25	-0.02
time (sec)	N/A	0.093	0.135	0.338	0.000	0.506	0.000	1.072	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	0	0	4891	0	557	-1
normalized size	1	1.00	1.00	0.00	0.00	58.93	0.00	6.71	-0.01
time (sec)	N/A	0.155	0.223	0.353	0.000	0.794	0.000	1.757	0.000
Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	161	0	0	2355	0	629	-1
normalized size	1	1.00	2.06	0.00	0.00	30.19	0.00	8.06	-0.01
time (sec)	N/A	0.146	6.218	0.348	0.000	0.599	0.000	2.215	0.000
Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	111	0	0	9642	0	947	-1
normalized size	1	1.00	0.92	0.00	0.00	79.69	0.00	7.83	-0.01
time (sec)	N/A	0.213	0.630	0.358	0.000	0.961	0.000	3.118	0.000
Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	86	593	0	4941	175	1063	112
normalized size	1	1.00	1.05	7.23	0.00	60.26	2.13	12.96	1.37
time (sec)	N/A	0.152	0.444	0.075	0.000	0.832	29.540	4.081	10.989
Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	584	633	0	0	0	949	-1
normalized size	1	1.00	4.75	5.15	0.00	0.00	0.00	7.72	-0.01
time (sec)	N/A	0.244	6.270	0.073	0.000	0.000	0.000	3.671	0.000
Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	59	578	0	2385	128	662	64
normalized size	1	1.00	0.94	9.17	0.00	37.86	2.03	10.51	1.02
time (sec)	N/A	0.101	0.175	0.059	0.000	2.124	17.584	2.591	3.711

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	161	578	0	4841	0	584	-1
normalized size	1	1.00	1.83	6.57	0.00	55.01	0.00	6.64	-0.01
time (sec)	N/A	0.085	0.367	0.089	0.000	3.055	0.000	1.997	0.000
Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	0	0	4039	0	433	-1
normalized size	1	1.00	1.00	0.00	0.00	56.89	0.00	6.10	-0.01
time (sec)	N/A	0.136	0.069	0.313	0.000	0.645	0.000	2.776	0.000
Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	197	0	0	3913	0	430	-1
normalized size	1	1.00	2.56	0.00	0.00	50.82	0.00	5.58	-0.01
time (sec)	N/A	0.127	3.005	0.300	0.000	0.641	0.000	2.795	0.000
Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	51	97	0	679	0	104	68
normalized size	1	1.00	1.65	3.13	0.00	21.90	0.00	3.35	2.19
time (sec)	N/A	0.027	0.063	0.135	0.000	0.458	0.000	0.154	0.235
Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	53	142	0	228	0	104	43
normalized size	1	1.00	1.18	3.16	0.00	5.07	0.00	2.31	0.96
time (sec)	N/A	0.035	0.037	0.123	0.000	0.427	0.000	0.166	1.343
Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	74	158	0	1027	0	202	78
normalized size	1	1.00	1.48	3.16	0.00	20.54	0.00	4.04	1.56
time (sec)	N/A	0.039	0.148	0.100	0.000	0.429	0.000	0.139	0.287

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	76	211	0	361	0	204	-1
normalized size	1	1.00	1.13	3.15	0.00	5.39	0.00	3.04	-0.01
time (sec)	N/A	0.047	0.075	0.106	0.000	0.405	0.000	0.172	0.000
Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	68	164	0	2827	0	0	65
normalized size	1	1.00	0.97	2.34	0.00	40.39	0.00	0.00	0.93
time (sec)	N/A	0.136	0.537	0.114	0.000	0.673	0.000	0.000	2.166
Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	208	178	0	5494	0	0	-1
normalized size	1	1.00	2.36	2.02	0.00	62.43	0.00	0.00	-0.01
time (sec)	N/A	0.123	4.900	0.128	0.000	0.769	0.000	0.000	0.000
Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	47	129	0	1625	0	0	39
normalized size	1	1.00	1.00	2.74	0.00	34.57	0.00	0.00	0.83
time (sec)	N/A	0.106	0.108	0.101	0.000	0.511	0.000	0.000	1.688
Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	101	137	0	3361	0	0	-1
normalized size	1	1.00	1.68	2.28	0.00	56.02	0.00	0.00	-0.02
time (sec)	N/A	0.093	0.512	0.102	0.000	0.668	0.000	0.000	0.000
Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	114	0	1361	31	0	23
normalized size	1	1.00	1.00	3.93	0.00	46.93	1.07	0.00	0.79
time (sec)	N/A	0.064	0.017	0.100	0.000	0.500	1.297	0.000	1.625

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	114	0	1287	0	188	25
normalized size	1	1.00	1.00	3.68	0.00	41.52	0.00	6.06	0.81
time (sec)	N/A	0.027	0.024	0.109	0.000	0.481	0.000	0.322	1.571
Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	0	0	3527	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	62.98	0.00	0.00	-0.02
time (sec)	N/A	0.113	0.050	0.335	0.000	0.611	0.000	0.000	0.000
Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	123	0	0	1565	0	0	-1
normalized size	1	1.00	2.41	0.00	0.00	30.69	0.00	0.00	-0.02
time (sec)	N/A	0.095	6.180	0.360	0.000	0.599	0.000	0.000	0.000
Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	107	0	0	5711	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	64.90	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.469	0.362	0.000	0.986	0.000	0.000	0.000
Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	67	322	0	3991	0	0	70
normalized size	1	1.00	0.93	4.47	0.00	55.43	0.00	0.00	0.97
time (sec)	N/A	0.163	0.113	0.084	0.000	0.864	0.000	0.000	2.521
Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	188	328	0	6973	0	0	-1
normalized size	1	1.00	2.24	3.90	0.00	83.01	0.00	0.00	-0.01
time (sec)	N/A	0.130	2.452	0.099	0.000	0.832	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	287	0	2525	0	0	45
normalized size	1	1.00	1.00	5.52	0.00	48.56	0.00	0.00	0.87
time (sec)	N/A	0.123	0.130	0.081	0.000	0.536	0.000	0.000	2.060
Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	112	289	0	2281	0	0	-1
normalized size	1	1.00	2.11	5.45	0.00	43.04	0.00	0.00	-0.02
time (sec)	N/A	0.099	1.833	0.084	0.000	0.553	0.000	0.000	0.000
Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	273	0	2277	51	0	41
normalized size	1	1.00	0.84	5.57	0.00	46.47	1.04	0.00	0.84
time (sec)	N/A	0.086	0.035	0.083	0.000	0.669	23.264	0.000	1.942
Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	223	272	0	2509	0	0	-1
normalized size	1	1.00	3.98	4.86	0.00	44.80	0.00	0.00	-0.02
time (sec)	N/A	0.043	4.600	0.092	0.000	0.716	0.000	0.000	0.000
Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	70	0	0	6955	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	89.17	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.077	0.332	0.000	1.007	0.000	0.000	0.000
Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	230	0	0	3929	0	0	-1
normalized size	1	1.00	2.71	0.00	0.00	46.22	0.00	0.00	-0.01
time (sec)	N/A	0.161	7.990	0.341	0.000	0.927	0.000	0.000	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	231	549	0	0	0	0	-1
normalized size	1	1.00	1.96	4.65	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.220	2.033	0.098	0.000	0.000	0.000	0.000	0.000
Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	68	469	0	7033	0	0	92
normalized size	1	1.00	0.81	5.58	0.00	83.73	0.00	0.00	1.10
time (sec)	N/A	0.181	0.116	0.101	0.000	1.579	0.000	0.000	4.013
Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	132	491	0	5719	0	0	-1
normalized size	1	1.00	1.47	5.46	0.00	63.54	0.00	0.00	-0.01
time (sec)	N/A	0.138	3.126	0.096	0.000	1.803	0.000	0.000	0.000
Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	63	435	0	6621	0	0	82
normalized size	1	1.00	0.85	5.88	0.00	89.47	0.00	0.00	1.11
time (sec)	N/A	0.144	0.093	0.083	0.000	1.431	0.000	0.000	3.824
Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	259	454	0	6507	0	0	-1
normalized size	1	1.00	2.94	5.16	0.00	73.94	0.00	0.00	-0.01
time (sec)	N/A	0.132	7.823	0.091	0.000	1.597	0.000	0.000	0.000
Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	43	420	0	5779	73	0	76
normalized size	1	1.00	0.61	6.00	0.00	82.56	1.04	0.00	1.09
time (sec)	N/A	0.103	0.047	0.084	0.000	1.396	30.763	0.000	3.556

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	976	420	0	6933	0	0	-1
normalized size	1	1.00	10.49	4.52	0.00	74.55	0.00	0.00	-0.01
time (sec)	N/A	0.085	7.711	0.095	0.000	1.482	0.000	0.000	0.000
Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	73	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.208	0.072	0.334	0.000	0.000	0.000	0.000	0.000
Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	246	0	0	10671	0	0	-1
normalized size	1	1.00	1.88	0.00	0.00	81.46	0.00	0.00	-0.01
time (sec)	N/A	0.242	7.766	0.350	0.000	2.491	0.000	0.000	0.000
Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	35	62	0	543	0	58	63
normalized size	1	1.00	1.40	2.48	0.00	21.72	0.00	2.32	2.52
time (sec)	N/A	0.019	0.023	0.131	0.000	0.446	0.000	0.119	0.171
Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	C	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	37	66	0	175	0	58	22
normalized size	1	1.00	1.37	2.44	0.00	6.48	0.00	2.15	0.81
time (sec)	N/A	0.020	0.021	0.125	0.000	0.407	0.000	0.137	1.200
Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	112	95	163	194	2074	100	142	91
normalized size	1	1.26	1.07	1.83	2.18	23.30	1.12	1.60	1.02
time (sec)	N/A	0.075	0.761	0.016	0.413	0.430	0.620	0.159	1.157

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	40	41	73	95	102	25	38
normalized size	1	1.00	1.05	1.08	1.92	2.50	2.68	0.66	1.00
time (sec)	N/A	0.066	0.072	0.064	0.412	0.527	0.559	0.114	0.097
Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	166	620	0	0	0	0	-1
normalized size	1	1.00	1.34	5.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.242	4.810	0.240	0.000	0.000	0.000	0.000	0.000
Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	86	116	0	5136	0	0	-1
normalized size	1	1.00	0.97	1.30	0.00	57.71	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.062	0.118	0.000	0.649	0.000	0.000	0.000
Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	37	0	1286	0	0	-1
normalized size	1	1.00	1.00	0.92	0.00	32.15	0.00	0.00	-0.02
time (sec)	N/A	0.078	0.018	0.128	0.000	0.639	0.000	0.000	0.000
Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	73	431	0	3914	0	0	-1
normalized size	1	1.00	0.99	5.82	0.00	52.89	0.00	0.00	-0.01
time (sec)	N/A	0.123	0.544	0.119	0.000	0.807	0.000	0.000	0.000
Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	113	637	0	0	0	0	-1
normalized size	1	1.00	0.96	5.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	0.853	0.137	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [258] had the largest ratio of [.6250]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	5	1.00	21	0.238
2	A	3	2	1.00	21	0.095
3	A	4	4	1.00	21	0.190
4	A	3	2	1.00	19	0.105
5	A	3	3	1.00	19	0.158
6	A	3	2	1.00	21	0.095
7	A	4	4	1.00	21	0.190
8	A	3	2	1.00	21	0.095
9	A	6	5	1.00	23	0.217
10	A	3	2	1.00	23	0.087
11	A	5	5	1.00	23	0.217
12	A	3	2	1.00	21	0.095
13	A	4	3	1.00	21	0.143
14	A	3	2	1.00	23	0.087
15	A	5	5	1.00	23	0.217
16	A	3	2	1.00	23	0.087
17	A	6	5	1.00	23	0.217
18	A	3	2	1.00	23	0.087
19	A	6	5	1.14	23	0.217
20	A	3	2	1.00	21	0.095
21	A	4	3	1.00	21	0.143
22	A	3	2	1.00	23	0.087
23	A	6	5	1.00	23	0.217
24	A	3	2	1.00	23	0.087
25	A	6	6	1.00	23	0.261
26	A	4	4	1.00	23	0.174
27	A	5	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
28	A	3	3	1.00	21	0.143
29	A	4	4	1.00	21	0.190
30	A	3	3	1.00	23	0.130
31	A	5	5	1.00	23	0.217
32	A	4	4	1.00	23	0.174
33	A	7	6	1.00	23	0.261
34	A	5	4	1.00	23	0.174
35	A	6	6	1.00	23	0.261
36	A	4	4	1.00	21	0.190
37	A	5	5	1.00	21	0.238
38	A	4	4	1.00	23	0.174
39	A	6	6	1.00	23	0.261
40	A	5	4	1.00	23	0.174
41	A	8	6	1.00	23	0.261
42	A	6	5	1.00	23	0.217
43	A	7	6	1.00	23	0.261
44	A	5	4	1.00	21	0.190
45	A	6	6	1.00	21	0.286
46	A	5	4	1.00	23	0.174
47	A	7	6	1.00	23	0.261
48	A	6	5	1.00	23	0.217
49	A	8	5	1.00	21	0.238
50	A	9	6	1.00	21	0.286
51	A	7	5	1.00	21	0.238
52	A	7	6	1.00	19	0.316
53	A	5	3	1.00	19	0.158
54	A	3	2	1.00	21	0.095
55	A	6	3	1.00	21	0.143
56	A	3	2	1.00	21	0.095
57	A	8	5	1.21	23	0.217
58	A	12	8	1.00	23	0.348
59	A	7	5	1.23	23	0.217
60	A	10	8	1.00	21	0.381
61	A	8	5	1.00	21	0.238
62	A	3	2	1.00	23	0.087
63	A	9	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	3	2	1.00	23	0.087
65	A	8	5	1.11	23	0.217
66	A	20	8	1.00	23	0.348
67	A	7	5	1.10	23	0.217
68	A	17	8	1.00	21	0.381
69	A	13	5	1.00	21	0.238
70	A	3	2	1.00	23	0.087
71	A	14	6	1.00	23	0.261
72	A	3	2	1.00	23	0.087
73	A	11	10	1.00	23	0.435
74	A	0	0	0.00	0	0.000
75	A	11	10	1.00	23	0.435
76	A	0	0	0.00	0	0.000
77	A	0	0	0.00	0	0.000
78	A	8	8	1.00	23	0.348
79	A	0	0	0.00	0	0.000
80	A	12	11	1.00	23	0.478
81	A	4	4	1.00	21	0.190
82	A	2	1	1.00	21	0.048
83	A	3	3	1.00	21	0.143
84	A	3	3	1.00	19	0.158
85	A	3	3	1.00	19	0.158
86	A	2	1	1.00	21	0.048
87	A	4	4	1.00	21	0.190
88	A	3	2	1.00	21	0.095
89	A	4	4	1.00	23	0.174
90	A	4	3	1.00	23	0.130
91	A	5	4	1.00	23	0.174
92	A	5	4	1.00	21	0.190
93	A	4	4	1.00	21	0.190
94	A	3	2	1.00	23	0.087
95	A	5	5	1.00	23	0.217
96	A	3	2	1.00	23	0.087
97	A	6	5	1.00	23	0.217
98	A	5	4	1.00	23	0.174
99	A	5	4	1.00	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
100	A	6	5	1.00	21	0.238
101	A	5	5	1.00	21	0.238
102	A	3	2	1.00	23	0.087
103	A	6	6	1.00	23	0.261
104	A	3	2	1.00	23	0.087
105	A	6	6	1.00	23	0.261
106	A	4	3	1.00	23	0.130
107	A	5	5	1.00	23	0.217
108	A	3	3	1.00	21	0.143
109	A	2	2	1.00	21	0.095
110	A	2	2	1.00	23	0.087
111	A	4	4	1.00	23	0.174
112	A	3	3	1.00	23	0.130
113	A	5	5	1.00	23	0.217
114	A	4	3	1.00	23	0.130
115	A	5	4	1.00	23	0.174
116	A	6	6	1.00	23	0.261
117	A	5	4	1.00	21	0.190
118	A	3	3	1.00	21	0.143
119	A	3	3	1.00	23	0.130
120	A	3	3	1.00	23	0.130
121	A	3	3	1.00	23	0.130
122	A	5	5	1.00	23	0.217
123	A	5	4	1.00	23	0.174
124	A	6	6	1.00	23	0.261
125	A	7	6	1.00	23	0.261
126	A	6	5	1.00	21	0.238
127	A	4	4	1.00	21	0.190
128	A	4	3	1.00	23	0.130
129	A	4	4	1.00	23	0.174
130	A	4	4	1.00	23	0.174
131	A	4	3	1.00	23	0.130
132	A	4	4	1.00	23	0.174
133	A	6	6	1.00	23	0.261
134	A	4	3	1.00	21	0.143
135	A	3	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
136	A	3	3	1.00	21	0.143
137	A	2	2	1.00	19	0.105
138	A	3	2	1.00	12	0.167
139	A	3	2	1.00	19	0.105
140	A	2	2	1.00	21	0.095
141	A	3	3	1.00	21	0.143
142	A	4	4	1.00	21	0.190
143	A	4	4	1.00	21	0.190
144	A	4	3	1.00	23	0.130
145	A	4	3	1.00	23	0.130
146	A	4	3	1.00	23	0.130
147	A	4	3	1.00	21	0.143
148	A	4	3	1.00	14	0.214
149	A	4	3	1.00	21	0.143
150	A	4	3	1.00	23	0.130
151	A	4	3	1.00	23	0.130
152	A	4	3	1.00	23	0.130
153	A	4	3	1.00	23	0.130
154	A	4	3	1.00	23	0.130
155	A	4	3	1.00	23	0.130
156	A	4	3	1.00	23	0.130
157	A	4	3	1.00	23	0.130
158	A	4	3	1.00	23	0.130
159	A	4	3	1.00	21	0.143
160	A	4	3	1.00	14	0.214
161	A	4	3	1.00	21	0.143
162	A	4	3	1.00	23	0.130
163	A	4	3	1.00	23	0.130
164	A	4	3	1.00	23	0.130
165	A	4	3	1.00	23	0.130
166	A	4	3	1.00	23	0.130
167	A	4	3	1.00	23	0.130
168	A	4	3	1.00	14	0.214
169	A	4	3	1.00	14	0.214
170	A	4	3	1.00	23	0.130
171	A	5	5	1.00	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
172	A	4	3	1.00	23	0.130
173	A	4	4	1.00	23	0.174
174	A	5	4	1.00	21	0.190
175	A	3	3	1.00	14	0.214
176	A	4	3	1.00	21	0.143
177	A	5	5	1.00	23	0.217
178	A	4	3	1.00	23	0.130
179	A	6	6	1.00	23	0.261
180	A	4	3	1.00	23	0.130
181	A	5	5	1.00	23	0.217
182	A	4	3	1.00	23	0.130
183	A	5	5	1.00	23	0.217
184	A	4	3	1.00	21	0.143
185	A	5	5	1.00	14	0.357
186	A	4	3	1.00	21	0.143
187	A	6	6	1.00	23	0.261
188	A	4	3	1.00	23	0.130
189	A	7	6	1.00	23	0.261
190	A	6	6	1.00	23	0.261
191	A	4	3	1.00	23	0.130
192	A	6	6	1.00	23	0.261
193	A	4	3	1.00	23	0.130
194	A	6	6	1.00	23	0.261
195	A	4	3	1.00	21	0.143
196	A	6	6	1.00	14	0.429
197	A	4	3	1.00	21	0.143
198	A	7	7	1.00	23	0.304
199	A	4	3	1.00	23	0.130
200	A	8	7	1.00	23	0.304
201	A	7	6	1.00	14	0.429
202	A	3	3	1.00	12	0.250
203	A	4	4	1.00	10	0.400
204	A	4	4	1.00	12	0.333
205	A	5	5	1.00	10	0.500
206	A	3	3	1.00	12	0.250
207	A	3	3	1.00	10	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
208	A	7	6	1.00	17	0.353
209	A	8	7	1.00	17	0.412
210	A	6	6	1.00	17	0.353
211	A	7	6	1.00	17	0.353
212	A	5	5	1.00	15	0.333
213	A	6	5	1.00	12	0.417
214	A	7	5	1.00	15	0.333
215	A	5	5	1.00	17	0.294
216	A	8	6	1.00	17	0.353
217	A	6	6	1.00	17	0.353
218	A	9	7	1.00	17	0.412
219	A	7	6	1.00	17	0.353
220	A	8	7	1.00	17	0.412
221	A	6	5	1.00	15	0.333
222	A	7	6	1.00	12	0.500
223	A	8	6	1.00	15	0.400
224	A	7	6	1.00	17	0.353
225	A	5	5	1.00	10	0.500
226	A	6	5	1.00	12	0.417
227	A	6	6	1.00	10	0.600
228	A	7	6	1.00	12	0.500
229	A	6	5	1.00	17	0.294
230	A	7	6	1.00	17	0.353
231	A	5	5	1.00	17	0.294
232	A	6	5	1.00	17	0.294
233	A	4	4	1.00	15	0.267
234	A	3	3	1.00	12	0.250
235	A	7	5	1.00	15	0.333
236	A	5	5	1.00	17	0.294
237	A	8	6	1.00	17	0.353
238	A	6	5	1.00	17	0.294
239	A	7	6	1.00	17	0.353
240	A	5	5	1.00	17	0.294
241	A	4	4	1.00	17	0.235
242	A	5	5	1.00	15	0.333
243	A	4	4	1.00	12	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
244	A	8	6	1.00	15	0.400
245	A	6	6	1.00	17	0.353
246	A	8	7	1.00	17	0.412
247	A	6	5	1.00	17	0.294
248	A	6	6	1.00	17	0.353
249	A	6	6	1.00	17	0.353
250	A	6	6	1.00	17	0.353
251	A	6	5	1.00	15	0.333
252	A	6	6	1.00	12	0.500
253	A	9	7	1.00	15	0.467
254	A	7	7	1.00	17	0.412
255	A	3	3	1.00	10	0.300
256	A	3	3	1.00	12	0.250
257	A	6	4	1.26	14	0.286
258	A	6	5	1.00	8	0.625
259	A	9	8	1.00	15	0.533
260	A	8	7	1.00	15	0.467
261	A	4	4	1.00	15	0.267
262	A	6	6	1.00	15	0.400
263	A	7	7	1.00	15	0.467

Chapter 3

Listing of integrals

3.1 $\int \sinh^4(c + dx) (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=73

$$\frac{(a + b) \sinh(c + dx) \cosh^3(c + dx)}{4d} - \frac{(5a + 9b) \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{3}{8} x^{(a+5b)} - \frac{b \tanh(c + dx)}{d}$$

[Out] 3/8*(a+5*b)*x-1/8*(5*a+9*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*(a+b)*cosh(d*x+c)^3*sinh(d*x+c)/d-b*tanh(d*x+c)/d

Rubi [A] time = 0.08, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3663, 455, 1157, 388, 206}

$$\frac{(a + b) \sinh(c + dx) \cosh^3(c + dx)}{4d} - \frac{(5a + 9b) \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{3}{8} x^{(a+5b)} - \frac{b \tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2), x]

[Out] (3*(a + 5*b)*x)/8 - ((5*a + 9*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + ((a + b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d) - (b*Tanh[c + d*x])/d

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 455

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] := Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)]/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&

(IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2,
x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x,
0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
&& IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned} \int \sinh^4(c + dx) (a + b \tanh^2(c + dx)) dx &= \frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{(a + b) \cosh^3(c + dx) \sinh(c + dx)}{4d} + \frac{\text{Subst}\left(\int \frac{-a-b-4(a+b)x^2-4bx^4}{(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{4d} \\ &= -\frac{(5a + 9b) \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{(a + b) \cosh^3(c + dx) \sinh(c + dx)}{4d} \\ &= -\frac{(5a + 9b) \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{(a + b) \cosh^3(c + dx) \sinh(c + dx)}{4d} \\ &= \frac{3}{8}(a + 5b)x - \frac{(5a + 9b) \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{(a + b) \cosh^3(c + dx) \sinh(c + dx)}{4d} \end{aligned}$$

Mathematica [A] time = 0.37, size = 56, normalized size = 0.77

$$\frac{12(a + 5b)(c + dx) - 8(a + 2b) \sinh(2(c + dx)) + (a + b) \sinh(4(c + dx)) - 32b \tanh(c + dx)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2), x]
```

```
[Out] (12*(a + 5*b)*(c + d*x) - 8*(a + 2*b)*Sinh[2*(c + d*x)] + (a + b)*Sinh[4*(c + d*x)] - 32*b*Tanh[c + d*x])/(32*d)
```

fricas [A] time = 0.51, size = 120, normalized size = 1.64

$$\frac{(a + b) \sinh(dx + c)^5 + (10(a + b) \cosh(dx + c)^2 - 7a - 15b) \sinh(dx + c)^3 + 8(3(a + 5b)dx + 8b) \cosh(dx + c)}{64d \cosh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2), x, algorithm="fricas")
```

[Out] $1/64*((a + b)*\sinh(d*x + c)^5 + (10*(a + b)*\cosh(d*x + c)^2 - 7*a - 15*b)*\sinh(d*x + c)^3 + 8*(3*(a + 5*b)*d*x + 8*b)*\cosh(d*x + c) + (5*(a + b)*\cosh(d*x + c)^4 - 3*(7*a + 15*b)*\cosh(d*x + c)^2 - 8*a - 80*b)*\sinh(d*x + c))/(d*\cosh(d*x + c))$

giac [B] time = 0.20, size = 145, normalized size = 1.99

$$\frac{24(a + 5b)dx - (18ae^{(4dx+4c)} + 90be^{(4dx+4c)} - 8ae^{(2dx+2c)} - 16be^{(2dx+2c)} + a + b)e^{(-4dx-4c)} + (ae^{(4dx+20c)} + \dots)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2), x, algorithm="giac")`

[Out] $1/64*(24*(a + 5*b)*d*x - (18*a*e^{(4*d*x + 4*c)} + 90*b*e^{(4*d*x + 4*c)} - 8*a*e^{(2*d*x + 2*c)} - 16*b*e^{(2*d*x + 2*c)} + a + b)*e^{(-4*d*x - 4*c)} + (a*e^{(4*d*x + 20*c)} + b*e^{(4*d*x + 20*c)} - 8*a*e^{(2*d*x + 18*c)} - 16*b*e^{(2*d*x + 18*c)})*e^{(-16*c)} + 128*b/(e^{(2*d*x + 2*c)} + 1))/d$

maple [A] time = 0.27, size = 96, normalized size = 1.32

$$\frac{a \left(\left(\frac{\sinh^3(dx+c)}{4} - \frac{3 \sinh(dx+c)}{8} \right) \cosh(dx+c) + \frac{3dx}{8} + \frac{3c}{8} \right) + b \left(\frac{\sinh^5(dx+c)}{4 \cosh(dx+c)} - \frac{5(\sinh^3(dx+c))}{8 \cosh(dx+c)} + \frac{15dx}{8} + \frac{15c}{8} - \frac{15 \tanh(dx+c)}{8} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2), x)`

[Out] $1/d*(a*((1/4*\sinh(d*x+c)^3-3/8*\sinh(d*x+c))*\cosh(d*x+c)+3/8*d*x+3/8*c)+b*(1/4*\sinh(d*x+c)^5/\cosh(d*x+c)-5/8*\sinh(d*x+c)^3/\cosh(d*x+c)+15/8*d*x+15/8*c-15/8*\tanh(d*x+c)))$

maxima [B] time = 1.13, size = 154, normalized size = 2.11

$$\frac{1}{64} a \left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) + \frac{1}{64} b \left(\frac{120(dx+c)}{d} + \frac{16e^{(-2dx-2c)} - e^{(-4dx-4c)}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2), x, algorithm="maxima")`

[Out] $1/64*a*(24*x + e^{(4*d*x + 4*c)}/d - 8*e^{(2*d*x + 2*c)}/d + 8*e^{(-2*d*x - 2*c)}/d - e^{(-4*d*x - 4*c)}/d) + 1/64*b*(120*(d*x + c)/d + (16*e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)})/d - (15*e^{(-2*d*x - 2*c)} + 144*e^{(-4*d*x - 4*c)} - 1)/(d*(e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)})))$

mupad [B] time = 0.24, size = 101, normalized size = 1.38

$$x \left(\frac{3a}{8} + \frac{15b}{8} \right) + \frac{2b}{d(e^{2c+2dx} + 1)} - \frac{e^{-4c-4dx}(a+b)}{64d} + \frac{e^{4c+4dx}(a+b)}{64d} + \frac{e^{-2c-2dx}(a+2b)}{8d} - \frac{e^{2c+2dx}(a+2b)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2), x)`

[Out] $x*((3*a)/8 + (15*b)/8) + (2*b)/(d*(\exp(2*c + 2*d*x) + 1)) - (\exp(-4*c - 4*d*x)*(a + b))/(64*d) + (\exp(4*c + 4*d*x)*(a + b))/(64*d) + (\exp(-2*c - 2*d*x)*(a + 2*b))/(8*d) - (\exp(2*c + 2*d*x)*(a + 2*b))/(8*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx)) \sinh^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*sinh(c + d*x)**4, x)
```


3.2 $\int \sinh^3(c + dx) (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=47

$$\frac{(a+b) \cosh^3(c+dx)}{3d} - \frac{(a+2b) \cosh(c+dx)}{d} - \frac{b \operatorname{sech}(c+dx)}{d}$$

[Out] $-(a+2*b)*\cosh(d*x+c)/d+1/3*(a+b)*\cosh(d*x+c)^3/d-b*\operatorname{sech}(d*x+c)/d$

Rubi [A] time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3664, 448}

$$\frac{(a+b) \cosh^3(c+dx)}{3d} - \frac{(a+2b) \cosh(c+dx)}{d} - \frac{b \operatorname{sech}(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[c + d*x]^3*(a + b*\text{Tanh}[c + d*x]^2), x]$

[Out] $-\frac{((a + 2*b)*\text{Cosh}[c + d*x])/d} + \frac{(a + b)*\text{Cosh}[c + d*x]^3}{(3*d)} - \frac{(b*\text{Sech}[c + d*x])/d}$

Rule 448

$\text{Int}[\frac{(e_{.})*(x_{.})^{(m_{.})}*((a_{.}) + (b_{.})*(x_{.})^{(n_{.})})^{(p_{.})}*((c_{.}) + (d_{.})*(x_{.})^{(n_{.})})^{(q_{.})}}{x_{\text{Symbol}}}] := \text{Int}[\text{ExpandIntegrand}[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;$ FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 3664

$\text{Int}[\sin[(e_{.}) + (f_{.})*(x_{.})]^{(m_{.})}*((a_{.}) + (b_{.})*\tan[(e_{.}) + (f_{.})*(x_{.})]^2)^{(p_{.})}, x_{\text{Symbol}}] := \text{With}[\{ff = \text{FreeFactors}[\text{Sec}[e + f*x], x]\}, \text{Dist}[1/(f*ff^m), \text{Subst}[\text{Int}[((-1 + ff^2*x^2)^{(m-1)/2}*(a - b + b*ff^2*x^2)^p)/x^{(m+1)}, x], x, \text{Sec}[e + f*x]/ff], x] /;$ FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \sinh^3(c + dx) (a + b \tanh^2(c + dx)) dx &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+b-bx^2)}{x^4} dx, x, \operatorname{sech}(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(-b + \frac{-a-b}{x^4} + \frac{a+2b}{x^2}\right) dx, x, \operatorname{sech}(c + dx)\right)}{d} \\ &= -\frac{(a+2b) \cosh(c + dx)}{d} + \frac{(a+b) \cosh^3(c + dx)}{3d} - \frac{b \operatorname{sech}(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.05, size = 73, normalized size = 1.55

$$-\frac{3a \cosh(c + dx)}{4d} + \frac{a \cosh(3(c + dx))}{12d} - \frac{7b \cosh(c + dx)}{4d} + \frac{b \cosh(3(c + dx))}{12d} - \frac{b \operatorname{sech}(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Sinh}[c + d*x]^3*(a + b*\text{Tanh}[c + d*x]^2), x]$

[Out] $(-3*a*\text{Cosh}[c + d*x])/(4*d) - (7*b*\text{Cosh}[c + d*x])/(4*d) + (a*\text{Cosh}[3*(c + d*x)])/(12*d) + (b*\text{Cosh}[3*(c + d*x)])/(12*d) - (b*\text{Sech}[c + d*x])/d$

fricas [B] time = 0.78, size = 91, normalized size = 1.94

$$\frac{(a+b)\cosh(dx+c)^4 + (a+b)\sinh(dx+c)^4 - 4(2a+5b)\cosh(dx+c)^2 + 2(3(a+b)\cosh(dx+c)^2 - 4a - 1)}{24d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")`

[Out] $1/24*((a+b)*\cosh(d*x+c)^4 + (a+b)*\sinh(d*x+c)^4 - 4*(2*a+5*b)*\cosh(d*x+c)^2 + 2*(3*(a+b)*\cosh(d*x+c)^2 - 4*a - 10*b)*\sinh(d*x+c)^2 - 9*a - 45*b)/(d*\cosh(d*x+c))$

giac [B] time = 0.17, size = 120, normalized size = 2.55

$$\frac{(9ae^{2dx+2c} + 21be^{2dx+2c} - a - b)e^{-3dx-3c} - (ae^{3dx+24c} + be^{3dx+24c} - 9ae^{dx+22c} - 21be^{dx+22c})e^{-21c} + 48be^{dx+c}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")`

[Out] $-1/24*((9*a*e^{2*d*x+2*c} + 21*b*e^{2*d*x+2*c} - a - b)*e^{-3*d*x-3*c} - (a*e^{3*d*x+24*c} + b*e^{3*d*x+24*c} - 9*a*e^{d*x+22*c} - 21*b*e^{d*x+22*c})*e^{-21*c} + 48*b*e^{d*x+c})/(e^{2*d*x+2*c} + 1)/d$

maple [A] time = 0.26, size = 75, normalized size = 1.60

$$\frac{a\left(-\frac{2}{3} + \frac{\sinh^2(dx+c)}{3}\right)\cosh(dx+c) + b\left(\frac{\sinh^4(dx+c)}{3\cosh(dx+c)} - \frac{4(\sinh^2(dx+c))}{3\cosh(dx+c)} - \frac{8}{3\cosh(dx+c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x)`

[Out] $1/d*(a*(-2/3+1/3*\sinh(d*x+c)^2)*\cosh(d*x+c)+b*(1/3*\sinh(d*x+c)^4/\cosh(d*x+c)-4/3*\sinh(d*x+c)^2/\cosh(d*x+c)-8/3/\cosh(d*x+c)))$

maxima [B] time = 0.39, size = 136, normalized size = 2.89

$$-\frac{1}{24}b\left(\frac{21e^{-dx-c} - e^{-3dx-3c}}{d} + \frac{20e^{-2dx-2c} + 69e^{-4dx-4c} - 1}{d(e^{-3dx-3c} + e^{-5dx-5c})}\right) + \frac{1}{24}a\left(\frac{e^{3dx+3c}}{d} - \frac{9e^{dx+c}}{d} - \frac{9e^{-dx-c}}{d} + \frac{e^{-3dx-3c}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")`

[Out] $-1/24*b*((21*e^{-d*x-c} - e^{-3*d*x-3*c})/d + (20*e^{-2*d*x-2*c} + 69*e^{-4*d*x-4*c} - 1)/(d*(e^{-3*d*x-3*c} + e^{-5*d*x-5*c}))) + 1/24*a*(e^{3*d*x+3*c}/d - 9*e^{d*x+c}/d - 9*e^{-d*x-c}/d + e^{-3*d*x-3*c}/d)$

mupad [B] time = 1.17, size = 99, normalized size = 2.11

$$\frac{e^{-3c-3dx}(a+b)}{24d} + \frac{e^{3c+3dx}(a+b)}{24d} - \frac{e^{c+dx}(3a+7b)}{8d} - \frac{e^{-c-dx}(3a+7b)}{8d} - \frac{2be^{c+dx}}{d(e^{2c+2dx}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] (exp(- 3*c - 3*d*x)*(a + b))/(24*d) + (exp(3*c + 3*d*x)*(a + b))/(24*d) - (
exp(c + d*x)*(3*a + 7*b))/(8*d) - (exp(- c - d*x)*(3*a + 7*b))/(8*d) - (2*b
*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1))
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int (a + b \tanh^2(c + dx)) \sinh^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*sinh(c + d*x)**3, x)
```

3.3 $\int \sinh^2(c + dx) (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=44

$$\frac{(a + b) \sinh(c + dx) \cosh(c + dx)}{2d} - \frac{1}{2}x(a + 3b) + \frac{b \tanh(c + dx)}{d}$$

[Out] $-1/2*(a+3*b)*x+1/2*(a+b)*\cosh(d*x+c)*\sinh(d*x+c)/d+b*\tanh(d*x+c)/d$

Rubi [A] time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3663, 455, 388, 206}

$$\frac{(a + b) \sinh(c + dx) \cosh(c + dx)}{2d} - \frac{1}{2}x(a + 3b) + \frac{b \tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2), x]

[Out] $-((a + 3*b)*x)/2 + ((a + b)*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])/(2*d) + (b*\text{Tanh}[c + d*x])/d$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 455

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] := Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)]/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 3663

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]

Rubi steps

$$\begin{aligned}
\int \sinh^2(c + dx) (a + b \tanh^2(c + dx)) dx &= \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)}{(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{(a + b) \cosh(c + dx) \sinh(c + dx)}{2d} - \frac{\text{Subst}\left(\int \frac{a+b+2bx^2}{1-x^2} dx, x, \tanh(c + dx)\right)}{2d} \\
&= \frac{(a + b) \cosh(c + dx) \sinh(c + dx)}{2d} + \frac{b \tanh(c + dx)}{d} - \frac{(a + 3b) \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{2d} \\
&= -\frac{1}{2}(a + 3b)x + \frac{(a + b) \cosh(c + dx) \sinh(c + dx)}{2d} + \frac{b \tanh(c + dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.21, size = 41, normalized size = 0.93

$$\frac{-2(a + 3b)(c + dx) + (a + b) \sinh(2(c + dx)) + 4b \tanh(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2), x]

[Out] (-2*(a + 3*b)*(c + d*x) + (a + b)*Sinh[2*(c + d*x)] + 4*b*Tanh[c + d*x])/(4*d)

fricas [A] time = 0.65, size = 71, normalized size = 1.61

$$\frac{(a + b) \sinh(dx + c)^3 - 4((a + 3b)dx + 2b) \cosh(dx + c) + (3(a + b) \cosh(dx + c)^2 + a + 9b) \sinh(dx + c)}{8d \cosh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] 1/8*((a + b)*sinh(d*x + c)^3 - 4*((a + 3*b)*d*x + 2*b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a + 9*b)*sinh(d*x + c))/(d*cosh(d*x + c))

giac [B] time = 0.18, size = 109, normalized size = 2.48

$$\frac{4(a + 3b)dx - (ae^{(2dx+8c)} + be^{(2dx+8c)})e^{(-6c)} - \frac{(ae^{(4dx+4c)} + 3be^{(4dx+4c)} - 14be^{(2dx+2c)} - a - b)e^{(-2c)}}{e^{(2dx)} + e^{(4dx+2c)}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2), x, algorithm="giac")

[Out] -1/8*(4*(a + 3*b)*d*x - (a*e^(2*d*x + 8*c) + b*e^(2*d*x + 8*c))*e^(-6*c) - (a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) - 14*b*e^(2*d*x + 2*c) - a - b)*e^(-2*c)/(e^(2*d*x) + e^(4*d*x + 2*c)))/d

maple [A] time = 0.18, size = 66, normalized size = 1.50

$$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2}\right) + b\left(\frac{\sinh^3(dx+c)}{2\cosh(dx+c)} - \frac{3dx}{2} - \frac{3c}{2} + \frac{3\tanh(dx+c)}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2), x)

[Out] $1/d*(a*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b*(1/2*sinh(d*x+c)^3/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c)))$

maxima [B] time = 1.38, size = 101, normalized size = 2.30

$$-\frac{1}{8}a\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{8}b\left(\frac{12(dx+c)}{d} + \frac{e^{(-2dx-2c)}}{d} - \frac{17e^{(-2dx-2c)} + 1}{d(e^{(-2dx-2c)} + e^{(-4dx-4c)})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")`

[Out] $-1/8*a*(4*x - e^{(2*d*x + 2*c)}/d + e^{(-2*d*x - 2*c)}/d) - 1/8*b*(12*(d*x + c)/d + e^{(-2*d*x - 2*c)}/d - (17*e^{(-2*d*x - 2*c)} + 1)/(d*(e^{(-2*d*x - 2*c)} + e^{(-4*d*x - 4*c)})))$

mupad [B] time = 0.15, size = 64, normalized size = 1.45

$$\frac{e^{2c+2dx}(a+b)}{8d} - \frac{2b}{d(e^{2c+2dx}+1)} - \frac{e^{-2c-2dx}(a+b)}{8d} - x\left(\frac{a}{2} + \frac{3b}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2),x)`

[Out] $(\exp(2*c + 2*d*x)*(a + b))/(8*d) - (2*b)/(d*(\exp(2*c + 2*d*x) + 1)) - (\exp(-2*c - 2*d*x)*(a + b))/(8*d) - x*(a/2 + (3*b)/2)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx)) \sinh^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)`

[Out] `Integral((a + b*tanh(c + d*x)**2)*sinh(c + d*x)**2, x)`

3.4 $\int \sinh(c + dx) (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=25

$$\frac{(a + b) \cosh(c + dx)}{d} + \frac{b \operatorname{sech}(c + dx)}{d}$$

[Out] (a+b)*cosh(d*x+c)/d+b*sech(d*x+c)/d

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3664, 14}

$$\frac{(a + b) \cosh(c + dx)}{d} + \frac{b \operatorname{sech}(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2), x]

[Out] ((a + b)*Cosh[c + d*x])/d + (b*Sech[c + d*x])/d

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 3664

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^(m - 1)/2)*(a - b + b*ff^2*x^2)^p]/x^(m + 1), x], x, Sec[e + f*x]/ff, x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \sinh(c + dx) (a + b \tanh^2(c + dx)) dx &= -\frac{\operatorname{Subst}\left(\int \frac{a+b-bx^2}{x^2} dx, x, \operatorname{sech}(c + dx)\right)}{d} \\ &= -\frac{\operatorname{Subst}\left(\int \left(-b + \frac{a+b}{x^2}\right) dx, x, \operatorname{sech}(c + dx)\right)}{d} \\ &= \frac{(a + b) \cosh(c + dx)}{d} + \frac{b \operatorname{sech}(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.06, size = 45, normalized size = 1.80

$$\frac{a \sinh(c) \sinh(dx)}{d} + \frac{a \cosh(c) \cosh(dx)}{d} + \frac{b \cosh(c + dx)}{d} + \frac{b \operatorname{sech}(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2), x]

[Out] (a*Cosh[c]*Cosh[d*x])/d + (b*Cosh[c + d*x])/d + (b*Sech[c + d*x])/d + (a*Sinh[c]*Sinh[d*x])/d

fricas [A] time = 0.56, size = 42, normalized size = 1.68

$$\frac{(a+b)\cosh(dx+c)^2 + (a+b)\sinh(dx+c)^2 + a + 3b}{2d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")

[Out] 1/2*((a+b)*cosh(d*x+c)^2 + (a+b)*sinh(d*x+c)^2 + a + 3*b)/(d*cosh(d*x+c))

giac [B] time = 0.15, size = 79, normalized size = 3.16

$$\frac{(ae^{(dx+6c)} + be^{(dx+6c)})e^{(-5c)} + \frac{(ae^{(2dx+2c)} + 5be^{(2dx+2c)} + a+b)e^{(-c)}}{e^{(3dx+2c)} + e^{(dx)}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] 1/2*((a*e^(d*x + 6*c) + b*e^(d*x + 6*c))*e^(-5*c) + (a*e^(2*d*x + 2*c) + 5*b*e^(2*d*x + 2*c) + a + b)*e^(-c)/(e^(3*d*x + 2*c) + e^(d*x)))/d

maple [A] time = 0.17, size = 44, normalized size = 1.76

$$\frac{a\cosh(dx+c) + b\left(\frac{\sinh^2(dx+c)}{\cosh(dx+c)} + \frac{2}{\cosh(dx+c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)*(a+b*tanh(d*x+c)^2),x)

[Out] 1/d*(a*cosh(d*x+c)+b*(sinh(d*x+c)^2/cosh(d*x+c)+2/cosh(d*x+c)))

maxima [B] time = 0.37, size = 67, normalized size = 2.68

$$\frac{1}{2}b\left(\frac{e^{(-dx-c)}}{d} + \frac{5e^{(-2dx-2c)} + 1}{d(e^{(-dx-c)} + e^{(-3dx-3c)})}\right) + \frac{a\cosh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] 1/2*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) + 1)/(d*(e^(-d*x - c) + e^(-3*d*x - 3*c)))) + a*cosh(d*x + c)/d

mupad [B] time = 0.12, size = 27, normalized size = 1.08

$$\frac{b}{d\cosh(c+dx)} + \frac{\cosh(c+dx)(a+b)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c+d*x)*(a+b*tanh(c+d*x)^2),x)

[Out] b/(d*cosh(c+d*x)) + (cosh(c+d*x)*(a+b))/d

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx)) \sinh(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*sinh(c + d*x), x)
```

3.5 $\int \operatorname{csch}(c + dx) \left(a + b \tanh^2(c + dx) \right) dx$

Optimal. Leaf size=26

$$-\frac{a \tanh^{-1}(\cosh(c + dx))}{d} - \frac{b \operatorname{sech}(c + dx)}{d}$$

[Out] $-a \operatorname{arctanh}(\cosh(d*x+c))/d - b \operatorname{sech}(d*x+c)/d$

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3664, 388, 207}

$$-\frac{a \tanh^{-1}(\cosh(c + dx))}{d} - \frac{b \operatorname{sech}(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]*(a + b*\operatorname{Tanh}[c + d*x]^2), x]$

[Out] $-((a*\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]])/d) - (b*\operatorname{Sech}[c + d*x])/d$

Rule 207

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Rt}[-a, 2]]/(\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 388

$\operatorname{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)*((c_ + (d_)*(x_)^{(n_)}))}, x_Symbol] \rightarrow \operatorname{Simp}[(d*x*(a + b*x^n)^{(p+1)})/(b*(n*(p+1)+1)), x] - \operatorname{Dist}[(a*d - b*c*(n*(p+1)+1))/(b*(n*(p+1)+1)), \operatorname{Int}[(a + b*x^n)^p, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p+1)+1, 0]

Rule 3664

$\operatorname{Int}[\sin[(e_ + (f_)*(x_))]^{(m_)*((a_ + (b_)*\tan[(e_ + (f_)*(x_))]^2)^{(p_)}), x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Sec}[e + f*x], x]\}, \operatorname{Dist}[1/(f*ff^m), \operatorname{Subst}[\operatorname{Int}[((-1 + ff^2*x^2)^{((m-1)/2)*(a-b+b*ff^2*x^2)^p})/x^{(m+1)}], x, \operatorname{Sec}[e + f*x]/ff], x] /;$ FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m-1)/2]

Rubi steps

$$\begin{aligned} \int \operatorname{csch}(c + dx) \left(a + b \tanh^2(c + dx) \right) dx &= \frac{\operatorname{Subst}\left(\int \frac{a+b-bx^2}{-1+x^2} dx, x, \operatorname{sech}(c + dx)\right)}{d} \\ &= -\frac{b \operatorname{sech}(c + dx)}{d} + \frac{a \operatorname{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \operatorname{sech}(c + dx)\right)}{d} \\ &= -\frac{a \tanh^{-1}(\cosh(c + dx))}{d} - \frac{b \operatorname{sech}(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.03, size = 52, normalized size = 2.00

$$\frac{a \log\left(\sinh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{a \log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{b \operatorname{sech}(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2), x]

[Out] -((a*Log[Cosh[c/2 + (d*x)/2]])/d) + (a*Log[Sinh[c/2 + (d*x)/2]])/d - (b*Sec h[c + d*x])/d

fricas [B] time = 0.74, size = 167, normalized size = 6.42

$$\frac{2b \cosh(dx + c) + (a \cosh(dx + c)^2 + 2a \cosh(dx + c) \sinh(dx + c) + a \sinh(dx + c)^2 + a) \log(\cosh(dx + c))}{d \cosh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] -(2*b*cosh(d*x + c) + (a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*b*sinh(d*x + c))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 + d)

giac [A] time = 0.16, size = 52, normalized size = 2.00

$$\frac{a \log(e^{(dx+c)} + 1) - a \log(|e^{(dx+c)} - 1|) + \frac{2be^{(dx+c)}}{e^{(2dx+2c)} + 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2), x, algorithm="giac")

[Out] -(a*log(e^(d*x + c) + 1) - a*log(abs(e^(d*x + c) - 1)) + 2*b*e^(d*x + c)/(e^(2*d*x + 2*c) + 1))/d

maple [A] time = 0.25, size = 27, normalized size = 1.04

$$\frac{-2a \operatorname{arctanh}(e^{dx+c}) - \frac{b}{\cosh(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)*(a+b*tanh(d*x+c)^2), x)

[Out] 1/d*(-2*a*arctanh(exp(d*x+c))-b/cosh(d*x+c))

maxima [A] time = 0.35, size = 40, normalized size = 1.54

$$\frac{a \log\left(\tanh\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}{d} - \frac{2b}{d(e^{(dx+c)} + e^{(-dx-c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2), x, algorithm="maxima")

[Out] a*log(tanh(1/2*d*x + 1/2*c))/d - 2*b/(d*(e^(d*x + c) + e^(-d*x - c)))

mupad [B] time = 0.12, size = 64, normalized size = 2.46

$$-\frac{2 \operatorname{atan}\left(\frac{a e^{dx} e^c \sqrt{-d^2}}{d \sqrt{a^2}}\right) \sqrt{a^2}}{\sqrt{-d^2}} - \frac{2b e^{c+dx}}{d(e^{2c+2dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)/sinh(c + d*x),x)
```

```
[Out] - (2*atan((a*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^2)^(1/2)))*(a^2)^(1/2))/(-
d^2)^(1/2) - (2*b*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1))
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int (a + b \tanh^2(c + dx)) \operatorname{csch}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*csch(c + d*x), x)
```

3.6 $\int \operatorname{csch}^2(c + dx) (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=24

$$\frac{b \tanh(c + dx)}{d} - \frac{a \operatorname{coth}(c + dx)}{d}$$

[Out] $-a \operatorname{coth}(d*x+c)/d + b \tanh(d*x+c)/d$

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3663, 14}

$$\frac{b \tanh(c + dx)}{d} - \frac{a \operatorname{coth}(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csch}[c + d*x]^2*(a + b*\text{Tanh}[c + d*x]^2), x]$

[Out] $-((a*\text{Coth}[c + d*x])/d) + (b*\text{Tanh}[c + d*x])/d$

Rule 14

$\text{Int}[(u_*)*((c_*)*(x_))^{(m_*)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(c*x)^{m*u}, x], x] /; \text{FreeQ}\{c, m\}, x \&\& \text{SumQ}[u] \&\& !\text{LinearQ}[u, x] \&\& !\text{MatchQ}[u, (a_*) + (b_*)*(v_)] /; \text{FreeQ}\{a, b\}, x \&\& \text{InverseFunctionQ}[v]$

Rule 3663

$\text{Int}[\sin[(e_*) + (f_*)*(x_)]^{(m_*)}*((a_*) + (b_*)*((c_*)*\tan[(e_*) + (f_*)*(x_)]))^{(n_*)}]^{(p_*)}, x_Symbol] \rightarrow \text{With}\{\text{ff} = \text{FreeFactors}[\text{Tan}[e + f*x], x]\}, \text{Dist}[(c*\text{ff}^{(m+1)})/f, \text{Subst}[\text{Int}[(x^m*(a + b*(\text{ff}*x)^n)^p]/(c^2 + \text{ff}^2*x^2)^{(m/2 + 1)}, x], x, (c*\text{Tan}[e + f*x])/\text{ff}], x] /; \text{FreeQ}\{a, b, c, e, f, n, p\}, x \&\& \text{IntegerQ}[m/2]$

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^2(c + dx) (a + b \tanh^2(c + dx)) dx &= \frac{\text{Subst}\left(\int \frac{a+bx^2}{x^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(b + \frac{a}{x^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{a \operatorname{coth}(c + dx)}{d} + \frac{b \tanh(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.02, size = 24, normalized size = 1.00

$$\frac{b \tanh(c + dx)}{d} - \frac{a \operatorname{coth}(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Csch}[c + d*x]^2*(a + b*\text{Tanh}[c + d*x]^2), x]$

[Out] $-((a*\text{Coth}[c + d*x])/d) + (b*\text{Tanh}[c + d*x])/d$

fricas [B] time = 0.47, size = 88, normalized size = 3.67

$$\frac{4(a \cosh(dx + c) + b \sinh(dx + c))}{d \cosh(dx + c)^3 + 3d \cosh(dx + c) \sinh(dx + c)^2 + d \sinh(dx + c)^3 - d \cosh(dx + c) + (3d \cosh(dx + c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")

[Out] $-4*(a*\cosh(d*x + c) + b*\sinh(d*x + c))/(d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c)*\sinh(d*x + c)^2 + d*\sinh(d*x + c)^3 - d*\cosh(d*x + c) + (3*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c))$

giac [A] time = 0.16, size = 45, normalized size = 1.88

$$\frac{2\left(ae^{2dx+2c} + be^{2dx+2c} + a - b\right)}{d\left(e^{4dx+4c} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] $-2*(a*e^{(2*d*x + 2*c)} + b*e^{(2*d*x + 2*c)} + a - b)/(d*(e^{(4*d*x + 4*c)} - 1))$

maple [A] time = 0.33, size = 23, normalized size = 0.96

$$\frac{-\coth(dx + c)a + b \tanh(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2),x)

[Out] $1/d*(-\coth(d*x+c)*a+b*\tanh(d*x+c))$

maxima [A] time = 0.34, size = 39, normalized size = 1.62

$$\frac{2b}{d(e^{(-2dx-2c)} + 1)} + \frac{2a}{d(e^{(-2dx-2c)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] $2*b/(d*(e^{(-2*d*x - 2*c)} + 1)) + 2*a/(d*(e^{(-2*d*x - 2*c)} - 1))$

mupad [B] time = 1.05, size = 43, normalized size = 1.79

$$-\frac{\frac{2(a-b)}{d} + \frac{2e^{2c+2dx}(a+b)}{d}}{e^{4c+4dx} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tanh(c + d*x)^2)/sinh(c + d*x)^2,x)

[Out] $-((2*(a - b))/d + (2*\exp(2*c + 2*d*x)*(a + b))/d)/(\exp(4*c + 4*d*x) - 1)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx)) \operatorname{csch}^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)

[Out] Integral((a + b*tanh(c + d*x)**2)*csch(c + d*x)**2, x)

3.7 $\int \operatorname{csch}^3(c + dx) (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=51

$$\frac{(a - 2b) \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + \frac{b \operatorname{sech}(c + dx)}{d}$$

[Out] 1/2*(a-2*b)*arctanh(cosh(d*x+c))/d-1/2*a*coth(d*x+c)*csch(d*x+c)/d+b*sech(d*x+c)/d

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3664, 455, 388, 207}

$$\frac{(a - 2b) \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + \frac{b \operatorname{sech}(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2), x]

[Out] ((a - 2*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - (a*Coth[c + d*x]*Csch[c + d*x])/(2*d) + (b*Sech[c + d*x])/d

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 455

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] := Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)]/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 3664

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p]/x^(m + 1), x], x, Sec[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^3(c+dx) (a+b \tanh^2(c+dx)) dx &= -\frac{\operatorname{Subst}\left(\int \frac{x^2(a+b-bx^2)}{(-1+x^2)^2} dx, x, \operatorname{sech}(c+dx)\right)}{d} \\
&= -\frac{a \coth(c+dx) \operatorname{csch}(c+dx)}{2d} + \frac{\operatorname{Subst}\left(\int \frac{-a+2bx^2}{-1+x^2} dx, x, \operatorname{sech}(c+dx)\right)}{2d} \\
&= -\frac{a \coth(c+dx) \operatorname{csch}(c+dx)}{2d} + \frac{b \operatorname{sech}(c+dx)}{d} - \frac{(a-2b) \operatorname{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \operatorname{sech}(c+dx)\right)}{2d} \\
&= \frac{(a-2b) \tanh^{-1}(\cosh(c+dx))}{2d} - \frac{a \coth(c+dx) \operatorname{csch}(c+dx)}{2d} + \frac{b \operatorname{sech}(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 87, normalized size = 1.71

$$-\frac{a \operatorname{acsch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a \operatorname{asech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a \log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{b \operatorname{sech}(c+dx)}{d} + \frac{b \log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2), x]

[Out] -1/8*(a*Csch[(c + d*x)/2]^2)/d - (a*Log[Tanh[(c + d*x)/2]])/(2*d) + (b*Log[Tanh[(c + d*x)/2]])/d - (a*Sech[(c + d*x)/2]^2)/(8*d) + (b*Sech[c + d*x])/d

fricas [B] time = 0.66, size = 924, normalized size = 18.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] -1/2*(2*(a - 2*b)*cosh(d*x + c)^5 + 10*(a - 2*b)*cosh(d*x + c)*sinh(d*x + c)^4 + 2*(a - 2*b)*sinh(d*x + c)^5 + 4*(a + 2*b)*cosh(d*x + c)^3 + 4*(5*(a - 2*b)*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^3 + 4*(5*(a - 2*b)*cosh(d*x + c)^3 + 3*(a + 2*b)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(a - 2*b)*cosh(d*x + c) - ((a - 2*b)*cosh(d*x + c)^6 + 6*(a - 2*b)*cosh(d*x + c)*sinh(d*x + c)^5 + (a - 2*b)*sinh(d*x + c)^6 - (a - 2*b)*cosh(d*x + c)^4 + (15*(a - 2*b)*cosh(d*x + c)^2 - a + 2*b)*sinh(d*x + c)^4 + 4*(5*(a - 2*b)*cosh(d*x + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a - 2*b)*cosh(d*x + c)^2 + (15*(a - 2*b)*cosh(d*x + c)^4 - 6*(a - 2*b)*cosh(d*x + c)^2 - a + 2*b)*sinh(d*x + c)^2 + 2*(3*(a - 2*b)*cosh(d*x + c)^5 - 2*(a - 2*b)*cosh(d*x + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + a - 2*b)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((a - 2*b)*cosh(d*x + c)^6 + 6*(a - 2*b)*cosh(d*x + c)*sinh(d*x + c)^5 + (a - 2*b)*sinh(d*x + c)^6 - (a - 2*b)*cosh(d*x + c)^4 + (15*(a - 2*b)*cosh(d*x + c)^2 - a + 2*b)*sinh(d*x + c)^4 + 4*(5*(a - 2*b)*cosh(d*x + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a - 2*b)*cosh(d*x + c)^2 + (15*(a - 2*b)*cosh(d*x + c)^4 - 6*(a - 2*b)*cosh(d*x + c)^2 - a + 2*b)*sinh(d*x + c)^2 + 2*(3*(a - 2*b)*cosh(d*x + c)^5 - 2*(a - 2*b)*cosh(d*x + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + a - 2*b)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(5*(a - 2*b)*cosh(d*x + c)^4 + 6*(a + 2*b)*cosh(d*x + c)^2 + a - 2*b)*sinh(d*x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 - d*cosh(d*x + c)^4 + (15*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c)^3 - d*cosh(d*x + c)^2 + (15*d*cosh(d*x + c)^4 - 6*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)

giac [B] time = 0.15, size = 113, normalized size = 2.22

$$\frac{(ae^c - 2be^c)e^{(-c)} \log(e^{(dx+c)} + 1) - (ae^c - 2be^c)e^{(-c)} \log(|e^{(dx+c)} - 1|) + \frac{4be^{(dx+c)}}{e^{(2dx+2c)} + 1} - \frac{2(ae^{(3dx+3c)} + ae^{(dx+c)})}{(e^{(2dx+2c)} - 1)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2), x, algorithm="giac")

[Out] 1/2*((a*e^c - 2*b*e^c)*e^(-c)*log(e^(d*x + c) + 1) - (a*e^c - 2*b*e^c)*e^(-c)*log(abs(e^(d*x + c) - 1)) + 4*b*e^(d*x + c)/(e^(2*d*x + 2*c) + 1) - 2*(a*e^(3*d*x + 3*c) + a*e^(d*x + c))/(e^(2*d*x + 2*c) - 1)^2)/d

maple [A] time = 0.34, size = 50, normalized size = 0.98

$$\frac{a \left(-\frac{\operatorname{csch}(dx+c) \operatorname{coth}(dx+c)}{2} + \operatorname{arctanh}(e^{dx+c}) \right) + b \left(\frac{1}{\operatorname{cosh}(dx+c)} - 2 \operatorname{arctanh}(e^{dx+c}) \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2), x)

[Out] 1/d*(a*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b*(1/cosh(d*x+c)-2*arctanh(exp(d*x+c))))

maxima [B] time = 0.38, size = 152, normalized size = 2.98

$$\frac{1}{2} a \left(\frac{\log(e^{(-dx-c)} + 1)}{d} - \frac{\log(e^{(-dx-c)} - 1)}{d} + \frac{2(e^{(-dx-c)} + e^{(-3dx-3c)})}{d(2e^{(-2dx-2c)} - e^{(-4dx-4c)} - 1)} \right) - b \left(\frac{\log(e^{(-dx-c)} + 1)}{d} - \frac{\log(e^{(-dx-c)} - 1)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2), x, algorithm="maxima")

[Out] 1/2*a*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/(d*(e^(-2*d*x - 2*c) + 1)))

mupad [B] time = 1.13, size = 156, normalized size = 3.06

$$\frac{\operatorname{atan}\left(\frac{e^{dx} e^c (a \sqrt{-d^2} - 2b \sqrt{-d^2})}{d \sqrt{a^2 - 4ab + 4b^2}}\right) \sqrt{a^2 - 4ab + 4b^2}}{\sqrt{-d^2}} - \frac{a e^{c+dx}}{d (e^{2c+2dx} - 1)} + \frac{2b e^{c+dx}}{d (e^{2c+2dx} + 1)} - \frac{2a e^{c+dx}}{d (e^{4c+4dx} - 2e^{2c+2dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tanh(c + d*x)^2)/sinh(c + d*x)^3, x)

[Out] (atan((exp(d*x)*exp(c)*(a*(-d^2)^(1/2) - 2*b*(-d^2)^(1/2)))/(d*(a^2 - 4*a*b + 4*b^2)^(1/2)))*(a^2 - 4*a*b + 4*b^2)^(1/2))/(-d^2)^(1/2) - (a*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) + (2*b*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)) - (2*a*exp(c + d*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx)) \operatorname{csch}^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**2), x)

[Out] Integral((a + b*tanh(c + d*x)**2)*csch(c + d*x)**3, x)

3.8 $\int \operatorname{csch}^4(c + dx) (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=44

$$\frac{(a - b) \operatorname{coth}(c + dx)}{d} - \frac{a \operatorname{coth}^3(c + dx)}{3d} - \frac{b \tanh(c + dx)}{d}$$

[Out] (a-b)*coth(d*x+c)/d-1/3*a*coth(d*x+c)^3/d-b*tanh(d*x+c)/d

Rubi [A] time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3663, 448}

$$\frac{(a - b) \operatorname{coth}(c + dx)}{d} - \frac{a \operatorname{coth}^3(c + dx)}{3d} - \frac{b \tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2), x]

[Out] ((a - b)*Coth[c + d*x])/d - (a*Coth[c + d*x]^3)/(3*d) - (b*Tanh[c + d*x])/d

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_.))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^4(c + dx) (a + b \tanh^2(c + dx)) dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(a+bx^2)}{x^4} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(-b + \frac{a}{x^4} + \frac{-a+b}{x^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{(a - b) \operatorname{coth}(c + dx)}{d} - \frac{a \operatorname{coth}^3(c + dx)}{3d} - \frac{b \tanh(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.07, size = 61, normalized size = 1.39

$$\frac{2a \operatorname{coth}(c + dx)}{3d} - \frac{a \operatorname{coth}(c + dx) \operatorname{csch}^2(c + dx)}{3d} - \frac{b \tanh(c + dx)}{d} - \frac{b \operatorname{coth}(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2), x]

[Out] $(2*a*Coth[c + d*x])/(3*d) - (b*Coth[c + d*x])/d - (a*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) - (b*Tanh[c + d*x])/d$

fricas [B] time = 0.72, size = 244, normalized size = 5.55

$$\frac{3(d \cosh(dx + c))^6 + 6d \cosh(dx + c) \sinh(dx + c)^5 + d \sinh(dx + c)^6 - 2d \cosh(dx + c)^4 + (15d \cosh(dx + c))^5 + d \sinh(dx + c)^5 - 2d \cosh(dx + c)^3 + (15d \cosh(dx + c))^4 + d \sinh(dx + c)^4 - 2d \cosh(dx + c)^2 + (15d \cosh(dx + c))^3 + d \sinh(dx + c)^3 - 2d \cosh(dx + c) + (15d \cosh(dx + c))^2 + d \sinh(dx + c)^2 - 2d \cosh(dx + c) + d \sinh(dx + c) + 2d}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")`

[Out] $-8/3*((a + 3*b)*\cosh(d*x + c)^2 + 4*a*\cosh(d*x + c)*\sinh(d*x + c) + (a + 3*b)*\sinh(d*x + c)^2 + a - 3*b)/(d*\cosh(d*x + c)^6 + 6*d*\cosh(d*x + c)*\sinh(d*x + c)^5 + d*\sinh(d*x + c)^6 - 2*d*\cosh(d*x + c)^4 + (15*d*\cosh(d*x + c))^2 - 2*d)*\sinh(d*x + c)^4 + 4*(5*d*\cosh(d*x + c)^3 - 2*d*\cosh(d*x + c))*\sinh(d*x + c)^3 - d*\cosh(d*x + c)^2 + (15*d*\cosh(d*x + c))^4 - 12*d*\cosh(d*x + c)^2 - d)*\sinh(d*x + c)^2 + 2*(3*d*\cosh(d*x + c)^5 - 4*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c) + 2*d)$

giac [A] time = 0.15, size = 80, normalized size = 1.82

$$\frac{2\left(\frac{3b}{e^{2dx+2c}+1} - \frac{3be^{4dx+4c}+6ae^{2dx+2c}-6be^{2dx+2c}-2a+3b}{(e^{2dx+2c}-1)^3}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")`

[Out] $2/3*(3*b/(e^{(2*d*x + 2*c)} + 1) - (3*b*e^{(4*d*x + 4*c)} + 6*a*e^{(2*d*x + 2*c)} - 6*b*e^{(2*d*x + 2*c)} - 2*a + 3*b)/(e^{(2*d*x + 2*c)} - 1)^3)/d$

maple [A] time = 0.37, size = 55, normalized size = 1.25

$$\frac{a\left(\frac{2}{3} - \frac{\operatorname{csch}(dx+c)^2}{3}\right)\operatorname{coth}(dx+c) + b\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)} - 2\tanh(dx+c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2),x)`

[Out] $1/d*(a*(2/3-1/3*\operatorname{csch}(d*x+c)^2)*\operatorname{coth}(d*x+c)+b*(-1/\sinh(d*x+c)/\cosh(d*x+c)-2*\tanh(d*x+c)))$

maxima [B] time = 0.36, size = 113, normalized size = 2.57

$$\frac{4}{3}a\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)} - \frac{1}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)}\right) + \frac{4b}{d(e^{(-4dx-4c)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")`

[Out] $4/3*a*(3*e^{(-2*d*x - 2*c)}/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} - 1)) - 1/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} - 1))) + 4*b/(d*(e^{(-4*d*x - 4*c)} - 1))$

mupad [B] time = 1.10, size = 173, normalized size = 3.93

$$\frac{2b}{d(e^{2c+2dx} + 1)} - \frac{\frac{2(2a-b)}{3d} + \frac{2be^{2c+2dx}}{3d}}{e^{4c+4dx} - 2e^{2c+2dx} + 1} - \frac{2b}{3d(e^{2c+2dx} - 1)} - \frac{\frac{2b}{3d} + \frac{2be^{4c+4dx}}{3d} + \frac{4e^{2c+2dx}(2a-b)}{3d}}{3e^{2c+2dx} - 3e^{4c+4dx} + e^{6c+6dx} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)/sinh(c + d*x)^4,x)
```

```
[Out] (2*b)/(d*(exp(2*c + 2*d*x) + 1)) - ((2*(2*a - b))/(3*d) + (2*b*exp(2*c + 2*d*x))/(3*d))/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - (2*b)/(3*d*(exp(2*c + 2*d*x) - 1)) - ((2*b)/(3*d) + (2*b*exp(4*c + 4*d*x))/(3*d) + (4*exp(2*c + 2*d*x)*(2*a - b))/(3*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int (a + b \tanh^2(c + dx)) \operatorname{csch}^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*csch(c + d*x)**4, x)
```

3.9 $\int \sinh^4(c + dx) (a + b \tanh^2(c + dx))^2 dx$

Optimal. Leaf size=118

$$\frac{(a^2 + 10ab + 13b^2) \tanh(c + dx)}{4d} + \frac{1}{8}x(3a^2 + 30ab + 35b^2) + \frac{(a + b)^2 \sinh^4(c + dx) \tanh(c + dx)}{4d} - \frac{(a + b)(a + b \tanh^2(c + dx)) \sinh^2(c + dx)}{4d}$$

[Out] 1/8*(3*a^2+30*a*b+35*b^2)*x-1/8*(a+b)*(a+9*b)*cosh(d*x+c)*sinh(d*x+c)/d-1/4*(a^2+10*a*b+13*b^2)*tanh(d*x+c)/d+1/4*(a+b)^2*sinh(d*x+c)^4*tanh(d*x+c)/d-1/3*b^2*tanh(d*x+c)^3/d

Rubi [A] time = 0.13, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3663, 463, 455, 1153, 206}

$$\frac{(a^2 + 10ab + 13b^2) \tanh(c + dx)}{4d} + \frac{1}{8}x(3a^2 + 30ab + 35b^2) + \frac{(a + b)^2 \sinh^4(c + dx) \tanh(c + dx)}{4d} - \frac{(a + b)(a + b \tanh^2(c + dx)) \sinh^2(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] ((3*a^2 + 30*a*b + 35*b^2)*x)/8 - ((a + b)*(a + 9*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) - ((a^2 + 10*a*b + 13*b^2)*Tanh[c + d*x])/(4*d) + ((a + b)^2*Sinh[c + d*x]^4*Tanh[c + d*x])/(4*d) - (b^2*Tanh[c + d*x]^3)/(3*d)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 455

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)]/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 463

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1]

Rule 1153

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 3663

$$54*a*b + 847*b^2)*\cosh(d*x + c)^2 - 15*a^2 - 190*a*b - 175*b^2)*\sinh(d*x + c))/(d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c)*\sinh(d*x + c)^2 + 3*d*\cosh(d*x + c))$$

giac [B] time = 0.46, size = 295, normalized size = 2.50

$$24(3a^2 + 30ab + 35b^2)dx - 3(18a^2e^{(4dx+4c)} + 180abe^{(4dx+4c)} + 210b^2e^{(4dx+4c)} - 8a^2e^{(2dx+2c)} - 32abe^{(2dx+2c)})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/192*(24*(3*a^2 + 30*a*b + 35*b^2)*d*x - 3*(18*a^2*e^(4*d*x + 4*c) + 180*a*b*e^(4*d*x + 4*c) + 210*b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) - 32*a*b*e^(2*d*x + 2*c) - 24*b^2*e^(2*d*x + 2*c) + a^2 + 2*a*b + b^2)*e^(-4*d*x - 4*c) + 3*(a^2*e^(4*d*x + 28*c) + 2*a*b*e^(4*d*x + 28*c) + b^2*e^(4*d*x + 28*c) - 8*a^2*e^(2*d*x + 26*c) - 32*a*b*e^(2*d*x + 26*c) - 24*b^2*e^(2*d*x + 26*c))*e^(-24*c) + 256*(3*a*b*e^(4*d*x + 4*c) + 6*b^2*e^(4*d*x + 4*c) + 6*a*b*e^(2*d*x + 2*c) + 9*b^2*e^(2*d*x + 2*c) + 3*a*b + 5*b^2)/(e^(2*d*x + 2*c) + 1)^3)/d

maple [A] time = 0.29, size = 166, normalized size = 1.41

$$a^2 \left(\left(\frac{\sinh^3(dx+c)}{4} - \frac{3\sinh(dx+c)}{8} \right) \cosh(dx+c) + \frac{3dx}{8} + \frac{3c}{8} \right) + 2ab \left(\frac{\sinh^5(dx+c)}{4\cosh(dx+c)} - \frac{5(\sinh^3(dx+c))}{8\cosh(dx+c)} + \frac{15dx}{8} + \frac{15c}{8} - \frac{15\tanh(dx+c)}{8} \right) \frac{1}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x)

[Out] 1/d*(a^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+2*a*b*(1/4*sinh(d*x+c)^5/cosh(d*x+c)-5/8*sinh(d*x+c)^3/cosh(d*x+c)+15/8*d*x+15/8*c-15/8*tanh(d*x+c))+b^2*(1/4*sinh(d*x+c)^7/cosh(d*x+c)^3-7/8*sinh(d*x+c)^5/cosh(d*x+c)^3+35/8*d*x+35/8*c-35/8*tanh(d*x+c)-35/24*tanh(d*x+c)^3))

maxima [B] time = 0.36, size = 295, normalized size = 2.50

$$\frac{1}{64} a^2 \left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) + \frac{1}{192} b^2 \left(\frac{840(dx+c)}{d} + \frac{3(24e^{(-2dx-2c)} - e^{(-4dx-4c)})}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] 1/64*a^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) + 1/192*b^2*(840*(d*x + c)/d + 3*(24*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (63*e^(-2*d*x - 2*c) + 1487*e^(-4*d*x - 4*c) + 2517*e^(-6*d*x - 6*c) + 1608*e^(-8*d*x - 8*c) - 3)/(d*(e^(-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) + 3*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c)))) + 1/32*a*b*(120*(d*x + c)/d + (16*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (15*e^(-2*d*x - 2*c) + 144*e^(-4*d*x - 4*c) - 1)/(d*(e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))))

mupad [B] time = 0.31, size = 293, normalized size = 2.48

$$\frac{4(b^2+ab)}{3d} + \frac{4e^{2c+2dx}(2b^2+ab)}{3d} + x \left(\frac{3a^2}{8} + \frac{15ab}{4} + \frac{35b^2}{8} \right) + \frac{4(2b^2+ab)}{3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1} + \frac{4}{3d} \left(\frac{24e^{(-2dx-2c)} - e^{(-4dx-4c)}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2,x)`

[Out]
$$\begin{aligned} & ((4*(a*b + b^2))/(3*d) + (4*\exp(2*c + 2*d*x)*(a*b + 2*b^2))/(3*d))/(2*\exp(2 \\ & *c + 2*d*x) + \exp(4*c + 4*d*x) + 1) + x*((15*a*b)/4 + (3*a^2)/8 + (35*b^2)/ \\ & 8) + ((4*(a*b + 2*b^2))/(3*d) + (8*\exp(2*c + 2*d*x)*(a*b + b^2))/(3*d) + (4 \\ & * \exp(4*c + 4*d*x)*(a*b + 2*b^2))/(3*d))/(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4 \\ & *d*x) + \exp(6*c + 6*d*x) + 1) + (4*(a*b + 2*b^2))/(3*d*(\exp(2*c + 2*d*x) + \\ & 1)) + (\exp(-2*c - 2*d*x)*(4*a*b + a^2 + 3*b^2))/(8*d) - (\exp(2*c + 2*d*x)* \\ & (4*a*b + a^2 + 3*b^2))/(8*d) - (\exp(-4*c - 4*d*x)*(a + b)^2)/(64*d) + (\exp \\ & (4*c + 4*d*x)*(a + b)^2)/(64*d) \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^2 \sinh^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)`

[Out] `Integral((a + b*tanh(c + d*x)**2)**2*sinh(c + d*x)**4, x)`

3.10 $\int \sinh^3(c + dx) \left(a + b \tanh^2(c + dx)\right)^2 dx$

Optimal. Leaf size=77

$$\frac{(a+b)^2 \cosh^3(c+dx)}{3d} - \frac{(a+b)(a+3b) \cosh(c+dx)}{d} - \frac{b(2a+3b) \operatorname{sech}(c+dx)}{d} + \frac{b^2 \operatorname{sech}^3(c+dx)}{3d}$$

[Out] $-(a+b)*(a+3*b)*\cosh(d*x+c)/d+1/3*(a+b)^2*\cosh(d*x+c)^3/d-b*(2*a+3*b)*\operatorname{sech}(d*x+c)/d+1/3*b^2*\operatorname{sech}(d*x+c)^3/d$

Rubi [A] time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3664, 448}

$$\frac{(a+b)^2 \cosh^3(c+dx)}{3d} - \frac{(a+b)(a+3b) \cosh(c+dx)}{d} - \frac{b(2a+3b) \operatorname{sech}(c+dx)}{d} + \frac{b^2 \operatorname{sech}^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[c + d*x]^3*(a + b*\text{Tanh}[c + d*x]^2)^2, x]$

[Out] $-(((a + b)*(a + 3*b)*\text{Cosh}[c + d*x])/d) + ((a + b)^2*\text{Cosh}[c + d*x]^3)/(3*d) - (b*(2*a + 3*b)*\text{Sech}[c + d*x])/d + (b^2*\text{Sech}[c + d*x]^3)/(3*d)$

Rule 448

$\text{Int}[(e_.*x_*)^{m_.*((a_.) + (b_.*x_*)^{n_*)^{p_.*((c_.) + (d_.*x_*)^{n_*)^{q_*.}})}, x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[p, 0] \&\& \text{IGtQ}[q, 0]$

Rule 3664

$\text{Int}[\sin[(e_.) + (f_.*x_*)^{m_.*((a_.) + (b_.*x_*)^{n_*)^{p_.*((c_.) + (d_.*x_*)^{n_*)^{q_*.}})}, x_Symbol] :> \text{With}\{\text{ff} = \text{FreeFactors}[\text{Sec}[e + f*x], x]\}, \text{Dist}[1/(f*\text{ff}^m), \text{Subst}[\text{Int}[((-1 + \text{ff}^2*x^2)^{(m-1)/2}*(a - b + b*\text{ff}^2*x^2)^p)/x^{m+1}, x], x, \text{Sec}[e + f*x]/\text{ff}], x] /; \text{FreeQ}\{a, b, e, f, p\}, x] \&\& \text{IntegerQ}[(m-1)/2]$

Rubi steps

$$\begin{aligned} \int \sinh^3(c + dx) \left(a + b \tanh^2(c + dx)\right)^2 dx &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+b-bx^2)^2}{x^4} dx, x, \operatorname{sech}(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(-b(2a+3b) - \frac{(a+b)^2}{x^4} + \frac{(a+b)(a+3b)}{x^2} + b^2x^2\right) dx, x, \operatorname{sech}(c + dx)\right)}{d} \\ &= -\frac{(a+b)(a+3b) \cosh(c+dx)}{d} + \frac{(a+b)^2 \cosh^3(c+dx)}{3d} - \frac{b(2a+3b) \operatorname{sech}(c+dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.54, size = 71, normalized size = 0.92

$$\frac{-3(3a^2 + 14ab + 11b^2) \cosh(c + dx) + (a + b)^2 \cosh(3(c + dx)) + 4b \operatorname{sech}(c + dx) (-6a + b \operatorname{sech}^2(c + dx) - 9b^2)}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] $(-3*(3*a^2 + 14*a*b + 11*b^2)*\text{Cosh}[c + d*x] + (a + b)^2*\text{Cosh}[3*(c + d*x)] + 4*b*\text{Sech}[c + d*x]*(-6*a - 9*b + b*\text{Sech}[c + d*x]^2))/(12*d)$

fricas [B] time = 0.52, size = 259, normalized size = 3.36

$$\frac{(a^2 + 2ab + b^2) \cosh(dx + c)^6 + (a^2 + 2ab + b^2) \sinh(dx + c)^6 - 6(a^2 + 6ab + 5b^2) \cosh(dx + c)^4 + 3(5(a^2 + 2ab + b^2) \cosh(dx + c)^2 - 2a^2 - 12ab - 10b^2) \sinh(dx + c)^4 - 3(11a^2 + 86ab + 91b^2) \cosh(dx + c)^2 + 3(5(a^2 + 2ab + b^2) \cosh(dx + c)^2 - 12(a^2 + 6ab + 5b^2) \cosh(dx + c)^2 - 11a^2 - 86ab - 91b^2) \sinh(dx + c)^2 - 26a^2 - 220ab - 210b^2}{(d \cosh(dx + c)^3 + 3d \cosh(dx + c) \sinh(dx + c)^2 + 3d \cosh(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] $1/24*((a^2 + 2*a*b + b^2)*\text{cosh}(d*x + c)^6 + (a^2 + 2*a*b + b^2)*\text{sinh}(d*x + c)^6 - 6*(a^2 + 6*a*b + 5*b^2)*\text{cosh}(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*\text{cosh}(d*x + c)^2 - 2*a^2 - 12*a*b - 10*b^2)*\text{sinh}(d*x + c)^4 - 3*(11*a^2 + 86*a*b + 91*b^2)*\text{cosh}(d*x + c)^2 + 3*(5*(a^2 + 2*a*b + b^2)*\text{cosh}(d*x + c)^2 - 12*(a^2 + 6*a*b + 5*b^2)*\text{cosh}(d*x + c)^2 - 11*a^2 - 86*a*b - 91*b^2)*\text{sinh}(d*x + c)^2 - 26*a^2 - 220*a*b - 210*b^2)/(d*\text{cosh}(d*x + c)^3 + 3*d*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^2 + 3*d*\text{cosh}(d*x + c))$

giac [B] time = 0.42, size = 290, normalized size = 3.77

$$\frac{(a^2 e^{3dx+36c} + 2abe^{3dx+36c} + b^2 e^{3dx+36c} - 9a^2 e^{(dx+34c)} - 42abe^{(dx+34c)} - 33b^2 e^{(dx+34c)})e^{-33c} - \frac{(9a^2 e^{(8dx+8c)} + 18ab e^{(8dx+8c)} + 9b^2 e^{(8dx+8c)})e^{-33c}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] $1/24*((a^2*e^{(3*d*x + 36*c)} + 2*a*b*e^{(3*d*x + 36*c)} + b^2*e^{(3*d*x + 36*c)} - 9*a^2*e^{(d*x + 34*c)} - 42*a*b*e^{(d*x + 34*c)} - 33*b^2*e^{(d*x + 34*c)})*e^{(-33*c)} - (9*a^2*e^{(8*d*x + 8*c)} + 138*a*b*e^{(8*d*x + 8*c)} + 177*b^2*e^{(8*d*x + 8*c)} + 26*a^2*e^{(6*d*x + 6*c)} + 316*a*b*e^{(6*d*x + 6*c)} + 322*b^2*e^{(6*d*x + 6*c)} + 24*a^2*e^{(4*d*x + 4*c)} + 216*a*b*e^{(4*d*x + 4*c)} + 240*b^2*e^{(4*d*x + 4*c)} + 6*a^2*e^{(2*d*x + 2*c)} + 36*a*b*e^{(2*d*x + 2*c)} + 30*b^2*e^{(2*d*x + 2*c)} - a^2 - 2*a*b - b^2)*e^{(-3*c)})/(e^{(3*d*x + 2*c)} + e^{(d*x)})^3/d$

maple [B] time = 0.29, size = 148, normalized size = 1.92

$$\frac{a^2 \left(-\frac{2}{3} + \frac{\sinh^2(dx+c)}{3} \right) \cosh(dx+c) + 2ab \left(\frac{\sinh^4(dx+c)}{3 \cosh(dx+c)} - \frac{4 \sinh^2(dx+c)}{3 \cosh(dx+c)} - \frac{8}{3 \cosh(dx+c)} \right) + b^2 \left(\frac{\sinh^6(dx+c)}{3 \cosh(dx+c)^3} - \frac{2 \sinh^4(dx+c)}{\cosh(dx+c)} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x)

[Out] $1/d*(a^2*(-2/3+1/3*\text{sinh}(d*x+c)^2)*\text{cosh}(d*x+c)+2*a*b*(1/3*\text{sinh}(d*x+c)^4/\text{cosh}(d*x+c)-4/3*\text{sinh}(d*x+c)^2/\text{cosh}(d*x+c)-8/3/\text{cosh}(d*x+c))+b^2*(1/3*\text{sinh}(d*x+c)^6/\text{cosh}(d*x+c)^3-2*\text{sinh}(d*x+c)^4/\text{cosh}(d*x+c)^3-8*\text{sinh}(d*x+c)^2/\text{cosh}(d*x+c)^3-16/3/\text{cosh}(d*x+c)^3))$

maxima [B] time = 0.37, size = 265, normalized size = 3.44

$$-\frac{1}{24} b^2 \left(\frac{33 e^{(-dx-c)} - e^{(-3dx-3c)}}{d} + \frac{30 e^{(-2dx-2c)} + 240 e^{(-4dx-4c)} + 322 e^{(-6dx-6c)} + 177 e^{(-8dx-8c)} - 1}{d(e^{(-3dx-3c)} + 3e^{(-5dx-5c)} + 3e^{(-7dx-7c)} + e^{(-9dx-9c)})} \right) - \frac{1}{12} ab \left(\frac{21 e^{(-dx-c)} - e^{(-3dx-3c)}}{d} + \frac{30 e^{(-2dx-2c)} + 240 e^{(-4dx-4c)} + 322 e^{(-6dx-6c)} + 177 e^{(-8dx-8c)} - 1}{d(e^{(-3dx-3c)} + 3e^{(-5dx-5c)} + 3e^{(-7dx-7c)} + e^{(-9dx-9c)})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out]
$$-1/24*b^2*((33*e^{(-d*x - c)} - e^{(-3*d*x - 3*c)})/d + (30*e^{(-2*d*x - 2*c)} + 240*e^{(-4*d*x - 4*c)} + 322*e^{(-6*d*x - 6*c)} + 177*e^{(-8*d*x - 8*c)} - 1)/(d*(e^{(-3*d*x - 3*c)} + 3*e^{(-5*d*x - 5*c)} + 3*e^{(-7*d*x - 7*c)} + e^{(-9*d*x - 9*c)}))) - 1/12*a*b*((21*e^{(-d*x - c)} - e^{(-3*d*x - 3*c)})/d + (20*e^{(-2*d*x - 2*c)} + 69*e^{(-4*d*x - 4*c)} - 1)/(d*(e^{(-3*d*x - 3*c)} + e^{(-5*d*x - 5*c)}))) + 1/24*a^2*(e^{(3*d*x + 3*c)}/d - 9*e^{(d*x + c)}/d - 9*e^{(-d*x - c)}/d + e^{(-3*d*x - 3*c)}/d)$$

mupad [B] time = 0.29, size = 215, normalized size = 2.79

$$\frac{e^{-3c-3dx} (a+b)^2}{24d} - \frac{e^{c+dx} (3a^2 + 14ab + 11b^2)}{8d} + \frac{e^{3c+3dx} (a+b)^2}{24d} - \frac{e^{-c-dx} (3a^2 + 14ab + 11b^2)}{8d} - \frac{1}{3d (3e^{2c+2dx})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2,x)

[Out]
$$(\exp(-3*c - 3*d*x)*(a + b)^2)/(24*d) - (\exp(c + d*x)*(14*a*b + 3*a^2 + 11*b^2))/(8*d) + (\exp(3*c + 3*d*x)*(a + b)^2)/(24*d) - (\exp(-c - d*x)*(14*a*b + 3*a^2 + 11*b^2))/(8*d) - (8*b^2*\exp(c + d*x))/(3*d*(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1)) - (2*\exp(c + d*x)*(2*a*b + 3*b^2))/(d*(\exp(2*c + 2*d*x) + 1)) + (8*b^2*\exp(c + d*x))/(3*d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1))$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^2 \sinh^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)

[Out] Integral((a + b*tanh(c + d*x)**2)**2*sinh(c + d*x)**3, x)

3.11 $\int \sinh^2(c + dx) (a + b \tanh^2(c + dx))^2 dx$

Optimal. Leaf size=79

$$\frac{(a+b)(a+5b)\tanh(c+dx)}{2d} + \frac{(a+b)^2\sinh^2(c+dx)\tanh(c+dx)}{2d} - \frac{1}{2}x^{(a+b)(a+5b)} + \frac{b^2\tanh^3(c+dx)}{3d}$$

[Out] $-1/2*(a+b)*(a+5*b)*x+1/2*(a+b)*(a+5*b)*\tanh(d*x+c)/d+1/2*(a+b)^2*\sinh(d*x+c)^2*\tanh(d*x+c)/d+1/3*b^2*\tanh(d*x+c)^3/d$

Rubi [A] time = 0.11, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3663, 463, 459, 321, 206}

$$\frac{(a+b)(a+5b)\tanh(c+dx)}{2d} + \frac{(a+b)^2\sinh^2(c+dx)\tanh(c+dx)}{2d} - \frac{1}{2}x^{(a+b)(a+5b)} + \frac{b^2\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] $-((a+b)*(a+5*b)*x)/2 + ((a+b)*(a+5*b)*\text{Tanh}[c+d*x])/(2*d) + ((a+b)^2*\text{Sinh}[c+d*x]^2*\text{Tanh}[c+d*x])/(2*d) + (b^2*\text{Tanh}[c+d*x]^3)/(3*d)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 321

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*(m+n*p+1)), x] - Dist[(a*c^n*(m-n+1))/(b*(m+n*p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 459

Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m+1)*(a+b*x^n)^(p+1))/(b*e*(m+n*(p+1)+1)), x] - Dist[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(b*(m+n*(p+1)+1)), Int[(e*x)^m*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m+n*(p+1)+1, 0]

Rule 463

Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^2, x_Symbol] := -Simp[((b*c - a*d)^2*(e*x)^(m+1)*(a+b*x^n)^(p+1))/(a*b^2*e*n*(p+1)), x] + Dist[1/(a*b^2*n*(p+1)), Int[(e*x)^m*(a+b*x^n)^(p+1)*Simp[(b*c - a*d)^2*(m+1) + b^2*c^2*n*(p+1) + a*b*d^2*n*(p+1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1]

Rule 3663

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m+1))/f, Subst[Int[(x^m*(a+b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/

$2 + 1), x], x, (c*\text{Tan}[e + f*x])/ff], x]] /; \text{FreeQ}[\{a, b, c, e, f, n, p\}, x]$
 $\&\& \text{IntegerQ}[m/2]$

Rubi steps

$$\begin{aligned} \int \sinh^2(c + dx) (a + b \tanh^2(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^2}{(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{(a + b)^2 \sinh^2(c + dx) \tanh(c + dx)}{2d} - \frac{\text{Subst}\left(\int \frac{x^2(a^2+6ab+3b^2+2b^2)}{1-x^2} dx, x, \tanh(c + dx)\right)}{2d} \\ &= \frac{(a + b)^2 \sinh^2(c + dx) \tanh(c + dx)}{2d} + \frac{b^2 \tanh^3(c + dx)}{3d} - \frac{(a + b)^2 \sinh^2(c + dx) \tanh(c + dx)}{2d} \\ &= \frac{(a + b)(a + 5b) \tanh(c + dx)}{2d} + \frac{(a + b)^2 \sinh^2(c + dx) \tanh(c + dx)}{2d} \\ &= -\frac{1}{2}(a + b)(a + 5b)x + \frac{(a + b)(a + 5b) \tanh(c + dx)}{2d} + \frac{(a + b)^2 \sinh^2(c + dx) \tanh(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.90, size = 70, normalized size = 0.89

$$\frac{-6(a^2 + 6ab + 5b^2)(c + dx) + 3(a + b)^2 \sinh(2(c + dx)) + 4b \tanh(c + dx) (6a - b \operatorname{sech}^2(c + dx) + 7b)}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (-6*(a^2 + 6*a*b + 5*b^2)*(c + d*x) + 3*(a + b)^2*Sinh[2*(c + d*x)] + 4*b*(6*a + 7*b - b*Sech[c + d*x]^2)*Tanh[c + d*x])/(12*d)

fricas [B] time = 0.48, size = 291, normalized size = 3.68

$$\frac{3(a^2 + 2ab + b^2) \sinh(dx + c)^5 - 4(3(a^2 + 6ab + 5b^2)dx + 12ab + 14b^2) \cosh(dx + c)^3 - 12(3(a^2 + 6ab + 5b^2)dx + 12ab + 14b^2) \cosh(dx + c)^2 + (30(a^2 + 2ab + b^2) \cosh(dx + c)^2 + 9a^2 + 66ab + 65b^2) \sinh(dx + c)^3 - 12(3(a^2 + 6ab + 5b^2)dx + 12ab + 14b^2) \cosh(dx + c) + 3(5(a^2 + 2ab + b^2) \cosh(dx + c)^4 + (9a^2 + 66ab + 65b^2) \cosh(dx + c)^2 + 2a^2 + 20ab + 10b^2) \sinh(dx + c)) / (d \cosh(dx + c)^3 + 3d \cosh(dx + c) \sinh(dx + c)^2 + 3d \cosh(dx + c))}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] 1/24*(3*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^5 - 4*(3*(a^2 + 6*a*b + 5*b^2)*d*x + 12*a*b + 14*b^2)*cosh(d*x + c)^3 - 12*(3*(a^2 + 6*a*b + 5*b^2)*d*x + 12*a*b + 14*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (30*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 9*a^2 + 66*a*b + 65*b^2)*sinh(d*x + c)^3 - 12*(3*(a^2 + 6*a*b + 5*b^2)*d*x + 12*a*b + 14*b^2)*cosh(d*x + c) + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + (9*a^2 + 66*a*b + 65*b^2)*cosh(d*x + c)^2 + 2*a^2 + 20*a*b + 10*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))

giac [B] time = 0.33, size = 215, normalized size = 2.72

$$12(a^2 + 6ab + 5b^2)dx - 3(2a^2e^{2dx+2c} + 12abe^{2dx+2c} + 10b^2e^{2dx+2c} - a^2 - 2ab - b^2)e^{(-2dx-2c)} - 3(a^2e^{2dx+2c} + 12abe^{2dx+2c} + 10b^2e^{2dx+2c} - a^2 - 2ab - b^2)e^{(-2dx-2c)} - 3(a^2e^{2dx+2c} + 12abe^{2dx+2c} + 10b^2e^{2dx+2c} - a^2 - 2ab - b^2)e^{(-2dx-2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c))^2,x, algorithm="giac")

[Out]
$$\frac{-1/24*(12*(a^2 + 6*a*b + 5*b^2)*d*x - 3*(2*a^2*e^{(2*d*x + 2*c)} + 12*a*b*e^{(2*d*x + 2*c)} + 10*b^2*e^{(2*d*x + 2*c)} - a^2 - 2*a*b - b^2)*e^{(-2*d*x - 2*c)} - 3*(a^2*e^{(2*d*x + 12*c)} + 2*a*b*e^{(2*d*x + 12*c)} + b^2*e^{(2*d*x + 12*c)})*e^{(-10*c)} + 16*(6*a*b*e^{(4*d*x + 4*c)} + 9*b^2*e^{(4*d*x + 4*c)} + 12*a*b*e^{(2*d*x + 2*c)} + 12*b^2*e^{(2*d*x + 2*c)} + 6*a*b + 7*b^2)/(e^{(2*d*x + 2*c)} + 1)^3)/d$$

maple [A] time = 0.21, size = 118, normalized size = 1.49

$$\frac{a^2 \left(\frac{\cosh(dx+c) \sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2} \right) + 2ab \left(\frac{\sinh^3(dx+c)}{2 \cosh(dx+c)} - \frac{3dx}{2} - \frac{3c}{2} + \frac{3 \tanh(dx+c)}{2} \right) + b^2 \left(\frac{\sinh^5(dx+c)}{2 \cosh(dx+c)^3} - \frac{5dx}{2} - \frac{5c}{2} + \frac{5 \tanh(dx+c)}{2} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^2*(a+b*tanh(d*x+c))^2,x)

[Out]
$$\frac{1}{d} * (a^2 * (1/2 * \cosh(d*x+c) * \sinh(d*x+c) - 1/2 * d*x - 1/2 * c) + 2 * a * b * (1/2 * \sinh(d*x+c)^3 / \cosh(d*x+c) - 3/2 * d*x - 3/2 * c + 3/2 * \tanh(d*x+c)) + b^2 * (1/2 * \sinh(d*x+c)^5 / \cosh(d*x+c)^3 - 5/2 * d*x - 5/2 * c + 5/2 * \tanh(d*x+c) + 5/6 * \tanh(d*x+c)^3))$$

maxima [B] time = 0.35, size = 217, normalized size = 2.75

$$-\frac{1}{8} a^2 \left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d} \right) - \frac{1}{24} b^2 \left(\frac{60(dx+c)}{d} + \frac{3e^{(-2dx-2c)}}{d} - \frac{121e^{(-2dx-2c)} + 201e^{(-4dx-4c)} + 147e^{(-6dx-6c)}}{d(e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + 3e^{(-6dx-6c)})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c))^2,x, algorithm="maxima")

[Out]
$$-1/8*a^2*(4*x - e^{(2*d*x + 2*c)}/d + e^{(-2*d*x - 2*c)}/d) - 1/24*b^2*(60*(d*x + c)/d + 3*e^{(-2*d*x - 2*c)}/d - (121*e^{(-2*d*x - 2*c)} + 201*e^{(-4*d*x - 4*c)} + 147*e^{(-6*d*x - 6*c)} + 3)/(d*(e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + 3*e^{(-6*d*x - 6*c)} + e^{(-8*d*x - 8*c)}))) - 1/4*a*b*(12*(d*x + c)/d + e^{(-2*d*x - 2*c)}/d - (17*e^{(-2*d*x - 2*c)} + 1)/(d*(e^{(-2*d*x - 2*c)} + e^{(-4*d*x - 4*c)})))$$

mupad [B] time = 1.18, size = 248, normalized size = 3.14

$$\frac{e^{2c+2dx} (a+b)^2}{8d} - x \left(\frac{a^2}{2} + 3ab + \frac{5b^2}{2} \right) - \frac{\frac{2(3b^2+2ab)}{3d} + \frac{4e^{2c+2dx}(b^2+2ab)}{3d} + \frac{2e^{4c+4dx}(3b^2+2ab)}{3d}}{3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1} - \frac{2(3b^2+2ab)}{3d(e^{2c+2dx} + 1)} - \frac{e^{2c+2dx}(a+b)^2}{8d} - \frac{e^{(-2dx-2c)}(a+b)^2}{8d} - \frac{e^{(-4dx-4c)}(a+b)^2}{8d} - \frac{e^{(-6dx-6c)}(a+b)^2}{8d} - \frac{e^{(-8dx-8c)}(a+b)^2}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^2*(a + b*tanh(c + d*x))^2,x)

[Out]
$$(exp(2*c + 2*d*x)*(a + b)^2)/(8*d) - x*(3*a*b + a^2/2 + (5*b^2)/2) - ((2*(2*a*b + 3*b^2))/(3*d) + (4*exp(2*c + 2*d*x)*(2*a*b + b^2))/(3*d) + (2*exp(4*c + 4*d*x)*(2*a*b + 3*b^2))/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (2*(2*a*b + 3*b^2))/(3*d*(exp(2*c + 2*d*x) + 1)) - (exp(-2*c - 2*d*x)*(a + b)^2)/(8*d) - ((2*(2*a*b + b^2))/(3*d) + (2*exp(2*c + 2*d*x)*(2*a*b + 3*b^2))/(3*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^2 \sinh^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*sinh(c + d*x)**2, x)
```

3.12 $\int \sinh(c + dx) (a + b \tanh^2(c + dx))^2 dx$

Optimal. Leaf size=49

$$\frac{(a+b)^2 \cosh(c+dx)}{d} + \frac{2b(a+b)\operatorname{sech}(c+dx)}{d} - \frac{b^2 \operatorname{sech}^3(c+dx)}{3d}$$

[Out] (a+b)^2*cosh(d*x+c)/d+2*b*(a+b)*sech(d*x+c)/d-1/3*b^2*sech(d*x+c)^3/d

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3664, 270}

$$\frac{(a+b)^2 \cosh(c+dx)}{d} + \frac{2b(a+b)\operatorname{sech}(c+dx)}{d} - \frac{b^2 \operatorname{sech}^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] ((a + b)^2*Cosh[c + d*x])/d + (2*b*(a + b)*Sech[c + d*x])/d - (b^2*Sech[c + d*x]^3)/(3*d)

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 3664

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p]/x^(m + 1), x], x, Sec[e + f*x]/ff, x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \sinh(c + dx) (a + b \tanh^2(c + dx))^2 dx &= -\frac{\operatorname{Subst}\left(\int \frac{(a+b-bx^2)^2}{x^2} dx, x, \operatorname{sech}(c + dx)\right)}{d} \\ &= -\frac{\operatorname{Subst}\left(\int \left(-2b(a+b) + \frac{(a+b)^2}{x^2} + b^2x^2\right) dx, x, \operatorname{sech}(c + dx)\right)}{d} \\ &= \frac{(a+b)^2 \cosh(c+dx)}{d} + \frac{2b(a+b)\operatorname{sech}(c+dx)}{d} - \frac{b^2 \operatorname{sech}^3(c+dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.34, size = 46, normalized size = 0.94

$$\frac{3(a+b)^2 \cosh(c+dx) + b \operatorname{sech}(c+dx) (6(a+b) - b \operatorname{sech}^2(c+dx))}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] $(3*(a + b)^2*\text{Cosh}[c + d*x] + b*\text{Sech}[c + d*x]*(6*(a + b) - b*\text{Sech}[c + d*x]^2))/ (3*d)$

fricas [B] time = 0.54, size = 167, normalized size = 3.41

$$\frac{3(a^2 + 2ab + b^2)\cosh(dx + c)^4 + 3(a^2 + 2ab + b^2)\sinh(dx + c)^4 + 12(a^2 + 4ab + 3b^2)\cosh(dx + c)^2 + 6(d\cosh(dx + c)^3 + 3d\cosh(dx + c)\sinh(dx + c))}{6(d\cosh(dx + c)^3 + 3d\cosh(dx + c)\sinh(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")`

[Out] $1/6*(3*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*\sinh(d*x + c)^4 + 12*(a^2 + 4*a*b + 3*b^2)*\cosh(d*x + c)^2 + 6*(3*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)^2 + 2*a^2 + 8*a*b + 6*b^2)*\sinh(d*x + c)^2 + 9*a^2 + 42*a*b + 25*b^2)/(d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c)*\sinh(d*x + c)^2 + 3*d*\cosh(d*x + c))$

giac [B] time = 0.22, size = 162, normalized size = 3.31

$$\frac{3(a^2 + 2ab + b^2)e^{(-dx-c)} + 3(a^2e^{(dx+10c)} + 2abe^{(dx+10c)} + b^2e^{(dx+10c)})e^{(-9c)} + \frac{8(3abe^{(5dx+5c)} + 3b^2e^{(5dx+5c)} + 6abe^{(3dx+5c)})}{e^{(2dx+5c)}}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")`

[Out] $1/6*(3*(a^2 + 2*a*b + b^2)*e^{(-d*x - c)} + 3*(a^2*e^{(d*x + 10*c)} + 2*a*b*e^{(d*x + 10*c)} + b^2*e^{(d*x + 10*c)})*e^{(-9*c)} + 8*(3*a*b*e^{(5*d*x + 5*c)} + 3*b^2*e^{(5*d*x + 5*c)} + 6*a*b*e^{(3*d*x + 3*c)} + 4*b^2*e^{(3*d*x + 3*c)} + 3*a*b*e^{(d*x + c)} + 3*b^2*e^{(d*x + c)})/(e^{(2*d*x + 2*c)} + 1)^3)/d$

maple [B] time = 0.21, size = 98, normalized size = 2.00

$$\frac{a^2 \cosh(dx + c) + 2ab \left(\frac{\sinh^2(dx+c)}{\cosh(dx+c)} + \frac{2}{\cosh(dx+c)} \right) + b^2 \left(\frac{\sinh^4(dx+c)}{\cosh(dx+c)^3} + \frac{4(\sinh^2(dx+c))}{\cosh(dx+c)^3} + \frac{8}{3\cosh(dx+c)^3} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x)`

[Out] $1/d*(a^2*\cosh(d*x+c)+2*a*b*(\sinh(d*x+c)^2/\cosh(d*x+c)+2/\cosh(d*x+c))+b^2*(\sinh(d*x+c)^4/\cosh(d*x+c)^3+4*\sinh(d*x+c)^2/\cosh(d*x+c)^3+8/3/\cosh(d*x+c)^3))$

maxima [B] time = 0.35, size = 171, normalized size = 3.49

$$\frac{1}{6}b^2 \left(\frac{3e^{(-dx-c)}}{d} + \frac{33e^{(-2dx-2c)} + 41e^{(-4dx-4c)} + 27e^{(-6dx-6c)} + 3}{d(e^{(-dx-c)} + 3e^{(-3dx-3c)} + 3e^{(-5dx-5c)} + e^{(-7dx-7c)})} \right) + ab \left(\frac{e^{(-dx-c)}}{d} + \frac{5e^{(-2dx-2c)} + 1}{d(e^{(-dx-c)} + e^{(-3dx-3c)})} \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] $1/6*b^2*(3*e^{(-d*x - c)}/d + (33*e^{(-2*d*x - 2*c)} + 41*e^{(-4*d*x - 4*c)} + 27*e^{(-6*d*x - 6*c)} + 3)/(d*(e^{(-d*x - c)} + 3*e^{(-3*d*x - 3*c)} + 3*e^{(-5*d*x - 5*c)} + e^{(-7*d*x - 7*c)}))) + a*b*(e^{(-d*x - c)}/d + (5*e^{(-2*d*x - 2*c)} + 1)/(d*(e^{(-d*x - c)} + e^{(-3*d*x - 3*c)}))) + a^2*\cosh(d*x + c)/d$

mupad [B] time = 0.19, size = 154, normalized size = 3.14

$$\frac{e^{c+dx}(a+b)^2}{2d} + \frac{e^{-c-dx}(a+b)^2}{2d} + \frac{8b^2 e^{c+dx}}{3d(3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1)} + \frac{4e^{c+dx}(b^2 + ab)}{d(e^{2c+2dx} + 1)} - \frac{8b^2 e^{c+dx}}{3d(2e^{2c+2dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^2,x)

[Out] (exp(c + d*x)*(a + b)^2)/(2*d) + (exp(-c - d*x)*(a + b)^2)/(2*d) + (8*b^2*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (4*exp(c + d*x)*(a*b + b^2))/(d*(exp(2*c + 2*d*x) + 1)) - (8*b^2*exp(c + d*x))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^2 \sinh(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)

[Out] Integral((a + b*tanh(c + d*x)**2)**2*sinh(c + d*x), x)

3.13 $\int \operatorname{csch}(c + dx) \left(a + b \tanh^2(c + dx) \right)^2 dx$

Optimal. Leaf size=51

$$-\frac{a^2 \tanh^{-1}(\cosh(c + dx))}{d} - \frac{b(2a + b)\operatorname{sech}(c + dx)}{d} + \frac{b^2 \operatorname{sech}^3(c + dx)}{3d}$$

[Out] $-a^2 \operatorname{arctanh}(\cosh(d*x+c))/d - b*(2*a+b)*\operatorname{sech}(d*x+c)/d + 1/3*b^2*\operatorname{sech}(d*x+c)^3/d$

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3664, 390, 207}

$$-\frac{a^2 \tanh^{-1}(\cosh(c + dx))}{d} - \frac{b(2a + b)\operatorname{sech}(c + dx)}{d} + \frac{b^2 \operatorname{sech}^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]*(a + b*\operatorname{Tanh}[c + d*x]^2)^2, x]$

[Out] $-((a^2*\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]])/d) - (b*(2*a + b)*\operatorname{Sech}[c + d*x])/d + (b^2*\operatorname{Sech}[c + d*x]^3)/(3*d)$

Rule 207

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Rt}[b, 2]*x]/\operatorname{Rt}[-a, 2]]/\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2], x] /;$ $\operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{LtQ}[a, 0] \ || \ \operatorname{GtQ}[b, 0])$

Rule 390

$\operatorname{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)*((c_ + (d_)*(x_)^{(n_)})^{(q_)})}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{PolynomialDivide}[(a + b*x^n)^p, (c + d*x^n)^{-q}, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{IGtQ}[n, 0] \ \&\& \operatorname{IGtQ}[p, 0] \ \&\& \operatorname{ILtQ}[q, 0] \ \&\& \operatorname{GeQ}[p, -q]$

Rule 3664

$\operatorname{Int}[\sin[(e_ + (f_)*(x_))]^{(m_)*((a_ + (b_)*\tan[(e_ + (f_)*(x_))]^2)^{(p_)}), x_Symbol] \rightarrow \operatorname{With}\{\operatorname{ff} = \operatorname{FreeFactors}[\operatorname{Sec}[e + f*x], x]\}, \operatorname{Dist}[1/(f*\operatorname{ff}^m), \operatorname{Subst}[\operatorname{Int}[((-1 + \operatorname{ff}^2*x^2)^{(m-1)/2}*(a - b + b*\operatorname{ff}^2*x^2)^p)/x^{(m+1)}, x], x, \operatorname{Sec}[e + f*x]/\operatorname{ff}], x] /;$ $\operatorname{FreeQ}\{a, b, e, f, p\}, x \ \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rubi steps

$$\begin{aligned} \int \operatorname{csch}(c + dx) \left(a + b \tanh^2(c + dx) \right)^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+b-x^2)^2}{-1+x^2} dx, x, \operatorname{sech}(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(-b(2a + b) + b^2x^2 + \frac{a^2}{-1+x^2}\right) dx, x, \operatorname{sech}(c + dx)\right)}{d} \\ &= -\frac{b(2a + b)\operatorname{sech}(c + dx)}{d} + \frac{b^2 \operatorname{sech}^3(c + dx)}{3d} + \frac{a^2 \operatorname{Subst}\left(\int \frac{1}{-1+x^2} dx\right)}{d} \\ &= -\frac{a^2 \tanh^{-1}(\cosh(c + dx))}{d} - \frac{b(2a + b)\operatorname{sech}(c + dx)}{d} + \frac{b^2 \operatorname{sech}^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.17, size = 50, normalized size = 0.98

$$\frac{3a^2 \log\left(\tanh\left(\frac{1}{2}(c + dx)\right)\right) - 3b(2a + b)\operatorname{sech}(c + dx) + b^2\operatorname{sech}^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (3*a^2*Log[Tanh[(c + d*x)/2]] - 3*b*(2*a + b)*Sech[c + d*x] + b^2*Sech[c + d*x]^3)/(3*d)

fricas [B] time = 0.90, size = 890, normalized size = 17.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -1/3*(6*(2*a*b + b^2)*cosh(d*x + c)^5 + 30*(2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^4 + 6*(2*a*b + b^2)*sinh(d*x + c)^5 + 4*(6*a*b + b^2)*cosh(d*x + c)^3 + 4*(15*(2*a*b + b^2)*cosh(d*x + c)^2 + 6*a*b + b^2)*sinh(d*x + c)^3 + 12*(5*(2*a*b + b^2)*cosh(d*x + c)^3 + (6*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 6*(2*a*b + b^2)*cosh(d*x + c) + 3*(a^2*cosh(d*x + c)^6 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + 3*a^2*cosh(d*x + c)^4 + 3*(5*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 3*a^2*cosh(d*x + c)^2 + 4*(5*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a^2*cosh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 + 6*(a^2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 3*(a^2*cosh(d*x + c)^6 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + 3*a^2*cosh(d*x + c)^4 + 3*(5*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 3*a^2*cosh(d*x + c)^2 + 4*(5*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a^2*cosh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 + 6*(a^2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 6*(5*(2*a*b + b^2)*cosh(d*x + c)^4 + 2*(6*a*b + b^2)*cosh(d*x + c)^2 + 2*a*b + b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 + 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 + 2*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)

giac [B] time = 0.23, size = 126, normalized size = 2.47

$$\frac{3a^2 \log\left(e^{(dx+c)} + 1\right) - 3a^2 \log\left(\left|e^{(dx+c)} - 1\right|\right) + \frac{2\left(6abe^{5dx+5c} + 3b^2e^{5dx+5c} + 12abe^{3dx+3c} + 2b^2e^{3dx+3c} + 6abe^{dx+c} + 3b^2e^{dx+c}\right)}{\left(e^{2dx+2c} + 1\right)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] -1/3*(3*a^2*log(e^(d*x + c) + 1) - 3*a^2*log(abs(e^(d*x + c) - 1)) + 2*(6*a*b*e^(5*d*x + 5*c) + 3*b^2*e^(5*d*x + 5*c) + 12*a*b*e^(3*d*x + 3*c) + 2*b^2*e^(3*d*x + 3*c) + 6*a*b*e^(d*x + c) + 3*b^2*e^(d*x + c)))/(e^(2*d*x + 2*c) + 1)^3)/d

maple [A] time = 0.24, size = 63, normalized size = 1.24

$$\frac{-2a^2 \operatorname{arctanh}\left(e^{dx+c}\right) - \frac{2ab}{\cosh(dx+c)} + b^2\left(-\frac{\sinh^2(dx+c)}{\cosh(dx+c)^3} - \frac{2}{3\cosh(dx+c)^3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)*(a+b*tanh(d*x+c)^2),x)`

[Out] $1/d*(-2*a^2*\operatorname{arctanh}(\exp(d*x+c))-2*a*b/\cosh(d*x+c)+b^2*(-\sinh(d*x+c)^2/\cosh(d*x+c)^3-2/3/\cosh(d*x+c)^3))$

maxima [B] time = 0.35, size = 196, normalized size = 3.84

$$-\frac{2}{3}b^2\left(\frac{3e^{(-dx-c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{2e^{(-3dx-3c)}}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}+\frac{1}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] $-2/3*b^2*(3*e^{(-d*x-c)}/(d*(3*e^{(-2*d*x-2*c)}+3*e^{(-4*d*x-4*c)}+e^{(-6*d*x-6*c)}+1))+2*e^{(-3*d*x-3*c)}/(d*(3*e^{(-2*d*x-2*c)}+3*e^{(-4*d*x-4*c)}+e^{(-6*d*x-6*c)}+1))+3*e^{(-5*d*x-5*c)}/(d*(3*e^{(-2*d*x-2*c)}+3*e^{(-4*d*x-4*c)}+e^{(-6*d*x-6*c)}+1)))+a^2*\log(\tanh(1/2*d*x+1/2*c))/d-4*a*b/(d*(e^{(d*x+c)}+e^{(-d*x-c)}))$

mupad [B] time = 0.15, size = 160, normalized size = 3.14

$$\frac{8b^2e^{c+dx}}{3d(2e^{2c+2dx}+e^{4c+4dx}+1)}-\frac{8b^2e^{c+dx}}{3d(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1)}-\frac{2e^{c+dx}(b^2+2ab)}{d(e^{2c+2dx}+1)}-\frac{2\operatorname{atan}\left(\frac{a^2e^{dx}}{d}\right)}{\sqrt{a^4+4a^2b^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tanh(c+d*x))^2/sinh(c+d*x),x)`

[Out] $(8*b^2*\exp(c+d*x))/(3*d*(2*\exp(2*c+2*d*x)+\exp(4*c+4*d*x)+1))- (8*b^2*\exp(c+d*x))/(3*d*(3*\exp(2*c+2*d*x)+3*\exp(4*c+4*d*x)+\exp(6*c+6*d*x)+1))- (2*\exp(c+d*x)*(2*a*b+b^2))/(d*(\exp(2*c+2*d*x)+1))- (2*\operatorname{atan}((a^2*\exp(d*x)*\exp(c)*(-d^2)^{(1/2)})/(d*(a^4)^{(1/2)}))*(a^4)^{(1/2)})/(-d^2)^{(1/2)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^2 \operatorname{csch}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)`

[Out] `Integral((a + b*tanh(c + d*x)**2)**2*csch(c + d*x), x)`

3.14 $\int \operatorname{csch}^2(c + dx) \left(a + b \tanh^2(c + dx)\right)^2 dx$

Optimal. Leaf size=46

$$-\frac{a^2 \operatorname{coth}(c + dx)}{d} + \frac{2ab \tanh(c + dx)}{d} + \frac{b^2 \tanh^3(c + dx)}{3d}$$

[Out] $-a^2 \operatorname{coth}(d*x+c)/d+2*a*b*\tanh(d*x+c)/d+1/3*b^2*\tanh(d*x+c)^3/d$

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3663, 270}

$$-\frac{a^2 \operatorname{coth}(c + dx)}{d} + \frac{2ab \tanh(c + dx)}{d} + \frac{b^2 \tanh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] $-((a^2*\operatorname{Coth}[c + d*x])/d) + (2*a*b*\operatorname{Tanh}[c + d*x])/d + (b^2*\operatorname{Tanh}[c + d*x]^3)/(3*d)$

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 3663

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_.))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^2(c + dx) \left(a + b \tanh^2(c + dx)\right)^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^2}{x^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(2ab + \frac{a^2}{x^2} + b^2x^2\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{a^2 \operatorname{coth}(c + dx)}{d} + \frac{2ab \tanh(c + dx)}{d} + \frac{b^2 \tanh^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.49, size = 43, normalized size = 0.93

$$\frac{b \tanh(c + dx) \left(6a - b \operatorname{sech}^2(c + dx) + b\right) - 3a^2 \operatorname{coth}(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] $(-3a^2 \operatorname{Coth}[c + dx] + b(6a + b - b \operatorname{Sech}[c + dx]^2) \operatorname{Tanh}[c + dx]) / (3d)$

fricas [B] time = 0.52, size = 264, normalized size = 5.74

$$\frac{4 \left((3a^2 + b^2) \cosh(dx + c)^3 + 3(3a^2 + b^2) \cosh(dx + c) \sinh(dx + c) \right)}{3 \left(d \cosh(dx + c)^5 + 5d \cosh(dx + c) \sinh(dx + c)^4 + d \sinh(dx + c)^5 + d \cosh(dx + c)^3 + (10d \cosh(dx + c) \sinh(dx + c))^2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")`

[Out] $-4/3 * ((3a^2 + b^2) \cosh(dx + c)^3 + 3(3a^2 + b^2) \cosh(dx + c) \sinh(dx + c)^2 + 2(3ab + b^2) \sinh(dx + c)^3 + (9a^2 - b^2) \cosh(dx + c) + 2(3(3ab + b^2) \cosh(dx + c)^2 + 3ab - b^2) \sinh(dx + c)) / (d \cosh(dx + c)^5 + 5d \cosh(dx + c) \sinh(dx + c)^4 + d \sinh(dx + c)^5 + d \cosh(dx + c)^3 + (10d \cosh(dx + c)^2 + 3d) \sinh(dx + c)^3 + (10d \cosh(dx + c) \sinh(dx + c))^2 + 3d \cosh(dx + c) \sinh(dx + c)^2 - 2d \cosh(dx + c) + (5d \cosh(dx + c)^4 + 9d \cosh(dx + c)^2 + 2d) \sinh(dx + c))$

giac [A] time = 0.26, size = 86, normalized size = 1.87

$$\frac{2 \left(\frac{3a^2}{e^{2dx+2c}-1} + \frac{6abe^{4dx+4c} + 3b^2e^{4dx+4c} + 12abe^{2dx+2c} + 6ab+b^2}{(e^{2dx+2c}+1)^3} \right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")`

[Out] $-2/3 * (3a^2 / (e^{2dx+2c} - 1) + (6ab * e^{4dx+4c} + 3b^2 * e^{4dx+4c} + 12ab * e^{2dx+2c} + 6ab + b^2) / (e^{2dx+2c} + 1)^3) / d$

maple [A] time = 0.43, size = 68, normalized size = 1.48

$$\frac{-a^2 \operatorname{coth}(dx + c) + 2ab \operatorname{tanh}(dx + c) + b^2 \left(-\frac{\sinh(dx + c)}{2 \cosh(dx + c)^3} + \frac{\left(\frac{2}{3} + \frac{\operatorname{sech}(dx + c)^2}{3} \right) \operatorname{tanh}(dx + c)}{2} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x)`

[Out] $1/d * (-a^2 \operatorname{coth}(dx + c) + 2ab \operatorname{tanh}(dx + c) + b^2 * (-1/2 * \sinh(dx + c) / \cosh(dx + c)^3 + 1/2 * (2/3 + 1/3 * \operatorname{sech}(dx + c)^2) * \operatorname{tanh}(dx + c)))$

maxima [B] time = 0.34, size = 136, normalized size = 2.96

$$\frac{2}{3} b^2 \left(\frac{3e^{(-4dx-4c)}}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)} + \frac{1}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)} \right) + \frac{4a}{d} \left(e^{(-2dx-2c)} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] $2/3 * b^2 * (3 * e^{(-4dx-4c)} / (d * (3 * e^{(-2dx-2c)} + 3 * e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)) + 1 / (d * (3 * e^{(-2dx-2c)} + 3 * e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1))) + 4 * a * b / (d * (e^{(-2dx-2c)} + 1)) + 2 * a^2 / (d * (e^{(-2dx-2c)} - 1))$

mupad [B] time = 1.16, size = 209, normalized size = 4.54

$$-\frac{\frac{2(2ab-b^2)}{3d} + \frac{2e^{2c+2dx}(b^2+2ab)}{3d}}{2e^{2c+2dx} + e^{4c+4dx} + 1} - \frac{\frac{2(b^2+2ab)}{3d} + \frac{2e^{4c+4dx}(b^2+2ab)}{3d} + \frac{4e^{2c+2dx}(2ab-b^2)}{3d}}{3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1} - \frac{2a^2}{d(e^{2c+2dx} - 1)} - \frac{2(b^2 + 2ab)}{3d(e^{2c+2dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tanh(c + d*x))^2/sinh(c + d*x)^2,x)

[Out] - ((2*(2*a*b - b^2))/(3*d) + (2*exp(2*c + 2*d*x)*(2*a*b + b^2))/(3*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*(2*a*b + b^2))/(3*d) + (2*exp(4*c + 4*d*x)*(2*a*b + b^2))/(3*d) + (4*exp(2*c + 2*d*x)*(2*a*b - b^2))/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - (2*a^2)/(d*(exp(2*c + 2*d*x) - 1)) - (2*(2*a*b + b^2))/(3*d*(exp(2*c + 2*d*x) + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^2 \operatorname{csch}^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)

[Out] Integral((a + b*tanh(c + d*x)**2)**2*csch(c + d*x)**2, x)

3.15 $\int \operatorname{csch}^3(c + dx) \left(a + b \tanh^2(c + dx) \right)^2 dx$

Optimal. Leaf size=82

$$\frac{a^2 \operatorname{csch}^2(c + dx) \operatorname{sech}(c + dx)}{2d} - \frac{a(a - 4b) \operatorname{sech}(c + dx)}{2d} + \frac{a(a - 4b) \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{b^2 \operatorname{sech}^3(c + dx)}{3d}$$

[Out] 1/2*a*(a-4*b)*arctanh(cosh(d*x+c))/d-1/2*a*(a-4*b)*sech(d*x+c)/d-1/2*a^2*csch(d*x+c)^2*sech(d*x+c)/d-1/3*b^2*sech(d*x+c)^3/d

Rubi [A] time = 0.12, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3664, 463, 459, 321, 207}

$$\frac{a^2 \operatorname{csch}^2(c + dx) \operatorname{sech}(c + dx)}{2d} - \frac{a(a - 4b) \operatorname{sech}(c + dx)}{2d} + \frac{a(a - 4b) \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{b^2 \operatorname{sech}^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2, x]

[Out] (a*(a - 4*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - (a*(a - 4*b)*Sech[c + d*x])/(2*d) - (a^2*Csch[c + d*x]^2*Sech[c + d*x])/(2*d) - (b^2*Sech[c + d*x]^3)/(3*d)

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 321

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 459

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]

Rule 463

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1]

Rule 3664

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^p_, x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^

m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1), x], x, Sec[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^3(c + dx) (a + b \tanh^2(c + dx))^2 dx &= -\frac{\operatorname{Subst}\left(\int \frac{x^2(a+b-bx^2)^2}{(-1+x^2)^2} dx, x, \operatorname{sech}(c + dx)\right)}{d} \\ &= -\frac{a^2 \operatorname{csch}^2(c + dx) \operatorname{sech}(c + dx)}{2d} - \frac{\operatorname{Subst}\left(\int \frac{x^2(3a^2 - 2(a+b)^2 + 2b^2x^2)}{-1+x^2} dx, x\right)}{2d} \\ &= -\frac{a^2 \operatorname{csch}^2(c + dx) \operatorname{sech}(c + dx)}{2d} - \frac{b^2 \operatorname{sech}^3(c + dx)}{3d} - \frac{(a(a - 4b)) \operatorname{Subst}\left(\int \frac{x}{-1+x^2} dx, x\right)}{2d} \\ &= -\frac{a(a - 4b) \operatorname{sech}(c + dx)}{2d} - \frac{a^2 \operatorname{csch}^2(c + dx) \operatorname{sech}(c + dx)}{2d} - \frac{b^2 \operatorname{sech}^3(c + dx)}{3d} \\ &= \frac{a(a - 4b) \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a(a - 4b) \operatorname{sech}(c + dx)}{2d} - \frac{a^2 \operatorname{csch}^2(c + dx) \operatorname{sech}(c + dx)}{2d} - \frac{b^2 \operatorname{sech}^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 1.59, size = 96, normalized size = 1.17

$$\frac{3a^2 \operatorname{csch}^2\left(\frac{1}{2}(c + dx)\right) + 3a^2 \operatorname{sech}^2\left(\frac{1}{2}(c + dx)\right) + 12a^2 \log\left(\tanh\left(\frac{1}{2}(c + dx)\right)\right) - 48ab \operatorname{sech}(c + dx) - 48ab \log\left(\tanh\left(\frac{1}{2}(c + dx)\right)\right)}{24d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2, x]

[Out] -1/24*(3*a^2*Csch[(c + d*x)/2]^2 + 12*a^2*Log[Tanh[(c + d*x)/2]] - 48*a*b*Log[Tanh[(c + d*x)/2]] + 3*a^2*Sech[(c + d*x)/2]^2 - 48*a*b*Sech[c + d*x] + 8*b^2*Sech[c + d*x]^3)/d

fricas [B] time = 0.51, size = 2462, normalized size = 30.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -1/6*(6*(a^2 - 4*a*b)*cosh(d*x + c)^9 + 54*(a^2 - 4*a*b)*cosh(d*x + c)*sinh(d*x + c)^8 + 6*(a^2 - 4*a*b)*sinh(d*x + c)^9 + 8*(3*a^2 + 2*b^2)*cosh(d*x + c)^7 + 8*(27*(a^2 - 4*a*b)*cosh(d*x + c)^2 + 3*a^2 + 2*b^2)*sinh(d*x + c)^7 + 56*(9*(a^2 - 4*a*b)*cosh(d*x + c)^3 + (3*a^2 + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 + 4*(9*a^2 + 12*a*b - 8*b^2)*cosh(d*x + c)^5 + 4*(189*(a^2 - 4*a*b)*cosh(d*x + c)^4 + 42*(3*a^2 + 2*b^2)*cosh(d*x + c)^2 + 9*a^2 + 12*a*b - 8*b^2)*sinh(d*x + c)^5 + 4*(189*(a^2 - 4*a*b)*cosh(d*x + c)^5 + 70*(3*a^2 + 2*b^2)*cosh(d*x + c)^3 + 5*(9*a^2 + 12*a*b - 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 8*(3*a^2 + 2*b^2)*cosh(d*x + c)^3 + 8*(63*(a^2 - 4*a*b)*cosh(d*x + c)^6 + 35*(3*a^2 + 2*b^2)*cosh(d*x + c)^4 + 5*(9*a^2 + 12*a*b - 8*b^2)*cosh(d*x + c)^2 + 3*a^2 + 2*b^2)*sinh(d*x + c)^3 + 8*(27*(a^2 - 4*a*b)*cosh(d*x + c)^7 + 21*(3*a^2 + 2*b^2)*cosh(d*x + c)^5 + 5*(9*a^2 + 12*a*b - 8*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 6*(a^2 - 4*a*b)*cosh(d*x + c) - 3*((a^2 - 4*a*b)*cosh(d*x + c)^10 + 10*(a^2 - 4*a*b)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^2 - 4*a*b)*sinh(d*x + c)^10 +

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(a^2 - 4*a*b)*cosh(d*x + c)^8 + (45*(a^2 - 4*a*b)*cosh(d*x + c)^2 + a^2 -
4*a*b)*sinh(d*x + c)^8 + 8*(15*(a^2 - 4*a*b)*cosh(d*x + c)^3 + (a^2 - 4*a*b
)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(a^2 - 4*a*b)*cosh(d*x + c)^6 + 2*(105
*(a^2 - 4*a*b)*cosh(d*x + c)^4 + 14*(a^2 - 4*a*b)*cosh(d*x + c)^2 - a^2 + 4
*a*b)*sinh(d*x + c)^6 + 4*(63*(a^2 - 4*a*b)*cosh(d*x + c)^5 + 14*(a^2 - 4*a
*b)*cosh(d*x + c)^3 - 3*(a^2 - 4*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a
^2 - 4*a*b)*cosh(d*x + c)^4 + 2*(105*(a^2 - 4*a*b)*cosh(d*x + c)^6 + 35*(a^
2 - 4*a*b)*cosh(d*x + c)^4 - 15*(a^2 - 4*a*b)*cosh(d*x + c)^2 - a^2 + 4*a*b
)*sinh(d*x + c)^4 + 8*(15*(a^2 - 4*a*b)*cosh(d*x + c)^7 + 7*(a^2 - 4*a*b)*c
osh(d*x + c)^5 - 5*(a^2 - 4*a*b)*cosh(d*x + c)^3 - (a^2 - 4*a*b)*cosh(d*x +
c))*sinh(d*x + c)^3 + (a^2 - 4*a*b)*cosh(d*x + c)^2 + (45*(a^2 - 4*a*b)*co
sh(d*x + c)^8 + 28*(a^2 - 4*a*b)*cosh(d*x + c)^6 - 30*(a^2 - 4*a*b)*cosh(d*
x + c)^4 - 12*(a^2 - 4*a*b)*cosh(d*x + c)^2 + a^2 - 4*a*b)*sinh(d*x + c)^2
+ a^2 - 4*a*b + 2*(5*(a^2 - 4*a*b)*cosh(d*x + c)^9 + 4*(a^2 - 4*a*b)*cosh(d
*x + c)^7 - 6*(a^2 - 4*a*b)*cosh(d*x + c)^5 - 4*(a^2 - 4*a*b)*cosh(d*x + c)
^3 + (a^2 - 4*a*b)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d
*x + c) + 1) + 3*((a^2 - 4*a*b)*cosh(d*x + c)^10 + 10*(a^2 - 4*a*b)*cosh(d*
x + c)*sinh(d*x + c)^9 + (a^2 - 4*a*b)*sinh(d*x + c)^10 + (a^2 - 4*a*b)*cos
h(d*x + c)^8 + (45*(a^2 - 4*a*b)*cosh(d*x + c)^2 + a^2 - 4*a*b)*sinh(d*x +
c)^8 + 8*(15*(a^2 - 4*a*b)*cosh(d*x + c)^3 + (a^2 - 4*a*b)*cosh(d*x + c))*s
inh(d*x + c)^7 - 2*(a^2 - 4*a*b)*cosh(d*x + c)^6 + 2*(105*(a^2 - 4*a*b)*cos
h(d*x + c)^4 + 14*(a^2 - 4*a*b)*cosh(d*x + c)^2 - a^2 + 4*a*b)*sinh(d*x + c
)^6 + 4*(63*(a^2 - 4*a*b)*cosh(d*x + c)^5 + 14*(a^2 - 4*a*b)*cosh(d*x + c)^
3 - 3*(a^2 - 4*a*b)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(a^2 - 4*a*b)*cosh(d
*x + c)^4 + 2*(105*(a^2 - 4*a*b)*cosh(d*x + c)^6 + 35*(a^2 - 4*a*b)*cosh(d*
x + c)^4 - 15*(a^2 - 4*a*b)*cosh(d*x + c)^2 - a^2 + 4*a*b)*sinh(d*x + c)^4
+ 8*(15*(a^2 - 4*a*b)*cosh(d*x + c)^7 + 7*(a^2 - 4*a*b)*cosh(d*x + c)^5 - 5
*(a^2 - 4*a*b)*cosh(d*x + c)^3 - (a^2 - 4*a*b)*cosh(d*x + c))*sinh(d*x + c)
^3 + (a^2 - 4*a*b)*cosh(d*x + c)^2 + (45*(a^2 - 4*a*b)*cosh(d*x + c)^8 + 28
*(a^2 - 4*a*b)*cosh(d*x + c)^6 - 30*(a^2 - 4*a*b)*cosh(d*x + c)^4 - 12*(a^2
- 4*a*b)*cosh(d*x + c)^2 + a^2 - 4*a*b)*sinh(d*x + c)^2 + a^2 - 4*a*b + 2*
(5*(a^2 - 4*a*b)*cosh(d*x + c)^9 + 4*(a^2 - 4*a*b)*cosh(d*x + c)^7 - 6*(a^2
- 4*a*b)*cosh(d*x + c)^5 - 4*(a^2 - 4*a*b)*cosh(d*x + c)^3 + (a^2 - 4*a*b)
*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(
27*(a^2 - 4*a*b)*cosh(d*x + c)^8 + 28*(3*a^2 + 2*b^2)*cosh(d*x + c)^6 + 10*
(9*a^2 + 12*a*b - 8*b^2)*cosh(d*x + c)^4 + 12*(3*a^2 + 2*b^2)*cosh(d*x + c)
^2 + 3*a^2 - 12*a*b)*sinh(d*x + c))/(d*cosh(d*x + c)^10 + 10*d*cosh(d*x + c
)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + d*cosh(d*x + c)^8 + (45*d*cosh(d*x
+ c)^2 + d)*sinh(d*x + c)^8 + 8*(15*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*s
inh(d*x + c)^7 - 2*d*cosh(d*x + c)^6 + 2*(105*d*cosh(d*x + c)^4 + 14*d*cosh
(d*x + c)^2 - d)*sinh(d*x + c)^6 + 4*(63*d*cosh(d*x + c)^5 + 14*d*cosh(d*x
+ c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 - 2*d*cosh(d*x + c)^4 + 2*(105*
d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 - 15*d*cosh(d*x + c)^2 - d)*sinh(d
*x + c)^4 + 8*(15*d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)^5 - 5*d*cosh(d*x +
c)^3 - d*cosh(d*x + c))*sinh(d*x + c)^3 + d*cosh(d*x + c)^2 + (45*d*cosh(d*
x + c)^8 + 28*d*cosh(d*x + c)^6 - 30*d*cosh(d*x + c)^4 - 12*d*cosh(d*x + c)
^2 + d)*sinh(d*x + c)^2 + 2*(5*d*cosh(d*x + c)^9 + 4*d*cosh(d*x + c)^7 - 6*
d*cosh(d*x + c)^5 - 4*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) +
d)

```

giac [B] time = 0.27, size = 168, normalized size = 2.05

$$\frac{3(a^2e^c - 4abe^c)e^{(-c)} \log(e^{(dx+c)} + 1) - 3(a^2e^c - 4abe^c)e^{(-c)} \log(|e^{(dx+c)} - 1|) - \frac{6(a^2e^{(3dx+3c)} + a^2e^{(dx+c)})}{(e^{(2dx+2c)} - 1)^2} + \frac{8(3abe^{(5dx+5c)} - 4a^2e^{(3dx+3c)} - 4a^2e^{(dx+c)})}{(e^{(2dx+2c)} - 1)^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] $\frac{1}{6} \cdot (3 \cdot (a^2 e^c - 4 a b e^c) \cdot e^{-c} \cdot \log(e^{(d x + c)} + 1) - 3 \cdot (a^2 e^c - 4 a b e^c) \cdot e^{-c} \cdot \log(\operatorname{abs}(e^{(d x + c)} - 1)) - 6 \cdot (a^2 e^{(3 d x + 3 c)} + a^2 e^{(d x + c)}) / (e^{(2 d x + 2 c)} - 1)^2 + 8 \cdot (3 a b e^{(5 d x + 5 c)} + 6 a b e^{(3 d x + 3 c)} - 2 b^2 e^{(3 d x + 3 c)} + 3 a b e^{(d x + c)}) / (e^{(2 d x + 2 c)} + 1)^3) / d$

maple [A] time = 0.38, size = 67, normalized size = 0.82

$$\frac{a^2 \left(-\frac{\operatorname{csch}(d x + c) \operatorname{coth}(d x + c)}{2} + \operatorname{arctanh}\left(e^{d x + c}\right) \right) + 2 a b \left(\frac{1}{\operatorname{cosh}(d x + c)} - 2 \operatorname{arctanh}\left(e^{d x + c}\right) \right) - \frac{b^2}{3 \operatorname{cosh}(d x + c)^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x)`

[Out] $\frac{1}{d} \cdot (a^2 \cdot (-1/2 \cdot \operatorname{csch}(d x + c) \cdot \operatorname{coth}(d x + c) + \operatorname{arctanh}(\exp(d x + c))) + 2 a b \cdot (1 / \operatorname{cosh}(d x + c) - 2 \operatorname{arctanh}(\exp(d x + c))) - 1/3 \cdot b^2 / \operatorname{cosh}(d x + c)^3)$

maxima [B] time = 0.32, size = 181, normalized size = 2.21

$$\frac{1}{2} a^2 \left(\frac{\log\left(e^{(-d x - c)} + 1\right)}{d} - \frac{\log\left(e^{(-d x - c)} - 1\right)}{d} + \frac{2\left(e^{(-d x - c)} + e^{(-3 d x - 3 c)}\right)}{d\left(2 e^{(-2 d x - 2 c)} - e^{(-4 d x - 4 c)} - 1\right)} \right) - 2 a b \left(\frac{\log\left(e^{(-d x - c)} + 1\right)}{d} - \frac{\log\left(e^{(-d x - c)} - 1\right)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] $\frac{1}{2} \cdot a^2 \cdot (\log(e^{(-d x - c)} + 1) / d - \log(e^{(-d x - c)} - 1) / d + 2 \cdot (e^{(-d x - c)} + e^{(-3 d x - 3 c)}) / (d \cdot (2 e^{(-2 d x - 2 c)} - e^{(-4 d x - 4 c)} - 1))) - 2 a b \cdot (\log(e^{(-d x - c)} + 1) / d - \log(e^{(-d x - c)} - 1) / d - 2 e^{(-d x - c)} / (d \cdot (e^{(-2 d x - 2 c)} + 1))) - 8/3 \cdot b^2 / (d \cdot (e^{(d x + c)} + e^{(-d x - c)})^3)$

mupad [B] time = 0.16, size = 261, normalized size = 3.18

$$\frac{\operatorname{atan}\left(\frac{e^{d x} e^c \left(a^2 \sqrt{-d^2} - 4 a b \sqrt{-d^2}\right)}{d \sqrt{a^4 - 8 a^3 b + 16 a^2 b^2}}\right) \sqrt{a^4 - 8 a^3 b + 16 a^2 b^2}}{\sqrt{-d^2}} + \frac{8 b^2 e^{c+d x}}{3 d \left(3 e^{2 c+2 d x} + 3 e^{4 c+4 d x} + e^{6 c+6 d x} + 1\right)} - \frac{a^2 e^{c+d x}}{d \left(e^{2 c+2 d x} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*tanh(c + d*x))^2/sinh(c + d*x)^3,x)`

[Out] $(\operatorname{atan}((\exp(d x) \cdot \exp(c) \cdot (a^2 \cdot (-d^2)^{(1/2)} - 4 a b \cdot (-d^2)^{(1/2)})) / (d \cdot (a^4 - 8 a^3 b + 16 a^2 b^2)^{(1/2)})) \cdot (a^4 - 8 a^3 b + 16 a^2 b^2)^{(1/2)}) / (-d^2)^{(1/2)} + (8 b^2 \cdot \exp(c + d x)) / (3 d \cdot (3 \exp(2 c + 2 d x) + 3 \exp(4 c + 4 d x) + \exp(6 c + 6 d x) + 1)) - (a^2 \cdot \exp(c + d x)) / (d \cdot (\exp(2 c + 2 d x) - 1)) - (2 a^2 \cdot \exp(c + d x)) / (d \cdot (\exp(4 c + 4 d x) - 2 \exp(2 c + 2 d x) + 1)) - (8 b^2 \cdot \exp(c + d x)) / (3 d \cdot (2 \exp(2 c + 2 d x) + \exp(4 c + 4 d x) + 1)) + (4 a b \cdot \exp(c + d x)) / (d \cdot (\exp(2 c + 2 d x) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + d x))^2 \operatorname{csch}^3(c + d x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)`

[Out] `Integral((a + b*tanh(c + d*x)**2)**2*csch(c + d*x)**3, x)`

3.16 $\int \operatorname{csch}^4(c + dx) \left(a + b \tanh^2(c + dx) \right)^2 dx$

Optimal. Leaf size=72

$$-\frac{a^2 \operatorname{coth}^3(c + dx)}{3d} - \frac{b(2a - b) \tanh(c + dx)}{d} + \frac{a(a - 2b) \operatorname{coth}(c + dx)}{d} - \frac{b^2 \tanh^3(c + dx)}{3d}$$

[Out] $a*(a-2*b)*\operatorname{coth}(d*x+c)/d-1/3*a^2*\operatorname{coth}(d*x+c)^3/d-(2*a-b)*b*\tanh(d*x+c)/d-1/3*b^2*\tanh(d*x+c)^3/d$

Rubi [A] time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3663, 448}

$$-\frac{a^2 \operatorname{coth}^3(c + dx)}{3d} - \frac{b(2a - b) \tanh(c + dx)}{d} + \frac{a(a - 2b) \operatorname{coth}(c + dx)}{d} - \frac{b^2 \tanh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]^4*(a + b*\operatorname{Tanh}[c + d*x]^2)^2, x]$

[Out] $(a*(a - 2*b)*\operatorname{Coth}[c + d*x])/d - (a^2*\operatorname{Coth}[c + d*x]^3)/(3*d) - ((2*a - b)*b*\operatorname{Tanh}[c + d*x])/d - (b^2*\operatorname{Tanh}[c + d*x]^3)/(3*d)$

Rule 448

$\operatorname{Int}[(e_*)*(x_*)^{(m_*)}*((a_*) + (b_*)*(x_*)^{(n_*)})^{(p_*)}*((c_*) + (d_*)*(x_*)^{(n_*)})^{(q_*)}, x_Symbol] :> \operatorname{Int}[\operatorname{ExpandIntegrand}[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, m, n\}, x] \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{IGtQ}[p, 0] \ \&\& \operatorname{IGtQ}[q, 0]$

Rule 3663

$\operatorname{Int}[\sin[(e_*) + (f_*)*(x_*)]^{(m_*)}*((a_*) + (b_*)*((c_*)*\tan[(e_*) + (f_*)*(x_*)])^{(n_*)})^{(p_*)}, x_Symbol] :> \operatorname{With}\{\operatorname{ff} = \operatorname{FreeFactors}[\tan[e + f*x], x]\}, \operatorname{Dist}[(c*\operatorname{ff}^{(m+1)})/f, \operatorname{Subst}[\operatorname{Int}[(x^m*(a + b*(\operatorname{ff}*x)^n)^p]/(c^2 + \operatorname{ff}^2*x^2)^{(m/2+1)}, x], x, (c*\tan[e + f*x])/ff], x] /;$ $\operatorname{FreeQ}\{a, b, c, e, f, n, p\}, x] \ \&\& \operatorname{IntegerQ}[m/2]$

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^4(c + dx) \left(a + b \tanh^2(c + dx) \right)^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(a+bx^2)^2}{x^4} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(b(-2a + b) + \frac{a^2}{x^4} - \frac{a(a-2b)}{x^2} - b^2x^2\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{a(a - 2b) \operatorname{coth}(c + dx)}{d} - \frac{a^2 \operatorname{coth}^3(c + dx)}{3d} - \frac{(2a - b)b \tanh(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.52, size = 59, normalized size = 0.82

$$\frac{b \tanh(c + dx) (-6a + b \operatorname{sech}^2(c + dx) + 2b) - a \operatorname{coth}(c + dx) (a \operatorname{csch}^2(c + dx) - 2a + 6b)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] $(- (a \operatorname{Coth}[c + d x] * (-2 a + 6 b + a \operatorname{Csch}[c + d x]^2)) + b * (-6 a + 2 b + b \operatorname{Sech}[c + d x]^2) * \operatorname{Tanh}[c + d x]) / (3 d)$

fricas [B] time = 0.71, size = 393, normalized size = 5.46

$$\frac{8 \left((a^2 + 6 a b + b^2) \cosh(dx + c)^4 + 8 (a^2 + b^2) \cosh(dx + c) \sinh(dx + c)^3 + (a^2 + 6 a b + b^2) \sinh(dx + c)^4 + 4 (a^2 - b^2) \cosh(dx + c)^2 + 2 (3 (a^2 + 6 a b + b^2) \cosh(dx + c)^2 + 2 a^2 - 2 b^2) \sinh(dx + c)^2 + 3 a^2 - 6 a b + 3 b^2 + 8 ((a^2 + b^2) \cosh(dx + c)^3 + (a^2 - b^2) \cosh(dx + c)) \sinh(dx + c) \right)}{3 \left(d \cosh(dx + c)^8 + 56 d \cosh(dx + c)^3 \sinh(dx + c)^5 + 28 d \cosh(dx + c)^2 \sinh(dx + c)^6 + 8 d \cosh(dx + c) \sinh(dx + c)^7 + d \sinh(dx + c)^8 - 4 d \cosh(dx + c)^4 + 2 (35 d \cosh(dx + c)^4 - 2 d) \sinh(dx + c)^4 + 8 (7 d \cosh(dx + c)^5 - d \cosh(dx + c)) \sinh(dx + c)^3 + 4 (7 d \cosh(dx + c)^6 - 6 d \cosh(dx + c)^2) \sinh(dx + c)^2 + 8 (d \cosh(dx + c)^7 - d \cosh(dx + c)^3) \sinh(dx + c) + 3 d \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] $-8/3 * ((a^2 + 6 a b + b^2) * \cosh(d x + c)^4 + 8 (a^2 + b^2) * \cosh(d x + c) * \sinh(d x + c)^3 + (a^2 + 6 a b + b^2) * \sinh(d x + c)^4 + 4 (a^2 - b^2) * \cosh(d x + c)^2 + 2 (3 (a^2 + 6 a b + b^2) * \cosh(d x + c)^2 + 2 a^2 - 2 b^2) * \sinh(d x + c)^2 + 3 a^2 - 6 a b + 3 b^2 + 8 ((a^2 + b^2) * \cosh(d x + c)^3 + (a^2 - b^2) * \cosh(d x + c)) * \sinh(d x + c)) / (d * \cosh(d x + c)^8 + 56 d * \cosh(d x + c)^3 * \sinh(d x + c)^5 + 28 d * \cosh(d x + c)^2 * \sinh(d x + c)^6 + 8 d * \cosh(d x + c) * \sinh(d x + c)^7 + d * \sinh(d x + c)^8 - 4 d * \cosh(d x + c)^4 + 2 (35 d * \cosh(d x + c)^4 - 2 d) * \sinh(d x + c)^4 + 8 (7 d * \cosh(d x + c)^5 - d * \cosh(d x + c)) * \sinh(d x + c)^3 + 4 (7 d * \cosh(d x + c)^6 - 6 d * \cosh(d x + c)^2) * \sinh(d x + c)^2 + 8 (d * \cosh(d x + c)^7 - d * \cosh(d x + c)^3) * \sinh(d x + c) + 3 d)$

giac [B] time = 0.26, size = 143, normalized size = 1.99

$$\frac{4 \left(3 a^2 e^{(8 d x + 8 c)} + 6 a b e^{(8 d x + 8 c)} + 3 b^2 e^{(8 d x + 8 c)} + 8 a^2 e^{(6 d x + 6 c)} - 8 b^2 e^{(6 d x + 6 c)} + 6 a^2 e^{(4 d x + 4 c)} - 12 a b e^{(4 d x + 4 c)} + 6 b^2 e^{(4 d x + 4 c)} \right)}{3 d \left(e^{(4 d x + 4 c)} - 1 \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] $-4/3 * (3 a^2 e^{(8 d x + 8 c)} + 6 a b e^{(8 d x + 8 c)} + 3 b^2 e^{(8 d x + 8 c)} + 8 a^2 e^{(6 d x + 6 c)} - 8 b^2 e^{(6 d x + 6 c)} + 6 a^2 e^{(4 d x + 4 c)} - 12 a b e^{(4 d x + 4 c)} + 6 b^2 e^{(4 d x + 4 c)} - a^2 + 6 a b - b^2) / (d * (e^{(4 d x + 4 c)} - 1)^3)$

maple [A] time = 0.46, size = 81, normalized size = 1.12

$$\frac{a^2 \left(\frac{2}{3} - \frac{\operatorname{csch}(dx+c)^2}{3} \right) \operatorname{coth}(dx + c) + 2 a b \left(-\frac{1}{\sinh(dx+c) \cosh(dx+c)} - 2 \tanh(dx + c) \right) + b^2 \left(\frac{2}{3} + \frac{\operatorname{sech}(dx+c)^2}{3} \right) \tanh(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x)

[Out] $1/d * (a^2 * (2/3 - 1/3 * \operatorname{csch}(d x + c)^2) * \operatorname{coth}(d x + c) + 2 a b * (-1/\sinh(d x + c)/\cosh(d x + c) - 2 * \tanh(d x + c)) + b^2 * (2/3 + 1/3 * \operatorname{sech}(d x + c)^2) * \tanh(d x + c))$

maxima [B] time = 0.32, size = 210, normalized size = 2.92

$$\frac{4}{3} b^2 \left(\frac{3 e^{(-2 d x - 2 c)}}{d \left(3 e^{(-2 d x - 2 c)} + 3 e^{(-4 d x - 4 c)} + e^{(-6 d x - 6 c)} + 1 \right)} + \frac{1}{d \left(3 e^{(-2 d x - 2 c)} + 3 e^{(-4 d x - 4 c)} + e^{(-6 d x - 6 c)} + 1 \right)} \right) + \frac{4}{3} a^2 \left(\frac{1}{d \left(3 e^{(-2 d x - 2 c)} + 3 e^{(-4 d x - 4 c)} + e^{(-6 d x - 6 c)} + 1 \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

```
[Out] 4/3*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 8*a*b/(d*(e^(-4*d*x - 4*c) - 1))
```

mupad [B] time = 1.08, size = 143, normalized size = 1.99

$$\frac{4 \left(6 a b - a^2 - b^2 + 6 a^2 e^{4c+4dx} + 8 a^2 e^{6c+6dx} + 3 a^2 e^{8c+8dx} + 6 b^2 e^{4c+4dx} - 8 b^2 e^{6c+6dx} + 3 b^2 e^{8c+8dx} \right)}{3 d \left(e^{4c+4dx} - 1 \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)^2/sinh(c + d*x)^4,x)
```

```
[Out] -(4*(6*a*b - a^2 - b^2 + 6*a^2*exp(4*c + 4*d*x) + 8*a^2*exp(6*c + 6*d*x) + 3*a^2*exp(8*c + 8*d*x) + 6*b^2*exp(4*c + 4*d*x) - 8*b^2*exp(6*c + 6*d*x) + 3*b^2*exp(8*c + 8*d*x) - 12*a*b*exp(4*c + 4*d*x) + 6*a*b*exp(8*c + 8*d*x)))/(3*d*(exp(4*c + 4*d*x) - 1)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^2 \operatorname{csch}^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*csch(c + d*x)**4, x)
```

3.17 $\int \sinh^4(c + dx) (a + b \tanh^2(c + dx))^3 dx$

Optimal. Leaf size=182

$$\frac{b(6a^2 + 35ab + 21b^2) \tanh^3(c + dx)}{8d} - \frac{3(a + b)(a^2 + 14ab + 21b^2) \tanh(c + dx)}{8d} + \frac{3}{8} x^{(a+b)} (a^2 + 14ab + 21b^2) - \dots$$

[Out] $\frac{3}{8}*(a+b)*(a^2+14*a*b+21*b^2)*x - \frac{3}{8}*(a+b)*(a^2+14*a*b+21*b^2)*\tanh(d*x+c)/d - \frac{1}{8}*b*(6*a^2+35*a*b+21*b^2)*\tanh(d*x+c)^3/d - \frac{3}{40}*b^2*(5*a+21*b)*\tanh(d*x+c)^5/d - \frac{3}{8}*(a+3*b)*\sinh(d*x+c)^2*\tanh(d*x+c)*(a+b*\tanh(d*x+c)^2)^2/d + \frac{1}{4}*\cosh(d*x+c)*\sinh(d*x+c)^3*(a+b*\tanh(d*x+c)^2)^3/d$

Rubi [A] time = 0.22, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3663, 467, 577, 570, 206}

$$\frac{b(6a^2 + 35ab + 21b^2) \tanh^3(c + dx)}{8d} - \frac{3(a + b)(a^2 + 14ab + 21b^2) \tanh(c + dx)}{8d} + \frac{3}{8} x^{(a+b)} (a^2 + 14ab + 21b^2) - \dots$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] $\frac{3*(a + b)*(a^2 + 14*a*b + 21*b^2)*x}{8} - \frac{3*(a + b)*(a^2 + 14*a*b + 21*b^2)*\text{Tanh}[c + d*x]}{8*d} - \frac{b*(6*a^2 + 35*a*b + 21*b^2)*\text{Tanh}[c + d*x]^3}{8*d} - \frac{3*b^2*(5*a + 21*b)*\text{Tanh}[c + d*x]^5}{40*d} - \frac{3*(a + 3*b)*\text{Sinh}[c + d*x]^2*\text{Tanh}[c + d*x]*(a + b*\text{Tanh}[c + d*x]^2)^2}{8*d} + \frac{(\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x]^3*(a + b*\text{Tanh}[c + d*x]^2)^3)}{4*d}$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 467

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q - 1) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 570

Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_))^(r_), x_Symbol] := Int[ExpandIntegrand[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]

Rule 577

Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n


```
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned} \int \sinh^4(c + dx) (a + b \tanh^2(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)^3}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\cosh(c + dx) \sinh^3(c + dx) (a + b \tanh^2(c + dx))^3}{4d} - \frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)^3}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{3(a + 3b) \sinh^2(c + dx) \tanh(c + dx) (a + b \tanh^2(c + dx))^2}{8d} + \frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)^3}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{3(a + 3b) \sinh^2(c + dx) \tanh(c + dx) (a + b \tanh^2(c + dx))^2}{8d} + \frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)^3}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{3(a + b)(a^2 + 14ab + 21b^2) \tanh(c + dx)}{8d} - \frac{b(6a^2 + 35ab + 21b^2)}{8d} \\ &= \frac{3}{8}(a + b)(a^2 + 14ab + 21b^2)x - \frac{3(a + b)(a^2 + 14ab + 21b^2) \tanh(c + dx)}{8d} \end{aligned}$$

Mathematica [A] time = 3.92, size = 125, normalized size = 0.69

$$\frac{-32b \tanh(c + dx) (15a^2 - b(5a + 7b) \text{sech}^2(c + dx) + 50ab + b^2 \text{sech}^4(c + dx) + 36b^2) + 60(a^3 + 15a^2b + 35ab^2 + 21b^3)(c + dx) - 40(a + b)^2(a + 4b) \sinh[2(c + dx)] + 5(a + b)^3 \sinh[4(c + dx)] - 32b(15a^2 + 50ab + 36b^2 - b(5a + 7b) \text{sech}[c + dx]^2 + b^2 \text{sech}[c + dx]^4) \tanh[c + dx]}{160d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (60*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*(c + d*x) - 40*(a + b)^2*(a + 4*b)*Sinh[2*(c + d*x)] + 5*(a + b)^3*Sinh[4*(c + d*x)] - 32*b*(15*a^2 + 50*a*b + 36*b^2 - b*(5*a + 7*b)*Sech[c + d*x]^2 + b^2*Sech[c + d*x]^4)*Tanh[c + d*x])/(160*d)
```

fricas [B] time = 0.55, size = 879, normalized size = 4.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/320*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^9 - 15*(a^3 + 11*a^2*b + 19*a*b^2 + 9*b^3 - 12*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)
```

```
*sinh(d*x + c)^7 + 8*(120*a^2*b + 400*a*b^2 + 288*b^3 + 15*(a^3 + 15*a^2*b
+ 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c)^5 + 40*(120*a^2*b + 400*a*b^2 + 288
*b^3 + 15*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c)*sinh(d*x
+ c)^4 + (630*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 - 150*a^3 - 2
010*a^2*b - 4850*a*b^2 - 3054*b^3 - 315*(a^3 + 11*a^2*b + 19*a*b^2 + 9*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 40*(120*a^2*b + 400*a*b^2 + 288*b^3 + 1
5*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c)^3 + 5*(84*(a^3 +
3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 - 105*(a^3 + 11*a^2*b + 19*a*b^2 +
9*b^3)*cosh(d*x + c)^4 - 62*a^3 - 978*a^2*b - 2282*a*b^2 - 1302*b^3 - 4*(7
5*a^3 + 1005*a^2*b + 2425*a*b^2 + 1527*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
3 + 40*(2*(120*a^2*b + 400*a*b^2 + 288*b^3 + 15*(a^3 + 15*a^2*b + 35*a*b^2
+ 21*b^3)*d*x)*cosh(d*x + c)^3 + 3*(120*a^2*b + 400*a*b^2 + 288*b^3 + 15*(a
^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 80
*(120*a^2*b + 400*a*b^2 + 288*b^3 + 15*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)
*d*x)*cosh(d*x + c) + 5*(9*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8
- 21*(a^3 + 11*a^2*b + 19*a*b^2 + 9*b^3)*cosh(d*x + c)^6 - 2*(75*a^3 + 1005
*a^2*b + 2425*a*b^2 + 1527*b^3)*cosh(d*x + c)^4 - 36*a^3 - 612*a^2*b - 1372
*a*b^2 - 924*b^3 - 6*(31*a^3 + 489*a^2*b + 1141*a*b^2 + 651*b^3)*cosh(d*x +
c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^
4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(
d*x + c)^2 + 10*d*cosh(d*x + c))
```

giac [B] time = 0.79, size = 507, normalized size = 2.79

$$120(a^3 + 15a^2b + 35ab^2 + 21b^3)dx - 5(18a^3e^{(4dx+4c)} + 270a^2be^{(4dx+4c)} + 630ab^2e^{(4dx+4c)} + 378b^3e^{(4dx+4c)} -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

```
[Out] 1/320*(120*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*d*x - 5*(18*a^3*e^(4*d*x +
4*c) + 270*a^2*b*e^(4*d*x + 4*c) + 630*a*b^2*e^(4*d*x + 4*c) + 378*b^3*e^(4
*d*x + 4*c) - 8*a^3*e^(2*d*x + 2*c) - 48*a^2*b*e^(2*d*x + 2*c) - 72*a*b^2*e
^(2*d*x + 2*c) - 32*b^3*e^(2*d*x + 2*c) + a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^
(-4*d*x - 4*c) + 5*(a^3*e^(4*d*x + 36*c) + 3*a^2*b*e^(4*d*x + 36*c) + 3*a*b
^2*e^(4*d*x + 36*c) + b^3*e^(4*d*x + 36*c) - 8*a^3*e^(2*d*x + 34*c) - 48*a^
2*b*e^(2*d*x + 34*c) - 72*a*b^2*e^(2*d*x + 34*c) - 32*b^3*e^(2*d*x + 34*c)
)*e^(-32*c) + 128*(15*a^2*b*e^(8*d*x + 8*c) + 60*a*b^2*e^(8*d*x + 8*c) + 50*
b^3*e^(8*d*x + 8*c) + 60*a^2*b*e^(6*d*x + 6*c) + 210*a*b^2*e^(6*d*x + 6*c)
+ 150*b^3*e^(6*d*x + 6*c) + 90*a^2*b*e^(4*d*x + 4*c) + 290*a*b^2*e^(4*d*x +
4*c) + 210*b^3*e^(4*d*x + 4*c) + 60*a^2*b*e^(2*d*x + 2*c) + 190*a*b^2*e^(2
*d*x + 2*c) + 130*b^3*e^(2*d*x + 2*c) + 15*a^2*b + 50*a*b^2 + 36*b^3)/(e^(2
*d*x + 2*c) + 1)^5)/d
```

maple [A] time = 0.36, size = 246, normalized size = 1.35

$$a^3 \left(\left(\frac{\sinh^3(dx+c)}{4} - \frac{3\sinh(dx+c)}{8} \right) \cosh(dx+c) + \frac{3dx}{8} + \frac{3c}{8} \right) + 3a^2b \left(\frac{\sinh^5(dx+c)}{4\cosh(dx+c)} - \frac{5(\sinh^3(dx+c))}{8\cosh(dx+c)} + \frac{15dx}{8} + \frac{15c}{8} - \frac{15\tanh(dx+c)}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x)

```
[Out] 1/d*(a^3*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+3*
a^2*b*(1/4*sinh(d*x+c)^5/cosh(d*x+c)-5/8*sinh(d*x+c)^3/cosh(d*x+c)+15/8*d*x
+15/8*c-15/8*tanh(d*x+c))+3*a*b^2*(1/4*sinh(d*x+c)^7/cosh(d*x+c)^3-7/8*sinh
(d*x+c)^5/cosh(d*x+c)^3+35/8*d*x+35/8*c-35/8*tanh(d*x+c)-35/24*tanh(d*x+c)^
3)+b^3*(1/4*sinh(d*x+c)^9/cosh(d*x+c)^5-9/8*sinh(d*x+c)^7/cosh(d*x+c)^5+63/
8*d*x+63/8*c-63/8*tanh(d*x+c)-21/8*tanh(d*x+c)^3-63/40*tanh(d*x+c)^5))
```

maxima [B] time = 0.34, size = 480, normalized size = 2.64

$$\frac{1}{64} a^3 \left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) + \frac{1}{320} b^3 \left(\frac{2520(dx+c)}{d} + \frac{5(32e^{(-2dx-2c)} - e^{(-4dx-4c)})}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 1/64*a^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) + 1/320*b^3*(2520*(d*x + c)/d + 5*(32*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (135*e^(-2*d*x - 2*c) + 5358*e^(-4*d*x - 4*c) + 18190*e^(-6*d*x - 6*c) + 28455*e^(-8*d*x - 8*c) + 19995*e^(-10*d*x - 10*c) + 6560*e^(-12*d*x - 12*c) - 5)/(d*(e^(-4*d*x - 4*c) + 5*e^(-6*d*x - 6*c) + 10*e^(-8*d*x - 8*c) + 10*e^(-10*d*x - 10*c) + 5*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c)))) + 1/64*a*b^2*(840*(d*x + c)/d + 3*(24*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (63*e^(-2*d*x - 2*c) + 1487*e^(-4*d*x - 4*c) + 2517*e^(-6*d*x - 6*c) + 1608*e^(-8*d*x - 8*c) - 3)/(d*(e^(-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) + 3*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c)))) + 3/64*a^2*b*(120*(d*x + c)/d + (16*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (15*e^(-2*d*x - 2*c) + 144*e^(-4*d*x - 4*c) - 1)/(d*(e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))))

mupad [B] time = 1.39, size = 730, normalized size = 4.01

$$\frac{\frac{2(3a^2b+12ab^2+10b^3)}{5d} + \frac{8e^{2c+2dx}(3a^2b+9ab^2+5b^3)}{5d} + \frac{12e^{4c+4dx}(3a^2b+8ab^2+6b^3)}{5d} + \frac{8e^{6c+6dx}(3a^2b+9ab^2+5b^3)}{5d} + \frac{2e^{8c+8dx}(3a^2b+9ab^2+5b^3)}{5d}}{5e^{2c+2dx} + 10e^{4c+4dx} + 10e^{6c+6dx} + 5e^{8c+8dx} + e^{10c+10dx} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3,x)

[Out] ((2*(12*a*b^2 + 3*a^2*b + 10*b^3))/(5*d) + (8*exp(2*c + 2*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (12*exp(4*c + 4*d*x)*(8*a*b^2 + 3*a^2*b + 6*b^3))/(5*d) + (8*exp(6*c + 6*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (2*exp(8*c + 8*d*x)*(12*a*b^2 + 3*a^2*b + 10*b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) + ((2*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (2*exp(2*c + 2*d*x)*(12*a*b^2 + 3*a^2*b + 10*b^3))/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + x*((105*a*b^2)/8 + (45*a^2*b)/8 + (3*a^3)/8 + (63*b^3)/8) + ((2*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (6*exp(2*c + 2*d*x)*(8*a*b^2 + 3*a^2*b + 6*b^3))/(5*d) + (6*exp(4*c + 4*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (2*exp(6*c + 6*d*x)*(12*a*b^2 + 3*a^2*b + 10*b^3))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) + ((2*(8*a*b^2 + 3*a^2*b + 6*b^3))/(5*d) + (4*exp(2*c + 2*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (2*exp(4*c + 4*d*x)*(12*a*b^2 + 3*a^2*b + 10*b^3))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + (2*(12*a*b^2 + 3*a^2*b + 10*b^3))/(5*d*(exp(2*c + 2*d*x) + 1)) - (exp(-4*c - 4*d*x)*(a + b)^3)/(64*d) + (exp(4*c + 4*d*x)*(a + b)^3)/(64*d) + (exp(-2*c - 2*d*x)*(a + b)^2*(a + 4*b))/(8*d) - (exp(2*c + 2*d*x)*(a + b)^2*(a + 4*b))/(8*d)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)

[Out] Timed out

3.18 $\int \sinh^3(c + dx) (a + b \tanh^2(c + dx))^3 dx$

Optimal. Leaf size=105

$$\frac{b^2(3a + 4b)\operatorname{sech}^3(c + dx)}{3d} + \frac{(a + b)^3 \cosh^3(c + dx)}{3d} - \frac{(a + b)^2(a + 4b) \cosh(c + dx)}{d} - \frac{3b(a + b)(a + 2b)\operatorname{sech}(c + dx)}{d}$$

[Out] $-(a+b)^2*(a+4*b)*\cosh(d*x+c)/d+1/3*(a+b)^3*\cosh(d*x+c)^3/d-3*b*(a+b)*(a+2*b)*\operatorname{sech}(d*x+c)/d+1/3*b^2*(3*a+4*b)*\operatorname{sech}(d*x+c)^3/d-1/5*b^3*\operatorname{sech}(d*x+c)^5/d$

Rubi [A] time = 0.12, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3664, 448}

$$\frac{b^2(3a + 4b)\operatorname{sech}^3(c + dx)}{3d} + \frac{(a + b)^3 \cosh^3(c + dx)}{3d} - \frac{(a + b)^2(a + 4b) \cosh(c + dx)}{d} - \frac{3b(a + b)(a + 2b)\operatorname{sech}(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[c + d*x]^3*(a + b*\text{Tanh}[c + d*x]^2)^3, x]$

[Out] $-(((a + b)^2*(a + 4*b)*\text{Cosh}[c + d*x])/d) + ((a + b)^3*\text{Cosh}[c + d*x]^3)/(3*d) - (3*b*(a + b)*(a + 2*b)*\text{Sech}[c + d*x])/d + (b^2*(3*a + 4*b)*\text{Sech}[c + d*x]^3)/(3*d) - (b^3*\text{Sech}[c + d*x]^5)/(5*d)$

Rule 448

$\text{Int}[(e_*)*(x_)^{(m_*)}((a_*) + (b_*)*(x_)^{(n_)})^{(p_*)}((c_*) + (d_*)*(x_)^{(n_)})^{(q_*)}, x_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; \text{FreeQ}\{a, b, c, d, e, m, n\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[p, 0] \&\& \text{IGtQ}[q, 0]$

Rule 3664

$\text{Int}[\sin[(e_*) + (f_*)*(x_)]^{(m_*)}((a_*) + (b_*)*\tan[(e_*) + (f_*)*(x_)]^2)^{(p_*)}, x_Symbol] :> \text{With}\{\{ff = \text{FreeFactors}[\text{Sec}[e + f*x], x]\}, \text{Dist}[1/(f*ff^m), \text{Subst}[\text{Int}[((-1 + ff^2*x^2)^{(m-1)/2}*(a - b + b*ff^2*x^2)^p)/x^{(m+1)}, x], x, \text{Sec}[e + f*x]/ff], x]\} /; \text{FreeQ}\{a, b, e, f, p\}, x\} \&\& \text{IntegerQ}[(m-1)/2]$

Rubi steps

$$\begin{aligned} \int \sinh^3(c + dx) (a + b \tanh^2(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{(-1+x^2)(a+b-bx^2)^3}{x^4} dx, x, \operatorname{sech}(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(3(-a-2b)b(a+b) - \frac{(a+b)^3}{x^4} + \frac{(a+b)^2(a+4b)}{x^2} + b^2(3a+4b)x^2\right) dx, x, \operatorname{sech}(c + dx)\right)}{d} \\ &= -\frac{(a+b)^2(a+4b) \cosh(c + dx)}{d} + \frac{(a+b)^3 \cosh^3(c + dx)}{3d} - \frac{3b(a+b)(a+2b) \operatorname{sech}(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.30, size = 91, normalized size = 0.87

$$\frac{20b^2(3a + 4b)\operatorname{sech}^3(c + dx) - 45(a + b)^2(a + 5b) \cosh(c + dx) + 5(a + b)^3 \cosh(3(c + dx)) - 180b(a + b)(a + 2b) \operatorname{sech}(c + dx)}{60d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (-45*(a + b)^2*(a + 5*b)*Cosh[c + d*x] + 5*(a + b)^3*Cosh[3*(c + d*x)] - 180*b*(a + b)*(a + 2*b)*Sech[c + d*x] + 20*b^2*(3*a + 4*b)*Sech[c + d*x]^3 - 12*b^3*Sech[c + d*x]^5)/(60*d)

fricas [B] time = 0.59, size = 540, normalized size = 5.14

$$5(a^3 + 3a^2b + 3ab^2 + b^3) \cosh(dx + c)^8 + 5(a^3 + 3a^2b + 3ab^2 + b^3) \sinh(dx + c)^8 - 20(a^3 + 12a^2b + 21ab^2 + 10b^3) \cosh(dx + c)^6 \sinh(dx + c)^2 - 20(a^3 + 12a^2b + 21ab^2 + 10b^3) \cosh(dx + c)^4 \sinh(dx + c)^4 - 20(a^3 + 12a^2b + 21ab^2 + 10b^3) \cosh(dx + c)^2 \sinh(dx + c)^6 - 20(11a^3 + 123a^2b + 249ab^2 + 137b^3) \cosh(dx + c)^4 + 10(35(a^3 + 3a^2b + 3ab^2 + b^3) \cosh(dx + c)^4 - 22a^3 - 246a^2b - 498ab^2 - 274b^3 - 30(a^3 + 12a^2b + 21ab^2 + 10b^3) \cosh(dx + c)^2) \sinh(dx + c)^4 - 425a^3 - 5235a^2b - 10395ab^2 - 5649b^3 - 20(31a^3 + 372a^2b + 747ab^2 + 390b^3) \cosh(dx + c)^2 + 20(7(a^3 + 3a^2b + 3ab^2 + b^3) \cosh(dx + c)^6 - 15(a^3 + 12a^2b + 21ab^2 + 10b^3) \cosh(dx + c)^4 - 31a^3 - 372a^2b - 747ab^2 - 390b^3 - 6(11a^3 + 123a^2b + 249ab^2 + 137b^3) \cosh(dx + c)^2) \sinh(dx + c)^2) / (d \cosh(dx + c)^5 + 5d \cosh(dx + c) \sinh(dx + c)^4 + 5d \cosh(dx + c)^3 + 5(2d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^2 + 10d \cosh(dx + c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/120*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^8 - 20*(a^3 + 12*a^2*b + 21*a*b^2 + 10*b^3)*cosh(d*x + c)^6 - 20*(a^3 + 12*a^2*b + 21*a*b^2 + 10*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 - 20*(11*a^3 + 123*a^2*b + 249*a*b^2 + 137*b^3)*cosh(d*x + c)^4 + 10*(35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 - 22*a^3 - 246*a^2*b - 498*a*b^2 - 274*b^3 - 30*(a^3 + 12*a^2*b + 21*a*b^2 + 10*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 425*a^3 - 5235*a^2*b - 10395*a*b^2 - 5649*b^3 - 20*(31*a^3 + 372*a^2*b + 747*a*b^2 + 390*b^3)*cosh(d*x + c)^2 + 20*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 - 15*(a^3 + 12*a^2*b + 21*a*b^2 + 10*b^3)*cosh(d*x + c)^4 - 31*a^3 - 372*a^2*b - 747*a*b^2 - 390*b^3 - 6*(11*a^3 + 123*a^2*b + 249*a*b^2 + 137*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2)/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))

giac [B] time = 0.66, size = 442, normalized size = 4.21

$$5(9a^3e^{(2dx+2c)} + 63a^2be^{(2dx+2c)} + 99ab^2e^{(2dx+2c)} + 45b^3e^{(2dx+2c)} - a^3 - 3a^2b - 3ab^2 - b^3)e^{(-3dx-3c)} - 5(a^3 + 3a^2b + 3ab^2 + b^3) \cosh(dx + c)^8 + 5(a^3 + 3a^2b + 3ab^2 + b^3) \sinh(dx + c)^8 - 20(a^3 + 12a^2b + 21ab^2 + 10b^3) \cosh(dx + c)^6 \sinh(dx + c)^2 - 20(a^3 + 12a^2b + 21ab^2 + 10b^3) \cosh(dx + c)^4 \sinh(dx + c)^4 - 20(11a^3 + 123a^2b + 249ab^2 + 137b^3) \cosh(dx + c)^4 + 10(35(a^3 + 3a^2b + 3ab^2 + b^3) \cosh(dx + c)^4 - 22a^3 - 246a^2b - 498ab^2 - 274b^3 - 30(a^3 + 12a^2b + 21ab^2 + 10b^3) \cosh(dx + c)^2) \sinh(dx + c)^4 - 425a^3 - 5235a^2b - 10395ab^2 - 5649b^3 - 20(31a^3 + 372a^2b + 747ab^2 + 390b^3) \cosh(dx + c)^2 + 20(7(a^3 + 3a^2b + 3ab^2 + b^3) \cosh(dx + c)^6 - 15(a^3 + 12a^2b + 21ab^2 + 10b^3) \cosh(dx + c)^4 - 31a^3 - 372a^2b - 747ab^2 - 390b^3 - 6(11a^3 + 123a^2b + 249ab^2 + 137b^3) \cosh(dx + c)^2) \sinh(dx + c)^2) / (d \cosh(dx + c)^5 + 5d \cosh(dx + c) \sinh(dx + c)^4 + 5d \cosh(dx + c)^3 + 5(2d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^2 + 10d \cosh(dx + c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out] -1/120*(5*(9*a^3*e^(2*d*x + 2*c) + 63*a^2*b*e^(2*d*x + 2*c) + 99*a*b^2*e^(2*d*x + 2*c) + 45*b^3*e^(2*d*x + 2*c) - a^3 - 3*a^2*b - 3*a*b^2 - b^3)*e^(-3*d*x - 3*c) - 5*(a^3*e^(3*d*x + 48*c) + 3*a^2*b*e^(3*d*x + 48*c) + 3*a*b^2*e^(3*d*x + 48*c) + b^3*e^(3*d*x + 48*c) - 9*a^3*e^(d*x + 46*c) - 63*a^2*b*e^(d*x + 46*c) - 99*a*b^2*e^(d*x + 46*c) - 45*b^3*e^(d*x + 46*c))*e^(-45*c) + 16*(45*a^2*b*e^(9*d*x + 9*c) + 135*a*b^2*e^(9*d*x + 9*c) + 90*b^3*e^(9*d*x + 9*c) + 180*a^2*b*e^(7*d*x + 7*c) + 480*a*b^2*e^(7*d*x + 7*c) + 280*b^3*e^(7*d*x + 7*c) + 270*a^2*b*e^(5*d*x + 5*c) + 690*a*b^2*e^(5*d*x + 5*c) + 428*b^3*e^(5*d*x + 5*c) + 180*a^2*b*e^(3*d*x + 3*c) + 480*a*b^2*e^(3*d*x + 3*c) + 280*b^3*e^(3*d*x + 3*c) + 45*a^2*b*e^(d*x + c) + 135*a*b^2*e^(d*x + c) + 90*b^3*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^5/d

maple [B] time = 0.32, size = 239, normalized size = 2.28

$$a^3 \left(-\frac{2}{3} + \frac{\sinh^2(dx+c)}{3} \right) \cosh(dx+c) + 3a^2b \left(\frac{\sinh^4(dx+c)}{3 \cosh(dx+c)} - \frac{4(\sinh^2(dx+c))}{3 \cosh(dx+c)} - \frac{8}{3 \cosh(dx+c)} \right) + 3ab^2 \left(\frac{\sinh^6(dx+c)}{3 \cosh(dx+c)^3} - \frac{2(\sinh^4(dx+c))}{3 \cosh(dx+c)^2} - \frac{4(\sinh^2(dx+c))}{3 \cosh(dx+c)} - \frac{8}{3 \cosh(dx+c)} \right) + b^3 \left(\frac{\sinh^8(dx+c)}{3 \cosh(dx+c)^3} - \frac{4(\sinh^6(dx+c))}{3 \cosh(dx+c)^2} - \frac{8(\sinh^4(dx+c))}{3 \cosh(dx+c)} - \frac{8}{3 \cosh(dx+c)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x)

[Out] $1/d*(a^3*(-2/3+1/3*\sinh(d*x+c)^2)*\cosh(d*x+c)+3*a^2*b*(1/3*\sinh(d*x+c)^4/\cosh(d*x+c)-4/3*\sinh(d*x+c)^2/\cosh(d*x+c)-8/3/\cosh(d*x+c))+3*a*b^2*(1/3*\sinh(d*x+c)^6/\cosh(d*x+c)^3-2*\sinh(d*x+c)^4/\cosh(d*x+c)^3-8*\sinh(d*x+c)^2/\cosh(d*x+c)^3-16/3/\cosh(d*x+c)^3)+b^3*(1/3*\sinh(d*x+c)^8/\cosh(d*x+c)^5-8/3*\sinh(d*x+c)^6/\cosh(d*x+c)^5-16*\sinh(d*x+c)^4/\cosh(d*x+c)^5-64/3*\sinh(d*x+c)^2/\cosh(d*x+c)^5-128/15/\cosh(d*x+c)^5))$

maxima [B] time = 0.35, size = 439, normalized size = 4.18

$$-\frac{1}{120} b^3 \left(\frac{5 \left(45 e^{(-dx-c)} - e^{(-3dx-3c)} \right)}{d} + \frac{200 e^{(-2dx-2c)} + 2515 e^{(-4dx-4c)} + 6680 e^{(-6dx-6c)} + 9073 e^{(-8dx-8c)} + 5600 e^{(-10dx-10c)} + 1665 e^{(-12dx-12c)} - 5}{d \left(e^{(-3dx-3c)} + 5 e^{(-5dx-5c)} + 10 e^{(-7dx-7c)} + 10 e^{(-9dx-9c)} + 5 e^{(-11dx-11c)} + e^{(-13dx-13c)} \right)} \right) - \frac{1}{8} a b^2 \left(\frac{33 e^{(-dx-c)} - e^{(-3dx-3c)}}{d} + \frac{30 e^{(-2dx-2c)} + 240 e^{(-4dx-4c)} + 322 e^{(-6dx-6c)} + 177 e^{(-8dx-8c)} - 1}{d \left(e^{(-3dx-3c)} + 3 e^{(-5dx-5c)} + 3 e^{(-7dx-7c)} + e^{(-9dx-9c)} \right)} \right) - \frac{1}{8} a^2 b \left(\frac{21 e^{(-dx-c)} - e^{(-3dx-3c)}}{d} + \frac{20 e^{(-2dx-2c)} + 69 e^{(-4dx-4c)} - 1}{d \left(e^{(-3dx-3c)} + e^{(-5dx-5c)} \right)} \right) + \frac{1}{24} a^3 \left(\frac{e^{(3dx+3c)}}{d} - 9 \frac{e^{(dx+c)}}{d} - 9 \frac{e^{(-dx-c)}}{d} + \frac{e^{(-3dx-3c)}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")`

[Out] $-1/120*b^3*(5*(45*e^{(-d*x - c)} - e^{(-3*d*x - 3*c)})/d + (200*e^{(-2*d*x - 2*c)} + 2515*e^{(-4*d*x - 4*c)} + 6680*e^{(-6*d*x - 6*c)} + 9073*e^{(-8*d*x - 8*c)} + 5600*e^{(-10*d*x - 10*c)} + 1665*e^{(-12*d*x - 12*c)} - 5)/(d*(e^{(-3*d*x - 3*c)} + 5*e^{(-5*d*x - 5*c)} + 10*e^{(-7*d*x - 7*c)} + 10*e^{(-9*d*x - 9*c)} + 5*e^{(-11*d*x - 11*c)} + e^{(-13*d*x - 13*c)})) - 1/8*a*b^2*((33*e^{(-d*x - c)} - e^{(-3*d*x - 3*c)})/d + (30*e^{(-2*d*x - 2*c)} + 240*e^{(-4*d*x - 4*c)} + 322*e^{(-6*d*x - 6*c)} + 177*e^{(-8*d*x - 8*c)} - 1)/(d*(e^{(-3*d*x - 3*c)} + 3*e^{(-5*d*x - 5*c)} + 3*e^{(-7*d*x - 7*c)} + e^{(-9*d*x - 9*c)}))) - 1/8*a^2*b*((21*e^{(-d*x - c)} - e^{(-3*d*x - 3*c)})/d + (20*e^{(-2*d*x - 2*c)} + 69*e^{(-4*d*x - 4*c)} - 1)/(d*(e^{(-3*d*x - 3*c)} + e^{(-5*d*x - 5*c)}))) + 1/24*a^3*(e^{(3*d*x + 3*c)}/d - 9*e^{(d*x + c)}/d - 9*e^{(-d*x - c)}/d + e^{(-3*d*x - 3*c)}/d)$

mupad [B] time = 0.41, size = 361, normalized size = 3.44

$$\frac{e^{-3c-3dx} (a+b)^3}{24d} + \frac{e^{3c+3dx} (a+b)^3}{24d} + \frac{8e^{c+dx} (4b^3 + 3ab^2)}{3d (2e^{2c+2dx} + e^{4c+4dx} + 1)} + \frac{64b^3 e^{c+dx}}{5d (4e^{2c+2dx} + 6e^{4c+4dx} + 4e^{6c+6dx} + e^{8c+8dx})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3,x)`

[Out] $(\exp(-3c - 3d*x)*(a + b)^3)/(24*d) + (\exp(3c + 3d*x)*(a + b)^3)/(24*d) + (8*\exp(c + d*x)*(3*a*b^2 + 4*b^3))/(3*d*(2*\exp(2c + 2*d*x) + \exp(4c + 4*d*x) + 1)) + (64*b^3*\exp(c + d*x))/(5*d*(4*\exp(2c + 2*d*x) + 6*\exp(4c + 4*d*x) + 4*\exp(6c + 6*d*x) + \exp(8c + 8*d*x) + 1)) - (8*\exp(c + d*x)*(15*a*b^2 + 32*b^3))/(15*d*(3*\exp(2c + 2*d*x) + 3*\exp(4c + 4*d*x) + \exp(6c + 6*d*x) + 1)) - (32*b^3*\exp(c + d*x))/(5*d*(5*\exp(2c + 2*d*x) + 10*\exp(4c + 4*d*x) + 10*\exp(6c + 6*d*x) + 5*\exp(8c + 8*d*x) + \exp(10c + 10*d*x) + 1)) - (3*\exp(c + d*x)*(a + b)^2*(a + 5*b))/(8*d) - (6*\exp(c + d*x)*(3*a*b^2 + a^2*b + 2*b^3))/(d*(\exp(2c + 2*d*x) + 1)) - (3*\exp(-c - d*x)*(a + b)^2*(a + 5*b))/(8*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^3 \sinh^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)`

[Out] `Integral((a + b*tanh(c + d*x)**2)**3*sinh(c + d*x)**3, x)`

3.19 $\int \sinh^2(c + dx) \left(a + b \tanh^2(c + dx)\right)^3 dx$

Optimal. Leaf size=122

$$\frac{b^2(3a + 2b) \tanh^3(c + dx)}{3d} + \frac{3b(a + b)^2 \tanh(c + dx)}{d} + \frac{(a + b)^3}{4d(1 - \tanh(c + dx))} - \frac{(a + b)^3}{4d(\tanh(c + dx) + 1)} - \frac{1}{2}x(a+b)^2$$

[Out] $-1/2*(a+b)^2*(a+7*b)*x+1/4*(a+b)^3/d/(1-\tanh(d*x+c))+3*b*(a+b)^2*\tanh(d*x+c)/d+1/3*b^2*(3*a+2*b)*\tanh(d*x+c)^3/d+1/5*b^3*\tanh(d*x+c)^5/d-1/4*(a+b)^3/d/(1+\tanh(d*x+c))$

Rubi [A] time = 0.19, antiderivative size = 139, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3663, 467, 528, 388, 206}

$$\frac{b(81a^2 + 190ab + 105b^2) \tanh(c + dx)}{30d} + \frac{7b \tanh(c + dx) (a + b \tanh^2(c + dx))^2}{10d} + \frac{b(33a + 35b) \tanh(c + dx)}{30d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] $-((a + b)^2*(a + 7*b)*x)/2 + (b*(81*a^2 + 190*a*b + 105*b^2)*Tanh[c + d*x])/ (30*d) + (b*(33*a + 35*b)*Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2))/ (30*d) + (7*b*Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2)/ (10*d) + (Cosh[c + d*x]*Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3)/ (2*d)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 467

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 528

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e - a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned} \int \sinh^2(c + dx) (a + b \tanh^2(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^3}{(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\cosh(c + dx) \sinh(c + dx) (a + b \tanh^2(c + dx))^3}{2d} - \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{1-x^2} dx, x, \tanh(c + dx)\right)}{2d} \\ &= \frac{7b \tanh(c + dx) (a + b \tanh^2(c + dx))^2}{10d} + \frac{\cosh(c + dx) \sinh(c + dx) (a + b \tanh^2(c + dx))^3}{2d} \\ &= \frac{b(33a + 35b) \tanh(c + dx) (a + b \tanh^2(c + dx))}{30d} + \frac{7b \tanh(c + dx) (a + b \tanh^2(c + dx))^3}{30d} \\ &= \frac{b(81a^2 + 190ab + 105b^2) \tanh(c + dx)}{30d} + \frac{b(33a + 35b) \tanh(c + dx) (a + b \tanh^2(c + dx))^3}{30d} \\ &= -\frac{1}{2}(a + b)^2(a + 7b)x + \frac{b(81a^2 + 190ab + 105b^2) \tanh(c + dx)}{30d} + \frac{b(33a + 35b) \tanh(c + dx) (a + b \tanh^2(c + dx))^3}{30d} \end{aligned}$$

Mathematica [A] time = 2.20, size = 95, normalized size = 0.78

$$\frac{4b \tanh(c + dx) (45a^2 - b(15a + 16b) \text{sech}^2(c + dx) + 105ab + 3b^2 \text{sech}^4(c + dx) + 58b^2) - 30(a + 7b)(a + b)^2(c + dx)}{60d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (-30*(a + b)^2*(a + 7*b)*(c + d*x) + 15*(a + b)^3*Sinh[2*(c + d*x)] + 4*b*(45*a^2 + 105*a*b + 58*b^2 - b*(15*a + 16*b)*Sech[c + d*x]^2 + 3*b^2*Sech[c + d*x]^4)*Tanh[c + d*x])/(60*d)

fricas [B] time = 0.87, size = 725, normalized size = 5.94

$$\frac{15(a^3 + 3a^2b + 3ab^2 + b^3) \sinh(dx + c)^7 - 4(90a^2b + 210ab^2 + 116b^3 + 15(a^3 + 9a^2b + 15ab^2 + 7b^3)dx) \cosh(dx + c) \sinh(dx + c)^6}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/120*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^7 - 4*(90*a^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*d*x)*cosh(d*x + c)^5 - 20*(90*a^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^4 + (75*a^3 + 585*a^2*b + 1065*a*b^2 + 539*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3)

$$+ c)^5 - 20*(90*a^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*d*x)*\cosh(d*x + c)^3 + 5*(105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^4 + 27*a^3 + 297*a^2*b + 489*a*b^2 + 203*b^3 + 2*(75*a^3 + 585*a^2*b + 1065*a*b^2 + 539*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^3 - 20*(2*(90*a^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*d*x)*\cosh(d*x + c)^3 + 3*(90*a^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^2 - 40*(90*a^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*d*x)*\cosh(d*x + c) + 5*(21*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^6 + (75*a^3 + 585*a^2*b + 1065*a*b^2 + 539*b^3)*\cosh(d*x + c)^4 + 15*a^3 + 189*a^2*b + 285*a*b^2 + 175*b^3 + 3*(27*a^3 + 297*a^2*b + 489*a*b^2 + 203*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c))/(d*\cosh(d*x + c)^5 + 5*d*\cosh(d*x + c)*\sinh(d*x + c)^4 + 5*d*\cosh(d*x + c)^3 + 5*(2*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^2 + 10*d*\cosh(d*x + c))$$

giac [B] time = 0.64, size = 395, normalized size = 3.24

$$60(a^3 + 9a^2b + 15ab^2 + 7b^3)dx - 15(2a^3e^{(2dx+2c)} + 18a^2be^{(2dx+2c)} + 30ab^2e^{(2dx+2c)} + 14b^3e^{(2dx+2c)} - a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c))^2)^3,x, algorithm="giac")

[Out] $-1/120*(60*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*d*x - 15*(2*a^3*e^{(2*d*x + 2*c)} + 18*a^2*b*e^{(2*d*x + 2*c)} + 30*a*b^2*e^{(2*d*x + 2*c)} + 14*b^3*e^{(2*d*x + 2*c)} - a^3 - 3*a^2*b - 3*a*b^2 - b^3)*e^{(-2*d*x - 2*c)} - 15*(a^3*e^{(2*d*x + 16*c)} + 3*a^2*b*e^{(2*d*x + 16*c)} + 3*a*b^2*e^{(2*d*x + 16*c)} + b^3*e^{(2*d*x + 16*c)})*e^{(-14*c)} + 16*(45*a^2*b*e^{(8*d*x + 8*c)} + 135*a*b^2*e^{(8*d*x + 8*c)} + 90*b^3*e^{(8*d*x + 8*c)} + 180*a^2*b*e^{(6*d*x + 6*c)} + 450*a*b^2*e^{(6*d*x + 6*c)} + 240*b^3*e^{(6*d*x + 6*c)} + 270*a^2*b*e^{(4*d*x + 4*c)} + 600*a*b^2*e^{(4*d*x + 4*c)} + 340*b^3*e^{(4*d*x + 4*c)} + 180*a^2*b*e^{(2*d*x + 2*c)} + 390*a*b^2*e^{(2*d*x + 2*c)} + 200*b^3*e^{(2*d*x + 2*c)} + 45*a^2*b + 105*a*b^2 + 58*b^3)/(e^{(2*d*x + 2*c)} + 1)^5)/d$

maple [A] time = 0.23, size = 180, normalized size = 1.48

$$a^3 \left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2} \right) + 3a^2b \left(\frac{\sinh^3(dx+c)}{2\cosh(dx+c)} - \frac{3dx}{2} - \frac{3c}{2} + \frac{3\tanh(dx+c)}{2} \right) + 3ab^2 \left(\frac{\sinh^5(dx+c)}{2\cosh(dx+c)^3} - \frac{5dx}{2} - \frac{5c}{2} + \frac{5\tanh(dx+c)}{2} \right) + \frac{15}{d} \left(\frac{420(dx+c)}{d} + \frac{15e^{(-2dx-2c)}}{d} - \frac{1003e^{(-2dx-2c)} + 3350e^{(-4dx-4c)}}{d(e^{(-2dx-2c)} + 5e^{(-4dx-4c)} + 10)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^2*(a+b*tanh(d*x+c))^2)^3,x)

[Out] $1/d*(a^3*(1/2*\cosh(d*x+c)*\sinh(d*x+c)-1/2*d*x-1/2*c)+3*a^2*b*(1/2*\sinh(d*x+c)^3/\cosh(d*x+c)-3/2*d*x-3/2*c+3/2*\tanh(d*x+c))+3*a*b^2*(1/2*\sinh(d*x+c)^5/\cosh(d*x+c)^3-5/2*d*x-5/2*c+5/2*\tanh(d*x+c)+5/6*\tanh(d*x+c)^3)+b^3*(1/2*\sinh(d*x+c)^7/\cosh(d*x+c)^5-7/2*d*x-7/2*c+7/2*\tanh(d*x+c)+7/6*\tanh(d*x+c)^3+7/10*\tanh(d*x+c)^5))$

maxima [B] time = 0.33, size = 377, normalized size = 3.09

$$-\frac{1}{8}a^3\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{120}b^3\left(\frac{420(dx+c)}{d} + \frac{15e^{(-2dx-2c)}}{d} - \frac{1003e^{(-2dx-2c)} + 3350e^{(-4dx-4c)}}{d(e^{(-2dx-2c)} + 5e^{(-4dx-4c)} + 10)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c))^2)^3,x, algorithm="maxima")

[Out] $-1/8*a^3*(4*x - e^{(2*d*x + 2*c)}/d + e^{(-2*d*x - 2*c)}/d) - 1/120*b^3*(420*(d*x + c)/d + 15*e^{(-2*d*x - 2*c)}/d - (1003*e^{(-2*d*x - 2*c)} + 3350*e^{(-4*d*x - 4*c)} + 5590*e^{(-6*d*x - 6*c)} + 3915*e^{(-8*d*x - 8*c)} + 1455*e^{(-10*d*x - 10*c)} + 15)/(d*(e^{(-2*d*x - 2*c)} + 5*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 10*e^{(-8*d*x - 8*c)} + 5*e^{(-10*d*x - 10*c)} + e^{(-12*d*x - 12*c)})) - 1/8*a*b^2*(60*(d*x + c)/d + 3*e^{(-2*d*x - 2*c)}/d - (121*e^{(-2*d*x - 2*c)} + 201*e^{(-4*d*x - 4*c)} + 147*e^{(-6*d*x - 6*c)} + 3)/(d*(e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + 3*e^{(-6*d*x - 6*c)} + e^{(-8*d*x - 8*c)})) - 3/8*a^2*b*(12*(d*x + c)/d + e^{(-2*d*x - 2*c)}/d - (17*e^{(-2*d*x - 2*c)} + 1)/(d*(e^{(-2*d*x - 2*c)} + e^{(-4*d*x - 4*c)}))$

mupad [B] time = 0.32, size = 668, normalized size = 5.48

$$\frac{e^{2c+2dx} (a+b)^3}{8d} - \frac{2(3a^2b+6ab^2+2b^3)}{5d} + \frac{6e^{2c+2dx}(a^2b+3ab^2+2b^3)}{5d} - \frac{2(3a^2b+6ab^2+2b^3)}{5d} + \frac{6e^{6c+6dx}(a^2b+3ab^2+2b^3)}{5d} + \frac{6e^{4c+4dx}}{4e^{2c+2dx} + 6e^{4c+4dx} + 4e^{6c+6dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(c + d*x)^2*(a + b*tanh(c + d*x))^3,x)`

[Out] $(\exp(2*c + 2*d*x)*(a + b)^3)/(8*d) - ((2*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (6*\exp(2*c + 2*d*x)*(3*a*b^2 + a^2*b + 2*b^3))/(5*d))/(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1) - ((2*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (6*\exp(6*c + 6*d*x)*(3*a*b^2 + a^2*b + 2*b^3))/(5*d) + (6*\exp(4*c + 4*d*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (2*\exp(2*c + 2*d*x)*(15*a*b^2 + 9*a^2*b + 10*b^3))/(5*d))/(4*\exp(2*c + 2*d*x) + 6*\exp(4*c + 4*d*x) + 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1) - ((2*(15*a*b^2 + 9*a^2*b + 10*b^3))/(15*d) + (6*\exp(4*c + 4*d*x)*(3*a*b^2 + a^2*b + 2*b^3))/(5*d) + (4*\exp(2*c + 2*d*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d))/(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1) - (6*(3*a*b^2 + a^2*b + 2*b^3))/(5*d*(\exp(2*c + 2*d*x) + 1)) - (\exp(-2*c - 2*d*x)*(a + b)^3)/(8*d) - ((6*(3*a*b^2 + a^2*b + 2*b^3))/(5*d) + (8*\exp(2*c + 2*d*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (6*\exp(8*c + 8*d*x)*(3*a*b^2 + a^2*b + 2*b^3))/(5*d) + (8*\exp(6*c + 6*d*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (4*\exp(4*c + 4*d*x)*(15*a*b^2 + 9*a^2*b + 10*b^3))/(5*d))/(5*\exp(2*c + 2*d*x) + 10*\exp(4*c + 4*d*x) + 10*\exp(6*c + 6*d*x) + 5*\exp(8*c + 8*d*x) + \exp(10*c + 10*d*x) + 1) - (x*(a + b)^2*(a + 7*b))/2$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^3 \sinh^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)`

[Out] `Integral((a + b*tanh(c + d*x)**2)**3*sinh(c + d*x)**2, x)`

3.20 $\int \sinh(c + dx) (a + b \tanh^2(c + dx))^3 dx$

Optimal. Leaf size=70

$$-\frac{b^2(a+b)\operatorname{sech}^3(c+dx)}{d} + \frac{(a+b)^3 \cosh(c+dx)}{d} + \frac{3b(a+b)^2 \operatorname{sech}(c+dx)}{d} + \frac{b^3 \operatorname{sech}^5(c+dx)}{5d}$$

[Out] $(a+b)^3 \cosh(d*x+c)/d + 3*b*(a+b)^2 \operatorname{sech}(d*x+c)/d - b^2*(a+b)*\operatorname{sech}(d*x+c)^3/d + 1/5*b^3*\operatorname{sech}(d*x+c)^5/d$

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3664, 270}

$$-\frac{b^2(a+b)\operatorname{sech}^3(c+dx)}{d} + \frac{(a+b)^3 \cosh(c+dx)}{d} + \frac{3b(a+b)^2 \operatorname{sech}(c+dx)}{d} + \frac{b^3 \operatorname{sech}^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3, x]

[Out] $((a+b)^3 \cosh[c+d*x])/d + (3*b*(a+b)^2 \operatorname{Sech}[c+d*x])/d - (b^2*(a+b)*\operatorname{Sech}[c+d*x]^3)/d + (b^3*\operatorname{Sech}[c+d*x]^5)/(5*d)$

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 3664

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m-1)/2)*(a - b + b*ff^2*x^2)^p]/x^(m+1), x], x, Sec[e + f*x]/ff, x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m-1)/2]

Rubi steps

$$\begin{aligned} \int \sinh(c + dx) (a + b \tanh^2(c + dx))^3 dx &= -\frac{\operatorname{Subst}\left(\int \frac{(a+b-bx^2)^3}{x^2} dx, x, \operatorname{sech}(c + dx)\right)}{d} \\ &= -\frac{\operatorname{Subst}\left(\int \left(-3b(a+b)^2 + \frac{(a+b)^3}{x^2} + 3b^2(a+b)x^2 - b^3x^4\right) dx, x, \operatorname{sech}(c + dx)\right)}{d} \\ &= \frac{(a+b)^3 \cosh(c + dx)}{d} + \frac{3b(a+b)^2 \operatorname{sech}(c + dx)}{d} - \frac{b^2(a+b)\operatorname{sech}^3(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.88, size = 63, normalized size = 0.90

$$\frac{b \operatorname{sech}(c + dx) \left(-5b(a+b)\operatorname{sech}^2(c + dx) + 15(a+b)^2 + b^2 \operatorname{sech}^4(c + dx)\right) + 5(a+b)^3 \cosh(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3, x]

[Out] $(5*(a + b)^3*\text{Cosh}[c + d*x] + b*\text{Sech}[c + d*x]*(15*(a + b)^2 - 5*b*(a + b)*\text{Sech}[c + d*x]^2 + b^2*\text{Sech}[c + d*x]^4))/(5*d)$

fricas [B] time = 0.64, size = 383, normalized size = 5.47

$$5(a^3 + 3a^2b + 3ab^2 + b^3)\cosh(dx + c)^6 + 5(a^3 + 3a^2b + 3ab^2 + b^3)\sinh(dx + c)^6 + 30(a^3 + 5a^2b + 7ab^2 + 3b^3)\cosh(dx + c)^4\sinh(dx + c)^2 + 15(2a^3 + 10a^2b + 14a*b^2 + 6b^3 + 5(a^3 + 3a^2b + 3a*b^2 + b^3)\cosh(dx + c)^2)\sinh(dx + c)^4 + 50a^3 + 330a^2b + 430a*b^2 + 182b^3 + 5(15a^3 + 93a^2b + 125a*b^2 + 47b^3)\cosh(dx + c)^2 + 5(15(a^3 + 3a^2b + 3a*b^2 + b^3)\cosh(dx + c)^4 + 15a^3 + 93a^2b + 125a*b^2 + 47b^3 + 36(a^3 + 5a^2b + 7a*b^2 + 3b^3)\cosh(dx + c)^2)\sinh(dx + c)^2)/(d*\cosh(dx + c)^5 + 5*d*\cosh(dx + c)*\sinh(dx + c)^4 + 5*d*\cosh(dx + c)^3 + 5*(2*d*\cosh(dx + c)^3 + 3*d*\cosh(dx + c))*\sinh(dx + c)^2 + 10*d*\cosh(dx + c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")`

[Out] $1/10*(5*(a^3 + 3a^2b + 3a*b^2 + b^3)*\cosh(d*x + c)^6 + 5*(a^3 + 3a^2b + 3a*b^2 + b^3)*\sinh(d*x + c)^6 + 30*(a^3 + 5a^2b + 7a*b^2 + 3b^3)*\cosh(d*x + c)^4 + 15*(2a^3 + 10a^2b + 14a*b^2 + 6b^3 + 5*(a^3 + 3a^2b + 3a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 50*a^3 + 330*a^2*b + 430*a*b^2 + 182*b^3 + 5*(15*a^3 + 93*a^2*b + 125*a*b^2 + 47*b^3)*\cosh(d*x + c)^2 + 5*(15*(a^3 + 3a^2b + 3a*b^2 + b^3)*\cosh(d*x + c)^4 + 15a^3 + 93a^2b + 125a*b^2 + 47b^3 + 36*(a^3 + 5a^2b + 7a*b^2 + 3b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2)/(d*\cosh(d*x + c)^5 + 5*d*\cosh(d*x + c)*\sinh(d*x + c)^4 + 5*d*\cosh(d*x + c)^3 + 5*(2*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^2 + 10*d*\cosh(d*x + c))$

giac [B] time = 0.39, size = 322, normalized size = 4.60

$$5(a^3 + 3a^2b + 3ab^2 + b^3)e^{(-dx-c)} + 5(a^3e^{(dx+14c)} + 3a^2be^{(dx+14c)} + 3ab^2e^{(dx+14c)} + b^3e^{(dx+14c)})e^{(-13c)} + \frac{4(15a^2be^{(dx+14c)} + 15a^3e^{(dx+14c)} + 15b^3e^{(dx+14c)})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")`

[Out] $1/10*(5*(a^3 + 3a^2b + 3a*b^2 + b^3)*e^{(-d*x - c)} + 5*(a^3*e^{(d*x + 14*c)} + 3a^2*b*e^{(d*x + 14*c)} + 3a*b^2*e^{(d*x + 14*c)} + b^3*e^{(d*x + 14*c)})*e^{(-13*c)} + 4*(15*a^2*b*e^{(9*d*x + 9*c)} + 30*a*b^2*e^{(9*d*x + 9*c)} + 15*b^3*e^{(9*d*x + 9*c)} + 60*a^2*b*e^{(7*d*x + 7*c)} + 100*a*b^2*e^{(7*d*x + 7*c)} + 40*b^3*e^{(7*d*x + 7*c)} + 90*a^2*b*e^{(5*d*x + 5*c)} + 140*a*b^2*e^{(5*d*x + 5*c)} + 66*b^3*e^{(5*d*x + 5*c)} + 60*a^2*b*e^{(3*d*x + 3*c)} + 100*a*b^2*e^{(3*d*x + 3*c)} + 40*b^3*e^{(3*d*x + 3*c)} + 15*a^2*b*e^{(d*x + c)} + 30*a*b^2*e^{(d*x + c)} + 15*b^3*e^{(d*x + c)})/(e^{(2*d*x + 2*c)} + 1)^5/d$

maple [B] time = 0.23, size = 170, normalized size = 2.43

$$a^3 \cosh(dx + c) + 3a^2b \left(\frac{\sinh^2(dx+c)}{\cosh(dx+c)} + \frac{2}{\cosh(dx+c)} \right) + 3ab^2 \left(\frac{\sinh^4(dx+c)}{\cosh(dx+c)^3} + \frac{4(\sinh^2(dx+c))}{\cosh(dx+c)^3} + \frac{8}{3\cosh(dx+c)^3} \right) + b^3 \left(\frac{\sinh^6(dx+c)}{\cosh(dx+c)^5} + \frac{6\sinh^4(dx+c)}{\cosh(dx+c)^5} + \frac{8\sinh^2(dx+c)}{\cosh(dx+c)^5} + \frac{4}{\cosh(dx+c)^5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x)`

[Out] $1/d*(a^3*\cosh(d*x+c)+3a^2*b*(\sinh(d*x+c)^2/\cosh(d*x+c)+2/\cosh(d*x+c))+3a*b^2*(\sinh(d*x+c)^4/\cosh(d*x+c)^3+4*\sinh(d*x+c)^2/\cosh(d*x+c)^3+8/3/\cosh(d*x+c)^3)+b^3*(\sinh(d*x+c)^6/\cosh(d*x+c)^5+6*\sinh(d*x+c)^4/\cosh(d*x+c)^5+8*\sinh(d*x+c)^2/\cosh(d*x+c)^5+16/5/\cosh(d*x+c)^5))$

maxima [B] time = 0.33, size = 321, normalized size = 4.59

$$\frac{1}{10}b^3 \left(\frac{5e^{(-dx-c)}}{d} + \frac{85e^{(-2dx-2c)} + 210e^{(-4dx-4c)} + 314e^{(-6dx-6c)} + 185e^{(-8dx-8c)} + 65e^{(-10dx-10c)} + 5}{d(e^{(-dx-c)} + 5e^{(-3dx-3c)} + 10e^{(-5dx-5c)} + 10e^{(-7dx-7c)} + 5e^{(-9dx-9c)} + e^{(-11dx-11c)})} \right) + \frac{1}{2}ab^2 \left(\frac{3e^{(-dx-c)}}{d} + \frac{15e^{(-2dx-2c)} + 45e^{(-4dx-4c)} + 105e^{(-6dx-6c)} + 135e^{(-8dx-8c)} + 75e^{(-10dx-10c)} + 15}{d(e^{(-dx-c)} + 5e^{(-3dx-3c)} + 10e^{(-5dx-5c)} + 10e^{(-7dx-7c)} + 5e^{(-9dx-9c)} + e^{(-11dx-11c)})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $\frac{1}{10}b^3(5e^{-(d*x - c)}/d + (85e^{(-2*d*x - 2*c)} + 210e^{(-4*d*x - 4*c)} + 314e^{(-6*d*x - 6*c)} + 185e^{(-8*d*x - 8*c)} + 65e^{(-10*d*x - 10*c)} + 5)/(d * (e^{-(d*x - c)} + 5e^{(-3*d*x - 3*c)} + 10e^{(-5*d*x - 5*c)} + 10e^{(-7*d*x - 7*c)} + 5e^{(-9*d*x - 9*c)} + e^{(-11*d*x - 11*c)}))) + 1/2*a*b^2*(3e^{-(d*x - c)}/d + (33e^{(-2*d*x - 2*c)} + 41e^{(-4*d*x - 4*c)} + 27e^{(-6*d*x - 6*c)} + 3)/(d*(e^{-(d*x - c)} + 3e^{(-3*d*x - 3*c)} + 3e^{(-5*d*x - 5*c)} + e^{(-7*d*x - 7*c)}))) + 3/2*a^2*b*(e^{-(d*x - c)}/d + (5e^{(-2*d*x - 2*c)} + 1)/(d*(e^{-(d*x - c)} + e^{(-3*d*x - 3*c)}))) + a^3*cosh(d*x + c)/d$

mupad [B] time = 1.25, size = 308, normalized size = 4.40

$$\frac{e^{c+dx}(a+b)^3}{2d} + \frac{e^{-c-dx}(a+b)^3}{2d} + \frac{6e^{c+dx}(a^2b + 2ab^2 + b^3)}{d(e^{2c+2dx} + 1)} - \frac{64b^3e^{c+dx}}{5d(4e^{2c+2dx} + 6e^{4c+4dx} + 4e^{6c+6dx} + e^{8c+8dx})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^3,x)

[Out] $(\exp(c + d*x)*(a + b)^3)/(2*d) + (\exp(-c - d*x)*(a + b)^3)/(2*d) + (6*\exp(c + d*x)*(2*a*b^2 + a^2*b + b^3))/(d*(\exp(2*c + 2*d*x) + 1)) - (64*b^3*\exp(c + d*x))/(5*d*(4*\exp(2*c + 2*d*x) + 6*\exp(4*c + 4*d*x) + 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1)) + (8*\exp(c + d*x)*(5*a*b^2 + 9*b^3))/(5*d*(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1)) + (32*b^3*\exp(c + d*x))/(5*d*(5*\exp(2*c + 2*d*x) + 10*\exp(4*c + 4*d*x) + 10*\exp(6*c + 6*d*x) + 5*\exp(8*c + 8*d*x) + \exp(10*c + 10*d*x) + 1)) - (8*\exp(c + d*x)*(a*b^2 + b^3))/(d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^3 \sinh(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)

[Out] Integral((a + b*tanh(c + d*x)**2)**3*sinh(c + d*x), x)

3.21 $\int \operatorname{csch}(c + dx) \left(a + b \tanh^2(c + dx) \right)^3 dx$

Optimal. Leaf size=84

$$\frac{a^3 \tanh^{-1}(\cosh(c + dx))}{d} - \frac{b(3a^2 + 3ab + b^2) \operatorname{sech}(c + dx)}{d} + \frac{b^2(3a + 2b) \operatorname{sech}^3(c + dx)}{3d} - \frac{b^3 \operatorname{sech}^5(c + dx)}{5d}$$

[Out] $-a^3 \operatorname{arctanh}(\cosh(dx+c))/d - b*(3*a^2+3*a*b+b^2)*\operatorname{sech}(dx+c)/d + 1/3*b^2*(3*a+2*b)*\operatorname{sech}(dx+c)^3/d - 1/5*b^3*\operatorname{sech}(dx+c)^5/d$

Rubi [A] time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3664, 390, 207}

$$\frac{b(3a^2 + 3ab + b^2) \operatorname{sech}(c + dx)}{d} - \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b^2(3a + 2b) \operatorname{sech}^3(c + dx)}{3d} - \frac{b^3 \operatorname{sech}^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]*(a + b*\operatorname{Tanh}[c + d*x]^2)^3, x]$

[Out] $-((a^3*\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]])/d) - (b*(3*a^2 + 3*a*b + b^2)*\operatorname{Sech}[c + d*x])/d + (b^2*(3*a + 2*b)*\operatorname{Sech}[c + d*x]^3)/(3*d) - (b^3*\operatorname{Sech}[c + d*x]^5)/(5*d)$

Rule 207

$\operatorname{Int}[(a_+) + (b_+)*(x_+)^2]^{-1}, x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Rt}[b, 2]*x]/\operatorname{Rt}[-a, 2]]/\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2], x] /; \operatorname{FreeQ}\{a, b, x\} \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{LtQ}[a, 0] \mid\mid \operatorname{GtQ}[b, 0])$

Rule 390

$\operatorname{Int}[(a_+) + (b_+)*(x_+)^{n_+}]^{p_+}*((c_+) + (d_+)*(x_+)^{n_+})^{q_+}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{PolynomialDivide}[(a + b*x^n)^p, (c + d*x^n)^{-q}], x] /; \operatorname{FreeQ}\{a, b, c, d, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{IGtQ}[n, 0] \&\& \operatorname{IGtQ}[p, 0] \&\& \operatorname{ILtQ}[q, 0] \&\& \operatorname{GeQ}[p, -q]$

Rule 3664

$\operatorname{Int}[\sin[(e_+) + (f_+)*(x_+)]^{m_+}*((a_+) + (b_+)*\tan[(e_+) + (f_+)*(x_+)]^2)^{p_+}, x_Symbol] \rightarrow \operatorname{With}\{ff = \operatorname{FreeFactors}[\operatorname{Sec}[e + f*x], x]\}, \operatorname{Dist}[1/(f*ff^m), \operatorname{Subst}[\operatorname{Int}[((-1 + ff^2*x^2)^{(m-1)/2}*(a - b + b*ff^2*x^2)^p)/x^{m+1}], x], x, \operatorname{Sec}[e + f*x]/ff], x] /; \operatorname{FreeQ}\{a, b, e, f, p, x\} \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rubi steps

$$\begin{aligned} \int \operatorname{csch}(c + dx) \left(a + b \tanh^2(c + dx) \right)^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+b-x^2)^3}{-1+x^2} dx, x, \operatorname{sech}(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(-b(3a^2 + 3ab + b^2) + b^2(3a + 2b)x^2 - b^3x^4 + \frac{a^3}{-1+x^2}\right) dx, x, \operatorname{sech}(c + dx)\right)}{d} \\ &= -\frac{b(3a^2 + 3ab + b^2) \operatorname{sech}(c + dx)}{d} + \frac{b^2(3a + 2b) \operatorname{sech}^3(c + dx)}{3d} - \frac{b^3 \operatorname{sech}^5(c + dx)}{5d} \\ &= -\frac{a^3 \tanh^{-1}(\cosh(c + dx))}{d} - \frac{b(3a^2 + 3ab + b^2) \operatorname{sech}(c + dx)}{d} + \frac{b^2(3a + 2b) \operatorname{sech}^3(c + dx)}{3d} - \frac{b^3 \operatorname{sech}^5(c + dx)}{5d} \end{aligned}$$

$$2 + 10*(a^3*\cosh(d*x + c)^9 + 4*a^3*\cosh(d*x + c)^7 + 6*a^3*\cosh(d*x + c)^5 + 4*a^3*\cosh(d*x + c)^3 + a^3*\cosh(d*x + c))*\sinh(d*x + c))\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) + 10*(27*(3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^8 + 28*(9*a^2*b + 6*a*b^2 + b^3)*\cosh(d*x + c)^6 + 2*(135*a^2*b + 75*a*b^2 + 29*b^3)*\cosh(d*x + c)^4 + 9*a^2*b + 9*a*b^2 + 3*b^3 + 12*(9*a^2*b + 6*a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c))/(d*\cosh(d*x + c)^10 + 10*d*\cosh(d*x + c)*\sinh(d*x + c)^9 + d*\sinh(d*x + c)^10 + 5*d*\cosh(d*x + c)^8 + 5*(9*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^8 + 40*(3*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 10*d*\cosh(d*x + c)^6 + 10*(21*d*\cosh(d*x + c)^4 + 14*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^6 + 4*(63*d*\cosh(d*x + c)^5 + 70*d*\cosh(d*x + c)^3 + 15*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 10*d*\cosh(d*x + c)^4 + 10*(21*d*\cosh(d*x + c)^6 + 35*d*\cosh(d*x + c)^4 + 15*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^4 + 40*(3*d*\cosh(d*x + c)^7 + 7*d*\cosh(d*x + c)^5 + 5*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 5*d*\cosh(d*x + c)^2 + 5*(9*d*\cosh(d*x + c)^8 + 28*d*\cosh(d*x + c)^6 + 30*d*\cosh(d*x + c)^4 + 12*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^2 + 10*(d*\cosh(d*x + c)^9 + 4*d*\cosh(d*x + c)^7 + 6*d*\cosh(d*x + c)^5 + 4*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c) + d)$$

giac [B] time = 0.37, size = 262, normalized size = 3.12

$$15 a^3 \log(e^{(dx+c)} + 1) - 15 a^3 \log(|e^{(dx+c)} - 1|) + \frac{2(45 a^2 b e^{(9 dx+9 c)} + 45 a b^2 e^{(9 dx+9 c)} + 15 b^3 e^{(9 dx+9 c)} + 180 a^2 b e^{(7 dx+7 c)} + 120 a b^2 e^{(7 dx+7 c)})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out] -1/15*(15*a^3*log(e^(d*x + c) + 1) - 15*a^3*log(abs(e^(d*x + c) - 1)) + 2*(45*a^2*b*e^(9*d*x + 9*c) + 45*a*b^2*e^(9*d*x + 9*c) + 15*b^3*e^(9*d*x + 9*c) + 180*a^2*b*e^(7*d*x + 7*c) + 120*a*b^2*e^(7*d*x + 7*c) + 20*b^3*e^(7*d*x + 7*c) + 270*a^2*b*e^(5*d*x + 5*c) + 150*a*b^2*e^(5*d*x + 5*c) + 58*b^3*e^(5*d*x + 5*c) + 180*a^2*b*e^(3*d*x + 3*c) + 120*a*b^2*e^(3*d*x + 3*c) + 20*b^3*e^(3*d*x + 3*c) + 45*a^2*b*e^(d*x + c) + 45*a*b^2*e^(d*x + c) + 15*b^3*e^(d*x + c)))/(e^(2*d*x + 2*c) + 1)^5)/d

maple [A] time = 0.24, size = 118, normalized size = 1.40

$$\frac{-2a^3 \operatorname{arctanh}(e^{dx+c}) - \frac{3a^2b}{\cosh(dx+c)} + 3ab^2 \left(-\frac{\sinh^2(dx+c)}{\cosh(dx+c)^3} - \frac{2}{3\cosh(dx+c)^3} \right) + b^3 \left(-\frac{\sinh^4(dx+c)}{\cosh(dx+c)^5} - \frac{4(\sinh^2(dx+c))}{3\cosh(dx+c)^5} - \frac{8}{15\cosh(dx+c)^5} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x)

[Out] 1/d*(-2*a^3*arctanh(exp(d*x+c))-3*a^2*b/cosh(d*x+c)+3*a*b^2*(-sinh(d*x+c)^2/cosh(d*x+c)^3-2/3/cosh(d*x+c)^3)+b^3*(-sinh(d*x+c)^4/cosh(d*x+c)^5-4/3*sinh(d*x+c)^2/cosh(d*x+c)^5-8/15/cosh(d*x+c)^5))

maxima [B] time = 0.33, size = 560, normalized size = 6.67

$$-\frac{2}{15} b^3 \left(\frac{15 e^{(-dx-c)}}{d(5 e^{(-2dx-2c)} + 10 e^{(-4dx-4c)} + 10 e^{(-6dx-6c)} + 5 e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} + \frac{1}{d(5 e^{(-2dx-2c)} + 10 e^{(-4dx-4c)} + 10 e^{(-6dx-6c)} + 5 e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")


```
[Out] -2/15*b^3*(15*e^(-d*x - c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 20*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 58*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 20*e^(-7*d*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-9*d*x - 9*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) - 2*a*b^2*(3*e^(-d*x - c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 2*e^(-3*d*x - 3*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 3*e^(-5*d*x - 5*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + a^3*log(tanh(1/2*d*x + 1/2*c))/d - 6*a^2*b/(d*(e^(d*x + c) + e^(-d*x - c)))
```

mupad [B] time = 0.25, size = 317, normalized size = 3.77

$$\frac{8e^{c+dx} (2b^3 + 3ab^2)}{3d (2e^{2c+2dx} + e^{4c+4dx} + 1)} - \frac{2 \operatorname{atan}\left(\frac{a^3 e^{dx} e^c \sqrt{-d^2}}{d \sqrt{a^6}}\right) \sqrt{a^6}}{\sqrt{-d^2}} - \frac{2e^{c+dx} (3a^2b + 3ab^2 + b^3)}{d (e^{2c+2dx} + 1)} + \frac{1}{5d (4e^{2c+2dx} + 6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x))^2)^3/sinh(c + d*x), x)
```

```
[Out] (8*exp(c + d*x)*(3*a*b^2 + 2*b^3))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (2*atan((a^3*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^6)^(1/2)))*(a^6)^(1/2))/(-d^2)^(1/2) - (2*exp(c + d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(d*(exp(2*c + 2*d*x) + 1)) + (64*b^3*exp(c + d*x))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (8*exp(c + d*x)*(15*a*b^2 + 22*b^3))/(15*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (32*b^3*exp(c + d*x))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^3 \operatorname{csch}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**2)**3, x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*csch(c + d*x), x)
```

3.22 $\int \operatorname{csch}^2(c + dx) \left(a + b \tanh^2(c + dx) \right)^3 dx$

Optimal. Leaf size=64

$$-\frac{a^3 \operatorname{coth}(c + dx)}{d} + \frac{3a^2 b \tanh(c + dx)}{d} + \frac{ab^2 \tanh^3(c + dx)}{d} + \frac{b^3 \tanh^5(c + dx)}{5d}$$

[Out] $-a^3 \operatorname{coth}(d*x+c)/d + 3*a^2*b*\tanh(d*x+c)/d + a*b^2*\tanh(d*x+c)^3/d + 1/5*b^3*\tanh(d*x+c)^5/d$

Rubi [A] time = 0.06, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3663, 270}

$$\frac{3a^2 b \tanh(c + dx)}{d} - \frac{a^3 \operatorname{coth}(c + dx)}{d} + \frac{ab^2 \tanh^3(c + dx)}{d} + \frac{b^3 \tanh^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] $-((a^3 \operatorname{Coth}[c + d*x])/d) + (3*a^2*b*\operatorname{Tanh}[c + d*x])/d + (a*b^2*\operatorname{Tanh}[c + d*x]^3)/d + (b^3*\operatorname{Tanh}[c + d*x]^5)/(5*d)$

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 3663

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_.)]^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^2(c + dx) \left(a + b \tanh^2(c + dx) \right)^3 dx &= \frac{\operatorname{Subst} \left(\int \frac{(a+bx^2)^3}{x^2} dx, x, \tanh(c + dx) \right)}{d} \\ &= \frac{\operatorname{Subst} \left(\int \left(3a^2b + \frac{a^3}{x^2} + 3ab^2x^2 + b^3x^4 \right) dx, x, \tanh(c + dx) \right)}{d} \\ &= -\frac{a^3 \operatorname{coth}(c + dx)}{d} + \frac{3a^2 b \tanh(c + dx)}{d} + \frac{ab^2 \tanh^3(c + dx)}{d} + \frac{b^3 \tanh^5(c + dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.71, size = 70, normalized size = 1.09

$$\frac{b \tanh(c + dx) \left(15a^2 - b(5a + 2b) \operatorname{sech}^2(c + dx) + 5ab + b^2 \operatorname{sech}^4(c + dx) + b^2 \right) - 5a^3 \operatorname{coth}(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] $(-5a^3 \operatorname{Coth}[c + dx] + b(15a^2 + 5ab + b^2 - b(5a + 2b) \operatorname{Sech}[c + dx])^2 + b^2 \operatorname{Sech}[c + dx]^4) \operatorname{Tanh}[c + dx] / (5d)$

fricas [B] time = 0.49, size = 572, normalized size = 8.94

$$\frac{4 \left((5a^3 + 5ab^2 + 2b^3) \cosh(dx + c)^5 + 5(5a^3 + 5ab^2 + 2b^3) \cosh(dx + c) \sinh(dx + c)^4 + (15a^2b + 10ab^2 + 3b^3) \sinh(dx + c)^5 + (25a^3 + 5ab^2 - 2b^3) \cosh(dx + c)^3 + (45a^2b + 10ab^2 - 3b^3 + 10(15a^2b + 10ab^2 + 3b^3) \cosh(dx + c)^2) \sinh(dx + c)^3 + (10(5a^3 + 5ab^2 + 2b^3) \cosh(dx + c)^3 + 3(25a^3 + 5ab^2 - 2b^3) \cosh(dx + c)) \sinh(dx + c)^2 + 10(5a^3 - ab^2) \cosh(dx + c) + (5(15a^2b + 10ab^2 + 3b^3) \cosh(dx + c)^4 + 30a^2b + 10b^3 + 3(45a^2b + 10ab^2 - 3b^3) \cosh(dx + c)^2) \sinh(dx + c) \right)}{5 \left(d \cosh(dx + c)^7 + 7d \cosh(dx + c) \sinh(dx + c)^6 + d \sinh(dx + c)^7 + 3d \cosh(dx + c)^5 + 21d \cosh(dx + c)^2 + 5d \right) \sinh(dx + c)^5 + 5(7d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^4 + d \cosh(dx + c)^3 + (35d \cosh(dx + c)^4 + 50d \cosh(dx + c)^2 + 9d) \sinh(dx + c)^3 + 3(7d \cosh(dx + c)^5 + 10d \cosh(dx + c)^3 + d \cosh(dx + c)) \sinh(dx + c)^2 - 5d \cosh(dx + c) + (7d \cosh(dx + c)^6 + 25d \cosh(dx + c)^4 + 27d \cosh(dx + c)^2 + 5d) \sinh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")`

[Out] $-4/5 * ((5a^3 + 5ab^2 + 2b^3) \cosh(dx + c)^5 + 5(5a^3 + 5ab^2 + 2b^3) \cosh(dx + c) \sinh(dx + c)^4 + (15a^2b + 10ab^2 + 3b^3) \sinh(dx + c)^5 + (25a^3 + 5ab^2 - 2b^3) \cosh(dx + c)^3 + (45a^2b + 10ab^2 - 3b^3 + 10(15a^2b + 10ab^2 + 3b^3) \cosh(dx + c)^2) \sinh(dx + c)^3 + (10(5a^3 + 5ab^2 + 2b^3) \cosh(dx + c)^3 + 3(25a^3 + 5ab^2 - 2b^3) \cosh(dx + c)) \sinh(dx + c)^2 + 10(5a^3 - ab^2) \cosh(dx + c) + (5(15a^2b + 10ab^2 + 3b^3) \cosh(dx + c)^4 + 30a^2b + 10b^3 + 3(45a^2b + 10ab^2 - 3b^3) \cosh(dx + c)^2) \sinh(dx + c)) / (d \cosh(dx + c)^7 + 7d \cosh(dx + c) \sinh(dx + c)^6 + d \sinh(dx + c)^7 + 3d \cosh(dx + c)^5 + (21d \cosh(dx + c)^2 + 5d) \sinh(dx + c)^5 + 5(7d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^4 + d \cosh(dx + c)^3 + (35d \cosh(dx + c)^4 + 50d \cosh(dx + c)^2 + 9d) \sinh(dx + c)^3 + 3(7d \cosh(dx + c)^5 + 10d \cosh(dx + c)^3 + d \cosh(dx + c)) \sinh(dx + c)^2 - 5d \cosh(dx + c) + (7d \cosh(dx + c)^6 + 25d \cosh(dx + c)^4 + 27d \cosh(dx + c)^2 + 5d) \sinh(dx + c))$

giac [B] time = 0.44, size = 202, normalized size = 3.16

$$\frac{2 \left(\frac{5a^3}{e^{(2dx+2c)-1}} + \frac{15a^2be^{(8dx+8c)} + 15ab^2e^{(8dx+8c)} + 5b^3e^{(8dx+8c)} + 60a^2be^{(6dx+6c)} + 30ab^2e^{(6dx+6c)} + 90a^2be^{(4dx+4c)} + 20ab^2e^{(4dx+4c)} + 10b^3e^{(4dx+4c)}}{(e^{(2dx+2c)+1})^5} \right)}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")`

[Out] $-2/5 * (5a^3 / (e^{(2dx+2c)} - 1) + (15a^2b e^{(8dx+8c)} + 15ab^2 e^{(8dx+8c)} + 5b^3 e^{(8dx+8c)} + 60a^2b e^{(6dx+6c)} + 30ab^2 e^{(6dx+6c)} + 90a^2b e^{(4dx+4c)} + 20ab^2 e^{(4dx+4c)} + 10b^3 e^{(4dx+4c)} + 60a^2b e^{(2dx+2c)} + 10ab^2 e^{(2dx+2c)} + 15a^2b + 5ab^2 + b^3) / (e^{(2dx+2c)} + 1)^5) / d$

maple [B] time = 0.51, size = 141, normalized size = 2.20

$$\frac{-a^3 \operatorname{coth}(dx + c) + 3a^2b \operatorname{tanh}(dx + c) + 3ab^2 \left(-\frac{\sinh(dx + c)}{2 \cosh(dx + c)^3} + \frac{\left(\frac{2}{3} + \frac{\operatorname{sech}(dx + c)^2}{3} \right) \operatorname{tanh}(dx + c)}{2} \right) + b^3 \left(-\frac{\sinh^3(dx + c)}{2 \cosh(dx + c)^5} - \frac{3}{8} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x)`

[Out] $1/d * (-a^3 \operatorname{coth}(dx + c) + 3a^2b \operatorname{tanh}(dx + c) + 3ab^2 * (-1/2 * \sinh(dx + c) / \cosh(dx + c)^3 + 1/2 * (2/3 + 1/3 * \operatorname{sech}(dx + c)^2) * \operatorname{tanh}(dx + c)) + b^3 * (-1/2 * \sinh(dx + c)^3 / \cosh(dx + c)^5 - 3/8 * \sinh(dx + c) / \cosh(dx + c)^5 + 3/8 * (8/15 + 1/5 * \operatorname{sech}(dx + c)^4 + 4/15 * \operatorname{sech}(dx + c)^2) * \operatorname{tanh}(dx + c)))$

3.23 $\int \operatorname{csch}^3(c + dx) \left(a + b \tanh^2(c + dx)\right)^3 dx$

Optimal. Leaf size=152

$$\frac{b(81a^2 - 28ab - 4b^2) \operatorname{sech}(c + dx)}{30d} + \frac{a^2(a - 6b) \tanh^{-1}(\cosh(c + dx))}{2d} + \frac{7b \operatorname{sech}(c + dx) (a - b \operatorname{sech}^2(c + dx))}{10d}$$

[Out] 1/2*a^2*(a-6*b)*arctanh(cosh(d*x+c))/d+1/30*b*(81*a^2-28*a*b-4*b^2)*sech(d*x+c)/d+1/30*(33*a-2*b)*b*sech(d*x+c)*(a+b-b*sech(d*x+c)^2)/d+7/10*b*sech(d*x+c)*(a+b-b*sech(d*x+c)^2)^2/d-1/2*coth(d*x+c)*csch(d*x+c)*(a+b-b*sech(d*x+c)^2)^3/d

Rubi [A] time = 0.23, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3664, 467, 528, 388, 207}

$$\frac{b(81a^2 - 28ab - 4b^2) \operatorname{sech}(c + dx)}{30d} + \frac{a^2(a - 6b) \tanh^{-1}(\cosh(c + dx))}{2d} + \frac{7b \operatorname{sech}(c + dx) (a - b \operatorname{sech}^2(c + dx))}{10d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (a^2*(a - 6*b)*ArcTanh[Cosh[c + d*x]]/(2*d) + (b*(81*a^2 - 28*a*b - 4*b^2)*Sech[c + d*x])/(30*d) + ((33*a - 2*b)*b*Sech[c + d*x]*(a + b - b*Sech[c + d*x]^2))/(30*d) + (7*b*Sech[c + d*x]*(a + b - b*Sech[c + d*x]^2)^2)/(10*d) - (Coth[c + d*x]*Csch[c + d*x]*(a + b - b*Sech[c + d*x]^2)^3)/(2*d)

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 467

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 528

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e - a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p]/x^(m + 1), x], x, Sec[e + f*x]/ff, x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^3(c + dx) (a + b \tanh^2(c + dx))^3 dx &= -\frac{\operatorname{Subst}\left(\int \frac{x^2(a+b-bx^2)^3}{(-1+x^2)^2} dx, x, \operatorname{sech}(c + dx)\right)}{d} \\ &= -\frac{\operatorname{coth}(c + dx)\operatorname{csch}(c + dx) (a + b - b\operatorname{sech}^2(c + dx))^3}{2d} - \frac{\operatorname{Subst}\left(\int \frac{x^2(a+b-bx^2)^3}{(-1+x^2)^2} dx, x, \operatorname{sech}(c + dx)\right)}{d} \\ &= \frac{7b\operatorname{sech}(c + dx) (a + b - b\operatorname{sech}^2(c + dx))^2}{10d} - \frac{\operatorname{coth}(c + dx)\operatorname{csch}(c + dx) (a + b - b\operatorname{sech}^2(c + dx))^3}{2d} \\ &= \frac{(33a - 2b)b\operatorname{sech}(c + dx) (a + b - b\operatorname{sech}^2(c + dx))}{30d} + \frac{7b\operatorname{sech}(c + dx) (a + b - b\operatorname{sech}^2(c + dx))^2}{10d} \\ &= \frac{b(81a^2 - 28ab - 4b^2)\operatorname{sech}(c + dx)}{30d} + \frac{(33a - 2b)b\operatorname{sech}(c + dx) (a + b - b\operatorname{sech}^2(c + dx))}{30d} \\ &= \frac{a^2(a - 6b)\tanh^{-1}(\cosh(c + dx))}{2d} + \frac{b(81a^2 - 28ab - 4b^2)\operatorname{sech}(c + dx)}{30d} \end{aligned}$$

Mathematica [A] time = 6.20, size = 127, normalized size = 0.84

$$-\frac{a^3\operatorname{csch}^2\left(\frac{1}{2}(c + dx)\right)}{8d} - \frac{a^3\operatorname{sech}^2\left(\frac{1}{2}(c + dx)\right)}{8d} + \frac{3a^2b\operatorname{sech}(c + dx)}{d} - \frac{a^2(a - 6b)\log\left(\tanh\left(\frac{1}{2}(c + dx)\right)\right)}{2d} - \frac{b^2(3a + b)\operatorname{sech}(c + dx)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] -1/8*(a^3*Csch[(c + d*x)/2]^2)/d - (a^2*(a - 6*b)*Log[Tanh[(c + d*x)/2]])/(2*d) - (a^3*Sech[(c + d*x)/2]^2)/(8*d) + (3*a^2*b*Sech[c + d*x])/d - (b^2*(3*a + b)*Sech[c + d*x]^3)/(3*d) + (b^3*Sech[c + d*x]^5)/(5*d)
```

fricas [B] time = 0.97, size = 5037, normalized size = 33.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/30*(30*(a^3 - 6*a^2*b)*cosh(d*x + c)^13 + 390*(a^3 - 6*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^12 + 30*(a^3 - 6*a^2*b)*sinh(d*x + c)^13 + 20*(9*a^3 - 18*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c)^11 + 20*(9*a^3 - 18*a^2*b + 12*a*b^2 + 4*b^3 + 117*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^11 + 220*(3*9*(a^3 - 6*a^2*b)*cosh(d*x + c)^3 + (9*a^3 - 18*a^2*b + 12*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^10 + 6*(75*a^3 + 30*a^2*b - 32*b^3)*cosh(d*x + c)^9 + 2*(10725*(a^3 - 6*a^2*b)*cosh(d*x + c)^4 + 225*a^3 + 90*a^2*b - 96*b^3)
```

$$\begin{aligned}
&^3 + 550*(9*a^3 - 18*a^2*b + 12*a*b^2 + 4*b^3)*\cosh(d*x + c)^2*\sinh(d*x + \\
&c)^9 + 6*(6435*(a^3 - 6*a^2*b)*\cosh(d*x + c)^5 + 550*(9*a^3 - 18*a^2*b + 12 \\
&*a*b^2 + 4*b^3)*\cosh(d*x + c)^3 + 9*(75*a^3 + 30*a^2*b - 32*b^3)*\cosh(d*x + \\
&c))*\sinh(d*x + c)^8 + 8*(75*a^3 + 90*a^2*b - 60*a*b^2 + 28*b^3)*\cosh(d*x + \\
&c)^7 + 8*(6435*(a^3 - 6*a^2*b)*\cosh(d*x + c)^6 + 825*(9*a^3 - 18*a^2*b + 1 \\
&2*a*b^2 + 4*b^3)*\cosh(d*x + c)^4 + 75*a^3 + 90*a^2*b - 60*a*b^2 + 28*b^3 + \\
&27*(75*a^3 + 30*a^2*b - 32*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^7 + 8*(6435* \\
&(a^3 - 6*a^2*b)*\cosh(d*x + c)^7 + 1155*(9*a^3 - 18*a^2*b + 12*a*b^2 + 4*b^3 \\
&)*\cosh(d*x + c)^5 + 63*(75*a^3 + 30*a^2*b - 32*b^3)*\cosh(d*x + c)^3 + 7*(75 \\
&*a^3 + 90*a^2*b - 60*a*b^2 + 28*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^6 + 6*(75 \\
&*a^3 + 30*a^2*b - 32*b^3)*\cosh(d*x + c)^5 + 6*(6435*(a^3 - 6*a^2*b)*\cosh(d* \\
&x + c)^8 + 1540*(9*a^3 - 18*a^2*b + 12*a*b^2 + 4*b^3)*\cosh(d*x + c)^6 + 126 \\
&*(75*a^3 + 30*a^2*b - 32*b^3)*\cosh(d*x + c)^4 + 75*a^3 + 30*a^2*b - 32*b^3 \\
&+ 28*(75*a^3 + 90*a^2*b - 60*a*b^2 + 28*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c) \\
&^5 + 2*(10725*(a^3 - 6*a^2*b)*\cosh(d*x + c)^9 + 3300*(9*a^3 - 18*a^2*b + 12 \\
&*a*b^2 + 4*b^3)*\cosh(d*x + c)^7 + 378*(75*a^3 + 30*a^2*b - 32*b^3)*\cosh(d*x \\
&+ c)^5 + 140*(75*a^3 + 90*a^2*b - 60*a*b^2 + 28*b^3)*\cosh(d*x + c)^3 + 15* \\
&(75*a^3 + 30*a^2*b - 32*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^4 + 20*(9*a^3 - 1 \\
&8*a^2*b + 12*a*b^2 + 4*b^3)*\cosh(d*x + c)^3 + 4*(2145*(a^3 - 6*a^2*b)*\cosh(\\
&d*x + c)^10 + 825*(9*a^3 - 18*a^2*b + 12*a*b^2 + 4*b^3)*\cosh(d*x + c)^8 + 1 \\
&26*(75*a^3 + 30*a^2*b - 32*b^3)*\cosh(d*x + c)^6 + 70*(75*a^3 + 90*a^2*b - 6 \\
&0*a*b^2 + 28*b^3)*\cosh(d*x + c)^4 + 45*a^3 - 90*a^2*b + 60*a*b^2 + 20*b^3 + \\
&15*(75*a^3 + 30*a^2*b - 32*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^3 + 4*(585* \\
&(a^3 - 6*a^2*b)*\cosh(d*x + c)^11 + 275*(9*a^3 - 18*a^2*b + 12*a*b^2 + 4*b^3 \\
&)*\cosh(d*x + c)^9 + 54*(75*a^3 + 30*a^2*b - 32*b^3)*\cosh(d*x + c)^7 + 42*(7 \\
&5*a^3 + 90*a^2*b - 60*a*b^2 + 28*b^3)*\cosh(d*x + c)^5 + 15*(75*a^3 + 30*a^2 \\
&*b - 32*b^3)*\cosh(d*x + c)^3 + 15*(9*a^3 - 18*a^2*b + 12*a*b^2 + 4*b^3)*\cos \\
&h(d*x + c))*\sinh(d*x + c)^2 + 30*(a^3 - 6*a^2*b)*\cosh(d*x + c) - 15*((a^3 - \\
&6*a^2*b)*\cosh(d*x + c)^14 + 14*(a^3 - 6*a^2*b)*\cosh(d*x + c)*\sinh(d*x + c) \\
&^13 + (a^3 - 6*a^2*b)*\sinh(d*x + c)^14 + 3*(a^3 - 6*a^2*b)*\cosh(d*x + c)^12 \\
&+ (3*a^3 - 18*a^2*b + 91*(a^3 - 6*a^2*b)*\cosh(d*x + c)^2)*\sinh(d*x + c)^12 \\
&+ 4*(91*(a^3 - 6*a^2*b)*\cosh(d*x + c)^3 + 9*(a^3 - 6*a^2*b)*\cosh(d*x + c)) \\
&*\sinh(d*x + c)^11 + (a^3 - 6*a^2*b)*\cosh(d*x + c)^10 + (1001*(a^3 - 6*a^2*b \\
&)*\cosh(d*x + c)^4 + a^3 - 6*a^2*b + 198*(a^3 - 6*a^2*b)*\cosh(d*x + c)^2)*\si \\
&nh(d*x + c)^10 + 2*(1001*(a^3 - 6*a^2*b)*\cosh(d*x + c)^5 + 330*(a^3 - 6*a^2 \\
&*b)*\cosh(d*x + c)^3 + 5*(a^3 - 6*a^2*b)*\cosh(d*x + c))*\sinh(d*x + c)^9 - 5* \\
&(a^3 - 6*a^2*b)*\cosh(d*x + c)^8 + (3003*(a^3 - 6*a^2*b)*\cosh(d*x + c)^6 + 1 \\
&485*(a^3 - 6*a^2*b)*\cosh(d*x + c)^4 - 5*a^3 + 30*a^2*b + 45*(a^3 - 6*a^2*b) \\
&*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 8*(429*(a^3 - 6*a^2*b)*\cosh(d*x + c)^7 \\
&+ 297*(a^3 - 6*a^2*b)*\cosh(d*x + c)^5 + 15*(a^3 - 6*a^2*b)*\cosh(d*x + c)^3 \\
&- 5*(a^3 - 6*a^2*b)*\cosh(d*x + c))*\sinh(d*x + c)^7 - 5*(a^3 - 6*a^2*b)*\cosh \\
&(d*x + c)^6 + (3003*(a^3 - 6*a^2*b)*\cosh(d*x + c)^8 + 2772*(a^3 - 6*a^2*b)* \\
&\cosh(d*x + c)^6 + 210*(a^3 - 6*a^2*b)*\cosh(d*x + c)^4 - 5*a^3 + 30*a^2*b - \\
&140*(a^3 - 6*a^2*b)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 2*(1001*(a^3 - 6*a^2 \\
&*b)*\cosh(d*x + c)^9 + 1188*(a^3 - 6*a^2*b)*\cosh(d*x + c)^7 + 126*(a^3 - 6*a \\
&^2*b)*\cosh(d*x + c)^5 - 140*(a^3 - 6*a^2*b)*\cosh(d*x + c)^3 - 15*(a^3 - 6*a \\
&^2*b)*\cosh(d*x + c))*\sinh(d*x + c)^5 + (a^3 - 6*a^2*b)*\cosh(d*x + c)^4 + (1 \\
&001*(a^3 - 6*a^2*b)*\cosh(d*x + c)^10 + 1485*(a^3 - 6*a^2*b)*\cosh(d*x + c)^8 \\
&+ 210*(a^3 - 6*a^2*b)*\cosh(d*x + c)^6 - 350*(a^3 - 6*a^2*b)*\cosh(d*x + c)^ \\
&4 + a^3 - 6*a^2*b - 75*(a^3 - 6*a^2*b)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 4 \\
&*(91*(a^3 - 6*a^2*b)*\cosh(d*x + c)^11 + 165*(a^3 - 6*a^2*b)*\cosh(d*x + c)^9 \\
&+ 30*(a^3 - 6*a^2*b)*\cosh(d*x + c)^7 - 70*(a^3 - 6*a^2*b)*\cosh(d*x + c)^5 \\
&- 25*(a^3 - 6*a^2*b)*\cosh(d*x + c)^3 + (a^3 - 6*a^2*b)*\cosh(d*x + c))*\sinh(\\
&d*x + c)^3 + a^3 - 6*a^2*b + 3*(a^3 - 6*a^2*b)*\cosh(d*x + c)^2 + (91*(a^3 - \\
&6*a^2*b)*\cosh(d*x + c)^12 + 198*(a^3 - 6*a^2*b)*\cosh(d*x + c)^10 + 45*(a^3 \\
&- 6*a^2*b)*\cosh(d*x + c)^8 - 140*(a^3 - 6*a^2*b)*\cosh(d*x + c)^6 - 75*(a^3 \\
&- 6*a^2*b)*\cosh(d*x + c)^4 + 3*a^3 - 18*a^2*b + 6*(a^3 - 6*a^2*b)*\cosh(d*x \\
&+ c)^2)*\sinh(d*x + c)^2 + 2*(7*(a^3 - 6*a^2*b)*\cosh(d*x + c)^13 + 18*(a^3 \\
&- 6*a^2*b)*\cosh(d*x + c)^11 + 5*(a^3 - 6*a^2*b)*\cosh(d*x + c)^9 - 20*(a^3 -
\end{aligned}$$

$*\cosh(d*x + c)^4 + 6*d*\cosh(d*x + c)^2 + 3*d)*\sinh(d*x + c)^2 + 2*(7*d*\cosh(d*x + c)^{13} + 18*d*\cosh(d*x + c)^{11} + 5*d*\cosh(d*x + c)^9 - 20*d*\cosh(d*x + c)^7 - 15*d*\cosh(d*x + c)^5 + 2*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c) + d$

giac [A] time = 0.50, size = 281, normalized size = 1.85

$$15(a^3 e^c - 6 a^2 b e^c) e^{(-c)} \log(e^{(dx+c)} + 1) - 15(a^3 e^c - 6 a^2 b e^c) e^{(-c)} \log(|e^{(dx+c)} - 1|) - \frac{30(a^3 e^{(3dx+3c)} + a^3 e^{(dx+c)})}{(e^{(2dx+2c)} - 1)^2} + \frac{4}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out] $\frac{1}{30}*(15*(a^3 e^c - 6 a^2 b e^c) e^{(-c)} * \log(e^{(d*x + c)} + 1) - 15*(a^3 e^c - 6 a^2 b e^c) e^{(-c)} * \log(\text{abs}(e^{(d*x + c)} - 1)) - 30*(a^3 e^{(3*d*x + 3*c)} + a^3 e^{(d*x + c)}) / (e^{(2*d*x + 2*c)} - 1)^2 + 4*(45*a^2*b*e^{(9*d*x + 9*c)} + 180*a^2*b*e^{(7*d*x + 7*c)} - 60*a*b^2*e^{(7*d*x + 7*c)} - 20*b^3*e^{(7*d*x + 7*c)} + 270*a^2*b*e^{(5*d*x + 5*c)} - 120*a*b^2*e^{(5*d*x + 5*c)} + 8*b^3*e^{(5*d*x + 5*c)} + 180*a^2*b*e^{(3*d*x + 3*c)} - 60*a*b^2*e^{(3*d*x + 3*c)} - 20*b^3*e^{(3*d*x + 3*c)} + 45*a^2*b*e^{(d*x + c)}) / (e^{(2*d*x + 2*c)} + 1)^5) / d$

maple [A] time = 0.39, size = 103, normalized size = 0.68

$$a^3 \left(-\frac{\text{csch}(dx+c) \text{coth}(dx+c)}{2} + \text{arctanh}(e^{dx+c}) \right) + 3a^2 b \left(\frac{1}{\cosh(dx+c)} - 2 \text{arctanh}(e^{dx+c}) \right) - \frac{ab^2}{\cosh(dx+c)^3} + b^3 \left(-\frac{\sinh^2(dx+c)}{3 \cosh(dx+c)} \right) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x)

[Out] $\frac{1}{d}*(a^3*(-1/2*\text{csch}(d*x+c)*\text{coth}(d*x+c)+\text{arctanh}(\exp(d*x+c)))+3*a^2*b*(1/\cosh(d*x+c)-2*\text{arctanh}(\exp(d*x+c)))-a*b^2/\cosh(d*x+c)^3+b^3*(-1/3*\sinh(d*x+c)^2/\cosh(d*x+c)^5-2/15/\cosh(d*x+c)^5))$

maxima [B] time = 0.33, size = 403, normalized size = 2.65

$$\frac{1}{2} a^3 \left(\frac{\log(e^{(-dx-c)} + 1)}{d} - \frac{\log(e^{(-dx-c)} - 1)}{d} + \frac{2(e^{(-dx-c)} + e^{(-3dx-3c)})}{d(2e^{(-2dx-2c)} - e^{(-4dx-4c)} - 1)} \right) - 3a^2 b \left(\frac{\log(e^{(-dx-c)} + 1)}{d} - \frac{\log(e^{(-dx-c)} - 1)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $\frac{1}{2} a^3 * (\log(e^{(-d*x - c)} + 1) / d - \log(e^{(-d*x - c)} - 1) / d + 2 * (e^{(-d*x - c)} + e^{(-3*d*x - 3*c)}) / (d * (2 * e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)} - 1))) - 3 * a^2 * b * (\log(e^{(-d*x - c)} + 1) / d - \log(e^{(-d*x - c)} - 1) / d - 2 * e^{(-d*x - c)} / (d * (e^{(-2*d*x - 2*c)} + 1))) - 8 / 15 * b^3 * (5 * e^{(-3*d*x - 3*c)} / (d * (5 * e^{(-2*d*x - 2*c)} + 10 * e^{(-4*d*x - 4*c)} + 10 * e^{(-6*d*x - 6*c)} + 5 * e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)) - 2 * e^{(-5*d*x - 5*c)} / (d * (5 * e^{(-2*d*x - 2*c)} + 10 * e^{(-4*d*x - 4*c)} + 10 * e^{(-6*d*x - 6*c)} + 5 * e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)) + 5 * e^{(-7*d*x - 7*c)} / (d * (5 * e^{(-2*d*x - 2*c)} + 10 * e^{(-4*d*x - 4*c)} + 10 * e^{(-6*d*x - 6*c)} + 5 * e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1))) - 8 * a * b^2 / (d * (e^{(d*x + c)} + e^{(-d*x - c)}))^3$

mupad [B] time = 1.26, size = 412, normalized size = 2.71

$$\frac{\text{atan}\left(\frac{e^{dx} e^c (a^3 \sqrt{-d^2} - 6 a^2 b \sqrt{-d^2})}{d \sqrt{a^6 - 12 a^5 b + 36 a^4 b^2}}\right) \sqrt{a^6 - 12 a^5 b + 36 a^4 b^2}}{\sqrt{-d^2}} - \frac{64 b^3 e^{c+dx}}{5 d (4 e^{2c+2dx} + 6 e^{4c+4dx} + 4 e^{6c+6dx} + e^{8c+8dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*tanh(c + d*x)^2)^3/sinh(c + d*x)^3,x)`

[Out] $(\operatorname{atan}((\exp(d*x)*\exp(c)*(a^3*(-d^2)^{(1/2)} - 6*a^2*b*(-d^2)^{(1/2)}))/(d*(a^6 - 12*a^5*b + 36*a^4*b^2)^{(1/2)}))*(a^6 - 12*a^5*b + 36*a^4*b^2)^{(1/2)})/(-d^2)^{(1/2)} - (64*b^3*\exp(c + d*x))/(5*d*(4*\exp(2*c + 2*d*x) + 6*\exp(4*c + 4*d*x) + 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1)) + (8*\exp(c + d*x)*(15*a*b^2 + 17*b^3))/(15*d*(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1)) + (32*b^3*\exp(c + d*x))/(5*d*(5*\exp(2*c + 2*d*x) + 10*\exp(4*c + 4*d*x) + 10*\exp(6*c + 6*d*x) + 5*\exp(8*c + 8*d*x) + \exp(10*c + 10*d*x) + 1)) - (a^3*\exp(c + d*x))/(d*(\exp(2*c + 2*d*x) - 1)) - (8*\exp(c + d*x)*(3*a*b^2 + b^3))/(3*d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1)) - (2*a^3*\exp(c + d*x))/(d*(\exp(4*c + 4*d*x) - 2*\exp(2*c + 2*d*x) + 1)) + (6*a^2*b*\exp(c + d*x))/(d*(\exp(2*c + 2*d*x) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^3 \operatorname{csch}^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)`

[Out] `Integral((a + b*tanh(c + d*x)**2)**3*csch(c + d*x)**3, x)`

3.24 $\int \operatorname{csch}^4(c + dx) \left(a + b \tanh^2(c + dx) \right)^3 dx$

Optimal. Leaf size=98

$$-\frac{a^3 \operatorname{coth}^3(c + dx)}{3d} + \frac{a^2(a - 3b) \operatorname{coth}(c + dx)}{d} - \frac{b^2(3a - b) \tanh^3(c + dx)}{3d} - \frac{3ab(a - b) \tanh(c + dx)}{d} - \frac{b^3 \tanh^5(c + dx)}{5d}$$

[Out] $a^2(a-3b)*\operatorname{coth}(d*x+c)/d-1/3*a^3*\operatorname{coth}(d*x+c)^3/d-3*a*(a-b)*b*\tanh(d*x+c)/d-1/3*(3*a-b)*b^2*\tanh(d*x+c)^3/d-1/5*b^3*\tanh(d*x+c)^5/d$

Rubi [A] time = 0.10, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3663, 448}

$$\frac{a^2(a - 3b) \operatorname{coth}(c + dx)}{d} - \frac{a^3 \operatorname{coth}^3(c + dx)}{3d} - \frac{b^2(3a - b) \tanh^3(c + dx)}{3d} - \frac{3ab(a - b) \tanh(c + dx)}{d} - \frac{b^3 \tanh^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3, x]

[Out] $(a^2*(a - 3*b)*\operatorname{Coth}[c + d*x])/d - (a^3*\operatorname{Coth}[c + d*x]^3)/(3*d) - (3*a*(a - b)*b*\operatorname{Tanh}[c + d*x])/d - ((3*a - b)*b^2*\operatorname{Tanh}[c + d*x]^3)/(3*d) - (b^3*\operatorname{Tanh}[c + d*x]^5)/(5*d)$

Rule 448

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 3663

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^4(c + dx) \left(a + b \tanh^2(c + dx) \right)^3 dx &= \frac{\operatorname{Subst} \left(\int \frac{(1-x^2)(a+bx^2)^3}{x^4} dx, x, \tanh(c + dx) \right)}{d} \\ &= \frac{\operatorname{Subst} \left(\int \left(-3a(a-b)b + \frac{a^3}{x^4} - \frac{a^2(a-3b)}{x^2} - (3a-b)b^2x^2 - b^3x^4 \right) dx \right)}{d} \\ &= \frac{a^2(a-3b) \operatorname{coth}(c + dx)}{d} - \frac{a^3 \operatorname{coth}^3(c + dx)}{3d} - \frac{3a(a-b)b \tanh(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 1.26, size = 87, normalized size = 0.89

$$\frac{b \tanh(c + dx) \left(-45a^2 + b(15a + b) \operatorname{sech}^2(c + dx) + 30ab - 3b^2 \operatorname{sech}^4(c + dx) + 2b^2 \right) - 5a^2 \operatorname{coth}(c + dx) \left(\operatorname{acsch}^4(c + dx) \right)}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] $(-5a^2\text{Coth}[c + d*x]*(-2a + 9b + a\text{Csch}[c + d*x]^2) + b*(-45a^2 + 30ab + 2b^2 + b(15a + b)\text{Sech}[c + d*x]^2 - 3b^2\text{Sech}[c + d*x]^4)\text{Tanh}[c + d*x])/(15d)$

fricas [B] time = 0.59, size = 925, normalized size = 9.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] $-8/15*((5a^3 + 45a^2b + 15ab^2 + 7b^3)\cosh(dx + c)^6 + 12*(5a^3 + 15ab^2 + 4b^3)\cosh(dx + c)\sinh(dx + c)^5 + (5a^3 + 45a^2b + 15ab^2 + 7b^3)\sinh(dx + c)^6 + 2*(15a^3 + 45a^2b - 15ab^2 - 13b^3)\cosh(dx + c)^4 + (30a^3 + 90a^2b - 30ab^2 - 26b^3 + 15(5a^3 + 45a^2b + 15ab^2 + 7b^3)\cosh(dx + c)^2)\sinh(dx + c)^4 + 8*(5(5a^3 + 15ab^2 + 4b^3)\cosh(dx + c)^3 + 4*(5a^3 - 3b^3)\cosh(dx + c))\sinh(dx + c)^3 + 50a^3 - 90a^2b + 30ab^2 - 22b^3 + (75a^3 - 45a^2b - 15ab^2 + 41b^3)\cosh(dx + c)^2 + (15(5a^3 + 45a^2b + 15ab^2 + 7b^3)\cosh(dx + c)^4 + 75a^3 - 45a^2b - 15ab^2 + 41b^3 + 12(15a^3 + 45a^2b - 15ab^2 - 13b^3)\cosh(dx + c)^2)\sinh(dx + c)^2 + 4*(3(5a^3 + 15ab^2 + 4b^3)\cosh(dx + c)^5 + 8*(5a^3 - 3b^3)\cosh(dx + c)^3 + (25a^3 - 45a^2b + 12b^3)\cosh(dx + c))\sinh(dx + c))/(d\cosh(dx + c)^{10} + 10d\cosh(dx + c)\sinh(dx + c)^9 + d\sinh(dx + c)^{10} + 2d\cosh(dx + c)^8 + (45d\cosh(dx + c)^2 + 2d)\sinh(dx + c)^8 + 8(15d\cosh(dx + c)^3 + 2d\cosh(dx + c))\sinh(dx + c)^7 - 3d\cosh(dx + c)^6 + (210d\cosh(dx + c)^4 + 56d\cosh(dx + c)^2 - 3d)\sinh(dx + c)^6 + 2(126d\cosh(dx + c)^5 + 56d\cosh(dx + c)^3 - 3d\cosh(dx + c))\sinh(dx + c)^5 - 8d\cosh(dx + c)^4 + (210d\cosh(dx + c)^6 + 140d\cosh(dx + c)^4 - 45d\cosh(dx + c)^2 - 8d)\sinh(dx + c)^4 + 4(30d\cosh(dx + c)^7 + 28d\cosh(dx + c)^5 - 5d\cosh(dx + c)^3 - 4d\cosh(dx + c))\sinh(dx + c)^3 + 2d\cosh(dx + c)^2 + (45d\cosh(dx + c)^8 + 56d\cosh(dx + c)^6 - 45d\cosh(dx + c)^4 - 48d\cosh(dx + c)^2 + 2d)\sinh(dx + c)^2 + 2(5d\cosh(dx + c)^9 + 8d\cosh(dx + c)^7 - 3d\cosh(dx + c)^5 - 8d\cosh(dx + c)^3 - 2d\cosh(dx + c))\sinh(dx + c) + 6d)$

giac [B] time = 0.46, size = 257, normalized size = 2.62

$$2 \left(\frac{5(9a^2be^{4dx+4c} + 6a^3e^{2dx+2c} - 18a^2be^{2dx+2c} - 2a^3 + 9a^2b)}{(e^{2dx+2c} - 1)^3} - \frac{45a^2be^{8dx+8c} + 180a^2be^{6dx+6c} - 90ab^2e^{6dx+6c} - 30b^3e^{6dx+6c} + 270a^2be^{4dx+4c}}{(e^{2dx+2c} - 1)^3} \right)$$

15d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out] $-2/15*(5*(9a^2b*e^{(4d*x + 4c)} + 6a^3*e^{(2d*x + 2c)} - 18a^2b*e^{(2d*x + 2c)} - 2a^3 + 9a^2b)/(e^{(2d*x + 2c)} - 1)^3 - (45a^2b*e^{(8d*x + 8c)} + 180a^2b*e^{(6d*x + 6c)} - 90a*b^2*e^{(6d*x + 6c)} - 30b^3*e^{(6d*x + 6c)} + 270a^2b*e^{(4d*x + 4c)} - 210a*b^2*e^{(4d*x + 4c)} + 10b^3*e^{(4d*x + 4c)} + 180a^2b*e^{(2d*x + 2c)} - 150a*b^2*e^{(2d*x + 2c)} - 10b^3*e^{(2d*x + 2c)} + 45a^2b - 30a*b^2 - 2b^3)/(e^{(2d*x + 2c)} + 1)^5)/d$

maple [A] time = 0.53, size = 136, normalized size = 1.39

$$a^3 \left(\frac{2}{3} - \frac{\text{csch}(dx+c)^2}{3} \right) \coth(dx + c) + 3a^2b \left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)} - 2 \tanh(dx + c) \right) + 3ab^2 \left(\frac{2}{3} + \frac{\text{sech}(dx+c)^2}{3} \right) \tanh(dx + c)$$

d

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x)`

[Out] $\frac{1}{d} \left(a^3 \left(\frac{2}{3} - \frac{1}{3} \operatorname{csch}(d*x+c)^2 \right) \coth(d*x+c) + 3a^2 b \left(-\frac{1}{\sinh(d*x+c)} \right) / \cosh(d*x+c) - 2 \tanh(d*x+c) \right) + 3a b^2 \left(\frac{2}{3} + \frac{1}{3} \operatorname{sech}(d*x+c)^2 \right) \tanh(d*x+c) + b^3 \left(-\frac{1}{4} \frac{\sinh(d*x+c)}{\cosh(d*x+c)^5} + \frac{1}{4} \left(\frac{8}{15} + \frac{1}{5} \operatorname{sech}(d*x+c)^4 + \frac{4}{15} \operatorname{sech}(d*x+c)^2 \right) \tanh(d*x+c) \right)$

maxima [B] time = 0.34, size = 493, normalized size = 5.03

$$\frac{4}{15} b^3 \left(\frac{5 e^{(-2dx-2c)}}{d \left(5 e^{(-2dx-2c)} + 10 e^{(-4dx-4c)} + 10 e^{(-6dx-6c)} + 5 e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1 \right)} - \frac{5 e^{(-4dx-4c)}}{d \left(5 e^{(-2dx-2c)} + 10 e^{(-4dx-4c)} + 10 e^{(-6dx-6c)} + 5 e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1 \right)} - \frac{5 e^{(-6dx-6c)}}{d \left(5 e^{(-2dx-2c)} + 10 e^{(-4dx-4c)} + 10 e^{(-6dx-6c)} + 5 e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1 \right)} - \frac{5 e^{(-8dx-8c)}}{d \left(5 e^{(-2dx-2c)} + 10 e^{(-4dx-4c)} + 10 e^{(-6dx-6c)} + 5 e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1 \right)} - \frac{e^{(-10dx-10c)}}{d \left(5 e^{(-2dx-2c)} + 10 e^{(-4dx-4c)} + 10 e^{(-6dx-6c)} + 5 e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1 \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")`

[Out] $\frac{4}{15} b^3 \left(\frac{5 e^{(-2dx-2c)}}{d \left(5 e^{(-2dx-2c)} + 10 e^{(-4dx-4c)} + 10 e^{(-6dx-6c)} + 5 e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1 \right)} - \frac{5 e^{(-4dx-4c)}}{d \left(5 e^{(-2dx-2c)} + 10 e^{(-4dx-4c)} + 10 e^{(-6dx-6c)} + 5 e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1 \right)} - \frac{5 e^{(-6dx-6c)}}{d \left(5 e^{(-2dx-2c)} + 10 e^{(-4dx-4c)} + 10 e^{(-6dx-6c)} + 5 e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1 \right)} - \frac{5 e^{(-8dx-8c)}}{d \left(5 e^{(-2dx-2c)} + 10 e^{(-4dx-4c)} + 10 e^{(-6dx-6c)} + 5 e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1 \right)} - \frac{e^{(-10dx-10c)}}{d \left(5 e^{(-2dx-2c)} + 10 e^{(-4dx-4c)} + 10 e^{(-6dx-6c)} + 5 e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1 \right)} \right) + 4 a b^2 \left(\frac{3 e^{(-2dx-2c)}}{d \left(3 e^{(-2dx-2c)} + 3 e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1 \right)} + \frac{1}{d \left(3 e^{(-2dx-2c)} + 3 e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1 \right)} \right) + \frac{4}{3} a^3 \left(\frac{3 e^{(-2dx-2c)}}{d \left(3 e^{(-2dx-2c)} - 3 e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1 \right)} - \frac{1}{d \left(3 e^{(-2dx-2c)} - 3 e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1 \right)} \right) + 12 a^2 b / \left(d \left(e^{(-4dx-4c)} - 1 \right) \right)$

mupad [B] time = 0.28, size = 622, normalized size = 6.35

$$\frac{\frac{2(9a^2b-12ab^2+4b^3)}{15d} - \frac{4e^{2c+2dx}(-3a^2b+3ab^2+b^3)}{5d} + \frac{6a^2be^{4c+4dx}}{5d}}{3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1} - \frac{\frac{2(-3a^2b+3ab^2+b^3)}{5d} + \frac{6e^{4c+4dx}(-3a^2b+3ab^2+b^3)}{5d} - \frac{2e^{2c+2dx}}{5d}}{4e^{2c+2dx} + 6e^{4c+4dx} + 4e^{6c+6dx} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*tanh(c + d*x))^3/sinh(c + d*x)^4,x)`

[Out] $\left(\frac{2(9a^2b - 12ab^2 + 4b^3)}{(15d)} - \frac{4 \exp(2c + 2d*x) (3a^2b^2 - 3a^2b + b^3)}{(5d)} + \frac{6a^2b \exp(4c + 4d*x)}{(5d)} \right) / \left(3 \exp(2c + 2d*x) + 3 \exp(4c + 4d*x) + \exp(6c + 6d*x) + 1 \right) - \left(\frac{2(3a^2b^2 - 3a^2b + b^3)}{(5d)} + \frac{6 \exp(4c + 4d*x) (3a^2b^2 - 3a^2b + b^3)}{(5d)} - \frac{2 \exp(2c + 2d*x) (9a^2b - 12ab^2 + 4b^3)}{(5d)} - \frac{6a^2b \exp(6c + 6d*x)}{(5d)} \right) / \left(4 \exp(2c + 2d*x) + 6 \exp(4c + 4d*x) + 4 \exp(6c + 6d*x) + \exp(8c + 8d*x) + 1 \right) + \left(\frac{6a^2b}{(5d)} - \frac{8 \exp(6c + 6d*x) (3a^2b^2 - 3a^2b + b^3)}{(5d)} - \frac{8 \exp(2c + 2d*x) (3a^2b^2 - 3a^2b + b^3)}{(5d)} \right) + \frac{4 \exp(4c + 4d*x) (9a^2b - 12ab^2 + 4b^3)}{(5d)} + \frac{6a^2b \exp(8c + 8d*x)}{(5d)} \right) / \left(5 \exp(2c + 2d*x) + 10 \exp(4c + 4d*x) + 10 \exp(6c + 6d*x) + 5 \exp(8c + 8d*x) + \exp(10c + 10d*x) + 1 \right) - \left(\frac{2(3a^2b^2 - 3a^2b + b^3)}{(5d)} - \frac{6a^2b \exp(2c + 2d*x)}{(5d)} \right) / \left(2 \exp(2c + 2d*x) + \exp(4c + 4d*x) + 1 \right) - \frac{4a^3}{d \left(\exp(4c + 4d*x) - 2 \exp(2c + 2d*x) + 1 \right)} - \frac{8a^3}{3d \left(3 \exp(2c + 2d*x) - 3 \exp(4c + 4d*x) + \exp(6c + 6d*x) - 1 \right)} - \frac{6a^2b}{d \left(\exp(2c + 2d*x) - 1 \right)} + \frac{6a^2b}{5d \left(\exp(2c + 2d*x) + 1 \right)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^3 \operatorname{csch}^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*csch(c + d*x)**4, x)
```

$$3.25 \quad \int \frac{\sinh^4(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=118

$$\frac{a^{3/2}\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{d(a+b)^3} + \frac{x(3a^2 - 6ab - b^2)}{8(a+b)^3} + \frac{\sinh(c+dx) \cosh^3(c+dx)}{4d(a+b)} - \frac{(5a+b) \sinh(c+dx) \cosh(c+dx)}{8d(a+b)^2}$$

[Out] 1/8*(3*a^2-6*a*b-b^2)*x/(a+b)^3-1/8*(5*a+b)*cosh(d*x+c)*sinh(d*x+c)/(a+b)^2/d+1/4*cosh(d*x+c)^3*sinh(d*x+c)/(a+b)/d+a^(3/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/(a+b)^3/d

Rubi [A] time = 0.17, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3663, 470, 527, 522, 206, 205}

$$\frac{x(3a^2 - 6ab - b^2)}{8(a+b)^3} + \frac{a^{3/2}\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{d(a+b)^3} + \frac{\sinh(c+dx) \cosh^3(c+dx)}{4d(a+b)} - \frac{(5a+b) \sinh(c+dx) \cosh(c+dx)}{8d(a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2), x]

[Out] ((3*a^2 - 6*a*b - b^2)*x)/(8*(a + b)^3) + (a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a + b)^3*d - ((5*a + b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*(a + b)^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*(a + b)*d)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x]
&& IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned} \int \frac{\sinh^4(c + dx)}{a + b \tanh^2(c + dx)} dx &= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^3(a+bx^2)} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\cosh^3(c + dx) \sinh(c + dx)}{4(a + b)d} - \frac{\text{Subst}\left(\int \frac{a+(4a+b)x^2}{(1-x^2)^2(a+bx^2)} dx, x, \tanh(c + dx)\right)}{4(a + b)d} \\ &= -\frac{(5a + b) \cosh(c + dx) \sinh(c + dx)}{8(a + b)^2d} + \frac{\cosh^3(c + dx) \sinh(c + dx)}{4(a + b)d} - \frac{\text{Subst}\left(\int \frac{-a(3a+bx^2)}{(1-x^2)^2(a+bx^2)} dx, x, \tanh(c + dx)\right)}{4(a + b)d} \\ &= -\frac{(5a + b) \cosh(c + dx) \sinh(c + dx)}{8(a + b)^2d} + \frac{\cosh^3(c + dx) \sinh(c + dx)}{4(a + b)d} + \frac{(a^2b) \text{Subst}\left(\int \frac{a+bx^2}{(1-x^2)^2(a+bx^2)} dx, x, \tanh(c + dx)\right)}{4(a + b)d} \\ &= \frac{(3a^2 - 6ab - b^2)x}{8(a + b)^3} + \frac{a^{3/2}\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{(a + b)^3d} - \frac{(5a + b) \cosh(c + dx) \sinh(c + dx)}{8(a + b)^2d} \end{aligned}$$

Mathematica [A] time = 0.27, size = 93, normalized size = 0.79

$$\frac{32a^{3/2}\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right) + 4(3a^2 - 6ab - b^2)(c + dx) + (a + b)^2 \sinh(4(c + dx)) - 8a(a + b) \sinh(2(c + dx))}{32d(a + b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2), x]
```

```
[Out] (4*(3*a^2 - 6*a*b - b^2)*(c + d*x) + 32*a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*Tan
h[c + d*x])/Sqrt[a]] - 8*a*(a + b)*Sinh[2*(c + d*x)] + (a + b)^2*Sinh[4*(c
+ d*x)])/(32*(a + b)^3*d)
```

fricas [B] time = 0.80, size = 2024, normalized size = 17.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")
```

```
[Out] [1/64*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 8*(3*a^2 - 6*
```



```

a*b - b^2)*d*x*cosh(d*x + c)^4 - 8*(a^2 + a*b)*cosh(d*x + c)^6 + 4*(7*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)^2 - 2*a^2 - 2*a*b)*sinh(d*x + c)^6 + 8*(7*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)^3 - 6*(a^2 + a*b)*cosh(d*x + c))*sinh(d*x + c
)^5 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(3*a^2 - 6*a*b - b^2)*d
*x - 60*(a^2 + a*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)^5 + 4*(3*a^2 - 6*a*b - b^2)*d*x*cosh(d*x + c) - 20*(a^2
+ a*b)*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 8*(a^2 + a*b)*cosh(d*x + c)^2 + 4
*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 12*(3*a^2 - 6*a*b - b^2)*d*x*cosh
(d*x + c)^2 - 30*(a^2 + a*b)*cosh(d*x + c)^4 + 2*a^2 + 2*a*b)*sinh(d*x + c)
^2 + 32*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)^3*sinh(d*x + c) + 6*a*cosh(d
*x + c)^2*sinh(d*x + c)^2 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x
+ c)^4)*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*
b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^
4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^
2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)
*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a + b)*co
sh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x +
c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c
)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2
*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x
+ c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - a^2 - 2*a*b - b^2
+ 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 4*(3*a^2 - 6*a*b - b^2)*d*x*cos
h(d*x + c)^3 - 6*(a^2 + a*b)*cosh(d*x + c)^5 + 2*(a^2 + a*b)*cosh(d*x + c))
*sinh(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^4 + 4*(a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*d*sinh(d*x + c)^4), 1/64*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(
d*x + c)^8 + 8*(3*a^2 - 6*a*b - b^2)*d*x*cosh(d*x + c)^4 - 8*(a^2 + a*b)*co
sh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - 2*a^2 - 2*a*b)*s
inh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - 6*(a^2 + a*b)*c
osh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 +
4*(3*a^2 - 6*a*b - b^2)*d*x - 60*(a^2 + a*b)*cosh(d*x + c)^2)*sinh(d*x + c
)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 4*(3*a^2 - 6*a*b - b^2)*d*
x*cosh(d*x + c) - 20*(a^2 + a*b)*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 8*(a^2
+ a*b)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 12*(3*a
^2 - 6*a*b - b^2)*d*x*cosh(d*x + c)^2 - 30*(a^2 + a*b)*cosh(d*x + c)^4 + 2*
a^2 + 2*a*b)*sinh(d*x + c)^2 + 64*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)^3*
sinh(d*x + c) + 6*a*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a*cosh(d*x + c)*sin
h(d*x + c)^3 + a*sinh(d*x + c)^4)*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d*x +
c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a
- b)*sqrt(a*b)/(a*b)) - a^2 - 2*a*b - b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^7 + 4*(3*a^2 - 6*a*b - b^2)*d*x*cosh(d*x + c)^3 - 6*(a^2 + a*b)*cosh(
d*x + c)^5 + 2*(a^2 + a*b)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*cosh(d*x + c)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh
(d*x + c)^3*sinh(d*x + c) + 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x +
c)^2*sinh(d*x + c)^2 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*sinh(d*x + c)^4)]

```

giac [B] time = 1.09, size = 301, normalized size = 2.55

$$\frac{64 a^2 b \arctan\left(\frac{a e^{(2 d x+2 c)}+b e^{(2 d x+2 c)}+a-b}{2 \sqrt{a b}}\right)}{\left(a^3+3 a^2 b+3 a b^2+b^3\right) \sqrt{a b}}+\frac{8\left(3 a^2-6 a b-b^2\right) d x}{a^3+3 a^2 b+3 a b^2+b^3}-\frac{\left(18 a^2 e^{(4 d x+4 c)}-36 a b e^{(4 d x+4 c)}-6 b^2 e^{(4 d x+4 c)}-8 a^2 e^{(2 d x+2 c)}-8 a b e^{(2 d x+2 c)}+8 b^2 e^{(2 d x+2 c)}\right)}{a^3 e^{(4 c)}+3 a^2 b e^{(4 c)}+3 a b^2 e^{(4 c)}+b^3 e^{(4 c)}}$$

64 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")

```
[Out] 1/64*(64*a^2*b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)) + 8*(3*a^2 - 6*a*b - b^2)*d*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (18*a^2*e^(4*d*x + 4*c) - 36*a*b*e^(4*d*x + 4*c) - 6*b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) - 8*a*b*e^(2*d*x + 2*c) + a^2 + 2*a*b + b^2)*e^(-4*d*x)/(a^3*e^(4*c) + 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c)) + (a*e^(4*d*x + 20*c) + b*e^(4*d*x + 20*c) - 8*a*e^(2*d*x + 18*c))/(a^2*e^(16*c) + 2*a*b*e^(16*c) + b^2*e^(16*c))/d
```

maple [B] time = 0.34, size = 865, normalized size = 7.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x)
```

```
[Out] 8/d/(32*a+32*b)/(tanh(1/2*d*x+1/2*c)-1)^4+32/d/(64*a+64*b)/(tanh(1/2*d*x+1/2*c)-1)^3-1/8/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^2*a+3/8/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^2*b-3/8/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)*a+1/8/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)*b-3/8/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)-1)*a^2+3/4/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)-1)*a*b+1/8/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)-1)*b^2-8/d/(32*a+32*b)/(tanh(1/2*d*x+1/2*c)+1)^4+32/d/(64*a+64*b)/(tanh(1/2*d*x+1/2*c)+1)^3+1/8/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)^2*a-3/8/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)^2*b-3/8/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)*a+1/8/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)*b+3/8/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)+1)*a^2-3/4/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)+1)*a*b-1/8/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)+1)*b^2-1/d*a^3*b/(a+b)^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*a^2*b/(a+b)^3/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/d*a^2*b^2/(a+b)^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/d*a^3*b/(a+b)^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/d*a^2*b/(a+b)^3/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/d*a^2*b^2/(a+b)^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))
```

maxima [B] time = 0.51, size = 514, normalized size = 4.36

$$-\frac{(ab - b^2)(dx + c)}{2(a^3 + 3a^2b + 3ab^2 + b^3)d} + \frac{(8be^{(-2dx-2c)} + a + b)e^{(4dx+4c)}}{64(a^2 + 2ab + b^2)d} - \frac{b \log((a + b)e^{(4dx+4c)} + 2(a - b)e^{(2dx+2c)} + a + b)}{4(a^2 + 2ab + b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(a*b - b^2)*(d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 1/64*(8*b*e^(-2*d*x - 2*c) + a + b)*e^(4*d*x + 4*c)/((a^2 + 2*a*b + b^2)*d) - 1/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^2 + 2*a*b + b^2)*d) + 1/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d) + 1/4*(a*b - b^2)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d) - 1/8*(a^2*b - 6*a*b^2 + b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)*d) - 1/4*(a*b - b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d) - 3/8*b*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((sqrt(a*b)*(a + b)*d) - 1/64*(8*b*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c))/((a^2 + 2*a*b + b^2)*d) + 3/8*(d*x + c)/((a + b)*d) - 1/8*e^(2*d*x + 2*c)/((a + b)*d) + 1/8*e^(-2*d*x - 2*c)/((a + b)*d)
```

mupad [B] time = 1.68, size = 250, normalized size = 2.12

$$\frac{e^{4c+4dx}}{64d(a+b)} - \frac{e^{-4c-4dx}}{64d(a+b)} - \frac{x(-3a^2 + 6ab + b^2)}{8(a+b)^3} + \frac{ae^{-2c-2dx}}{8d(a+b)^2} - \frac{ae^{2c+2dx}}{8d(a+b)^2} + \frac{(-a)^{3/2} \sqrt{b} \ln((-a)^{3/2} b^{3/2} (e^{2c+2dx} - 1))}{8d(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2), x)

[Out] exp(4*c + 4*d*x)/(64*d*(a + b)) - exp(- 4*c - 4*d*x)/(64*d*(a + b)) - (x*(6*a*b - 3*a^2 + b^2))/(8*(a + b)^3) + (a*exp(- 2*c - 2*d*x))/(8*d*(a + b)^2) - (a*exp(2*c + 2*d*x))/(8*d*(a + b)^2) + ((-a)^(3/2)*b^(1/2)*log((-a)^(3/2)*b^(3/2)*(exp(2*c + 2*d*x) - 1) - 2*a^2*b*exp(2*c + 2*d*x) + (-a)^(5/2)*b^(1/2)*(exp(2*c + 2*d*x) + 1)))/(2*d*(a + b)^3) - ((-a)^(3/2)*b^(1/2)*log(2*a^2*b*exp(2*c + 2*d*x) + (-a)^(3/2)*b^(3/2)*(exp(2*c + 2*d*x) - 1) + (-a)^(5/2)*b^(1/2)*(exp(2*c + 2*d*x) + 1)))/(2*d*(a + b)^3)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^4(c + dx)}{a + b \tanh^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**4/(a+b*tanh(d*x+c)**2), x)

[Out] Integral(sinh(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)

$$3.26 \quad \int \frac{\sinh^3(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=75

$$\frac{\cosh^3(c+dx)}{3d(a+b)} - \frac{a \cosh(c+dx)}{d(a+b)^2} + \frac{a\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{5/2}}$$

[Out] $-a*\cosh(d*x+c)/(a+b)^2/d+1/3*\cosh(d*x+c)^3/(a+b)/d+a*\operatorname{arctanh}(\operatorname{sech}(d*x+c)*b^{1/2})/(a+b)^{1/2})*b^{1/2}/(a+b)^{5/2}/d$

Rubi [A] time = 0.13, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3664, 453, 325, 208}

$$\frac{\cosh^3(c+dx)}{3d(a+b)} - \frac{a \cosh(c+dx)}{d(a+b)^2} + \frac{a\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2), x]

[Out] $(a*\sqrt{b}*\operatorname{ArcTanh}[(\sqrt{b}*\operatorname{Sech}[c + d*x])/(\sqrt{a + b})])/((a + b)^{5/2}*d) - (a*\operatorname{Cosh}[c + d*x])/((a + b)^2*d) + \operatorname{Cosh}[c + d*x]^3/(3*(a + b)*d)$

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 325

Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 453

Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m+1)*(a+b*x^n)^(p+1))/(a*e*(m+1)), x] + Dist[(a*d*(m+1) - b*c*(m+n*(p+1)+1))/(a*e^n*(m+1)), Int[(e*x)^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]

Rule 3664

Int[sin[(e_)+(f_)*(x_)]^(m_)*((a_) + (b_)*tan[(e_)+(f_)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m-1)/2)*(a-b+b*ff^2*x^2)^p)/x^(m+1), x], x, Sec[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m-1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(c+dx)}{a+b \tanh^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4(a+b-bx^2)} dx, x, \text{sech}(c+dx)\right)}{d} \\
&= \frac{\cosh^3(c+dx)}{3(a+b)d} + \frac{a \text{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)} dx, x, \text{sech}(c+dx)\right)}{(a+b)d} \\
&= -\frac{a \cosh(c+dx)}{(a+b)^2 d} + \frac{\cosh^3(c+dx)}{3(a+b)d} + \frac{(ab) \text{Subst}\left(\int \frac{1}{a+b-bx^2} dx, x, \text{sech}(c+dx)\right)}{(a+b)^2 d} \\
&= \frac{a\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \text{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2} d} - \frac{a \cosh(c+dx)}{(a+b)^2 d} + \frac{\cosh^3(c+dx)}{3(a+b)d}
\end{aligned}$$

Mathematica [C] time = 0.58, size = 135, normalized size = 1.80

$$\frac{(a+b)^{3/2} \cosh(3(c+dx)) - 3(3a-b)\sqrt{a+b} \cosh(c+dx) + 12ia\sqrt{b} \left(\tan^{-1}\left(\frac{-\sqrt{a} \tanh\left(\frac{1}{2}(c+dx)\right) - i\sqrt{a+b}}{\sqrt{b}}\right) \right) + \tan^{-1}\left(\frac{-\sqrt{a} \tanh\left(\frac{1}{2}(c+dx)\right) - i\sqrt{a+b}}{\sqrt{b}}\right)}{12d(a+b)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2), x]

[Out] ((12*I)*a*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]]) - 3*(3*a - b)*Sqrt[a + b]*Cosh[c + d*x] + (a + b)^(3/2)*Cosh[3*(c + d*x)]/(12*(a + b)^(5/2)*d)

fricas [B] time = 0.65, size = 1367, normalized size = 18.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] [1/24*((a + b)*cosh(d*x + c)^6 + 6*(a + b)*cosh(d*x + c)*sinh(d*x + c)^5 + (a + b)*sinh(d*x + c)^6 - 3*(3*a - b)*cosh(d*x + c)^4 + 3*(5*(a + b)*cosh(d*x + c)^2 - 3*a + b)*sinh(d*x + c)^4 + 4*(5*(a + b)*cosh(d*x + c)^3 - 3*(3*a - b)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(3*a - b)*cosh(d*x + c)^2 + 3*(5*(a + b)*cosh(d*x + c)^4 - 6*(3*a - b)*cosh(d*x + c)^2 - 3*a + b)*sinh(d*x + c)^2 + 12*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)^2*sinh(d*x + c) + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3)*sqrt(b/(a + b))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b) + 6*((a + b)*cosh(d*x + c)^5 - 2*(3*a - b)*cosh(d*x + c)^3 - (3*a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)/((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c)^2 + (a^2 + 2*a*b +

$b^2) * d * \sinh(dx + c)^3$, $1/24 * ((a + b) * \cosh(dx + c)^6 + 6 * (a + b) * \cosh(dx + c) * \sinh(dx + c)^5 + (a + b) * \sinh(dx + c)^6 - 3 * (3 * a - b) * \cosh(dx + c)^4 + 3 * (5 * (a + b) * \cosh(dx + c)^2 - 3 * a + b) * \sinh(dx + c)^4 + 4 * (5 * (a + b) * \cosh(dx + c)^3 - 3 * (3 * a - b) * \cosh(dx + c)) * \sinh(dx + c)^3 - 3 * (3 * a - b) * \cosh(dx + c)^2 + 3 * (5 * (a + b) * \cosh(dx + c)^4 - 6 * (3 * a - b) * \cosh(dx + c)^2 - 3 * a + b) * \sinh(dx + c)^2 + 24 * (a * \cosh(dx + c)^3 + 3 * a * \cosh(dx + c)^2 * \sinh(dx + c) + 3 * a * \cosh(dx + c) * \sinh(dx + c)^2 + a * \sinh(dx + c)^3) * \sqrt{-b / (a + b)} * \arctan(1/2 * ((a + b) * \cosh(dx + c)^3 + 3 * (a + b) * \cosh(dx + c) * \sinh(dx + c)^2 + (a + b) * \sinh(dx + c)^3 + (a - 3 * b) * \cosh(dx + c) + (3 * (a + b) * \cosh(dx + c)^2 + a - 3 * b) * \sinh(dx + c)) * \sqrt{-b / (a + b)}) / b - 24 * (a * \cosh(dx + c)^3 + 3 * a * \cosh(dx + c)^2 * \sinh(dx + c) + 3 * a * \cosh(dx + c) * \sinh(dx + c)^2 + a * \sinh(dx + c)^3) * \sqrt{-b / (a + b)} * \arctan(1/2 * ((a + b) * \cosh(dx + c) + (a + b) * \sinh(dx + c)) * \sqrt{-b / (a + b)}) / b + 6 * ((a + b) * \cosh(dx + c)^5 - 2 * (3 * a - b) * \cosh(dx + c)^3 - (3 * a - b) * \cosh(dx + c)) * \sinh(dx + c) + a + b) / ((a^2 + 2 * a * b + b^2) * d * \cosh(dx + c)^3 + 3 * (a^2 + 2 * a * b + b^2) * d * \cosh(dx + c)^2 * \sinh(dx + c) + 3 * (a^2 + 2 * a * b + b^2) * d * \cosh(dx + c) * \sinh(dx + c)^2 + (a^2 + 2 * a * b + b^2) * d * \sinh(dx + c)^3)]$

giac [B] time = 0.49, size = 810, normalized size = 10.80

$$24 \sqrt{a^2 - b^2 + 2 \sqrt{-ab} (a+b)} (ab - \sqrt{-ab} a) | a e^{(2c)} + b e^{(2c)} | \arctan \left(\frac{e^{(dx)}}{\sqrt{\frac{a^3 e^{(2c)} + a^2 b e^{(2c)} - a b^2 e^{(2c)} - b^3 e^{(2c)} + \sqrt{(a^3 e^{(2c)} + a^2 b e^{(2c)} - a b^2 e^{(2c)} - b^3 e^{(2c)})^2 - (a^3 e^{(4c)} + 3 a^2 b e^{(4c)} - a^3 e^{(4c)} + 3 a^2 b e^{(4c)} + 3 a b^2 e^{(4c)} + b^3 e^{(4c)})}}{a^5 + 3 a^4 b + 2 a^3 b^2 - 2 a^2 b^3 - 3 a b^4 - b^5 + 2 (a^4 + 4 a^3 b + 6 a^2 b^2 + 4 a b^3 + b^4) \sqrt{-ab}}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)^3/(a+b*tanh(dx+c)^2),x, algorithm="giac")

[Out] $-1/24 * (24 * \sqrt{a^2 - b^2 + 2 * \sqrt{-a * b} * (a + b)} * (a * b - \sqrt{-a * b} * a) * \text{abs}(a * e^{(2 * c)} + b * e^{(2 * c)}) * \arctan(e^{(d * x)} / \sqrt{(a^3 * e^{(2 * c)} + a^2 * b * e^{(2 * c)} - a * b^2 * e^{(2 * c)} - b^3 * e^{(2 * c)} - \sqrt{(a^3 * e^{(2 * c)} + a^2 * b * e^{(2 * c)} - a * b^2 * e^{(2 * c)} - b^3 * e^{(2 * c)})^2 - (a^3 * e^{(4 * c)} + 3 * a^2 * b * e^{(4 * c)} + 3 * a * b^2 * e^{(4 * c)} + b^3 * e^{(4 * c)}) * (a^3 + 3 * a^2 * b + 3 * a * b^2 + b^3)})} / (a^3 * e^{(4 * c)} + 3 * a^2 * b * e^{(4 * c)} + 3 * a * b^2 * e^{(4 * c)} + b^3 * e^{(4 * c)})) * e^{(-2 * c)} / (a^5 + 3 * a^4 * b + 2 * a^3 * b^2 - 2 * a^2 * b^3 - 3 * a * b^4 - b^5 + 2 * (a^4 + 4 * a^3 * b + 6 * a^2 * b^2 + 4 * a * b^3 + b^4) * \sqrt{-a * b}) + 24 * (3 * a^2 * b - a * b^2 + (a^2 - 3 * a * b) * \sqrt{-a * b}) * \text{abs}(a * e^{(2 * c)} + b * e^{(2 * c)}) * \arctan(e^{(d * x)} / \sqrt{(a^3 * e^{(2 * c)} + a^2 * b * e^{(2 * c)} - a * b^2 * e^{(2 * c)} - b^3 * e^{(2 * c)} - \sqrt{(a^3 * e^{(2 * c)} + a^2 * b * e^{(2 * c)} - a * b^2 * e^{(2 * c)} - b^3 * e^{(2 * c)})^2 - (a^3 * e^{(4 * c)} + 3 * a^2 * b * e^{(4 * c)} + 3 * a * b^2 * e^{(4 * c)} + b^3 * e^{(4 * c)}) * (a^3 + 3 * a^2 * b + 3 * a * b^2 + b^3)})} / (a^3 * e^{(4 * c)} + 3 * a^2 * b * e^{(4 * c)} + 3 * a * b^2 * e^{(4 * c)} + b^3 * e^{(4 * c)})) * e^{(-2 * c)} / ((a^4 + 2 * a^3 * b - 2 * a * b^3 - b^4 - 2 * (a^3 + 3 * a^2 * b + 3 * a * b^2 + b^3) * \sqrt{-a * b})) * \sqrt{a^2 - b^2 - 2 * \sqrt{-a * b} * (a + b)}) + (9 * a * e^{(2 * d * x + 2 * c)} - 3 * b * e^{(2 * d * x + 2 * c)} - a - b) * e^{(-3 * d * x)} / (a^2 * e^{(3 * c)} + 2 * a * b * e^{(3 * c)} + b^2 * e^{(3 * c)}) - (a^2 * e^{(3 * d * x + 24 * c)} + 2 * a * b * e^{(3 * d * x + 24 * c)} + b^2 * e^{(3 * d * x + 24 * c)} - 9 * a^2 * e^{(d * x + 22 * c)} - 6 * a * b * e^{(d * x + 22 * c)} + 3 * b^2 * e^{(d * x + 22 * c)}) / (a^3 * e^{(21 * c)} + 3 * a^2 * b * e^{(21 * c)} + 3 * a * b^2 * e^{(21 * c)} + b^3 * e^{(21 * c)})) / d$

maple [B] time = 0.30, size = 202, normalized size = 2.69

$$\frac{16}{3 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^3 (16a + 16b)} - \frac{8}{(16a + 16b) \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^2} - \frac{-a+b}{2(a+b)^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)} - \frac{8}{(16a + 16b) \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^2} + \frac{16}{3 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^3} + \frac{1}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x)

[Out] $\frac{1}{d} \left(-\frac{16}{3} \frac{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)-1)^3}{(16*a+16*b)} - \frac{8}{(16*a+16*b)} \frac{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)-1)^2}{(a+b)^2} - \frac{1}{2} \frac{(-a+b)}{(a+b)^2} \frac{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)-1)}{(16*a+16*b)} - \frac{8}{(16*a+16*b)} \frac{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+1)^2}{(16*a+16*b)} + \frac{16}{3} \frac{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+1)^3}{(16*a+16*b)} - \frac{1}{2} \frac{(a-b)}{(a+b)^2} \frac{(\tanh(\frac{1}{2}d*x+\frac{1}{2}c)+1)}{(16*a+16*b)} + \frac{a*b}{(a+b)^2} \frac{1}{(a*b+b^2)^{1/2}} \operatorname{arctanh}\left(\frac{1}{4} \frac{(2*\tanh(\frac{1}{2}d*x+\frac{1}{2}c)^2*a+2*a+4*b)}{(a*b+b^2)^{1/2}}\right) \right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left((ae^{6c} + be^{6c})e^{6dx} - 3(3ae^{4c} - be^{4c})e^{4dx} - 3(3ae^{2c} - be^{2c})e^{2dx} + a + b \right) e^{-3dx}}{24(a^2de^{3c} + 2abde^{3c} + b^2de^{3c})} - \frac{1}{8} \int \frac{1}{a^3 + 3a^2b + \dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] $\frac{1}{24} \left((a*e^{6c} + b*e^{6c})e^{6dx} - 3(3*a*e^{4c} - b*e^{4c})e^{4dx} - 3(3*a*e^{2c} - b*e^{2c})e^{2dx} + a + b \right) e^{-3dx} / (a^2*d*e^{3c} + 2*a*b*d*e^{3c} + b^2*d*e^{3c}) - \frac{1}{8} \operatorname{integrate}\left(\frac{16*(a*b*e^{3dx} + 3*c) - a*b*e^{(d*x+c)}}{a^3 + 3*a^2*b + 3*a*b^2 + b^3 + (a^3*e^{4c} + 3*a^2*b*e^{4c} + 3*a*b^2*e^{4c} + b^3*e^{4c})e^{4dx} + 2*(a^3*e^{2c} + a^2*b*e^{2c} - a*b^2*e^{2c} - b^3*e^{2c})e^{2dx}} \right), x$

mupad [B] time = 2.65, size = 955, normalized size = 12.73

$$\frac{e^{-3c-3dx}}{24d(a+b)} + \frac{e^{3c+3dx}}{24d(a+b)} - \frac{\sqrt{a^2b} \left(2 \operatorname{atan} \left(\frac{e^{dx} e^c \left(\frac{4(2a^2b^3d\sqrt{a^2b} + 4a^3b^2d\sqrt{a^2b} + 2a^4bd\sqrt{a^2b})}{a(a+b)\sqrt{-d^2(a+b)^5(a^2+2ab+b^2)}(a^3+3a^2b+3ab^2+b^3)\sqrt{-a^5d^2-5a^4bd^2-10a^3b^2d^2-10a^2b^3d^2-5a*b^4d^2-5a^4*b*d^2-10a^2*b^3*d^2-10a^3*b^2*d^2} \right)}{a^3+3a^2b+3ab^2+b^3} \right)}{\dots} \right)}{24d(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^2),x)

[Out] $\frac{\exp(-3c-3d*x)}{(24*d*(a+b))} + \frac{\exp(3c+3d*x)}{(24*d*(a+b))} - \left((a^2*b)^{1/2} \left(2*\operatorname{atan}\left(\frac{(\exp(d*x)*\exp(c))*\left((4*(2*a^2*b^3*d*(a^2*b)^{1/2} + 4*a^3*b^2*d*(a^2*b)^{1/2} + 2*a^4*b*d*(a^2*b)^{1/2}) \right)}{a*(a+b)*(-d^2*(a+b)^5)^{1/2}} \right) \right) \right) \left(2*a*b + a^2 + b^2 \right) \left(3*a*b^2 + 3*a^2*b + a^3 + b^3 \right) \left(-a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2 \right)^{1/2} + \left(2*a^3*b \right) / \left(d*(a+b)^3*(a^2*b)^{1/2} \left(2*a*b + a^2 + b^2 \right) \left(3*a*b^2 + 3*a^2*b + a^3 + b^3 \right) \right) + \left(2*a^3*b*\exp(3c)*\exp(3d*x) \right) / \left(d*(a+b)^3*(a^2*b)^{1/2} \left(2*a*b + a^2 + b^2 \right) \left(3*a*b^2 + 3*a^2*b + a^3 + b^3 \right) \right) \left(a^6 \left(-a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2 \right)^{1/2} + b^6 \left(-a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2 \right)^{1/2} + 15*a^2*b^4 \left(-a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2 \right)^{1/2} + 20*a^3*b^3 \left(-a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2 \right)^{1/2} + 15*a^4*b^2 \left(-a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2 \right)^{1/2} + 6*a*b^5 \left(-a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2 \right)^{1/2} + 6*a^5*b \left(-a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2 \right)^{1/2} \right) / \left(4*a^2*b \right) - 2*\operatorname{atan}\left(\frac{a*\exp(d*x)*\exp(c)*(-d^2*(a+b)^5)^{1/2}}{2*d*(a+b)^2*(a^2*b)^{1/2}} \right) \right) / \left(2 \left(-a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2 \right)^{1/2} \right) - \left(\exp(-c-d*x) \right) \left(3*a - b \right) / \left(8*d*(a+b)^2 \right) - \left(\exp(c+d*x) \right) \left(3*a - b \right) / \left(8*d*(a+b)^2 \right)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^3(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*tanh(d*x+c)**2), x)
```

```
[Out] Integral(sinh(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```


$$3.27 \quad \int \frac{\sinh^2(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=78

$$-\frac{\sqrt{a} \sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{d(a+b)^2} + \frac{\sinh(c+dx) \cosh(c+dx)}{2d(a+b)} - \frac{x(a-b)}{2(a+b)^2}$$

[Out] $-1/2*(a-b)*x/(a+b)^2+1/2*\cosh(d*x+c)*\sinh(d*x+c)/(a+b)/d-\arctan(b^{(1/2)}*\tanh(d*x+c)/a^{(1/2)})*a^{(1/2)}*b^{(1/2)}/(a+b)^2/d$

Rubi [A] time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3663, 471, 522, 206, 205}

$$-\frac{\sqrt{a} \sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{d(a+b)^2} + \frac{\sinh(c+dx) \cosh(c+dx)}{2d(a+b)} - \frac{x(a-b)}{2(a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]

[Out] $-\frac{(a-b)x}{2(a+b)^2} - \frac{(\sqrt{a}\sqrt{b}\text{ArcTan}[\frac{\sqrt{b}\text{Tanh}[c+d*x]}{\sqrt{a}}])}{(a+b)^2d} + \frac{(\text{Cosh}[c+d*x]\text{Sinh}[c+d*x])}{2(a+b)d}$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 471

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n-1)*(e*x)^(m-n+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(n*(b*c-a*d)*(p+1)), x] - Dist[e^n/(n*(b*c-a*d)*(p+1)), Int[(e*x)^(m-n)*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[c*(m-n+1)+d*(m+n*(p+q+1)+1]*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m-n+1] && GtQ[m-n+1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e-a*f)/(b*c-a*d), Int[1/(a+b*x^n), x], x] - Dist[(d*e-c*f)/(b*c-a*d), Int[1/(c+d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 3663

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m+1))/f, Subst[Int[(x^m*(a+b*(ff*x)^n)^p]/(c^2+ff^2*x^2)^(m/2+1), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x]

&& IntegerQ[m/2]

Rubi steps

$$\begin{aligned}
 \int \frac{\sinh^2(c+dx)}{a+b \tanh^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+bx^2)} dx, x, \tanh(c+dx)\right)}{d} \\
 &= \frac{\cosh(c+dx) \sinh(c+dx)}{2(a+b)d} - \frac{\text{Subst}\left(\int \frac{a-bx^2}{(1-x^2)(a+bx^2)} dx, x, \tanh(c+dx)\right)}{2(a+b)d} \\
 &= \frac{\cosh(c+dx) \sinh(c+dx)}{2(a+b)d} - \frac{(a-b) \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c+dx)\right)}{2(a+b)^2 d} - \frac{(ab) \text{Subst}\left(\int \frac{x}{1-x^2} dx, x, \tanh(c+dx)\right)}{2(a+b)^2 d} \\
 &= -\frac{(a-b)x}{2(a+b)^2} - \frac{\sqrt{a} \sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{(a+b)^2 d} + \frac{\cosh(c+dx) \sinh(c+dx)}{2(a+b)d}
 \end{aligned}$$

Mathematica [A] time = 0.15, size = 67, normalized size = 0.86

$$\frac{-2(a-b)(c+dx) + (a+b) \sinh(2(c+dx)) - 4\sqrt{a} \sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{4d(a+b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2), x]

[Out] (-2*(a - b)*(c + d*x) - 4*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] + (a + b)*Sinh[2*(c + d*x)])/(4*(a + b)^2*d)

fricas [B] time = 0.57, size = 916, normalized size = 11.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] [-1/8*(4*(a - b)*d*x*cosh(d*x + c)^2 - (a + b)*cosh(d*x + c)^4 - 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 - (a + b)*sinh(d*x + c)^4 + 2*(2*(a - b)*d*x - 3*(a + b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 4*sqrt(-a*b)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b) + 4*(2*(a - b)*d*x*cosh(d*x + c) - (a + b)*cosh(d*x + c)^3)*sinh(d*x + c) + a + b)/((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^2), -1/8*(4*(a - b)*d*x*cosh(d*x + c)^2 - (a + b)*cosh(d*x + c)^4 - 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 - (a + b)*sinh(d*x + c)^4 + 2*(2*(a - b)*d*x - 3*(a + b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*sqrt(a*b)*(cosh(d*x

+ c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b)) + 4*(2*(a - b)*d*x*cosh(d*x + c) - (a + b)*cosh(d*x + c)^3)*sinh(d*x + c) + a + b)/((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^2)]

giac [B] time = 0.59, size = 176, normalized size = 2.26

$$\frac{\frac{4(a-b)dx}{a^2+2ab+b^2} + \frac{8ab \arctan\left(\frac{ae^{2dx+2c}+be^{2dx+2c}+a-b}{2\sqrt{ab}}\right)}{(a^2+2ab+b^2)\sqrt{ab}} - \frac{(2ae^{2dx+2c}-2be^{2dx+2c}-a-b)e^{-2dx}}{a^2e^{2c}+2abe^{2c}+b^2e^{2c}} - \frac{e^{2dx+8c}}{ae^{6c}+be^{6c}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] -1/8*(4*(a - b)*d*x/(a^2 + 2*a*b + b^2) + 8*a*b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b)))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) - (2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) - a - b)*e^(-2*d*x)/(a^2*e^(2*c) + 2*a*b*e^(2*c) + b^2*e^(2*c)) - e^(2*d*x + 8*c)/(a*e^(6*c) + b*e^(6*c)))/d

maple [B] time = 0.32, size = 605, normalized size = 7.76

$$\frac{4}{d(8a+8b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2} + \frac{8}{d(16a+16b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} + \frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a}{2d(a+b)^2} - \frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)b}{2d(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x)

[Out] 4/d/(8*a+8*b)/(tanh(1/2*d*x+1/2*c)-1)^2+8/d/(16*a+16*b)/(tanh(1/2*d*x+1/2*c)-1)+1/2/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)-1)*a-1/2/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)-1)*b-4/d/(8*a+8*b)/(tanh(1/2*d*x+1/2*c)+1)^2+8/d/(16*a+16*b)/(tanh(1/2*d*x+1/2*c)+1)-1/2/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)+1)*a+1/2/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)+1)*b+1/d*a^2*b/(a+b)^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/d*a*b/(a+b)^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*a*b^2/(a+b)^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*a^2*b/(a+b)^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/d*a*b/(a+b)^2/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/d*a*b^2/(a+b)^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))

maxima [B] time = 0.46, size = 316, normalized size = 4.05

$$\frac{b \log\left((a+b)e^{4dx+4c} + 2(a-b)e^{2dx+2c} + a+b\right)}{4(a^2+2ab+b^2)d} - \frac{b \log\left(2(a-b)e^{-2dx-2c} + (a+b)e^{-4dx-4c} + a+b\right)}{4(a^2+2ab+b^2)d} - \frac{(ab)^2}{4(a^2+2ab+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

```
[Out] 1/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/4*(a*b - b^2)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d) + 1/4*(a*b - b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d) + 1/2*b*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a + b)*d) - 1/2*(d*x + c)/((a + b)*d) + 1/8*e^(2*d*x + 2*c)/((a + b)*d) - 1/8*e^(-2*d*x - 2*c)/((a + b)*d)
```

mupad [B] time = 1.51, size = 198, normalized size = 2.54

$$\frac{e^{2c+2dx}}{8d(a+b)} - \frac{e^{-2c-2dx}}{8d(a+b)} - \frac{x(a-b)}{2(a+b)^2} - \frac{\sqrt{-a}\sqrt{b}\ln(\sqrt{-a}b^{3/2}(e^{2c+2dx}-1) + (-a)^{3/2}\sqrt{b}(e^{2c+2dx}+1) - 2abe^{2c+2dx})}{2d(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2), x)
```

```
[Out] exp(2*c + 2*d*x)/(8*d*(a + b)) - exp(- 2*c - 2*d*x)/(8*d*(a + b)) - (x*(a - b))/(2*(a + b)^2) - ((-a)^(1/2)*b^(1/2)*log((-a)^(1/2)*b^(3/2)*(exp(2*c + 2*d*x) - 1) + (-a)^(3/2)*b^(1/2)*(exp(2*c + 2*d*x) + 1) - 2*a*b*exp(2*c + 2*d*x)))/(2*d*(a + b)^2) + ((-a)^(1/2)*b^(1/2)*log((-a)^(1/2)*b^(3/2)*(exp(2*c + 2*d*x) - 1) + (-a)^(3/2)*b^(1/2)*(exp(2*c + 2*d*x) + 1) + 2*a*b*exp(2*c + 2*d*x)))/(2*d*(a + b)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh^2(c + dx)}{a + b \tanh^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*tanh(d*x+c)**2), x)
```

```
[Out] Integral(sinh(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

$$3.28 \quad \int \frac{\sinh(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=53

$$\frac{\cosh(c+dx)}{d(a+b)} - \frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}$$

[Out] cosh(d*x+c)/(a+b)/d-arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/(a+b)^(3/2)/d

Rubi [A] time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3664, 325, 208}

$$\frac{\cosh(c+dx)}{d(a+b)} - \frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2), x]

[Out] -((Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/((a + b)^(3/2)*d) + Cosh[c + d*x]/((a + b)*d)

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 325

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 3664

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m-1)/2)*(a - b + b*ff^2*x^2)^p]/x^(m+1), x], x, Sec[e + f*x]/ff, x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m-1)/2]

Rubi steps

$$\begin{aligned} \int \frac{\sinh(c+dx)}{a+b \tanh^2(c+dx)} dx &= -\frac{\operatorname{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)} dx, x, \operatorname{sech}(c+dx)\right)}{d} \\ &= \frac{\cosh(c+dx)}{(a+b)d} - \frac{b \operatorname{Subst}\left(\int \frac{1}{a+b-bx^2} dx, x, \operatorname{sech}(c+dx)\right)}{(a+b)d} \\ &= -\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d} + \frac{\cosh(c+dx)}{(a+b)d} \end{aligned}$$

Mathematica [C] time = 0.27, size = 107, normalized size = 2.02

$$\frac{\sqrt{a+b} \cosh(c+dx) - i\sqrt{b} \left(\tan^{-1} \left(\frac{-\sqrt{a} \tanh\left(\frac{1}{2}(c+dx)\right) - i\sqrt{a+b}}{\sqrt{b}} \right) + \tan^{-1} \left(\frac{\sqrt{a} \tanh\left(\frac{1}{2}(c+dx)\right) - i\sqrt{a+b}}{\sqrt{b}} \right) \right)}{d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2), x]

[Out] ((-I)*Sqrt[b]*(ArcTan[(-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2]]/Sqrt[b]) + ArcTan[(-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2]]/Sqrt[b]) + Sqrt[a + b]*Cosh[c + d*x]/((a + b)^(3/2)*d)

fricas [B] time = 0.92, size = 666, normalized size = 12.57

$$\left[\sqrt{\frac{b}{a+b}} (\cosh(dx+c) + \sinh(dx+c)) \log \left(\frac{(a+b) \cosh(dx+c)^4 + 4(a+b) \cosh(dx+c) \sinh(dx+c)^3 + (a+b) \sinh(dx+c)^4 + 2(a+3b) \cosh(dx+c) \sinh(dx+c)^2 + (a+b) \sinh(dx+c)^2}{(a+b) \cosh(dx+c)^2} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] [1/2*(sqrt(b/(a + b))*(cosh(d*x + c) + sinh(d*x + c))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 1)/((a + b)*d*cosh(d*x + c) + (a + b)*d*sinh(d*x + c)), -1/2*(2*sqrt(-b/(a + b))*(cosh(d*x + c) + sinh(d*x + c))*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x + c))*sqrt(-b/(a + b))/b) - 2*sqrt(-b/(a + b))*(cosh(d*x + c) + sinh(d*x + c))*arctan(1/2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-b/(a + b))/b) - cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) - sinh(d*x + c)^2 - 1)/((a + b)*d*cosh(d*x + c) + (a + b)*d*sinh(d*x + c))]

giac [B] time = 0.25, size = 1065, normalized size = 20.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2), x, algorithm="giac")

[Out] -1/2*(2*(2*(2*a*b^2 - (a*b - b^2)*sqrt(-a*b))*sqrt(a^2 - b^2 + 2*sqrt(-a*b)*(a + b))*(a*e^(2*c) + b*e^(2*c))^2*abs(a*e^(2*c) + b*e^(2*c)) + (a^3*b + a^2*b^2 - a*b^3 - b^4 + 2*(a^2*b + 2*a*b^2 + b^3)*sqrt(-a*b))*sqrt(a^2 - b^2 + 2*sqrt(-a*b)*(a + b))*abs(a*e^(2*c) + b*e^(2*c))*abs(-a*e^(2*c) - b*e^(2*c))*e^(2*c) + (2*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 - (a^4 - 2*a^2*b^2 + b^4)*sqrt(-a*b))*sqrt(a^2 - b^2 + 2*sqrt(-a*b)*(a + b))*abs(a*e^(2*c) + b*e^(2*c))*e^(4*c))*arctan(e^(d*x)/sqrt((a^2*e^(2*c) - b^2*e^(2*c) - sqrt(a^2 - b^2 + 2*sqrt(-a*b)*(a + b))*sqrt(a^2 - b^2 + 2*sqrt(-a*b)*(a + b))))))

$$\frac{(a^2 e^{2c} - b^2 e^{2c})^2 - (a^2 e^{4c} + 2ab e^{4c} + b^2 e^{4c}) \cdot (a^2 + 2ab + b^2)}{(a^2 e^{4c} + 2ab e^{4c} + b^2 e^{4c})} e^{-4c} / \left((a^7 + 5a^6 b + 9a^5 b^2 + 5a^4 b^3 - 5a^3 b^4 - 9a^2 b^5 - 5ab^6 - b^7 + 2(a^6 + 6a^5 b + 15a^4 b^2 + 20a^3 b^3 + 15a^2 b^4 + 6ab^5 + b^6) \sqrt{-ab}) \cdot \text{abs}(-a e^{2c} - b e^{2c}) \right) + 2 \cdot \left(2(4a^2 b^2 - 4ab^3 + (a^2 b - 6ab^2 + b^3) \sqrt{-ab}) \cdot (a e^{2c} + b e^{2c})^2 \cdot \text{abs}(a e^{2c} + b e^{2c}) + (a^4 b - 4a^3 b^2 - 10a^2 b^3 - 4ab^4 + b^5 - 4(a^3 b + a^2 b^2 - ab^3 - b^4) \sqrt{-ab}) \cdot \text{abs}(a e^{2c} + b e^{2c}) \cdot \text{abs}(-a e^{2c} - b e^{2c}) \cdot e^{2c} + (4a^5 b - 8a^3 b^3 + 4ab^5 + (a^5 - 5a^4 b - 6a^3 b^2 + 6a^2 b^3 + 5ab^4 - b^5) \sqrt{-ab}) \cdot \text{abs}(a e^{2c} + b e^{2c}) \cdot e^{4c} \right) \cdot \arctan(e^{dx} / \sqrt{(a^2 e^{2c} - b^2 e^{2c})^2 - (a^2 e^{4c} + 2ab e^{4c} + b^2 e^{4c}) \cdot (a^2 + 2ab + b^2)}) / (a^2 e^{4c} + 2ab e^{4c} + b^2 e^{4c})) \cdot e^{-4c} / \left((a^6 + 4a^5 b + 5a^4 b^2 - 5a^2 b^4 - 4ab^5 - b^6 - 2(a^5 + 5a^4 b + 10a^3 b^2 + 10a^2 b^3 + 5ab^4 + b^5) \sqrt{-ab}) \cdot \sqrt{a^2 - b^2 - 2 \sqrt{-ab} \cdot (a + b) \cdot \text{abs}(-a e^{2c} - b e^{2c})} \right) - e^{(dx + 6c)} / (a e^{5c} + b e^{5c}) - e^{-dx} / (a e^c + b e^c) / d$$

maple [B] time = 0.22, size = 104, normalized size = 1.96

$$\frac{\frac{4}{(4a+4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)} - \frac{b \operatorname{arctanh}\left(\frac{2\left(\tanh^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+2a+4b}{4\sqrt{ab+b^2}}\right)}{(a+b)\sqrt{ab+b^2}} - \frac{4}{(4a+4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)/(a+b*tanh(d*x+c)^2), x)

[Out] 1/d*(4/(4*a+4*b)/(tanh(1/2*d*x+1/2*c)+1)-b/(a+b)/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))-4/(4*a+4*b)/(tanh(1/2*d*x+1/2*c)-1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(e^{2dx+2c} + 1)e^{-dx}}{2(ade^c + bde^c)} + \frac{1}{2} \int \frac{4(b e^{3dx+3c} - b e^{(dx+c)})}{a^2 + 2ab + b^2 + (a^2 e^{4c} + 2abe^{4c} + b^2 e^{4c})e^{4dx} + 2(a^2 e^{2c} - b^2 e^{2c})e^{2dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2), x, algorithm="maxima")

[Out] 1/2*(e^(2*d*x + 2*c) + 1)*e^(-d*x)/(a*d*e^c + b*d*e^c) + 1/2*integrate(4*(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^2 + 2*a*b + b^2 + (a^2*e^(4*c) + 2*a*b*e^(4*c) + b^2*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) - b^2*e^(2*c))*e^(2*d*x)), x)

mupad [B] time = 2.00, size = 520, normalized size = 9.81

$$\frac{e^{c+dx}}{2d(a+b)} + \frac{e^{-c-dx}}{2d(a+b)} - \frac{\sqrt{b} \left(2 \operatorname{atan}\left(\frac{e^{dx} e^c \sqrt{-d^2(a+b)^3}}{2\sqrt{b} d(a+b)}\right) - 2 \operatorname{atan}\left(\frac{e^{dx} e^c \left(\frac{2a\sqrt{b}}{d(a+b)^2(a^3+3a^2b+3ab^2+b^3)} + \frac{1}{(a+b)\sqrt{-d^2(a+b)^3 - a^2}}\right)}{d(a+b)^2(a^3+3a^2b+3ab^2+b^3)} + \frac{1}{(a+b)\sqrt{-d^2(a+b)^3 - a^2}}\right)}{\sqrt{b} \left(2 \operatorname{atan}\left(\frac{e^{dx} e^c \sqrt{-d^2(a+b)^3}}{2\sqrt{b} d(a+b)}\right) - 2 \operatorname{atan}\left(\frac{e^{dx} e^c \left(\frac{2a\sqrt{b}}{d(a+b)^2(a^3+3a^2b+3ab^2+b^3)} + \frac{1}{(a+b)\sqrt{-d^2(a+b)^3 - a^2}}\right)}{d(a+b)^2(a^3+3a^2b+3ab^2+b^3)} + \frac{1}{(a+b)\sqrt{-d^2(a+b)^3 - a^2}}\right)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2), x)

[Out] exp(c + d*x)/(2*d*(a + b)) + exp(-c - d*x)/(2*d*(a + b)) - (b^(1/2)*(2*atan((exp(d*x)*exp(c))*(-d^2*(a + b)^3)^(1/2))/(2*b^(1/2)*d*(a + b))) - 2*atan((exp(d*x)*exp(c))*(-d^2*(a + b)^3)^(1/2))/(2*b^(1/2)*d*(a + b))

```
((exp(d*x)*exp(c)*((2*a*b^(1/2))/(d*(a + b)^2*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + (4*(2*a^2*b^(3/2)*d + 2*a*b^(5/2)*d))/((a + b)*(-d^2*(a + b)^3)^(1/2) *(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))) + (2*a*b^(1/2)*exp(3*c)*exp(3*d*x))/(d*(a + b)^2*(3*a*b^2 + 3*a^2*b + a^3 + b^3)))*(a^4*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + b^4*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + 4*a*b^3*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + 4*a^3*b*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + 6*a^2*b^2*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)))/(4*a*b)))/(2*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(c + dx)}{a + b \tanh^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)**2),x)

[Out] Integral(sinh(c + d*x)/(a + b*tanh(c + d*x)**2), x)

$$3.29 \quad \int \frac{\operatorname{csch}(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=55

$$\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{ad\sqrt{a+b}} - \frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

[Out] $-\operatorname{arctanh}(\cosh(d*x+c))/a/d+\operatorname{arctanh}(\operatorname{sech}(d*x+c)*b^{(1/2)/(a+b)^{(1/2)})}*b^{(1/2)}/a/d/(a+b)^{(1/2)}$

Rubi [A] time = 0.08, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3664, 391, 207, 208}

$$\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{ad\sqrt{a+b}} - \frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]/(a + b*\operatorname{Tanh}[c + d*x]^2), x]$

[Out] $-(\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]/(a*d)) + (\operatorname{Sqrt}[b]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Sech}[c + d*x])/ \operatorname{Sqrt}[a + b]])/(a*\operatorname{Sqrt}[a + b]*d)$

Rule 207

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Rt}[-a, 2]]/(\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{LtQ}[a, 0] \ || \ \operatorname{GtQ}[b, 0])$

Rule 208

$\operatorname{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b\}, x] \ \&\& \operatorname{NegQ}[a/b]$

Rule 391

$\operatorname{Int}[1/(((a_ + (b_)*(x_)^{(n_)})*((c_ + (d_)*(x_)^{(n_)}))), x_Symbol] \rightarrow \operatorname{Dist}[b/(b*c - a*d), \operatorname{Int}[1/(a + b*x^n), x], x] - \operatorname{Dist}[d/(b*c - a*d), \operatorname{Int}[1/(c + d*x^n), x], x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \operatorname{NeQ}[b*c - a*d, 0]$

Rule 3664

$\operatorname{Int}[\sin[(e_ + (f_)*(x_)]^{(m_)}*((a_ + (b_)*\tan[(e_ + (f_)*(x_)]^2)^{(p_)}), x_Symbol] \rightarrow \operatorname{With}\{ff = \operatorname{FreeFactors}[\operatorname{Sec}[e + f*x], x]\}, \operatorname{Dist}[1/(f*ff^m), \operatorname{Subst}[\operatorname{Int}[((-1 + ff^2*x^2)^{((m - 1)/2)*(a - b + b*ff^2*x^2)^p})/x^{(m + 1)}], x], x, \operatorname{Sec}[e + f*x]/ff], x] /; \operatorname{FreeQ}\{a, b, e, f, p\}, x] \ \&\& \operatorname{IntegerQ}[(m - 1)/2]$

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{a+b \tanh^2(c+dx)} dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(-1+x^2)(a+b-bx^2)} dx, x, \operatorname{sech}(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \operatorname{sech}(c+dx)\right)}{ad} + \frac{b \operatorname{Subst}\left(\int \frac{1}{a+b-bx^2} dx, x, \operatorname{sech}(c+dx)\right)}{ad}$$

$$= -\frac{\tanh^{-1}(\operatorname{cosh}(c+dx))}{ad} + \frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}d}$$

Mathematica [C] time = 0.21, size = 123, normalized size = 2.24

$$\frac{\sqrt{a+b} \log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + i\sqrt{b} \tan^{-1}\left(\frac{-\sqrt{a} \tanh\left(\frac{1}{2}(c+dx)\right) - i\sqrt{a+b}}{\sqrt{b}}\right) + i\sqrt{b} \tan^{-1}\left(\frac{\sqrt{a} \tanh\left(\frac{1}{2}(c+dx)\right) - i\sqrt{a+b}}{\sqrt{b}}\right)}{ad\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2), x]

[Out] (I*Sqrt[b]*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]] + I*Sqrt[b]*ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]] + Sqrt[a + b]*Log[Tanh[(c + d*x)/2]]/(a*Sqrt[a + b]*d)

fricas [B] time = 0.55, size = 587, normalized size = 10.67

$$\left[\frac{\sqrt{\frac{b}{a+b}} \log\left(\frac{(a+b) \cosh(dx+c)^4 + 4(a+b) \cosh(dx+c) \sinh(dx+c)^3 + (a+b) \sinh(dx+c)^4 + 2(a+3b) \cosh(dx+c)^2 + 2(3(a+b) \cosh(dx+c)^2 + a+3b) \sinh(dx+c)}{(a+b) \cosh(dx+c)^4 + 4(a+b) \cosh(dx+c) \sinh(dx+c)^3 + (a+b) \sinh(dx+c)^4}\right)}{ad} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] [1/2*(sqrt(b/(a + b))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c)))*sqrt(b/(a + b)) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b) - 2*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 2*log(cosh(d*x + c) + sinh(d*x + c) - 1))/(a*d), (sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x + c))*sqrt(-b/(a + b)))/b - sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-b/(a + b)))/b - log(cosh(d*x + c) + sinh(d*x + c) + 1) + log(cosh(d*x + c) + sinh(d*x + c) - 1))/(a*d)]

giac [B] time = 0.21, size = 958, normalized size = 17.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out]
$$-\left(\left(2\left(4a^2b^2 - 4ab^3 - (a^2b - 6ab^2 + b^3)\sqrt{-ab}\right)\sqrt{a^2 - b^2} + 2\sqrt{-ab}(a+b)\right)a^2\operatorname{abs}(ae^{2c} + be^{2c}) - (a^4b - 5a^3b^2 - 5a^2b^3 + ab^4 + 4(a^3b - ab^3)\sqrt{-ab})\sqrt{a^2 - b^2} + 2\sqrt{-ab}(a+b)\operatorname{abs}(ae^{2c} + be^{2c})\operatorname{abs}(a) + (4a^5b - 8a^4b^2 + 4a^3b^3 - (a^5 - 7a^4b + 7a^3b^2 - a^2b^3)\sqrt{-ab})\sqrt{a^2 - b^2} + 2\sqrt{-ab}(a+b)\operatorname{abs}(ae^{2c} + be^{2c})\arctan(e^{dx})/\sqrt{(a^2e^{2c} - abe^{2c} + \sqrt{(a^2e^{2c} - abe^{2c})^2 - (a^2e^{4c} + abe^{4c})(a^2 + ab)})}/(a^2e^{4c} + abe^{4c}))\right)e^{-2c}/((a^8 - 2a^7b - 17a^6b^2 - 28a^5b^3 - 17a^4b^4 - 2a^3b^5 + a^2b^6 + 4(a^7 + 3a^6b + 2a^5b^2 - 2a^4b^3 - 3a^3b^4 - a^2b^5)\sqrt{-ab})\operatorname{abs}(a) + (2(4a^2b^2 - 4ab^3 + (a^2b - 6ab^2 + b^3)\sqrt{-ab})\sqrt{a^2 - b^2} - 2\sqrt{-ab}(a+b))a^2\operatorname{abs}(ae^{2c} + be^{2c}) - (a^4b - 5a^3b^2 - 5a^2b^3 + ab^4 - 4(a^3b - ab^3)\sqrt{-ab})\sqrt{a^2 - b^2} - 2\sqrt{-ab}(a+b)\operatorname{abs}(ae^{2c} + be^{2c})\operatorname{abs}(a) + (4a^5b - 8a^4b^2 + 4a^3b^3 + (a^5 - 7a^4b + 7a^3b^2 - a^2b^3)\sqrt{-ab})\sqrt{a^2 - b^2} - 2\sqrt{-ab}(a+b)\operatorname{abs}(ae^{2c} + be^{2c})\arctan(e^{dx})/\sqrt{(a^2e^{2c} - abe^{2c} - \sqrt{(a^2e^{2c} - abe^{2c})^2 - (a^2e^{4c} + abe^{4c})(a^2 + ab)})}/(a^2e^{4c} + abe^{4c}))\right)e^{-2c}/((a^8 - 2a^7b - 17a^6b^2 - 28a^5b^3 - 17a^4b^4 - 2a^3b^5 + a^2b^6 - 4(a^7 + 3a^6b + 2a^5b^2 - 2a^4b^3 - 3a^3b^4 - a^2b^5)\sqrt{-ab})\operatorname{abs}(a) + \log(e^{dx+c} + 1)/a - \log(\operatorname{abs}(e^{dx+c} - 1))/a)/d$$

maple [A] time = 0.39, size = 69, normalized size = 1.25

$$\frac{b \operatorname{arctanh}\left(\frac{2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)_{a+2a+4b}}{4\sqrt{ab+b^2}}\right)}{da\sqrt{ab+b^2}} + \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)/(a+b*tanh(d*x+c)^2),x)

[Out]
$$1/d/a*b/(a+b^2)^{(1/2)}*\operatorname{arctanh}(1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a+b^2)^{(1/2}))+1/d/a*\ln(\tanh(1/2*d*x+1/2*c))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\log\left(\left(e^{(dx+c)} + 1\right)e^{(-c)}\right)}{ad} + \frac{\log\left(\left(e^{(dx+c)} - 1\right)e^{(-c)}\right)}{ad} - 2 \int \frac{be^{(3dx+3c)} - be^{(dx+c)}}{a^2 + ab + (a^2e^{4c} + abe^{4c})e^{4dx} + 2(a^2e^{2c} - abe^{2c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out]
$$-\log((e^{dx+c} + 1)*e^{(-c)})/(a*d) + \log((e^{dx+c} - 1)*e^{(-c)})/(a*d) - 2*\operatorname{integrate}((b*e^{(3*d*x + 3*c)} - b*e^{(d*x + c)})/(a^2 + a*b + (a^2*e^{(4*c)} + a*b*e^{(4*c)})*e^{(4*d*x)} + 2*(a^2*e^{(2*c)} - a*b*e^{(2*c)})*e^{(2*d*x)}), x)$$

mupad [B] time = 1.92, size = 284, normalized size = 5.16

$$\frac{2 \operatorname{atan}\left(\frac{e^{dx} e^c (9b^4 \sqrt{-a^2 d^2} + 16a^2 b^2 \sqrt{-a^2 d^2} + 24ab^3 \sqrt{-a^2 d^2})}{16da^3 b^2 + 24da^2 b^3 + 9da b^4}\right)}{\sqrt{-a^2 d^2}} \sqrt{b} \left(2 \operatorname{atan}\left(\frac{e^{dx} e^c \sqrt{-a^3 d^2 - b a^2 d^2} \sqrt{-a^2 d^2 (a+b)} + e^{3c} e^{3dx} \sqrt{-a^2 d^2}}{2a \sqrt{b} d \sqrt{-a^2 d^2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)),x)`

[Out]
$$-\frac{(2*\operatorname{atan}(\frac{\exp(d*x)*\exp(c)*(9*b^4*(-a^2*d^2)^{(1/2)} + 16*a^2*b^2*(-a^2*d^2)^{(1/2)} + 24*a*b^3*(-a^2*d^2)^{(1/2)})}{24*a^2*b^3*d + 16*a^3*b^2*d + 9*a*b^4*d}))/(-a^2*d^2)^{(1/2)} - (b^{(1/2)}*(2*\operatorname{atan}(\frac{\exp(d*x)*\exp(c)*(-a^3*d^2 - a^2*b*d^2)^{(1/2)}*(-a^2*d^2*(a + b))^{(1/2)} + \exp(3*c)*\exp(3*d*x)*(-a^3*d^2 - a^2*b*d^2)^{(1/2)}*(-a^2*d^2*(a + b))^{(1/2)} + 4*a^2*b*d^2*\exp(d*x)*\exp(c))}{2*a*b^{(1/2)*d*(-a^2*d^2*(a + b))^{(1/2)}})) - 2*\operatorname{atan}(\frac{\exp(d*x)*\exp(c)*(-a^2*d^2*(a + b))^{(1/2)}}{2*a*b^{(1/2)*d}}))}{2*(-a^3*d^2 - a^2*b*d^2)^{(1/2)}}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(c + dx)}{a + b \tanh^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)/(a+b*tanh(d*x+c)**2),x)`

[Out] `Integral(csch(c + d*x)/(a + b*tanh(c + d*x)**2), x)`

$$3.30 \quad \int \frac{\operatorname{csch}^2(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=48

$$-\frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\operatorname{coth}(c+dx)}{ad}$$

[Out] $-\operatorname{coth}(d*x+c)/a/d-\arctan(b^{(1/2)}*\tanh(d*x+c)/a^{(1/2)})*b^{(1/2)}/a^{(3/2)}/d$

Rubi [A] time = 0.06, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3663, 325, 205}

$$-\frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\operatorname{coth}(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2), x]

[Out] $-\left(\frac{\sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Tanh}[c + d*x]}{\sqrt{a}}\right]}{a^{3/2}d}\right) - \frac{\operatorname{Coth}[c + d*x]}{a*d}$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 325

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 3663

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m+1))/f, Subst[Int[(x^m*(a+b*(ff*x)^n)^p]/(c^2+ff^2*x^2)^(m/2+1), x], x, (c*Tan[e+f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{csch}^2(c+dx)}{a+b \tanh^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx^2)} dx, x, \tanh(c+dx)\right)}{d} \\ &= -\frac{\operatorname{coth}(c+dx)}{ad} - \frac{b \operatorname{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \tanh(c+dx)\right)}{ad} \\ &= -\frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\operatorname{coth}(c+dx)}{ad} \end{aligned}$$

Mathematica [A] time = 0.13, size = 48, normalized size = 1.00

$$\frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2), x]

[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*d)) - Coth[c + d*x]/(a*d)

fricas [B] time = 0.54, size = 618, normalized size = 12.88

$$\left[\frac{(\cosh(dx+c)^2 + 2 \cosh(dx+c) \sinh(dx+c) + \sinh(dx+c)^2 - 1) \sqrt{-\frac{b}{a}} \log\left(\frac{(a^2+2ab+b^2) \cosh(dx+c)^4 + 4(a^2+2ab+b^2)}{\dots}\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] [1/2*((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c)*sinh(d*x + c) + a + b)) - 4)/(a*d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2 - a*d), -((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) + 2)/(a*d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2 - a*d)]

giac [A] time = 0.35, size = 69, normalized size = 1.44

$$\frac{b \arctan\left(\frac{ae^{(2dx+2c)+be^{(2dx+2c)+a-b}}}{2\sqrt{ab}}\right)}{\sqrt{ab}a} + \frac{2}{a(e^{(2dx+2c)-1})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2), x, algorithm="giac")

[Out] -(b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b)))/(sqrt(a*b)*a) + 2/(a*(e^(2*d*x + 2*c) - 1))/d

maple [B] time = 0.36, size = 413, normalized size = 8.60

$$\frac{\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{2da} - \frac{1}{2da \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)} + \frac{b \operatorname{arctanh}\left(\frac{a \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}}\right)}{d\sqrt{b(a+b)} \sqrt{(2\sqrt{b(a+b)} - a - 2b)a}} - \frac{b \operatorname{arctanh}\left(\frac{a \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}}\right)}{da\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2),x)`

[Out]
$$-1/2/d/a*\tanh(1/2*d*x+1/2*c)-1/2/d/a/\tanh(1/2*d*x+1/2*c)+1/d*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/d*b/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))*b^2+1/d*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/d*b/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))*b^2$$

maxima [A] time = 0.43, size = 62, normalized size = 1.29

$$\frac{b \operatorname{arctan}\left(\frac{(a+b)e^{(-2dx-2c)+a-b}}{2\sqrt{ab}}\right)}{\sqrt{ab} ad} + \frac{2}{(ae^{(-2dx-2c)} - a)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")`

[Out]
$$b*\operatorname{arctan}(1/2*((a + b)*e^{(-2*d*x - 2*c)} + a - b)/\operatorname{sqrt}(a*b))/(\operatorname{sqrt}(a*b)*a*d) + 2/((a*e^{(-2*d*x - 2*c)} - a)*d)$$

mupad [B] time = 1.31, size = 136, normalized size = 2.83

$$\frac{2}{ad - a d e^{2c+2dx}} - \frac{\sqrt{b} \operatorname{atan}\left(\frac{\sqrt{a^3 d^2}}{2a \sqrt{b} d} - \frac{\sqrt{b} \sqrt{a^3 d^2}}{2a^2 d} + \frac{e^{2c} e^{2dx} \sqrt{a^3 d^2}}{2a \sqrt{b} d} + \frac{\sqrt{b} e^{2c} e^{2dx} \sqrt{a^3 d^2}}{2a^2 d}\right)}{\sqrt{a^3 d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)),x)`

[Out]
$$2/(a*d - a*d*\exp(2*c + 2*d*x)) - (b^(1/2)*\operatorname{atan}((a^3*d^2)^(1/2)/(2*a*b^(1/2)*d) - (b^(1/2)*(a^3*d^2)^(1/2))/(2*a^2*d) + (\exp(2*c)*\exp(2*d*x)*(a^3*d^2)^(1/2))/(2*a*b^(1/2)*d) + (b^(1/2)*\exp(2*c)*\exp(2*d*x)*(a^3*d^2)^(1/2))/(2*a^2*d))/((a^3*d^2)^(1/2))$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(c + dx)}{a + b \tanh^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)`

[Out] `Integral(csch(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)`

$$3.31 \quad \int \frac{\operatorname{csch}^3(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=85

$$\frac{(a+2b) \tanh^{-1}(\cosh(c+dx))}{2a^2d} - \frac{\sqrt{b} \sqrt{a+b} \tanh^{-1}\left(\frac{\sqrt{b} \operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{a^2d} - \frac{\coth(c+dx) \operatorname{csch}(c+dx)}{2ad}$$

[Out] 1/2*(a+2*b)*arctanh(cosh(d*x+c))/a^2/d-1/2*coth(d*x+c)*csch(d*x+c)/a/d-arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)*(a+b)^(1/2)/a^2/d

Rubi [A] time = 0.12, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3664, 471, 522, 207, 208}

$$\frac{(a+2b) \tanh^{-1}(\cosh(c+dx))}{2a^2d} - \frac{\sqrt{b} \sqrt{a+b} \tanh^{-1}\left(\frac{\sqrt{b} \operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{a^2d} - \frac{\coth(c+dx) \operatorname{csch}(c+dx)}{2ad}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2), x]

[Out] ((a + 2*b)*ArcTanh[Cosh[c + d*x]])/(2*a^2*d) - (Sqrt[b]*Sqrt[a + b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(a^2*d) - (Coth[c + d*x]*Csch[c + d*x])/(2*a*d)

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 471

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Simp[(e^(n-1)*(e*x)^(m-n+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(n*(b*c-a*d)*(p+1)), x] - Dist[e^n/(n*(b*c-a*d)*(p+1)), Int[(e*x)^(m-n)*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[c*(m-n+1)+d*(m+n*(p+q+1)+1]*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m-n+1] && GtQ[m-n+1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_.))/(((a_) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.))), x_Symbol] := Dist[(b*e-a*f)/(b*c-a*d), Int[1/(a+b*x^n), x], x] - Dist[(d*e-c*f)/(b*c-a*d), Int[1/(c+d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 3664

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m-1)/2)*(a-b+b*ff^2*x^2)^p]/x^(m+1)

), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{csch}^3(c+dx)}{a+b \tanh^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{x^2}{(-1+x^2)^2(a+bx^2)} dx, x, \operatorname{sech}(c+dx)\right)}{d} \\ &= \frac{\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{\operatorname{Subst}\left(\int \frac{a+bx^2}{(-1+x^2)(a+bx^2)} dx, x, \operatorname{sech}(c+dx)\right)}{2ad} \\ &= \frac{\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{(b(a+b))\operatorname{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \operatorname{sech}(c+dx)\right)}{a^2d} \\ &= \frac{(a+2b)\tanh^{-1}(\cosh(c+dx))}{2a^2d} - \frac{\sqrt{b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{a^2d} - \frac{\operatorname{coth}(c+dx)}{2a^2d} \end{aligned}$$

Mathematica [C] time = 0.70, size = 170, normalized size = 2.00

$$\frac{8i\sqrt{b}\sqrt{a+b}\tan^{-1}\left(\frac{-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a+b}}{\sqrt{b}}\right) + 8i\sqrt{b}\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a+b}}{\sqrt{b}}\right) + \operatorname{acsch}^2\left(\frac{1}{2}(c+dx)\right)}{8a^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2), x]

[Out] $-\frac{1}{8}((8I)\sqrt{b}\sqrt{a+b}\operatorname{ArcTan}((-I)\sqrt{a+b} - \sqrt{a}\operatorname{Tanh}[(c+d*x)/2])/\sqrt{b} + (8I)\sqrt{b}\sqrt{a+b}\operatorname{ArcTan}((-I)\sqrt{a+b} + \sqrt{a}\operatorname{Tanh}[(c+d*x)/2])/\sqrt{b} + a\operatorname{Csch}[(c+d*x)/2]^2 + 4a\operatorname{Log}[\operatorname{Tanh}[(c+d*x)/2]] + 8b\operatorname{Log}[\operatorname{Tanh}[(c+d*x)/2]] + a\operatorname{Sech}[(c+d*x)/2]^2)/(a^2d)$

fricas [B] time = 1.27, size = 1790, normalized size = 21.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] $[-1/2*(2*a*\cosh(d*x+c)^3 + 6*a*\cosh(d*x+c)*\sinh(d*x+c)^2 + 2*a*\sinh(d*x+c)^3 - (\cosh(d*x+c)^4 + 4*\cosh(d*x+c)*\sinh(d*x+c)^3 + \sinh(d*x+c)^4 + 2*(3*\cosh(d*x+c)^2 - 1)*\sinh(d*x+c)^2 - 2*\cosh(d*x+c)^2 + 4*(\cosh(d*x+c)^3 - \cosh(d*x+c))*\sinh(d*x+c) + 1)*\sqrt{a*b+b^2}*\log(((a+b)*\cosh(d*x+c)^4 + 4*(a+b)*\cosh(d*x+c)*\sinh(d*x+c)^3 + (a+b)*\sinh(d*x+c)^4 + 2*(a+3*b)*\cosh(d*x+c)^2 + 2*(3*(a+b)*\cosh(d*x+c)^2 + a+3*b)*\sinh(d*x+c)^2 + 4*((a+b)*\cosh(d*x+c)^3 + (a+3*b)*\cosh(d*x+c))*\sinh(d*x+c) - 4*(\cosh(d*x+c)^3 + 3*\cosh(d*x+c)*\sinh(d*x+c)^2 + \sinh(d*x+c)^3 + (3*\cosh(d*x+c)^2 + 1)*\sinh(d*x+c) + \cosh(d*x+c))*\sqrt{a*b+b^2} + a+b)/((a+b)*\cosh(d*x+c)^4 + 4*(a+b)*\cosh(d*x+c)*\sinh(d*x+c)^3 + (a+b)*\sinh(d*x+c)^4 + 2*(a-b)*\cosh(d*x+c)^2 + 2*(3*(a+b)*\cosh(d*x+c)^2 + a-b)*\sinh(d*x+c)^2 + 4*((a+b)*\cosh(d*x+c)^3 + (a-b)*\cosh(d*x+c))*\sinh(d*x+c) + a+b)] + 2*a*\cosh(d*x+c) - ((a+2*b)*\cosh(d*x+c)^4 + 4*(a+2*b)*\cosh(d*x+c)*\sinh(d*x+c)^3 + (a+2*b)*\sinh(d*x+c)^4 - 2*(a+2*b)*\cosh(d*x+c)^2 + 2*(3*(a+2*b)*\cosh(d*x+c)^2 - a-2*b)*\sinh(d*x+c)^2 + 4*((a+2*b)*\cosh(d*x+c)^3$

```

- (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) + sin
h(d*x + c) + 1) + ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*si
nh(d*x + c)^3 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2
*(3*(a + 2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cos
h(d*x + c)^3 - (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d
*x + c) + sinh(d*x + c) - 1) + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))/
(a^2*d*cosh(d*x + c)^4 + 4*a^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*d*sinh(
d*x + c)^4 - 2*a^2*d*cosh(d*x + c)^2 + a^2*d + 2*(3*a^2*d*cosh(d*x + c)^2 -
a^2*d)*sinh(d*x + c)^2 + 4*(a^2*d*cosh(d*x + c)^3 - a^2*d*cosh(d*x + c))*s
inh(d*x + c)), -1/2*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x + c)^
2 + 2*a*sinh(d*x + c)^3 + 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c
)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(
d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-a
*b - b^2)*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sin
h(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c) + (3*(a +
b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x + c))*sqrt(-a*b - b^2)/(a*b + b^2))
- 2*(cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 +
2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x
+ c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(-a*b - b^2)*arctan(1/2*sqrt
(-a*b - b^2)*(cosh(d*x + c) + sinh(d*x + c))/b) + 2*a*cosh(d*x + c) - ((a
+ 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + 2
*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*(a + 2*b)*cosh(d*x
+ c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cosh(d*x + c)^3 - (a + 2*
b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) + sinh(d*x + c
) + 1) + ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*sinh(d*x +
c)^3 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*(a +
2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cosh(d*x + c
)^3 - (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) +
sinh(d*x + c) - 1) + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))/(a^2*d*cos
h(d*x + c)^4 + 4*a^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*d*sinh(d*x + c)^
4 - 2*a^2*d*cosh(d*x + c)^2 + a^2*d + 2*(3*a^2*d*cosh(d*x + c)^2 - a^2*d)*s
inh(d*x + c)^2 + 4*(a^2*d*cosh(d*x + c)^3 - a^2*d*cosh(d*x + c))*sinh(d*x +
c)))]

```

giac [B] time = 0.38, size = 475, normalized size = 5.59

$$\frac{2(3ab - b^2 - \sqrt{-ab}(a - 3b))|ae^{2c} + be^{2c}| \arctan \left(\frac{e^{dx}}{\sqrt{\frac{a^3 e^{2c} - a^2 b e^{2c} + \sqrt{(a^3 e^{2c} - a^2 b e^{2c})^2 - (a^3 e^{4c} + a^2 b e^{4c})(a^3 + a^2 b)}}{a^3 e^{4c} + a^2 b e^{4c}}}} \right) e^{-2c}}{(a^3 - a^2 b + 2\sqrt{-ab}a^2)\sqrt{a^2 - b^2 + 2\sqrt{-ab}(a + b)}} + \frac{2(3ab - b^2 + \sqrt{-ab}(a - 3b))}{(a^3 - a^2 b + 2\sqrt{-ab}a^2)\sqrt{a^2 - b^2 + 2\sqrt{-ab}(a + b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] 1/2*(2*(3*a*b - b^2 - sqrt(-a*b)*(a - 3*b))*abs(a*e^(2*c) + b*e^(2*c))*arct
an(e^(d*x)/sqrt((a^3*e^(2*c) - a^2*b*e^(2*c) + sqrt((a^3*e^(2*c) - a^2*b*e^(
2*c))^2 - (a^3*e^(4*c) + a^2*b*e^(4*c))*(a^3 + a^2*b)))/(a^3*e^(4*c) + a^2
*b*e^(4*c))))*e^(-2*c)/((a^3 - a^2*b + 2*sqrt(-a*b)*a^2)*sqrt(a^2 - b^2 + 2
*sqrt(-a*b)*(a + b))) + 2*(3*a*b - b^2 + sqrt(-a*b)*(a - 3*b))*abs(a*e^(2*c
) + b*e^(2*c))*arctan(e^(d*x)/sqrt((a^3*e^(2*c) - a^2*b*e^(2*c) - sqrt((a^3
*e^(2*c) - a^2*b*e^(2*c))^2 - (a^3*e^(4*c) + a^2*b*e^(4*c))*(a^3 + a^2*b)))/
(a^3*e^(4*c) + a^2*b*e^(4*c))))*e^(-2*c)/((a^3 - a^2*b - 2*sqrt(-a*b)*a^2)
*sqrt(a^2 - b^2 - 2*sqrt(-a*b)*(a + b))) + (a*e^c + 2*b*e^c)*e^(-c)*log(e^(
d*x + c) + 1)/a^2 - (a*e^c + 2*b*e^c)*e^(-c)*log(abs(e^(d*x + c) - 1))/a^2
- 2*(e^(3*d*x + 3*c) + e^(d*x + c))/(a*(e^(2*d*x + 2*c) - 1)^2))/d

```

maple [B] time = 0.39, size = 181, normalized size = 2.13

$$\frac{\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right) b \operatorname{arctanh}\left(\frac{2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{a+2a+4b}}{4\sqrt{ab+b^2}}\right) b^2 \operatorname{arctanh}\left(\frac{2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{a+2a+4b}}{4\sqrt{ab+b^2}}\right)}{8da} - \frac{1}{8da \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2), x)

[Out] 1/8/d/a*tanh(1/2*d*x+1/2*c)^2-1/d/a*b/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))-1/d*b^2/a^2/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))-1/8/d/a/tanh(1/2*d*x+1/2*c)^2-1/2/d/a*ln(tanh(1/2*d*x+1/2*c))-1/d/a^2*b*ln(tanh(1/2*d*x+1/2*c))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{e^{(3dx+3c)} + e^{(dx+c)}}{ade^{(4dx+4c)} - 2ade^{(2dx+2c)} + ad} + \frac{(a+2b) \log\left(\left(e^{(dx+c)} + 1\right)e^{(-c)}\right)}{2a^2d} - \frac{(a+2b) \log\left(\left(e^{(dx+c)} - 1\right)e^{(-c)}\right)}{2a^2d} + 8 \int \frac{1}{4(a+b \tanh(dx+c))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2), x, algorithm="maxima")

[Out] -(e^(3*d*x + 3*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4*c) - 2*a*d*e^(2*d*x + 2*c) + a*d) + 1/2*(a + 2*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) - 1/2*(a + 2*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^2*d) + 8*integrate(1/4*((a*b*e^(3*c) + b^2*e^(3*c))*e^(3*d*x) - (a*b*e^c + b^2*e^c)*e^(d*x))/(a^3 + a^2*b + (a^3*e^(4*c) + a^2*b*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(2*c) - a^2*b*e^(2*c))*e^(2*d*x)), x)

mapad [B] time = 1.80, size = 787, normalized size = 9.26

$$\frac{\operatorname{atan}\left(\frac{e^{dx} e^c \left(18 b^7 \sqrt{-a^4 d^2} + 48 a^2 b^5 \sqrt{-a^4 d^2} + 27 a^3 b^4 \sqrt{-a^4 d^2} + 8 a^4 b^3 \sqrt{-a^4 d^2} + a^5 b^2 \sqrt{-a^4 d^2} + 45 a b^6 \sqrt{-a^4 d^2}\right)}{9 a^2 b^6 d \sqrt{a^2+4 a b+4 b^2}+18 a^3 b^5 d \sqrt{a^2+4 a b+4 b^2}+15 a^4 b^4 d \sqrt{a^2+4 a b+4 b^2}+6 a^5 b^3 d \sqrt{a^2+4 a b+4 b^2}+a^6 b^2 d \sqrt{a^2+4 a b+4 b^2}}\right)}{\sqrt{-a^4 d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)), x)

[Out] (atan((exp(d*x)*exp(c)*(18*b^7*(-a^4*d^2)^(1/2) + 48*a^2*b^5*(-a^4*d^2)^(1/2) + 27*a^3*b^4*(-a^4*d^2)^(1/2) + 8*a^4*b^3*(-a^4*d^2)^(1/2) + a^5*b^2*(-a^4*d^2)^(1/2) + 45*a*b^6*(-a^4*d^2)^(1/2)))/(9*a^2*b^6*d*(4*a*b + a^2 + 4*b^2)^(1/2) + 18*a^3*b^5*d*(4*a*b + a^2 + 4*b^2)^(1/2) + 15*a^4*b^4*d*(4*a*b + a^2 + 4*b^2)^(1/2) + 6*a^5*b^3*d*(4*a*b + a^2 + 4*b^2)^(1/2) + a^6*b^2*d*(4*a*b + a^2 + 4*b^2)^(1/2)))*(4*a*b + a^2 + 4*b^2)^(1/2))/(-a^4*d^2)^(1/2) - ((2*atan((exp(d*x)*exp(c)*(a + b)*(-a^4*d^2)^(1/2))/(2*a^2*d*(b*(a + b))^(1/2))) + 2*atan((exp(d*x)*exp(c)*((64*(2*a^4*b*d*(a*b + b^2)^(1/2) + 6*a^2*b^3*d*(a*b + b^2)^(1/2) + 6*a^3*b^2*d*(a*b + b^2)^(1/2)))/(a^9*d^2*(a + b)^2*(2*a*b + a^2 + b^2)) - (32*(3*b^4*(-a^4*d^2)^(1/2) + 4*a^2*b^2*(-a^4*d^2)^(1/2) + 6*a*b^3*(-a^4*d^2)^(1/2) + a^3*b*(-a^4*d^2)^(1/2)))/(a^7*d*(a + b)*(-a^4*d^2)^(1/2)*(b*(a + b))^(1/2)*(2*a*b + a^2 + b^2))) - (32*exp(3*c)*exp(3*d*x)*(3*b^4*(-a^4*d^2)^(1/2) + 4*a^2*b^2*(-a^4*d^2)^(1/2) + 6*a*b^3*(-a^4*d^2)^(1/2) + a^3*b*(-a^4*d^2)^(1/2)))/(a^7*d*(a + b)*(-a^4*d^2)^(1/2))

```

*(b*(a + b))^(1/2)*(2*a*b + a^2 + b^2))*(a^8*(-a^4*d^2)^(1/2) + a^5*b^3*(-
a^4*d^2)^(1/2) + 3*a^6*b^2*(-a^4*d^2)^(1/2) + 3*a^7*b*(-a^4*d^2)^(1/2)))/(1
92*a*b^2 + 64*a^2*b + 192*b^3))*(a*b + b^2)^(1/2))/(2*(-a^4*d^2)^(1/2)) -
exp(c + d*x)/(a*d*(exp(2*c + 2*d*x) - 1)) - (2*exp(c + d*x))/(a*d*(exp(4*c
+ 4*d*x) - 2*exp(2*c + 2*d*x) + 1))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(c + dx)}{a + b \tanh^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**3/(a+b*tanh(d*x+c)**2), x)

[Out] Integral(csch(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)

$$3.32 \quad \int \frac{\operatorname{csch}^4(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=70

$$\frac{\sqrt{b}(a+b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{(a+b) \operatorname{coth}(c+dx)}{a^2d} - \frac{\operatorname{coth}^3(c+dx)}{3ad}$$

[Out] (a+b)*coth(d*x+c)/a^2/d-1/3*coth(d*x+c)^3/a/d+(a+b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/b^(1/2)/a^(5/2)/d

Rubi [A] time = 0.09, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3663, 453, 325, 205}

$$\frac{(a+b) \operatorname{coth}(c+dx)}{a^2d} + \frac{\sqrt{b}(a+b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d} - \frac{\operatorname{coth}^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2), x]

[Out] (Sqrt[b]*(a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*d) + ((a + b)*Coth[c + d*x])/(a^2*d) - Coth[c + d*x]^3/(3*a*d)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 325

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 453

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rule 3663

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^4(c+dx)}{a+b \tanh^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^4(a+bx^2)} dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{\operatorname{coth}^3(c+dx)}{3ad} - \frac{(a+b) \operatorname{Subst}\left(\int \frac{1}{x^2(a+bx^2)} dx, x, \tanh(c+dx)\right)}{ad} \\
&= \frac{(a+b) \operatorname{coth}(c+dx)}{a^2d} - \frac{\operatorname{coth}^3(c+dx)}{3ad} + \frac{(b(a+b)) \operatorname{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \tanh(c+dx)\right)}{a^2d} \\
&= \frac{\sqrt{b}(a+b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{(a+b) \operatorname{coth}(c+dx)}{a^2d} - \frac{\operatorname{coth}^3(c+dx)}{3ad}
\end{aligned}$$

Mathematica [A] time = 0.32, size = 71, normalized size = 1.01

$$\frac{3\sqrt{b}(a+b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right) + \sqrt{a} \operatorname{coth}(c+dx) (-\operatorname{acsch}^2(c+dx) + 2a + 3b)}{3a^{5/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2), x]

[Out] (3*Sqrt[b]*(a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*Coth[c + d*x]*(2*a + 3*b - a*Csch[c + d*x]^2))/(3*a^(5/2)*d)

fricas [B] time = 0.62, size = 1628, normalized size = 23.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] [1/6*(12*b*cosh(d*x + c)^4 + 48*b*cosh(d*x + c)*sinh(d*x + c)^3 + 12*b*sinh(d*x + c)^4 - 24*(a + b)*cosh(d*x + c)^2 + 24*(3*b*cosh(d*x + c)^2 - a - b)*sinh(d*x + c)^2 + 3*((a + b)*cosh(d*x + c)^6 + 6*(a + b)*cosh(d*x + c)*sinh(d*x + c)^5 + (a + b)*sinh(d*x + c)^6 - 3*(a + b)*cosh(d*x + c)^4 + 3*(5*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c)^4 + 4*(5*(a + b)*cosh(d*x + c)^3 - 3*(a + b)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)^2 + 3*(5*(a + b)*cosh(d*x + c)^4 - 6*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c)^2 + 6*((a + b)*cosh(d*x + c)^5 - 2*(a + b)*cosh(d*x + c)^3 + (a + b)*cosh(d*x + c))*sinh(d*x + c) - a - b)*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 48*(b*cosh(d*x + c)^3 - (a + b)*cosh(d*x + c))*sinh(d*x + c) + 8*a + 12*b)/(a^2*d*cosh(d*x + c)^6 + 6*a^2*d*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*d*sinh(d*x + c)^6 - 3*a^2*d*cosh(d*x + c)^4 + 3*a^2*d*cosh(d*x + c)^2 + 3*(5*a^2*d*cosh(d*x + c)^2 - a^2*d)*sinh(d*x + c)^4 + 4*(5*a^2*d*cosh(d*x + c)^3 - 3*a^2*d*cosh(d*x + c))*sinh(d*x + c)^3 - a^2*d + 3*(5*a^2*d*cosh(d*x + c)^4 - 6*a^2*d*cosh(d*x + c)^2 + a^2*d)*sinh(d*x +

$c)^2 + 6*(a^2*d*cosh(d*x + c))^5 - 2*a^2*d*cosh(d*x + c)^3 + a^2*d*cosh(d*x + c))*sinh(d*x + c)), 1/3*(6*b*cosh(d*x + c)^4 + 24*b*cosh(d*x + c)*sinh(d*x + c)^3 + 6*b*sinh(d*x + c)^4 - 12*(a + b)*cosh(d*x + c)^2 + 12*(3*b*cosh(d*x + c)^2 - a - b)*sinh(d*x + c)^2 + 3*((a + b)*cosh(d*x + c)^6 + 6*(a + b)*cosh(d*x + c)*sinh(d*x + c)^5 + (a + b)*sinh(d*x + c)^6 - 3*(a + b)*cosh(d*x + c)^4 + 3*(5*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c)^4 + 4*(5*(a + b)*cosh(d*x + c)^3 - 3*(a + b)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)^2 + 3*(5*(a + b)*cosh(d*x + c)^4 - 6*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c)^2 + 6*((a + b)*cosh(d*x + c)^5 - 2*(a + b)*cosh(d*x + c)^3 + (a + b)*cosh(d*x + c))*sinh(d*x + c) - a - b)*sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) + 24*(b*cosh(d*x + c)^3 - (a + b)*cosh(d*x + c))*sinh(d*x + c) + 4*a + 6*b)/(a^2*d*cosh(d*x + c)^6 + 6*a^2*d*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*d*sinh(d*x + c)^6 - 3*a^2*d*cosh(d*x + c)^4 + 3*a^2*d*cosh(d*x + c)^2 + 3*(5*a^2*d*cosh(d*x + c)^2 - a^2*d)*sinh(d*x + c)^4 + 4*(5*a^2*d*cosh(d*x + c)^3 - 3*a^2*d*cosh(d*x + c))*sinh(d*x + c)^3 - a^2*d + 3*(5*a^2*d*cosh(d*x + c)^4 - 6*a^2*d*cosh(d*x + c)^2 + a^2*d)*sinh(d*x + c)^2 + 6*(a^2*d*cosh(d*x + c))^5 - 2*a^2*d*cosh(d*x + c)^3 + a^2*d*cosh(d*x + c))*sinh(d*x + c))]$

giac [B] time = 0.36, size = 132, normalized size = 1.89

$$\frac{3 \left(a b e^{2c} + b^2 e^{2c} \right) \arctan \left(\frac{a e^{2dx+2c} + b e^{2dx+2c} + a - b}{2 \sqrt{ab}} \right) e^{-2c}}{\sqrt{ab} a^2} + \frac{2 \left(3 b e^{4dx+4c} - 6 a e^{2dx+2c} - 6 b e^{2dx+2c} + 2 a + 3 b \right)}{a^2 \left(e^{2dx+2c} - 1 \right)^3}$$

$$3d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] 1/3*(3*(a*b*e^(2*c) + b^2*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))*e^(-2*c)/(sqrt(a*b)*a^2) + 2*(3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) - 6*b*e^(2*d*x + 2*c) + 2*a + 3*b)/(a^2*(e^(2*d*x + 2*c) - 1)^3))/d

maple [B] time = 0.37, size = 750, normalized size = 10.71

$$-\frac{\tanh^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{24da} + \frac{3 \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{8da} + \frac{\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)b}{2d a^2} - \frac{b \operatorname{arctanh}\left(\frac{a \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}}\right)}{d\sqrt{b(a+b)}\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}} + \frac{b \operatorname{arctanh}\left(\frac{a \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(2\sqrt{b(a+b)} + a + 2b)a}}\right)}{da\sqrt{(2\sqrt{b(a+b)} + a + 2b)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2),x)

[Out] -1/24/d/a*tanh(1/2*d*x+1/2*c)^3+3/8/d/a*tanh(1/2*d*x+1/2*c)+1/2/d/a^2*tanh(1/2*d*x+1/2*c)*b-1/d*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*b/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-2/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/d*b/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/d*b/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-2/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))*b^2+1/d/a^2*b^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/d/a^2*b^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))

$2*b)*a)^{(1/2)}-1/d/a^2*b^2/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}-1/d/a^2*b^3/(b*(a+b))^{(1/2)}/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}-1/24/d/a/\tanh(1/2*d*x+1/2*c)^3+3/8/d/a/\tanh(1/2*d*x+1/2*c)+1/2/d*b/a^2/\tanh(1/2*d*x+1/2*c)$

maxima [B] time = 0.46, size = 134, normalized size = 1.91

$$\frac{2(6(a+b)e^{(-2dx-2c)} - 3be^{(-4dx-4c)} - 2a - 3b)}{3(a^2e^{(-2dx-2c)} - 3a^2e^{(-4dx-4c)} + a^2e^{(-6dx-6c)} - a^2)d} - \frac{(ab + b^2) \arctan\left(\frac{(a+b)e^{(-2dx-2c)+a-b}}{2\sqrt{ab}}\right)}{\sqrt{ab} a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2), x, algorithm="maxima")

[Out] $2/3*(6*(a + b)*e^{(-2*d*x - 2*c)} - 3*b*e^{(-4*d*x - 4*c)} - 2*a - 3*b)/((3*a^2 *e^{(-2*d*x - 2*c)} - 3*a^2*e^{(-4*d*x - 4*c)} + a^2*e^{(-6*d*x - 6*c)} - a^2)*d) - (a*b + b^2)*\arctan(1/2*((a + b)*e^{(-2*d*x - 2*c)} + a - b)/\sqrt{a*b})/(\sqrt{a*b}*a^2*d)$

mupad [B] time = 1.44, size = 254, normalized size = 3.63

$$\frac{2b}{a^2 d (e^{2c+2dx} - 1)} - \frac{8}{3 a d (3 e^{2c+2dx} - 3 e^{4c+4dx} + e^{6c+6dx} - 1)} - \frac{4}{a d (e^{4c+4dx} - 2 e^{2c+2dx} + 1)} + \frac{\sqrt{-b} \ln\left(-\frac{4be^2}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)), x)

[Out] $(2*b)/(a^2*d*(\exp(2*c + 2*d*x) - 1)) - 8/(3*a*d*(3*\exp(2*c + 2*d*x) - 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) - 1)) - 4/(a*d*(\exp(4*c + 4*d*x) - 2*\exp(2*c + 2*d*x) + 1)) + ((-b)^{(1/2)}*\log(- (4*b*\exp(2*c + 2*d*x))/a^2 - (2*(-b)^{(1/2)}*(a*d + b*d + a*d*\exp(2*c + 2*d*x) - b*d*\exp(2*c + 2*d*x)))/(a^{(5/2)*d})*(a + b)))/(2*a^{(5/2)*d} - ((-b)^{(1/2)}*\log((2*(-b)^{(1/2)}*(a*d + b*d + a*d*\exp(2*c + 2*d*x) - b*d*\exp(2*c + 2*d*x)))/(a^{(5/2)*d} - (4*b*\exp(2*c + 2*d*x))/a^2)*(a + b)))/(2*a^{(5/2)*d}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(c + dx)}{a + b \tanh^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**4/(a+b*tanh(d*x+c)**2), x)

[Out] Integral(csch(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)

$$3.33 \quad \int \frac{\sinh^4(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=192

$$\frac{3x(a^2 - 6ab + b^2)}{8(a+b)^4} + \frac{3b(3a-b) \tanh(c+dx)}{8d(a+b)^3(a+b \tanh^2(c+dx))} + \frac{3\sqrt{a}\sqrt{b}(a-b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2d(a+b)^4} + \frac{\sinh(c+dx) \cosh(c+dx)}{4d(a+b)(a+b \tanh^2(c+dx))}$$

[Out] 3/8*(a^2-6*a*b+b^2)*x/(a+b)^4+3/2*(a-b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*a^(1/2)*b^(1/2)/(a+b)^4/d-1/8*(5*a-b)*cosh(d*x+c)*sinh(d*x+c)/(a+b)^2/d/(a+b*tanh(d*x+c)^2)+1/4*cosh(d*x+c)^3*sinh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c)^2)+3/8*(3*a-b)*b*tanh(d*x+c)/(a+b)^3/d/(a+b*tanh(d*x+c)^2)

Rubi [A] time = 0.25, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3663, 470, 527, 522, 206, 205}

$$\frac{3x(a^2 - 6ab + b^2)}{8(a+b)^4} + \frac{3b(3a-b) \tanh(c+dx)}{8d(a+b)^3(a+b \tanh^2(c+dx))} + \frac{3\sqrt{a}\sqrt{b}(a-b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2d(a+b)^4} + \frac{\sinh(c+dx) \cosh(c+dx)}{4d(a+b)(a+b \tanh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (3*(a^2 - 6*a*b + b^2)*x)/(8*(a + b)^4) + (3*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*(a + b)^4*d) - ((5*a - b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*(a + b)^2*d*(a + b*Tanh[c + d*x]^2)) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*(a + b)*d*(a + b*Tanh[c + d*x]^2)) + (3*(3*a - b)*b*Tanh[c + d*x])/(8*(a + b)^3*d*(a + b*Tanh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,

c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sinh^4(c + dx)}{(a + b \tanh^2(c + dx))^2} dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^3(a+bx^2)^2} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{\cosh^3(c + dx) \sinh(c + dx)}{4(a + b)d(a + b \tanh^2(c + dx))} - \frac{\text{Subst}\left(\int \frac{a+(4a-b)x^2}{(1-x^2)^2(a+bx^2)^2} dx, x, \tanh(c + dx)\right)}{4(a + b)d}$$

$$= -\frac{(5a - b) \cosh(c + dx) \sinh(c + dx)}{8(a + b)^2d(a + b \tanh^2(c + dx))} + \frac{\cosh^3(c + dx) \sinh(c + dx)}{4(a + b)d(a + b \tanh^2(c + dx))} - \frac{\text{Subst}\left(\int \frac{3(3a-b)x^2}{(1-x^2)^2(a+bx^2)^2} dx, x, \tanh(c + dx)\right)}{8(a + b)d}$$

$$= -\frac{(5a - b) \cosh(c + dx) \sinh(c + dx)}{8(a + b)^2d(a + b \tanh^2(c + dx))} + \frac{\cosh^3(c + dx) \sinh(c + dx)}{4(a + b)d(a + b \tanh^2(c + dx))} + \frac{3(3a - b)}{8(a + b)d}$$

$$= -\frac{(5a - b) \cosh(c + dx) \sinh(c + dx)}{8(a + b)^2d(a + b \tanh^2(c + dx))} + \frac{\cosh^3(c + dx) \sinh(c + dx)}{4(a + b)d(a + b \tanh^2(c + dx))} + \frac{3(3a - b)}{8(a + b)d}$$

$$= \frac{3(a^2 - 6ab + b^2)x}{8(a + b)^4} + \frac{3\sqrt{a}(a - b)\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2(a + b)^4d} - \frac{(5a - b) \cosh(c + dx) \sinh(c + dx)}{8(a + b)^2d(a + b \tanh^2(c + dx))}$$

Mathematica [A] time = 1.04, size = 132, normalized size = 0.69

$$\frac{12(a^2 - 6ab + b^2)(c + dx) + (a + b)^2 \sinh(4(c + dx)) - 8(a - b)(a + b) \sinh(2(c + dx)) + 48\sqrt{a} \sqrt{b} (a - b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{32d(a + b)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
[Out] (12*(a^2 - 6*a*b + b^2)*(c + d*x) + 48*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] - 8*(a - b)*(a + b)*Sinh[2*(c + d*x)] + (16*a*b
```

$(a + b) \cdot \text{Sinh}[2 \cdot (c + d \cdot x)] / (a - b + (a + b) \cdot \text{Cosh}[2 \cdot (c + d \cdot x)]) + (a + b)^2 \cdot \text{Sinh}[4 \cdot (c + d \cdot x)] / (32 \cdot (a + b)^4 \cdot d)$

fricas [B] time = 1.08, size = 7366, normalized size = 38.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] $[1/64 \cdot ((a^3 + 3a^2b + 3ab^2 + b^3) \cdot \cosh(dx + c)^{12} + 12(a^3 + 3a^2b + 3ab^2 + b^3) \cdot \cosh(dx + c) \cdot \sinh(dx + c)^{11} + (a^3 + 3a^2b + 3ab^2 + b^3) \cdot \sinh(dx + c)^{12} - 6(a^3 + a^2b - ab^2 - b^3) \cdot \cosh(dx + c)^{10} - 6(a^3 + a^2b - ab^2 - b^3 - 11(a^3 + 3a^2b + 3ab^2 + b^3) \cdot \cosh(dx + c)^2) \cdot \sinh(dx + c)^{10} + 20(11(a^3 + 3a^2b + 3ab^2 + b^3) \cdot \cosh(dx + c)^3 - 3(a^3 + a^2b - ab^2 - b^3) \cdot \cosh(dx + c)) \cdot \sinh(dx + c)^9 - (15a^3 - 19a^2b - 19ab^2 + 15b^3 - 24(a^3 - 5a^2b - 5ab^2 + b^3) \cdot dx) \cdot \cosh(dx + c)^8 + (495(a^3 + 3a^2b + 3ab^2 + b^3) \cdot \cosh(dx + c)^4 - 15a^3 + 19a^2b + 19ab^2 - 15b^3 + 24(a^3 - 5a^2b - 5ab^2 + b^3) \cdot dx) \cdot \sinh(dx + c)^8 + 8(99(a^3 + 3a^2b + 3ab^2 + b^3) \cdot \cosh(dx + c)^5 - 90(a^3 + a^2b - ab^2 - b^3) \cdot \cosh(dx + c)^3 - (15a^3 - 19a^2b - 19ab^2 + 15b^3 - 24(a^3 - 5a^2b - 5ab^2 + b^3) \cdot dx) \cdot \cosh(dx + c)) \cdot \sinh(dx + c)^7 - 16(4a^2b - 4ab^2 - 3(a^3 - 7a^2b + 7ab^2 - b^3) \cdot dx) \cdot \cosh(dx + c)^6 + 4(231(a^3 + 3a^2b + 3ab^2 + b^3) \cdot \cosh(dx + c)^6 - 315(a^3 + a^2b - ab^2 - b^3) \cdot \cosh(dx + c)^4 - 16a^2b + 16ab^2 + 12(a^3 - 7a^2b + 7ab^2 - b^3) \cdot dx - 7(15a^3 - 19a^2b - 19ab^2 + 15b^3 - 24(a^3 - 5a^2b - 5ab^2 + b^3) \cdot dx) \cdot \cosh(dx + c)^2) \cdot \sinh(dx + c)^6 + 8(99(a^3 + 3a^2b + 3ab^2 + b^3) \cdot \cosh(dx + c)^7 - 189(a^3 + a^2b - ab^2 - b^3) \cdot \cosh(dx + c)^5 - 7(15a^3 - 19a^2b - 19ab^2 + 15b^3 - 24(a^3 - 5a^2b - 5ab^2 + b^3) \cdot dx) \cdot \cosh(dx + c)^3 - 12(4a^2b - 4ab^2 - 3(a^3 - 7a^2b + 7ab^2 - b^3) \cdot dx) \cdot \cosh(dx + c)) \cdot \sinh(dx + c)^5 + (15a^3 - 83a^2b - 83ab^2 + 15b^3 + 24(a^3 - 5a^2b - 5ab^2 + b^3) \cdot dx) \cdot \cosh(dx + c)^4 + (495(a^3 + 3a^2b + 3ab^2 + b^3) \cdot \cosh(dx + c)^8 - 1260(a^3 + a^2b - ab^2 - b^3) \cdot \cosh(dx + c)^6 - 70(15a^3 - 19a^2b - 19ab^2 + 15b^3 - 24(a^3 - 5a^2b - 5ab^2 + b^3) \cdot dx) \cdot \cosh(dx + c)^4 + 15a^3 - 83a^2b - 83ab^2 + 15b^3 + 24(a^3 - 5a^2b - 5ab^2 + b^3) \cdot dx) \cdot \sinh(dx + c)^4 + 4(55(a^3 + 3a^2b + 3ab^2 + b^3) \cdot \cosh(dx + c)^9 - 180(a^3 + a^2b - ab^2 - b^3) \cdot \cosh(dx + c)^7 - 14(15a^3 - 19a^2b - 19ab^2 + 15b^3 - 24(a^3 - 5a^2b - 5ab^2 + b^3) \cdot dx) \cdot \cosh(dx + c)^5 - 80(4a^2b - 4ab^2 - 3(a^3 - 7a^2b + 7ab^2 - b^3) \cdot dx) \cdot \cosh(dx + c)^3 + (15a^3 - 83a^2b - 83ab^2 + 15b^3 + 24(a^3 - 5a^2b - 5ab^2 + b^3) \cdot dx) \cdot \cosh(dx + c)) \cdot \sinh(dx + c)^3 - a^3 - 3a^2b - 3ab^2 - b^3 + 6(a^3 + a^2b - ab^2 - b^3) \cdot \cosh(dx + c)^2 + 2(33(a^3 + 3a^2b + 3ab^2 + b^3) \cdot \cosh(dx + c)^{10} - 135(a^3 + a^2b - ab^2 - b^3) \cdot \cosh(dx + c)^8 - 14(15a^3 - 19a^2b - 19ab^2 + 15b^3 - 24(a^3 - 5a^2b - 5ab^2 + b^3) \cdot dx) \cdot \cosh(dx + c)^6 - 120(4a^2b - 4ab^2 - 3(a^3 - 7a^2b + 7ab^2 - b^3) \cdot dx) \cdot \cosh(dx + c)^4 + 3a^3 + 3a^2b - 3ab^2 - 3b^3 + 3(15a^3 - 83a^2b - 83ab^2 + 15b^3 + 24(a^3 - 5a^2b - 5ab^2 + b^3) \cdot dx) \cdot \cosh(dx + c)^2) \cdot \sinh(dx + c)^2 - 48((a^2 - b^2) \cdot \cosh(dx + c)^8 + 8(a^2 - b^2) \cdot \cosh(dx + c) \cdot \sinh(dx + c)^7 + (a^2 - b^2) \cdot \sinh(dx + c)^8 + 2(a^2 - 2ab + b^2) \cdot \cosh(dx + c)^6 + 2(14(a^2 - b^2) \cdot \cosh(dx + c)^2 + a^2 - 2ab + b^2) \cdot \sinh(dx + c)^6 + 4(14(a^2 - b^2) \cdot \cosh(dx + c)^3 + 3(a^2 - 2ab + b^2) \cdot \cosh(dx + c)) \cdot \sinh(dx + c)^5 + (a^2 - b^2) \cdot \cosh(dx + c)^4 + (70(a^2 - b^2) \cdot \cosh(dx + c)^4 + 30(a^2 - 2ab + b^2) \cdot \cosh(dx + c)^2 + a^2 - b^2) \cdot \sinh(dx + c)^4 + 4(14(a^2 - b^2) \cdot \cosh(dx + c)^5 + 10(a^2 - 2ab + b^2) \cdot \cosh(dx + c)^3 + (a^2 - b^2) \cdot \cosh(dx + c)) \cdot \sinh(dx + c)^3 + 2(14(a^2 - b^2) \cdot \cosh(dx + c)^6 + 15(a^2 - 2ab + b^2) \cdot \cosh(dx + c)^4 + 3(a^2 - b^2) \cdot \cosh(dx + c)^2) \cdot \sinh(dx + c)^2$

$$\begin{aligned}
& + 4*(2*(a^2 - b^2)*\cosh(d*x + c)^7 + 3*(a^2 - 2*a*b + b^2)*\cosh(d*x + c)^5 \\
& + (a^2 - b^2)*\cosh(d*x + c)^3*\sinh(d*x + c))*\sqrt{-a*b}*\log(((a^2 + 2*a*b \\
& + b^2)*\cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 \\
& + (a^2 + 2*a*b + b^2)*\sinh(d*x + c)^4 + 2*(a^2 - b^2)*\cosh(d*x + c)^2 + 2 \\
& *(3*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)^2 + a^2 - b^2)*\sinh(d*x + c)^2 + a^2 \\
& - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*\cosh(d*x + c)^3 + (a^2 - b^2)*\cosh(d \\
& *x + c))*\sinh(d*x + c) - 4*((a + b)*\cosh(d*x + c)^2 + 2*(a + b)*\cosh(d*x + \\
& c)*\sinh(d*x + c) + (a + b)*\sinh(d*x + c)^2 + a - b)*\sqrt{-a*b}))/((a + b)*\co \\
& sh(d*x + c)^4 + 4*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a + b)*\sinh(d*x \\
& + c)^4 + 2*(a - b)*\cosh(d*x + c)^2 + 2*(3*(a + b)*\cosh(d*x + c)^2 + a - b)* \\
& \sinh(d*x + c)^2 + 4*((a + b)*\cosh(d*x + c)^3 + (a - b)*\cosh(d*x + c))*\sinh(\\
& d*x + c) + a + b)) + 4*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^11 \\
& - 15*(a^3 + a^2*b - a*b^2 - b^3)*\cosh(d*x + c)^9 - 2*(15*a^3 - 19*a^2*b - 1 \\
& 9*a*b^2 + 15*b^3 - 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*\cosh(d*x + c)^7 \\
& - 24*(4*a^2*b - 4*a*b^2 - 3*(a^3 - 7*a^2*b + 7*a*b^2 - b^3)*d*x)*\cosh(d*x + \\
& c)^5 + (15*a^3 - 83*a^2*b - 83*a*b^2 + 15*b^3 + 24*(a^3 - 5*a^2*b - 5*a*b^ \\
& 2 + b^3)*d*x)*\cosh(d*x + c)^3 + 3*(a^3 + a^2*b - a*b^2 - b^3)*\cosh(d*x + c) \\
&)*\sinh(d*x + c))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) \\
& *d*\cosh(d*x + c)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + \\
& b^5)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^ \\
& 2*b^3 + 5*a*b^4 + b^5)*d*\sinh(d*x + c)^8 + 2*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2 \\
& *a^2*b^3 - 3*a*b^4 - b^5)*d*\cosh(d*x + c)^6 + 2*(14*(a^5 + 5*a^4*b + 10*a^3 \\
& *b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(d*x + c)^2 + (a^5 + 3*a^4*b + 2*a \\
& ^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d)*\sinh(d*x + c)^6 + (a^5 + 5*a^4*b + 1 \\
& 0*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(d*x + c)^4 + 4*(14*(a^5 + 5* \\
& a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(d*x + c)^3 + 3*(a^5 \\
& + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*\cosh(d*x + c))*\sinh(d \\
& *x + c)^5 + (70*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d \\
& *\cosh(d*x + c)^4 + 30*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^ \\
& 5)*d*\cosh(d*x + c)^2 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + \\
& b^5)*d)*\sinh(d*x + c)^4 + 4*(14*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + \\
& 5*a*b^4 + b^5)*d*\cosh(d*x + c)^5 + 10*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b \\
& ^3 - 3*a*b^4 - b^5)*d*\cosh(d*x + c)^3 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^ \\
& 2*b^3 + 5*a*b^4 + b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 2*(14*(a^5 + 5*a^ \\
& 4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(d*x + c)^6 + 15*(a^5 \\
& + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*\cosh(d*x + c)^4 + 3*(a \\
& ^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(d*x + c)^2)* \\
& \sinh(d*x + c)^2 + 4*(2*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + \\
& b^5)*d*\cosh(d*x + c)^7 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^ \\
& 4 - b^5)*d*\cosh(d*x + c)^5 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a \\
& *b^4 + b^5)*d*\cosh(d*x + c)^3)*\sinh(d*x + c)), 1/64*((a^3 + 3*a^2*b + 3*a*b \\
& ^2 + b^3)*\cosh(d*x + c)^12 + 12*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + \\
& c)*\sinh(d*x + c)^11 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\sinh(d*x + c)^12 - 6* \\
& (a^3 + a^2*b - a*b^2 - b^3)*\cosh(d*x + c)^10 - 6*(a^3 + a^2*b - a*b^2 - b^3 \\
& - 11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^10 + 2 \\
& 0*(11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^3 - 3*(a^3 + a^2*b - a* \\
& b^2 - b^3)*\cosh(d*x + c))*\sinh(d*x + c)^9 - (15*a^3 - 19*a^2*b - 19*a*b^2 + \\
& 15*b^3 - 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*\cosh(d*x + c)^8 + (495*(a \\
& ^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^4 - 15*a^3 + 19*a^2*b + 19*a*b^ \\
& 2 - 15*b^3 + 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x - 270*(a^3 + a^2*b - a* \\
& b^2 - b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 8*(99*(a^3 + 3*a^2*b + 3*a*b^ \\
& 2 + b^3)*\cosh(d*x + c)^5 - 90*(a^3 + a^2*b - a*b^2 - b^3)*\cosh(d*x + c)^3 - \\
& (15*a^3 - 19*a^2*b - 19*a*b^2 + 15*b^3 - 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3) \\
&)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^7 - 16*(4*a^2*b - 4*a*b^2 - 3*(a^3 - 7* \\
& a^2*b + 7*a*b^2 - b^3)*d*x)*\cosh(d*x + c)^6 + 4*(231*(a^3 + 3*a^2*b + 3*a*b \\
& ^2 + b^3)*\cosh(d*x + c)^6 - 315*(a^3 + a^2*b - a*b^2 - b^3)*\cosh(d*x + c)^4 \\
& - 16*a^2*b + 16*a*b^2 + 12*(a^3 - 7*a^2*b + 7*a*b^2 - b^3)*d*x - 7*(15*a^3 \\
& - 19*a^2*b - 19*a*b^2 + 15*b^3 - 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*d*x)*c \\
& osh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(99*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cos
\end{aligned}$$

$$\begin{aligned}
& h(dx + c)^7 - 189*(a^3 + a^2*b - a*b^2 - b^3)*\cosh(dx + c)^5 - 7*(15*a^3 \\
& - 19*a^2*b - 19*a*b^2 + 15*b^3 - 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*dx)*\co \\
& sh(dx + c)^3 - 12*(4*a^2*b - 4*a*b^2 - 3*(a^3 - 7*a^2*b + 7*a*b^2 - b^3)*d \\
& *x)*\cosh(dx + c)*\sinh(dx + c)^5 + (15*a^3 - 83*a^2*b - 83*a*b^2 + 15*b^3 \\
& + 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*dx)*\cosh(dx + c)^4 + (495*(a^3 + 3* \\
& a^2*b + 3*a*b^2 + b^3)*\cosh(dx + c)^8 - 1260*(a^3 + a^2*b - a*b^2 - b^3)*c \\
& osh(dx + c)^6 - 70*(15*a^3 - 19*a^2*b - 19*a*b^2 + 15*b^3 - 24*(a^3 - 5*a^ \\
& 2*b - 5*a*b^2 + b^3)*dx)*\cosh(dx + c)^4 + 15*a^3 - 83*a^2*b - 83*a*b^2 + \\
& 15*b^3 + 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*dx - 240*(4*a^2*b - 4*a*b^2 - \\
& 3*(a^3 - 7*a^2*b + 7*a*b^2 - b^3)*dx)*\cosh(dx + c)^2*\sinh(dx + c)^4 + 4 \\
& *(55*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(dx + c)^9 - 180*(a^3 + a^2*b - a \\
& *b^2 - b^3)*\cosh(dx + c)^7 - 14*(15*a^3 - 19*a^2*b - 19*a*b^2 + 15*b^3 - 2 \\
& 4*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*dx)*\cosh(dx + c)^5 - 80*(4*a^2*b - 4*a* \\
& b^2 - 3*(a^3 - 7*a^2*b + 7*a*b^2 - b^3)*dx)*\cosh(dx + c)^3 + (15*a^3 - 83 \\
& *a^2*b - 83*a*b^2 + 15*b^3 + 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*dx)*\cosh(d \\
& *x + c))*\sinh(dx + c)^3 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 6*(a^3 + a^2*b - \\
& a*b^2 - b^3)*\cosh(dx + c)^2 + 2*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(\\
& dx + c)^10 - 135*(a^3 + a^2*b - a*b^2 - b^3)*\cosh(dx + c)^8 - 14*(15*a^3 \\
& - 19*a^2*b - 19*a*b^2 + 15*b^3 - 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*dx)*\co \\
& sh(dx + c)^6 - 120*(4*a^2*b - 4*a*b^2 - 3*(a^3 - 7*a^2*b + 7*a*b^2 - b^3)* \\
& dx)*\cosh(dx + c)^4 + 3*a^3 + 3*a^2*b - 3*a*b^2 - 3*b^3 + 3*(15*a^3 - 83*a \\
& ^2*b - 83*a*b^2 + 15*b^3 + 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*dx)*\cosh(dx \\
& + c)^2*\sinh(dx + c)^2 + 96*((a^2 - b^2)*\cosh(dx + c)^8 + 8*(a^2 - b^2)* \\
& cosh(dx + c)*\sinh(dx + c)^7 + (a^2 - b^2)*\sinh(dx + c)^8 + 2*(a^2 - 2*a* \\
& b + b^2)*\cosh(dx + c)^6 + 2*(14*(a^2 - b^2)*\cosh(dx + c)^2 + a^2 - 2*a*b \\
& + b^2)*\sinh(dx + c)^6 + 4*(14*(a^2 - b^2)*\cosh(dx + c)^3 + 3*(a^2 - 2*a*b \\
& + b^2)*\cosh(dx + c))*\sinh(dx + c)^5 + (a^2 - b^2)*\cosh(dx + c)^4 + (70* \\
& (a^2 - b^2)*\cosh(dx + c)^4 + 30*(a^2 - 2*a*b + b^2)*\cosh(dx + c)^2 + a^2 \\
& - b^2)*\sinh(dx + c)^4 + 4*(14*(a^2 - b^2)*\cosh(dx + c)^5 + 10*(a^2 - 2*a* \\
& b + b^2)*\cosh(dx + c)^3 + (a^2 - b^2)*\cosh(dx + c))*\sinh(dx + c)^3 + 2*(\\
& 14*(a^2 - b^2)*\cosh(dx + c)^6 + 15*(a^2 - 2*a*b + b^2)*\cosh(dx + c)^4 + 3 \\
& *(a^2 - b^2)*\cosh(dx + c)^2)*\sinh(dx + c)^2 + 4*(2*(a^2 - b^2)*\cosh(dx + \\
& c)^7 + 3*(a^2 - 2*a*b + b^2)*\cosh(dx + c)^5 + (a^2 - b^2)*\cosh(dx + c)^3 \\
&)*\sinh(dx + c))*\sqrt{a*b}*\arctan(1/2*((a + b)*\cosh(dx + c)^2 + 2*(a + b)* \\
& cosh(dx + c)*\sinh(dx + c) + (a + b)*\sinh(dx + c)^2 + a - b)*\sqrt{a*b}/(a \\
& *b)) + 4*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(dx + c)^11 - 15*(a^3 + a^ \\
& 2*b - a*b^2 - b^3)*\cosh(dx + c)^9 - 2*(15*a^3 - 19*a^2*b - 19*a*b^2 + 15*b \\
& ^3 - 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*dx)*\cosh(dx + c)^7 - 24*(4*a^2*b \\
& - 4*a*b^2 - 3*(a^3 - 7*a^2*b + 7*a*b^2 - b^3)*dx)*\cosh(dx + c)^5 + (15*a^ \\
& 3 - 83*a^2*b - 83*a*b^2 + 15*b^3 + 24*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*dx)* \\
& cosh(dx + c)^3 + 3*(a^3 + a^2*b - a*b^2 - b^3)*\cosh(dx + c))*\sinh(dx + c \\
&))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(dx + \\
& c)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(d \\
& *x + c)*\sinh(dx + c)^7 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^ \\
& 4 + b^5)*d*\sinh(dx + c)^8 + 2*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a \\
& *b^4 - b^5)*d*\cosh(dx + c)^6 + 2*(14*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2* \\
& b^3 + 5*a*b^4 + b^5)*d*\cosh(dx + c)^2 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2 \\
& *b^3 - 3*a*b^4 - b^5)*d)*\sinh(dx + c)^6 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10 \\
& *a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(dx + c)^4 + 4*(14*(a^5 + 5*a^4*b + 10*a^3 \\
& *b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(dx + c)^3 + 3*(a^5 + 3*a^4*b + 2 \\
& *a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*\cosh(dx + c))*\sinh(dx + c)^5 + (7 \\
& 0*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(dx + c) \\
& ^4 + 30*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*\cosh(dx \\
& + c)^2 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d)*\sinh(\\
& dx + c)^4 + 4*(14*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5 \\
&)*d*\cosh(dx + c)^5 + 10*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - \\
& b^5)*d*\cosh(dx + c)^3 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^ \\
& 4 + b^5)*d*\cosh(dx + c))*\sinh(dx + c)^3 + 2*(14*(a^5 + 5*a^4*b + 10*a^3*b \\
& ^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(dx + c)^6 + 15*(a^5 + 3*a^4*b + 2*
\end{aligned}$$

$$a^3b^2 - 2a^2b^3 - 3ab^4 - b^5) * d * \cosh(dx + c)^4 + 3(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5) * d * \cosh(dx + c)^2 * \sinh(dx + c)^2 + 4(2(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5) * d * \cosh(dx + c)^7 + 3(a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5) * d * \cosh(dx + c)^5 + (a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5) * d * \cosh(dx + c)^3 * \sinh(dx + c))]$$

giac [B] time = 1.91, size = 525, normalized size = 2.73

$$\frac{24(a^2 - 6ab + b^2)dx}{a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4} + \frac{96(a^2be^{2c} - ab^2e^{2c}) \arctan\left(\frac{ae^{2dx+2c} + be^{2dx+2c} + a - b}{2\sqrt{ab}}\right) e^{-2c}}{(a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4)\sqrt{ab}} - \frac{(18a^2e^{4dx+4c} - 108abe^{4dx+4c} + 18b^2e^{4dx+4c} - 8a^4e^{4c} + 4a^3be^{4c} + 6a^2b^2e^{4c})}{a^4e^{4c} + 4a^3be^{4c} + 6a^2b^2e^{4c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)^4/(a+b*tanh(dx+c)^2)^2,x, algorithm="giac")

[Out] 1/64*(24*(a^2 - 6*a*b + b^2)*d*x/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 96*(a^2*b*e^(2*c) - a*b^2*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))*e^(-2*c)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt(a*b)) - (18*a^2*e^(4*d*x + 4*c) - 108*a*b*e^(4*d*x + 4*c) + 18*b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) + 8*b^2*e^(2*d*x + 2*c) + a^2 + 2*a*b + b^2)*e^(-4*d*x)/(a^4*e^(4*c) + 4*a^3*b*e^(4*c) + 6*a^2*b^2*e^(4*c) + 4*a*b^3*e^(4*c) + b^4*e^(4*c)) + (a^2*e^(4*d*x + 28*c) + 2*a*b*e^(4*d*x + 28*c) + b^2*e^(4*d*x + 28*c) - 8*a^2*e^(2*d*x + 26*c) + 8*b^2*e^(2*d*x + 26*c))/(a^4*e^(24*c) + 4*a^3*b*e^(24*c) + 6*a^2*b^2*e^(24*c) + 4*a*b^3*e^(24*c) + b^4*e^(24*c)) - 64*(a^2*b*e^(2*d*x + 2*c) - a*b^2*e^(2*d*x + 2*c) + a^2*b + a*b^2)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b))/d

maple [B] time = 0.35, size = 1246, normalized size = 6.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(dx+c)^4/(a+b*tanh(dx+c)^2)^2,x)

[Out] -3/2/d*a^3*b/(a+b)^4/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+3/2/d*a*b^3/(a+b)^4/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/2/d*a*b^3/(a+b)^4/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/8/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^2*a+7/8/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^2*b-3/8/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)*a+5/8/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)*b-3/8/d/(a+b)^4*ln(tanh(1/2*d*x+1/2*c)-1)*a^2-3/8/d/(a+b)^4*ln(tanh(1/2*d*x+1/2*c)-1)*b^2+1/8/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)^2*a-3/2/d*a^3*b/(a+b)^4/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-3/2/d*a*b^2/(a+b)^4/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/2/d*a*b^2/(a+b)^4/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-7/8/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)^2*b-3/8/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)*a+5/8/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)*b+3/8/d/(a+b)^4*ln(tanh(1/2*d*x+1/2*c)+1)*a^2+3/8/d/(a+b)^4*ln(tanh(1/2*d*x+1/2*c)+1)*b^2+1/d*a^2*b/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)^3+1/d*a*b^2/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)^3+1/d*a^2*b/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2

```
*d*x+1/2*c)+1/d*a*b^2/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)
)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)+3/2/d*a^2*b/(a+b)^4/
((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a
+b))^(1/2)-a-2*b)*a)^(1/2))-3/2/d*a^2*b/(a+b)^4/((2*(b*(a+b))^(1/2)+a+2*b)*
a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+
1/4/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^4+1/2/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-
1)^3-1/4/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)^4+1/2/d/(a+b)^2/(tanh(1/2*d*x+1/
2*c)+1)^3+9/4/d/(a+b)^4*ln(tanh(1/2*d*x+1/2*c)-1)*a*b-9/4/d/(a+b)^4*ln(tanh
(1/2*d*x+1/2*c)+1)*a*b
```

maxima [B] time = 0.72, size = 1690, normalized size = 8.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(a*b - 2*b^2)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c)
+ a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 1/2*b*log((a + b
)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*d) + 1/4*(a*b - 2*b^2)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b
)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d)
+ 1/2*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)
/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 1/32*(3*a^3*b - 33*a^2*b^2 + 13*a*b^
3 + b^4)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^5 + 4*
a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*sqrt(a*b)*d) + 1/8*(3*a^2*b - 6*a*b^
2 - b^3)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^4 + 3*
a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) - 1/32*(3*a^3*b - 33*a^2*b^2 + 13*a
*b^3 + b^4)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^5
+ 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*sqrt(a*b)*d) - 1/8*(3*a^2*b - 6*
a*b^2 - b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^4
+ 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) - 3/16*(3*a*b + b^2)*arctan(1/
2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sq
rt(a*b)*d) - 1/16*(a^3*b - 5*a^2*b^2 - 5*a*b^3 + b^4 + (a^3*b - 15*a^2*b^2
+ 15*a*b^3 - b^4)*e^(2*d*x + 2*c))/((a^6 + 5*a^5*b + 10*a^4*b^2 + 10*a^3*b^
3 + 5*a^2*b^4 + a*b^5 + (a^6 + 5*a^5*b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^
4 + a*b^5)*e^(4*d*x + 4*c) + 2*(a^6 + 3*a^5*b + 2*a^4*b^2 - 2*a^3*b^3 - 3*a
^2*b^4 - a*b^5)*e^(2*d*x + 2*c))*d) + 1/16*(a^3*b - 5*a^2*b^2 - 5*a*b^3 + b
^4 + (a^3*b - 15*a^2*b^2 + 15*a*b^3 - b^4)*e^(-2*d*x - 2*c))/((a^6 + 5*a^5*
b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5 + 2*(a^6 + 3*a^5*b + 2*a^4*
b^2 - 2*a^3*b^3 - 3*a^2*b^4 - a*b^5)*e^(-2*d*x - 2*c) + (a^6 + 5*a^5*b + 10
*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*e^(-4*d*x - 4*c))*d) - 1/4*(a^2*
b - b^3 + (a^2*b - 6*a*b^2 + b^3)*e^(2*d*x + 2*c))/((a^5 + 4*a^4*b + 6*a^3*
b^2 + 4*a^2*b^3 + a*b^4 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*e
^(4*d*x + 4*c) + 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*e^(2*d*x + 2*c))*d)
+ 1/4*(a^2*b - b^3 + (a^2*b - 6*a*b^2 + b^3)*e^(-2*d*x - 2*c))/((a^5 + 4*a^
4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)
*e^(-2*d*x - 2*c) + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*e^(-4*d
*x - 4*c))*d) + 3/8*(a*b + b^2 + (a*b - b^2)*e^(-2*d*x - 2*c))/((a^4 + 3*a^
3*b + 3*a^2*b^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*e^(-2*d*x - 2*c
) + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*e^(-4*d*x - 4*c))*d) + 3/8*(d*x + c
)/((a^2 + 2*a*b + b^2)*d) + 1/64*((a + b)*e^(4*d*x + 4*c) + 16*b*e^(2*d*x +
2*c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/64*(16*b*e^(-2*d*x - 2*c) +
(a + b)*e^(-4*d*x - 4*c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/8*e^(2*d*
x + 2*c)/((a^2 + 2*a*b + b^2)*d) + 1/8*e^(-2*d*x - 2*c)/((a^2 + 2*a*b + b^2
)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(c + dx)^4}{(b \tanh(c + dx)^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^2, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```


$$3.34 \quad \int \frac{\sinh^3(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=124

$$\frac{\cosh^3(c+dx)}{3d(a+b)^2} - \frac{(a-b)\cosh(c+dx)}{d(a+b)^3} + \frac{ab\operatorname{sech}(c+dx)}{2d(a+b)^3(a-b\operatorname{sech}^2(c+dx)+b)} + \frac{\sqrt{b}(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{7/2}}$$

[Out] $-(a-b)*\cosh(d*x+c)/(a+b)^{3/d+1/3}*\cosh(d*x+c)^3/(a+b)^{2/d+1/2}*a*b*\operatorname{sech}(d*x+c)/(a+b)^{3/d}/(a+b*b*\operatorname{sech}(d*x+c)^2)+1/2*(3*a-2*b)*\operatorname{arctanh}(\operatorname{sech}(d*x+c)*b^{(1/2)})/(a+b)^{(1/2)}*b^{(1/2)}/(a+b)^{(7/2)}/d$

Rubi [A] time = 0.22, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3664, 456, 1261, 208}

$$\frac{\cosh^3(c+dx)}{3d(a+b)^2} - \frac{(a-b)\cosh(c+dx)}{d(a+b)^3} + \frac{ab\operatorname{sech}(c+dx)}{2d(a+b)^3(a-b\operatorname{sech}^2(c+dx)+b)} + \frac{\sqrt{b}(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2, x]

[Out] $((3*a - 2*b)*\operatorname{Sqrt}[b]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Sech}[c + d*x])/(\operatorname{Sqrt}[a + b])])/(2*(a + b)^{(7/2)*d}) - ((a - b)*\operatorname{Cosh}[c + d*x])/((a + b)^{3*d}) + \operatorname{Cosh}[c + d*x]^3/(3*(a + b)^{2*d}) + (a*b*\operatorname{Sech}[c + d*x])/((2*(a + b)^{3*d}*(a + b - b*\operatorname{Sech}[c + d*x]^2))$

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 456

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)*x^(-m + 2)]/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 1261

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 3664

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p]/x^(m + 1), x], x, Sec[e + f*x]/ff, x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(c+dx)}{(a+b \tanh^2(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4(a+b-bx^2)^2} dx, x, \text{sech}(c+dx)\right)}{d} \\
&= \frac{ab \text{sech}(c+dx)}{2(a+b)^3 d (a+b-b \text{sech}^2(c+dx))} + \frac{b \text{Subst}\left(\int \frac{-\frac{2}{b(a+b)} + \frac{2ax^2}{b(a+b)^2} + \frac{ax^4}{(a+b)^3}}{x^4(a+b-bx^2)} dx, x, \text{sech}(c+dx)\right)}{2d} \\
&= \frac{ab \text{sech}(c+dx)}{2(a+b)^3 d (a+b-b \text{sech}^2(c+dx))} + \frac{b \text{Subst}\left(\int \left(-\frac{2}{b(a+b)^2 x^4} + \frac{2(a-b)}{b(a+b)^3 x^2} + \frac{3a-b}{(a+b)^3}\right) dx, x, \text{sech}(c+dx)\right)}{2d} \\
&= -\frac{(a-b) \cosh(c+dx)}{(a+b)^3 d} + \frac{\cosh^3(c+dx)}{3(a+b)^2 d} + \frac{ab \text{sech}(c+dx)}{2(a+b)^3 d (a+b-b \text{sech}^2(c+dx))} + \frac{(3a-2b)\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \text{sech}(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{7/2} d} - \frac{(a-b) \cosh(c+dx)}{(a+b)^3 d} + \frac{\cosh^3(c+dx)}{3(a+b)^2 d} + \frac{ab \text{sech}(c+dx)}{2(a+b)^3 d (a+b-b \text{sech}^2(c+dx))}
\end{aligned}$$

Mathematica [C] time = 1.37, size = 160, normalized size = 1.29

$$\frac{3 \cosh(c+dx) \left(a \left(\frac{4b}{(a+b) \cosh(2(c+dx)) + a - b} - 3 \right) + 5b \right)}{(a+b)^3} + \frac{\cosh(3(c+dx))}{(a+b)^2} + \frac{6i\sqrt{b}(3a-2b) \left(\tan^{-1}\left(\frac{-\sqrt{a} \tanh\left(\frac{1}{2}(c+dx)\right) - i\sqrt{a+b}}{\sqrt{b}}\right) + \tan^{-1}\left(\frac{\sqrt{a} \tanh\left(\frac{1}{2}(c+dx)\right) - i\sqrt{a+b}}{\sqrt{b}}\right) \right)}{(a+b)^{7/2}}$$

12d

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2, x]

[Out] (((6*I)*(3*a - 2*b)*Sqrt[b]*(ArcTan[(-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2]]/Sqrt[b]] + ArcTan[(-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2]]/Sqrt[b]))/(a + b)^(7/2) + (3*Cosh[c + d*x]*(5*b + a*(-3 + (4*b)/(a - b + (a + b)*Cosh[2*(c + d*x)])))/(a + b)^3 + Cosh[3*(c + d*x)]/(a + b)^2)/(12*d)

fricas [B] time = 0.75, size = 5025, normalized size = 40.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2, x, algorithm="fricas")

[Out] [1/24*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 + 10*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^10 - (7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^8 + (45*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - 7*a^2 + 6*a*b + 13*b^2)*sinh(d*x + c)^8 + 8*(15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - (7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c)^6 + 2*(105*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 - 14*(7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^2 - 13*a^2 + 40*a*b - 7*b^2)*sinh(d*x + c)^6 + 4*(63*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 - 14*(7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^3 - 3*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c)^4 + 2*(105*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 - 35*(7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^4 - 15*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c)^2 - 13*a^2 + 40*a*b

$$\begin{aligned}
& - 7*b^2)*\sinh(d*x + c)^4 + 8*(15*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)^7 - 7*(7 \\
& *a^2 - 6*a*b - 13*b^2)*\cosh(d*x + c)^5 - 5*(13*a^2 - 40*a*b + 7*b^2)*\cosh(d \\
& *x + c)^3 - (13*a^2 - 40*a*b + 7*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 - (7*a \\
& ^2 - 6*a*b - 13*b^2)*\cosh(d*x + c)^2 + (45*(a^2 + 2*a*b + b^2)*\cosh(d*x + c \\
&)^8 - 28*(7*a^2 - 6*a*b - 13*b^2)*\cosh(d*x + c)^6 - 30*(13*a^2 - 40*a*b + 7 \\
& *b^2)*\cosh(d*x + c)^4 - 12*(13*a^2 - 40*a*b + 7*b^2)*\cosh(d*x + c)^2 - 7*a^ \\
& 2 + 6*a*b + 13*b^2)*\sinh(d*x + c)^2 - 6*((3*a^2 + a*b - 2*b^2)*\cosh(d*x + c \\
&)^7 + 7*(3*a^2 + a*b - 2*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^6 + (3*a^2 + a*b \\
& - 2*b^2)*\sinh(d*x + c)^7 + 2*(3*a^2 - 5*a*b + 2*b^2)*\cosh(d*x + c)^5 + (21* \\
& (3*a^2 + a*b - 2*b^2)*\cosh(d*x + c)^2 + 6*a^2 - 10*a*b + 4*b^2)*\sinh(d*x + \\
& c)^5 + 5*(7*(3*a^2 + a*b - 2*b^2)*\cosh(d*x + c)^3 + 2*(3*a^2 - 5*a*b + 2*b^ \\
& 2)*\cosh(d*x + c))*\sinh(d*x + c)^4 + (3*a^2 + a*b - 2*b^2)*\cosh(d*x + c)^3 + \\
& (35*(3*a^2 + a*b - 2*b^2)*\cosh(d*x + c)^4 + 20*(3*a^2 - 5*a*b + 2*b^2)*\cos \\
& h(d*x + c)^2 + 3*a^2 + a*b - 2*b^2)*\sinh(d*x + c)^3 + (21*(3*a^2 + a*b - 2* \\
& b^2)*\cosh(d*x + c)^5 + 20*(3*a^2 - 5*a*b + 2*b^2)*\cosh(d*x + c)^3 + 3*(3*a^ \\
& 2 + a*b - 2*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^2 + (7*(3*a^2 + a*b - 2*b^2)* \\
& \cosh(d*x + c)^6 + 10*(3*a^2 - 5*a*b + 2*b^2)*\cosh(d*x + c)^4 + 3*(3*a^2 + a \\
& *b - 2*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c))*\sqrt{b/(a + b)}*\log(((a + b)*\co \\
& sh(d*x + c)^4 + 4*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a + b)*\sinh(d*x \\
& + c)^4 + 2*(a + 3*b)*\cosh(d*x + c)^2 + 2*(3*(a + b)*\cosh(d*x + c)^2 + a + 3 \\
& *b)*\sinh(d*x + c)^2 + 4*((a + b)*\cosh(d*x + c)^3 + (a + 3*b)*\cosh(d*x + c)) \\
& *\sinh(d*x + c) - 4*((a + b)*\cosh(d*x + c)^3 + 3*(a + b)*\cosh(d*x + c)*\sinh(\\
& d*x + c)^2 + (a + b)*\sinh(d*x + c)^3 + (a + b)*\cosh(d*x + c) + (3*(a + b)*c \\
& osh(d*x + c)^2 + a + b)*\sinh(d*x + c))*\sqrt{b/(a + b)} + a + b)/((a + b)*\co \\
& sh(d*x + c)^4 + 4*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a + b)*\sinh(d*x \\
& + c)^4 + 2*(a - b)*\cosh(d*x + c)^2 + 2*(3*(a + b)*\cosh(d*x + c)^2 + a - b)* \\
& \sinh(d*x + c)^2 + 4*((a + b)*\cosh(d*x + c)^3 + (a - b)*\cosh(d*x + c))*\sinh(\\
& d*x + c) + a + b)) + a^2 + 2*a*b + b^2 + 2*(5*(a^2 + 2*a*b + b^2)*\cosh(d*x \\
& + c)^9 - 4*(7*a^2 - 6*a*b - 13*b^2)*\cosh(d*x + c)^7 - 6*(13*a^2 - 40*a*b + \\
& 7*b^2)*\cosh(d*x + c)^5 - 4*(13*a^2 - 40*a*b + 7*b^2)*\cosh(d*x + c)^3 - (7*a \\
& ^2 - 6*a*b - 13*b^2)*\cosh(d*x + c))*\sinh(d*x + c))/((a^4 + 4*a^3*b + 6*a^2* \\
& b^2 + 4*a*b^3 + b^4)*d*\cosh(d*x + c)^7 + 7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a \\
& *b^3 + b^4)*d*\cosh(d*x + c)*\sinh(d*x + c)^6 + (a^4 + 4*a^3*b + 6*a^2*b^2 + \\
& 4*a*b^3 + b^4)*d*\sinh(d*x + c)^7 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*\cosh \\
& (d*x + c)^5 + (21*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*\cosh(d*x + \\
& c)^2 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d)*\sinh(d*x + c)^5 + (a^4 + 4*a^3* \\
& b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*\cosh(d*x + c)^3 + 5*(7*(a^4 + 4*a^3*b + 6* \\
& a^2*b^2 + 4*a*b^3 + b^4)*d*\cosh(d*x + c)^3 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b \\
& ^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^4 + (35*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a \\
& *b^3 + b^4)*d*\cosh(d*x + c)^4 + 20*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*\cosh(d \\
& *x + c)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d)*\sinh(d*x + c)^3 \\
& + (21*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*\cosh(d*x + c)^5 + 20*(a \\
& ^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*\cosh(d*x + c)^3 + 3*(a^4 + 4*a^3*b + 6*a^2* \\
& b^2 + 4*a*b^3 + b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^2 + (7*(a^4 + 4*a^3*b + \\
& 6*a^2*b^2 + 4*a*b^3 + b^4)*d*\cosh(d*x + c)^6 + 10*(a^4 + 2*a^3*b - 2*a*b^3 \\
& - b^4)*d*\cosh(d*x + c)^4 + 3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d \\
& *\cosh(d*x + c)^2)*\sinh(d*x + c)), 1/24*((a^2 + 2*a*b + b^2)*\cosh(d*x + c)^1 \\
& 0 + 10*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)*\sinh(d*x + c)^9 + (a^2 + 2*a*b + b \\
& ^2)*\sinh(d*x + c)^10 - (7*a^2 - 6*a*b - 13*b^2)*\cosh(d*x + c)^8 + (45*(a^2 \\
& + 2*a*b + b^2)*\cosh(d*x + c)^2 - 7*a^2 + 6*a*b + 13*b^2)*\sinh(d*x + c)^8 + \\
& 8*(15*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)^3 - (7*a^2 - 6*a*b - 13*b^2)*\cosh(d \\
& *x + c))*\sinh(d*x + c)^7 - 2*(13*a^2 - 40*a*b + 7*b^2)*\cosh(d*x + c)^6 + 2* \\
& (105*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)^4 - 14*(7*a^2 - 6*a*b - 13*b^2)*\cosh \\
& (d*x + c)^2 - 13*a^2 + 40*a*b - 7*b^2)*\sinh(d*x + c)^6 + 4*(63*(a^2 + 2*a*b \\
& + b^2)*\cosh(d*x + c)^5 - 14*(7*a^2 - 6*a*b - 13*b^2)*\cosh(d*x + c)^3 - 3*(\\
& 13*a^2 - 40*a*b + 7*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2*(13*a^2 - 40*a* \\
& b + 7*b^2)*\cosh(d*x + c)^4 + 2*(105*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)^6 - 3 \\
& 5*(7*a^2 - 6*a*b - 13*b^2)*\cosh(d*x + c)^4 - 15*(13*a^2 - 40*a*b + 7*b^2)*c \\
& osh(d*x + c)^2 - 13*a^2 + 40*a*b - 7*b^2)*\sinh(d*x + c)^4 + 8*(15*(a^2 + 2*
\end{aligned}$$

```

a*b + b^2)*cosh(d*x + c)^7 - 7*(7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^5 - 5
*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c)^3 - (13*a^2 - 40*a*b + 7*b^2)*cosh
(d*x + c))*sinh(d*x + c)^3 - (7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^2 + (45
*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 - 28*(7*a^2 - 6*a*b - 13*b^2)*cosh(d*x
+ c)^6 - 30*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x + c)^4 - 12*(13*a^2 - 40*a*
b + 7*b^2)*cosh(d*x + c)^2 - 7*a^2 + 6*a*b + 13*b^2)*sinh(d*x + c)^2 + 12*(
(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^7 + 7*(3*a^2 + a*b - 2*b^2)*cosh(d*x +
c))*sinh(d*x + c)^6 + (3*a^2 + a*b - 2*b^2)*sinh(d*x + c)^7 + 2*(3*a^2 - 5*a
*b + 2*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^2 + 6
*a^2 - 10*a*b + 4*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 + a*b - 2*b^2)*cosh(d*
x + c)^3 + 2*(3*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + (3*a^
2 + a*b - 2*b^2)*cosh(d*x + c)^3 + (35*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^
4 + 20*(3*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 3*a^2 + a*b - 2*b^2)*sinh(
d*x + c)^3 + (21*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^5 + 20*(3*a^2 - 5*a*b
+ 2*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x
+ c)^2 + (7*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^6 + 10*(3*a^2 - 5*a*b + 2*b
^2)*cosh(d*x + c)^4 + 3*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c
))*sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*
x + c))*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c)
+ (3*(a + b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x + c))*sqrt(-b/(a + b))/b)
- 12*((3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^7 + 7*(3*a^2 + a*b - 2*b^2)*cosh(
d*x + c))*sinh(d*x + c)^6 + (3*a^2 + a*b - 2*b^2)*sinh(d*x + c)^7 + 2*(3*a^2
- 5*a*b + 2*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)
^2 + 6*a^2 - 10*a*b + 4*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 + a*b - 2*b^2)*c
osh(d*x + c)^3 + 2*(3*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 +
(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^3 + (35*(3*a^2 + a*b - 2*b^2)*cosh(d*x
+ c)^4 + 20*(3*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 3*a^2 + a*b - 2*b^2)
*sinh(d*x + c)^3 + (21*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^5 + 20*(3*a^2 -
5*a*b + 2*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c))*sin
h(d*x + c)^2 + (7*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^6 + 10*(3*a^2 - 5*a*b
+ 2*b^2)*cosh(d*x + c)^4 + 3*(3*a^2 + a*b - 2*b^2)*cosh(d*x + c)^2)*sinh(d
*x + c))*sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x + c) + (a + b)*sinh(
d*x + c))*sqrt(-b/(a + b))/b) + a^2 + 2*a*b + b^2 + 2*(5*(a^2 + 2*a*b + b^2)
)*cosh(d*x + c)^9 - 4*(7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c)^7 - 6*(13*a^2
- 40*a*b + 7*b^2)*cosh(d*x + c)^5 - 4*(13*a^2 - 40*a*b + 7*b^2)*cosh(d*x +
c)^3 - (7*a^2 - 6*a*b - 13*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 4*a^3
*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^7 + 7*(a^4 + 4*a^3*b + 6*a^
2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c))*sinh(d*x + c)^6 + (a^4 + 4*a^3*b + 6
*a^2*b^2 + 4*a*b^3 + b^4)*d*sinh(d*x + c)^7 + 2*(a^4 + 2*a^3*b - 2*a*b^3 -
b^4)*d*cosh(d*x + c)^5 + (21*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*
cosh(d*x + c)^2 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d)*sinh(d*x + c)^5 + (a
^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^3 + 5*(7*(a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^3 + 2*(a^4 + 2*a^3*b -
2*a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^4 + (35*(a^4 + 4*a^3*b + 6*a^
2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^4 + 20*(a^4 + 2*a^3*b - 2*a*b^3 - b^
4)*d*cosh(d*x + c)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d)*sinh(
d*x + c)^3 + (21*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c
)^5 + 20*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c)^3 + 3*(a^4 + 4*a^3
*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(a^4
+ 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^6 + 10*(a^4 + 2*a^3*
b - 2*a*b^3 - b^4)*d*cosh(d*x + c)^4 + 3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b
^3 + b^4)*d*cosh(d*x + c)^2)*sinh(d*x + c))]

```

giac [B] time = 0.94, size = 1929, normalized size = 15.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

```
[Out] -1/24*((9*a*e^(2*d*x + 2*c) - 15*b*e^(2*d*x + 2*c) - a - b)*e^(-3*d*x)/(a^3
*e^(3*c) + 3*a^2*b*e^(3*c) + 3*a*b^2*e^(3*c) + b^3*e^(3*c)) - 12*(2*(6*a^2*
b^2 - 4*a*b^3 + (3*a^2*b - 5*a*b^2 + 2*b^3)*sqrt(-a*b))*(a^3*e^(2*c) + 3*a^
2*b*e^(2*c) + 3*a*b^2*e^(2*c) + b^3*e^(2*c))^2*abs(a*e^(2*c) + b*e^(2*c)) +
(3*a^6*b + 7*a^5*b^2 - 10*a^3*b^4 - 5*a^2*b^5 + 3*a*b^6 + 2*b^7 - 2*(3*a^5
*b + 10*a^4*b^2 + 10*a^3*b^3 - 5*a*b^5 - 2*b^6)*sqrt(-a*b))*abs(-a^3*e^(2*c
) - 3*a^2*b*e^(2*c) - 3*a*b^2*e^(2*c) - b^3*e^(2*c))*abs(a*e^(2*c) + b*e^(2
*c))*e^(2*c) + (6*a^9*b + 26*a^8*b^2 + 34*a^7*b^3 - 6*a^6*b^4 - 50*a^5*b^5
- 34*a^4*b^6 + 6*a^3*b^7 + 14*a^2*b^8 + 4*a*b^9 + (3*a^9 + 10*a^8*b + 4*a^7
*b^2 - 20*a^6*b^3 - 22*a^5*b^4 + 8*a^4*b^5 + 20*a^3*b^6 + 4*a^2*b^7 - 5*a*b
^8 - 2*b^9)*sqrt(-a*b))*abs(a*e^(2*c) + b*e^(2*c))*e^(4*c))*arctan(e^(d*x)/
sqrt((a^4*e^(2*c) + 2*a^3*b*e^(2*c) - 2*a*b^3*e^(2*c) - b^4*e^(2*c)) + sqrt(
(a^4*e^(2*c) + 2*a^3*b*e^(2*c) - 2*a*b^3*e^(2*c) - b^4*e^(2*c))^2 - (a^4*e^
(4*c) + 4*a^3*b*e^(4*c) + 6*a^2*b^2*e^(4*c) + 4*a*b^3*e^(4*c) + b^4*e^(4*c)
)*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)))/(a^4*e^(4*c) + 4*a^3*b*e^(4
*c) + 6*a^2*b^2*e^(4*c) + 4*a*b^3*e^(4*c) + b^4*e^(4*c)))e^(-4*c)/((a^9 +
9*a^8*b + 36*a^7*b^2 + 84*a^6*b^3 + 126*a^5*b^4 + 126*a^4*b^5 + 84*a^3*b^6
+ 36*a^2*b^7 + 9*a*b^8 + b^9)*sqrt(a^2 - b^2 - 2*sqrt(-a*b)*(a + b))*abs(-
a^3*e^(2*c) - 3*a^2*b*e^(2*c) - 3*a*b^2*e^(2*c) - b^3*e^(2*c))) - 12*(2*(6*
a^2*b^2 - 4*a*b^3 - (3*a^2*b - 5*a*b^2 + 2*b^3)*sqrt(-a*b))*(a^3*e^(2*c) +
3*a^2*b*e^(2*c) + 3*a*b^2*e^(2*c) + b^3*e^(2*c))^2*abs(a*e^(2*c) + b*e^(2*c
)) + (3*a^6*b + 7*a^5*b^2 - 10*a^3*b^4 - 5*a^2*b^5 + 3*a*b^6 + 2*b^7 + 2*(3
*a^5*b + 10*a^4*b^2 + 10*a^3*b^3 - 5*a*b^5 - 2*b^6)*sqrt(-a*b))*abs(-a^3*e^
(2*c) - 3*a^2*b*e^(2*c) - 3*a*b^2*e^(2*c) - b^3*e^(2*c))*abs(a*e^(2*c) + b*
e^(2*c))*e^(2*c) + (6*a^9*b + 26*a^8*b^2 + 34*a^7*b^3 - 6*a^6*b^4 - 50*a^5*
b^5 - 34*a^4*b^6 + 6*a^3*b^7 + 14*a^2*b^8 + 4*a*b^9 - (3*a^9 + 10*a^8*b + 4
*a^7*b^2 - 20*a^6*b^3 - 22*a^5*b^4 + 8*a^4*b^5 + 20*a^3*b^6 + 4*a^2*b^7 - 5
*a*b^8 - 2*b^9)*sqrt(-a*b))*abs(a*e^(2*c) + b*e^(2*c))*e^(4*c))*arctan(e^(d
*x)/sqrt((a^4*e^(2*c) + 2*a^3*b*e^(2*c) - 2*a*b^3*e^(2*c) - b^4*e^(2*c) - s
qrt((a^4*e^(2*c) + 2*a^3*b*e^(2*c) - 2*a*b^3*e^(2*c) - b^4*e^(2*c))^2 - (a^
4*e^(4*c) + 4*a^3*b*e^(4*c) + 6*a^2*b^2*e^(4*c) + 4*a*b^3*e^(4*c) + b^4*e^
(4*c))*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)))/(a^4*e^(4*c) + 4*a^3*b*
e^(4*c) + 6*a^2*b^2*e^(4*c) + 4*a*b^3*e^(4*c) + b^4*e^(4*c)))e^(-4*c)/((a
^9 + 9*a^8*b + 36*a^7*b^2 + 84*a^6*b^3 + 126*a^5*b^4 + 126*a^4*b^5 + 84*a^3
*b^6 + 36*a^2*b^7 + 9*a*b^8 + b^9)*sqrt(a^2 - b^2 + 2*sqrt(-a*b)*(a + b))*a
bs(-a^3*e^(2*c) - 3*a^2*b*e^(2*c) - 3*a*b^2*e^(2*c) - b^3*e^(2*c))) - (a^4*
e^(3*d*x + 36*c) + 4*a^3*b*e^(3*d*x + 36*c) + 6*a^2*b^2*e^(3*d*x + 36*c) +
4*a*b^3*e^(3*d*x + 36*c) + b^4*e^(3*d*x + 36*c) - 9*a^4*e^(d*x + 34*c) - 12
*a^3*b*e^(d*x + 34*c) + 18*a^2*b^2*e^(d*x + 34*c) + 36*a*b^3*e^(d*x + 34*c)
+ 15*b^4*e^(d*x + 34*c))/(a^6*e^(33*c) + 6*a^5*b*e^(33*c) + 15*a^4*b^2*e^(
33*c) + 20*a^3*b^3*e^(33*c) + 15*a^2*b^4*e^(33*c) + 6*a*b^5*e^(33*c) + b^6*
e^(33*c)) - 24*(a*b*e^(3*d*x + 3*c) + a*b*e^(d*x + c))/((a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) -
2*b*e^(2*d*x + 2*c) + a + b))/d
```

maple [B] time = 0.32, size = 267, normalized size = 2.15

$$\frac{1}{3(a+b)^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^3} - \frac{1}{2(a+b)^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^2} - \frac{-a+3b}{2(a+b)^3 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)} + \frac{1}{3(a+b)^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^3} - \frac{1}{2(a+b)^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(-1/3/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c)
)-1)^2-1/2/(a+b)^3*(-a+3*b)/(tanh(1/2*d*x+1/2*c)-1)+1/3/(a+b)^2/(tanh(1/2*d
```


$$3.35 \quad \int \frac{\sinh^2(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=132

$$\frac{b \tanh(c+dx)}{d(a+b)^2(a+b \tanh^2(c+dx))} - \frac{\sqrt{b}(3a-b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}d(a+b)^3} + \frac{\sinh(c+dx) \cosh(c+dx)}{2d(a+b)(a+b \tanh^2(c+dx))} - \frac{x(a-3b)}{2(a+b)}$$

[Out] $-1/2*(a-3*b)*x/(a+b)^3 - 1/2*(3*a-b)*\arctan(b^{(1/2)}*\tanh(d*x+c)/a^{(1/2)})*b^{(1/2)}/(a+b)^3/d/a^{(1/2)} + 1/2*\cosh(d*x+c)*\sinh(d*x+c)/(a+b)/d/(a+b*\tanh(d*x+c)^2) - b*\tanh(d*x+c)/(a+b)^2/d/(a+b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.17, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3663, 471, 527, 522, 206, 205}

$$\frac{b \tanh(c+dx)}{d(a+b)^2(a+b \tanh^2(c+dx))} - \frac{\sqrt{b}(3a-b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}d(a+b)^3} + \frac{\sinh(c+dx) \cosh(c+dx)}{2d(a+b)(a+b \tanh^2(c+dx))} - \frac{x(a-3b)}{2(a+b)}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2, x]

[Out] $-((a - 3*b)*x)/(2*(a + b)^3) - ((3*a - b)*\text{Sqrt}[b]*\text{ArcTan}[(\text{Sqrt}[b]*\text{Tanh}[c + d*x])/\text{Sqrt}[a]])/(2*\text{Sqrt}[a]*(a + b)^3*d) + (\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])/(2*(a + b)*d*(a + b*\text{Tanh}[c + d*x]^2)) - (b*\text{Tanh}[c + d*x])/((a + b)^2*d*(a + b*\text{Tanh}[c + d*x]^2))$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 471

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n-1)*(e*x)^(m-n+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(n*(b*c-a*d)*(p+1)), x] - Dist[e^n/(n*(b*c-a*d)*(p+1)), Int[(e*x)^(m-n)*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[c*(m-n+1)+d*(m+n*(p+q+1)+1]*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m-n+1] && GtQ[m-n+1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]
&& IntegerQ[m/2]
```

Rubi steps

$$\int \frac{\sinh^2(c + dx)}{(a + b \tanh^2(c + dx))^2} dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+bx^2)^2} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{\cosh(c + dx) \sinh(c + dx)}{2(a + b)d(a + b \tanh^2(c + dx))} - \frac{\text{Subst}\left(\int \frac{a-3bx^2}{(1-x^2)(a+bx^2)^2} dx, x, \tanh(c + dx)\right)}{2(a + b)d}$$

$$= \frac{\cosh(c + dx) \sinh(c + dx)}{2(a + b)d(a + b \tanh^2(c + dx))} - \frac{b \tanh(c + dx)}{(a + b)^2 d(a + b \tanh^2(c + dx))} + \frac{\text{Subst}\left(\int \frac{-2}{(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{(a + b)d}$$

$$= \frac{\cosh(c + dx) \sinh(c + dx)}{2(a + b)d(a + b \tanh^2(c + dx))} - \frac{b \tanh(c + dx)}{(a + b)^2 d(a + b \tanh^2(c + dx))} - \frac{(a - 3b) \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{(a + b)d}$$

$$= \frac{(a - 3b)x}{2(a + b)^3} - \frac{(3a - b)\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c + dx)}{\sqrt{a}}\right)}{2\sqrt{a}(a + b)^3 d} + \frac{\cosh(c + dx) \sinh(c + dx)}{2(a + b)d(a + b \tanh^2(c + dx))}$$

Mathematica [A] time = 0.74, size = 105, normalized size = 0.80

$$\frac{-2(a - 3b)(c + dx) + (a + b) \sinh(2(c + dx)) + \frac{2\sqrt{b}(b-3a) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{2b(a+b) \sinh(2(c+dx))}{(a+b) \cosh(2(c+dx))+a-b}}{4d(a + b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2), x]
```

```
[Out] (-2*(a - 3*b)*(c + d*x) + (2*sqrt[b]*(-3*a + b)*ArcTan[(sqrt[b]*Tanh[c + d*
x])/sqrt[a]])/sqrt[a] + (a + b)*Sinh[2*(c + d*x)] - (2*b*(a + b)*Sinh[2*(c
+ d*x)]/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(4*(a + b)^3*d)
```

fricas [B] time = 0.81, size = 3918, normalized size = 29.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```



```
[Out] [1/8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 - 2*(2*(a^2 - 2*
a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^6 - 2*(2*(a^2 - 2*a*b - 3*b^2)*
d*x - 14*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - a^2 + b^2)*sinh(d*x + c)^6 +
4*(14*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - 3*(2*(a^2 - 2*a*b - 3*b^2)*d*x
- a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*((a^2 - 4*a*b + 3*b^2)*d*x
- a*b + b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 -
4*(a^2 - 4*a*b + 3*b^2)*d*x - 15*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)
*cosh(d*x + c)^2 + 4*a*b - 4*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2
)*cosh(d*x + c)^5 - 5*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x +
c)^3 - 4*((a^2 - 4*a*b + 3*b^2)*d*x - a*b + b^2)*cosh(d*x + c))*sinh(d*x +
c)^3 - 2*(2*(a^2 - 2*a*b - 3*b^2)*d*x + a^2 - 4*a*b - 5*b^2)*cosh(d*x + c)^
2 + 2*(14*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 - 15*(2*(a^2 - 2*a*b - 3*b^2)
*d*x - a^2 + b^2)*cosh(d*x + c)^4 - 2*(a^2 - 2*a*b - 3*b^2)*d*x - 24*((a^2
- 4*a*b + 3*b^2)*d*x - a*b + b^2)*cosh(d*x + c)^2 - a^2 + 4*a*b + 5*b^2)*si
nh(d*x + c)^2 - 2*((3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^6 + 6*(3*a^2 + 2*a*b
- b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^2 + 2*a*b - b^2)*sinh(d*x + c)
^6 + 2*(3*a^2 - 4*a*b + b^2)*cosh(d*x + c)^4 + (15*(3*a^2 + 2*a*b - b^2)*co
sh(d*x + c)^2 + 6*a^2 - 8*a*b + 2*b^2)*sinh(d*x + c)^4 + 4*(5*(3*a^2 + 2*a*
b - b^2)*cosh(d*x + c)^3 + 2*(3*a^2 - 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x
+ c)^3 + (3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^2 + (15*(3*a^2 + 2*a*b - b^2)*
cosh(d*x + c)^4 + 12*(3*a^2 - 4*a*b + b^2)*cosh(d*x + c)^2 + 3*a^2 + 2*a*b
- b^2)*sinh(d*x + c)^2 + 2*(3*(3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^5 + 4*(3*
a^2 - 4*a*b + b^2)*cosh(d*x + c)^3 + (3*a^2 + 2*a*b - b^2)*cosh(d*x + c))*s
inh(d*x + c))*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2
+ 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x
+ c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2
+ a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2
+ a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 +
4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a -
b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2
+ 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b
)) - a^2 - 2*a*b - b^2 + 4*(2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 - 3*(2*(a
^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^5 - 8*((a^2 - 4*a*b + 3*
b^2)*d*x - a*b + b^2)*cosh(d*x + c)^3 - (2*(a^2 - 2*a*b - 3*b^2)*d*x + a^2
- 4*a*b - 5*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 + 4*a^3*b + 6*a^2*b^2
+ 4*a*b^3 + b^4)*d*cosh(d*x + c)^6 + 6*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3
+ b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*
b^3 + b^4)*d*sinh(d*x + c)^6 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x
+ c)^4 + (15*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^2
+ 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d)*sinh(d*x + c)^4 + (a^4 + 4*a^3*b +
6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^2 + 4*(5*(a^4 + 4*a^3*b + 6*a^2*
b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^3 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*
d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3
+ b^4)*d*cosh(d*x + c)^4 + 12*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x +
c)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d)*sinh(d*x + c)^2 + 2*
(3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^5 + 4*(a^4 +
2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c)^3 + (a^4 + 4*a^3*b + 6*a^2*b^2 +
4*a*b^3 + b^4)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*((a^2 + 2*a*b + b^2)*co
sh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2
+ 2*a*b + b^2)*sinh(d*x + c)^8 - 2*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)
*cosh(d*x + c)^6 - 2*(2*(a^2 - 2*a*b - 3*b^2)*d*x - 14*(a^2 + 2*a*b + b^2)
*cosh(d*x + c)^2 - a^2 + b^2)*sinh(d*x + c)^6 + 4*(14*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^3 - 3*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x + c)
)*sinh(d*x + c)^5 - 8*((a^2 - 4*a*b + 3*b^2)*d*x - a*b + b^2)*cosh(d*x + c)^
4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 - 4*(a^2 - 4*a*b + 3*b^2)*d*x
- 15*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^2 + 4*a*b - 4
```

```

*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 - 5*(2*(a^
2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^3 - 4*((a^2 - 4*a*b + 3*b
^2)*d*x - a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(2*(a^2 - 2*a*b - 3
*b^2)*d*x + a^2 - 4*a*b - 5*b^2)*cosh(d*x + c)^2 + 2*(14*(a^2 + 2*a*b + b^2
))*cosh(d*x + c)^6 - 15*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b^2)*cosh(d*x +
c)^4 - 2*(a^2 - 2*a*b - 3*b^2)*d*x - 24*((a^2 - 4*a*b + 3*b^2)*d*x - a*b +
b^2)*cosh(d*x + c)^2 - a^2 + 4*a*b + 5*b^2)*sinh(d*x + c)^2 - 4*((3*a^2 +
2*a*b - b^2)*cosh(d*x + c)^6 + 6*(3*a^2 + 2*a*b - b^2)*cosh(d*x + c)*sinh(d
*x + c)^5 + (3*a^2 + 2*a*b - b^2)*sinh(d*x + c)^6 + 2*(3*a^2 - 4*a*b + b^2)
*cosh(d*x + c)^4 + (15*(3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^2 + 6*a^2 - 8*a*
b + 2*b^2)*sinh(d*x + c)^4 + 4*(5*(3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^3 + 2
*(3*a^2 - 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (3*a^2 + 2*a*b - b^
2)*cosh(d*x + c)^2 + (15*(3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^4 + 12*(3*a^2
- 4*a*b + b^2)*cosh(d*x + c)^2 + 3*a^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 2*(
3*(3*a^2 + 2*a*b - b^2)*cosh(d*x + c)^5 + 4*(3*a^2 - 4*a*b + b^2)*cosh(d*x
+ c)^3 + (3*a^2 + 2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arct
an(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (
a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) - a^2 - 2*a*b - b^2 + 4*(2*(a^
2 + 2*a*b + b^2)*cosh(d*x + c)^7 - 3*(2*(a^2 - 2*a*b - 3*b^2)*d*x - a^2 + b
^2)*cosh(d*x + c)^5 - 8*((a^2 - 4*a*b + 3*b^2)*d*x - a*b + b^2)*cosh(d*x +
c)^3 - (2*(a^2 - 2*a*b - 3*b^2)*d*x + a^2 - 4*a*b - 5*b^2)*cosh(d*x + c))*s
inh(d*x + c))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^
6 + 6*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)*sinh(d*x
+ c)^5 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*sinh(d*x + c)^6 + 2*
(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c)^4 + (15*(a^4 + 4*a^3*b + 6*
a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^2 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b
^4)*d)*sinh(d*x + c)^4 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh
(d*x + c)^2 + 4*(5*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x +
c)^3 + 2*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3
+ (15*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c)^4 + 12*(a
^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(d*x + c)^2 + (a^4 + 4*a^3*b + 6*a^2*b^
2 + 4*a*b^3 + b^4)*d)*sinh(d*x + c)^2 + 2*(3*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4
*a*b^3 + b^4)*d*cosh(d*x + c)^5 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*d*cosh(
d*x + c)^3 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cosh(d*x + c))*s
inh(d*x + c))]

```

giac [B] time = 1.13, size = 400, normalized size = 3.03

$$\frac{12(a-3b)dx}{a^3+3a^2b+3ab^2+b^3} + \frac{12(3abe^{2c}-b^2e^{2c})\arctan\left(\frac{ae^{2dx+2c}+be^{2dx+2c}+a-b}{2\sqrt{ab}}\right)e^{(-2c)}}{(a^3+3a^2b+3ab^2+b^3)\sqrt{ab}} - \frac{3e^{2dx+12c}}{a^2e^{10c}+2abe^{10c}+b^2e^{10c}} - \frac{2a^2e^{6dx+6c}-4abe^{6dx+6c}}{(a^3e^{10c}+3a^2be^{10c}+3ab^2e^{10c}+b^3e^{10c})}$$

24 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c))^2,x, algorithm="giac")

```

[Out] -1/24*(12*(a - 3*b)*d*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 12*(3*a*b*e^(2*c)
- b^2*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/
sqrt(a*b))*e^(-2*c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)) - 3*e^(2*d*
x + 12*c)/(a^2*e^(10*c) + 2*a*b*e^(10*c) + b^2*e^(10*c)) - (2*a^2*e^(6*d*x
+ 6*c) - 4*a*b*e^(6*d*x + 6*c) - 6*b^2*e^(6*d*x + 6*c) + a^2*e^(4*d*x + 4*c
) + 2*a*b*e^(4*d*x + 4*c) - 15*b^2*e^(4*d*x + 4*c) - 4*a^2*e^(2*d*x + 2*c)
+ 20*a*b*e^(2*d*x + 2*c) + 24*b^2*e^(2*d*x + 2*c) - 3*a^2 - 6*a*b - 3*b^2)/
((a^3*e^(2*c) + 3*a^2*b*e^(2*c) + 3*a*b^2*e^(2*c) + b^3*e^(2*c))*(a*e^(2*d*
x) + b*e^(2*d*x) + a*e^(6*d*x + 4*c) + b*e^(6*d*x + 4*c) + 2*a*e^(4*d*x + 2
*c) - 2*b*e^(4*d*x + 2*c)))/d

```

maple [B] time = 0.34, size = 1128, normalized size = 8.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sinh(dx+c)^2/(a+b*\tanh(dx+c))^2, x)$

[Out] $\frac{1}{2} \frac{d}{(a+b)^2} \frac{1}{(\tanh(\frac{1}{2}dx+\frac{1}{2}c)-1)^2} + \frac{1}{2} \frac{d}{(a+b)^2} \frac{1}{(\tanh(\frac{1}{2}dx+\frac{1}{2}c)+1)^2} + \frac{1}{2} \frac{d}{(a+b)^3} \ln(\tanh(\frac{1}{2}dx+\frac{1}{2}c)-1) * a - \frac{3}{2} \frac{d}{(a+b)^3} \ln(\tanh(\frac{1}{2}dx+\frac{1}{2}c)+1) * b - \frac{1}{2} \frac{d}{(a+b)^2} \frac{1}{(\tanh(\frac{1}{2}dx+\frac{1}{2}c)+1)^2} + \frac{1}{2} \frac{d}{(a+b)^2} \frac{1}{(\tanh(\frac{1}{2}dx+\frac{1}{2}c)-1)^2} - \frac{1}{2} \frac{d}{(a+b)^3} \ln(\tanh(\frac{1}{2}dx+\frac{1}{2}c)+1) * a + \frac{3}{2} \frac{d}{(a+b)^3} \ln(\tanh(\frac{1}{2}dx+\frac{1}{2}c)-1) * b - \frac{1}{d} \frac{b}{(a+b)^3} \frac{1}{(\tanh(\frac{1}{2}dx+\frac{1}{2}c))^4} * a + 2 * \tanh(\frac{1}{2}dx+\frac{1}{2}c)^2 * a + 4 * \tanh(\frac{1}{2}dx+\frac{1}{2}c)^2 * b + a * \tanh(\frac{1}{2}dx+\frac{1}{2}c)^3 * a - \frac{1}{d} \frac{b^2}{(a+b)^3} \frac{1}{(\tanh(\frac{1}{2}dx+\frac{1}{2}c))^4} * a + 2 * \tanh(\frac{1}{2}dx+\frac{1}{2}c)^2 * a + 4 * \tanh(\frac{1}{2}dx+\frac{1}{2}c)^2 * b + a * \tanh(\frac{1}{2}dx+\frac{1}{2}c)^3 - \frac{1}{d} \frac{b}{(a+b)^3} \frac{1}{(\tanh(\frac{1}{2}dx+\frac{1}{2}c))^4} * a + 2 * \tanh(\frac{1}{2}dx+\frac{1}{2}c)^2 * a + 4 * \tanh(\frac{1}{2}dx+\frac{1}{2}c)^2 * b + a * \tanh(\frac{1}{2}dx+\frac{1}{2}c)^3 - \frac{1}{d} \frac{b^2}{(a+b)^3} \frac{1}{(\tanh(\frac{1}{2}dx+\frac{1}{2}c))^4} * a + 2 * \tanh(\frac{1}{2}dx+\frac{1}{2}c)^2 * a + 4 * \tanh(\frac{1}{2}dx+\frac{1}{2}c)^2 * b + a * \tanh(\frac{1}{2}dx+\frac{1}{2}c)^3 + \frac{3}{2} \frac{d}{(a+b)^3} \frac{a^2}{(b*(a+b))^{1/2}} / ((2*(b*(a+b))^{1/2} - a - 2*b)*a)^{1/2} * \arctanh(a*\tanh(\frac{1}{2}dx+\frac{1}{2}c)) / ((2*(b*(a+b))^{1/2} - a - 2*b)*a)^{1/2} - \frac{3}{2} \frac{d}{(a+b)^3} \frac{a}{((2*(b*(a+b))^{1/2} - a - 2*b)*a)^{1/2}} * \arctanh(a*\tanh(\frac{1}{2}dx+\frac{1}{2}c)) / ((2*(b*(a+b))^{1/2} - a - 2*b)*a)^{1/2} + \frac{1}{d} \frac{b^2}{(a+b)^3} \frac{a}{(b*(a+b))^{1/2}} / ((2*(b*(a+b))^{1/2} - a - 2*b)*a)^{1/2} * \arctanh(a*\tanh(\frac{1}{2}dx+\frac{1}{2}c)) / ((2*(b*(a+b))^{1/2} - a - 2*b)*a)^{1/2} + \frac{3}{2} \frac{d}{(a+b)^3} \frac{a^2}{(b*(a+b))^{1/2}} / ((2*(b*(a+b))^{1/2} + a + 2*b)*a)^{1/2} * \arctan(a*\tanh(\frac{1}{2}dx+\frac{1}{2}c)) / ((2*(b*(a+b))^{1/2} + a + 2*b)*a)^{1/2} + \frac{3}{2} \frac{d}{(a+b)^3} \frac{a}{((2*(b*(a+b))^{1/2} + a + 2*b)*a)^{1/2}} * \arctan(a*\tanh(\frac{1}{2}dx+\frac{1}{2}c)) / ((2*(b*(a+b))^{1/2} + a + 2*b)*a)^{1/2} + \frac{1}{d} \frac{b^2}{(a+b)^3} \frac{a}{(b*(a+b))^{1/2}} / ((2*(b*(a+b))^{1/2} + a + 2*b)*a)^{1/2} * \arctan(a*\tanh(\frac{1}{2}dx+\frac{1}{2}c)) / ((2*(b*(a+b))^{1/2} + a + 2*b)*a)^{1/2} + \frac{1}{2} \frac{d}{(a+b)^3} \frac{1}{((2*(b*(a+b))^{1/2} - a - 2*b)*a)^{1/2}} * \arctanh(a*\tanh(\frac{1}{2}dx+\frac{1}{2}c)) / ((2*(b*(a+b))^{1/2} - a - 2*b)*a)^{1/2} - \frac{1}{2} \frac{d}{(a+b)^3} \frac{1}{(b*(a+b))^{1/2}} / ((2*(b*(a+b))^{1/2} - a - 2*b)*a)^{1/2} * \arctanh(a*\tanh(\frac{1}{2}dx+\frac{1}{2}c)) / ((2*(b*(a+b))^{1/2} - a - 2*b)*a)^{1/2} - \frac{1}{2} \frac{d}{(a+b)^3} \frac{1}{((2*(b*(a+b))^{1/2} + a + 2*b)*a)^{1/2}} * \arctan(a*\tanh(\frac{1}{2}dx+\frac{1}{2}c)) / ((2*(b*(a+b))^{1/2} + a + 2*b)*a)^{1/2} + \frac{1}{2} \frac{d}{(a+b)^3} \frac{1}{(b*(a+b))^{1/2}} / ((2*(b*(a+b))^{1/2} + a + 2*b)*a)^{1/2} * \arctan(a*\tanh(\frac{1}{2}dx+\frac{1}{2}c)) / ((2*(b*(a+b))^{1/2} + a + 2*b)*a)^{1/2}$

maxima [B] time = 0.59, size = 840, normalized size = 6.36

$$\frac{b \log((a+b)e^{4dx+4c} + 2(a-b)e^{2dx+2c} + a+b)}{2(a^3 + 3a^2b + 3ab^2 + b^3)d} - \frac{b \log(2(a-b)e^{-2dx-2c} + (a+b)e^{-4dx-4c} + a+b)}{2(a^3 + 3a^2b + 3ab^2 + b^3)d} - \frac{3}{2} \frac{d}{(a+b)^3} \frac{1}{((2*(b*(a+b))^{1/2} - a - 2*b)*a)^{1/2}} * \arctanh(a*\tanh(\frac{1}{2}dx+\frac{1}{2}c)) / ((2*(b*(a+b))^{1/2} - a - 2*b)*a)^{1/2} + \frac{1}{d} \frac{b^2}{(a+b)^3} \frac{a}{(b*(a+b))^{1/2}} / ((2*(b*(a+b))^{1/2} - a - 2*b)*a)^{1/2} * \arctanh(a*\tanh(\frac{1}{2}dx+\frac{1}{2}c)) / ((2*(b*(a+b))^{1/2} - a - 2*b)*a)^{1/2} + \frac{3}{2} \frac{d}{(a+b)^3} \frac{a^2}{(b*(a+b))^{1/2}} / ((2*(b*(a+b))^{1/2} + a + 2*b)*a)^{1/2} * \arctan(a*\tanh(\frac{1}{2}dx+\frac{1}{2}c)) / ((2*(b*(a+b))^{1/2} + a + 2*b)*a)^{1/2} + \frac{3}{2} \frac{d}{(a+b)^3} \frac{a}{((2*(b*(a+b))^{1/2} + a + 2*b)*a)^{1/2}} * \arctan(a*\tanh(\frac{1}{2}dx+\frac{1}{2}c)) / ((2*(b*(a+b))^{1/2} + a + 2*b)*a)^{1/2} + \frac{1}{d} \frac{b^2}{(a+b)^3} \frac{a}{(b*(a+b))^{1/2}} / ((2*(b*(a+b))^{1/2} + a + 2*b)*a)^{1/2} * \arctan(a*\tanh(\frac{1}{2}dx+\frac{1}{2}c)) / ((2*(b*(a+b))^{1/2} + a + 2*b)*a)^{1/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\sinh(dx+c)^2/(a+b*\tanh(dx+c))^2, x, \text{algorithm}="maxima")$

[Out] $\frac{1}{2} * b * \log((a+b)*e^{(4dx+4c)} + 2*(a-b)*e^{(2dx+2c)} + a+b) / ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - \frac{1}{2} * b * \log(2*(a-b)*e^{(-2dx-2c)} + (a+b)*e^{(-4dx-4c)} + a+b) / ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - \frac{1}{8} * (3*a^2*b - 6*a*b^2 - b^3) * \arctan(1/2*((a+b)*e^{(2dx+2c)} + a-b) / \sqrt{a*b}) / ((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3) * \sqrt{a*b}) * d + \frac{1}{8} * (3*a^2*b - 6*a*b^2 - b^3) * \arctan(1/2*((a+b)*e^{(-2dx-2c)} + a-b) / \sqrt{a*b}) / ((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3) * \sqrt{a*b}) * d + \frac{1}{4} * (3*a*b + b^2) * \arctan(1/2*((a+b)*e^{(-2dx-2c)} + a-b) / \sqrt{a*b}) / ((a^3 + 2*a^2*b + a*b^2) * \sqrt{a*b}) * d + \frac{1}{4} * (a^2*b - b^3 + (a^2*b - 6*a*b^2 + b^3) * e^{(2dx+2c)}) / ((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4) * e^{(4dx+4c)} + 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4) * e^{(2dx+2c)}) * d) - \frac{1}{4} * (a^2*b - b^3 + (a^2*b - 6*a*b^2 + b^3) * e^{(-2dx-2c)}) / ((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4) * e^{(-2dx-2c)} + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4) * e^{(-4dx-4c)}) * d) - \frac{1}{2} * (a*b + b^2 + (a*b - b^2) * e^{(-2dx-2c)}) / ((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3) * e^{(-2dx-2c)} + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3) * e^{(-4dx-4c)}) * d)$

$c)) * d) - 1/2 * (d * x + c) / ((a^2 + 2 * a * b + b^2) * d) + 1/8 * e^{(2 * d * x + 2 * c)} / ((a^2 + 2 * a * b + b^2) * d) - 1/8 * e^{(-2 * d * x - 2 * c)} / ((a^2 + 2 * a * b + b^2) * d)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(c + dx)^2}{(b \tanh(c + dx)^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2,x)

[Out] int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)

[Out] Timed out

$$3.36 \quad \int \frac{\sinh(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=92

$$\frac{3 \cosh(c+dx)}{2d(a+b)^2} - \frac{\cosh(c+dx)}{2d(a+b)(a-b \operatorname{sech}^2(c+dx)+b)} - \frac{3\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{5/2}}$$

[Out] 3/2*cosh(d*x+c)/(a+b)^2/d-1/2*cosh(d*x+c)/(a+b)/d/(a+b-b*sech(d*x+c)^2)-3/2*arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/(a+b)^(5/2)/d

Rubi [A] time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3664, 290, 325, 208}

$$\frac{3 \cosh(c+dx)}{2d(a+b)^2} - \frac{\cosh(c+dx)}{2d(a+b)(a-b \operatorname{sech}^2(c+dx)+b)} - \frac{3\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2d(a+b)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2), x]

[Out] (-3*sqrt[b]*ArcTanh[(sqrt[b]*Sech[c + d*x])/sqrt[a + b]])/(2*(a + b)^(5/2)*d) + (3*Cosh[c + d*x])/(2*(a + b)^2*d) - Cosh[c + d*x]/(2*(a + b)*d*(a + b - b*Sech[c + d*x]^2))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*n*(p+1)), x] + Dist[(m+n*(p+1)+1)/(a*n*(p+1)), Int[(c*x)^(m*(a+b*x^n)^(p+1)), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 325

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c*n*(m+1)), Int[(c*x)^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 3664

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^(m-1)/2)*(a-b+b*ff^2*x^2)^p]/x^(m+1), x], x, Sec[e + f*x]/ff, x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m-1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(c+dx)}{(a+b \tanh^2(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)^2} dx, x, \text{sech}(c+dx)\right)}{d} \\
&= \frac{\cosh(c+dx)}{2(a+b)d(a+b-b \text{sech}^2(c+dx))} - \frac{3 \text{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)} dx, x, \text{sech}(c+dx)\right)}{2(a+b)d} \\
&= \frac{3 \cosh(c+dx)}{2(a+b)^2 d} - \frac{\cosh(c+dx)}{2(a+b)d(a+b-b \text{sech}^2(c+dx))} - \frac{(3b) \text{Subst}\left(\int \frac{1}{a+b-bx^2} dx, x, \text{sech}(c+dx)\right)}{2(a+b)^2 d} \\
&= -\frac{3\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \text{sech}(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{5/2} d} + \frac{3 \cosh(c+dx)}{2(a+b)^2 d} - \frac{\cosh(c+dx)}{2(a+b)d(a+b-b \text{sech}^2(c+dx))}
\end{aligned}$$

Mathematica [C] time = 0.77, size = 133, normalized size = 1.45

$$\frac{2 \cosh(c+dx) \left(1 - \frac{b}{(a+b) \cosh(2(c+dx))+a-b}\right)}{(a+b)^2} - \frac{3i\sqrt{b} \left(\tan^{-1}\left(\frac{-\sqrt{a} \tanh\left(\frac{1}{2}(c+dx)\right) - i\sqrt{a+b}}{\sqrt{b}}\right) + \tan^{-1}\left(\frac{\sqrt{a} \tanh\left(\frac{1}{2}(c+dx)\right) - i\sqrt{a+b}}{\sqrt{b}}\right)\right)}{(a+b)^{5/2}}}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2), x]

[Out] (((-3*I)*Sqrt[b]*(ArcTan[(-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2]]/Sqrt[b]] + ArcTan[(-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2]]/Sqrt[b]))/(a + b)^(5/2) + (2*Cosh[c + d*x]*(1 - b/(a - b + (a + b)*Cosh[2*(c + d*x)])))/(a + b)^2/(2*d)

fricas [B] time = 0.61, size = 2252, normalized size = 24.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] [1/4*(2*(a + b)*cosh(d*x + c)^6 + 12*(a + b)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(a + b)*sinh(d*x + c)^6 + 6*(a - b)*cosh(d*x + c)^4 + 6*(5*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^4 + 8*(5*(a + b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(a - b)*cosh(d*x + c)^2 + 6*(5*(a + b)*cosh(d*x + c)^4 + 6*(a - b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 3*((a + b)*cosh(d*x + c)^5 + 5*(a + b)*cosh(d*x + c)*sinh(d*x + c)^4 + (a + b)*sinh(d*x + c)^5 + 2*(a - b)*cosh(d*x + c)^3 + 2*(5*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^3 + 2*(5*(a + b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c))*sinh(d*x + c)^2 + (a + b)*cosh(d*x + c) + (5*(a + b)*cosh(d*x + c)^4 + 6*(a - b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)

$$\begin{aligned}
& c)^2 + a - b) \sinh(dx + c)^2 + 4*((a + b) \cosh(dx + c)^3 + (a - b) \cosh(dx + c) \sinh(dx + c) + a + b)) + 12*((a + b) \cosh(dx + c)^5 + 2*(a - b) \cosh(dx + c)^3 + (a - b) \cosh(dx + c) \sinh(dx + c) + 2*a + 2*b) / ((a^3 + 3*a^2*b + 3*a*b^2 + b^3) * d * \cosh(dx + c)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * d * \cosh(dx + c) \sinh(dx + c)^4 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3) * d * \sinh(dx + c)^5 + 2*(a^3 + a^2*b - a*b^2 - b^3) * d * \cosh(dx + c)^3 + 2*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * d * \cosh(dx + c)^2 + (a^3 + a^2*b - a*b^2 - b^3) * d) * \sinh(dx + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3) * d * \cosh(dx + c) + 2*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * d * \cosh(dx + c)^3 + 3*(a^3 + a^2*b - a*b^2 - b^3) * d * \cosh(dx + c)) * \sinh(dx + c)^2 + (5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * d * \cosh(dx + c)^4 + 6*(a^3 + a^2*b - a*b^2 - b^3) * d * \cosh(dx + c)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3) * d) * \sinh(dx + c)), 1/2*((a + b) \cosh(dx + c)^6 + 6*(a + b) \cosh(dx + c) \sinh(dx + c)^5 + (a + b) \sinh(dx + c)^6 + 3*(a - b) \cosh(dx + c)^4 + 3*(5*(a + b) \cosh(dx + c)^2 + a - b) \sinh(dx + c)^4 + 4*(5*(a + b) \cosh(dx + c)^3 + 3*(a - b) \cosh(dx + c) \sinh(dx + c)^3 + 3*(a - b) \cosh(dx + c)^2 + 3*(5*(a + b) \cosh(dx + c)^4 + 6*(a - b) \cosh(dx + c)^2 + a - b) \sinh(dx + c)^2 - 3*((a + b) \cosh(dx + c)^5 + 5*(a + b) \cosh(dx + c) \sinh(dx + c)^4 + (a + b) \sinh(dx + c)^5 + 2*(a - b) \cosh(dx + c)^3 + 2*(5*(a + b) \cosh(dx + c)^2 + a - b) \sinh(dx + c)^3 + 2*(5*(a + b) \cosh(dx + c)^3 + 3*(a - b) \cosh(dx + c) \sinh(dx + c)^2 + (a + b) \cosh(dx + c) + (5*(a + b) \cosh(dx + c)^4 + 6*(a - b) \cosh(dx + c)^2 + a + b) \sinh(dx + c)) * \sqrt{-b/(a + b)} * \arctan(1/2*((a + b) \cosh(dx + c)^3 + 3*(a + b) \cosh(dx + c) \sinh(dx + c)^2 + (a + b) \sinh(dx + c)^3 + (a - 3*b) \cosh(dx + c) + (3*(a + b) \cosh(dx + c)^2 + a - 3*b) \sinh(dx + c)) * \sqrt{-b/(a + b)})/b) + 3*((a + b) \cosh(dx + c)^5 + 5*(a + b) \cosh(dx + c) \sinh(dx + c)^4 + (a + b) \sinh(dx + c)^5 + 2*(a - b) \cosh(dx + c)^3 + 2*(5*(a + b) \cosh(dx + c)^2 + a - b) \sinh(dx + c)^3 + 2*(5*(a + b) \cosh(dx + c)^3 + 3*(a - b) \cosh(dx + c) \sinh(dx + c)^2 + (a + b) \cosh(dx + c) + (5*(a + b) \cosh(dx + c)^4 + 6*(a - b) \cosh(dx + c)^2 + a + b) \sinh(dx + c)) * \sqrt{-b/(a + b)} * \arctan(1/2*((a + b) \cosh(dx + c) + (a + b) \sinh(dx + c)) * \sqrt{-b/(a + b)})/b) + 6*((a + b) \cosh(dx + c)^5 + 2*(a - b) \cosh(dx + c)^3 + (a - b) \cosh(dx + c) \sinh(dx + c) + a + b) / ((a^3 + 3*a^2*b + 3*a*b^2 + b^3) * d * \cosh(dx + c)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * d * \cosh(dx + c) \sinh(dx + c)^4 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3) * d * \sinh(dx + c)^5 + 2*(a^3 + a^2*b - a*b^2 - b^3) * d * \cosh(dx + c)^3 + 2*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * d * \cosh(dx + c)^2 + (a^3 + a^2*b - a*b^2 - b^3) * d) * \sinh(dx + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3) * d * \cosh(dx + c) + 2*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * d * \cosh(dx + c)^3 + 3*(a^3 + a^2*b - a*b^2 - b^3) * d * \cosh(dx + c)) * \sinh(dx + c)^2 + (5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * d * \cosh(dx + c)^4 + 6*(a^3 + a^2*b - a*b^2 - b^3) * d * \cosh(dx + c)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3) * d) * \sinh(dx + c))]
\end{aligned}$$

giac [B] time = 0.41, size = 924, normalized size = 10.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)/(a+b*tanh(dx+c)^2)^2,x, algorithm="giac")

[Out] $1/2*(3*(5*a^2*b - 10*a*b^2 + b^3 - (a^2 - 10*a*b + 5*b^2)*\sqrt{-a*b})*\sqrt{a^2 - b^2 + 2*\sqrt{-a*b}*(a + b)}*abs(a*e^{(2*c)} + b*e^{(2*c)})*\arctan(e^{(d*x)}/\sqrt{(a^3*e^{(2*c)} + a^2*b*e^{(2*c)} - a*b^2*e^{(2*c)} - b^3*e^{(2*c)} + \sqrt{(a^3*e^{(2*c)} + a^2*b*e^{(2*c)} - a*b^2*e^{(2*c)} - b^3*e^{(2*c)})^2 - (a^3*e^{(4*c)} + 3*a^2*b*e^{(4*c)} + 3*a*b^2*e^{(4*c)} + b^3*e^{(4*c)})*(a^3 + 3*a^2*b + 3*a*b^2 + b^3))})/(a^3*e^{(4*c)} + 3*a^2*b*e^{(4*c)} + 3*a*b^2*e^{(4*c)} + b^3*e^{(4*c)})))*e^{(-2*c)}/(a^7 - 11*a^6*b - 39*a^5*b^2 - 27*a^4*b^3 + 27*a^3*b^4 + 39*a^2*b^5 + 11*a*b^6 - b^7 + 2*(3*a^6 + 2*a^5*b - 19*a^4*b^2 - 36*a^3*b^3 - 19*a^2*b^4 + 2*a*b^5 + 3*b^6)*\sqrt{-a*b})) + 3*(5*a^2*b - 10*a*b^2 + b^3 + (a^2 - 10*a*b + 5*b^2)*\sqrt{-a*b})*\sqrt{a^2 - b^2 - 2*\sqrt{-a*b}*(a + b)}*abs(a*e^{(2*c)} + b*e^{(2*c)})*\arctan(e^{(d*x)}/\sqrt{(a^3*e^{(2*c)} + a^2*b*e^{(2*c)} - a*b^2*e^{(2*c)} - b^3*e^{(2*c)} + \sqrt{(a^3*e^{(2*c)} + a^2*b*e^{(2*c)} - a*b^2*e^{(2*c)} - b^3*e^{(2*c)})^2 - (a^3*e^{(4*c)} + 3*a^2*b*e^{(4*c)} + 3*a*b^2*e^{(4*c)} + b^3*e^{(4*c)})*(a^3 + 3*a^2*b + 3*a*b^2 + b^3))})/(a^3*e^{(4*c)} + 3*a^2*b*e^{(4*c)} + 3*a*b^2*e^{(4*c)} + b^3*e^{(4*c)})))*e^{(-2*c)}/(a^7 - 11*a^6*b - 39*a^5*b^2 - 27*a^4*b^3 + 27*a^3*b^4 + 39*a^2*b^5 + 11*a*b^6 - b^7 + 2*(3*a^6 + 2*a^5*b - 19*a^4*b^2 - 36*a^3*b^3 - 19*a^2*b^4 + 2*a*b^5 + 3*b^6)*\sqrt{-a*b}))$

$e^{(2*c)} - b^3e^{(2*c)} - \sqrt{((a^3e^{(2*c)} + a^2*b*e^{(2*c)} - a*b^2e^{(2*c)} - b^3e^{(2*c)})^2 - (a^3e^{(4*c)} + 3*a^2*b*e^{(4*c)} + 3*a*b^2e^{(4*c)} + b^3e^{(4*c)})) * (a^3 + 3*a^2*b + 3*a*b^2 + b^3))} / (a^3e^{(4*c)} + 3*a^2*b*e^{(4*c)} + 3*a*b^2e^{(4*c)} + b^3e^{(4*c)}) * e^{(-2*c)} / (a^7 - 11*a^6*b - 39*a^5*b^2 - 27*a^4*b^3 + 27*a^3*b^4 + 39*a^2*b^5 + 11*a*b^6 - b^7 - 2*(3*a^6 + 2*a^5*b - 19*a^4*b^2 - 36*a^3*b^3 - 19*a^2*b^4 + 2*a*b^5 + 3*b^6) * \sqrt{-a*b}) + e^{(d*x + 10*c)} / (a^2e^{(9*c)} + 2*a*b*e^{(9*c)} + b^2e^{(9*c)}) + (a*e^{(4*d*x + 4*c)} - b*e^{(4*d*x + 4*c)} + 2*a*e^{(2*d*x + 2*c)} - 4*b*e^{(2*d*x + 2*c)} + a + b) / ((a^2e^c + 2*a*b*e^c + b^2e^c) * (a*e^{(5*d*x + 4*c)} + b*e^{(5*d*x + 4*c)} + 2*a*e^{(3*d*x + 2*c)} - 2*b*e^{(3*d*x + 2*c)} + a*e^{(d*x)} + b*e^{(d*x)})) / d$

maple [B] time = 0.32, size = 167, normalized size = 1.82

$$\frac{\frac{1}{(a+b)^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} + \frac{1}{(a+b)^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} + \frac{2b \left[\frac{(a+2b) \left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \frac{1}{2}}{2a} - \frac{3 \operatorname{arctanh}\left(\frac{2 \left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{a+2a+4b}}{4 \sqrt{ab+b^2}}\right)}{4 \sqrt{ab+b^2}} \right]}{(a+b)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x)`

[Out] $1/d * (-1/(a+b)^2 / (\tanh(1/2*d*x+1/2*c) - 1) + 1/(a+b)^2 / (\tanh(1/2*d*x+1/2*c) + 1) + 2*b/(a+b)^2 * ((-1/2*(a+2*b)/a * \tanh(1/2*d*x+1/2*c)^2 - 1/2) / (\tanh(1/2*d*x+1/2*c)^4 * a + 2 * \tanh(1/2*d*x+1/2*c)^2 * a + 4 * \tanh(1/2*d*x+1/2*c)^2 * b + a) - 3/4 / (a*b + b^2)^{(1/2)} * \operatorname{arctanh}(1/4 * (2 * \tanh(1/2*d*x+1/2*c)^2 * a + 2 * a + 4 * b) / (a*b + b^2)^{(1/2)}))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(ae^{(6c)} + be^{(6c)})e^{(6dx)} + 3(ae^{(4c)} - be^{(4c)})e^{(4dx)} + 3(ae^{(2c)} - be^{(2c)})e^{(2dx)} + a}{2 \left((a^3de^{(5c)} + 3a^2bde^{(5c)} + 3ab^2de^{(5c)} + b^3de^{(5c)})e^{(5dx)} + 2(a^3de^{(3c)} + a^2bde^{(3c)} - ab^2de^{(3c)} - b^3de^{(3c)})e^{(3dx)} + (a^3e^{(4c)} + 3a^2b^2e^{(4c)} + 3a^2b^2e^{(4c)} + b^3e^{(4c)})e^{(4dx)} + 2(a^3e^{(2c)} + a^2b^2e^{(2c)} - a^2b^2e^{(2c)} - b^3e^{(2c)})e^{(2dx)} \right), x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] $1/2 * ((a*e^{(6*c)} + b*e^{(6*c)}) * e^{(6*d*x)} + 3 * (a*e^{(4*c)} - b*e^{(4*c)}) * e^{(4*d*x)} + 3 * (a*e^{(2*c)} - b*e^{(2*c)}) * e^{(2*d*x)} + a + b) / ((a^3*d*e^{(5*c)} + 3*a^2*b*d*e^{(5*c)} + 3*a*b^2*d*e^{(5*c)} + b^3*d*e^{(5*c)}) * e^{(5*d*x)} + 2 * (a^3*d*e^{(3*c)} + a^2*b*d*e^{(3*c)} - a*b^2*d*e^{(3*c)} - b^3*d*e^{(3*c)}) * e^{(3*d*x)} + (a^3*d*e^c + 3*a^2*b*d*e^c + 3*a*b^2*d*e^c + b^3*d*e^c) * e^{(d*x)}) + 1/2 * \operatorname{integrate}(6 * (b*e^{(3*d*x + 3*c)} - b*e^{(d*x + c)}) / (a^3 + 3*a^2*b + 3*a*b^2 + b^3 + (a^3e^{(4*c)} + 3*a^2*b^2e^{(4*c)} + 3*a*b^2e^{(4*c)} + b^3e^{(4*c)}) * e^{(4*d*x)} + 2 * (a^3e^{(2*c)} + a^2*b^2e^{(2*c)} - a^2b^2e^{(2*c)} - b^3e^{(2*c)}) * e^{(2*d*x)}), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(c + dx)}{(b \tanh(c + dx)^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2)^2,x)`

[Out] `int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2)^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sinh(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral(sinh(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)
```

$$3.37 \quad \int \frac{\operatorname{csch}(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=103

$$\frac{\sqrt{b}(3a+2b) \tanh^{-1}\left(\frac{\sqrt{b} \operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{3/2}} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^2d} + \frac{b \operatorname{sech}(c+dx)}{2ad(a+b)(a-b \operatorname{sech}^2(c+dx)+b)}$$

[Out] $-\operatorname{arctanh}(\cosh(d*x+c))/a^2/d+1/2*b*\operatorname{sech}(d*x+c)/a/(a+b)/d/(a+b-b*\operatorname{sech}(d*x+c)^2)+1/2*(3*a+2*b)*\operatorname{arctanh}(\operatorname{sech}(d*x+c)*b^{(1/2)/(a+b)^{(1/2)})}*b^{(1/2)/a^2/(a+b)^{(3/2)}/d}$

Rubi [A] time = 0.14, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3664, 414, 522, 207, 208}

$$\frac{\sqrt{b}(3a+2b) \tanh^{-1}\left(\frac{\sqrt{b} \operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{3/2}} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^2d} + \frac{b \operatorname{sech}(c+dx)}{2ad(a+b)(a-b \operatorname{sech}^2(c+dx)+b)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c+d*x]/(a+b*\operatorname{Tanh}[c+d*x]^2), x]$

[Out] $-(\operatorname{ArcTanh}[\operatorname{Cosh}[c+d*x]]/(a^2*d)) + (\operatorname{Sqrt}[b]*(3*a+2*b)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Sech}[c+d*x])/\operatorname{Sqrt}[a+b]])/(2*a^2*(a+b)^{(3/2)*d}) + (b*\operatorname{Sech}[c+d*x])/(2*a*(a+b)*d*(a+b-b*\operatorname{Sech}[c+d*x]^2))$

Rule 207

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Rt}[-a, 2]]/(\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2]), x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{NegQ}[a/b] \ \&\& \ (\operatorname{LtQ}[a, 0] \ || \ \operatorname{GtQ}[b, 0])$

Rule 208

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(\operatorname{Rt}[-(a/b), 2]*\operatorname{ArcTanh}[x/\operatorname{Rt}[-(a/b), 2]])/a, x] /; \operatorname{FreeQ}\{a, b, x\} \ \&\& \ \operatorname{NegQ}[a/b]$

Rule 414

$\operatorname{Int}[(a_+ + (b_+)*(x_+)^{n_+})^{p_+}*((c_+ + (d_+)*(x_+)^{n_+})^{q_+}), x_Symbol] \rightarrow -\operatorname{Simp}[(b*x*(a+b*x^n)^{(p+1)}*(c+d*x^n)^{(q+1)})/(a*n*(p+1)*(b*c-a*d)), x] + \operatorname{Dist}[1/(a*n*(p+1)*(b*c-a*d)), \operatorname{Int}[(a+b*x^n)^{(p+1)}*(c+d*x^n)^q*\operatorname{Simp}[b*c+n*(p+1)*(b*c-a*d)+d*b*(n*(p+q+2)+1)*x^n, x], x] /; \operatorname{FreeQ}\{a, b, c, d, n, q\}, x \ \&\& \ \operatorname{NeQ}[b*c-a*d, 0] \ \&\& \ \operatorname{LtQ}[p, -1] \ \&\& \ !(\ !\operatorname{IntegerQ}[p] \ \&\& \ \operatorname{IntegerQ}[q] \ \&\& \ \operatorname{LtQ}[q, -1]) \ \&\& \ \operatorname{IntBinomialQ}[a, b, c, d, n, p, q, x]$

Rule 522

$\operatorname{Int}[(e_+ + (f_+)*(x_+)^{n_+})/((a_+ + (b_+)*(x_+)^{n_+})*((c_+ + (d_+)*(x_+)^{n_+}))), x_Symbol] \rightarrow \operatorname{Dist}[(b*e-a*f)/(b*c-a*d), \operatorname{Int}[1/(a+b*x^n), x], x] - \operatorname{Dist}[(d*e-c*f)/(b*c-a*d), \operatorname{Int}[1/(c+d*x^n), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, n\}, x]$

Rule 3664

$\operatorname{Int}[\sin[(e_+ + (f_+)*(x_+)]^{m_+}*((a_+ + (b_+)*\tan[(e_+ + (f_+)*(x_+)]^2)^{p_+}), x_Symbol] \rightarrow \operatorname{With}\{ff = \operatorname{FreeFactors}[\operatorname{Sec}[e+f*x], x]\}, \operatorname{Dist}[1/(f*ff^$

m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p)/x^(m + 1), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{csch}(c+dx)}{(a+b \tanh^2(c+dx))^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(-1+x^2)(a+b-bx^2)^2} dx, x, \operatorname{sech}(c+dx)\right)}{d} \\ &= \frac{b \operatorname{sech}(c+dx)}{2a(a+b)d(a+b-b \operatorname{sech}^2(c+dx))} + \frac{\operatorname{Subst}\left(\int \frac{2a+b+bx^2}{(-1+x^2)(a+b-bx^2)} dx, x, \operatorname{sech}(c+dx)\right)}{2a(a+b)d} \\ &= \frac{b \operatorname{sech}(c+dx)}{2a(a+b)d(a+b-b \operatorname{sech}^2(c+dx))} + \frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \operatorname{sech}(c+dx)\right)}{a^2 d} + \dots \\ &= -\frac{\tanh^{-1}(\cosh(c+dx))}{a^2 d} + \frac{\sqrt{b}(3a+2b) \tanh^{-1}\left(\frac{\sqrt{b} \operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2} d} + \frac{b \operatorname{sech}(c+dx)}{2a(a+b)d} \end{aligned}$$

Mathematica [C] time = 0.75, size = 175, normalized size = 1.70

$$\frac{2ab \cosh(c+dx)}{(a+b)((a+b) \cosh(2(c+dx))+a-b)} + \frac{i\sqrt{b}(3a+2b) \tan^{-1}\left(\frac{-\sqrt{a} \tanh\left(\frac{1}{2}(c+dx)\right) - i\sqrt{a+b}}{\sqrt{b}}\right)}{(a+b)^{3/2}} + \frac{i\sqrt{b}(3a+2b) \tan^{-1}\left(\frac{\sqrt{a} \tanh\left(\frac{1}{2}(c+dx)\right) - i\sqrt{a+b}}{\sqrt{b}}\right)}{(a+b)^{3/2}} + 2 \log\left(\frac{\dots}{2a^2 d}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2)^2, x]

[Out] ((I*Sqrt[b]*(3*a + 2*b)*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])/(a + b)^(3/2) + (I*Sqrt[b]*(3*a + 2*b)*ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])/(a + b)^(3/2) + (2*a*b*Cosh[c + d*x])/((a + b)*(a - b + (a + b)*Cosh[2*(c + d*x)]) + 2*Log[Tanh[(c + d*x)/2]])/(2*a^2*d)

fricas [B] time = 1.09, size = 2614, normalized size = 25.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2, x, algorithm="fricas")

[Out] [1/4*(4*a*b*cosh(d*x + c)^3 + 12*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 4*a*b*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + ((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^4 + 4*(3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + 5*a*b + 2*b^2)*sinh(d*x + c)^4 + 2*(3*a^2 - a*b - 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 3*a^2 - a*b - 2*b^2)*sinh(d*x + c)^2 + 3*a^2 + 5*a*b + 2*b^2 + 4*((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^3 + (3*a^2 - a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d

```

*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c) +
(3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) + a + b
)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a +
b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c
)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*
x + c))*sinh(d*x + c) + a + b)) - 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 +
4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*s
inh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 +
2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*l
og(cosh(d*x + c) + sinh(d*x + c) + 1) + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c
)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b +
b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 4*(
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x +
c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(3*a*b*cosh(d*x + c)^2 + a*
b)*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 + 2
*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*b^
2)*d*sinh(d*x + c)^4 + 2*(a^4 - a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*
a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 - a^2*b^2)*d)*sinh(d*x + c)^2 + (
a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^3
+ (a^4 - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*a*b*cosh(d*x + c
)^3 + 6*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*b*sinh(d*x + c)^3 + 2*a*b*c
osh(d*x + c) + ((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^4 + 4*(3*a^2 + 5*a*b
+ 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + 5*a*b + 2*b^2)*sinh(d*x +
c)^4 + 2*(3*a^2 - a*b - 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + 5*a*b + 2*b
^2)*cosh(d*x + c)^2 + 3*a^2 - a*b - 2*b^2)*sinh(d*x + c)^2 + 3*a^2 + 5*a*b
+ 2*b^2 + 4*((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^3 + (3*a^2 - a*b - 2*b^2
)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d
*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)
^3 + (a - 3*b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d
*x + c))*sqrt(-b/(a + b))/b) - ((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^4 + 4
*(3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + 5*a*b + 2
*b^2)*sinh(d*x + c)^4 + 2*(3*a^2 - a*b - 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a
^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 3*a^2 - a*b - 2*b^2)*sinh(d*x + c)^2
+ 3*a^2 + 5*a*b + 2*b^2 + 4*((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^3 + (3*a
^2 - a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2
*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))*sqrt(-b/(a + b))/b) - 2*((
a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*
x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x +
c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 -
b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1)
+ 2*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*c
osh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh
(d*x + c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 +
(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c)
- 1) + 2*(3*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c))/((a^4 + 2*a^3*b + a^
2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh
(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*b^2)*d*sinh(d*x + c)^4 + 2*(a^4 - a^2*b^
2)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (
a^4 - a^2*b^2)*d)*sinh(d*x + c)^2 + (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 +
2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^3 + (a^4 - a^2*b^2)*d*cosh(d*x + c))*si
nh(d*x + c))]

```

giac [B] time = 0.37, size = 1405, normalized size = 13.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] $\frac{1}{2} \left((2(6a^2b^2 + 4ab^3 - (3a^2b - ab^2 - 2b^3))\sqrt{-ab})(a^3e^{2c} + a^2be^{2c})^2\sqrt{a^2 - b^2 + 2\sqrt{-ab}(a+b)}\operatorname{abs}(ae^{2c} + be^{2c}) + (3a^6b + 5a^5b^2 - a^4b^3 - 5a^3b^4 - 2a^2b^5 + 2(3a^5b + 8a^4b^2 + 7a^3b^3 + 2a^2b^4))\sqrt{-ab})\sqrt{a^2 - b^2 + 2\sqrt{-ab}(a+b)}\operatorname{abs}(-a^3e^{2c} - a^2be^{2c})\operatorname{abs}(ae^{2c} + be^{2c})e^{2c} + (6a^9b + 10a^8b^2 - 2a^7b^3 - 10a^6b^4 - 4a^5b^5 - (3a^9 + 2a^8b - 6a^7b^2 - 4a^6b^3 + 3a^5b^4 + 2a^4b^5))\sqrt{-ab})\sqrt{a^2 - b^2 + 2\sqrt{-ab}(a+b)}\operatorname{abs}(ae^{2c} + be^{2c})e^{4c} \right) \arctan\left(\frac{e^{dx}}{\sqrt{(a^4e^{2c} - a^2b^2e^{2c}) - \sqrt{(a^4e^{2c} - a^2b^2e^{2c})^2 - (a^4e^{4c} + 2a^3be^{4c} + a^2b^2e^{4c})(a^4 + 2a^3b + a^2b^2)}}}\right) / \left((a^4e^{4c} + 2a^3be^{4c} + a^2b^2e^{4c}) \right) e^{-4c} / \left((a^{11} + 5a^{10}b + 9a^9b^2 + 5a^8b^3 - 5a^7b^4 - 9a^6b^5 - 5a^5b^6 - a^4b^7 + 2(a^{10} + 6a^9b + 15a^8b^2 + 20a^7b^3 + 15a^6b^4 + 6a^5b^5 + a^4b^6))\sqrt{-ab})\operatorname{abs}(-a^3e^{2c} - a^2be^{2c}) \right) + (2(12a^3b^2 - 4a^2b^3 - 8ab^4 + (3a^3b - 16a^2b^2 - 9ab^3 + 2b^4))\sqrt{-ab})(a^3e^{2c} + a^2be^{2c})^2\operatorname{abs}(ae^{2c} + be^{2c}) + (3a^7b - 10a^6b^2 - 38a^5b^3 - 32a^4b^4 - 5a^3b^5 + 2a^2b^6 - 4(3a^6b + 5a^5b^2 - a^4b^3 - 5a^3b^4 - 2a^2b^5))\sqrt{-ab})\operatorname{abs}(-a^3e^{2c} - a^2be^{2c})\operatorname{abs}(ae^{2c} + be^{2c})e^{2c} + (12a^{10}b + 8a^9b^2 - 24a^8b^3 - 16a^7b^4 + 12a^6b^5 + 8a^5b^6 + (3a^{10} - 13a^9b - 28a^8b^2 + 6a^7b^3 + 27a^6b^4 + 7a^5b^5 - 2a^4b^6))\sqrt{-ab})\operatorname{abs}(ae^{2c} + be^{2c})e^{4c} \right) \arctan\left(\frac{e^{dx}}{\sqrt{(a^4e^{2c} - a^2b^2e^{2c}) + \sqrt{(a^4e^{2c} - a^2b^2e^{2c})^2 - (a^4e^{4c} + 2a^3be^{4c} + a^2b^2e^{4c})(a^4 + 2a^3b + a^2b^2)}}}\right) / \left((a^4e^{4c} + 2a^3be^{4c} + a^2b^2e^{4c}) \right) e^{-4c} / \left((a^{10} + 4a^9b + 5a^8b^2 - 5a^6b^4 - 4a^5b^5 - a^4b^6 - 2(a^9 + 5a^8b + 10a^7b^2 + 10a^6b^3 + 5a^5b^4 + a^4b^5))\sqrt{-ab})\sqrt{a^2 - b^2 - 2\sqrt{-ab}(a+b)}\operatorname{abs}(-a^3e^{2c} - a^2be^{2c}) \right) + 2(b e^{3dx + 3c} + b e^{dx + c}) / \left((a^2 + ab)(a e^{4dx + 4c} + b e^{4dx + 4c} + 2a e^{2dx + 2c} - 2b e^{2dx + 2c} + a + b) - 2\log(e^{dx + c} + 1) / a^2 + 2\log(\operatorname{abs}(e^{dx + c} - 1)) / a^2 \right) / d$

maple [B] time = 0.47, size = 331, normalized size = 3.21

$$\frac{b \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{da \left(\left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a + 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a + 4 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b + a \right) (a+b) + da^2 \left(\left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x)

[Out] $\frac{1}{d} \frac{a b}{(a+b) \tanh(1/2 dx + 1/2 c)^4} + \frac{2}{d} \frac{a^2 b^2}{(a+b) \tanh(1/2 dx + 1/2 c)^2} + \frac{4}{d} \frac{a^2 \tanh(1/2 dx + 1/2 c)^2 b + a}{(a+b) \tanh(1/2 dx + 1/2 c)^2} + \frac{1}{d} \frac{a b}{(a+b) \tanh(1/2 dx + 1/2 c)^4} + \frac{2}{d} \frac{a^2 \tanh(1/2 dx + 1/2 c)^2 b + a}{(a+b) \tanh(1/2 dx + 1/2 c)^2} + \frac{3}{2} \frac{1}{d} \frac{a b}{(a+b) \sqrt{a b + b^2}} \operatorname{arctanh} \left(\frac{1}{4} \frac{2 \tanh(1/2 dx + 1/2 c)^2 a + 2 a + 4 b}{(a b + b^2)^{1/2}} \right) + \frac{1}{d} \frac{a^2 b^2}{(a+b) \sqrt{a b + b^2}} \operatorname{arctanh} \left(\frac{1}{4} \frac{2 \tanh(1/2 dx + 1/2 c)^2 a + 2 a + 4 b}{(a b + b^2)^{1/2}} \right) + \frac{1}{d} \frac{a^2 \ln(\tanh(1/2 dx + 1/2 c))}{(a+b)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b e^{3dx+3c} + b e^{dx+c}}{a^3 d + 2 a^2 b d + a b^2 d + (a^3 d e^{4c} + 2 a^2 b d e^{4c} + a b^2 d e^{4c}) e^{4dx} + 2 (a^3 d e^{2c} - a b^2 d e^{2c}) e^{2dx}} \frac{\log \left(\left(e^{dx+c} + \right. \right.}{a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] (b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^3*d + 2*a^2*b*d + a*b^2*d + (a^3*d*e^(4*c) + 2*a^2*b*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(a^3*d*e^(2*c) - a*b^2*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) + log((e^(d*x + c) - 1)*e^(-c))/(a^2*d) - 2*integrate(1/2*((3*a*b*e^(3*c) + 2*b^2*e^(3*c))*e^(3*d*x) - (3*a*b*e^c + 2*b^2*e^c)*e^(d*x))/(a^4 + 2*a^3*b + a^2*b^2 + (a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) - a^2*b^2*e^(2*c))*e^(2*d*x)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh(c + dx) (b \tanh(c + dx)^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^2), x)

[Out] int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)

[Out] Integral(csch(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)

$$3.38 \quad \int \frac{\operatorname{csch}^2(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=82

$$-\frac{3\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d} - \frac{3 \operatorname{coth}(c+dx)}{2a^2d} + \frac{\operatorname{coth}(c+dx)}{2ad(a+b \tanh^2(c+dx))}$$

[Out] $-3/2*\operatorname{coth}(d*x+c)/a^2/d-3/2*\arctan(b^{(1/2)}*\tanh(d*x+c)/a^{(1/2)})*b^{(1/2)}/a^{(5/2)}/d+1/2*\operatorname{coth}(d*x+c)/a/d/(a+b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3663, 290, 325, 205}

$$-\frac{3\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d} - \frac{3 \operatorname{coth}(c+dx)}{2a^2d} + \frac{\operatorname{coth}(c+dx)}{2ad(a+b \tanh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]

[Out] $(-3*\sqrt{b}*\operatorname{ArcTan}[(\sqrt{b}*\operatorname{Tanh}[c + d*x])/ \sqrt{a}])/(2*a^{(5/2)*d}) - (3*\operatorname{Coth}[c + d*x])/(2*a^2*d) + \operatorname{Coth}[c + d*x]/(2*a*d*(a + b*\operatorname{Tanh}[c + d*x]^2))$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*n*(p+1)), x] + Dist[(m+n*(p+1)+1)/(a*n*(p+1)), Int[(c*x)^m*(a+b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 325

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 3663

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m+1))/f, Subst[Int[(x^m*(a+b*(ff*x)^n)^p]/(c^2+ff^2*x^2)^(m/2+1), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(c+dx)}{(a+b \tanh^2(c+dx))^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\operatorname{coth}(c+dx)}{2ad(a+b \tanh^2(c+dx))} + \frac{3 \operatorname{Subst}\left(\int \frac{1}{x^2(a+bx^2)} dx, x, \tanh(c+dx)\right)}{2ad} \\
&= -\frac{3 \operatorname{coth}(c+dx)}{2a^2d} + \frac{\operatorname{coth}(c+dx)}{2ad(a+b \tanh^2(c+dx))} - \frac{(3b) \operatorname{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \tanh(c+dx)\right)}{2a^2d} \\
&= -\frac{3\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d} - \frac{3 \operatorname{coth}(c+dx)}{2a^2d} + \frac{\operatorname{coth}(c+dx)}{2ad(a+b \tanh^2(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.50, size = 86, normalized size = 1.05

$$\frac{-3\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right) - \frac{\sqrt{a} b \sinh(2(c+dx))}{(a+b) \cosh(2(c+dx))+a-b} - 2\sqrt{a} \operatorname{coth}(c+dx)}{2a^{5/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (-3*sqrt[b]*ArcTan[(sqrt[b]*Tanh[c + d*x])/sqrt[a]] - 2*sqrt[a]*Coth[c + d*x] - (sqrt[a]*b*Sinh[2*(c + d*x)]/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(2*a^(5/2)*d)

fricas [B] time = 0.63, size = 2562, normalized size = 31.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [-1/4*(4*(2*a^2 + 3*a*b + 3*b^2)*cosh(d*x + c)^4 + 16*(2*a^2 + 3*a*b + 3*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(2*a^2 + 3*a*b + 3*b^2)*sinh(d*x + c)^4 + 8*(2*a^2 - 3*b^2)*cosh(d*x + c)^2 + 8*(3*(2*a^2 + 3*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*b^2)*sinh(d*x + c)^2 - 3*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + (a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^4 + (15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - 2*a*b - 3*b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - 2*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^2 - a^2 + 2*a*b + 3*b^2)*sinh(d*x + c)^2 - a^2 - 2*a*b - b^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 2*(a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a)]/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)

$$\begin{aligned} &^2 + a - b) \sinh(dx + c)^2 + 4*((a + b) \cosh(dx + c)^3 + (a - b) \cosh(dx \\ &+ c)) \sinh(dx + c) + a + b)) + 8a^2 + 20ab + 12b^2 + 16*((2a^2 + 3a \\ &*b + 3b^2) \cosh(dx + c)^3 + (2a^2 - 3b^2) \cosh(dx + c)) \sinh(dx + c)) \\ &/((a^4 + 2a^3b + a^2b^2) d \cosh(dx + c)^6 + 6(a^4 + 2a^3b + a^2b^2) \\ &*d \cosh(dx + c) \sinh(dx + c)^5 + (a^4 + 2a^3b + a^2b^2) d \sinh(dx + c \\ &)^6 + (a^4 - 2a^3b - 3a^2b^2) d \cosh(dx + c)^4 + (15(a^4 + 2a^3b + \\ &a^2b^2) d \cosh(dx + c)^2 + (a^4 - 2a^3b - 3a^2b^2) d) \sinh(dx + c)^4 \\ &- (a^4 - 2a^3b - 3a^2b^2) d \cosh(dx + c)^2 + 4*(5(a^4 + 2a^3b + a^2 \\ &b^2) d \cosh(dx + c)^3 + (a^4 - 2a^3b - 3a^2b^2) d \cosh(dx + c)) \sin \\ &h(dx + c)^3 + (15(a^4 + 2a^3b + a^2b^2) d \cosh(dx + c)^4 + 6(a^4 - 2 \\ &a^3b - 3a^2b^2) d \cosh(dx + c)^2 - (a^4 - 2a^3b - 3a^2b^2) d) \sinh \\ &(dx + c)^2 - (a^4 + 2a^3b + a^2b^2) d + 2*(3(a^4 + 2a^3b + a^2b^2) * \\ &d \cosh(dx + c)^5 + 2(a^4 - 2a^3b - 3a^2b^2) d \cosh(dx + c)^3 - (a^4 \\ &- 2a^3b - 3a^2b^2) d \cosh(dx + c)) \sinh(dx + c)), -1/2*(2*(2a^2 + 3* \\ &a*b + 3b^2) \cosh(dx + c)^4 + 8*(2a^2 + 3a*b + 3b^2) \cosh(dx + c) \sinh \\ &(dx + c)^3 + 2*(2a^2 + 3a*b + 3b^2) \sinh(dx + c)^4 + 4*(2a^2 - 3b^2) \\ &* \cosh(dx + c)^2 + 4*(3*(2a^2 + 3a*b + 3b^2) \cosh(dx + c)^2 + 2a^2 - 3 \\ &b^2) \sinh(dx + c)^2 + 3*((a^2 + 2a*b + b^2) \cosh(dx + c)^6 + 6(a^2 + 2 \\ &a*b + b^2) \cosh(dx + c) \sinh(dx + c)^5 + (a^2 + 2a*b + b^2) \sinh(dx + \\ &c)^6 + (a^2 - 2a*b - 3b^2) \cosh(dx + c)^4 + (15(a^2 + 2a*b + b^2) \cosh \\ &(dx + c)^2 + a^2 - 2a*b - 3b^2) \sinh(dx + c)^4 + 4*(5(a^2 + 2a*b + b^ \\ &2) \cosh(dx + c)^3 + (a^2 - 2a*b - 3b^2) \cosh(dx + c)) \sinh(dx + c)^3 - \\ &(a^2 - 2a*b - 3b^2) \cosh(dx + c)^2 + (15(a^2 + 2a*b + b^2) \cosh(dx + \\ &c)^4 + 6(a^2 - 2a*b - 3b^2) \cosh(dx + c)^2 - a^2 + 2a*b + 3b^2) \sinh \\ &(dx + c)^2 - a^2 - 2a*b - b^2 + 2*(3(a^2 + 2a*b + b^2) \cosh(dx + c)^5 \\ &+ 2(a^2 - 2a*b - 3b^2) \cosh(dx + c)^3 - (a^2 - 2a*b - 3b^2) \cosh(dx \\ &+ c)) \sinh(dx + c)) \sqrt{b/a} \arctan(1/2*((a + b) \cosh(dx + c)^2 + 2(a + \\ &b) \cosh(dx + c) \sinh(dx + c) + (a + b) \sinh(dx + c)^2 + a - b) \sqrt{b/a} \\ &)/b) + 4a^2 + 10ab + 6b^2 + 8*((2a^2 + 3a*b + 3b^2) \cosh(dx + c)^3 \\ &+ (2a^2 - 3b^2) \cosh(dx + c)) \sinh(dx + c))/((a^4 + 2a^3b + a^2b^2) * \\ &d \cosh(dx + c)^6 + 6(a^4 + 2a^3b + a^2b^2) d \cosh(dx + c) \sinh(dx + \\ &c)^5 + (a^4 + 2a^3b + a^2b^2) d \sinh(dx + c)^6 + (a^4 - 2a^3b - 3a^2 \\ &b^2) d \cosh(dx + c)^4 + (15(a^4 + 2a^3b + a^2b^2) d \cosh(dx + c)^2 + \\ &(a^4 - 2a^3b - 3a^2b^2) d) \sinh(dx + c)^4 - (a^4 - 2a^3b - 3a^2b^2 \\ &2) d \cosh(dx + c)^2 + 4*(5(a^4 + 2a^3b + a^2b^2) d \cosh(dx + c)^3 + (\\ &a^4 - 2a^3b - 3a^2b^2) d \cosh(dx + c)) \sinh(dx + c)^3 + (15(a^4 + 2* \\ &a^3b + a^2b^2) d \cosh(dx + c)^4 + 6(a^4 - 2a^3b - 3a^2b^2) d \cosh(dx \\ &*x + c)^2 - (a^4 - 2a^3b - 3a^2b^2) d) \sinh(dx + c)^2 - (a^4 + 2a^3b \\ &+ a^2b^2) d + 2*(3(a^4 + 2a^3b + a^2b^2) d \cosh(dx + c)^5 + 2(a^4 - \\ &2a^3b - 3a^2b^2) d \cosh(dx + c)^3 - (a^4 - 2a^3b - 3a^2b^2) d \cos \\ &h(dx + c)) \sinh(dx + c))] \end{aligned}$$

giac [B] time = 0.62, size = 227, normalized size = 2.77

$$\frac{3b \arctan\left(\frac{ae^{(2dx+2c)} + be^{(2dx+2c)+a-b}}{2\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{2(2a^2e^{(4dx+4c)} + 3abe^{(4dx+4c)} + 3b^2e^{(4dx+4c)} + 4a^2e^{(2dx+2c)} - 6b^2e^{(2dx+2c)} + 2a^2 + 5ab + 3b^2)}{(a^3 + a^2b)(ae^{(6dx+6c)} + be^{(6dx+6c)} + ae^{(4dx+4c)} - 3be^{(4dx+4c)} - ae^{(2dx+2c)} + 3be^{(2dx+2c)} - a - b)}$$

2d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)^2/(a+b*tanh(dx+c)^2),x, algorithm="giac")

[Out] -1/2*(3*b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/sqrt(a*b)*a^2) + 2*(2*a^2*e^(4*d*x + 4*c) + 3*a*b*e^(4*d*x + 4*c) + 3*b^2*e^(4*d*x + 4*c) + 4*a^2*e^(2*d*x + 2*c) - 6*b^2*e^(2*d*x + 2*c) + 2*a^2 + 5*a*b + 3*b^2)/((a^3 + a^2*b)*(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) + a*e^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) - a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c) - a - b))/d

maple [B] time = 0.46, size = 552, normalized size = 6.73

$$\frac{\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{2d a^2} \frac{b\left(\tanh^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d a^2 \left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + 4\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b + a\right) d a^2 \left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + 4\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x)

[Out]
$$-1/2/d/a^2*\tanh(1/2*d*x+1/2*c)-1/d/a^2*b/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)*\tanh(1/2*d*x+1/2*c)^3-1/d/a^2*b/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)*\tanh(1/2*d*x+1/2*c)+3/2/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))*b-3/2/d/a^2*b/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/2/d/a^2*b^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/2/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))*b+3/2/d/a^2*b/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+3/2/d/a^2*b^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/2/d/a^2/\tanh(1/2*d*x+1/2*c)$$

maxima [B] time = 0.50, size = 212, normalized size = 2.59

$$\frac{2a^2 + 5ab + 3b^2 + 2(2a^2 - 3b^2)e^{(-2dx-2c)} + (2a^2 + 3ab + 3b^2)e^{(-4dx-4c)}}{(a^4 + 2a^3b + a^2b^2 + (a^4 - 2a^3b - 3a^2b^2)e^{(-2dx-2c)} - (a^4 - 2a^3b - 3a^2b^2)e^{(-4dx-4c)} - (a^4 + 2a^3b + a^2b^2)e^{(-6dx-6c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out]
$$-(2*a^2 + 5*a*b + 3*b^2 + 2*(2*a^2 - 3*b^2))*e^{(-2*d*x - 2*c)} + (2*a^2 + 3*a*b + 3*b^2)*e^{(-4*d*x - 4*c)}/((a^4 + 2*a^3*b + a^2*b^2 + (a^4 - 2*a^3*b - 3*a^2*b^2))*e^{(-2*d*x - 2*c)} - (a^4 - 2*a^3*b - 3*a^2*b^2))*e^{(-4*d*x - 4*c)} - (a^4 + 2*a^3*b + a^2*b^2)*e^{(-6*d*x - 6*c))*d) + 3/2*b*\arctan(1/2*((a + b)*e^{(-2*d*x - 2*c)} + a - b)/\sqrt{a*b})/(\sqrt{a*b}*a^2*d)$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh(c + dx)^2 (b \tanh(c + dx)^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2),x)

[Out] int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)

[Out] Integral(csch(c + d*x)**2/(a + b*tanh(c + d*x)**2)**2, x)

$$3.39 \quad \int \frac{\operatorname{csch}^3(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=141

$$\frac{(a+4b) \tanh^{-1}(\cosh(c+dx))}{2a^3d} - \frac{\sqrt{b}(3a+4b) \tanh^{-1}\left(\frac{\sqrt{b} \operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^3d\sqrt{a+b}} - \frac{b \operatorname{sech}(c+dx)}{a^2d(a-b \operatorname{sech}^2(c+dx)+b)} - \frac{\coth(c+dx)}{2ad(a-b \operatorname{sech}^2(c+dx)+b)}$$

[Out] 1/2*(a+4*b)*arctanh(cosh(d*x+c))/a^3/d-1/2*coth(d*x+c)*csch(d*x+c)/a/d/(a+b-b*sech(d*x+c)^2)-b*sech(d*x+c)/a^2/d/(a+b-b*sech(d*x+c)^2)-1/2*(3*a+4*b)*a*rctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a^3/d/(a+b)^(1/2)

Rubi [A] time = 0.21, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3664, 471, 527, 522, 207, 208}

$$-\frac{b \operatorname{sech}(c+dx)}{a^2d(a-b \operatorname{sech}^2(c+dx)+b)} + \frac{(a+4b) \tanh^{-1}(\cosh(c+dx))}{2a^3d} - \frac{\sqrt{b}(3a+4b) \tanh^{-1}\left(\frac{\sqrt{b} \operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^3d\sqrt{a+b}} - \frac{\coth(c+dx)}{2ad(a-b \operatorname{sech}^2(c+dx)+b)}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2, x]

[Out] ((a + 4*b)*ArcTanh[Cosh[c + d*x]])/(2*a^3*d) - (Sqrt[b]*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(2*a^3*Sqrt[a + b]*d) - (Coth[c + d*x]*Csch[c + d*x])/(2*a*d*(a + b - b*Sech[c + d*x]^2)) - (b*Sech[c + d*x])/(a^2*d*(a + b - b*Sech[c + d*x]^2))

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 471

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n-1)*(e*x)^(m-n+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(n*(b*c-a*d)*(p+1)), x] - Dist[e^n/(n*(b*c-a*d)*(p+1)), Int[(e*x)^(m-n)*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[c*(m-n+1)+d*(m+n*(p+q+1)+1]*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m-n+1] && GtQ[m-n+1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e-a*f)/(b*c-a*d), Int[1/(a+b*x^n), x], x] - Dist[(d*e-c*f)/(b*c-a*d), Int[1/(c+d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3664

```
Int[sin[(e_) + (f_)*(x_)]^(m_)*((a_) + (b_)*tan[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] := With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p]/x^(m + 1), x], x, Sec[e + f*x]/ff, x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c + dx)}{(a + b \tanh^2(c + dx))^2} dx = -\frac{\operatorname{Subst}\left(\int \frac{x^2}{(-1+x^2)^2(a+b-bx^2)^2} dx, x, \operatorname{sech}(c + dx)\right)}{d}$$

$$= -\frac{\operatorname{coth}(c + dx)\operatorname{csch}(c + dx)}{2ad(a + b - b\operatorname{sech}^2(c + dx))} - \frac{\operatorname{Subst}\left(\int \frac{a+b+3bx^2}{(-1+x^2)(a+b-bx^2)^2} dx, x, \operatorname{sech}(c + dx)\right)}{2ad}$$

$$= -\frac{\operatorname{coth}(c + dx)\operatorname{csch}(c + dx)}{2ad(a + b - b\operatorname{sech}^2(c + dx))} - \frac{b\operatorname{sech}(c + dx)}{a^2d(a + b - b\operatorname{sech}^2(c + dx))} - \frac{\operatorname{Subst}\left(\int \frac{2(a+b)}{(-1+x^2)} dx, x, \operatorname{sech}(c + dx)\right)}{(a + 4b)\operatorname{Subst}\left(\int \frac{2(a+b)}{(-1+x^2)} dx, x, \operatorname{sech}(c + dx)\right)}$$

$$= \frac{(a + 4b)\tanh^{-1}(\cosh(c + dx))}{2a^3d} - \frac{\sqrt{b}(3a + 4b)\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c + dx)}{\sqrt{a+b}}\right)}{2a^3\sqrt{a + b}d} - \frac{\operatorname{coth}(c + dx)}{2ad(a + b)}$$

Mathematica [C] time = 4.35, size = 203, normalized size = 1.44

$$\frac{8ab \cosh(c+dx)}{(a+b) \cosh(2(c+dx))+a-b} + 4(a + 4b) \log\left(\tanh\left(\frac{1}{2}(c + dx)\right)\right) + \frac{4i\sqrt{b}(3a+4b) \tan^{-1}\left(\frac{-\sqrt{a} \tanh\left(\frac{1}{2}(c+dx)\right) - i\sqrt{a+b}}{\sqrt{b}}\right)}{\sqrt{a+b}} + \frac{4i\sqrt{b}(3a+4b) \tan^{-1}\left(\frac{-\sqrt{a} \tanh\left(\frac{1}{2}(c+dx)\right) - i\sqrt{a+b}}{\sqrt{b}}\right)}{\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2), x]
```

```
[Out] -1/8*(((4*I)*Sqrt[b]*(3*a + 4*b)*ArcTan[(-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2]]/Sqrt[b])/Sqrt[a + b] + ((4*I)*Sqrt[b]*(3*a + 4*b)*ArcTan[(-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2]]/Sqrt[b])/Sqrt[a + b] + (8*a*b*Cosh[c + d*x])/(a - b + (a + b)*Cosh[2*(c + d*x)]) + a*Csch[(c + d*x)/2]^2 + 4*(a + 4*b)*Log[Tanh[(c + d*x)/2]] + a*Sech[(c + d*x)/2]^2/(a^3*d)
```

fricas [B] time = 0.73, size = 6335, normalized size = 44.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/4*(4*(a^2 + 2*a*b)*\cosh(d*x + c)^7 + 28*(a^2 + 2*a*b)*\cosh(d*x + c)*\sinh \\ & (d*x + c)^6 + 4*(a^2 + 2*a*b)*\sinh(d*x + c)^7 + 4*(3*a^2 - 2*a*b)*\cosh(d*x \\ & + c)^5 + 4*(21*(a^2 + 2*a*b)*\cosh(d*x + c)^2 + 3*a^2 - 2*a*b)*\sinh(d*x + c) \\ & ^5 + 20*(7*(a^2 + 2*a*b)*\cosh(d*x + c)^3 + (3*a^2 - 2*a*b)*\cosh(d*x + c))*\sinh(d*x + c)^4 \\ & + 4*(3*a^2 - 2*a*b)*\cosh(d*x + c)^3 + 4*(35*(a^2 + 2*a*b)*\cosh(d*x + c)^4 + 10*(3*a^2 - 2*a*b)*\cosh(d*x + c)^2 + 3*a^2 - 2*a*b)*\sinh(d*x + c)^3 + 4*(21*(a^2 + 2*a*b)*\cosh(d*x + c)^5 + 10*(3*a^2 - 2*a*b)*\cosh(d*x + c)^3 + 3*(3*a^2 - 2*a*b)*\cosh(d*x + c))*\sinh(d*x + c)^2 - ((3*a^2 + 7*a*b + 4*b^2)*\cosh(d*x + c)^8 + 8*(3*a^2 + 7*a*b + 4*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (3*a^2 + 7*a*b + 4*b^2)*\sinh(d*x + c)^8 - 4*(3*a*b + 4*b^2)*\cosh(d*x + c)^6 + 4*(7*(3*a^2 + 7*a*b + 4*b^2)*\cosh(d*x + c)^2 - 3*a*b - 4*b^2)*\sinh(d*x + c)^6 + 8*(7*(3*a^2 + 7*a*b + 4*b^2)*\cosh(d*x + c)^3 - 3*(3*a*b + 4*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2*(3*a^2 - 5*a*b - 12*b^2)*\cosh(d*x + c)^4 + 2*(35*(3*a^2 + 7*a*b + 4*b^2)*\cosh(d*x + c)^4 - 30*(3*a*b + 4*b^2)*\cosh(d*x + c)^2 - 3*a^2 + 5*a*b + 12*b^2)*\sinh(d*x + c)^4 + 8*(7*(3*a^2 + 7*a*b + 4*b^2)*\cosh(d*x + c)^5 - 10*(3*a*b + 4*b^2)*\cosh(d*x + c)^3 - (3*a^2 - 5*a*b - 12*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 4*(3*a*b + 4*b^2)*\cosh(d*x + c)^2 + 4*(7*(3*a^2 + 7*a*b + 4*b^2)*\cosh(d*x + c)^6 - 15*(3*a*b + 4*b^2)*\cosh(d*x + c)^4 - 3*(3*a^2 - 5*a*b - 12*b^2)*\cosh(d*x + c)^2 - 3*a*b - 4*b^2)*\sinh(d*x + c)^2 + 3*a^2 + 7*a*b + 4*b^2 + 8*((3*a^2 + 7*a*b + 4*b^2)*\cosh(d*x + c)^7 - 3*(3*a*b + 4*b^2)*\cosh(d*x + c)^5 - (3*a^2 - 5*a*b - 12*b^2)*\cosh(d*x + c)^3 - (3*a*b + 4*b^2)*\cosh(d*x + c))*\sinh(d*x + c)) * \sqrt{b/(a + b)} * \log(((a + b)*\cosh(d*x + c)^4 + 4*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a + b)*\sinh(d*x + c)^4 + 2*(a + 3*b)*\cosh(d*x + c)^2 + 2*(3*(a + b)*\cosh(d*x + c)^2 + a + 3*b)*\sinh(d*x + c)^2 + 4*((a + b)*\cosh(d*x + c)^2 + (a + 3*b)*\cosh(d*x + c))*\sinh(d*x + c) - 4*((a + b)*\cosh(d*x + c)^3 + 3*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a + b)*\sinh(d*x + c)^3 + (a + b)*\cosh(d*x + c) + (3*(a + b)*\cosh(d*x + c)^2 + a + b)*\sinh(d*x + c))*\sqrt{b/(a + b)} + a + b)/((a + b)*\cosh(d*x + c)^4 + 4*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a + b)*\sinh(d*x + c)^4 + 2*(a - b)*\cosh(d*x + c)^2 + 2*(3*(a + b)*\cosh(d*x + c)^2 + a - b)*\sinh(d*x + c)^2 + 4*((a + b)*\cosh(d*x + c)^3 + (a - b)*\cosh(d*x + c))*\sinh(d*x + c) + a + b)) + 4*(a^2 + 2*a*b)*\cosh(d*x + c) - 2*((a^2 + 5*a*b + 4*b^2)*\cosh(d*x + c)^8 + 8*(a^2 + 5*a*b + 4*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^2 + 5*a*b + 4*b^2)*\sinh(d*x + c)^8 - 4*(a*b + 4*b^2)*\cosh(d*x + c)^6 + 4*(7*(a^2 + 5*a*b + 4*b^2)*\cosh(d*x + c)^2 - a*b - 4*b^2)*\sinh(d*x + c)^6 + 8*(7*(a^2 + 5*a*b + 4*b^2)*\cosh(d*x + c)^3 - 3*(a*b + 4*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2*(a^2 + a*b - 12*b^2)*\cosh(d*x + c)^4 + 2*(35*(a^2 + 5*a*b + 4*b^2)*\cosh(d*x + c)^4 - 30*(a*b + 4*b^2)*\cosh(d*x + c)^2 - a^2 - a*b + 12*b^2)*\sinh(d*x + c)^4 + 8*(7*(a^2 + 5*a*b + 4*b^2)*\cosh(d*x + c)^5 - 10*(a*b + 4*b^2)*\cosh(d*x + c)^3 - (a^2 + a*b - 12*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 4*(a*b + 4*b^2)*\cosh(d*x + c)^2 + 4*(7*(a^2 + 5*a*b + 4*b^2)*\cosh(d*x + c)^6 - 15*(a*b + 4*b^2)*\cosh(d*x + c)^4 - 3*(a^2 + a*b - 12*b^2)*\cosh(d*x + c)^2 - a*b - 4*b^2)*\sinh(d*x + c)^2 + a^2 + 5*a*b + 4*b^2 + 8*((a^2 + 5*a*b + 4*b^2)*\cosh(d*x + c)^7 - 3*(a*b + 4*b^2)*\cosh(d*x + c)^5 - (a^2 + a*b - 12*b^2)*\cosh(d*x + c)^3 - (a*b + 4*b^2)*\cosh(d*x + c))*\sinh(d*x + c)) * \log(\cosh(d*x + c) + \sinh(d*x + c) + 1) + 2*((a^2 + 5*a*b + 4*b^2)*\cosh(d*x + c)^8 + 8*(a^2 + 5*a*b + 4*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^2 + 5*a*b + 4*b^2)*\sinh(d*x + c)^8 - 4*(a*b + 4*b^2)*\cosh(d*x + c)^6 + 4*(7*(a^2 + 5*a*b + 4*b^2)*\cosh(d*x + c)^2 - a*b - 4*b^2)*\sinh(d*x + c)^6 + 8*(7*(a^2 + 5*a*b + 4*b^2)*\cosh(d*x + c)^3 - 3*(a*b + 4*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2*(a^2 + a*b - 12*b^2)*\cosh(d*x + c)^4 + 2*(35*(a^2 + 5*a*b + 4*b^2)*\cosh(d*x + c)^4 - 30*(a*b + 4*b^2)*\cosh(d*x + c)^2 - a^2 - a*b + 12*b^2)*\sinh(d*x + c)^4 + 8*(7*(a^2 + 5*a*b + 4*b^2)*\cosh(d*x + c)^5 - 10*(a*b + 4*b^2)*\cosh(d*x + c)^3 - (a^2 + a*b - 12*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 4*(a*b + 4*b^2)*\cosh(d*x + c)^2 + 4*(7*(a^2 + 5*a*b + 4*b^2)*\cosh(d*x + c)^6 - 15*(a*b + 4*b^2)*\cosh(d*x + c) \end{aligned}$$

$$\begin{aligned}
&)^4 - 3*(a^2 + a*b - 12*b^2)*\cosh(d*x + c)^2 - a*b - 4*b^2)*\sinh(d*x + c)^2 \\
&+ a^2 + 5*a*b + 4*b^2 + 8*((a^2 + 5*a*b + 4*b^2)*\cosh(d*x + c)^7 - 3*(a*b \\
&+ 4*b^2)*\cosh(d*x + c)^5 - (a^2 + a*b - 12*b^2)*\cosh(d*x + c)^3 - (a*b + 4* \\
&b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) + \\
&4*(7*(a^2 + 2*a*b)*\cosh(d*x + c)^6 + 5*(3*a^2 - 2*a*b)*\cosh(d*x + c)^4 + 3 \\
&)*(3*a^2 - 2*a*b)*\cosh(d*x + c)^2 + a^2 + 2*a*b)*\sinh(d*x + c))/(4*a^3*b*d*c \\
&\cosh(d*x + c)^6 - (a^4 + a^3*b)*d*\cosh(d*x + c)^8 - 8*(a^4 + a^3*b)*d*\cosh(d \\
&*x + c)*\sinh(d*x + c)^7 - (a^4 + a^3*b)*d*\sinh(d*x + c)^8 + 4*a^3*b*d*\cosh(\\
&d*x + c)^2 + 4*(a^3*b*d - 7*(a^4 + a^3*b)*d*\cosh(d*x + c)^2)*\sinh(d*x + c)^ \\
&6 + 2*(a^4 - 3*a^3*b)*d*\cosh(d*x + c)^4 + 8*(3*a^3*b*d*\cosh(d*x + c) - 7*(a \\
&^4 + a^3*b)*d*\cosh(d*x + c)^3)*\sinh(d*x + c)^5 + 2*(30*a^3*b*d*\cosh(d*x + c \\
&)^2 - 35*(a^4 + a^3*b)*d*\cosh(d*x + c)^4 + (a^4 - 3*a^3*b)*d)*\sinh(d*x + c) \\
&^4 + 8*(10*a^3*b*d*\cosh(d*x + c)^3 - 7*(a^4 + a^3*b)*d*\cosh(d*x + c)^5 + (a \\
&^4 - 3*a^3*b)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(15*a^3*b*d*\cosh(d*x + c \\
&)^4 - 7*(a^4 + a^3*b)*d*\cosh(d*x + c)^6 + a^3*b*d + 3*(a^4 - 3*a^3*b)*d*\cos \\
&h(d*x + c)^2)*\sinh(d*x + c)^2 - (a^4 + a^3*b)*d + 8*(3*a^3*b*d*\cosh(d*x + c \\
&)^5 - (a^4 + a^3*b)*d*\cosh(d*x + c)^7 + a^3*b*d*\cosh(d*x + c) + (a^4 - 3*a^ \\
&3*b)*d*\cosh(d*x + c)^3)*\sinh(d*x + c)), 1/2*(2*(a^2 + 2*a*b)*\cosh(d*x + c)^ \\
&7 + 14*(a^2 + 2*a*b)*\cosh(d*x + c)*\sinh(d*x + c)^6 + 2*(a^2 + 2*a*b)*\sinh(d \\
&*x + c)^7 + 2*(3*a^2 - 2*a*b)*\cosh(d*x + c)^5 + 2*(21*(a^2 + 2*a*b)*\cosh(d \\
&x + c)^2 + 3*a^2 - 2*a*b)*\sinh(d*x + c)^5 + 10*(7*(a^2 + 2*a*b)*\cosh(d*x + \\
&c)^3 + (3*a^2 - 2*a*b)*\cosh(d*x + c))*\sinh(d*x + c)^4 + 2*(3*a^2 - 2*a*b)*\c \\
&osh(d*x + c)^3 + 2*(35*(a^2 + 2*a*b)*\cosh(d*x + c)^4 + 10*(3*a^2 - 2*a*b)*\c \\
&osh(d*x + c)^2 + 3*a^2 - 2*a*b)*\sinh(d*x + c)^3 + 2*(21*(a^2 + 2*a*b)*\cosh(\\
&d*x + c)^5 + 10*(3*a^2 - 2*a*b)*\cosh(d*x + c)^3 + 3*(3*a^2 - 2*a*b)*\cosh(d \\
&x + c))*\sinh(d*x + c)^2 + ((3*a^2 + 7*a*b + 4*b^2)*\cosh(d*x + c)^8 + 8*(3*a \\
&^2 + 7*a*b + 4*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (3*a^2 + 7*a*b + 4*b^2) \\
&*\sinh(d*x + c)^8 - 4*(3*a*b + 4*b^2)*\cosh(d*x + c)^6 + 4*(7*(3*a^2 + 7*a*b \\
&+ 4*b^2)*\cosh(d*x + c)^2 - 3*a*b - 4*b^2)*\sinh(d*x + c)^6 + 8*(7*(3*a^2 + 7 \\
&a*b + 4*b^2)*\cosh(d*x + c)^3 - 3*(3*a*b + 4*b^2)*\cosh(d*x + c))*\sinh(d*x + \\
&c)^5 - 2*(3*a^2 - 5*a*b - 12*b^2)*\cosh(d*x + c)^4 + 2*(35*(3*a^2 + 7*a*b + \\
&4*b^2)*\cosh(d*x + c)^4 - 30*(3*a*b + 4*b^2)*\cosh(d*x + c)^2 - 3*a^2 + 5*a* \\
&b + 12*b^2)*\sinh(d*x + c)^4 + 8*(7*(3*a^2 + 7*a*b + 4*b^2)*\cosh(d*x + c)^5 \\
&- 10*(3*a*b + 4*b^2)*\cosh(d*x + c)^3 - (3*a^2 - 5*a*b - 12*b^2)*\cosh(d*x + \\
&c))*\sinh(d*x + c)^3 - 4*(3*a*b + 4*b^2)*\cosh(d*x + c)^2 + 4*(7*(3*a^2 + 7*a \\
&b + 4*b^2)*\cosh(d*x + c)^6 - 15*(3*a*b + 4*b^2)*\cosh(d*x + c)^4 - 3*(3*a^2 \\
&- 5*a*b - 12*b^2)*\cosh(d*x + c)^2 - 3*a*b - 4*b^2)*\sinh(d*x + c)^2 + 3*a^2 \\
&+ 7*a*b + 4*b^2 + 8*((3*a^2 + 7*a*b + 4*b^2)*\cosh(d*x + c)^7 - 3*(3*a*b + \\
&4*b^2)*\cosh(d*x + c)^5 - (3*a^2 - 5*a*b - 12*b^2)*\cosh(d*x + c)^3 - (3*a*b \\
&+ 4*b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-b/(a + b)}*\arctan(1/2*((a + b) \\
&*\cosh(d*x + c)^3 + 3*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a + b)*\sinh(d \\
&*x + c)^3 + (a - 3*b)*\cosh(d*x + c) + (3*(a + b)*\cosh(d*x + c)^2 + a - 3*b) \\
&*\sinh(d*x + c))*\sqrt{-b/(a + b)})/b - ((3*a^2 + 7*a*b + 4*b^2)*\cosh(d*x + c \\
&)^8 + 8*(3*a^2 + 7*a*b + 4*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (3*a^2 + 7* \\
&a*b + 4*b^2)*\sinh(d*x + c)^8 - 4*(3*a*b + 4*b^2)*\cosh(d*x + c)^6 + 4*(7*(3* \\
&a^2 + 7*a*b + 4*b^2)*\cosh(d*x + c)^2 - 3*a*b - 4*b^2)*\sinh(d*x + c)^6 + 8*(\\
&7*(3*a^2 + 7*a*b + 4*b^2)*\cosh(d*x + c)^3 - 3*(3*a*b + 4*b^2)*\cosh(d*x + c) \\
&)*\sinh(d*x + c)^5 - 2*(3*a^2 - 5*a*b - 12*b^2)*\cosh(d*x + c)^4 + 2*(35*(3*a \\
&^2 + 7*a*b + 4*b^2)*\cosh(d*x + c)^4 - 30*(3*a*b + 4*b^2)*\cosh(d*x + c)^2 - \\
&3*a^2 + 5*a*b + 12*b^2)*\sinh(d*x + c)^4 + 8*(7*(3*a^2 + 7*a*b + 4*b^2)*\cosh \\
&(d*x + c)^5 - 10*(3*a*b + 4*b^2)*\cosh(d*x + c)^3 - (3*a^2 - 5*a*b - 12*b^2) \\
&*\cosh(d*x + c))*\sinh(d*x + c)^3 - 4*(3*a*b + 4*b^2)*\cosh(d*x + c)^2 + 4*(7* \\
&(3*a^2 + 7*a*b + 4*b^2)*\cosh(d*x + c)^6 - 15*(3*a*b + 4*b^2)*\cosh(d*x + c)^ \\
&4 - 3*(3*a^2 - 5*a*b - 12*b^2)*\cosh(d*x + c)^2 - 3*a*b - 4*b^2)*\sinh(d*x + \\
&c)^2 + 3*a^2 + 7*a*b + 4*b^2 + 8*((3*a^2 + 7*a*b + 4*b^2)*\cosh(d*x + c)^7 - \\
&3*(3*a*b + 4*b^2)*\cosh(d*x + c)^5 - (3*a^2 - 5*a*b - 12*b^2)*\cosh(d*x + c) \\
&^3 - (3*a*b + 4*b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-b/(a + b)}*\arctan(\\
&1/2*((a + b)*\cosh(d*x + c) + (a + b)*\sinh(d*x + c))*\sqrt{-b/(a + b)})/b + 2 \\
&*(a^2 + 2*a*b)*\cosh(d*x + c) - ((a^2 + 5*a*b + 4*b^2)*\cosh(d*x + c)^8 + 8*(
\end{aligned}$$

$$\begin{aligned}
& a^2 + 5ab + 4b^2) \cosh(dx + c) \sinh(dx + c)^7 + (a^2 + 5ab + 4b^2) \sinh(dx + c)^8 - 4(ab + 4b^2) \cosh(dx + c)^6 + 4(7(a^2 + 5ab + 4b^2) \cosh(dx + c)^2 - ab - 4b^2) \sinh(dx + c)^6 + 8(7(a^2 + 5ab + 4b^2) \cosh(dx + c)^3 - 3(ab + 4b^2) \cosh(dx + c)) \sinh(dx + c)^5 - 2(a^2 + ab - 12b^2) \cosh(dx + c)^4 + 2(35(a^2 + 5ab + 4b^2) \cosh(dx + c)^4 - 30(ab + 4b^2) \cosh(dx + c)^2 - a^2 - ab + 12b^2) \sinh(dx + c)^4 + 8(7(a^2 + 5ab + 4b^2) \cosh(dx + c)^5 - 10(ab + 4b^2) \cosh(dx + c)^3 - (a^2 + ab - 12b^2) \cosh(dx + c)) \sinh(dx + c)^3 - 4(ab + 4b^2) \cosh(dx + c)^2 + 4(7(a^2 + 5ab + 4b^2) \cosh(dx + c)^6 - 15(ab + 4b^2) \cosh(dx + c)^4 - 3(a^2 + ab - 12b^2) \cosh(dx + c)^2 - ab - 4b^2) \sinh(dx + c)^2 + a^2 + 5ab + 4b^2 + 8((a^2 + 5ab + 4b^2) \cosh(dx + c)^7 - 3(ab + 4b^2) \cosh(dx + c)^5 - (a^2 + ab - 12b^2) \cosh(dx + c)^3 - (ab + 4b^2) \cosh(dx + c)) \sinh(dx + c) \log(\cosh(dx + c) + \sinh(dx + c) + 1) + ((a^2 + 5ab + 4b^2) \cosh(dx + c)^8 + 8(a^2 + 5ab + 4b^2) \cosh(dx + c) \sinh(dx + c)^7 + (a^2 + 5ab + 4b^2) \sinh(dx + c)^8 - 4(ab + 4b^2) \cosh(dx + c)^6 + 4(7(a^2 + 5ab + 4b^2) \cosh(dx + c)^2 - ab - 4b^2) \sinh(dx + c)^6 + 8(7(a^2 + 5ab + 4b^2) \cosh(dx + c)^3 - 3(ab + 4b^2) \cosh(dx + c)) \sinh(dx + c)^5 - 2(a^2 + ab - 12b^2) \cosh(dx + c)^4 + 2(35(a^2 + 5ab + 4b^2) \cosh(dx + c)^4 - 30(ab + 4b^2) \cosh(dx + c)^2 - a^2 - ab + 12b^2) \sinh(dx + c)^4 + 8(7(a^2 + 5ab + 4b^2) \cosh(dx + c)^5 - 10(ab + 4b^2) \cosh(dx + c)^3 - (a^2 + ab - 12b^2) \cosh(dx + c)) \sinh(dx + c)^3 - 4(ab + 4b^2) \cosh(dx + c)^2 + 4(7(a^2 + 5ab + 4b^2) \cosh(dx + c)^6 - 15(ab + 4b^2) \cosh(dx + c)^4 - 3(a^2 + ab - 12b^2) \cosh(dx + c)^2 - ab - 4b^2) \sinh(dx + c)^2 + a^2 + 5ab + 4b^2 + 8((a^2 + 5ab + 4b^2) \cosh(dx + c)^7 - 3(ab + 4b^2) \cosh(dx + c)^5 - (a^2 + ab - 12b^2) \cosh(dx + c)^3 - (ab + 4b^2) \cosh(dx + c)) \sinh(dx + c) \log(\cosh(dx + c) + \sinh(dx + c) - 1) + 2(7(a^2 + 2ab) \cosh(dx + c)^6 + 5(3a^2 - 2ab) \cosh(dx + c)^4 + 3(3a^2 - 2ab) \cosh(dx + c)^2 + a^2 + 2ab) \sinh(dx + c) / (4a^3b d \cosh(dx + c)^6 - (a^4 + a^3b) d \cosh(dx + c)^8 - 8(a^4 + a^3b) d \cosh(dx + c) \sinh(dx + c)^7 - (a^4 + a^3b) d \sinh(dx + c)^8 + 4a^3b d \cosh(dx + c)^2 + 4(a^3b d - 7(a^4 + a^3b) d \cosh(dx + c)^2) \sinh(dx + c)^6 + 2(a^4 - 3a^3b) d \cosh(dx + c)^4 + 8(3a^3b d \cosh(dx + c) - 7(a^4 + a^3b) d \cosh(dx + c)^3) \sinh(dx + c)^5 + 2(30a^3b d \cosh(dx + c)^2 - 35(a^4 + a^3b) d \cosh(dx + c)^4 + (a^4 - 3a^3b) d) \sinh(dx + c)^4 + 8(10a^3b d \cosh(dx + c)^3 - 7(a^4 + a^3b) d \cosh(dx + c)^5 + (a^4 - 3a^3b) d \cosh(dx + c)) \sinh(dx + c)^3 + 4(15a^3b d \cosh(dx + c)^4 - 7(a^4 + a^3b) d \cosh(dx + c)^6 + a^3b d + 3(a^4 - 3a^3b) d \cosh(dx + c)^2) \sinh(dx + c)^2 - (a^4 + a^3b) d + 8(3a^3b d \cosh(dx + c)^5 - (a^4 + a^3b) d \cosh(dx + c)^7 + a^3b d \cosh(dx + c) + (a^4 - 3a^3b) d \cosh(dx + c)^3) \sinh(dx + c))]
\end{aligned}$$

giac [B] time = 0.71, size = 1074, normalized size = 7.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)^3/(a+b*tanh(dx+c)^2)^2,x, algorithm="giac")

[Out]
$$\begin{aligned}
& -1/2 * ((2 * (12 * a^3 * b^2 + 4 * a^2 * b^3 - 16 * a * b^4 + (3 * a^3 * b - 14 * a^2 * b^2 - 21 * a * b^3 + 4 * b^4) * \sqrt{-a * b}) * a^2 * \text{abs}(a * e^{(2 * c)} + b * e^{(2 * c)}) + (3 * a^5 * b - 11 * a^4 * b^2 - 35 * a^3 * b^3 - 17 * a^2 * b^4 + 4 * a * b^5 - 4 * (3 * a^4 * b + 4 * a^3 * b^2 - 3 * a^2 * b^3 - 4 * a * b^4) * \sqrt{-a * b})) * \text{abs}(a * e^{(2 * c)} + b * e^{(2 * c)}) * \text{abs}(a) + (12 * a^6 * b - 8 * a^5 * b^2 - 20 * a^4 * b^3 + 16 * a^3 * b^4 + (3 * a^6 - 17 * a^5 * b - 7 * a^4 * b^2 + 25 * a^3 * b^3 - 4 * a^2 * b^4) * \sqrt{-a * b})) * \text{abs}(a * e^{(2 * c)} + b * e^{(2 * c)})) * \arctan(e^{(d * x)} / \sqrt{(a^4 * e^{(2 * c)} - a^3 * b * e^{(2 * c)} + \sqrt{(a^4 * e^{(2 * c)} - a^3 * b * e^{(2 * c)})^2 - (a^4 * e^{(4 * c)} + a^3 * b * e^{(4 * c)}) * (a^4 + a^3 * b)})}) / (a^4 * e^{(4 * c)} + a^3 * b * e^{(4 * c)})}) * e^{(-2 * c)} / ((a^8 + 2 * a^7 * b - 2 * a^5 * b^3 - a^4 * b^4 - 2 * (a^7 + 3 * a^6 * b + 3 * a^5 * b^2 + a^4 * b^3) * \sqrt{-a * b}) * \sqrt{a^2 - b^2 - 2 * \sqrt{-a * b} * (a + b)} * \text{abs}(a)) + (2 * (12 * a^3 * b^2 + 4 * a^2 * b^3 - 16 * a * b^4 - (3 * a^3 * b - 14 * a^2 * b^2 - 21 * a * b^3 +
\end{aligned}$$

$4*b^4*\sqrt{-a*b})*a^2*abs(a*e^{(2*c)} + b*e^{(2*c)}) + (3*a^5*b - 11*a^4*b^2 - 35*a^3*b^3 - 17*a^2*b^4 + 4*a*b^5 + 4*(3*a^4*b + 4*a^3*b^2 - 3*a^2*b^3 - 4*a*b^4)*\sqrt{-a*b})*abs(a*e^{(2*c)} + b*e^{(2*c)})*abs(a) + (12*a^6*b - 8*a^5*b^2 - 20*a^4*b^3 + 16*a^3*b^4 - (3*a^6 - 17*a^5*b - 7*a^4*b^2 + 25*a^3*b^3 - 4*a^2*b^4)*\sqrt{-a*b})*abs(a*e^{(2*c)} + b*e^{(2*c)})*\arctan(e^{(d*x)}/\sqrt{(a^4*e^{(2*c)} - a^3*b*e^{(2*c)} - \sqrt{(a^4*e^{(2*c)} - a^3*b*e^{(2*c)})^2 - (a^4*e^{(4*c)} + a^3*b*e^{(4*c)})*(a^4 + a^3*b)})))/(a^4*e^{(4*c)} + a^3*b*e^{(4*c)}))*e^{(-2*c)}/((a^8 + 2*a^7*b - 2*a^5*b^3 - a^4*b^4 + 2*(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*\sqrt{-a*b})*\sqrt{a^2 - b^2 + 2*\sqrt{-a*b}*(a + b)})*abs(a)) - (a*e^c + 4*b*e^c)*e^{-c}*log(e^{(d*x + c)} + 1)/a^3 + (a*e^c + 4*b*e^c)*e^{-c}*log(abs(e^{(d*x + c)} - 1))/a^3 + 2*(b*e^{(3*d*x + 3*c)} + b*e^{(d*x + c)})/((a*e^{(4*d*x + 4*c)} + b*e^{(4*d*x + 4*c)} + 2*a*e^{(2*d*x + 2*c)} - 2*b*e^{(2*d*x + 2*c)} + a + b)*a^2) + 2*(e^{(3*d*x + 3*c)} + e^{(d*x + c)})/(a^2*(e^{(2*d*x + 2*c)} - 1)^2))/d$

maple [B] time = 0.46, size = 367, normalized size = 2.60

$$\frac{\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)}{8d a^2} \frac{b\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d a^2 \left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a + 4\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) b + a\right)} d a^3 \left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a + 4\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) b + a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x)

[Out] 1/8/d*tanh(1/2*d*x+1/2*c)^2/a^2-1/d*b/a^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)^2-2/d*b^2/a^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)^2-1/d*b/a^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)-3/2/d*b/a^2/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))-2/d*b^2/a^3/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))-1/8/d/a^2/tanh(1/2*d*x+1/2*c)^2-1/2/d/a^2*ln(tanh(1/2*d*x+1/2*c))-2/d/a^3*ln(tanh(1/2*d*x+1/2*c))*b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(ae^{(7c)} + 2be^{(7c)})e^{(7dx)} + (3ae^{(5c)} - 2be^{(5c)})e^{(5dx)} + (3ae^{(3c)} - 2be^{(3c)})e^{(3dx)} + (ae^c + 2be^c)e^{(dx)}}{4a^2bde^{(6dx+6c)} + 4a^2bde^{(2dx+2c)} - a^3d - a^2bd - (a^3de^{(8c)} + a^2bde^{(8c)})e^{(8dx)} + 2(a^3de^{(4c)} - 3a^2bde^{(4c)})e^{(4dx)}} + \frac{(a - b)e^{(7dx)}}{4a^2bde^{(6dx+6c)} + 4a^2bde^{(2dx+2c)} - a^3d - a^2bd - (a^3de^{(8c)} + a^2bde^{(8c)})e^{(8dx)} + 2(a^3de^{(4c)} - 3a^2bde^{(4c)})e^{(4dx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] ((a*e^{(7*c)} + 2*b*e^{(7*c)})*e^{(7*d*x)} + (3*a*e^{(5*c)} - 2*b*e^{(5*c)})*e^{(5*d*x)} + (3*a*e^{(3*c)} - 2*b*e^{(3*c)})*e^{(3*d*x)} + (a*e^c + 2*b*e^c)*e^{(d*x)})/(4*a^2*b*d*e^{(6*d*x + 6*c)} + 4*a^2*b*d*e^{(2*d*x + 2*c)} - a^3*d - a^2*b*d - (a^3*d*e^{(8*c)} + a^2*b*d*e^{(8*c)})*e^{(8*d*x)} + 2*(a^3*d*e^{(4*c)} - 3*a^2*b*d*e^{(4*c)})*e^{(4*d*x)}) + 1/2*(a + 4*b)*log((e^{(d*x + c)} + 1)*e^{(-c)})/(a^3*d) - 1/2*(a + 4*b)*log((e^{(d*x + c)} - 1)*e^{(-c)})/(a^3*d) + 8*integrate(1/8*((3*a*b*e^{(3*c)} + 4*b^2*e^{(3*c)})*e^{(3*d*x)} - (3*a*b*e^c + 4*b^2*e^c)*e^{(d*x)})/(a^4 + a^3*b + (a^4*e^{(4*c)} + a^3*b*e^{(4*c)})*e^{(4*d*x)} + 2*(a^4*e^{(2*c)} - a^3*b*e^{(2*c)})*e^{(2*d*x)}), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh(c + dx)^3 (b \tanh(c + dx)^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2), x)`

[Out] `int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2, x)`

[Out] `Integral(csch(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)`

$$3.40 \quad \int \frac{\operatorname{csch}^4(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=113

$$\frac{\sqrt{b}(3a+5b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d} + \frac{b(a+b) \tanh(c+dx)}{2a^3d(a+b \tanh^2(c+dx))} + \frac{(a+2b) \operatorname{coth}(c+dx)}{a^3d} - \frac{\operatorname{coth}^3(c+dx)}{3a^2d}$$

[Out] (a+2*b)*coth(d*x+c)/a^3/d-1/3*coth(d*x+c)^3/a^2/d+1/2*(3*a+5*b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/a^(7/2)/d+1/2*b*(a+b)*tanh(d*x+c)/a^3/d/(a+b*tanh(d*x+c)^2)

Rubi [A] time = 0.15, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3663, 456, 1261, 205}

$$\frac{b(a+b) \tanh(c+dx)}{2a^3d(a+b \tanh^2(c+dx))} + \frac{(a+2b) \operatorname{coth}(c+dx)}{a^3d} + \frac{\sqrt{b}(3a+5b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d} - \frac{\operatorname{coth}^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (Sqrt[b]*(3*a + 5*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(7/2)*d) + ((a + 2*b)*Coth[c + d*x])/(a^3*d) - Coth[c + d*x]^3/(3*a^2*d) + (b*(a + b)*Tanh[c + d*x])/(2*a^3*d*(a + b*Tanh[c + d*x]^2))

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 456

Int[(x_)^(m_)*((a_) + (b_)*(x_)^2)^(p_)*((c_) + (d_)*(x_)^2), x_Symbol] :> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)*x^(-m + 2)]/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 1261

Int[((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 3663

Int[sin[(e_)+(f_)*(x_)]^(m_)*((a_) + (b_)*((c_)*tan[(e_)+(f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^4(c+dx)}{(a+b \tanh^2(c+dx))^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^4(a+bx^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{b(a+b) \tanh(c+dx)}{2a^3d(a+b \tanh^2(c+dx))} - \frac{b \operatorname{Subst}\left(\int \frac{-\frac{2}{ab} + \frac{2(a+b)x^2}{a^2b} - \frac{(a+b)x^4}{a^3}}{x^4(a+bx^2)} dx, x, \tanh(c+dx)\right)}{2d} \\
&= \frac{b(a+b) \tanh(c+dx)}{2a^3d(a+b \tanh^2(c+dx))} - \frac{b \operatorname{Subst}\left(\int \left(-\frac{2}{a^2bx^4} + \frac{2(a+2b)}{a^3bx^2} + \frac{-3a-5b}{a^3(a+bx^2)}\right) dx, x, \tanh(c+dx)\right)}{2d} \\
&= \frac{(a+2b) \operatorname{coth}(c+dx)}{a^3d} - \frac{\operatorname{coth}^3(c+dx)}{3a^2d} + \frac{b(a+b) \tanh(c+dx)}{2a^3d(a+b \tanh^2(c+dx))} + \frac{(b(3a+5b))}{6a^{7/2}d} \\
&= \frac{\sqrt{b}(3a+5b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d} + \frac{(a+2b) \operatorname{coth}(c+dx)}{a^3d} - \frac{\operatorname{coth}^3(c+dx)}{3a^2d} + \frac{(b(3a+5b))}{6a^{7/2}d}
\end{aligned}$$

Mathematica [A] time = 0.93, size = 114, normalized size = 1.01

$$\frac{3\sqrt{b}(3a+5b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right) + \frac{3\sqrt{a}b(a+b) \sinh(2(c+dx))}{(a+b) \cosh(2(c+dx))+a-b} + 2\sqrt{a} \operatorname{coth}(c+dx) (-\operatorname{acsch}^2(c+dx) + 2a + 6b)}{6a^{7/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (3*sqrt[b]*(3*a + 5*b)*ArcTan[(sqrt[b]*Tanh[c + d*x])/sqrt[a]] + 2*sqrt[a]*Coth[c + d*x]*(2*a + 6*b - a*Csch[c + d*x]^2) + (3*sqrt[a]*b*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(6*a^(7/2)*d)

fricas [B] time = 0.86, size = 5062, normalized size = 44.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/12*(12*(3*a*b + 5*b^2)*cosh(d*x + c)^8 + 96*(3*a*b + 5*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + 12*(3*a*b + 5*b^2)*sinh(d*x + c)^8 - 24*(2*a^2 + a*b + 10*b^2)*cosh(d*x + c)^6 + 24*(14*(3*a*b + 5*b^2)*cosh(d*x + c)^2 - 2*a^2 - a*b - 10*b^2)*sinh(d*x + c)^6 + 48*(14*(3*a*b + 5*b^2)*cosh(d*x + c)^3 - 3*(2*a^2 + a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*(10*a^2 - 2*a*b - 45*b^2)*cosh(d*x + c)^4 + 8*(105*(3*a*b + 5*b^2)*cosh(d*x + c)^4 - 45*(2*a^2 + a*b + 10*b^2)*cosh(d*x + c)^2 - 10*a^2 + 2*a*b + 45*b^2)*sinh(d*x + c)^4 + 32*(21*(3*a*b + 5*b^2)*cosh(d*x + c)^5 - 15*(2*a^2 + a*b + 10*b^2)*cosh(d*x + c)^3 - (10*a^2 - 2*a*b - 45*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*(2*a^2 + 13*a*b + 30*b^2)*cosh(d*x + c)^2 + 8*(42*(3*a*b + 5*b^2)*cosh(d*x + c)^6 - 45*(2*a^2 + a*b + 10*b^2)*cosh(d*x + c)^4 - 6*(10*a^2 - 2*a*b - 45*b^2)*cosh(d*x + c)^2 - 2*a^2 - 13*a*b - 30*b^2)*sinh(d*x + c)^2 + 3*((3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)^10 + 10*(3*a^2 + 8*a*b + 5*b^2)*cosh(d*x + c)*sinh(d*x + c)^9 + (3*a^2 + 8*a*b + 5*b^2)*sinh(d*x + c)^10 - (3*a^2 +

$$\begin{aligned}
& 20*a*b + 25*b^2) * \cosh(d*x + c)^8 + (45*(3*a^2 + 8*a*b + 5*b^2) * \cosh(d*x + c) \\
&)^2 - 3*a^2 - 20*a*b - 25*b^2) * \sinh(d*x + c)^8 + 8*(15*(3*a^2 + 8*a*b + 5*b \\
& ^2) * \cosh(d*x + c)^3 - (3*a^2 + 20*a*b + 25*b^2) * \cosh(d*x + c)) * \sinh(d*x + c \\
&)^7 - 2*(3*a^2 - 10*a*b - 25*b^2) * \cosh(d*x + c)^6 + 2*(105*(3*a^2 + 8*a*b + \\
& 5*b^2) * \cosh(d*x + c)^4 - 14*(3*a^2 + 20*a*b + 25*b^2) * \cosh(d*x + c)^2 - 3* \\
& a^2 + 10*a*b + 25*b^2) * \sinh(d*x + c)^6 + 4*(63*(3*a^2 + 8*a*b + 5*b^2) * \cosh \\
& (d*x + c)^5 - 14*(3*a^2 + 20*a*b + 25*b^2) * \cosh(d*x + c)^3 - 3*(3*a^2 - 10* \\
& a*b - 25*b^2) * \cosh(d*x + c)) * \sinh(d*x + c)^5 + 2*(3*a^2 - 10*a*b - 25*b^2) * \\
& \cosh(d*x + c)^4 + 2*(105*(3*a^2 + 8*a*b + 5*b^2) * \cosh(d*x + c)^6 - 35*(3*a^ \\
& 2 + 20*a*b + 25*b^2) * \cosh(d*x + c)^4 - 15*(3*a^2 - 10*a*b - 25*b^2) * \cosh(d* \\
& x + c)^2 + 3*a^2 - 10*a*b - 25*b^2) * \sinh(d*x + c)^4 + 8*(15*(3*a^2 + 8*a*b \\
& + 5*b^2) * \cosh(d*x + c)^7 - 7*(3*a^2 + 20*a*b + 25*b^2) * \cosh(d*x + c)^5 - 5* \\
& (3*a^2 - 10*a*b - 25*b^2) * \cosh(d*x + c)^3 + (3*a^2 - 10*a*b - 25*b^2) * \cosh(\\
& d*x + c)) * \sinh(d*x + c)^3 + (3*a^2 + 20*a*b + 25*b^2) * \cosh(d*x + c)^2 + (45 \\
& *(3*a^2 + 8*a*b + 5*b^2) * \cosh(d*x + c)^8 - 28*(3*a^2 + 20*a*b + 25*b^2) * \cos \\
& h(d*x + c)^6 - 30*(3*a^2 - 10*a*b - 25*b^2) * \cosh(d*x + c)^4 + 12*(3*a^2 - 1 \\
& 0*a*b - 25*b^2) * \cosh(d*x + c)^2 + 3*a^2 + 20*a*b + 25*b^2) * \sinh(d*x + c)^2 \\
& - 3*a^2 - 8*a*b - 5*b^2 + 2*(5*(3*a^2 + 8*a*b + 5*b^2) * \cosh(d*x + c)^9 - 4* \\
& (3*a^2 + 20*a*b + 25*b^2) * \cosh(d*x + c)^7 - 6*(3*a^2 - 10*a*b - 25*b^2) * \cos \\
& h(d*x + c)^5 + 4*(3*a^2 - 10*a*b - 25*b^2) * \cosh(d*x + c)^3 + (3*a^2 + 20*a* \\
& b + 25*b^2) * \cosh(d*x + c)) * \sinh(d*x + c)) * \sqrt{-b/a} * \log(((a^2 + 2*a*b + b^ \\
& 2) * \cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2) * \cosh(d*x + c) * \sinh(d*x + c)^3 + \\
& (a^2 + 2*a*b + b^2) * \sinh(d*x + c)^4 + 2*(a^2 - b^2) * \cosh(d*x + c)^2 + 2*(3* \\
& (a^2 + 2*a*b + b^2) * \cosh(d*x + c)^2 + a^2 - b^2) * \sinh(d*x + c)^2 + a^2 - 6* \\
& a*b + b^2 + 4*((a^2 + 2*a*b + b^2) * \cosh(d*x + c)^3 + (a^2 - b^2) * \cosh(d*x + \\
& c)) * \sinh(d*x + c) + 4*((a^2 + a*b) * \cosh(d*x + c)^2 + 2*(a^2 + a*b) * \cosh(d* \\
& x + c) * \sinh(d*x + c) + (a^2 + a*b) * \sinh(d*x + c)^2 + a^2 - a*b) * \sqrt{-b/a}) \\
& / ((a + b) * \cosh(d*x + c)^4 + 4*(a + b) * \cosh(d*x + c) * \sinh(d*x + c)^3 + (a + \\
& b) * \sinh(d*x + c)^4 + 2*(a - b) * \cosh(d*x + c)^2 + 2*(3*(a + b) * \cosh(d*x + c) \\
& ^2 + a - b) * \sinh(d*x + c)^2 + 4*((a + b) * \cosh(d*x + c)^3 + (a - b) * \cosh(d*x \\
& + c)) * \sinh(d*x + c) + a + b)) + 16*a^2 + 76*a*b + 60*b^2 + 16*(6*(3*a*b + \\
& 5*b^2) * \cosh(d*x + c)^7 - 9*(2*a^2 + a*b + 10*b^2) * \cosh(d*x + c)^5 - 2*(10*a \\
& ^2 - 2*a*b - 45*b^2) * \cosh(d*x + c)^3 - (2*a^2 + 13*a*b + 30*b^2) * \cosh(d*x + \\
& c)) * \sinh(d*x + c)) / ((a^4 + a^3*b) * d * \cosh(d*x + c)^10 + 10*(a^4 + a^3*b) * d * \\
& \cosh(d*x + c) * \sinh(d*x + c)^9 + (a^4 + a^3*b) * d * \sinh(d*x + c)^10 - (a^4 + 5 \\
& *a^3*b) * d * \cosh(d*x + c)^8 + (45*(a^4 + a^3*b) * d * \cosh(d*x + c)^2 - (a^4 + 5* \\
& a^3*b) * d) * \sinh(d*x + c)^8 - 2*(a^4 - 5*a^3*b) * d * \cosh(d*x + c)^6 + 8*(15*(a^ \\
& 4 + a^3*b) * d * \cosh(d*x + c)^3 - (a^4 + 5*a^3*b) * d * \cosh(d*x + c)) * \sinh(d*x + \\
& c)^7 + 2*(105*(a^4 + a^3*b) * d * \cosh(d*x + c)^4 - 14*(a^4 + 5*a^3*b) * d * \cosh(d \\
& *x + c)^2 - (a^4 - 5*a^3*b) * d) * \sinh(d*x + c)^6 + 2*(a^4 - 5*a^3*b) * d * \cosh(d \\
& *x + c)^4 + 4*(63*(a^4 + a^3*b) * d * \cosh(d*x + c)^5 - 14*(a^4 + 5*a^3*b) * d * \cos \\
& h(d*x + c)^3 - 3*(a^4 - 5*a^3*b) * d * \cosh(d*x + c)) * \sinh(d*x + c)^5 + 2*(105 \\
& *(a^4 + a^3*b) * d * \cosh(d*x + c)^6 - 35*(a^4 + 5*a^3*b) * d * \cosh(d*x + c)^4 - 1 \\
& 5*(a^4 - 5*a^3*b) * d * \cosh(d*x + c)^2 + (a^4 - 5*a^3*b) * d) * \sinh(d*x + c)^4 + \\
& (a^4 + 5*a^3*b) * d * \cosh(d*x + c)^2 + 8*(15*(a^4 + a^3*b) * d * \cosh(d*x + c)^7 - \\
& 7*(a^4 + 5*a^3*b) * d * \cosh(d*x + c)^5 - 5*(a^4 - 5*a^3*b) * d * \cosh(d*x + c)^3 \\
& + (a^4 - 5*a^3*b) * d * \cosh(d*x + c)) * \sinh(d*x + c)^3 + (45*(a^4 + a^3*b) * d * \cos \\
& h(d*x + c)^8 - 28*(a^4 + 5*a^3*b) * d * \cosh(d*x + c)^6 - 30*(a^4 - 5*a^3*b) * d \\
& * \cosh(d*x + c)^4 + 12*(a^4 - 5*a^3*b) * d * \cosh(d*x + c)^2 + (a^4 + 5*a^3*b) * d \\
&) * \sinh(d*x + c)^2 - (a^4 + a^3*b) * d + 2*(5*(a^4 + a^3*b) * d * \cosh(d*x + c)^9 \\
& - 4*(a^4 + 5*a^3*b) * d * \cosh(d*x + c)^7 - 6*(a^4 - 5*a^3*b) * d * \cosh(d*x + c)^5 \\
& + 4*(a^4 - 5*a^3*b) * d * \cosh(d*x + c)^3 + (a^4 + 5*a^3*b) * d * \cosh(d*x + c)) * \s \\
& \sinh(d*x + c)), 1/6*(6*(3*a*b + 5*b^2) * \cosh(d*x + c)^8 + 48*(3*a*b + 5*b^2) * \\
& \cosh(d*x + c) * \sinh(d*x + c)^7 + 6*(3*a*b + 5*b^2) * \sinh(d*x + c)^8 - 12*(2*a \\
& ^2 + a*b + 10*b^2) * \cosh(d*x + c)^6 + 12*(14*(3*a*b + 5*b^2) * \cosh(d*x + c)^2 \\
& - 2*a^2 - a*b - 10*b^2) * \sinh(d*x + c)^6 + 24*(14*(3*a*b + 5*b^2) * \cosh(d*x \\
& + c)^3 - 3*(2*a^2 + a*b + 10*b^2) * \cosh(d*x + c)) * \sinh(d*x + c)^5 - 4*(10*a^ \\
& 2 - 2*a*b - 45*b^2) * \cosh(d*x + c)^4 + 4*(105*(3*a*b + 5*b^2) * \cosh(d*x + c)^ \\
& 4 - 45*(2*a^2 + a*b + 10*b^2) * \cosh(d*x + c)^2 - 10*a^2 + 2*a*b + 45*b^2) * \si
\end{aligned}$$

$$\begin{aligned} & \text{nh}(d*x + c)^4 + 16*(21*(3*a*b + 5*b^2)*\cosh(d*x + c)^5 - 15*(2*a^2 + a*b + \\ & 10*b^2)*\cosh(d*x + c)^3 - (10*a^2 - 2*a*b - 45*b^2)*\cosh(d*x + c))*\sinh(d*x \\ & + c)^3 - 4*(2*a^2 + 13*a*b + 30*b^2)*\cosh(d*x + c)^2 + 4*(42*(3*a*b + 5*b^2) \\ & 2)*\cosh(d*x + c)^6 - 45*(2*a^2 + a*b + 10*b^2)*\cosh(d*x + c)^4 - 6*(10*a^2 \\ & - 2*a*b - 45*b^2)*\cosh(d*x + c)^2 - 2*a^2 - 13*a*b - 30*b^2)*\sinh(d*x + c)^ \\ & 2 + 3*((3*a^2 + 8*a*b + 5*b^2)*\cosh(d*x + c)^{10} + 10*(3*a^2 + 8*a*b + 5*b^2) \\ &)*\cosh(d*x + c)*\sinh(d*x + c)^9 + (3*a^2 + 8*a*b + 5*b^2)*\sinh(d*x + c)^{10} \\ & - (3*a^2 + 20*a*b + 25*b^2)*\cosh(d*x + c)^8 + (45*(3*a^2 + 8*a*b + 5*b^2)*\cosh(d*x + c)^2 - 3*a^2 - 20*a*b - 25*b^2)*\sinh(d*x + c)^8 + 8*(15*(3*a^2 + 8*a*b + 5*b^2)*\cosh(d*x + c)^3 - (3*a^2 + 20*a*b + 25*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^7 - 2*(3*a^2 - 10*a*b - 25*b^2)*\cosh(d*x + c)^6 + 2*(105*(3*a^2 + 8*a*b + 5*b^2)*\cosh(d*x + c)^4 - 14*(3*a^2 + 20*a*b + 25*b^2)*\cosh(d*x + c)^2 - 3*a^2 + 10*a*b + 25*b^2)*\sinh(d*x + c)^6 + 4*(63*(3*a^2 + 8*a*b + 5*b^2)*\cosh(d*x + c)^5 - 14*(3*a^2 + 20*a*b + 25*b^2)*\cosh(d*x + c)^3 - 3*(3*a^2 - 10*a*b - 25*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(3*a^2 - 10*a*b - 25*b^2)*\cosh(d*x + c)^4 + 2*(105*(3*a^2 + 8*a*b + 5*b^2)*\cosh(d*x + c)^6 - 35*(3*a^2 + 20*a*b + 25*b^2)*\cosh(d*x + c)^4 - 15*(3*a^2 - 10*a*b - 25*b^2)*\cosh(d*x + c)^2 + 3*a^2 - 10*a*b - 25*b^2)*\sinh(d*x + c)^4 + 8*(15*(3*a^2 + 8*a*b + 5*b^2)*\cosh(d*x + c)^7 - 7*(3*a^2 + 20*a*b + 25*b^2)*\cosh(d*x + c)^5 - 5*(3*a^2 - 10*a*b - 25*b^2)*\cosh(d*x + c)^3 + (3*a^2 - 10*a*b - 25*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + (3*a^2 + 20*a*b + 25*b^2)*\cosh(d*x + c)^2 + (45*(3*a^2 + 8*a*b + 5*b^2)*\cosh(d*x + c)^8 - 28*(3*a^2 + 20*a*b + 25*b^2)*\cosh(d*x + c)^6 - 30*(3*a^2 - 10*a*b - 25*b^2)*\cosh(d*x + c)^4 + 12*(3*a^2 - 10*a*b - 25*b^2)*\cosh(d*x + c)^2 + 3*a^2 + 20*a*b + 25*b^2)*\sinh(d*x + c)^2 - 3*a^2 - 8*a*b - 5*b^2 + 2*(5*(3*a^2 + 8*a*b + 5*b^2)*\cosh(d*x + c)^9 - 4*(3*a^2 + 20*a*b + 25*b^2)*\cosh(d*x + c)^7 - 6*(3*a^2 - 10*a*b - 25*b^2)*\cosh(d*x + c)^5 + 4*(3*a^2 - 10*a*b - 25*b^2)*\cosh(d*x + c)^3 + (3*a^2 + 20*a*b + 25*b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{b/a}*\arctan(1/2*((a + b)*\cosh(d*x + c)^2 + 2*(a + b)*\cosh(d*x + c)*\sinh(d*x + c) + (a + b)*\sinh(d*x + c)^2 + a - b)*\sqrt{b/a}/b) + 8*a^2 + 38*a*b + 30*b^2 + 8*(6*(3*a*b + 5*b^2)*\cosh(d*x + c)^7 - 9*(2*a^2 + a*b + 10*b^2)*\cosh(d*x + c)^5 - 2*(10*a^2 - 2*a*b - 45*b^2)*\cosh(d*x + c)^3 - (2*a^2 + 13*a*b + 30*b^2)*\cosh(d*x + c))*\sinh(d*x + c))/((a^4 + a^3*b)*d*\cosh(d*x + c)^{10} + 10*(a^4 + a^3*b)*d*\cosh(d*x + c)*\sinh(d*x + c)^9 + (a^4 + a^3*b)*d*\sinh(d*x + c)^{10} - (a^4 + 5*a^3*b)*d*\cosh(d*x + c)^8 + (45*(a^4 + a^3*b)*d*\cosh(d*x + c)^2 - (a^4 + 5*a^3*b)*d)*\sinh(d*x + c)^8 - 2*(a^4 - 5*a^3*b)*d*\cosh(d*x + c)^6 + 8*(15*(a^4 + a^3*b)*d*\cosh(d*x + c)^3 - (a^4 + 5*a^3*b)*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(105*(a^4 + a^3*b)*d*\cosh(d*x + c)^4 - 14*(a^4 + 5*a^3*b)*d*\cosh(d*x + c)^2 - (a^4 - 5*a^3*b)*d)*\sinh(d*x + c)^6 + 2*(a^4 - 5*a^3*b)*d*\cosh(d*x + c)^4 + 4*(63*(a^4 + a^3*b)*d*\cosh(d*x + c)^5 - 14*(a^4 + 5*a^3*b)*d*\cosh(d*x + c)^3 - 3*(a^4 - 5*a^3*b)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(105*(a^4 + a^3*b)*d*\cosh(d*x + c)^6 - 35*(a^4 + 5*a^3*b)*d*\cosh(d*x + c)^4 - 15*(a^4 - 5*a^3*b)*d*\cosh(d*x + c)^2 + (a^4 - 5*a^3*b)*d)*\sinh(d*x + c)^4 + (a^4 + 5*a^3*b)*d*\cosh(d*x + c)^2 + 8*(15*(a^4 + a^3*b)*d*\cosh(d*x + c)^7 - 7*(a^4 + 5*a^3*b)*d*\cosh(d*x + c)^5 - 5*(a^4 - 5*a^3*b)*d*\cosh(d*x + c)^3 + (a^4 - 5*a^3*b)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + (45*(a^4 + a^3*b)*d*\cosh(d*x + c)^8 - 28*(a^4 + 5*a^3*b)*d*\cosh(d*x + c)^6 - 30*(a^4 - 5*a^3*b)*d*\cosh(d*x + c)^4 + 12*(a^4 - 5*a^3*b)*d*\cosh(d*x + c)^2 + (a^4 + 5*a^3*b)*d)*\sinh(d*x + c)^2 - (a^4 + a^3*b)*d + 2*(5*(a^4 + a^3*b)*d*\cosh(d*x + c)^9 - 4*(a^4 + 5*a^3*b)*d*\cosh(d*x + c)^7 - 6*(a^4 - 5*a^3*b)*d*\cosh(d*x + c)^5 + 4*(a^4 - 5*a^3*b)*d*\cosh(d*x + c)^3 + (a^4 + 5*a^3*b)*d*\cosh(d*x + c))*\sinh(d*x + c)] \end{aligned}$$

giac [B] time = 0.56, size = 221, normalized size = 1.96

$$\frac{3(3abe^{2c} + 5b^2e^{2c})\arctan\left(\frac{ae^{2dx+2c} + be^{2dx+2c} + a - b}{2\sqrt{ab}}\right)e^{-2c}}{\sqrt{ab}a^3} - \frac{6(abe^{2dx+2c} - b^2e^{2dx+2c} + ab + b^2)}{(ae^{4dx+4c} + be^{4dx+4c} + 2ae^{2dx+2c} - 2be^{2dx+2c} + a + b)a^3} + \frac{8(3be^{4dx+4c})}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] $\frac{1}{6} \cdot (3 \cdot (3 \cdot a \cdot b \cdot e^{(2 \cdot c)} + 5 \cdot b^2 \cdot e^{(2 \cdot c)}) \cdot \arctan\left(\frac{1}{2} \cdot (a \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + b \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + a - b) / \sqrt{a \cdot b}\right) \cdot e^{(-2 \cdot c)} / (\sqrt{a \cdot b}) \cdot a^3 - 6 \cdot (a \cdot b \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} - b^2 \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + a \cdot b + b^2) / ((a \cdot e^{(4 \cdot d \cdot x + 4 \cdot c)} + b \cdot e^{(4 \cdot d \cdot x + 4 \cdot c)} + 2 \cdot a \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} - 2 \cdot b \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + a + b) \cdot a^3) + 8 \cdot (3 \cdot b \cdot e^{(4 \cdot d \cdot x + 4 \cdot c)} - 3 \cdot a \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} - 6 \cdot b \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + a + 3 \cdot b) / (a^3 \cdot (e^{(2 \cdot d \cdot x + 2 \cdot c)} - 1)^3)) / d$

maple [B] time = 0.47, size = 1012, normalized size = 8.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x)

[Out] $-\frac{1}{24} \cdot \frac{d}{a^2} \cdot \frac{\tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)^3 + \frac{3}{8} \cdot \frac{d}{a^2} \cdot \frac{\tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right) + \frac{1}{d} \cdot a^{-3} \cdot \tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right) \cdot b + \frac{1}{d} \cdot a^{-2} \cdot b}{\left(\tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)\right)^4 \cdot a + 2 \cdot \tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)^2 \cdot a + 4 \cdot \tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)^2 \cdot b + a} \cdot \frac{\tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)^3 + \frac{1}{d} \cdot a^{-3} \cdot b^2}{\left(\tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)\right)^4 \cdot a + 2 \cdot \tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)^2 \cdot a + 4 \cdot \tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)^2 \cdot b + a} \cdot \frac{\tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)^3 + \frac{1}{d} \cdot a^{-2} \cdot b}{\left(\tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)\right)^4 \cdot a + 2 \cdot \tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)^2 \cdot a + 4 \cdot \tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)^2 \cdot b + a} \cdot \frac{\tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right) + \frac{1}{d} \cdot a^{-3} \cdot b^2}{\left(\tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)\right)^4 \cdot a + 2 \cdot \tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)^2 \cdot a + 4 \cdot \tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)^2 \cdot b + a} \cdot \frac{\tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right) - \frac{3}{2} \cdot \frac{d}{(b \cdot (a+b))^{(1/2)}} \cdot \frac{1}{a} \cdot \left(\left(2 \cdot (b \cdot (a+b))^{(1/2)} - a - 2 \cdot b\right) \cdot a\right)^{(1/2)}}{\left(\left(2 \cdot (b \cdot (a+b))^{(1/2)} - a - 2 \cdot b\right) \cdot a\right)^{(1/2)}} \cdot \frac{\operatorname{arctanh}\left(a \cdot \tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)\right)}{\left(\left(2 \cdot (b \cdot (a+b))^{(1/2)} - a - 2 \cdot b\right) \cdot a\right)^{(1/2)}} \cdot b + \frac{3}{2} \cdot \frac{d}{a^2} \cdot \frac{b}{\left(\left(2 \cdot (b \cdot (a+b))^{(1/2)} - a - 2 \cdot b\right) \cdot a\right)^{(1/2)}} \cdot \frac{\operatorname{arctanh}\left(a \cdot \tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)\right)}{\left(\left(2 \cdot (b \cdot (a+b))^{(1/2)} - a - 2 \cdot b\right) \cdot a\right)^{(1/2)}} - \frac{4}{d} \cdot \frac{a^{-2} \cdot b^2}{(b \cdot (a+b))^{(1/2)}} \cdot \frac{1}{\left(\left(2 \cdot (b \cdot (a+b))^{(1/2)} - a - 2 \cdot b\right) \cdot a\right)^{(1/2)}} \cdot \frac{\operatorname{arctanh}\left(a \cdot \tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)\right)}{\left(\left(2 \cdot (b \cdot (a+b))^{(1/2)} - a - 2 \cdot b\right) \cdot a\right)^{(1/2)}} - \frac{3}{2} \cdot \frac{d}{(b \cdot (a+b))^{(1/2)}} \cdot \frac{1}{a} \cdot \left(\left(2 \cdot (b \cdot (a+b))^{(1/2)} + a + 2 \cdot b\right) \cdot a\right)^{(1/2)}}{\left(\left(2 \cdot (b \cdot (a+b))^{(1/2)} + a + 2 \cdot b\right) \cdot a\right)^{(1/2)}} \cdot \frac{\operatorname{arctan}\left(a \cdot \tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)\right)}{\left(\left(2 \cdot (b \cdot (a+b))^{(1/2)} + a + 2 \cdot b\right) \cdot a\right)^{(1/2)}} \cdot b - \frac{3}{2} \cdot \frac{d}{a^2} \cdot \frac{b}{\left(\left(2 \cdot (b \cdot (a+b))^{(1/2)} + a + 2 \cdot b\right) \cdot a\right)^{(1/2)}} \cdot \frac{\operatorname{arctan}\left(a \cdot \tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)\right)}{\left(\left(2 \cdot (b \cdot (a+b))^{(1/2)} + a + 2 \cdot b\right) \cdot a\right)^{(1/2)}} - \frac{4}{d} \cdot \frac{a^{-2} \cdot b^2}{(b \cdot (a+b))^{(1/2)}} \cdot \frac{1}{\left(\left(2 \cdot (b \cdot (a+b))^{(1/2)} + a + 2 \cdot b\right) \cdot a\right)^{(1/2)}} \cdot \frac{\operatorname{arctan}\left(a \cdot \tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)\right)}{\left(\left(2 \cdot (b \cdot (a+b))^{(1/2)} + a + 2 \cdot b\right) \cdot a\right)^{(1/2)}} + \frac{5}{2} \cdot \frac{d}{a^3} \cdot \frac{b^2}{\left(\left(2 \cdot (b \cdot (a+b))^{(1/2)} - a - 2 \cdot b\right) \cdot a\right)^{(1/2)}} \cdot \frac{\operatorname{arctanh}\left(a \cdot \tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)\right)}{\left(\left(2 \cdot (b \cdot (a+b))^{(1/2)} - a - 2 \cdot b\right) \cdot a\right)^{(1/2)}} - \frac{5}{2} \cdot \frac{d}{a^3} \cdot \frac{b^3}{(b \cdot (a+b))^{(1/2)}} \cdot \frac{1}{\left(\left(2 \cdot (b \cdot (a+b))^{(1/2)} - a - 2 \cdot b\right) \cdot a\right)^{(1/2)}} \cdot \frac{\operatorname{arctanh}\left(a \cdot \tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)\right)}{\left(\left(2 \cdot (b \cdot (a+b))^{(1/2)} - a - 2 \cdot b\right) \cdot a\right)^{(1/2)}} - \frac{5}{2} \cdot \frac{d}{a^3} \cdot \frac{b^2}{\left(\left(2 \cdot (b \cdot (a+b))^{(1/2)} + a + 2 \cdot b\right) \cdot a\right)^{(1/2)}} \cdot \frac{\operatorname{arctan}\left(a \cdot \tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)\right)}{\left(\left(2 \cdot (b \cdot (a+b))^{(1/2)} + a + 2 \cdot b\right) \cdot a\right)^{(1/2)}} - \frac{5}{2} \cdot \frac{d}{a^3} \cdot \frac{b^3}{(b \cdot (a+b))^{(1/2)}} \cdot \frac{1}{\left(\left(2 \cdot (b \cdot (a+b))^{(1/2)} + a + 2 \cdot b\right) \cdot a\right)^{(1/2)}} \cdot \frac{\operatorname{arctan}\left(a \cdot \tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)\right)}{\left(\left(2 \cdot (b \cdot (a+b))^{(1/2)} + a + 2 \cdot b\right) \cdot a\right)^{(1/2)}} - \frac{1}{24} \cdot \frac{d}{a^2} \cdot \frac{\tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)^3 + \frac{3}{8} \cdot \frac{d}{a^2} \cdot \frac{\tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right) + \frac{1}{d} \cdot a^{-3} \cdot \tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right) \cdot b}{\left(\tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)\right)^4 \cdot a + 2 \cdot \tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)^2 \cdot a + 4 \cdot \tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)^2 \cdot b + a} \cdot b$

maxima [B] time = 0.55, size = 282, normalized size = 2.50

$$\frac{4a^2 + 19ab + 15b^2 - 2(2a^2 + 13ab + 30b^2)e^{(-2dx-2c)} - 2(10a^2 - 2ab - 45b^2)e^{(-4dx-4c)} - 6(2a^2 + ab + 10b^2)}{3(a^4 + a^3b - (a^4 + 5a^3b)e^{(-2dx-2c)} - 2(a^4 - 5a^3b)e^{(-4dx-4c)} + 2(a^4 - 5a^3b)e^{(-6dx-6c)} + (a^4 + 5a^3b)e^{(-8dx-8c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $\frac{1}{3} \cdot (4 \cdot a^2 + 19 \cdot a \cdot b + 15 \cdot b^2 - 2 \cdot (2 \cdot a^2 + 13 \cdot a \cdot b + 30 \cdot b^2) \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} - 2 \cdot (10 \cdot a^2 - 2 \cdot a \cdot b - 45 \cdot b^2) \cdot e^{(-4 \cdot d \cdot x - 4 \cdot c)} - 6 \cdot (2 \cdot a^2 + a \cdot b + 10 \cdot b^2) \cdot e^{(-6 \cdot d \cdot x - 6 \cdot c)} + 3 \cdot (3 \cdot a \cdot b + 5 \cdot b^2) \cdot e^{(-8 \cdot d \cdot x - 8 \cdot c)}) / ((a^4 + a^3 \cdot b - (a^4 + 5 \cdot a^3 \cdot b) \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} - 2 \cdot (a^4 - 5 \cdot a^3 \cdot b) \cdot e^{(-4 \cdot d \cdot x - 4 \cdot c)} + 2 \cdot (a^4 - 5 \cdot a^3 \cdot b) \cdot e^{(-6 \cdot d \cdot x - 6 \cdot c)} + (a^4 + 5 \cdot a^3 \cdot b) \cdot e^{(-8 \cdot d \cdot x - 8 \cdot c)} - (a^4 + a^3 \cdot b) \cdot e^{(-10 \cdot d \cdot x - 10 \cdot c)}) \cdot d) - \frac{1}{2} \cdot (3 \cdot a \cdot b + 5 \cdot b^2) \cdot \arctan\left(\frac{1}{2} \cdot ((a + b) \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} + a - b) / \sqrt{a \cdot b}\right) / (\sqrt{a \cdot b}) \cdot a^3 \cdot d$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh(c + dx)^4 (b \tanh(c + dx)^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2), x)

[Out] int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2, x)

[Out] Integral(csch(c + d*x)**4/(a + b*tanh(c + d*x)**2)**2, x)

$$3.41 \quad \int \frac{\sinh^4(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=240

$$\frac{3\sqrt{b} (5a^2 - 10ab + b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a} d(a+b)^5} + \frac{3x(a^2 - 10ab + 5b^2)}{8(a+b)^5} + \frac{3b(a-b) \tanh(c+dx)}{2d(a+b)^4(a+b \tanh^2(c+dx))} + \frac{b(7a^2 - 10ab + 5b^2)}{8d(a+b)^5}$$

[Out] 3/8*(a^2-10*a*b+5*b^2)*x/(a+b)^5+3/8*(5*a^2-10*a*b+b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/(a+b)^5/d/a^(1/2)-1/8*(5*a-3*b)*cosh(d*x+c)*sinh(d*x+c)/(a+b)^2/d/(a+b*tanh(d*x+c)^2)+1/4*cosh(d*x+c)^3*sinh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c)^2)+1/8*(7*a-5*b)*b*tanh(d*x+c)/(a+b)^3/d/(a+b*tanh(d*x+c)^2)+3/2*(a-b)*b*tanh(d*x+c)/(a+b)^4/d/(a+b*tanh(d*x+c)^2)

Rubi [A] time = 0.35, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3663, 470, 527, 522, 206, 205}

$$\frac{3\sqrt{b} (5a^2 - 10ab + b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a} d(a+b)^5} + \frac{3x(a^2 - 10ab + 5b^2)}{8(a+b)^5} + \frac{3b(a-b) \tanh(c+dx)}{2d(a+b)^4(a+b \tanh^2(c+dx))} + \frac{b(7a^2 - 10ab + 5b^2)}{8d(a+b)^5}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (3*(a^2 - 10*a*b + 5*b^2)*x)/(8*(a + b)^5) + (3*sqrt[b]*(5*a^2 - 10*a*b + b^2)*ArcTan[(sqrt[b]*Tanh[c + d*x])/sqrt[a]])/(8*sqrt[a]*(a + b)^5*d) - ((5*a - 3*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*(a + b)^2*d*(a + b*Tanh[c + d*x]^2)^2) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) + ((7*a - 5*b)*b*Tanh[c + d*x])/(8*(a + b)^3*d*(a + b*Tanh[c + d*x]^2)^2) + (3*(a - b)*b*Tanh[c + d*x])/(2*(a + b)^4*d*(a + b*Tanh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]

- Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

Rule 3663

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]

Rubi steps

$$\begin{aligned} \int \frac{\sinh^4(c + dx)}{(a + b \tanh^2(c + dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^3(a+bx^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\cosh^3(c + dx) \sinh(c + dx)}{4(a + b)d(a + b \tanh^2(c + dx))^2} - \frac{\text{Subst}\left(\int \frac{a+(4a-3b)x^2}{(1-x^2)^2(a+bx^2)^3} dx, x, \tanh(c + dx)\right)}{4(a + b)d} \\ &= -\frac{(5a - 3b) \cosh(c + dx) \sinh(c + dx)}{8(a + b)^2 d (a + b \tanh^2(c + dx))^2} + \frac{\cosh^3(c + dx) \sinh(c + dx)}{4(a + b)d(a + b \tanh^2(c + dx))^2} - \frac{\text{Subst}\left(\int \frac{a+(4a-3b)x^2}{(1-x^2)^2(a+bx^2)^3} dx, x, \tanh(c + dx)\right)}{4(a + b)d} \\ &= -\frac{(5a - 3b) \cosh(c + dx) \sinh(c + dx)}{8(a + b)^2 d (a + b \tanh^2(c + dx))^2} + \frac{\cosh^3(c + dx) \sinh(c + dx)}{4(a + b)d(a + b \tanh^2(c + dx))^2} + \frac{\text{Subst}\left(\int \frac{a+(4a-3b)x^2}{(1-x^2)^2(a+bx^2)^3} dx, x, \tanh(c + dx)\right)}{4(a + b)d} \\ &= -\frac{(5a - 3b) \cosh(c + dx) \sinh(c + dx)}{8(a + b)^2 d (a + b \tanh^2(c + dx))^2} + \frac{\cosh^3(c + dx) \sinh(c + dx)}{4(a + b)d(a + b \tanh^2(c + dx))^2} + \frac{\text{Subst}\left(\int \frac{a+(4a-3b)x^2}{(1-x^2)^2(a+bx^2)^3} dx, x, \tanh(c + dx)\right)}{4(a + b)d} \\ &= -\frac{(5a - 3b) \cosh(c + dx) \sinh(c + dx)}{8(a + b)^2 d (a + b \tanh^2(c + dx))^2} + \frac{\cosh^3(c + dx) \sinh(c + dx)}{4(a + b)d(a + b \tanh^2(c + dx))^2} + \frac{\text{Subst}\left(\int \frac{a+(4a-3b)x^2}{(1-x^2)^2(a+bx^2)^3} dx, x, \tanh(c + dx)\right)}{4(a + b)d} \\ &= \frac{3(a^2 - 10ab + 5b^2)x}{8(a + b)^5} + \frac{3\sqrt{b}(5a^2 - 10ab + b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c + dx)}{\sqrt{a}}\right)}{8\sqrt{a}(a + b)^5 d} - \frac{(5a - 3b) \cosh^3(c + dx) \sinh(c + dx)}{8(a + b)d(a + b \tanh^2(c + dx))^2} \end{aligned}$$

Mathematica [A] time = 0.86, size = 184, normalized size = 0.77

$$\frac{12(a^2 - 10ab + 5b^2)(c + dx) + \frac{12\sqrt{b}(5a^2 - 10ab + b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c + dx)}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{16ab^2(a+b) \sinh(2(c + dx))}{((a+b) \cosh(2(c + dx)) + a - b)^2} - 8(a - 2b)(a + b) \sinh(c + dx)}{32d(a + b)^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (12*(a^2 - 10*a*b + 5*b^2)*(c + d*x) + (12*Sqrt[b]*(5*a^2 - 10*a*b + b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a] - 8*(a - 2*b)*(a + b)*Sinh[2*(c + d*x)] + (16*a*b^2*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)])^2 + (4*(9*a - 5*b)*b*(a + b)*Sinh[2*(c + d*x)]/(a - b + (a + b)*Cosh[2*(c + d*x)]) + (a + b)^2*Sinh[4*(c + d*x)]/(32*(a + b)^5*d)
```

```
fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

```
giac [B] time = 3.00, size = 923, normalized size = 3.85
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/64*(24*(a^2 - 10*a*b + 5*b^2)*d*x/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) + 24*(5*a^2*b*e^(2*c) - 10*a*b^2*e^(2*c) + b^3*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))*e^(-2*c)/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sqrt(a*b)) + (a^3*e^(4*d*x + 36*c) + 3*a^2*b*e^(4*d*x + 36*c) + 3*a*b^2*e^(4*d*x + 36*c) + b^3*e^(4*d*x + 36*c) - 8*a^3*e^(2*d*x + 34*c) + 24*a*b^2*e^(2*d*x + 34*c) + 16*b^3*e^(2*d*x + 34*c))/(a^6*e^(32*c) + 6*a^5*b*e^(32*c) + 15*a^4*b^2*e^(32*c) + 20*a^3*b^3*e^(32*c) + 15*a^2*b^4*e^(32*c) + 6*a*b^5*e^(32*c) + b^6*e^(32*c)) - (6*a^4*e^(12*d*x + 12*c) - 48*a^3*b*e^(12*d*x + 12*c) - 84*a^2*b^2*e^(12*d*x + 12*c) + 30*b^4*e^(12*d*x + 12*c) + 16*a^4*e^(10*d*x + 10*c) - 104*a^3*b*e^(10*d*x + 10*c) - 24*a^2*b^2*e^(10*d*x + 10*c) + 72*a*b^3*e^(10*d*x + 10*c) - 24*b^4*e^(10*d*x + 10*c) + 5*a^4*e^(8*d*x + 8*c) + 84*a^3*b*e^(8*d*x + 8*c) + 30*a^2*b^2*e^(8*d*x + 8*c) + 84*a*b^3*e^(8*d*x + 8*c) - 123*b^4*e^(8*d*x + 8*c) - 20*a^4*e^(6*d*x + 6*c) + 280*a^3*b*e^(6*d*x + 6*c) - 64*a^2*b^2*e^(6*d*x + 6*c) - 152*a*b^3*e^(6*d*x + 6*c) + 212*b^4*e^(6*d*x + 6*c) - 20*a^4*e^(4*d*x + 4*c) + 136*a^3*b*e^(4*d*x + 4*c) + 224*a^2*b^2*e^(4*d*x + 4*c) - 40*a*b^3*e^(4*d*x + 4*c) - 108*b^4*e^(4*d*x + 4*c) - 4*a^4*e^(2*d*x + 2*c) + 24*a^2*b^2*e^(2*d*x + 2*c) + 32*a*b^3*e^(2*d*x + 2*c) + 12*b^4*e^(2*d*x + 2*c) + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)/(a^5*e^(4*c) + 5*a^4*b*e^(4*c) + 10*a^3*b^2*e^(4*c) + 10*a^2*b^3*e^(4*c) + 5*a*b^4*e^(4*c) + b^5*e^(4*c))*(a*e^(2*d*x) + b*e^(2*d*x) + a*e^(6*d*x + 4*c) + b*e^(6*d*x + 4*c) + 2*a*e^(4*d*x + 2*c) - 2*b*e^(4*d*x + 2*c))^2)/d
```

```
maple [B] time = 0.39, size = 2366, normalized size = 9.86
```

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] -1/4/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)^4+1/4/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^4+1/2/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)^3-15/8/d*b/(a+b)^5/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c))/((2*(
```

$$\begin{aligned}
& b*(a+b)^{(1/2)-a-2*b}*a^{(1/2)}*a^3-15/8/d*b/(a+b)^5/(b*(a+b))^{(1/2)/((2*(b*(a+b))^{(1/2)+a+2*b}*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)+a+2*b}*a)^{(1/2)}*a^3+15/8/d*b^2/(a+b)^5/(b*(a+b))^{(1/2)/((2*(b*(a+b))^{(1/2)-a-2*b}*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)-a-2*b}*a)^{(1/2)}*a^2+27/8/d*b^3/(a+b)^5*a/(b*(a+b))^{(1/2)/((2*(b*(a+b))^{(1/2)-a-2*b}*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)-a-2*b}*a)^{(1/2)}+15/8/d*b^2/(a+b)^5/(b*(a+b))^{(1/2)/((2*(b*(a+b))^{(1/2)+a+2*b}*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)+a+2*b}*a)^{(1/2)}*a^2+27/8/d*b^3/(a+b)^5*a/(b*(a+b))^{(1/2)/((2*(b*(a+b))^{(1/2)+a+2*b}*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)+a+2*b}*a)^{(1/2)}+1/2/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^3-1/8/d/(a+b)^4/(tanh(1/2*d*x+1/2*c)-1)^2*a+11/8/d/(a+b)^4/(tanh(1/2*d*x+1/2*c)-1)^2*b-3/8/d/(a+b)^4/(tanh(1/2*d*x+1/2*c)-1)*a+9/8/d/(a+b)^4/(tanh(1/2*d*x+1/2*c)-1)*b-3/8/d/(a+b)^5*\ln(tanh(1/2*d*x+1/2*c)-1)*a^2-15/8/d/(a+b)^5*\ln(tanh(1/2*d*x+1/2*c)-1)*b^2+1/8/d/(a+b)^4/(tanh(1/2*d*x+1/2*c)+1)^2*a-11/8/d/(a+b)^4/(tanh(1/2*d*x+1/2*c)+1)^2*b-3/8/d/(a+b)^4/(tanh(1/2*d*x+1/2*c)+1)*a+9/8/d/(a+b)^4/(tanh(1/2*d*x+1/2*c)+1)*b+3/8/d/(a+b)^5*\ln(tanh(1/2*d*x+1/2*c)+1)*a^2+15/8/d/(a+b)^5*\ln(tanh(1/2*d*x+1/2*c)+1)*b^2-5/d*b^4/(a+b)^5/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5-5/d*b^4/(a+b)^5/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3+3/8/d*b^3/(a+b)^5/((2*(b*(a+b))^{(1/2)-a-2*b}*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)-a-2*b}*a)^{(1/2)}))-3/8/d*b^3/(a+b)^5/((2*(b*(a+b))^{(1/2)+a+2*b}*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)+a+2*b}*a)^{(1/2)}))+15/4/d/(a+b)^5*\ln(tanh(1/2*d*x+1/2*c)-1)*a*b-15/4/d/(a+b)^5*\ln(tanh(1/2*d*x+1/2*c)+1)*a*b-15/8/d*b/(a+b)^5*a^2/((2*(b*(a+b))^{(1/2)+a+2*b}*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)+a+2*b}*a)^{(1/2)}))+9/4/d*b/(a+b)^5/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7*a^3+9/4/d*b/(a+b)^5/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)*a^3+27/4/d*b/(a+b)^5/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3*a^3-15/4/d*b^2/(a+b)^5*a/((2*(b*(a+b))^{(1/2)-a-2*b}*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)-a-2*b}*a)^{(1/2)}))+3/2/d*b^2/(a+b)^5/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7*a^2-3/4/d*b^3/(a+b)^5/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7*a+23/2/d*b^2/(a+b)^5/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3*a^2-1/4/d*b^3/(a+b)^5/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)*a^2-3/4/d*b^3/(a+b)^5/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)*a+15/8/d*b/(a+b)^5*a^2/((2*(b*(a+b))^{(1/2)-a-2*b}*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)-a-2*b}*a)^{(1/2)}))-3/8/d*b^4/(a+b)^5/(b*(a+b))^{(1/2)/((2*(b*(a+b))^{(1/2)-a-2*b}*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)-a-2*b}*a)^{(1/2)}))-3/8/d*b^4/(a+b)^5/(b*(a+b))^{(1/2)/((2*(b*(a+b))^{(1/2)+a+2*b}*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)+a+2*b}*a)^{(1/2)}))+15/4/d*b^2/(a+b)^5*a/((2*(b*(a+b))^{(1/2)+a+2*b}*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)+a+2*b}*a)^{(1/2)}))
\end{aligned}$$

maxima [B] time = 1.12, size = 3392, normalized size = 14.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.


```

7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*e^(-4*
d*x - 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6
)*e^(-6*d*x - 6*c) + (a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4
+ 6*a^3*b^5 + a^2*b^6)*e^(-8*d*x - 8*c))*d) + 3/32*(9*a^3*b + 21*a^2*b^2 +
15*a*b^3 + 3*b^4 + (27*a^3*b + 13*a^2*b^2 - 23*a*b^3 - 9*b^4)*e^(-2*d*x - 2
*c) + 3*(9*a^3*b - 3*a^2*b^2 + 7*a*b^3 + 3*b^4)*e^(-4*d*x - 4*c) + (9*a^3*b
- a^2*b^2 - 13*a*b^3 - 3*b^4)*e^(-6*d*x - 6*c))/((a^7 + 5*a^6*b + 10*a^5*b
^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^
4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 7*a^6*b + 6*a^5*
b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*e^(-4*d*x - 4*c) + 4*(a^7 + 3*a^6*
b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(-6*d*x - 6*c) + (a^7 +
5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*e^(-8*d*x - 8*c))*
d) + 3/8*(d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 1/64*((a + b)*e^(4
*d*x + 4*c) + 24*b*e^(2*d*x + 2*c))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
b^4)*d) - 1/64*(24*b*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c))/((a^4 +
4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 1/8*e^(2*d*x + 2*c)/((a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*d) + 1/8*e^(-2*d*x - 2*c)/((a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*d)

```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sinh(c + dx)^4}{(b \tanh(c + dx)^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

$$3.42 \quad \int \frac{\sinh^3(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=166

$$\frac{\cosh^3(c+dx)}{3d(a+b)^3} - \frac{(a-2b)\cosh(c+dx)}{d(a+b)^4} + \frac{b(7a-4b)\operatorname{sech}(c+dx)}{8d(a+b)^4(a-b\operatorname{sech}^2(c+dx)+b)} + \frac{ab\operatorname{sech}(c+dx)}{4d(a+b)^3(a-b\operatorname{sech}^2(c+dx)+b)}$$

[Out] $-(a-2*b)*\cosh(d*x+c)/(a+b)^4/d+1/3*\cosh(d*x+c)^3/(a+b)^3/d+1/4*a*b*\operatorname{sech}(d*x+c)/(a+b)^3/d/(a+b-b*\operatorname{sech}(d*x+c)^2)^2+1/8*(7*a-4*b)*b*\operatorname{sech}(d*x+c)/(a+b)^4/d/(a+b-b*\operatorname{sech}(d*x+c)^2)+5/8*(3*a-4*b)*\operatorname{arctanh}(\operatorname{sech}(d*x+c)*b^{1/2}/(a+b)^{1/2})*b^{1/2}/(a+b)^{9/2}/d$

Rubi [A] time = 0.29, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3664, 456, 1259, 1261, 208}

$$\frac{\cosh^3(c+dx)}{3d(a+b)^3} - \frac{(a-2b)\cosh(c+dx)}{d(a+b)^4} + \frac{b(7a-4b)\operatorname{sech}(c+dx)}{8d(a+b)^4(a-b\operatorname{sech}^2(c+dx)+b)} + \frac{ab\operatorname{sech}(c+dx)}{4d(a+b)^3(a-b\operatorname{sech}^2(c+dx)+b)}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] $(5*(3*a - 4*b)*\operatorname{Sqrt}[b]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Sech}[c + d*x])/\operatorname{Sqrt}[a + b]])/(8*(a + b)^{9/2}*d) - ((a - 2*b)*\operatorname{Cosh}[c + d*x])/((a + b)^4*d) + \operatorname{Cosh}[c + d*x]^3/(3*(a + b)^3*d) + (a*b*\operatorname{Sech}[c + d*x])/(4*(a + b)^3*d*(a + b - b*\operatorname{Sech}[c + d*x]^2)^2) + ((7*a - 4*b)*b*\operatorname{Sech}[c + d*x])/(8*(a + b)^4*d*(a + b - b*\operatorname{Sech}[c + d*x]^2))$

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 456

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)*x^(-m + 2)]/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 1259

Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4))^p - ((c*d^2 - b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)]]/(d + e*x^2)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m/2, 0]

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 3664

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^m), Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p]/x^(m + 1), x], x, Sec[e + f*x]/ff, x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned} \int \frac{\sinh^3(c + dx)}{(a + b \tanh^2(c + dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{-1+x^2}{x^4(a+b-bx^2)^3} dx, x, \text{sech}(c + dx)\right)}{d} \\ &= \frac{ab \text{sech}(c + dx)}{4(a + b)^3 d (a + b - b \text{sech}^2(c + dx))^2} + \frac{b \text{Subst}\left(\int \frac{\frac{4}{b(a+b)} + \frac{4ax^2}{b(a+b)^2} + \frac{3ax^4}{(a+b)^3}}{x^4(a+b-bx^2)^2} dx, x, \text{sech}(c + dx)\right)}{4d} \\ &= \frac{ab \text{sech}(c + dx)}{4(a + b)^3 d (a + b - b \text{sech}^2(c + dx))^2} + \frac{(7a - 4b)b \text{sech}(c + dx)}{8(a + b)^4 d (a + b - b \text{sech}^2(c + dx))} + \\ &= \frac{ab \text{sech}(c + dx)}{4(a + b)^3 d (a + b - b \text{sech}^2(c + dx))^2} + \frac{(7a - 4b)b \text{sech}(c + dx)}{8(a + b)^4 d (a + b - b \text{sech}^2(c + dx))} + \\ &= -\frac{(a - 2b) \cosh(c + dx)}{(a + b)^4 d} + \frac{\cosh^3(c + dx)}{3(a + b)^3 d} + \frac{ab \text{sech}(c + dx)}{4(a + b)^3 d (a + b - b \text{sech}^2(c + dx))} \\ &= \frac{5(3a - 4b)\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \text{sech}(c + dx)}{\sqrt{a + b}}\right)}{8(a + b)^{9/2} d} - \frac{(a - 2b) \cosh(c + dx)}{(a + b)^4 d} + \frac{\cosh^3(c + dx)}{3(a + b)^3 d} \end{aligned}$$

Mathematica [C] time = 2.00, size = 227, normalized size = 1.37

$$\frac{-\frac{6 \cosh(c + dx)(3a^3 - 24a^2b + (6a^3 - 27a^2b - 11ab^2 + 22b^3) \cosh(2(c + dx)) + 30ab^2 + 3(a - 3b)(a + b)^2 \cosh^2(2(c + dx)) - 13b^3)}{(a + b)^4((a + b) \cosh(2(c + dx)) + a - b)^2} + \frac{2 \cosh(3(c + dx))}{(a + b)^3} + \frac{15i\sqrt{b} \text{ArcTan}\left[\frac{-i\sqrt{a + b} - \sqrt{a} \tanh\left(\frac{c + dx}{2}\right)}{\sqrt{b}}\right] + \text{ArcTan}\left[\frac{-i\sqrt{a + b} + \sqrt{a} \tanh\left(\frac{c + dx}{2}\right)}{\sqrt{b}}\right]}{24d}}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3, x]
```

```
[Out] (((15*I)*(3*a - 4*b)*Sqrt[b]*(ArcTan[(-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2]]/Sqrt[b]] + ArcTan[(-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2]]/Sqrt[b]))/(a + b)^(9/2) - (6*Cosh[c + d*x]*(3*a^3 - 24*a^2*b + 30*a*b^2 - 13*b^3) + (6*a^3 - 27*a^2*b - 11*a*b^2 + 22*b^3)*Cosh[2*(c + d*x)] + 3*(a - 3*b)*(a + b)^2*Cosh[2*(c + d*x)]^2)/((a + b)^4*(a - b + (a + b)*Cosh[2*(c + d*x)]^2) + (2*Cosh[3*(c + d*x)])/(a + b)^3)/(24*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] Timed out

giac [B] time = 1.64, size = 1748, normalized size = 10.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out]
$$-1/24*(15*(15*a^3*b - 50*a^2*b^2 + 43*a*b^3 - 4*b^4 - (3*a^3 - 34*a^2*b + 55*a*b^2 - 20*b^3)*\sqrt{-a*b})*\sqrt{a^2 - b^2 + 2*\sqrt{-a*b}*(a + b)}*abs(a*e^{(2*c)} + b*e^{(2*c)})*\arctan(e^{(d*x)}/\sqrt{(a^5*e^{(2*c)} + 3*a^4*b*e^{(2*c)} + 2*a^3*b^2*e^{(2*c)} - 2*a^2*b^3*e^{(2*c)} - 3*a*b^4*e^{(2*c)} - b^5*e^{(2*c)} + \sqrt{(a^5*e^{(2*c)} + 3*a^4*b*e^{(2*c)} + 2*a^3*b^2*e^{(2*c)} - 2*a^2*b^3*e^{(2*c)} - 3*a*b^4*e^{(2*c)} - b^5*e^{(2*c)})^2 - (a^5*e^{(4*c)} + 5*a^4*b*e^{(4*c)} + 10*a^3*b^2*e^{(4*c)} + 10*a^2*b^3*e^{(4*c)} + 5*a*b^4*e^{(4*c)} + b^5*e^{(4*c)})*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)))/\sqrt{(a^5*e^{(4*c)} + 5*a^4*b*e^{(4*c)} + 10*a^3*b^2*e^{(4*c)} + 10*a^2*b^3*e^{(4*c)} + 5*a*b^4*e^{(4*c)} + b^5*e^{(4*c)})))*e^{(-2*c)}/(a^9 - 9*a^8*b - 60*a^7*b^2 - 116*a^6*b^3 - 66*a^5*b^4 + 66*a^4*b^5 + 116*a^3*b^6 + 60*a^2*b^7 + 9*a*b^8 - b^9 + 2*(3*a^8 + 8*a^7*b - 12*a^6*b^2 - 72*a^5*b^3 - 110*a^4*b^4 - 72*a^3*b^5 - 12*a^2*b^6 + 8*a*b^7 + 3*b^8)*\sqrt{-a*b}) + 15*(15*a^3*b - 50*a^2*b^2 + 43*a*b^3 - 4*b^4 + (3*a^3 - 34*a^2*b + 55*a*b^2 - 20*b^3)*\sqrt{-a*b})*\sqrt{a^2 - b^2 - 2*\sqrt{-a*b}*(a + b)}*abs(a*e^{(2*c)} + b*e^{(2*c)})*\arctan(e^{(d*x)}/\sqrt{(a^5*e^{(2*c)} + 3*a^4*b*e^{(2*c)} + 2*a^3*b^2*e^{(2*c)} - 2*a^2*b^3*e^{(2*c)} - 3*a*b^4*e^{(2*c)} - b^5*e^{(2*c)} - \sqrt{(a^5*e^{(2*c)} + 3*a^4*b*e^{(2*c)} + 2*a^3*b^2*e^{(2*c)} - 2*a^2*b^3*e^{(2*c)} - 3*a*b^4*e^{(2*c)} - b^5*e^{(2*c)})^2 - (a^5*e^{(4*c)} + 5*a^4*b*e^{(4*c)} + 10*a^3*b^2*e^{(4*c)} + 10*a^2*b^3*e^{(4*c)} + 5*a*b^4*e^{(4*c)} + b^5*e^{(4*c)})*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)))/\sqrt{(a^5*e^{(4*c)} + 5*a^4*b*e^{(4*c)} + 10*a^3*b^2*e^{(4*c)} + 10*a^2*b^3*e^{(4*c)} + 5*a*b^4*e^{(4*c)} + b^5*e^{(4*c)})))*e^{(-2*c)}/(a^9 - 9*a^8*b - 60*a^7*b^2 - 116*a^6*b^3 - 66*a^5*b^4 + 66*a^4*b^5 + 116*a^3*b^6 + 60*a^2*b^7 + 9*a*b^8 - b^9 - 2*(3*a^8 + 8*a^7*b - 12*a^6*b^2 - 72*a^5*b^3 - 110*a^4*b^4 - 72*a^3*b^5 - 12*a^2*b^6 + 8*a*b^7 + 3*b^8)*\sqrt{-a*b}) + (9*a*e^{(2*d*x + 2*c)} - 27*b*e^{(2*d*x + 2*c)} - a - b)*e^{(-3*d*x)}/(a^4*e^{(3*c)} + 4*a^3*b*e^{(3*c)} + 6*a^2*b^2*e^{(3*c)} + 4*a*b^3*e^{(3*c)} + b^4*e^{(3*c)}) - (a^6*e^{(3*d*x + 48*c)} + 6*a^5*b*e^{(3*d*x + 48*c)} + 15*a^4*b^2*e^{(3*d*x + 48*c)} + 20*a^3*b^3*e^{(3*d*x + 48*c)} + 15*a^2*b^4*e^{(3*d*x + 48*c)} + 6*a*b^5*e^{(3*d*x + 48*c)} + b^6*e^{(3*d*x + 48*c)} - 9*a^6*e^{(d*x + 46*c)} - 18*a^5*b*e^{(d*x + 46*c)} + 45*a^4*b^2*e^{(d*x + 46*c)} + 180*a^3*b^3*e^{(d*x + 46*c)} + 225*a^2*b^4*e^{(d*x + 46*c)} + 126*a*b^5*e^{(d*x + 46*c)} + 27*b^6*e^{(d*x + 46*c)})/(a^9*e^{(45*c)} + 9*a^8*b*e^{(45*c)} + 36*a^7*b^2*e^{(45*c)} + 84*a^6*b^3*e^{(45*c)} + 126*a^5*b^4*e^{(45*c)} + 126*a^4*b^5*e^{(45*c)} + 84*a^3*b^6*e^{(45*c)} + 36*a^2*b^7*e^{(45*c)} + 9*a*b^8*e^{(45*c)} + b^9*e^{(45*c)}) - 6*(9*a^2*b*e^{(7*d*x + 7*c)} + 5*a*b^2*e^{(7*d*x + 7*c)} - 4*b^3*e^{(7*d*x + 7*c)} + 27*a^2*b*e^{(5*d*x + 5*c)} - 13*a*b^2*e^{(5*d*x + 5*c)} + 4*b^3*e^{(5*d*x + 5*c)} + 27*a^2*b*e^{(3*d*x + 3*c)} - 13*a*b^2*e^{(3*d*x + 3*c)} + 4*b^3*e^{(3*d*x + 3*c)} + 9*a^2*b*e^{(d*x + c)} + 5*a*b^2*e^{(d*x + c)} - 4*b^3*e^{(d*x + c)})/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(a*e^{(4*d*x + 4*c)} + b*e^{(4*d*x + 4*c)} + 2*a*e^{(2*d*x + 2*c)} - 2*b*e^{(2*d*x + 2*c)} + a + b)^2))/d$$

maple [B] time = 0.34, size = 341, normalized size = 2.05

$$-\frac{1}{3(a+b)^3 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^3} - \frac{1}{2(a+b)^3 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^2} - \frac{-a+5b}{2(a+b)^4 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)} + \frac{1}{3(a+b)^3 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^3} - \frac{1}{2(a+b)^3 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x)

[Out] 1/d*(-1/3/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/(a+b)^4*(-a+5*b)/(tanh(1/2*d*x+1/2*c)-1)+1/3/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)^3-1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)^2-1/2*(a-5*b)/(a+b)^4/(tanh(1/2*d*x+1/2*c)+1)-2*b/(a+b)^4*((-1/8*(9*a+20*b)*a*tanh(1/2*d*x+1/2*c)^6-1/8*(27*a^3+66*a^2*b+56*a*b^2-16*b^3)/a*tanh(1/2*d*x+1/2*c)^4+(-27/8*a^2-1/2*a*b+2*b^2)*tanh(1/2*d*x+1/2*c)^2-9/8*a^2+1/4*a*b)/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2-5/16*(3*a-4*b)/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is undefined.

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(c + dx)^3}{(b \tanh(c + dx)^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^3,x)

[Out] int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)

[Out] Timed out

$$3.43 \quad \int \frac{\sinh^2(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=185

$$\frac{\sqrt{b} (15a^2 - 10ab - b^2) \tan^{-1} \left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}} \right)}{8a^{3/2}d(a+b)^4} - \frac{b(11a-b) \tanh(c+dx)}{8ad(a+b)^3 (a+b \tanh^2(c+dx))} - \frac{3b \tanh(c+dx)}{4d(a+b)^2 (a+b \tanh^2(c+dx))}$$

[Out] $-1/2*(a-5*b)*x/(a+b)^4-1/8*(15*a^2-10*a*b-b^2)*\arctan(b^{(1/2)}*\tanh(d*x+c)/a^{(1/2)})*b^{(1/2)}/a^{(3/2)}/(a+b)^4/d+1/2*\cosh(d*x+c)*\sinh(d*x+c)/(a+b)/d/(a+b*\tanh(d*x+c)^2)^2-3/4*b*\tanh(d*x+c)/(a+b)^2/d/(a+b*\tanh(d*x+c)^2)^2-1/8*(11*a-b)*b*\tanh(d*x+c)/a/(a+b)^3/d/(a+b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.25, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3663, 471, 527, 522, 206, 205}

$$\frac{\sqrt{b} (15a^2 - 10ab - b^2) \tan^{-1} \left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}} \right)}{8a^{3/2}d(a+b)^4} - \frac{b(11a-b) \tanh(c+dx)}{8ad(a+b)^3 (a+b \tanh^2(c+dx))} - \frac{3b \tanh(c+dx)}{4d(a+b)^2 (a+b \tanh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] $-((a-5*b)*x)/(2*(a+b)^4) - (\text{Sqrt}[b]*(15*a^2-10*a*b-b^2)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Tanh}[c+d*x])/\text{Sqrt}[a]])/(8*a^{(3/2)}*(a+b)^4*d) + (\text{Cosh}[c+d*x]*\text{Sinh}[c+d*x])/(2*(a+b)*d*(a+b*\text{Tanh}[c+d*x]^2)^2) - (3*b*\text{Tanh}[c+d*x])/(4*(a+b)^2*d*(a+b*\text{Tanh}[c+d*x]^2)^2) - ((11*a-b)*b*\text{Tanh}[c+d*x])/(8*a*(a+b)^3*d*(a+b*\text{Tanh}[c+d*x]^2))$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 471

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n-1)*(e*x)^(m-n+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(n*(b*c-a*d)*(p+1)), x] - Dist[e^n/(n*(b*c-a*d)*(p+1)), Int[(e*x)^(m-n)*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[c*(m-n+1)+d*(m+n*(p+q+1)+1]*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m-n+1] && GtQ[m-n+1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e-a*f)/(b*c-a*d), Int[1/(a+b*x^n), x], x] - Dist[(d*e-c*f)/(b*c-a*d), Int[1/(c+d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned} \int \frac{\sinh^2(c + dx)}{(a + b \tanh^2(c + dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+bx^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\cosh(c + dx) \sinh(c + dx)}{2(a + b)d (a + b \tanh^2(c + dx))^2} - \frac{\text{Subst}\left(\int \frac{a-5bx^2}{(1-x^2)(a+bx^2)^3} dx, x, \tanh(c + dx)\right)}{2(a + b)d} \\ &= \frac{\cosh(c + dx) \sinh(c + dx)}{2(a + b)d (a + b \tanh^2(c + dx))^2} - \frac{3b \tanh(c + dx)}{4(a + b)^2d (a + b \tanh^2(c + dx))^2} + \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^2)^3} dx, x, \tanh(c + dx)\right)}{8a(a + b)d} \\ &= \frac{\cosh(c + dx) \sinh(c + dx)}{2(a + b)d (a + b \tanh^2(c + dx))^2} - \frac{3b \tanh(c + dx)}{4(a + b)^2d (a + b \tanh^2(c + dx))^2} - \frac{1}{8a(a + b)d} \\ &= \frac{\cosh(c + dx) \sinh(c + dx)}{2(a + b)d (a + b \tanh^2(c + dx))^2} - \frac{3b \tanh(c + dx)}{4(a + b)^2d (a + b \tanh^2(c + dx))^2} - \frac{1}{8a(a + b)d} \\ &= -\frac{(a - 5b)x}{2(a + b)^4} - \frac{\sqrt{b} (15a^2 - 10ab - b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}(a + b)^4d} + \frac{\cosh(c + dx) \sinh(c + dx)}{2(a + b)d (a + b \tanh^2(c + dx))^2} \end{aligned}$$

Mathematica [A] time = 1.34, size = 158, normalized size = 0.85

$$\frac{\sqrt{b}(-15a^2 + 10ab + b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} - \frac{4b^2(a+b) \sinh(2(c+dx))}{((a+b) \cosh(2(c+dx)) + a - b)^2} - \frac{4(a - 5b)(c + dx) + 2(a + b) \sinh(2(c + dx))}{8d(a + b)^4} - \frac{b^9}{a((a + b) \cosh(2(c + dx)) + a - b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3, x]

[Out] (-4*(a - 5*b)*(c + d*x) + (Sqrt[b]*(-15*a^2 + 10*a*b + b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2) + 2*(a + b)*Sinh[2*(c + d*x)] - (4*b^2*(a + b)*Sinh[2*(c + d*x)]/(a - b + (a + b)*Cosh[2*(c + d*x)])^2 - ((9*a - b)

$$\frac{b(a+b)\sinh[2(c+dx)]}{(a(a-b+(a+b)\cosh[2(c+dx)]))} / (8(a+b)^4d)$$

fricas [B] time = 1.71, size = 12965, normalized size = 70.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/16*(2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*\cosh(d*x + c)^{12} + 24*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^{11} + 2*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*\sinh(d*x + c)^{12} + 8*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*\cosh(d*x + c)^{10} + 4*(2*a^4 + 2*a^3*b - 2*a^2*b^2 - 2*a*b^3 - 2*(a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x + 33*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{10} + 40*(11*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*\cosh(d*x + c)^3 + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^9 + 2*(5*a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*\cosh(d*x + c)^8 + 2*(495*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*\cosh(d*x + c)^4 + 5*a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x + 180*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 16*(99*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*\cosh(d*x + c)^5 + 60*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*\cosh(d*x + c)^3 + (5*a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 4*(27*a^3*b - 21*a^2*b^2 + 29*a*b^3 - 3*b^4 - 4*(3*a^4 - 17*a^3*b + 13*a^2*b^2 - 15*a*b^3)*d*x)*\cosh(d*x + c)^6 + 4*(462*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*\cosh(d*x + c)^6 + 420*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*\cosh(d*x + c)^4 + 27*a^3*b - 21*a^2*b^2 + 29*a*b^3 - 3*b^4 - 4*(3*a^4 - 17*a^3*b + 13*a^2*b^2 - 15*a*b^3)*d*x + 14*(5*a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(198*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*\cosh(d*x + c)^7 + 252*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*\cosh(d*x + c)^5 + 14*(5*a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*\cosh(d*x + c)^3 + 3*(27*a^3*b - 21*a^2*b^2 + 29*a*b^3 - 3*b^4 - 4*(3*a^4 - 17*a^3*b + 13*a^2*b^2 - 15*a*b^3)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2*(5*a^4 - 55*a^3*b - 3*a^2*b^2 + 51*a*b^3 - 6*b^4 + 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*\cosh(d*x + c)^4 + 2*(495*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*\cosh(d*x + c)^8 + 840*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*\cosh(d*x + c)^6 + 70*(5*a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*\cosh(d*x + c)^4 - 5*a^4 + 55*a^3*b + 3*a^2*b^2 - 51*a*b^3 + 6*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x + 30*(27*a^3*b - 21*a^2*b^2 + 29*a*b^3 - 3*b^4 - 4*(3*a^4 - 17*a^3*b + 13*a^2*b^2 - 15*a*b^3)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 - 2*a^4 - 6*a^3*b - 6*a^2*b^2 - 2*a*b^3 + 8*(55*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*\cosh(d*x + c)^9 + 120*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*\cosh(d*x + c)^7 + 14*(5*a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*\cosh(d*x + c)^5 + 10*(27*a^3*b - 21*a^2*b^2 + 29*a*b^3 - 3*b^4 - 4*(3*a^4 - 17*a^3*b + 13*a^2*b^2 - 15*a*b^3)*d*x)*\cosh(d*x + c)^3 - (5*a^4 - 55*a^3*b - 3*a^2*b^2 + 51*a*b^3 - 6*b^4 + 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 4*(2*a^4 - 7*a^3*b - 19*a^2*b^2 - 9*a*b^3 + b^4 + 2*(a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*\cosh(d*x + c)^2 + 4*(33*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*\cosh(d*x + c)^{10} + 90*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*\cosh(d*x + c)^8 + 14*(5*a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x) \end{aligned}$$

$$\begin{aligned}
& x) * \cosh(dx + c)^6 + 15 * (27 * a^3 * b - 21 * a^2 * b^2 + 29 * a * b^3 - 3 * b^4 - 4 * (3 * a^4 \\
& - 17 * a^3 * b + 13 * a^2 * b^2 - 15 * a * b^3) * dx) * \cosh(dx + c)^4 - 2 * a^4 + 7 * a^3 * \\
& b + 19 * a^2 * b^2 + 9 * a * b^3 - b^4 - 2 * (a^4 - 3 * a^3 * b - 9 * a^2 * b^2 - 5 * a * b^3) * d * \\
& x - 3 * (5 * a^4 - 55 * a^3 * b - 3 * a^2 * b^2 + 51 * a * b^3 - 6 * b^4 + 16 * (a^4 - 5 * a^3 * b \\
& - a^2 * b^2 + 5 * a * b^3) * dx) * \cosh(dx + c)^2 * \sinh(dx + c)^2 - ((15 * a^4 + 20 * \\
& a^3 * b - 6 * a^2 * b^2 - 12 * a * b^3 - b^4) * \cosh(dx + c)^{10} + 10 * (15 * a^4 + 20 * a^3 * \\
& b - 6 * a^2 * b^2 - 12 * a * b^3 - b^4) * \cosh(dx + c) * \sinh(dx + c)^9 + (15 * a^4 + 2 \\
& 0 * a^3 * b - 6 * a^2 * b^2 - 12 * a * b^3 - b^4) * \sinh(dx + c)^{10} + 4 * (15 * a^4 - 10 * a^3 \\
& * b - 16 * a^2 * b^2 + 10 * a * b^3 + b^4) * \cosh(dx + c)^8 + (60 * a^4 - 40 * a^3 * b - 64 \\
& * a^2 * b^2 + 40 * a * b^3 + 4 * b^4 + 45 * (15 * a^4 + 20 * a^3 * b - 6 * a^2 * b^2 - 12 * a * b^3 \\
& - b^4) * \cosh(dx + c)^2 * \sinh(dx + c)^8 + 8 * (15 * (15 * a^4 + 20 * a^3 * b - 6 * a^2 * \\
& b^2 - 12 * a * b^3 - b^4) * \cosh(dx + c)^3 + 4 * (15 * a^4 - 10 * a^3 * b - 16 * a^2 * b^2 + \\
& 10 * a * b^3 + b^4) * \cosh(dx + c)) * \sinh(dx + c)^7 + 2 * (45 * a^4 - 60 * a^3 * b + 62 \\
& * a^2 * b^2 - 28 * a * b^3 - 3 * b^4) * \cosh(dx + c)^6 + 2 * (105 * (15 * a^4 + 20 * a^3 * b - \\
& 6 * a^2 * b^2 - 12 * a * b^3 - b^4) * \cosh(dx + c)^4 + 45 * a^4 - 60 * a^3 * b + 62 * a^2 * b^2 \\
& - 28 * a * b^3 - 3 * b^4 + 56 * (15 * a^4 - 10 * a^3 * b - 16 * a^2 * b^2 + 10 * a * b^3 + b^4) \\
& * \cosh(dx + c)^2 * \sinh(dx + c)^6 + 4 * (63 * (15 * a^4 + 20 * a^3 * b - 6 * a^2 * b^2 - \\
& 12 * a * b^3 - b^4) * \cosh(dx + c)^5 + 56 * (15 * a^4 - 10 * a^3 * b - 16 * a^2 * b^2 + 10 * a \\
& * b^3 + b^4) * \cosh(dx + c)^3 + 3 * (45 * a^4 - 60 * a^3 * b + 62 * a^2 * b^2 - 28 * a * b^3 \\
& - 3 * b^4) * \cosh(dx + c)) * \sinh(dx + c)^5 + 4 * (15 * a^4 - 10 * a^3 * b - 16 * a^2 * b^2 \\
& + 10 * a * b^3 + b^4) * \cosh(dx + c)^4 + 2 * (105 * (15 * a^4 + 20 * a^3 * b - 6 * a^2 * b^2 \\
& - 12 * a * b^3 - b^4) * \cosh(dx + c)^6 + 140 * (15 * a^4 - 10 * a^3 * b - 16 * a^2 * b^2 + 1 \\
& 0 * a * b^3 + b^4) * \cosh(dx + c)^4 + 30 * a^4 - 20 * a^3 * b - 32 * a^2 * b^2 + 20 * a * b^3 \\
& + 2 * b^4 + 15 * (45 * a^4 - 60 * a^3 * b + 62 * a^2 * b^2 - 28 * a * b^3 - 3 * b^4) * \cosh(dx + \\
& c)^2 * \sinh(dx + c)^4 + 8 * (15 * (15 * a^4 + 20 * a^3 * b - 6 * a^2 * b^2 - 12 * a * b^3 - \\
& b^4) * \cosh(dx + c)^7 + 28 * (15 * a^4 - 10 * a^3 * b - 16 * a^2 * b^2 + 10 * a * b^3 + b^4) \\
& * \cosh(dx + c)^5 + 5 * (45 * a^4 - 60 * a^3 * b + 62 * a^2 * b^2 - 28 * a * b^3 - 3 * b^4) * \co \\
& sh(dx + c)^3 + 2 * (15 * a^4 - 10 * a^3 * b - 16 * a^2 * b^2 + 10 * a * b^3 + b^4) * \cosh(dx \\
& x + c)) * \sinh(dx + c)^3 + (15 * a^4 + 20 * a^3 * b - 6 * a^2 * b^2 - 12 * a * b^3 - b^4) * \\
& \cosh(dx + c)^2 + (45 * (15 * a^4 + 20 * a^3 * b - 6 * a^2 * b^2 - 12 * a * b^3 - b^4) * \cosh \\
& (dx + c)^8 + 112 * (15 * a^4 - 10 * a^3 * b - 16 * a^2 * b^2 + 10 * a * b^3 + b^4) * \cosh(dx \\
& x + c)^6 + 30 * (45 * a^4 - 60 * a^3 * b + 62 * a^2 * b^2 - 28 * a * b^3 - 3 * b^4) * \cosh(dx \\
& + c)^4 + 15 * a^4 + 20 * a^3 * b - 6 * a^2 * b^2 - 12 * a * b^3 - b^4 + 24 * (15 * a^4 - 10 * a \\
& ^3 * b - 16 * a^2 * b^2 + 10 * a * b^3 + b^4) * \cosh(dx + c)^2 * \sinh(dx + c)^2 + 2 * (5 \\
& * (15 * a^4 + 20 * a^3 * b - 6 * a^2 * b^2 - 12 * a * b^3 - b^4) * \cosh(dx + c)^9 + 16 * (15 * \\
& a^4 - 10 * a^3 * b - 16 * a^2 * b^2 + 10 * a * b^3 + b^4) * \cosh(dx + c)^7 + 6 * (45 * a^4 - \\
& 60 * a^3 * b + 62 * a^2 * b^2 - 28 * a * b^3 - 3 * b^4) * \cosh(dx + c)^5 + 8 * (15 * a^4 - 10 \\
& * a^3 * b - 16 * a^2 * b^2 + 10 * a * b^3 + b^4) * \cosh(dx + c)^3 + (15 * a^4 + 20 * a^3 * b \\
& - 6 * a^2 * b^2 - 12 * a * b^3 - b^4) * \cosh(dx + c)) * \sinh(dx + c)) * \sqrt{-b/a} * \log(\\
& ((a^2 + 2 * a * b + b^2) * \cosh(dx + c)^4 + 4 * (a^2 + 2 * a * b + b^2) * \cosh(dx + c) * \\
& \sinh(dx + c)^3 + (a^2 + 2 * a * b + b^2) * \sinh(dx + c)^4 + 2 * (a^2 - b^2) * \cosh(\\
& dx + c)^2 + 2 * (3 * (a^2 + 2 * a * b + b^2) * \cosh(dx + c)^2 + a^2 - b^2) * \sinh(dx \\
& + c)^2 + a^2 - 6 * a * b + b^2 + 4 * ((a^2 + 2 * a * b + b^2) * \cosh(dx + c)^3 + (a^2 \\
& - b^2) * \cosh(dx + c)) * \sinh(dx + c) + 4 * ((a^2 + a * b) * \cosh(dx + c)^2 + 2 * (\\
& a^2 + a * b) * \cosh(dx + c) * \sinh(dx + c) + (a^2 + a * b) * \sinh(dx + c)^2 + a^2 \\
& - a * b) * \sqrt{-b/a}) / ((a + b) * \cosh(dx + c)^4 + 4 * (a + b) * \cosh(dx + c) * \sinh(\\
& dx + c)^3 + (a + b) * \sinh(dx + c)^4 + 2 * (a - b) * \cosh(dx + c)^2 + 2 * (3 * (a \\
& + b) * \cosh(dx + c)^2 + a - b) * \sinh(dx + c)^2 + 4 * ((a + b) * \cosh(dx + c)^3 \\
& + (a - b) * \cosh(dx + c)) * \sinh(dx + c) + a + b)) + 8 * (3 * (a^4 + 3 * a^3 * b + 3 * \\
& a^2 * b^2 + a * b^3) * \cosh(dx + c)^{11} + 10 * (a^4 + a^3 * b - a^2 * b^2 - a * b^3 - (a^4 \\
& - 3 * a^3 * b - 9 * a^2 * b^2 - 5 * a * b^3) * dx) * \cosh(dx + c)^9 + 2 * (5 * a^4 + 17 * a^3 \\
& * b - 11 * a^2 * b^2 - 21 * a * b^3 + 2 * b^4 - 16 * (a^4 - 5 * a^3 * b - a^2 * b^2 + 5 * a * b^3) \\
& * dx) * \cosh(dx + c)^7 + 3 * (27 * a^3 * b - 21 * a^2 * b^2 + 29 * a * b^3 - 3 * b^4 - 4 * (3 * \\
& a^4 - 17 * a^3 * b + 13 * a^2 * b^2 - 15 * a * b^3) * dx) * \cosh(dx + c)^5 - (5 * a^4 - 55 * \\
& a^3 * b - 3 * a^2 * b^2 + 51 * a * b^3 - 6 * b^4 + 16 * (a^4 - 5 * a^3 * b - a^2 * b^2 + 5 * a * b^ \\
& 3) * dx) * \cosh(dx + c)^3 - (2 * a^4 - 7 * a^3 * b - 19 * a^2 * b^2 - 9 * a * b^3 + b^4 + 2 \\
& * (a^4 - 3 * a^3 * b - 9 * a^2 * b^2 - 5 * a * b^3) * dx) * \cosh(dx + c)) * \sinh(dx + c)) / (\\
& (a^7 + 6 * a^6 * b + 15 * a^5 * b^2 + 20 * a^4 * b^3 + 15 * a^3 * b^4 + 6 * a^2 * b^5 + a * b^6) * \\
& d * \cosh(dx + c)^{10} + 10 * (a^7 + 6 * a^6 * b + 15 * a^5 * b^2 + 20 * a^4 * b^3 + 15 * a^3 * b
\end{aligned}$$

$$\begin{aligned}
&^4 + 6a^2b^5 + ab^6) * d * \cosh(dx + c) * \sinh(dx + c)^9 + (a^7 + 6a^6b + \\
&15a^5b^2 + 20a^4b^3 + 15a^3b^4 + 6a^2b^5 + ab^6) * d * \sinh(dx + c)^{10} + 4 * (a^7 + 4a^6b + 5a^5b^2 - 5a^3b^4 - 4a^2b^5 - ab^6) * d * \cosh(dx \\
&x + c)^8 + (45 * (a^7 + 6a^6b + 15a^5b^2 + 20a^4b^3 + 15a^3b^4 + 6a^2b^5 + ab^6) * d * \cosh(dx + c)^2 + 4 * (a^7 + 4a^6b + 5a^5b^2 - 5a^3b^4 \\
&- 4a^2b^5 - ab^6) * d) * \sinh(dx + c)^8 + 2 * (3a^7 + 10a^6b + 13a^5b^2 \\
&+ 12a^4b^3 + 13a^3b^4 + 10a^2b^5 + 3ab^6) * d * \cosh(dx + c)^6 + 8 * (1 \\
&5 * (a^7 + 6a^6b + 15a^5b^2 + 20a^4b^3 + 15a^3b^4 + 6a^2b^5 + ab^6) \\
&) * d * \cosh(dx + c)^3 + 4 * (a^7 + 4a^6b + 5a^5b^2 - 5a^3b^4 - 4a^2b^5 \\
&- ab^6) * d * \cosh(dx + c) * \sinh(dx + c)^7 + 2 * (105 * (a^7 + 6a^6b + 15a^5b^2 \\
&b^2 + 20a^4b^3 + 15a^3b^4 + 6a^2b^5 + ab^6) * d * \cosh(dx + c)^4 + 56 * (\\
&a^7 + 4a^6b + 5a^5b^2 - 5a^3b^4 - 4a^2b^5 - ab^6) * d * \cosh(dx + c)^2 + (3a^7 + 10a^6b + 13a^5b^2 + 12a^4b^3 + 13a^3b^4 + 10a^2b^5 + \\
&3ab^6) * d) * \sinh(dx + c)^6 + 4 * (a^7 + 4a^6b + 5a^5b^2 - 5a^3b^4 - 4 \\
&a^2b^5 - ab^6) * d * \cosh(dx + c)^4 + 4 * (63 * (a^7 + 6a^6b + 15a^5b^2 + 2 \\
&0a^4b^3 + 15a^3b^4 + 6a^2b^5 + ab^6) * d * \cosh(dx + c)^5 + 56 * (a^7 + 4 \\
&a^6b + 5a^5b^2 - 5a^3b^4 - 4a^2b^5 - ab^6) * d * \cosh(dx + c)^3 + 3 * (\\
&3a^7 + 10a^6b + 13a^5b^2 + 12a^4b^3 + 13a^3b^4 + 10a^2b^5 + 3ab^6) * d * \cosh(dx + c) * \sinh(dx + c)^5 + 2 * (105 * (a^7 + 6a^6b + 15a^5b^2 \\
&+ 20a^4b^3 + 15a^3b^4 + 6a^2b^5 + ab^6) * d * \cosh(dx + c)^6 + 140 * (a^7 \\
&+ 4a^6b + 5a^5b^2 - 5a^3b^4 - 4a^2b^5 - ab^6) * d * \cosh(dx + c)^4 + \\
&15 * (3a^7 + 10a^6b + 13a^5b^2 + 12a^4b^3 + 13a^3b^4 + 10a^2b^5 + \\
&3ab^6) * d * \cosh(dx + c)^2 + 2 * (a^7 + 4a^6b + 5a^5b^2 - 5a^3b^4 - 4 \\
&a^2b^5 - ab^6) * d) * \sinh(dx + c)^4 + (a^7 + 6a^6b + 15a^5b^2 + 20a^4b^3 \\
&b^3 + 15a^3b^4 + 6a^2b^5 + ab^6) * d * \cosh(dx + c)^2 + 8 * (15 * (a^7 + 6a^6b \\
&+ 15a^5b^2 + 20a^4b^3 + 15a^3b^4 + 6a^2b^5 + ab^6) * d * \cosh(dx \\
&+ c)^7 + 28 * (a^7 + 4a^6b + 5a^5b^2 - 5a^3b^4 - 4a^2b^5 - ab^6) * d * c \\
&>osh(dx + c)^5 + 5 * (3a^7 + 10a^6b + 13a^5b^2 + 12a^4b^3 + 13a^3b^4 \\
&+ 10a^2b^5 + 3ab^6) * d * \cosh(dx + c)^3 + 2 * (a^7 + 4a^6b + 5a^5b^2 - \\
&5a^3b^4 - 4a^2b^5 - ab^6) * d * \cosh(dx + c) * \sinh(dx + c)^3 + (45 * (a^7 \\
&+ 6a^6b + 15a^5b^2 + 20a^4b^3 + 15a^3b^4 + 6a^2b^5 + ab^6) * d * co \\
&sh(dx + c)^8 + 112 * (a^7 + 4a^6b + 5a^5b^2 - 5a^3b^4 - 4a^2b^5 - a \\
&b^6) * d * \cosh(dx + c)^6 + 30 * (3a^7 + 10a^6b + 13a^5b^2 + 12a^4b^3 + 1 \\
&3a^3b^4 + 10a^2b^5 + 3ab^6) * d * \cosh(dx + c)^4 + 24 * (a^7 + 4a^6b + 5 \\
&a^5b^2 - 5a^3b^4 - 4a^2b^5 - ab^6) * d * \cosh(dx + c)^2 + (a^7 + 6a^6b \\
&b + 15a^5b^2 + 20a^4b^3 + 15a^3b^4 + 6a^2b^5 + ab^6) * d) * \sinh(dx + \\
&c)^2 + 2 * (5 * (a^7 + 6a^6b + 15a^5b^2 + 20a^4b^3 + 15a^3b^4 + 6a^2b^5 \\
&b^5 + ab^6) * d * \cosh(dx + c)^9 + 16 * (a^7 + 4a^6b + 5a^5b^2 - 5a^3b^4 \\
&- 4a^2b^5 - ab^6) * d * \cosh(dx + c)^7 + 6 * (3a^7 + 10a^6b + 13a^5b^2 + \\
&12a^4b^3 + 13a^3b^4 + 10a^2b^5 + 3ab^6) * d * \cosh(dx + c)^5 + 8 * (a^7 \\
&+ 4a^6b + 5a^5b^2 - 5a^3b^4 - 4a^2b^5 - ab^6) * d * \cosh(dx + c)^3 + \\
&(a^7 + 6a^6b + 15a^5b^2 + 20a^4b^3 + 15a^3b^4 + 6a^2b^5 + ab^6) \\
&* d * \cosh(dx + c) * \sinh(dx + c)), 1/8 * ((a^4 + 3a^3b + 3a^2b^2 + ab^3) * \\
&cosh(dx + c)^{12} + 12 * (a^4 + 3a^3b + 3a^2b^2 + ab^3) * cosh(dx + c) * sin \\
&h(dx + c)^{11} + (a^4 + 3a^3b + 3a^2b^2 + ab^3) * \sinh(dx + c)^{12} + 4 * (a \\
&^4 + a^3b - a^2b^2 - ab^3 - (a^4 - 3a^3b - 9a^2b^2 - 5ab^3) * dx) * c \\
&>osh(dx + c)^{10} + 2 * (2a^4 + 2a^3b - 2a^2b^2 - 2ab^3 - 2 * (a^4 - 3a^3 \\
&>* b - 9a^2b^2 - 5ab^3) * dx + 33 * (a^4 + 3a^3b + 3a^2b^2 + ab^3) * cosh \\
&(dx + c)^2) * \sinh(dx + c)^{10} + 20 * (11 * (a^4 + 3a^3b + 3a^2b^2 + ab^3) * \\
&cosh(dx + c)^3 + 2 * (a^4 + a^3b - a^2b^2 - ab^3 - (a^4 - 3a^3b - 9a^2 \\
&>* b^2 - 5ab^3) * dx) * cosh(dx + c) * \sinh(dx + c)^9 + (5a^4 + 17a^3b - 1 \\
&1a^2b^2 - 21ab^3 + 2b^4 - 16 * (a^4 - 5a^3b - a^2b^2 + 5ab^3) * dx) * \\
&>cosh(dx + c)^8 + (495 * (a^4 + 3a^3b + 3a^2b^2 + ab^3) * cosh(dx + c)^4 \\
&+ 5a^4 + 17a^3b - 11a^2b^2 - 21ab^3 + 2b^4 - 16 * (a^4 - 5a^3b - a^2 \\
&b^2 + 5ab^3) * dx + 180 * (a^4 + a^3b - a^2b^2 - ab^3 - (a^4 - 3a^3b \\
&- 9a^2b^2 - 5ab^3) * dx) * cosh(dx + c)^2) * \sinh(dx + c)^8 + 8 * (99 * (a^4 + \\
&3a^3b + 3a^2b^2 + ab^3) * cosh(dx + c)^5 + 60 * (a^4 + a^3b - a^2b^2 - \\
&ab^3 - (a^4 - 3a^3b - 9a^2b^2 - 5ab^3) * dx) * cosh(dx + c)^3 + (5a^4 \\
&+ 17a^3b - 11a^2b^2 - 21ab^3 + 2b^4 - 16 * (a^4 - 5a^3b - a^2b^2
\end{aligned}$$

$$\begin{aligned}
& + 5*a*b^3)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(27*a^3*b - 21*a^2*b^2 + \\
& 29*a*b^3 - 3*b^4 - 4*(3*a^4 - 17*a^3*b + 13*a^2*b^2 - 15*a*b^3)*d*x)*\cosh(\\
& d*x + c)^6 + 2*(462*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*\cosh(d*x + c)^6 + 4 \\
& 20*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d \\
& *x)*\cosh(d*x + c)^4 + 27*a^3*b - 21*a^2*b^2 + 29*a*b^3 - 3*b^4 - 4*(3*a^4 - \\
& 17*a^3*b + 13*a^2*b^2 - 15*a*b^3)*d*x + 14*(5*a^4 + 17*a^3*b - 11*a^2*b^2 \\
& - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*\cosh(d*x + \\
& c)^2)*\sinh(d*x + c)^6 + 4*(198*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*\cosh(d \\
& x + c)^7 + 252*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 \\
& - 5*a*b^3)*d*x)*\cosh(d*x + c)^5 + 14*(5*a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a* \\
& b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*\cosh(d*x + c)^3 + \\
& 3*(27*a^3*b - 21*a^2*b^2 + 29*a*b^3 - 3*b^4 - 4*(3*a^4 - 17*a^3*b + 13*a^2 \\
& *b^2 - 15*a*b^3)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^5 - (5*a^4 - 55*a^3*b - \\
& 3*a^2*b^2 + 51*a*b^3 - 6*b^4 + 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)* \\
& \cosh(d*x + c)^4 + (495*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*\cosh(d*x + c)^8 \\
& + 840*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3 \\
&)*d*x)*\cosh(d*x + c)^6 + 70*(5*a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b \\
& ^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*\cosh(d*x + c)^4 - 5*a^4 + \\
& 55*a^3*b + 3*a^2*b^2 - 51*a*b^3 + 6*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a \\
& *b^3)*d*x + 30*(27*a^3*b - 21*a^2*b^2 + 29*a*b^3 - 3*b^4 - 4*(3*a^4 - 17*a^ \\
& 3*b + 13*a^2*b^2 - 15*a*b^3)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 - a^4 - \\
& 3*a^3*b - 3*a^2*b^2 - a*b^3 + 4*(55*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*\cos \\
& h(d*x + c)^9 + 120*(a^4 + a^3*b - a^2*b^2 - a*b^3 - (a^4 - 3*a^3*b - 9*a^2* \\
& b^2 - 5*a*b^3)*d*x)*\cosh(d*x + c)^7 + 14*(5*a^4 + 17*a^3*b - 11*a^2*b^2 - 2 \\
& 1*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*\cosh(d*x + c) \\
& ^5 + 10*(27*a^3*b - 21*a^2*b^2 + 29*a*b^3 - 3*b^4 - 4*(3*a^4 - 17*a^3*b + 1 \\
& 3*a^2*b^2 - 15*a*b^3)*d*x)*\cosh(d*x + c)^3 - (5*a^4 - 55*a^3*b - 3*a^2*b^2 \\
& + 51*a*b^3 - 6*b^4 + 16*(a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*\cosh(d*x + \\
& c))*\sinh(d*x + c)^3 - 2*(2*a^4 - 7*a^3*b - 19*a^2*b^2 - 9*a*b^3 + b^4 + 2* \\
& (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*\cosh(d*x + c)^2 + 2*(33*(a^4 + 3 \\
& *a^3*b + 3*a^2*b^2 + a*b^3)*\cosh(d*x + c)^10 + 90*(a^4 + a^3*b - a^2*b^2 - \\
& a*b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*\cosh(d*x + c)^8 + 14*(5* \\
& a^4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^ \\
& 2 + 5*a*b^3)*d*x)*\cosh(d*x + c)^6 + 15*(27*a^3*b - 21*a^2*b^2 + 29*a*b^3 - \\
& 3*b^4 - 4*(3*a^4 - 17*a^3*b + 13*a^2*b^2 - 15*a*b^3)*d*x)*\cosh(d*x + c)^4 - \\
& 2*a^4 + 7*a^3*b + 19*a^2*b^2 + 9*a*b^3 - b^4 - 2*(a^4 - 3*a^3*b - 9*a^2*b^ \\
& 2 - 5*a*b^3)*d*x - 3*(5*a^4 - 55*a^3*b - 3*a^2*b^2 + 51*a*b^3 - 6*b^4 + 16* \\
& (a^4 - 5*a^3*b - a^2*b^2 + 5*a*b^3)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 - \\
& ((15*a^4 + 20*a^3*b - 6*a^2*b^2 - 12*a*b^3 - b^4)*\cosh(d*x + c)^10 + 10*(1 \\
& 5*a^4 + 20*a^3*b - 6*a^2*b^2 - 12*a*b^3 - b^4)*\cosh(d*x + c)*\sinh(d*x + c)^ \\
& 9 + (15*a^4 + 20*a^3*b - 6*a^2*b^2 - 12*a*b^3 - b^4)*\sinh(d*x + c)^10 + 4*(\\
& 15*a^4 - 10*a^3*b - 16*a^2*b^2 + 10*a*b^3 + b^4)*\cosh(d*x + c)^8 + (60*a^4 \\
& - 40*a^3*b - 64*a^2*b^2 + 40*a*b^3 + 4*b^4 + 45*(15*a^4 + 20*a^3*b - 6*a^2* \\
& b^2 - 12*a*b^3 - b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 8*(15*(15*a^4 + 20 \\
& *a^3*b - 6*a^2*b^2 - 12*a*b^3 - b^4)*\cosh(d*x + c)^3 + 4*(15*a^4 - 10*a^3*b \\
& - 16*a^2*b^2 + 10*a*b^3 + b^4)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(45*a^4 \\
& - 60*a^3*b + 62*a^2*b^2 - 28*a*b^3 - 3*b^4)*\cosh(d*x + c)^6 + 2*(105*(15*a^ \\
& 4 + 20*a^3*b - 6*a^2*b^2 - 12*a*b^3 - b^4)*\cosh(d*x + c)^4 + 45*a^4 - 60*a^ \\
& 3*b + 62*a^2*b^2 - 28*a*b^3 - 3*b^4 + 56*(15*a^4 - 10*a^3*b - 16*a^2*b^2 + \\
& 10*a*b^3 + b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 4*(63*(15*a^4 + 20*a^3*b \\
& - 6*a^2*b^2 - 12*a*b^3 - b^4)*\cosh(d*x + c)^5 + 56*(15*a^4 - 10*a^3*b - 16 \\
& *a^2*b^2 + 10*a*b^3 + b^4)*\cosh(d*x + c)^3 + 3*(45*a^4 - 60*a^3*b + 62*a^2* \\
& b^2 - 28*a*b^3 - 3*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 4*(15*a^4 - 10*a^3 \\
& *b - 16*a^2*b^2 + 10*a*b^3 + b^4)*\cosh(d*x + c)^4 + 2*(105*(15*a^4 + 20*a^3 \\
& *b - 6*a^2*b^2 - 12*a*b^3 - b^4)*\cosh(d*x + c)^6 + 140*(15*a^4 - 10*a^3*b - \\
& 16*a^2*b^2 + 10*a*b^3 + b^4)*\cosh(d*x + c)^4 + 30*a^4 - 20*a^3*b - 32*a^2* \\
& b^2 + 20*a*b^3 + 2*b^4 + 15*(45*a^4 - 60*a^3*b + 62*a^2*b^2 - 28*a*b^3 - 3* \\
& b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(15*(15*a^4 + 20*a^3*b - 6*a^2*b^ \\
& 2 - 12*a*b^3 - b^4)*\cosh(d*x + c)^7 + 28*(15*a^4 - 10*a^3*b - 16*a^2*b^2 +
\end{aligned}$$

$$\begin{aligned}
& 10*a*b^3 + b^4)*\cosh(d*x + c)^5 + 5*(45*a^4 - 60*a^3*b + 62*a^2*b^2 - 28*a* \\
& b^3 - 3*b^4)*\cosh(d*x + c)^3 + 2*(15*a^4 - 10*a^3*b - 16*a^2*b^2 + 10*a*b^3 \\
& + b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + (15*a^4 + 20*a^3*b - 6*a^2*b^2 - 1 \\
& 2*a*b^3 - b^4)*\cosh(d*x + c)^2 + (45*(15*a^4 + 20*a^3*b - 6*a^2*b^2 - 12*a* \\
& b^3 - b^4)*\cosh(d*x + c)^8 + 112*(15*a^4 - 10*a^3*b - 16*a^2*b^2 + 10*a*b^3 \\
& + b^4)*\cosh(d*x + c)^6 + 30*(45*a^4 - 60*a^3*b + 62*a^2*b^2 - 28*a*b^3 - 3 \\
& *b^4)*\cosh(d*x + c)^4 + 15*a^4 + 20*a^3*b - 6*a^2*b^2 - 12*a*b^3 - b^4 + 24 \\
& *(15*a^4 - 10*a^3*b - 16*a^2*b^2 + 10*a*b^3 + b^4)*\cosh(d*x + c)^2)*\sinh(d* \\
& x + c)^2 + 2*(5*(15*a^4 + 20*a^3*b - 6*a^2*b^2 - 12*a*b^3 - b^4)*\cosh(d*x + \\
& c)^9 + 16*(15*a^4 - 10*a^3*b - 16*a^2*b^2 + 10*a*b^3 + b^4)*\cosh(d*x + c)^ \\
& 7 + 6*(45*a^4 - 60*a^3*b + 62*a^2*b^2 - 28*a*b^3 - 3*b^4)*\cosh(d*x + c)^5 + \\
& 8*(15*a^4 - 10*a^3*b - 16*a^2*b^2 + 10*a*b^3 + b^4)*\cosh(d*x + c)^3 + (15* \\
& a^4 + 20*a^3*b - 6*a^2*b^2 - 12*a*b^3 - b^4)*\cosh(d*x + c))*\sinh(d*x + c))* \\
& \sqrt{b/a}*\arctan(1/2*((a + b)*\cosh(d*x + c)^2 + 2*(a + b)*\cosh(d*x + c)*\sin \\
& h(d*x + c) + (a + b)*\sinh(d*x + c)^2 + a - b)*\sqrt{b/a}/b) + 4*(3*(a^4 + 3* \\
& a^3*b + 3*a^2*b^2 + a*b^3)*\cosh(d*x + c)^11 + 10*(a^4 + a^3*b - a^2*b^2 - a \\
& *b^3 - (a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*\cosh(d*x + c)^9 + 2*(5*a^ \\
& 4 + 17*a^3*b - 11*a^2*b^2 - 21*a*b^3 + 2*b^4 - 16*(a^4 - 5*a^3*b - a^2*b^2 \\
& + 5*a*b^3)*d*x)*\cosh(d*x + c)^7 + 3*(27*a^3*b - 21*a^2*b^2 + 29*a*b^3 - 3*b \\
& ^4 - 4*(3*a^4 - 17*a^3*b + 13*a^2*b^2 - 15*a*b^3)*d*x)*\cosh(d*x + c)^5 - (5 \\
& *a^4 - 55*a^3*b - 3*a^2*b^2 + 51*a*b^3 - 6*b^4 + 16*(a^4 - 5*a^3*b - a^2*b^ \\
& 2 + 5*a*b^3)*d*x)*\cosh(d*x + c)^3 - (2*a^4 - 7*a^3*b - 19*a^2*b^2 - 9*a*b^3 \\
& + b^4 + 2*(a^4 - 3*a^3*b - 9*a^2*b^2 - 5*a*b^3)*d*x)*\cosh(d*x + c))*\sinh(d \\
& *x + c))/((a^7 + 6*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 \\
& + a*b^6)*d*\cosh(d*x + c)^10 + 10*(a^7 + 6*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 \\
& + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)*d*\cosh(d*x + c)*\sinh(d*x + c)^9 + (a^7 + \\
& 6*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)*d*\sinh(\\
& d*x + c)^10 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6) \\
& *d*\cosh(d*x + c)^8 + (45*(a^7 + 6*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3* \\
& b^4 + 6*a^2*b^5 + a*b^6)*d*\cosh(d*x + c)^2 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 - \\
& 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*d)*\sinh(d*x + c)^8 + 2*(3*a^7 + 10*a^6*b + \\
& 13*a^5*b^2 + 12*a^4*b^3 + 13*a^3*b^4 + 10*a^2*b^5 + 3*a*b^6)*d*\cosh(d*x + c \\
&)^6 + 8*(15*(a^7 + 6*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b \\
& ^5 + a*b^6)*d*\cosh(d*x + c)^3 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - \\
& 4*a^2*b^5 - a*b^6)*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(105*(a^7 + 6*a^6*b \\
& + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)*d*\cosh(d*x + c \\
&)^4 + 56*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*d*\cosh \\
& (d*x + c)^2 + (3*a^7 + 10*a^6*b + 13*a^5*b^2 + 12*a^4*b^3 + 13*a^3*b^4 + 10 \\
& *a^2*b^5 + 3*a*b^6)*d)*\sinh(d*x + c)^6 + 4*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a \\
& ^3*b^4 - 4*a^2*b^5 - a*b^6)*d*\cosh(d*x + c)^4 + 4*(63*(a^7 + 6*a^6*b + 15*a \\
& ^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)*d*\cosh(d*x + c)^5 + 5 \\
& 6*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*d*\cosh(d*x + \\
& c)^3 + 3*(3*a^7 + 10*a^6*b + 13*a^5*b^2 + 12*a^4*b^3 + 13*a^3*b^4 + 10*a^2* \\
& b^5 + 3*a*b^6)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(105*(a^7 + 6*a^6*b + 1 \\
& 5*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)*d*\cosh(d*x + c)^6 \\
& + 140*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*d*\cosh(d* \\
& x + c)^4 + 15*(3*a^7 + 10*a^6*b + 13*a^5*b^2 + 12*a^4*b^3 + 13*a^3*b^4 + 10 \\
& *a^2*b^5 + 3*a*b^6)*d*\cosh(d*x + c)^2 + 2*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^ \\
& 3*b^4 - 4*a^2*b^5 - a*b^6)*d)*\sinh(d*x + c)^4 + (a^7 + 6*a^6*b + 15*a^5*b^2 \\
& + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)*d*\cosh(d*x + c)^2 + 8*(15*(\\
& a^7 + 6*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a*b^6)*d \\
& *\cosh(d*x + c)^7 + 28*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - \\
& a*b^6)*d*\cosh(d*x + c)^5 + 5*(3*a^7 + 10*a^6*b + 13*a^5*b^2 + 12*a^4*b^3 + \\
& 13*a^3*b^4 + 10*a^2*b^5 + 3*a*b^6)*d*\cosh(d*x + c)^3 + 2*(a^7 + 4*a^6*b + 5 \\
& *a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 \\
& + (45*(a^7 + 6*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 + 6*a^2*b^5 + a \\
& *b^6)*d*\cosh(d*x + c)^8 + 112*(a^7 + 4*a^6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^ \\
& 2*b^5 - a*b^6)*d*\cosh(d*x + c)^6 + 30*(3*a^7 + 10*a^6*b + 13*a^5*b^2 + 12*a \\
& ^4*b^3 + 13*a^3*b^4 + 10*a^2*b^5 + 3*a*b^6)*d*\cosh(d*x + c)^4 + 24*(a^7 + 4
\end{aligned}$$

$a^6b + 5a^5b^2 - 5a^3b^4 - 4a^2b^5 - ab^6) * d * \cosh(dx + c)^2 + (a^7 + 6a^6b + 15a^5b^2 + 20a^4b^3 + 15a^3b^4 + 6a^2b^5 + ab^6) * d * \sinh(dx + c)^2 + 2 * (5(a^7 + 6a^6b + 15a^5b^2 + 20a^4b^3 + 15a^3b^4 + 6a^2b^5 + ab^6) * d * \cosh(dx + c)^9 + 16(a^7 + 4a^6b + 5a^5b^2 - 5a^3b^4 - 4a^2b^5 - ab^6) * d * \cosh(dx + c)^7 + 6(3a^7 + 10a^6b + 13a^5b^2 + 12a^4b^3 + 13a^3b^4 + 10a^2b^5 + 3ab^6) * d * \cosh(dx + c)^5 + 8(a^7 + 4a^6b + 5a^5b^2 - 5a^3b^4 - 4a^2b^5 - ab^6) * d * \cosh(dx + c)^3 + (a^7 + 6a^6b + 15a^5b^2 + 20a^4b^3 + 15a^3b^4 + 6a^2b^5 + ab^6) * d * \cosh(dx + c)) * \sinh(dx + c)]$

giac [B] time = 1.94, size = 584, normalized size = 3.16

$$\frac{4(a-5b)dx}{a^4+4a^3b+6a^2b^2+4ab^3+b^4} + \frac{(15a^2be^{2c}-10ab^2e^{2c}-b^3e^{2c})\arctan\left(\frac{ae^{2dx+2c}+be^{2dx+2c}+a-b}{2\sqrt{ab}}\right)e^{-2c}}{(a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4)\sqrt{ab}} - \frac{(2ae^{2dx+2c}-10be^{2dx+2c}-a^4e^{2c}+4a^3be^{2c}+6a^2b^2e^{2c}+4ab^3e^{2c})}{a^4e^{2c}+4a^3be^{2c}+6a^2b^2e^{2c}+4ab^3e^{2c}+b^4e^{2c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)^2/(a+b*tanh(dx+c)^2)^3,x, algorithm="giac")

[Out] $-1/8 * (4 * (a - 5 * b) * d * x / (a^4 + 4 * a^3 * b + 6 * a^2 * b^2 + 4 * a * b^3 + b^4) + (15 * a^2 * b * e^{(2 * c)} - 10 * a * b^2 * e^{(2 * c)} - b^3 * e^{(2 * c)}) * \arctan(1/2 * (a * e^{(2 * d * x + 2 * c)} + b * e^{(2 * d * x + 2 * c)} + a - b) / \sqrt{a * b}) * e^{(-2 * c)} / ((a^5 + 4 * a^4 * b + 6 * a^3 * b^2 + 4 * a^2 * b^3 + a * b^4) * \sqrt{a * b})) - (2 * a * e^{(2 * d * x + 2 * c)} - 10 * b * e^{(2 * d * x + 2 * c)} - a - b) * e^{(-2 * d * x)} / (a^4 * e^{(2 * c)} + 4 * a^3 * b * e^{(2 * c)} + 6 * a^2 * b^2 * e^{(2 * c)} + 4 * a * b^3 * e^{(2 * c)} + b^4 * e^{(2 * c)}) - e^{(2 * d * x + 16 * c)} / (a^3 * e^{(14 * c)} + 3 * a^2 * b * e^{(14 * c)} + 3 * a * b^2 * e^{(14 * c)} + b^3 * e^{(14 * c)}) - 2 * (9 * a^3 * b * e^{(6 * d * x + 6 * c)} - 5 * a^2 * b^2 * e^{(6 * d * x + 6 * c)} - 13 * a * b^3 * e^{(6 * d * x + 6 * c)} + b^4 * e^{(6 * d * x + 6 * c)}) + 27 * a^3 * b * e^{(4 * d * x + 4 * c)} - 21 * a^2 * b^2 * e^{(4 * d * x + 4 * c)} + 29 * a * b^3 * e^{(4 * d * x + 4 * c)} - 3 * b^4 * e^{(4 * d * x + 4 * c)} + 27 * a^3 * b * e^{(2 * d * x + 2 * c)} + a^2 * b^2 * e^{(2 * d * x + 2 * c)} - 23 * a * b^3 * e^{(2 * d * x + 2 * c)} + 3 * b^4 * e^{(2 * d * x + 2 * c)} + 9 * a^3 * b + 17 * a^2 * b^2 + 7 * a * b^3 - b^4) / ((a^5 + 4 * a^4 * b + 6 * a^3 * b^2 + 4 * a^2 * b^3 + a * b^4) * (a * e^{(4 * d * x + 4 * c)} + b * e^{(4 * d * x + 4 * c)} + 2 * a * e^{(2 * d * x + 2 * c)} - 2 * b * e^{(2 * d * x + 2 * c)} + a + b)^2) / d$

maple [B] time = 0.36, size = 2110, normalized size = 11.41

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(dx+c)^2/(a+b*tanh(dx+c)^2)^3,x)

[Out] $-1/2/d/(a+b)^4 * \ln(\tanh(1/2 * d * x + 1/2 * c) + 1) * a + 1/d * b^4/(a+b)^4 / (\tanh(1/2 * d * x + 1/2 * c)^4 * a + 2 * \tanh(1/2 * d * x + 1/2 * c)^2 * a + 4 * \tanh(1/2 * d * x + 1/2 * c)^2 * b + a)^2 / a * \tanh(1/2 * d * x + 1/2 * c)^5 - 29/2/d * b^2/(a+b)^4 / (\tanh(1/2 * d * x + 1/2 * c)^4 * a + 2 * \tanh(1/2 * d * x + 1/2 * c)^2 * a + 4 * \tanh(1/2 * d * x + 1/2 * c)^2 * b + a)^2 * a * \tanh(1/2 * d * x + 1/2 * c)^3 - 5/2/d * b^2/(a+b)^4 / (\tanh(1/2 * d * x + 1/2 * c)^4 * a + 2 * \tanh(1/2 * d * x + 1/2 * c)^2 * a + 4 * \tanh(1/2 * d * x + 1/2 * c)^2 * b + a)^2 * \tanh(1/2 * d * x + 1/2 * c) * a - 27/4/d * b^3/(a+b)^4 / (\tanh(1/2 * d * x + 1/2 * c)^4 * a + 2 * \tanh(1/2 * d * x + 1/2 * c)^2 * a + 4 * \tanh(1/2 * d * x + 1/2 * c)^2 * b + a)^2 * \tanh(1/2 * d * x + 1/2 * c)^5 - 27/4/d * b^3/(a+b)^4 / (\tanh(1/2 * d * x + 1/2 * c)^4 * a + 2 * \tanh(1/2 * d * x + 1/2 * c)^2 * a + 4 * \tanh(1/2 * d * x + 1/2 * c)^2 * b + a)^2 * \tanh(1/2 * d * x + 1/2 * c)^3 - 5/4/d * b^2/(a+b)^4 / ((2 * (b * (a + b))^(1/2) + a + 2 * b) * a)^(1/2) * \arctan(a * \tanh(1/2 * d * x + 1/2 * c)) / ((2 * (b * (a + b))^(1/2) + a + 2 * b) * a)^(1/2)) + 5/4/d * b^2/(a+b)^4 / ((2 * (b * (a + b))^(1/2) - a - 2 * b) * a)^(1/2) * \operatorname{arctanh}(a * \tanh(1/2 * d * x + 1/2 * c)) / ((2 * (b * (a + b))^(1/2) - a - 2 * b) * a)^(1/2)) - 1/4/d * b^3/(a+b)^4 / (\tanh(1/2 * d * x + 1/2 * c)^4 * a + 2 * \tanh(1/2 * d * x + 1/2 * c)^2 * a + 4 * \tanh(1/2 * d * x + 1/2 * c)^2 * b + a)^2 * \tanh(1/2 * d * x + 1/2 * c) - 1/4/d * b^3/(a+b)^4 / (\tanh(1/2 * d * x + 1/2 * c)^4 * a + 2 * \tanh(1/2 * d * x + 1/2 * c)^2 * a + 4 * \tanh(1/2 * d * x + 1/2 * c)^2 * b + a)^2 * \tanh(1/2 * d * x + 1/2 * c)^7 + 5/2/d/(a+b)^4 * \ln(\tanh(1/2 * d * x + 1/2 * c) + 1) * b + 1/2/d/(a+b)^4 * \ln(\tanh(1/2 * d * x + 1/2 * c) - 1) * a - 5/2/d/(a+b)^4 * \ln(\tanh(1/2 * d * x + 1/2 * c) - 1) * b - 11/8/d * b^3/(a+b)^4 / (b * (a + b))^(1/2) / ((2 * (b * (a + b))^(1/2) + a + 2 * b) * a)^(1/2) * \arctan(a * \tanh(1/2 * d * x + 1/2 * c))$

$$\begin{aligned} & h(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}-5/2/d*b^2/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)^7*a-1/8/d*b^3/(a+b)^4/a/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)})-11/8/d*b^3/(a+b)^4/(b*(a+b))^{(1/2)}/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)})-9/4/d*b/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)^7*a^2+1/2/d/(a+b)^3/(\tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/(a+b)^3/(\tanh(1/2*d*x+1/2*c)-1)-1/2/d/(a+b)^3/(\tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/(a+b)^3/(\tanh(1/2*d*x+1/2*c)+1)-1/8/d*b^4/(a+b)^4/(b*(a+b))^{(1/2)}/a/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)})-1/8/d*b^4/(a+b)^4/(b*(a+b))^{(1/2)}/a/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)})+15/8/d*b/(a+b)^4*a^2/(b*(a+b))^{(1/2)}/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)})+15/8/d*b/(a+b)^4*a^2/(b*(a+b))^{(1/2)}/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)})+5/8/d*b^2/(a+b)^4*a/(b*(a+b))^{(1/2)}/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)})+5/8/d*b^2/(a+b)^4*a/(b*(a+b))^{(1/2)}/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)})-27/4/d*b/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*a^2*\tanh(1/2*d*x+1/2*c)^5-27/4/d*b/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*a^2*\tanh(1/2*d*x+1/2*c)^3+1/d*b^4/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*\tanh(1/2*d*x+1/2*c)^3-15/8/d*b/(a+b)^4*a/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)})-29/2/d*b^2/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*\tanh(1/2*d*x+1/2*c)^5+1/8/d*b^3/(a+b)^4/a/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)})-9/4/d*b/(a+b)^4/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)*a^2+15/8/d*b/(a+b)^4*a/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}) \end{aligned}$$

maxima [B] time = 0.82, size = 1806, normalized size = 9.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c))^2)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned} & 3/4*b*\log((a+b)*e^{(4*d*x+4*c)}+2*(a-b)*e^{(2*d*x+2*c)}+a+b)/((a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*d)-3/4*b*\log(2*(a-b)*e^{(-2*d*x-2*c)}+(a+b)*e^{(-4*d*x-4*c)}+a+b)/((a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*d)-3/32*(5*a^3*b-15*a^2*b^2-5*a*b^3-b^4)*\arctan(1/2*((a+b)*e^{(2*d*x+2*c)}+a-b)/\sqrt{a*b})/((a^6+4*a^5*b+6*a^4*b^2+4*a^3*b^3+a^2*b^4)*\sqrt{a*b}*d)+3/32*(5*a^3*b-15*a^2*b^2-5*a*b^3-b^4)*\arctan(1/2*((a+b)*e^{(-2*d*x-2*c)}+a-b)/\sqrt{a*b})/((a^6+4*a^5*b+6*a^4*b^2+4*a^3*b^3+a^2*b^4)*\sqrt{a*b}*d)+1/16*(15*a^2*b+10*a*b^2+3*b^3)*\arctan(1/2*((a+b)*e^{(-2*d*x-2*c)}+a-b)/\sqrt{a*b})/((a^5+3*a^4*b+3*a^3*b^2+a^2*b^3)*\sqrt{a*b}*d)+1/16*(9*a^4*b+4*a^3*b^2-22*a^2*b^3-20*a*b^4-3*b^5+3*(3*a^4*b-22*a^3*b^2-20*a^2*b^3+6*a*b^4+b^5))*e^{(6*d*x+6*c)}+(27*a^4*b-156*a^3*b^2+110*a^2*b^3-36*a*b^4-9*b^5)*e^{(4*d*x+4*c)}+(27*a^4*b-86*a^3*b^2-84*a^2*b^3+38*a*b^4+9*b^5)*e^{(2*d*x+2*c)}/((a^8+6*a^7*b+15*a^6*b^2+20*a^5*b^3+15*a^4*b^4+6*a^3*b^5+a^2*b^6+(a^8+6*a^7*b+15*a^6*b^2+20*a^5*b^3+15*a^4*b^4+6*a^3*b^5+a^2*b^6))*e^{(8*d*x+8*c)}+4*(a^8+4*a^7*b+5*a^6*b^2-5*a^4*b^4-4*a^3*b^5-a^2*b^6))*e^{(6*d*x+6*c)}+2*(3*a^8+10*a^7*b+13*a^6*b^2+12*a^5*b^3+13*a^4*b^4+10*a^3*b^5+3*a^2*b^6)*e^{(4*d*x+4*c)} \end{aligned}$$

$$\begin{aligned}
& d*x + 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6) \\
& *e^{(2*d*x + 2*c)}*d) - 1/16*(9*a^4*b + 4*a^3*b^2 - 22*a^2*b^3 - 20*a*b^4 - \\
& 3*b^5 + (27*a^4*b - 86*a^3*b^2 - 84*a^2*b^3 + 38*a*b^4 + 9*b^5)*e^{(-2*d*x \\
& - 2*c)} + (27*a^4*b - 156*a^3*b^2 + 110*a^2*b^3 - 36*a*b^4 - 9*b^5)*e^{(-4*d*x \\
& x - 4*c)} + 3*(3*a^4*b - 22*a^3*b^2 - 20*a^2*b^3 + 6*a*b^4 + b^5)*e^{(-6*d*x \\
& - 6*c)})/((a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 \\
& + a^2*b^6 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6) \\
& *e^{(-2*d*x - 2*c)} + 2*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4* \\
& b^4 + 10*a^3*b^5 + 3*a^2*b^6)*e^{(-4*d*x - 4*c)} + 4*(a^8 + 4*a^7*b + 5*a^6*b \\
& ^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*e^{(-6*d*x - 6*c)} + (a^8 + 6*a^7*b + 1 \\
& 5*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*e^{(-8*d*x - 8*c)} \\
&)*d) - 1/8*(9*a^3*b + 21*a^2*b^2 + 15*a*b^3 + 3*b^4 + (27*a^3*b + 13*a^2*b^2 \\
& 2 - 23*a*b^3 - 9*b^4)*e^{(-2*d*x - 2*c)} + 3*(9*a^3*b - 3*a^2*b^2 + 7*a*b^3 + \\
& 3*b^4)*e^{(-4*d*x - 4*c)} + (9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4)*e^{(-6*d*x \\
& - 6*c)})/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5 + \\
& 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^{(-2*d*x - \\
& 2*c)} + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5) \\
& *e^{(-4*d*x - 4*c)} + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - \\
& a^2*b^5)*e^{(-6*d*x - 6*c)} + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^ \\
& 3*b^4 + a^2*b^5)*e^{(-8*d*x - 8*c)})*d) - 1/2*(d*x + c)/((a^3 + 3*a^2*b + 3*a \\
& *b^2 + b^3)*d) + 1/8*e^{(2*d*x + 2*c)}/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - \\
& 1/8*e^{(-2*d*x - 2*c)}/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
\end{aligned}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(c + dx)^2}{(b \tanh(c + dx)^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3, x)

[Out] int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3, x)

[Out] Timed out

$$3.44 \quad \int \frac{\sinh(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=126

$$\frac{15 \cosh(c+dx)}{8d(a+b)^3} - \frac{5 \cosh(c+dx)}{8d(a+b)^2 (a - b \operatorname{sech}^2(c+dx) + b)} - \frac{\cosh(c+dx)}{4d(a+b) (a - b \operatorname{sech}^2(c+dx) + b)^2} - \frac{15\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \operatorname{sech}(c+dx)}{\sqrt{a+b \tanh^2(c+dx)}}\right)}{8d(a+b)^{7/2}}$$

[Out] 15/8*cosh(d*x+c)/(a+b)^3/d-1/4*cosh(d*x+c)/(a+b)/d/(a+b-b*sech(d*x+c)^2)^2-5/8*cosh(d*x+c)/(a+b)^2/d/(a+b-b*sech(d*x+c)^2)-15/8*arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/(a+b)^(7/2)/d

Rubi [A] time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3664, 290, 325, 208}

$$\frac{15 \cosh(c+dx)}{8d(a+b)^3} - \frac{5 \cosh(c+dx)}{8d(a+b)^2 (a - b \operatorname{sech}^2(c+dx) + b)} - \frac{\cosh(c+dx)}{4d(a+b) (a - b \operatorname{sech}^2(c+dx) + b)^2} - \frac{15\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \operatorname{sech}(c+dx)}{\sqrt{a+b \tanh^2(c+dx)}}\right)}{8d(a+b)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (-15*sqrt[b]*ArcTanh[(sqrt[b]*Sech[c + d*x])/sqrt[a + b]])/(8*(a + b)^(7/2)*d) + (15*Cosh[c + d*x])/(8*(a + b)^3*d) - Cosh[c + d*x]/(4*(a + b)*d*(a + b - b*Sech[c + d*x]^2)^2) - (5*Cosh[c + d*x])/(8*(a + b)^2*d*(a + b - b*Sech[c + d*x]^2))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1) + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 325

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 3664

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p]/x^(m + 1), x], x, Sec[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(c+dx)}{(a+b \tanh^2(c+dx))^3} dx &= -\frac{\text{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)^3} dx, x, \text{sech}(c+dx)\right)}{d} \\
&= -\frac{\cosh(c+dx)}{4(a+b)d(a+b-b \text{sech}^2(c+dx))^2} - \frac{5 \text{Subst}\left(\int \frac{1}{x^2(a+b-bx^2)^2} dx, x, \text{sech}(c+dx)\right)}{4(a+b)d} \\
&= -\frac{\cosh(c+dx)}{4(a+b)d(a+b-b \text{sech}^2(c+dx))^2} - \frac{5 \cosh(c+dx)}{8(a+b)^2 d(a+b-b \text{sech}^2(c+dx))} \\
&= \frac{15 \cosh(c+dx)}{8(a+b)^3 d} - \frac{\cosh(c+dx)}{4(a+b)d(a+b-b \text{sech}^2(c+dx))^2} - \frac{5 \cosh(c+dx)}{8(a+b)^2 d(a+b-b \text{sech}^2(c+dx))} \\
&= -\frac{15\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \text{sech}(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{7/2} d} + \frac{15 \cosh(c+dx)}{8(a+b)^3 d} - \frac{\cosh(c+dx)}{4(a+b)d(a+b-b \text{sech}^2(c+dx))^2}
\end{aligned}$$

Mathematica [C] time = 1.83, size = 157, normalized size = 1.25

$$\frac{2 \cosh(c+dx) \left(-\frac{4b^2}{((a+b) \cosh(2(c+dx))+a-b)^2} - \frac{9b}{(a+b) \cosh(2(c+dx))+a-b} + 4 \right)}{(a+b)^3} - \frac{15i\sqrt{b} \left(\tan^{-1}\left(\frac{-\sqrt{a} \tanh\left(\frac{1}{2}(c+dx)\right) - i\sqrt{a+b}}{\sqrt{b}}\right) + \tan^{-1}\left(\frac{\sqrt{a} \tanh\left(\frac{1}{2}(c+dx)\right) - i\sqrt{a+b}}{\sqrt{b}}\right) \right)}{(a+b)^{7/2}}}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3, x]

[Out] (((-15*I)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]]))/(a + b)^(7/2) + (2*Cosh[c + d*x]*(4 - (4*b^2)/(a - b + (a + b)*Cosh[2*(c + d*x)]))^2 - (9*b)/(a - b + (a + b)*Cosh[2*(c + d*x)])))/(a + b)^3/(8*d)

fricas [B] time = 1.38, size = 7119, normalized size = 56.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^3, x, algorithm="fricas")

[Out] [1/16*(8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 + 80*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^9 + 8*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^10 + 20*(2*a^2 - a*b - 3*b^2)*cosh(d*x + c)^8 + 20*(18*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 - a*b - 3*b^2)*sinh(d*x + c)^8 + 160*(6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (2*a^2 - a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(4*a^2 - 7*a*b + b^2)*cosh(d*x + c)^6 + 20*(84*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 28*(2*a^2 - a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a^2 - 7*a*b + b^2)*sinh(d*x + c)^6 + 8*(252*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 140*(2*a^2 - a*b - 3*b^2)*cosh(d*x + c)^3 + 15*(4*a^2 - 7*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 20*(4*a^2 - 7*a*b + b^2)*cosh(d*x + c)^4 + 20*(84*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 70*(2*a^2 - a*b - 3*b^2)*cosh(d*x + c)^4 + 15*(4*a^2 - 7*a*b + b^2)*cosh(d*x + c)^2 + 4*a^2 - 7*a*b + b^2)*sinh(d*x + c)^4 +

$$\begin{aligned}
& 80*(12*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)^7 + 14*(2*a^2 - a*b - 3*b^2)*\cosh \\
& (d*x + c)^5 + 5*(4*a^2 - 7*a*b + b^2)*\cosh(d*x + c)^3 + (4*a^2 - 7*a*b + b^ \\
& 2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 20*(2*a^2 - a*b - 3*b^2)*\cosh(d*x + c)^ \\
& 2 + 20*(18*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)^8 + 28*(2*a^2 - a*b - 3*b^2)*c \\
& osh(d*x + c)^6 + 15*(4*a^2 - 7*a*b + b^2)*\cosh(d*x + c)^4 + 6*(4*a^2 - 7*a* \\
& b + b^2)*\cosh(d*x + c)^2 + 2*a^2 - a*b - 3*b^2)*\sinh(d*x + c)^2 + 15*((a^2 \\
& + 2*a*b + b^2)*\cosh(d*x + c)^9 + 9*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)*\sinh(d \\
& *x + c)^8 + (a^2 + 2*a*b + b^2)*\sinh(d*x + c)^9 + 4*(a^2 - b^2)*\cosh(d*x + \\
& c)^7 + 4*(9*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)^2 + a^2 - b^2)*\sinh(d*x + c)^ \\
& 7 + 28*(3*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)^3 + (a^2 - b^2)*\cosh(d*x + c))* \\
& \sinh(d*x + c)^6 + 2*(3*a^2 - 2*a*b + 3*b^2)*\cosh(d*x + c)^5 + 2*(63*(a^2 + \\
& 2*a*b + b^2)*\cosh(d*x + c)^4 + 42*(a^2 - b^2)*\cosh(d*x + c)^2 + 3*a^2 - 2*a \\
& *b + 3*b^2)*\sinh(d*x + c)^5 + 2*(63*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)^5 + 7 \\
& 0*(a^2 - b^2)*\cosh(d*x + c)^3 + 5*(3*a^2 - 2*a*b + 3*b^2)*\cosh(d*x + c))*\si \\
& nh(d*x + c)^4 + 4*(a^2 - b^2)*\cosh(d*x + c)^3 + 4*(21*(a^2 + 2*a*b + b^2)*c \\
& osh(d*x + c)^6 + 35*(a^2 - b^2)*\cosh(d*x + c)^4 + 5*(3*a^2 - 2*a*b + 3*b^2) \\
& *\cosh(d*x + c)^2 + a^2 - b^2)*\sinh(d*x + c)^3 + 4*(9*(a^2 + 2*a*b + b^2)*co \\
& sh(d*x + c)^7 + 21*(a^2 - b^2)*\cosh(d*x + c)^5 + 5*(3*a^2 - 2*a*b + 3*b^2)* \\
& cosh(d*x + c)^3 + 3*(a^2 - b^2)*\cosh(d*x + c))*\sinh(d*x + c)^2 + (a^2 + 2*a \\
& *b + b^2)*\cosh(d*x + c) + (9*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)^8 + 28*(a^2 \\
& - b^2)*\cosh(d*x + c)^6 + 10*(3*a^2 - 2*a*b + 3*b^2)*\cosh(d*x + c)^4 + 12*(a \\
& ^2 - b^2)*\cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*\sinh(d*x + c))*\sqrt{b/(a + b \\
&)}*\log(((a + b)*\cosh(d*x + c)^4 + 4*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + \\
& (a + b)*\sinh(d*x + c)^4 + 2*(a + 3*b)*\cosh(d*x + c)^2 + 2*(3*(a + b)*\cosh(\\
& d*x + c)^2 + a + 3*b)*\sinh(d*x + c)^2 + 4*((a + b)*\cosh(d*x + c)^3 + (a + 3 \\
& *b)*\cosh(d*x + c))*\sinh(d*x + c) - 4*((a + b)*\cosh(d*x + c)^3 + 3*(a + b)*c \\
& osh(d*x + c)*\sinh(d*x + c)^2 + (a + b)*\sinh(d*x + c)^3 + (a + b)*\cosh(d*x + \\
& c) + (3*(a + b)*\cosh(d*x + c)^2 + a + b)*\sinh(d*x + c))*\sqrt{b/(a + b)} + \\
& a + b)/((a + b)*\cosh(d*x + c)^4 + 4*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + \\
& (a + b)*\sinh(d*x + c)^4 + 2*(a - b)*\cosh(d*x + c)^2 + 2*(3*(a + b)*\cosh(d* \\
& x + c)^2 + a - b)*\sinh(d*x + c)^2 + 4*((a + b)*\cosh(d*x + c)^3 + (a - b)*co \\
& sh(d*x + c))*\sinh(d*x + c) + a + b)) + 8*a^2 + 16*a*b + 8*b^2 + 40*(2*(a^2 \\
& + 2*a*b + b^2)*\cosh(d*x + c)^9 + 4*(2*a^2 - a*b - 3*b^2)*\cosh(d*x + c)^7 + \\
& 3*(4*a^2 - 7*a*b + b^2)*\cosh(d*x + c)^5 + 2*(4*a^2 - 7*a*b + b^2)*\cosh(d*x \\
& + c)^3 + (2*a^2 - a*b - 3*b^2)*\cosh(d*x + c))*\sinh(d*x + c))/((a^5 + 5*a^4*b \\
& + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(d*x + c)^9 + 9*(a^5 + 5 \\
& *a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(d*x + c)*\sinh(d*x \\
& + c)^8 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\sinh(d \\
& *x + c)^9 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cos \\
& h(d*x + c)^7 + 4*(9*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^ \\
& 5)*d*\cosh(d*x + c)^2 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b \\
& ^5)*d)*\sinh(d*x + c)^7 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b \\
& ^4 + 3*b^5)*d*\cosh(d*x + c)^5 + 28*(3*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2* \\
& b^3 + 5*a*b^4 + b^5)*d*\cosh(d*x + c)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2 \\
& *b^3 - 3*a*b^4 - b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^6 + 2*(63*(a^5 + 5*a^4 \\
& *b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(d*x + c)^4 + 42*(a^5 + \\
& 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*\cosh(d*x + c)^2 + (3*a^ \\
& 5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d)*\sinh(d*x + c)^5 + \\
& 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*\cosh(d*x + c)^ \\
& 3 + 2*(63*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(\\
& d*x + c)^5 + 70*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*c \\
& osh(d*x + c)^3 + 5*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b \\
& ^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^4 + 4*(21*(a^5 + 5*a^4*b + 10*a^3*b^2 + \\
& 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(d*x + c)^6 + 35*(a^5 + 3*a^4*b + 2*a^3*b \\
& ^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*\cosh(d*x + c)^4 + 5*(3*a^5 + 7*a^4*b + 6* \\
& a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*\cosh(d*x + c)^2 + (a^5 + 3*a^4*b + \\
& 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d)*\sinh(d*x + c)^3 + (a^5 + 5*a^4*b \\
& + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(d*x + c) + 4*(9*(a^5 + 5 \\
& *a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(d*x + c)^7 + 21*(a
\end{aligned}$$

$$\begin{aligned}
&^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5) * d * \cosh(dx + c)^5 + 5 \\
& * (3a^5 + 7a^4b + 6a^3b^2 + 6a^2b^3 + 7ab^4 + 3b^5) * d * \cosh(dx + c \\
&)^3 + 3(a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5) * d * \cosh(dx \\
& + c) * \sinh(dx + c)^2 + (9(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 \\
& + b^5) * d * \cosh(dx + c)^8 + 28(a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3 \\
& * ab^4 - b^5) * d * \cosh(dx + c)^6 + 10(3a^5 + 7a^4b + 6a^3b^2 + 6a^2b^3 \\
& + 7ab^4 + 3b^5) * d * \cosh(dx + c)^4 + 12(a^5 + 3a^4b + 2a^3b^2 - 2 \\
& * a^2b^3 - 3ab^4 - b^5) * d * \cosh(dx + c)^2 + (a^5 + 5a^4b + 10a^3b^2 + \\
& 10a^2b^3 + 5ab^4 + b^5) * d) * \sinh(dx + c)), 1/8(4(a^2 + 2ab + b^2) * \\
& \cosh(dx + c)^{10} + 40(a^2 + 2ab + b^2) * \cosh(dx + c) * \sinh(dx + c)^9 + 4 \\
& * (a^2 + 2ab + b^2) * \sinh(dx + c)^{10} + 10(2a^2 - ab - 3b^2) * \cosh(dx + \\
& c)^8 + 10(18(a^2 + 2ab + b^2) * \cosh(dx + c)^2 + 2a^2 - ab - 3b^2) * s \\
& \sinh(dx + c)^8 + 80(6(a^2 + 2ab + b^2) * \cosh(dx + c)^3 + (2a^2 - ab - \\
& 3b^2) * \cosh(dx + c)) * \sinh(dx + c)^7 + 10(4a^2 - 7ab + b^2) * \cosh(dx \\
& + c)^6 + 10(84(a^2 + 2ab + b^2) * \cosh(dx + c)^4 + 28(2a^2 - ab - 3b^2 \\
& ^2) * \cosh(dx + c)^2 + 4a^2 - 7ab + b^2) * \sinh(dx + c)^6 + 4(252(a^2 + \\
& 2ab + b^2) * \cosh(dx + c)^5 + 140(2a^2 - ab - 3b^2) * \cosh(dx + c)^3 + \\
& 15(4a^2 - 7ab + b^2) * \cosh(dx + c)) * \sinh(dx + c)^5 + 10(4a^2 - 7ab \\
& + b^2) * \cosh(dx + c)^4 + 10(84(a^2 + 2ab + b^2) * \cosh(dx + c)^6 + 70(\\
& 2a^2 - ab - 3b^2) * \cosh(dx + c)^4 + 15(4a^2 - 7ab + b^2) * \cosh(dx + \\
& c)^2 + 4a^2 - 7ab + b^2) * \sinh(dx + c)^4 + 40(12(a^2 + 2ab + b^2) * co \\
& sh(dx + c)^7 + 14(2a^2 - ab - 3b^2) * \cosh(dx + c)^5 + 5(4a^2 - 7ab \\
& + b^2) * \cosh(dx + c)^3 + (4a^2 - 7ab + b^2) * \cosh(dx + c)) * \sinh(dx + c \\
&)^3 + 10(2a^2 - ab - 3b^2) * \cosh(dx + c)^2 + 10(18(a^2 + 2ab + b^2) \\
& * \cosh(dx + c)^8 + 28(2a^2 - ab - 3b^2) * \cosh(dx + c)^6 + 15(4a^2 - 7 \\
& * ab + b^2) * \cosh(dx + c)^4 + 6(4a^2 - 7ab + b^2) * \cosh(dx + c)^2 + 2a \\
& ^2 - ab - 3b^2) * \sinh(dx + c)^2 - 15((a^2 + 2ab + b^2) * \cosh(dx + c)^9 \\
& + 9(a^2 + 2ab + b^2) * \cosh(dx + c) * \sinh(dx + c)^8 + (a^2 + 2ab + b^2 \\
&) * \sinh(dx + c)^9 + 4(a^2 - b^2) * \cosh(dx + c)^7 + 4(9(a^2 + 2ab + b^2 \\
&) * \cosh(dx + c)^2 + a^2 - b^2) * \sinh(dx + c)^7 + 28(3(a^2 + 2ab + b^2) * \\
& \cosh(dx + c)^3 + (a^2 - b^2) * \cosh(dx + c)) * \sinh(dx + c)^6 + 2(3a^2 - 2 \\
& * ab + 3b^2) * \cosh(dx + c)^5 + 2(63(a^2 + 2ab + b^2) * \cosh(dx + c)^4 + \\
& 42(a^2 - b^2) * \cosh(dx + c)^2 + 3a^2 - 2ab + 3b^2) * \sinh(dx + c)^5 + \\
& 2(63(a^2 + 2ab + b^2) * \cosh(dx + c)^5 + 70(a^2 - b^2) * \cosh(dx + c)^3 \\
& + 5(3a^2 - 2ab + 3b^2) * \cosh(dx + c)) * \sinh(dx + c)^4 + 4(a^2 - b^2) * \\
& \cosh(dx + c)^3 + 4(21(a^2 + 2ab + b^2) * \cosh(dx + c)^6 + 35(a^2 - b^2 \\
&) * \cosh(dx + c)^4 + 5(3a^2 - 2ab + 3b^2) * \cosh(dx + c)^2 + a^2 - b^2) * \\
& \sinh(dx + c)^3 + 4(9(a^2 + 2ab + b^2) * \cosh(dx + c)^7 + 21(a^2 - b^2) \\
& * \cosh(dx + c)^5 + 5(3a^2 - 2ab + 3b^2) * \cosh(dx + c)^3 + 3(a^2 - b^2 \\
&) * \cosh(dx + c)) * \sinh(dx + c)^2 + (a^2 + 2ab + b^2) * \cosh(dx + c) + (9(\\
& a^2 + 2ab + b^2) * \cosh(dx + c)^8 + 28(a^2 - b^2) * \cosh(dx + c)^6 + 10(3 \\
& * a^2 - 2ab + 3b^2) * \cosh(dx + c)^4 + 12(a^2 - b^2) * \cosh(dx + c)^2 + a^ \\
& 2 + 2ab + b^2) * \sinh(dx + c)) * \sqrt{-b/(a + b)} * \arctan(1/2((a + b) * \cosh(d \\
& * x + c)^3 + 3(a + b) * \cosh(dx + c) * \sinh(dx + c)^2 + (a + b) * \sinh(dx + c) \\
& ^3 + (a - 3b) * \cosh(dx + c) + (3(a + b) * \cosh(dx + c)^2 + a - 3b) * \sinh(d \\
& * x + c)) * \sqrt{-b/(a + b)})/b) + 15((a^2 + 2ab + b^2) * \cosh(dx + c)^9 + 9 \\
& (a^2 + 2ab + b^2) * \cosh(dx + c) * \sinh(dx + c)^8 + (a^2 + 2ab + b^2) * sin \\
& h(dx + c)^9 + 4(a^2 - b^2) * \cosh(dx + c)^7 + 4(9(a^2 + 2ab + b^2) * cos \\
& h(dx + c)^2 + a^2 - b^2) * \sinh(dx + c)^7 + 28(3(a^2 + 2ab + b^2) * \cosh(\\
& dx + c)^3 + (a^2 - b^2) * \cosh(dx + c)) * \sinh(dx + c)^6 + 2(3a^2 - 2ab \\
& + 3b^2) * \cosh(dx + c)^5 + 2(63(a^2 + 2ab + b^2) * \cosh(dx + c)^4 + 42(\\
& a^2 - b^2) * \cosh(dx + c)^2 + 3a^2 - 2ab + 3b^2) * \sinh(dx + c)^5 + 2(63 \\
& * (a^2 + 2ab + b^2) * \cosh(dx + c)^5 + 70(a^2 - b^2) * \cosh(dx + c)^3 + 5(\\
& 3a^2 - 2ab + 3b^2) * \cosh(dx + c)) * \sinh(dx + c)^4 + 4(a^2 - b^2) * \cosh(\\
& dx + c)^3 + 4(21(a^2 + 2ab + b^2) * \cosh(dx + c)^6 + 35(a^2 - b^2) * cos \\
& h(dx + c)^4 + 5(3a^2 - 2ab + 3b^2) * \cosh(dx + c)^2 + a^2 - b^2) * \sinh(\\
& dx + c)^3 + 4(9(a^2 + 2ab + b^2) * \cosh(dx + c)^7 + 21(a^2 - b^2) * \cosh \\
& (dx + c)^5 + 5(3a^2 - 2ab + 3b^2) * \cosh(dx + c)^3 + 3(a^2 - b^2) * cos \\
& h(dx + c)) * \sinh(dx + c)^2 + (a^2 + 2ab + b^2) * \cosh(dx + c) + (9(a^2 +
\end{aligned}$$

$$\begin{aligned}
& 2*a*b + b^2)*\cosh(d*x + c)^8 + 28*(a^2 - b^2)*\cosh(d*x + c)^6 + 10*(3*a^2 \\
& - 2*a*b + 3*b^2)*\cosh(d*x + c)^4 + 12*(a^2 - b^2)*\cosh(d*x + c)^2 + a^2 + 2 \\
& *a*b + b^2)*\sinh(d*x + c))*\sqrt{-b/(a + b))*\arctan(1/2*((a + b)*\cosh(d*x + \\
& c) + (a + b)*\sinh(d*x + c))*\sqrt{-b/(a + b)})/b) + 4*a^2 + 8*a*b + 4*b^2 + 2 \\
& 0*(2*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)^9 + 4*(2*a^2 - a*b - 3*b^2)*\cosh(d*x \\
& + c)^7 + 3*(4*a^2 - 7*a*b + b^2)*\cosh(d*x + c)^5 + 2*(4*a^2 - 7*a*b + b^2) \\
& *\cosh(d*x + c)^3 + (2*a^2 - a*b - 3*b^2)*\cosh(d*x + c))*\sinh(d*x + c))/((a^ \\
& 5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(d*x + c)^9 + \\
& 9*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(d*x + c) \\
& *\sinh(d*x + c)^8 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5 \\
&)*d*\sinh(d*x + c)^9 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - \\
& b^5)*d*\cosh(d*x + c)^7 + 4*(9*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5* \\
& a*b^4 + b^5)*d*\cosh(d*x + c)^2 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3 \\
& *a*b^4 - b^5)*d)*\sinh(d*x + c)^7 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b \\
& ^3 + 7*a*b^4 + 3*b^5)*d*\cosh(d*x + c)^5 + 28*(3*(a^5 + 5*a^4*b + 10*a^3*b^2 \\
& + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(d*x + c)^3 + (a^5 + 3*a^4*b + 2*a^3*b \\
& ^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^6 + 2*(63*(a \\
& ^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(d*x + c)^4 + \\
& 42*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*\cosh(d*x + c) \\
& ^2 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d)*\sinh(d* \\
& x + c)^5 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*\cosh \\
& (d*x + c)^3 + 2*(63*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^ \\
& 5)*d*\cosh(d*x + c)^5 + 70*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 \\
& - b^5)*d*\cosh(d*x + c)^3 + 5*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a \\
& *b^4 + 3*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^4 + 4*(21*(a^5 + 5*a^4*b + 10* \\
& a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(d*x + c)^6 + 35*(a^5 + 3*a^4*b \\
& + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*\cosh(d*x + c)^4 + 5*(3*a^5 + 7* \\
& a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*\cosh(d*x + c)^2 + (a^5 + \\
& 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d)*\sinh(d*x + c)^3 + (a^5 \\
& + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(d*x + c) + 4*(\\
& 9*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(d*x + c) \\
& ^7 + 21*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*\cosh(d*x \\
& + c)^5 + 5*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*\co \\
& sh(d*x + c)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d \\
& *\cosh(d*x + c))*\sinh(d*x + c)^2 + (9*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b \\
& ^3 + 5*a*b^4 + b^5)*d*\cosh(d*x + c)^8 + 28*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a \\
& ^2*b^3 - 3*a*b^4 - b^5)*d*\cosh(d*x + c)^6 + 10*(3*a^5 + 7*a^4*b + 6*a^3*b^2 \\
& + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*\cosh(d*x + c)^4 + 12*(a^5 + 3*a^4*b + 2*a \\
& ^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*\cosh(d*x + c)^2 + (a^5 + 5*a^4*b + 10 \\
& *a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d)*\sinh(d*x + c))]
\end{aligned}$$

giac [B] time = 0.62, size = 1718, normalized size = 13.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c))^2)^3,x, algorithm="giac")

[Out]
$$\begin{aligned}
& -1/8*(15*(2*(a^3*e^{(2*c)} + 3*a^2*b*e^{(2*c)} + 3*a*b^2*e^{(2*c)} + b^3*e^{(2*c)}) \\
& ^2*(2*a*b^2 + (a*b - b^2)*\sqrt{-a*b}))*\text{abs}(a*e^{(2*c)} + b*e^{(2*c)}) + (a^5*b + \\
& 3*a^4*b^2 + 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6 - 2*(a^4*b + 4*a^3*b^2 + \\
& 6*a^2*b^3 + 4*a*b^4 + b^5)*\sqrt{-a*b}))*\text{abs}(-a^3*e^{(2*c)} - 3*a^2*b*e^{(2*c)} \\
& - 3*a*b^2*e^{(2*c)} - b^3*e^{(2*c)})*\text{abs}(a*e^{(2*c)} + b*e^{(2*c)})*e^{(2*c)} + (2*a^ \\
& 8*b + 10*a^7*b^2 + 18*a^6*b^3 + 10*a^5*b^4 - 10*a^4*b^5 - 18*a^3*b^6 - 10*a \\
& ^2*b^7 - 2*a*b^8 + (a^8 + 4*a^7*b + 4*a^6*b^2 - 4*a^5*b^3 - 10*a^4*b^4 - 4* \\
& a^3*b^5 + 4*a^2*b^6 + 4*a*b^7 + b^8)*\sqrt{-a*b}))*\text{abs}(a*e^{(2*c)} + b*e^{(2*c)}) \\
& *e^{(4*c)}*\arctan(e^{(d*x)}/\sqrt{(a^4*e^{(2*c)} + 2*a^3*b*e^{(2*c)} - 2*a*b^3*e^{(2* \\
& c)} - b^4*e^{(2*c)})^2 - (a^4*e^{(4*c)} + 4*a^3*b*e^{(4*c)} + 6*a^2*b^2*e^{(4*c)} + 4*a* \\
& b^3*e^{(4*c)} + b^4*e^{(4*c)})*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)))/(a
\end{aligned}$$

$$\begin{aligned} & \left(a^4 e^{4c} + 4a^3 b e^{4c} + 6a^2 b^2 e^{4c} + 4a b^3 e^{4c} + b^4 e^{4c} \right) e^{-4c} / \left((a^9 + 9a^8 b + 36a^7 b^2 + 84a^6 b^3 + 126a^5 b^4 + 126a^4 b^5 + 84a^3 b^6 + 36a^2 b^7 + 9a b^8 + b^9) \sqrt{a^2 - b^2 - 2\sqrt{-ab}(a+b)} \right) \cdot \left(-a^3 e^{2c} - 3a^2 b e^{2c} - 3a b^2 e^{2c} - b^3 e^{2c} \right) \\ & + 15 \left(2(a^3 e^{2c} + 3a^2 b e^{2c} + 3a b^2 e^{2c} + b^3 e^{2c})^2 (2a b^2 - (a b - b^2) \sqrt{-ab}) \right) \cdot \left(a e^{2c} + b e^{2c} \right) \\ & + (a^5 b + 3a^4 b^2 + 2a^3 b^3 - 2a^2 b^4 - 3a b^5 - b^6 + 2(a^4 b + 4a^3 b^2 + 6a^2 b^3 + 4a b^4 + b^5) \sqrt{-ab}) \cdot \left(-a^3 e^{2c} - 3a^2 b e^{2c} - 3a b^2 e^{2c} - b^3 e^{2c} \right) \cdot \left(a e^{2c} + b e^{2c} \right) e^{2c} \\ & + (2a^8 b + 10a^7 b^2 + 18a^6 b^3 + 10a^5 b^4 - 10a^4 b^5 - 18a^3 b^6 - 10a^2 b^7 - 2a b^8 - (a^8 + 4a^7 b + 4a^6 b^2 - 4a^5 b^3 - 10a^4 b^4 - 4a^3 b^5 + 4a^2 b^6 + 4a b^7 + b^8) \sqrt{-ab}) \cdot \left(a e^{2c} + b e^{2c} \right) e^{4c} \\ & \cdot \arctan \left(e^{dx} / \sqrt{(a^4 e^{2c} + 2a^3 b e^{2c} - 2a b^3 e^{2c} - b^4 e^{2c}) - \sqrt{(a^4 e^{2c} + 2a^3 b e^{2c} - 2a b^3 e^{2c} - b^4 e^{2c})^2 - (a^4 e^{4c} + 4a^3 b e^{4c} + 6a^2 b^2 e^{4c} + 4a b^3 e^{4c} + b^4 e^{4c})}} \right) \cdot (a^4 + 4a^3 b + 6a^2 b^2 + 4a b^3 + b^4) \\ & / (a^4 e^{4c} + 4a^3 b e^{4c} + 6a^2 b^2 e^{4c} + 4a b^3 e^{4c} + b^4 e^{4c}) e^{-4c} / \left((a^9 + 9a^8 b + 36a^7 b^2 + 84a^6 b^3 + 126a^5 b^4 + 126a^4 b^5 + 84a^3 b^6 + 36a^2 b^7 + 9a b^8 + b^9) \sqrt{a^2 - b^2 + 2\sqrt{-ab}(a+b)} \right) \cdot \left(-a^3 e^{2c} - 3a^2 b e^{2c} - 3a b^2 e^{2c} - b^3 e^{2c} \right) \\ & - 4e^{dx+14c} / (a^3 e^{13c} + 3a^2 b e^{13c} + 3a b^2 e^{13c} + b^3 e^{13c}) - 4e^{-dx} / (a^3 e^c + 3a^2 b e^c + 3a b^2 e^c + b^3 e^c) + 2(9a b e^{7dx+7c} + 9b^2 e^{7dx+7c} + 27a b e^{5dx+5c} - b^2 e^{5dx+5c} + 27a b e^{3dx+3c} - b^2 e^{3dx+3c} + 9a b e^{dx+c} + 9b^2 e^{dx+c}) / ((a^3 + 3a^2 b + 3a b^2 + b^3) (a e^{4dx+4c} + b e^{4dx+4c} + 2a e^{2dx+2c} - 2b e^{2dx+2c} + a + b)^2) / d \end{aligned}$$

maple [B] time = 0.30, size = 252, normalized size = 2.00

$$\frac{1}{(a+b)^3 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)} + \frac{1}{(a+b)^3 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)} + \frac{2b \left(\frac{(9a^2 + 24ab + 8b^2) \left(\tanh^6\left(\frac{dx}{2} + \frac{c}{2}\right) \right) (27a^3 + 78a^2 b + 88a b^2 + 16b^3) \left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right) \right) (27a^2 + 56ab + 27b^2)}{8a} - \frac{(27a^3 + 78a^2 b + 88a b^2 + 16b^3) \left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right) \right) (27a^2 + 56ab + 27b^2)}{8a^2} \right)}{\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right) \right)_{a+2} \left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right)_{a+4} \left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right)_{b+a} \right)^2}}{(a+b)^3} d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x)

[Out] $\frac{1}{d} \left(-\frac{1}{(a+b)^3} \left(\frac{1}{\tanh\left(\frac{1}{2}d*x + \frac{1}{2}c\right) - 1} + \frac{1}{\tanh\left(\frac{1}{2}d*x + \frac{1}{2}c\right) + 1} \right) + \frac{2b}{(a+b)^3} \left(-\frac{1}{8} \frac{(9a^2 + 24ab + 8b^2)}{a \tanh\left(\frac{1}{2}d*x + \frac{1}{2}c\right)} \left(\tanh\left(\frac{1}{2}d*x + \frac{1}{2}c\right) - 1 \right)^6 - \frac{1}{8} \frac{1}{a^2} (27a^3 + 78a^2 b + 88a b^2 + 16b^3) \tanh\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^4 - \frac{1}{8} (27a^2 + 56ab + 8b^2) \frac{1}{a \tanh\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2} - \frac{9}{8} \frac{a - 1/4 b}{\left(\tanh\left(\frac{1}{2}d*x + \frac{1}{2}c\right) - 1 \right)^4} + \frac{2 \tanh\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2 a + 4 \tanh\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2 b + a^2 - 15/16}{(a+b+b^2)^{(1/2)}} \arctan\left(\frac{1/4 (2 \tanh\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2 a + 2a + 4b)}{(a+b+b^2)^{(1/2)}}\right) \right) \right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2a^2 + 4ab + \dots}{4 \left((a^5 d e^{9c} + 5a^4 b d e^{9c} + 10a^3 b^2 d e^{9c} + 10a^2 b^3 d e^{9c} + 5ab^4 d e^{9c} + b^5 d e^{9c}) e^{9dx} + 4(a^5 d e^{7c} + 3a^4 b d e^{7c} + \dots) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $\frac{1}{4} (2a^2 + 4ab + 2b^2 + 2(a^2 e^{10c} + 2ab e^{10c} + b^2 e^{10c})) e^{10dx} + 5(2a^2 e^{8c} - ab e^{8c} - 3b^2 e^{8c}) e^{8dx} + 5(4a^2 e^{6c} - 7ab e^{6c} + b^2 e^{6c}) e^{6dx} + 5(4a^2 e^{4c} - 7ab e^{4c} + b^2 e^{4c}) e^{4dx} + 5(a^2 e^{2c} - ab e^{2c} + b^2 e^{2c}) e^{2dx} + 5(a^2 e^c - ab e^c + b^2 e^c) e^c$

$c) - 7*a*b*e^{(4*c)} + b^2*e^{(4*c)})*e^{(4*d*x)} + 5*(2*a^2*e^{(2*c)} - a*b*e^{(2*c)} - 3*b^2*e^{(2*c)})*e^{(2*d*x)})/((a^5*d*e^{(9*c)} + 5*a^4*b*d*e^{(9*c)} + 10*a^3*b^2*d*e^{(9*c)} + 10*a^2*b^3*d*e^{(9*c)} + 5*a*b^4*d*e^{(9*c)} + b^5*d*e^{(9*c)})*e^{(9*d*x)} + 4*(a^5*d*e^{(7*c)} + 3*a^4*b*d*e^{(7*c)} + 2*a^3*b^2*d*e^{(7*c)} - 2*a^2*b^3*d*e^{(7*c)} - 3*a*b^4*d*e^{(7*c)} - b^5*d*e^{(7*c)})*e^{(7*d*x)} + 2*(3*a^5*d*e^{(5*c)} + 7*a^4*b*d*e^{(5*c)} + 6*a^3*b^2*d*e^{(5*c)} + 6*a^2*b^3*d*e^{(5*c)} + 7*a*b^4*d*e^{(5*c)} + 3*b^5*d*e^{(5*c)})*e^{(5*d*x)} + 4*(a^5*d*e^{(3*c)} + 3*a^4*b*d*e^{(3*c)} + 2*a^3*b^2*d*e^{(3*c)} - 2*a^2*b^3*d*e^{(3*c)} - 3*a*b^4*d*e^{(3*c)} - b^5*d*e^{(3*c)})*e^{(3*d*x)} + (a^5*d*e^c + 5*a^4*b*d*e^c + 10*a^3*b^2*d*e^c + 10*a^2*b^3*d*e^c + 5*a*b^4*d*e^c + b^5*d*e^c)*e^{(d*x)}) + 1/2*integrate(15/2*(b*e^{(3*d*x + 3*c)} - b*e^{(d*x + c)})/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 + (a^4*e^{(4*c)} + 4*a^3*b*e^{(4*c)} + 6*a^2*b^2*e^{(4*c)} + 4*a*b^3*e^{(4*c)} + b^4*e^{(4*c)})*e^{(4*d*x)} + 2*(a^4*e^{(2*c)} + 2*a^3*b*e^{(2*c)} - 2*a*b^3*e^{(2*c)} - b^4*e^{(2*c)})*e^{(2*d*x)}), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sinh(c + dx)}{(b \tanh(c + dx)^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2)^3,x)

[Out] int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2)^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)

[Out] Timed out

$$3.45 \quad \int \frac{\operatorname{csch}(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=156

$$\frac{\tanh^{-1}(\cosh(c+dx))}{a^3 d} + \frac{b(7a+4b)\operatorname{sech}(c+dx)}{8a^2 d(a+b)^2(a-b\operatorname{sech}^2(c+dx)+b)} + \frac{\sqrt{b}(15a^2+20ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{8a^3 d(a+b)^{5/2}}$$

[Out] $-\operatorname{arctanh}(\cosh(d*x+c))/a^{3/d+1/4}*b*\operatorname{sech}(d*x+c)/a/(a+b)/d/(a+b-b*\operatorname{sech}(d*x+c)^2)^{2+1/8}*b*(7*a+4*b)*\operatorname{sech}(d*x+c)/a^2/(a+b)^2/d/(a+b-b*\operatorname{sech}(d*x+c)^2)+1/8*(15*a^2+20*a*b+8*b^2)*\operatorname{arctanh}(\operatorname{sech}(d*x+c)*b^{1/2}/(a+b)^{1/2})*b^{1/2}/a^3/(a+b)^{5/2}/d$

Rubi [A] time = 0.25, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3664, 414, 527, 522, 207, 208}

$$\frac{\sqrt{b}(15a^2+20ab+8b^2)\tanh^{-1}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{8a^3 d(a+b)^{5/2}} + \frac{b(7a+4b)\operatorname{sech}(c+dx)}{8a^2 d(a+b)^2(a-b\operatorname{sech}^2(c+dx)+b)} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^3 d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2)^3, x]

[Out] $-(\operatorname{ArcTanh}[\operatorname{Cosh}[c+d*x]]/(a^3*d)) + (\operatorname{Sqrt}[b]*(15*a^2+20*a*b+8*b^2)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Sech}[c+d*x])/(\operatorname{Sqrt}[a+b])]/(8*a^3*(a+b)^{5/2}*d) + (b*\operatorname{Sech}[c+d*x])/(4*a*(a+b)*d*(a+b-b*\operatorname{Sech}[c+d*x]^2)^2) + (b*(7*a+4*b)*\operatorname{Sech}[c+d*x])/(8*a^2*(a+b)^2*d*(a+b-b*\operatorname{Sech}[c+d*x]^2))$

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3664

```
Int[sin[(e_) + (f_)*(x_)]^(m_)*((a_) + (b_)*tan[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p]/x^(m + 1), x], x, Sec[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c + dx)}{(a + b \tanh^2(c + dx))^3} dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(-1+x^2)(a+b-bx^2)^3} dx, x, \operatorname{sech}(c + dx)\right)}{d}$$

$$= \frac{b \operatorname{sech}(c + dx)}{4a(a + b)d (a + b - b \operatorname{sech}^2(c + dx))^2} + \frac{\operatorname{Subst}\left(\int \frac{4a+b+3bx^2}{(-1+x^2)(a+b-bx^2)^2} dx, x, \operatorname{sech}(c + dx)\right)}{4a(a + b)d}$$

$$= \frac{b \operatorname{sech}(c + dx)}{4a(a + b)d (a + b - b \operatorname{sech}^2(c + dx))^2} + \frac{b(7a + 4b) \operatorname{sech}(c + dx)}{8a^2(a + b)^2d (a + b - b \operatorname{sech}^2(c + dx))} + \dots$$

$$= \frac{b \operatorname{sech}(c + dx)}{4a(a + b)d (a + b - b \operatorname{sech}^2(c + dx))^2} + \frac{b(7a + 4b) \operatorname{sech}(c + dx)}{8a^2(a + b)^2d (a + b - b \operatorname{sech}^2(c + dx))} + \dots$$

$$= -\frac{\tanh^{-1}(\cosh(c + dx))}{a^3d} + \frac{\sqrt{b} (15a^2 + 20ab + 8b^2) \tanh^{-1}\left(\frac{\sqrt{b} \operatorname{sech}(c + dx)}{\sqrt{a + b}}\right)}{8a^3(a + b)^{5/2}d} + \frac{1}{4a(a + b)}$$

Mathematica [C] time = 1.51, size = 236, normalized size = 1.51

$$\frac{8a^2b^2 \cosh(c + dx)}{(a + b)^2((a + b) \cosh(2(c + dx)) + a - b)^2} + \frac{i\sqrt{b} (15a^2 + 20ab + 8b^2) \tan^{-1}\left(\frac{-\sqrt{a} \tanh\left(\frac{1}{2}(c + dx)\right) - i\sqrt{a + b}}{\sqrt{b}}\right)}{(a + b)^{5/2}} + \frac{i\sqrt{b} (15a^2 + 20ab + 8b^2) \tan^{-1}\left(\frac{\sqrt{a} \tanh\left(\frac{1}{2}(c + dx)\right) - i\sqrt{a + b}}{\sqrt{b}}\right)}{(a + b)^{5/2}}$$

$$8a^3d$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2)^3, x]
[Out] ((I*Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])/(a + b)^(5/2) + (I*Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])/(a + b)^(5/2) + (8*a^2*b^2*Cosh[c + d*x])/((a + b)^2*(a - b + (a + b)*Cosh[2*(c + d*x)])^2) + (2*a*b*(9*a + 4*b)*Cosh[c + d*x])/((a + b)^2*(a - b + (a + b)*Cosh[2*(c + d*x)])) + 8*Log[Tanh[(c + d*x)/2]]/(8*a^3*d)
```

fricas [B] time = 1.77, size = 10716, normalized size = 68.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(4*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^7 + 28*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*sinh(d*x + c)^7 + 4*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^5 + 4*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3 + 21*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^3 + (27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^3 + 4*(35*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^4 + 27*a^3*b + 11*a^2*b^2 - 4*a*b^3 + 10*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(21*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^5 + 10*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))^sinh(d*x + c)^2 + ((15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^8 + 8*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*sinh(d*x + c)^8 + 4*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c)^6 + 4*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4 + 7*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^3 + 3*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(45*a^4 + 30*a^3*b + 29*a^2*b^2 + 44*a*b^3 + 24*b^4)*cosh(d*x + c)^4 + 2*(35*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^4 + 45*a^4 + 30*a^3*b + 29*a^2*b^2 + 44*a*b^3 + 24*b^4 + 30*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4 + 8*(7*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^5 + 10*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c)^3 + (45*a^4 + 30*a^3*b + 29*a^2*b^2 + 44*a*b^3 + 24*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c)^2 + 4*(7*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^6 + 15*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c)^4 + 15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4 + 3*(45*a^4 + 30*a^3*b + 29*a^2*b^2 + 44*a*b^3 + 24*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^7 + 3*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c)^5 + (45*a^4 + 30*a^3*b + 29*a^2*b^2 + 44*a*b^3 + 24*b^4)*cosh(d*x + c)^3 + (15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a + b))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 4*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*cosh(d*x + c) - 16*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^8 + 8*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sinh(d*x + c)^8 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4 + 7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*c

$$\begin{aligned}
& \text{osh}(d*x + c)^3 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c)*\sinh(d*x \\
& + c)^5 + 2*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*\text{cosh}(d*x + c)^4 \\
& + 2*(35*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\text{cosh}(d*x + c)^4 + 3*a^4 \\
& + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4 + 30*(a^4 + 2*a^3*b - 2*a*b^3 - b^4) \\
& *\text{cosh}(d*x + c)^2)*\sinh(d*x + c)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + \\
& b^4 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\text{cosh}(d*x + c)^5 + 1 \\
& 0*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c)^3 + (3*a^4 + 4*a^3*b + 2*a^2 \\
& *b^2 + 4*a*b^3 + 3*b^4)*\text{cosh}(d*x + c))*\sinh(d*x + c)^3 + 4*(a^4 + 2*a^3*b \\
& - 2*a*b^3 - b^4)*\text{cosh}(d*x + c)^2 + 4*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 \\
& + b^4)*\text{cosh}(d*x + c)^6 + 15*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c) \\
& ^4 + a^4 + 2*a^3*b - 2*a*b^3 - b^4 + 3*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b \\
& ^3 + 3*b^4)*\text{cosh}(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((a^4 + 4*a^3*b + 6*a^2*b^2 \\
& + 4*a*b^3 + b^4)*\text{cosh}(d*x + c)^7 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh} \\
& (d*x + c)^5 + (3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*\text{cosh}(d*x + c) \\
& ^3 + (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c))*\sinh(d*x + c))*\log(\text{cosh} \\
& (d*x + c) + \sinh(d*x + c) + 1) + 16*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + \\
& b^4)*\text{cosh}(d*x + c)^8 + 8*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\text{cosh} \\
& (d*x + c)*\sinh(d*x + c)^7 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\sinh \\
& (d*x + c)^8 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c)^6 + 4*(a^4 + \\
& 2*a^3*b - 2*a*b^3 - b^4 + 7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\cos \\
& h(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + \\
& b^4)*\text{cosh}(d*x + c)^3 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c))*\si \\
& nh(d*x + c)^5 + 2*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*\text{cosh}(d*x \\
& + c)^4 + 2*(35*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\text{cosh}(d*x + c)^4 \\
& + 3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4 + 30*(a^4 + 2*a^3*b - 2*a*b \\
& ^3 - b^4)*\text{cosh}(d*x + c)^2)*\sinh(d*x + c)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4* \\
& a*b^3 + b^4 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\text{cosh}(d*x + c) \\
&)^5 + 10*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c)^3 + (3*a^4 + 4*a^3*b \\
& + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*\text{cosh}(d*x + c))*\sinh(d*x + c)^3 + 4*(a^4 + 2 \\
& *a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c)^2 + 4*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + \\
& 4*a*b^3 + b^4)*\text{cosh}(d*x + c)^6 + 15*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d \\
& *x + c)^4 + a^4 + 2*a^3*b - 2*a*b^3 - b^4 + 3*(3*a^4 + 4*a^3*b + 2*a^2*b^2 \\
& + 4*a*b^3 + 3*b^4)*\text{cosh}(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((a^4 + 4*a^3*b + 6 \\
& *a^2*b^2 + 4*a*b^3 + b^4)*\text{cosh}(d*x + c)^7 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4) \\
& *\text{cosh}(d*x + c)^5 + (3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*\text{cosh}(d \\
& *x + c)^3 + (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c))*\sinh(d*x + c))*\l \\
& og(\text{cosh}(d*x + c) + \sinh(d*x + c) - 1) + 4*(7*(9*a^3*b + 13*a^2*b^2 + 4*a*b^ \\
& 3)*\text{cosh}(d*x + c)^6 + 5*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*\text{cosh}(d*x + c)^4 + \\
& 9*a^3*b + 13*a^2*b^2 + 4*a*b^3 + 3*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*\text{cosh}(d \\
& *x + c)^2)*\sinh(d*x + c))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4) \\
&)*d*\text{cosh}(d*x + c)^8 + 8*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d \\
& *\text{cosh}(d*x + c)*\sinh(d*x + c)^7 + (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a \\
& ^3*b^4)*d*\sinh(d*x + c)^8 + 4*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*\text{cosh} \\
& (d*x + c)^6 + 4*(7*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\text{cosh} \\
& (d*x + c)^2 + (a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d)*\sinh(d*x + c)^6 + 2*(\\
& 3*a^7 + 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)*d*\text{cosh}(d*x + c)^4 + 8* \\
& (7*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\text{cosh}(d*x + c)^3 + 3* \\
& (a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*\text{cosh}(d*x + c))*\sinh(d*x + c)^5 + 2* \\
& (35*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\text{cosh}(d*x + c)^4 + 3 \\
& 0*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*\text{cosh}(d*x + c)^2 + (3*a^7 + 4*a^6*b \\
& + 2*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)*d)*\sinh(d*x + c)^4 + 4*(a^7 + 2*a^6*b \\
& b - 2*a^4*b^3 - a^3*b^4)*d*\text{cosh}(d*x + c)^2 + 8*(7*(a^7 + 4*a^6*b + 6*a^5*b^ \\
& 2 + 4*a^4*b^3 + a^3*b^4)*d*\text{cosh}(d*x + c)^5 + 10*(a^7 + 2*a^6*b - 2*a^4*b^3 \\
& - a^3*b^4)*d*\text{cosh}(d*x + c)^3 + (3*a^7 + 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^3 + 3 \\
& *a^3*b^4)*d*\text{cosh}(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^7 + 4*a^6*b + 6*a^5*b^ \\
& 2 + 4*a^4*b^3 + a^3*b^4)*d*\text{cosh}(d*x + c)^6 + 15*(a^7 + 2*a^6*b - 2*a^4*b^3 \\
& - a^3*b^4)*d*\text{cosh}(d*x + c)^4 + 3*(3*a^7 + 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^3 + \\
& 3*a^3*b^4)*d*\text{cosh}(d*x + c)^2 + (a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d)*\si \\
& nh(d*x + c)^2 + (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d + 8*((a
\end{aligned}$$

$$\begin{aligned}
& ^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^7 + 3*(a^7 \\
& + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^5 + (3*a^7 + 4*a^6*b + 2*a \\
& ^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^3 + (a^7 + 2*a^6*b - 2*a^4* \\
& b^3 - a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(2*(9*a^3*b + 13*a^2*b^ \\
& 2 + 4*a*b^3)*cosh(d*x + c)^7 + 14*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*cosh(d*x \\
& + c)*sinh(d*x + c)^6 + 2*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*sinh(d*x + c)^7 \\
& + 2*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^5 + 2*(27*a^3*b + 11*a^ \\
& 2*b^2 - 4*a*b^3 + 21*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^2)*sinh \\
& (d*x + c)^5 + 10*(7*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^3 + (27* \\
& a^3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 2*(27*a^3*b \\
& + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^3 + 2*(35*(9*a^3*b + 13*a^2*b^2 + 4*a \\
& *b^3)*cosh(d*x + c)^4 + 27*a^3*b + 11*a^2*b^2 - 4*a*b^3 + 10*(27*a^3*b + 11 \\
& *a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(21*(9*a^3*b + 13* \\
& a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^5 + 10*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*c \\
& osh(d*x + c)^3 + 3*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*cosh(d*x + c))*sinh(d* \\
& x + c)^2 + ((15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + \\
& c)^8 + 8*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)* \\
& sinh(d*x + c)^7 + (15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*sinh(\\
& d*x + c)^8 + 4*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x \\
& + c)^6 + 4*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4 + 7*(15*a^4 + \\
& 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 \\
& + 8*(7*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^3 \\
& + 3*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(\\
& d*x + c)^5 + 2*(45*a^4 + 30*a^3*b + 29*a^2*b^2 + 44*a*b^3 + 24*b^4)*cosh(d* \\
& x + c)^4 + 2*(35*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d \\
& *x + c)^4 + 45*a^4 + 30*a^3*b + 29*a^2*b^2 + 44*a*b^3 + 24*b^4 + 30*(15*a^4 \\
& + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^ \\
& 4 + 15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4 + 8*(7*(15*a^4 + 50*a \\
& ^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^5 + 10*(15*a^4 + 20*a^3 \\
& *b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c)^3 + (45*a^4 + 30*a^3*b + 2 \\
& 9*a^2*b^2 + 44*a*b^3 + 24*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^4 + \\
& 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c)^2 + 4*(7*(15*a^4 + \\
& 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^6 + 15*(15*a^4 + 20 \\
& *a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c)^4 + 15*a^4 + 20*a^3*b \\
& - 7*a^2*b^2 - 20*a*b^3 - 8*b^4 + 3*(45*a^4 + 30*a^3*b + 29*a^2*b^2 + 44*a*b \\
& ^3 + 24*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((15*a^4 + 50*a^3*b + 63* \\
& a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^7 + 3*(15*a^4 + 20*a^3*b - 7*a^2* \\
& b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c)^5 + (45*a^4 + 30*a^3*b + 29*a^2*b^2 + \\
& 44*a*b^3 + 24*b^4)*cosh(d*x + c)^3 + (15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a \\
& *b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a + b))*arctan(1/2*((a \\
& + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*s \\
& inh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a - \\
& 3*b)*sinh(d*x + c))*sqrt(-b/(a + b))/b) - ((15*a^4 + 50*a^3*b + 63*a^2*b^2 \\
& + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^8 + 8*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + \\
& 36*a*b^3 + 8*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (15*a^4 + 50*a^3*b + 63*a \\
& ^2*b^2 + 36*a*b^3 + 8*b^4)*sinh(d*x + c)^8 + 4*(15*a^4 + 20*a^3*b - 7*a^2*b \\
& ^2 - 20*a*b^3 - 8*b^4)*cosh(d*x + c)^6 + 4*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - \\
& 20*a*b^3 - 8*b^4 + 7*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*c \\
& osh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36 \\
& *a*b^3 + 8*b^4)*cosh(d*x + c)^3 + 3*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b \\
& ^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(45*a^4 + 30*a^3*b + 29*a^2* \\
& b^2 + 44*a*b^3 + 24*b^4)*cosh(d*x + c)^4 + 2*(35*(15*a^4 + 50*a^3*b + 63*a^ \\
& 2*b^2 + 36*a*b^3 + 8*b^4)*cosh(d*x + c)^4 + 45*a^4 + 30*a^3*b + 29*a^2*b^2 \\
& + 44*a*b^3 + 24*b^4 + 30*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4) \\
& *cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b \\
& ^3 + 8*b^4 + 8*(7*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*cosh(\\
& d*x + c)^5 + 10*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*cosh(d*x \\
& + c)^3 + (45*a^4 + 30*a^3*b + 29*a^2*b^2 + 44*a*b^3 + 24*b^4)*cosh(d*x + c \\
&))*sinh(d*x + c)^3 + 4*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*c
\end{aligned}$$

$$\begin{aligned}
& \text{osh}(d*x + c)^2 + 4*(7*(15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*\text{cosh}(d*x + c)^6 + 15*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*\text{cosh}(d*x + c)^4 + 15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4 + 3*(45*a^4 + 30*a^3*b + 29*a^2*b^2 + 44*a*b^3 + 24*b^4)*\text{cosh}(d*x + c)^2)*\text{sinh}(d*x + c)^2 + 8*((15*a^4 + 50*a^3*b + 63*a^2*b^2 + 36*a*b^3 + 8*b^4)*\text{cosh}(d*x + c)^7 + 3*(15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*\text{cosh}(d*x + c)^5 + (45*a^4 + 30*a^3*b + 29*a^2*b^2 + 44*a*b^3 + 24*b^4)*\text{cosh}(d*x + c)^3 + (15*a^4 + 20*a^3*b - 7*a^2*b^2 - 20*a*b^3 - 8*b^4)*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)) * \text{sqrt}(-b/(a + b)) * \arctan(1/2*((a + b)*\text{cosh}(d*x + c) + (a + b)*\text{sinh}(d*x + c)) * \text{sqrt}(-b/(a + b))/b) + 2*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*\text{cosh}(d*x + c) - 8*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\text{cosh}(d*x + c)^8 + 8*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^7 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\text{sinh}(d*x + c)^8 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c)^6 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4 + 7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\text{cosh}(d*x + c)^2)*\text{sinh}(d*x + c)^6 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\text{cosh}(d*x + c)^3 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)^5 + 2*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*\text{cosh}(d*x + c)^4 + 2*(35*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\text{cosh}(d*x + c)^4 + 3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4 + 30*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c)^2)*\text{sinh}(d*x + c)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\text{cosh}(d*x + c)^5 + 10*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c)^3 + (3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)^3 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c)^2 + 4*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\text{cosh}(d*x + c)^6 + 15*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c)^4 + a^4 + 2*a^3*b - 2*a*b^3 - b^4 + 3*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*\text{cosh}(d*x + c)^2)*\text{sinh}(d*x + c)^2 + 8*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\text{cosh}(d*x + c)^7 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c)^5 + (3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*\text{cosh}(d*x + c)^3 + (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)) * \log(\cosh(d*x + c) + \sinh(d*x + c) + 1) + 8*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\text{cosh}(d*x + c)^8 + 8*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^7 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\text{sinh}(d*x + c)^8 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c)^6 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4 + 7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\text{cosh}(d*x + c)^2)*\text{sinh}(d*x + c)^6 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\text{cosh}(d*x + c)^3 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)^5 + 2*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*\text{cosh}(d*x + c)^4 + 2*(35*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\text{cosh}(d*x + c)^4 + 3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4 + 30*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c)^2)*\text{sinh}(d*x + c)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\text{cosh}(d*x + c)^5 + 10*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c)^3 + (3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)^3 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c)^2 + 4*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\text{cosh}(d*x + c)^6 + 15*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c)^4 + a^4 + 2*a^3*b - 2*a*b^3 - b^4 + 3*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*\text{cosh}(d*x + c)^2)*\text{sinh}(d*x + c)^2 + 8*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\text{cosh}(d*x + c)^7 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c)^5 + (3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*\text{cosh}(d*x + c)^3 + (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)) * \log(\cosh(d*x + c) + \sinh(d*x + c) - 1) + 2*(7*(9*a^3*b + 13*a^2*b^2 + 4*a*b^3)*\text{cosh}(d*x + c)^6 + 5*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*\text{cosh}(d*x + c)^4 + 9*a^3*b + 13*a^2*b^2 + 4*a*b^3 + 3*(27*a^3*b + 11*a^2*b^2 - 4*a*b^3)*\text{cosh}(d*x + c)^2)*\text{sinh}(d*x + c))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\text{cosh}(d*x + c)^8 + 8*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^7 + (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\text{sinh}(d*x + c)^8 + 4*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*\text{cosh}(d*x + c)^6 + 4*(7*(a^7 + 4*a
\end{aligned}$$

$$\begin{aligned} & ^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4) * d * \cosh(dx + c)^2 + (a^7 + 2a^6b \\ & - 2a^4b^3 - a^3b^4) * d) * \sinh(dx + c)^6 + 2 * (3a^7 + 4a^6b + 2a^5b^2 \\ & + 4a^4b^3 + 3a^3b^4) * d * \cosh(dx + c)^4 + 8 * (7 * (a^7 + 4a^6b + 6a^5b^2 \\ & + 4a^4b^3 + a^3b^4) * d * \cosh(dx + c)^3 + 3 * (a^7 + 2a^6b - 2a^4b^3 - \\ & a^3b^4) * d * \cosh(dx + c)) * \sinh(dx + c)^5 + 2 * (35 * (a^7 + 4a^6b + 6a^5b^2 \\ & + 4a^4b^3 + a^3b^4) * d * \cosh(dx + c)^4 + 30 * (a^7 + 2a^6b - 2a^4b^3 \\ & - a^3b^4) * d * \cosh(dx + c)^2 + (3a^7 + 4a^6b + 2a^5b^2 + 4a^4b^3 + \\ & 3a^3b^4) * d) * \sinh(dx + c)^4 + 4 * (a^7 + 2a^6b - 2a^4b^3 - a^3b^4) * d * \c \\ & \cosh(dx + c)^2 + 8 * (7 * (a^7 + 4a^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4) * d * \c \\ & \cosh(dx + c)^5 + 10 * (a^7 + 2a^6b - 2a^4b^3 - a^3b^4) * d * \cosh(dx + c)^3 \\ & + (3a^7 + 4a^6b + 2a^5b^2 + 4a^4b^3 + 3a^3b^4) * d * \cosh(dx + c)) * \s \\ & \sinh(dx + c)^3 + 4 * (7 * (a^7 + 4a^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4) * d * \c \\ & \cosh(dx + c)^6 + 15 * (a^7 + 2a^6b - 2a^4b^3 - a^3b^4) * d * \cosh(dx + c)^4 \\ & + 3 * (3a^7 + 4a^6b + 2a^5b^2 + 4a^4b^3 + 3a^3b^4) * d * \cosh(dx + c)^2 \\ & + (a^7 + 2a^6b - 2a^4b^3 - a^3b^4) * d) * \sinh(dx + c)^2 + (a^7 + 4a^6 \\ & * b + 6a^5b^2 + 4a^4b^3 + a^3b^4) * d + 8 * ((a^7 + 4a^6b + 6a^5b^2 + 4 \\ & * a^4b^3 + a^3b^4) * d * \cosh(dx + c)^7 + 3 * (a^7 + 2a^6b - 2a^4b^3 - a^3 \\ & b^4) * d * \cosh(dx + c)^5 + (3a^7 + 4a^6b + 2a^5b^2 + 4a^4b^3 + 3a^3b^4) \\ & * d * \cosh(dx + c)^3 + (a^7 + 2a^6b - 2a^4b^3 - a^3b^4) * d * \cosh(dx + \\ & c)) * \sinh(dx + c))] \end{aligned}$$

giac [B] time = 0.50, size = 2033, normalized size = 13.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)/(a+b*tanh(dx+c)^2)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/8 * ((2 * (a^5 * e^{(2*c)} + 2 * a^4 * b * e^{(2*c)} + a^3 * b^2 * e^{(2*c)})^2 * (30 * a^3 * b^2 + \\ & 40 * a^2 * b^3 + 16 * a * b^4 - (15 * a^3 * b + 5 * a^2 * b^2 - 12 * a * b^3 - 8 * b^4) * \sqrt{-a * b} \\ &)) * \sqrt{a^2 - b^2 + 2 * \sqrt{-a * b} * (a + b)} * \operatorname{abs}(a * e^{(2*c)} + b * e^{(2*c)}) - (15 * \\ & a^9 * b + 50 * a^8 * b^2 + 48 * a^7 * b^3 - 14 * a^6 * b^4 - 55 * a^5 * b^5 - 36 * a^4 * b^6 - 8 * \\ & a^3 * b^7 + 2 * (15 * a^8 * b + 65 * a^7 * b^2 + 113 * a^6 * b^3 + 99 * a^5 * b^4 + 44 * a^4 * b^5 \\ & + 8 * a^3 * b^6) * \sqrt{-a * b}) * \sqrt{a^2 - b^2 + 2 * \sqrt{-a * b} * (a + b)} * \operatorname{abs}(a^5 * e^{(\\ & 2*c)} + 2 * a^4 * b * e^{(2*c)} + a^3 * b^2 * e^{(2*c)}) * \operatorname{abs}(a * e^{(2*c)} + b * e^{(2*c)}) * e^{(2*c)} \\ &) + (30 * a^{14} * b + 130 * a^{13} * b^2 + 196 * a^{12} * b^3 + 68 * a^{11} * b^4 - 138 * a^{10} * b^5 - \\ & 182 * a^9 * b^6 - 88 * a^8 * b^7 - 16 * a^7 * b^8 - (15 * a^{14} + 50 * a^{13} * b + 33 * a^{12} * b^2 \\ & - 64 * a^{11} * b^3 - 103 * a^{10} * b^4 - 22 * a^9 * b^5 + 47 * a^8 * b^6 + 36 * a^7 * b^7 + 8 * a^6 \\ & * b^8) * \sqrt{-a * b}) * \sqrt{a^2 - b^2 + 2 * \sqrt{-a * b} * (a + b)} * \operatorname{abs}(a * e^{(2*c)} + b \\ & * e^{(2*c)}) * e^{(4*c)} * \arctan(e^{(d*x)} / \sqrt{(a^6 * e^{(2*c)} + a^5 * b * e^{(2*c)} - a^4 * b^2 * \\ & e^{(2*c)} - a^3 * b^3 * e^{(2*c)})^2 - (a^6 * e^{(4*c)} + 3 * a^5 * b * e^{(4*c)} + 3 * a^4 * b^2 * e^{ \\ & (4*c)} + a^3 * b^3 * e^{(4*c)}) * (a^6 + 3 * a^5 * b + 3 * a^4 * b^2 + a^3 * b^3)})) / (a^6 * e^{(4 \\ & *c)} + 3 * a^5 * b * e^{(4*c)} + 3 * a^4 * b^2 * e^{(4*c)} + a^3 * b^3 * e^{(4*c)})) * e^{(-4*c)} / ((a \\ & ^{15} + 7 * a^{14} * b + 20 * a^{13} * b^2 + 28 * a^{12} * b^3 + 14 * a^{11} * b^4 - 14 * a^{10} * b^5 - 28 \\ & * a^9 * b^6 - 20 * a^8 * b^7 - 7 * a^7 * b^8 - a^6 * b^9 + 2 * (a^{14} + 8 * a^{13} * b + 28 * a^{12} * \\ & b^2 + 56 * a^{11} * b^3 + 70 * a^{10} * b^4 + 56 * a^9 * b^5 + 28 * a^8 * b^6 + 8 * a^7 * b^7 + a^6 \\ & * b^8) * \sqrt{-a * b}) * \operatorname{abs}(a^5 * e^{(2*c)} + 2 * a^4 * b * e^{(2*c)} + a^3 * b^2 * e^{(2*c)})) + (\\ & 2 * (60 * a^4 * b^2 + 20 * a^3 * b^3 - 48 * a^2 * b^4 - 32 * a * b^5 + (15 * a^4 * b - 70 * a^3 * b^2 \\ & - 97 * a^2 * b^3 - 28 * a * b^4 + 8 * b^5) * \sqrt{-a * b})) * (a^5 * e^{(2*c)} + 2 * a^4 * b * e^{(2*c)} \\ &) + a^3 * b^2 * e^{(2*c)})^2 * \operatorname{abs}(a * e^{(2*c)} + b * e^{(2*c)}) - (15 * a^{10} * b - 25 * a^9 * b^2 \\ & - 262 * a^8 * b^3 - 514 * a^7 * b^4 - 437 * a^6 * b^5 - 157 * a^5 * b^6 - 4 * a^4 * b^7 + 8 * a^3 \\ & * b^8 - 4 * (15 * a^9 * b + 50 * a^8 * b^2 + 48 * a^7 * b^3 - 14 * a^6 * b^4 - 55 * a^5 * b^5 - 3 \\ & 6 * a^4 * b^6 - 8 * a^3 * b^7) * \sqrt{-a * b}) * \operatorname{abs}(a^5 * e^{(2*c)} + 2 * a^4 * b * e^{(2*c)} + a^3 * \\ & b^2 * e^{(2*c)}) * \operatorname{abs}(a * e^{(2*c)} + b * e^{(2*c)}) * e^{(2*c)} + (60 * a^{15} * b + 200 * a^{14} * b^2 \\ & + 132 * a^{13} * b^3 - 256 * a^{12} * b^4 - 412 * a^{11} * b^5 - 88 * a^{10} * b^6 + 188 * a^9 * b^7 + \\ & 144 * a^8 * b^8 + 32 * a^7 * b^9 + (15 * a^{15} - 25 * a^{14} * b - 277 * a^{13} * b^2 - 489 * a^{12} * \\ & b^3 - 175 * a^{11} * b^4 + 357 * a^{10} * b^5 + 433 * a^9 * b^6 + 165 * a^8 * b^7 + 4 * a^7 * b^8 - \\ & 8 * a^6 * b^9) * \sqrt{-a * b}) * \operatorname{abs}(a * e^{(2*c)} + b * e^{(2*c)}) * e^{(4*c)} * \arctan(e^{(d*x)} / \\ & \sqrt{(a^6 * e^{(2*c)} + a^5 * b * e^{(2*c)} - a^4 * b^2 * e^{(2*c)} - a^3 * b^3 * e^{(2*c)} - \operatorname{sqr} \end{aligned}$$

$$\frac{t((a^6e^{(2c)} + a^5b^2e^{(2c)} - a^4b^2e^{(2c)} - a^3b^3e^{(2c)})^2 - (a^6e^{(4c)} + 3a^5b^2e^{(4c)} + 3a^4b^2e^{(4c)} + a^3b^3e^{(4c)})(a^6 + 3a^5b + 3a^4b^2 + a^3b^3))}{(a^6e^{(4c)} + 3a^5b^2e^{(4c)} + 3a^4b^2e^{(4c)} + a^3b^3e^{(4c)})}e^{(-4c)} / ((a^{14} + 6a^{13}b + 14a^{12}b^2 + 14a^{11}b^3 - 14a^9b^5 - 14a^8b^6 - 6a^7b^7 - a^6b^8 - 2(a^{13} + 7a^{12}b + 21a^{11}b^2 + 35a^{10}b^3 + 35a^9b^4 + 21a^8b^5 + 7a^7b^6 + a^6b^7) \sqrt{-ab}) \sqrt{a^2 - b^2 - 2\sqrt{-ab}(a+b)} \operatorname{abs}(a^5e^{(2c)} + 2a^4b^2e^{(2c)} + a^3b^2e^{(2c)})) - 2(9a^2b^2e^{(7d*x+7c)} + 13a^2b^2e^{(7d*x+7c)} + 4b^3e^{(7d*x+7c)} + 27a^2b^2e^{(5d*x+5c)} + 11a^2b^2e^{(5d*x+5c)} - 4b^3e^{(5d*x+5c)} + 27a^2b^2e^{(3d*x+3c)} + 11a^2b^2e^{(3d*x+3c)} - 4b^3e^{(3d*x+3c)} + 9a^2b^2e^{(d*x+c)} + 13a^2b^2e^{(d*x+c)} + 4b^3e^{(d*x+c)}) / ((a^4 + 2a^3b + a^2b^2)(ae^{(4d*x+4c)} + be^{(4d*x+4c)} + 2ae^{(2d*x+2c)} - 2be^{(2d*x+2c)} + a+b)^2) + 8\log(e^{(d*x+c)} + 1)/a^3 - 8\log(\operatorname{abs}(e^{(d*x+c)} - 1))/a^3)/d$$

maple [B] time = 0.45, size = 1132, normalized size = 7.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)/(a+b*tanh(d*x+c))^2)^3,x`

[Out] $\frac{9}{4} \frac{d}{b} \frac{(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{4a+2} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2a+4} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2b+a})^2}{(a^2+2a*b+b^2) \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^6} + \frac{7}{d} \frac{a*b^2}{(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{4a+2} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2a+4} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2b+a})^2} + \frac{2}{(a^2+2a*b+b^2) \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^6} + \frac{4}{d} \frac{a^2b^3}{(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{4a+2} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2a+4} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2b+a})^2} + \frac{2}{(a^2+2a*b+b^2) \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^6} + \frac{27}{4} \frac{d}{b} \frac{(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{4a+2} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2a+4} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2b+a})^2}{(a^2+2a*b+b^2) \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^6} + \frac{45}{2} \frac{d}{a*b^2} \frac{(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{4a+2} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2a+4} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2b+a})^2}{(a^2+2a*b+b^2) \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^6} + \frac{30}{d} \frac{a^2b^3}{(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{4a+2} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2a+4} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2b+a})^2} + \frac{12}{d} \frac{a^3b^4}{(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{4a+2} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2a+4} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2b+a})^2} + \frac{27}{4} \frac{d}{b} \frac{(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{4a+2} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2a+4} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2b+a})^2}{(a^2+2a*b+b^2) \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^6} + \frac{17}{d} \frac{a*b^2}{(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{4a+2} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2a+4} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2b+a})^2} + \frac{8}{d} \frac{a^2b^3}{(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{4a+2} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2a+4} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2b+a})^2} + \frac{9}{4} \frac{d}{b} \frac{(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{4a+2} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2a+4} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2b+a})^2}{(a^2+2a*b+b^2) \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^6} + \frac{3}{2} \frac{d}{a*b^2} \frac{(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{4a+2} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2a+4} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2b+a})^2}{(a^2+2a*b+b^2) \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^6} + \frac{15}{8} \frac{d}{a*b} \frac{(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{4a+2} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2a+4} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2b+a})^2}{(a^2+2a*b+b^2) \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^6} + \frac{5}{2} \frac{d}{a^2b^2} \frac{(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{4a+2} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2a+4} \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^{2b+a})^2}{(a^2+2a*b+b^2) \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^6} + \frac{1}{d} \frac{a^3b^3}{(a^2+2a*b+b^2) \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^6} + \frac{1}{d} \frac{a^3 \ln(\tanh(\frac{1}{2}d*x + \frac{1}{2}c))}{(a^2+2a*b+b^2) \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^6}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$(9a^2be^{(7c)} + 13ab^2e^{(7c)} + 4b^3e^{(7c)})e^{(7c)}$$

$$4(a^6d + 4a^5bd + 6a^4b^2d + 4a^3b^3d + a^2b^4d + (a^6de^{(8c)} + 4a^5bde^{(8c)} + 6a^4b^2de^{(8c)} + 4a^3b^3de^{(8c)} + a^2b^4de^{(8c)})e^{(8c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)/(a+b*tanh(d*x+c))^2)^3,x, algorithm="maxima"`

[Out] $\frac{1}{4} * ((9a^2b^2e^{(7c)} + 13a^2b^2e^{(7c)} + 4b^3e^{(7c)})e^{(7d*x)} + (27a^2b^2e^{(5c)} + 11a^2b^2e^{(5c)} - 4b^3e^{(5c)})e^{(5d*x)} + (27a^2b^2e^{(3c)} + 11a^2b^2e^{(3c)} - 4b^3e^{(3c)})e^{(d*x)} + (27a^2b^2e^{(c)} + 11a^2b^2e^{(c)} - 4b^3e^{(c)})e^{(0)}$

*c) + 11*a*b^2*e^(3*c) - 4*b^3*e^(3*c))*e^(3*d*x) + (9*a^2*b*e^c + 13*a*b^2*e^c + 4*b^3*e^c)*e^(d*x))/(a^6*d + 4*a^5*b*d + 6*a^4*b^2*d + 4*a^3*b^3*d + a^2*b^4*d + (a^6*d*e^(8*c) + 4*a^5*b*d*e^(8*c) + 6*a^4*b^2*d*e^(8*c) + 4*a^3*b^3*d*e^(8*c) + a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(a^6*d*e^(6*c) + 2*a^5*b*d*e^(6*c) - 2*a^3*b^3*d*e^(6*c) - a^2*b^4*d*e^(6*c))*e^(6*d*x) + 2*(3*a^6*d*e^(4*c) + 4*a^5*b*d*e^(4*c) + 2*a^4*b^2*d*e^(4*c) + 4*a^3*b^3*d*e^(4*c) + 3*a^2*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^6*d*e^(2*c) + 2*a^5*b*d*e^(2*c) - 2*a^3*b^3*d*e^(2*c) - a^2*b^4*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^3*d) + log((e^(d*x + c) - 1)*e^(-c))/(a^3*d) - 2*integrate(1/8*(15*a^2*b*e^(3*c) + 20*a*b^2*e^(3*c) + 8*b^3*e^(3*c))*e^(3*d*x) - (15*a^2*b*e^c + 20*a*b^2*e^c + 8*b^3*e^c)*e^(d*x))/(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 + (a^6*e^(4*c) + 3*a^5*b*e^(4*c) + 3*a^4*b^2*e^(4*c) + a^3*b^3*e^(4*c))*e^(4*d*x) + 2*(a^6*e^(2*c) + a^5*b*e^(2*c) - a^4*b^2*e^(2*c) - a^3*b^3*e^(2*c))*e^(2*d*x)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh(c + dx) (b \tanh(c + dx)^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^3), x)

[Out] int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(c + dx)}{(a + b \tanh^2(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)**2)**3, x)

[Out] Integral(csch(c + d*x)/(a + b*tanh(c + d*x)**2)**3, x)

$$3.46 \quad \int \frac{\operatorname{csch}^2(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=112

$$-\frac{15\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d} - \frac{15 \operatorname{coth}(c+dx)}{8a^3d} + \frac{5 \operatorname{coth}(c+dx)}{8a^2d(a+b \tanh^2(c+dx))} + \frac{\operatorname{coth}(c+dx)}{4ad(a+b \tanh^2(c+dx))^2}$$

[Out] $-15/8*\operatorname{coth}(d*x+c)/a^{3/d}-15/8*\arctan(b^{(1/2)}*\tanh(d*x+c)/a^{(1/2)})*b^{(1/2)}/a^{(7/2)/d}+1/4*\operatorname{coth}(d*x+c)/a/d/(a+b*\tanh(d*x+c)^2)^2+5/8*\operatorname{coth}(d*x+c)/a^2/d/(a+b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.09, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3663, 290, 325, 205}

$$-\frac{15\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d} + \frac{5 \operatorname{coth}(c+dx)}{8a^2d(a+b \tanh^2(c+dx))} - \frac{15 \operatorname{coth}(c+dx)}{8a^3d} + \frac{\operatorname{coth}(c+dx)}{4ad(a+b \tanh^2(c+dx))^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csch}[c + d*x]^2/(a + b*\text{Tanh}[c + d*x]^2)^3, x]$

[Out] $(-15*\text{Sqrt}[b]*\text{ArcTan}[(\text{Sqrt}[b]*\text{Tanh}[c + d*x])/(\text{Sqrt}[a])]/(8*a^{(7/2)*d}) - (15*\operatorname{coth}[c + d*x])/(8*a^3*d) + \operatorname{Coth}[c + d*x]/(4*a*d*(a + b*\text{Tanh}[c + d*x]^2)^2) + (5*\operatorname{Coth}[c + d*x])/(8*a^2*d*(a + b*\text{Tanh}[c + d*x]^2)))$

Rule 205

$\text{Int}[(a + (b*x)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /;$ $\text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{PosQ}[a/b]$

Rule 290

$\text{Int}[(c*x)^m*(a + b*x^n)^{p+1}/(a*c*n*(p+1)), x] + \text{Dist}[(m + n*(p+1) + 1)/(a*n*(p+1)), \text{Int}[(c*x)^m*(a + b*x^n)^{p+1}, x], x] /;$ $\text{FreeQ}\{a, b, c, m\}, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 325

$\text{Int}[(c*x)^m*(a + b*x^n)^{p+1}/(a*c*(m+1)), x] - \text{Dist}[(b*(m + n*(p+1) + 1))/(a*c^n*(m+1)), \text{Int}[(c*x)^{m+n}*(a + b*x^n)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, p\}, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{IntBinomialQ}[a, b, c, n, m, p, x]$

Rule 3663

$\text{Int}[(\sin[e + f*x])^m*(a + b*(\sin[e + f*x])^n)^{p+1}/(c^2 + f^2*x^2)^{m/2+1}, x], x, (c*\text{Tan}[e + f*x])/ff], x] /;$ $\text{FreeQ}\{a, b, c, e, f, n, p\}, x\} \ \&\& \ \text{IntegerQ}[m/2]$

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(c+dx)}{(a+b \tanh^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\operatorname{coth}(c+dx)}{4ad(a+b \tanh^2(c+dx))^2} + \frac{5 \operatorname{Subst}\left(\int \frac{1}{x^2(a+bx^2)^2} dx, x, \tanh(c+dx)\right)}{4ad} \\
&= \frac{\operatorname{coth}(c+dx)}{4ad(a+b \tanh^2(c+dx))^2} + \frac{5 \operatorname{coth}(c+dx)}{8a^2d(a+b \tanh^2(c+dx))} + \frac{15 \operatorname{Subst}\left(\int \frac{1}{x^2(a+bx^2)} dx, x, \tanh(c+dx)\right)}{8a^2d} \\
&= -\frac{15 \operatorname{coth}(c+dx)}{8a^3d} + \frac{\operatorname{coth}(c+dx)}{4ad(a+b \tanh^2(c+dx))^2} + \frac{5 \operatorname{coth}(c+dx)}{8a^2d(a+b \tanh^2(c+dx))} \\
&= -\frac{15\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d} - \frac{15 \operatorname{coth}(c+dx)}{8a^3d} + \frac{\operatorname{coth}(c+dx)}{4ad(a+b \tanh^2(c+dx))^2}
\end{aligned}$$

Mathematica [A] time = 1.06, size = 109, normalized size = 0.97

$$\frac{-15\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right) - \frac{\sqrt{a} b \sinh(2(c+dx))((9a+7b) \cosh(2(c+dx))+9a-7b)}{((a+b) \cosh(2(c+dx))+a-b)^2} - 8\sqrt{a} \operatorname{coth}(c+dx)}{8a^{7/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (-15*sqrt[b]*ArcTan[(sqrt[b]*Tanh[c + d*x])/sqrt[a]] - 8*sqrt[a]*Coth[c + d*x] - (sqrt[a]*b*(9*a - 7*b + (9*a + 7*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)]/(a - b + (a + b)*Cosh[2*(c + d*x)]^2)/(8*a^(7/2)*d)

fricas [B] time = 0.67, size = 8312, normalized size = 74.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [-1/16*(4*(8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*cosh(d*x + c)^8 + 32*(8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + 4*(8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*sinh(d*x + c)^8 + 8*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4)*cosh(d*x + c)^6 + 8*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4 + 14*(8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 16*(14*(8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*cosh(d*x + c)^3 + 3*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 8*(24*a^4 + 32*a^3*b + 5*a^2*b^2 + 50*a*b^3 + 45*b^4)*cosh(d*x + c)^4 + 8*(35*(8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*cosh(d*x + c)^4 + 24*a^4 + 32*a^3*b + 5*a^2*b^2 + 50*a*b^3 + 45*b^4 + 15*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4))*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 32*a^4 + 164*a^3*b + 292*a^2*b^2 + 220*a*b^3 + 60*b^4 + 32*(7*(8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*cosh(d*x + c)^5 + 5*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4)*cosh(d*x + c)^3 + (24*a^4 + 32*a^3*b + 5*a^2*b^2 + 50*a*b^3 + 45*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 8*(16*a^4 + 41*a^3*b - 55*a*b^3 - 30*b^4)*cosh(d*x + c)^2 + 8

$$\begin{aligned}
&*(14*(8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*\cosh(d*x + c)^6 + \\
&15*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4)*\cosh(d*x + c)^4 + 16*a^4 + 41*a^3*b - 55*a*b^3 - 30*b^4 + 6*(24*a^4 + 32*a^3*b + 5*a^2*b^2 + 50*a*b^3 + 45*b^4)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 - 15*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\cosh(d*x + c)^10 + 10*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\cosh(d*x + c)*\sinh(d*x + c)^9 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\sinh(d*x + c)^10 + (3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*\cosh(d*x + c)^8 + (3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4 + 45*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 8*(15*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\cosh(d*x + c)^3 + (3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*\cosh(d*x + c)^6 + 2*(105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\cosh(d*x + c)^4 + a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4 + 14*(3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 4*(63*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\cosh(d*x + c)^5 + 14*(3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*\cosh(d*x + c)^3 + 3*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*\cosh(d*x + c)^4 + 2*(105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\cosh(d*x + c)^6 + 35*(3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*\cosh(d*x + c)^4 - a^4 - 2*a^2*b^2 - 8*a*b^3 - 5*b^4 + 15*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 - a^4 - 4*a^3*b - 6*a^2*b^2 - 4*a*b^3 - b^4 + 8*(15*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\cosh(d*x + c)^7 + 7*(3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*\cosh(d*x + c)^5 + 5*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*\cosh(d*x + c)^3 - (a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 - (3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*\cosh(d*x + c)^2 + (45*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\cosh(d*x + c)^8 + 2*8*(3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*\cosh(d*x + c)^6 + 30*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*\cosh(d*x + c)^4 - 3*a^4 - 4*a^3*b + 6*a^2*b^2 + 12*a*b^3 + 5*b^4 - 12*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 2*(5*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\cosh(d*x + c)^9 + 4*(3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*\cosh(d*x + c)^7 + 6*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*\cosh(d*x + c)^5 - 4*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*\cosh(d*x + c)^3 - (3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-b/a}*\log(((a^2 + 2*a*b + b^2)*\cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*\sinh(d*x + c)^4 + 2*(a^2 - b^2)*\cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)^2 + a^2 - b^2)*\sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*\cosh(d*x + c)^3 + (a^2 - b^2)*\cosh(d*x + c))*\sinh(d*x + c) - 4*((a^2 + a*b)*\cosh(d*x + c)^2 + 2*(a^2 + a*b)*\cosh(d*x + c)*\sinh(d*x + c) + (a^2 + a*b)*\sinh(d*x + c)^2 + a^2 - a*b)*\sqrt{-b/a})/((a + b)*\cosh(d*x + c)^4 + 4*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a + b)*\sinh(d*x + c)^4 + 2*(a - b)*\cosh(d*x + c)^2 + 2*(3*(a + b)*\cosh(d*x + c)^2 + a - b)*\sinh(d*x + c)^2 + 4*((a + b)*\cosh(d*x + c)^3 + (a - b)*\cosh(d*x + c))*\sinh(d*x + c) + a + b)) + 16*(2*(8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*\cosh(d*x + c)^7 + 3*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4)*\cosh(d*x + c)^5 + 2*(24*a^4 + 32*a^3*b + 5*a^2*b^2 + 50*a*b^3 + 45*b^4)*\cosh(d*x + c)^3 + (16*a^4 + 41*a^3*b - 55*a*b^3 - 30*b^4)*\cosh(d*x + c))*\sinh(d*x + c))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^10 + 10*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)*\sinh(d*x + c)^9 + (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\sinh(d*x + c)^10 + (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d*\cosh(d*x + c)^8 + (45*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^2 + (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d)*\sinh(d*x + c)^8 + 2*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d*\cosh(d*x + c)^6 + 8*(15*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^3 + (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(105*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^4 + 14*(3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3
\end{aligned}$$

$$\begin{aligned}
& 3 - 5a^3b^4) * d * \cosh(dx + c)^2 + (a^7 + 2a^5b^2 + 8a^4b^3 + 5a^3b^4) * d * \sinh(dx + c)^6 - 2 * (a^7 + 2a^5b^2 + 8a^4b^3 + 5a^3b^4) * d * \cosh(dx + c)^4 + 4 * (63 * (a^7 + 4a^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4) * d * \cosh(dx + c)^5 + 14 * (3a^7 + 4a^6b - 6a^5b^2 - 12a^4b^3 - 5a^3b^4) * d * \cosh(dx + c)^3 + 3 * (a^7 + 2a^5b^2 + 8a^4b^3 + 5a^3b^4) * d * \cosh(dx + c) * \sinh(dx + c)^5 + 2 * (105 * (a^7 + 4a^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4) * d * \cosh(dx + c)^6 + 35 * (3a^7 + 4a^6b - 6a^5b^2 - 12a^4b^3 - 5a^3b^4) * d * \cosh(dx + c)^4 + 15 * (a^7 + 2a^5b^2 + 8a^4b^3 + 5a^3b^4) * d * \cosh(dx + c)^2 - (a^7 + 2a^5b^2 + 8a^4b^3 + 5a^3b^4) * d * \sinh(dx + c)^4 - (3a^7 + 4a^6b - 6a^5b^2 - 12a^4b^3 - 5a^3b^4) * d * \cosh(dx + c)^2 + 8 * (15 * (a^7 + 4a^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4) * d * \cosh(dx + c)^7 + 7 * (3a^7 + 4a^6b - 6a^5b^2 - 12a^4b^3 - 5a^3b^4) * d * \cosh(dx + c)^5 + 5 * (a^7 + 2a^5b^2 + 8a^4b^3 + 5a^3b^4) * d * \cosh(dx + c)^3 - (a^7 + 2a^5b^2 + 8a^4b^3 + 5a^3b^4) * d * \cosh(dx + c) * \sinh(dx + c)^3 + (45 * (a^7 + 4a^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4) * d * \cosh(dx + c)^8 + 28 * (3a^7 + 4a^6b - 6a^5b^2 - 12a^4b^3 - 5a^3b^4) * d * \cosh(dx + c)^6 + 30 * (a^7 + 2a^5b^2 + 8a^4b^3 + 5a^3b^4) * d * \cosh(dx + c)^4 - 12 * (a^7 + 2a^5b^2 + 8a^4b^3 + 5a^3b^4) * d * \cosh(dx + c)^2 - (3a^7 + 4a^6b - 6a^5b^2 - 12a^4b^3 - 5a^3b^4) * d * \sinh(dx + c)^2 - (a^7 + 4a^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4) * d + 2 * (5 * (a^7 + 4a^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4) * d * \cosh(dx + c)^9 + 4 * (3a^7 + 4a^6b - 6a^5b^2 - 12a^4b^3 - 5a^3b^4) * d * \cosh(dx + c)^7 + 6 * (a^7 + 2a^5b^2 + 8a^4b^3 + 5a^3b^4) * d * \cosh(dx + c)^5 - 4 * (a^7 + 2a^5b^2 + 8a^4b^3 + 5a^3b^4) * d * \cosh(dx + c)^3 - (3a^7 + 4a^6b - 6a^5b^2 - 12a^4b^3 - 5a^3b^4) * d * \cosh(dx + c) * \sinh(dx + c)), -1/8 * (2 * (8a^4 + 23a^3b + 45a^2b^2 + 45ab^3 + 15b^4) * \cosh(dx + c)^8 + 16 * (8a^4 + 23a^3b + 45a^2b^2 + 45ab^3 + 15b^4) * \cosh(dx + c) * \sinh(dx + c)^7 + 2 * (8a^4 + 23a^3b + 45a^2b^2 + 45ab^3 + 15b^4) * \sinh(dx + c)^8 + 4 * (16a^4 + 23a^3b - 45ab^3 - 30b^4) * \cosh(dx + c)^6 + 4 * (16a^4 + 23a^3b - 45ab^3 - 30b^4 + 14 * (8a^4 + 23a^3b + 45a^2b^2 + 45ab^3 + 15b^4) * \cosh(dx + c)^2) * \sinh(dx + c)^6 + 8 * (14 * (8a^4 + 23a^3b + 45a^2b^2 + 45ab^3 + 15b^4) * \cosh(dx + c)^3 + 3 * (16a^4 + 23a^3b - 45ab^3 - 30b^4) * \cosh(dx + c) * \sinh(dx + c)^5 + 4 * (24a^4 + 32a^3b + 5a^2b^2 + 50ab^3 + 45b^4) * \cosh(dx + c)^4 + 4 * (35 * (8a^4 + 23a^3b + 45a^2b^2 + 45ab^3 + 15b^4) * \cosh(dx + c)^4 + 24a^4 + 32a^3b + 5a^2b^2 + 50ab^3 + 45b^4 + 15 * (16a^4 + 23a^3b - 45ab^3 - 30b^4) * \cosh(dx + c)^2) * \sinh(dx + c)^4 + 16a^4 + 82a^3b + 146a^2b^2 + 110ab^3 + 30b^4 + 16 * (7 * (8a^4 + 23a^3b + 45a^2b^2 + 45ab^3 + 15b^4) * \cosh(dx + c)^5 + 5 * (16a^4 + 23a^3b - 45ab^3 - 30b^4) * \cosh(dx + c)^3 + (24a^4 + 32a^3b + 5a^2b^2 + 50ab^3 + 45b^4) * \cosh(dx + c) * \sinh(dx + c)^3 + 4 * (16a^4 + 41a^3b - 55ab^3 - 30b^4) * \cosh(dx + c)^2 + 4 * (14 * (8a^4 + 23a^3b + 45a^2b^2 + 45ab^3 + 15b^4) * \cosh(dx + c)^6 + 15 * (16a^4 + 23a^3b - 45ab^3 - 30b^4) * \cosh(dx + c)^4 + 16a^4 + 41a^3b - 55ab^3 - 30b^4 + 6 * (24a^4 + 32a^3b + 5a^2b^2 + 50ab^3 + 45b^4) * \cosh(dx + c)^2) * \sinh(dx + c)^2 + 15 * ((a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4) * \cosh(dx + c)^10 + 10 * (a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4) * \cosh(dx + c) * \sinh(dx + c)^9 + (a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4) * \sinh(dx + c)^10 + (3a^4 + 4a^3b - 6a^2b^2 - 12ab^3 - 5b^4) * \cosh(dx + c)^8 + (3a^4 + 4a^3b - 6a^2b^2 - 12ab^3 - 5b^4 + 45 * (a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4) * \cosh(dx + c)^2) * \sinh(dx + c)^8 + 8 * (15 * (a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4) * \cosh(dx + c)^3 + (3a^4 + 4a^3b - 6a^2b^2 - 12ab^3 - 5b^4) * \cosh(dx + c) * \sinh(dx + c)^7 + 2 * (a^4 + 2a^2b^2 + 8ab^3 + 5b^4) * \cosh(dx + c)^6 + 2 * (105 * (a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4) * \cosh(dx + c)^4 + a^4 + 2a^2b^2 + 8ab^3 + 5b^4 + 14 * (3a^4 + 4a^3b - 6a^2b^2 - 12ab^3 - 5b^4) * \cosh(dx + c)^2) * \sinh(dx + c)^6 + 4 * (63 * (a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4) * \cosh(dx + c)^5 + 14 * (3a^4 + 4a^3b - 6a^2b^2 - 12ab^3 - 5b^4) * \cosh(dx + c)^3 + 3 * (a^4 + 2a^2b^2 + 8ab^3 + 5b^4) * \cosh(dx + c) * \sinh(dx + c)^5 - 2 * (a^4 + 2a^2b^2 + 8ab^3 + 5b^4) * \cosh(dx + c)^4 + 2 * (105 * (a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4) * \cosh(dx + c)^6
\end{aligned}$$

$$\begin{aligned}
& + 35*(3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*\cosh(d*x + c)^4 - a^4 - 2*a^2*b^2 - 8*a*b^3 - 5*b^4 + 15*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*\cosh(d*x + c)^2*\sinh(d*x + c)^4 - a^4 - 4*a^3*b - 6*a^2*b^2 - 4*a*b^3 - b^4 \\
& + 8*(15*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\cosh(d*x + c)^7 + 7*(3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*\cosh(d*x + c)^5 + 5*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*\cosh(d*x + c)^3 - (a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 - (3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*\cosh(d*x + c)^2 + (45*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\cosh(d*x + c)^8 + 28*(3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*\cosh(d*x + c)^6 + 30*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*\cosh(d*x + c)^4 - 3*a^4 - 4*a^3*b + 6*a^2*b^2 + 12*a*b^3 + 5*b^4 - 12*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 2*(5*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\cosh(d*x + c)^9 + 4*(3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*\cosh(d*x + c)^7 + 6*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*\cosh(d*x + c)^5 - 4*(a^4 + 2*a^2*b^2 + 8*a*b^3 + 5*b^4)*\cosh(d*x + c)^3 - (3*a^4 + 4*a^3*b - 6*a^2*b^2 - 12*a*b^3 - 5*b^4)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{b/a}*\arctan(1/2*((a + b)*\cosh(d*x + c)^2 + 2*(a + b)*\cosh(d*x + c)*\sinh(d*x + c) + (a + b)*\sinh(d*x + c)^2 + a - b)*\sqrt{b/a}/b) + 8*(2*(8*a^4 + 2*3*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*\cosh(d*x + c)^7 + 3*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4)*\cosh(d*x + c)^5 + 2*(24*a^4 + 32*a^3*b + 5*a^2*b^2 + 50*a*b^3 + 45*b^4)*\cosh(d*x + c)^3 + (16*a^4 + 41*a^3*b - 55*a*b^3 - 30*b^4)*\cosh(d*x + c))*\sinh(d*x + c))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^10 + 10*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)*\sinh(d*x + c)^9 + (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\sinh(d*x + c)^10 + (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d*\cosh(d*x + c)^8 + (45*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^2 + (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d)*\sinh(d*x + c)^8 + 2*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d*\cosh(d*x + c)^6 + 8*(15*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^3 + (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(105*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^4 + 14*(3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d*\cosh(d*x + c)^2 + (a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d)*\sinh(d*x + c)^6 - 2*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d*\cosh(d*x + c)^4 + 4*(63*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^5 + 14*(3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d*\cosh(d*x + c)^3 + 3*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(105*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^6 + 35*(3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d*\cosh(d*x + c)^4 + 15*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d*\cosh(d*x + c)^2 - (a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d)*\sinh(d*x + c)^4 - (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d*\cosh(d*x + c)^2 + 8*(15*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^7 + 7*(3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d*\cosh(d*x + c)^5 + 5*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d*\cosh(d*x + c)^3 - (a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + (45*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^8 + 28*(3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d*\cosh(d*x + c)^6 + 30*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d*\cosh(d*x + c)^4 - 12*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d*\cosh(d*x + c)^2 - (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d)*\sinh(d*x + c)^2 - (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d + 2*(5*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^9 + 4*(3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d*\cosh(d*x + c)^7 + 6*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d*\cosh(d*x + c)^5 - 4*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*d*\cosh(d*x + c)^3 - (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c))]
\end{aligned}$$

giac [B] time = 0.80, size = 351, normalized size = 3.13

$$\frac{15 b \arctan\left(\frac{ae^{(2 dx+2 c)}+be^{(2 dx+2 c)}+a-b}{2 \sqrt{ab}}\right)}{\sqrt{ab} a^3} - \frac{2\left(9 a^3 b e^{(6 dx+6 c)}+3 a^2 b^2 e^{(6 dx+6 c)}-13 a b^3 e^{(6 dx+6 c)}-7 b^4 e^{(6 dx+6 c)}+27 a^3 b e^{(4 dx+4 c)}+3 a^2 b^2 e^{(4 dx+4 c)}+\dots\right)}{\left(a^5+2 a^4 b+a^3 b^2\right)\left(a e^{(4 d x+4 c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^3) - 2*(9*a^3*b*e^(6*d*x + 6*c) + 3*a^2*b^2*e^(6*d*x + 6*c) - 13*a*b^3*e^(6*d*x + 6*c) - 7*b^4*e^(6*d*x + 6*c) + 27*a^3*b*e^(4*d*x + 4*c) + 3*a^2*b^2*e^(4*d*x + 4*c) + 13*a*b^3*e^(4*d*x + 4*c) + 21*b^4*e^(4*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + 25*a^2*b^2*e^(2*d*x + 2*c) - 23*a*b^3*e^(2*d*x + 2*c) - 21*b^4*e^(2*d*x + 2*c) + 9*a^3*b + 25*a^2*b^2 + 23*a*b^3 + 7*b^4)/((a^5 + 2*a^4*b + a^3*b^2)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2) + 16/(a^3*(e^(2*d*x + 2*c) - 1)))/d
```

maple [B] time = 0.44, size = 816, normalized size = 7.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] -1/2/d/a^3*tanh(1/2*d*x+1/2*c)-9/4/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7-27/4/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5-7/d/a^3*b^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5-27/4/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3-7/d/a^3*b^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3-9/4/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)+15/8/d/a^2*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-15/8/d/a^3*b/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+15/8/d/a^3*b^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+15/8/d/a^2*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+15/8/d/a^3*b/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/2/d/a^3/tanh(1/2*d*x+1/2*c)
```

maxima [B] time = 0.62, size = 478, normalized size = 4.27

$$\frac{8 a^4 + 41 a^3 b + 73 a^2 b^2 + 55 a b^3 + 15 b^4 + 2\left(16 a^4 + 41 a^3 b - 55 a b^3 - 30 b^4\right) e^{(-2 d x-2 c)}}{4\left(a^7 + 4 a^6 b + 6 a^5 b^2 + 4 a^4 b^3 + a^3 b^4 + \left(3 a^7 + 4 a^6 b - 6 a^5 b^2 - 12 a^4 b^3 - 5 a^3 b^4\right) e^{(-2 d x-2 c)} + 2\left(a^7 + 2 a^5 b^2 - \dots\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(8*a^4 + 41*a^3*b + 73*a^2*b^2 + 55*a*b^3 + 15*b^4 + 2*(16*a^4 + 41*a^3*b - 55*a*b^3 - 30*b^4)*e^(-2*d*x - 2*c) + 2*(24*a^4 + 32*a^3*b + 5*a^2*b^2 + 50*a*b^3 + 45*b^4)*e^(-4*d*x - 4*c) + 2*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4)*e^(-6*d*x - 6*c) + (8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*e^(-8*d*x - 8*c))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4 + (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*e^(-2*d*x - 2*c) + 2*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*e^(-4*d*x - 4*c) - 2*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*e^(-6*d*x - 6*c) - (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*e^(-8*d*x - 8*c) - (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*e^(-10*d*x - 10*c))*d) + 15/8*b*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^3*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh(c + dx)^2 (b \tanh(c + dx)^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3), x)
```

```
[Out] int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(c + dx)}{(a + b \tanh^2(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3, x)
```

```
[Out] Integral(csch(c + d*x)**2/(a + b*tanh(c + d*x)**2)**3, x)
```

$$3.47 \quad \int \frac{\operatorname{csch}^3(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=196

$$\frac{(a+6b) \tanh^{-1}(\cosh(c+dx))}{2a^4d} - \frac{b(11a+12b) \operatorname{sech}(c+dx)}{8a^3d(a+b)(a-b \operatorname{sech}^2(c+dx)+b)} - \frac{3b \operatorname{sech}(c+dx)}{4a^2d(a-b \operatorname{sech}^2(c+dx)+b)^2} - \frac{\sqrt{b}}{1}$$

[Out] 1/2*(a+6*b)*arctanh(cosh(d*x+c))/a^4/d-1/2*coth(d*x+c)*csch(d*x+c)/a/d/(a+b-b*sech(d*x+c)^2)^2-3/4*b*sech(d*x+c)/a^2/d/(a+b-b*sech(d*x+c)^2)^2-1/8*b*(11*a+12*b)*sech(d*x+c)/a^3/(a+b)/d/(a+b-b*sech(d*x+c)^2)-1/8*(15*a^2+40*a*b+24*b^2)*arctanh(sech(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a^4/(a+b)^(3/2)/d

Rubi [A] time = 0.33, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3664, 471, 527, 522, 207, 208}

$$\frac{\sqrt{b} (15a^2 + 40ab + 24b^2) \tanh^{-1} \left(\frac{\sqrt{b} \operatorname{sech}(c+dx)}{\sqrt{a+b}} \right)}{8a^4d(a+b)^{3/2}} - \frac{b(11a+12b) \operatorname{sech}(c+dx)}{8a^3d(a+b)(a-b \operatorname{sech}^2(c+dx)+b)} - \frac{3b \operatorname{sech}(c+dx)}{4a^2d(a-b \operatorname{sech}^2(c+dx)+b)^2}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] ((a + 6*b)*ArcTanh[Cosh[c + d*x]])/(2*a^4*d) - (Sqrt[b]*(15*a^2 + 40*a*b + 24*b^2)*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(8*a^4*(a + b)^(3/2)*d) - (Coth[c + d*x]*Csch[c + d*x])/(2*a*d*(a + b - b*Sech[c + d*x]^2)^2) - (3*b*Sech[c + d*x])/(4*a^2*d*(a + b - b*Sech[c + d*x]^2)^2) - (b*(11*a + 12*b)*Sech[c + d*x])/(8*a^3*(a + b)*d*(a + b - b*Sech[c + d*x]^2))

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 471

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Simp[(e^(n-1)*(e*x)^(m-n+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(n*(b*c-a*d)*(p+1)), x] - Dist[e^n/(n*(b*c-a*d)*(p+1)), Int[(e*x)^(m-n)*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[c*(m-n+1)+d*(m+n*(p+q+1)+1]*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m-n+1] && GtQ[m-n+1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_.))/(((a_) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.))), x_Symbol] := Dist[(b*e-a*f)/(b*c-a*d), Int[1/(a+b*x^n), x], x] - Dist[(d*e-c*f)/(b*c-a*d), Int[1/(c+d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3664

```
Int[sin[(e_) + (f_)*(x_)]^(m_)*((a_) + (b_)*tan[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Dist[1/(f*ff^m), Subst[Int[((-1 + ff^2*x^2)^((m - 1)/2)*(a - b + b*ff^2*x^2)^p]/x^(m + 1), x], x, Sec[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c + dx)}{(a + b \tanh^2(c + dx))^3} dx = -\frac{\operatorname{Subst}\left(\int \frac{x^2}{(-1+x^2)^2(a+b-bx^2)^3} dx, x, \operatorname{sech}(c + dx)\right)}{d}$$

$$= -\frac{\operatorname{coth}(c + dx)\operatorname{csch}(c + dx)}{2ad(a + b - b\operatorname{sech}^2(c + dx))^2} - \frac{\operatorname{Subst}\left(\int \frac{a+b+5bx^2}{(-1+x^2)(a+b-bx^2)^3} dx, x, \operatorname{sech}(c + dx)\right)}{2ad}$$

$$= -\frac{\operatorname{coth}(c + dx)\operatorname{csch}(c + dx)}{2ad(a + b - b\operatorname{sech}^2(c + dx))^2} - \frac{3b\operatorname{sech}(c + dx)}{4a^2d(a + b - b\operatorname{sech}^2(c + dx))^2} - \frac{\operatorname{Subst}\left(\int \frac{2(a-b)}{(-1+x^2)(a+b-bx^2)^3} dx, x, \operatorname{sech}(c + dx)\right)}{2ad}$$

$$= -\frac{\operatorname{coth}(c + dx)\operatorname{csch}(c + dx)}{2ad(a + b - b\operatorname{sech}^2(c + dx))^2} - \frac{3b\operatorname{sech}(c + dx)}{4a^2d(a + b - b\operatorname{sech}^2(c + dx))^2} - \frac{b(11a + 6b)}{8a^3(a + b)d}$$

$$= -\frac{\operatorname{coth}(c + dx)\operatorname{csch}(c + dx)}{2ad(a + b - b\operatorname{sech}^2(c + dx))^2} - \frac{3b\operatorname{sech}(c + dx)}{4a^2d(a + b - b\operatorname{sech}^2(c + dx))^2} - \frac{b(11a + 6b)}{8a^3(a + b)d}$$

$$= \frac{(a + 6b) \tanh^{-1}(\cosh(c + dx))}{2a^4d} - \frac{\sqrt{b} (15a^2 + 40ab + 24b^2) \tanh^{-1}\left(\frac{\sqrt{b} \operatorname{sech}(c + dx)}{\sqrt{a + b}}\right)}{8a^4(a + b)^{3/2}d}$$

Mathematica [C] time = 4.45, size = 269, normalized size = 1.37

$$\frac{8a^2b^2 \cosh(c + dx)}{(a + b)((a + b) \cosh(2(c + dx)) + a - b)^2} + \frac{i\sqrt{b} (15a^2 + 40ab + 24b^2) \tan^{-1}\left(\frac{-\sqrt{a} \tanh\left(\frac{1}{2}(c + dx)\right) - i\sqrt{a + b}}{\sqrt{b}}\right)}{(a + b)^{3/2}} + \frac{i\sqrt{b} (15a^2 + 40ab + 24b^2) \tan^{-1}\left(\frac{\sqrt{a} \tanh\left(\frac{1}{2}(c + dx)\right)}{\sqrt{b}}\right)}{(a + b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3, x]
```

```
[Out] -1/8*((I*Sqrt[b]*(15*a^2 + 40*a*b + 24*b^2)*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])/(a + b)^(3/2) + (I*Sqrt[b]*(15*a^2 + 40*a*
```

$$b + 24*b^2)*\text{ArcTan}[((-I)*\text{Sqrt}[a + b] + \text{Sqrt}[a]*\text{Tanh}[(c + d*x)/2])/\text{Sqrt}[b]] / (a + b)^{(3/2)} + (8*a^2*b^2*\text{Cosh}[c + d*x])/((a + b)*(a - b + (a + b)*\text{Cosh}[2*(c + d*x)]))^2 + (2*a*b*(9*a + 8*b)*\text{Cosh}[c + d*x])/((a + b)*(a - b + (a + b)*\text{Cosh}[2*(c + d*x)])) + a*\text{Csch}[(c + d*x)/2]^2 + 4*(a + 6*b)*\text{Log}[\text{Tanh}[(c + d*x)/2]] + a*\text{Sech}[(c + d*x)/2]^2/(a^4*d)$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] Timed out

giac [B] time = 1.03, size = 1877, normalized size = 9.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out]
$$-1/8*((2*(a^5*e^{(2*c)} + a^4*b*e^{(2*c)})^2*(30*a^3*b^2 + 80*a^2*b^3 + 48*a*b^4 - (15*a^3*b + 25*a^2*b^2 - 16*a*b^3 - 24*b^4)*\text{sqrt}(-a*b))*\text{sqrt}(a^2 - b^2 + 2*\text{sqrt}(-a*b)*(a + b))*\text{abs}(a*e^{(2*c)} + b*e^{(2*c)}) + (15*a^9*b + 55*a^8*b^2 + 49*a^7*b^3 - 31*a^6*b^4 - 64*a^5*b^5 - 24*a^4*b^6 + 2*(15*a^8*b + 70*a^7*b^2 + 119*a^6*b^3 + 88*a^5*b^4 + 24*a^4*b^5)*\text{sqrt}(-a*b))*\text{sqrt}(a^2 - b^2 + 2*\text{sqrt}(-a*b)*(a + b))*\text{abs}(-a^5*e^{(2*c)} - a^4*b*e^{(2*c)})*\text{abs}(a*e^{(2*c)} + b*e^{(2*c)})*e^{(2*c)} + (30*a^{14}*b + 110*a^{13}*b^2 + 98*a^{12}*b^3 - 62*a^{11}*b^4 - 128*a^{10}*b^5 - 48*a^9*b^6 - (15*a^{14} + 40*a^{13}*b - 6*a^{12}*b^2 - 80*a^{11}*b^3 - 33*a^{10}*b^4 + 40*a^9*b^5 + 24*a^8*b^6)*\text{sqrt}(-a*b))*\text{sqrt}(a^2 - b^2 + 2*\text{sqrt}(-a*b)*(a + b))*\text{abs}(a*e^{(2*c)} + b*e^{(2*c)})*e^{(4*c)})*\text{arctan}(e^{(d*x)}/\text{sqrt}((a^6*e^{(2*c)} - a^4*b^2*e^{(2*c)})^2 - (a^6*e^{(4*c)} + 2*a^5*b*e^{(4*c)} + a^4*b^2*e^{(4*c)})*(a^6 + 2*a^5*b + a^4*b^2))))/(a^6*e^{(4*c)} + 2*a^5*b*e^{(4*c)} + a^4*b^2*e^{(4*c)})))*e^{(-4*c)}/((a^{15} + 5*a^{14}*b + 9*a^{13}*b^2 + 5*a^{12}*b^3 - 5*a^{11}*b^4 - 9*a^{10}*b^5 - 5*a^9*b^6 - a^8*b^7 + 2*(a^{14} + 6*a^{13}*b + 15*a^{12}*b^2 + 20*a^{11}*b^3 + 15*a^{10}*b^4 + 6*a^9*b^5 + a^8*b^6)*\text{sqrt}(-a*b))*\text{abs}(-a^5*e^{(2*c)} - a^4*b*e^{(2*c)})) + (2*(60*a^4*b^2 + 100*a^3*b^3 - 64*a^2*b^4 - 96*a*b^5 + (15*a^4*b - 50*a^3*b^2 - 201*a^2*b^3 - 104*a*b^4 + 24*b^5)*\text{sqrt}(-a*b))*\text{abs}(a^5*e^{(2*c)} + a^4*b*e^{(2*c)})^2*\text{abs}(a*e^{(2*c)} + b*e^{(2*c)}) + (15*a^{10}*b - 20*a^9*b^2 - 286*a^8*b^3 - 556*a^7*b^4 - 385*a^6*b^5 - 56*a^5*b^6 + 24*a^4*b^7 - 4*(15*a^9*b + 55*a^8*b^2 + 49*a^7*b^3 - 31*a^6*b^4 - 64*a^5*b^5 - 24*a^4*b^6)*\text{sqrt}(-a*b))*\text{abs}(-a^5*e^{(2*c)} - a^4*b*e^{(2*c)})*\text{abs}(a*e^{(2*c)} + b*e^{(2*c)})*e^{(2*c)} + (60*a^{15}*b + 160*a^{14}*b^2 - 24*a^{13}*b^3 - 320*a^{12}*b^4 - 132*a^{11}*b^5 + 160*a^{10}*b^6 + 96*a^9*b^7 + (15*a^{15} - 35*a^{14}*b - 266*a^{13}*b^2 - 270*a^{12}*b^3 + 171*a^{11}*b^4 + 329*a^{10}*b^5 + 80*a^9*b^6 - 24*a^8*b^7)*\text{sqrt}(-a*b))*\text{abs}(a*e^{(2*c)} + b*e^{(2*c)})*e^{(4*c)})*\text{arctan}(e^{(d*x)}/\text{sqrt}((a^6*e^{(2*c)} - a^4*b^2*e^{(2*c)})^2 - (a^6*e^{(4*c)} + 2*a^5*b*e^{(4*c)} + a^4*b^2*e^{(4*c)})*(a^6 + 2*a^5*b + a^4*b^2))))/(a^6*e^{(4*c)} + 2*a^5*b*e^{(4*c)} + a^4*b^2*e^{(4*c)})))*e^{(-4*c)}/((a^{14} + 4*a^{13}*b + 5*a^{12}*b^2 - 5*a^{10}*b^4 - 4*a^9*b^5 - a^8*b^6 - 2*(a^{13} + 5*a^{12}*b + 10*a^{11}*b^2 + 10*a^{10}*b^3 + 5*a^9*b^4 + a^8*b^5)*\text{sqrt}(-a*b))*\text{sqrt}(a^2 - b^2 - 2*\text{sqrt}(-a*b)*(a + b))*\text{abs}(-a^5*e^{(2*c)} - a^4*b*e^{(2*c)})) - 4*(a*e^c + 6*b*e^c)*e^{(-c)}*\text{log}(e^{(d*x + c)} + 1)/a^4 + 4*(a*e^c + 6*b*e^c)*e^{(-c)}*\text{log}(\text{abs}(-e^{(d*x + c)} + 1))/a^4 + 2*(4*a^3*e^{(11*d*x + 11*c)} + 21*a^2*b*e^{(11*d*x + 11*c)} + 29*a*b^2*e^{(11*d*x + 11*c)} + 12*b^3*e^{(11*d*x + 11*c)} + 20*a^3*e^{(9*d*x + 9*c)} + 37*a^2*b*e^{(9*d*x + 9*c)} - 15*a*b^2*e^{(9*d*x + 9*c)} - 36*b^3*e^{(9*d*x + 9*c)} + 40*a^3*e^{(7*d*x + 7*c)} + 6*a^2*b*e^{(7*d*x + 7*c)} - 14*a*b^2*e^{(7*d*x + 7*c)} + 24*b^3*e^{(7*d*x + 7*c)} + 40*a^3*e^{(5*d*x + 5*c)} + 6*a^2*b*e^{(5*d*x + 5*c)} - 14*a*b^2*e^{(5*d*x + 5*c)} +$$

$$5*c) + 24*b^3*e^{(5*d*x + 5*c)} + 20*a^3*e^{(3*d*x + 3*c)} + 37*a^2*b*e^{(3*d*x + 3*c)} - 15*a*b^2*e^{(3*d*x + 3*c)} - 36*b^3*e^{(3*d*x + 3*c)} + 4*a^3*e^{(d*x + c)} + 21*a^2*b*e^{(d*x + c)} + 29*a*b^2*e^{(d*x + c)} + 12*b^3*e^{(d*x + c)})/((a^4 + a^3*b)*(a*e^{(6*d*x + 6*c)} + b*e^{(6*d*x + 6*c)} + a*e^{(4*d*x + 4*c)} - 3*b*e^{(4*d*x + 4*c)} - a*e^{(2*d*x + 2*c)} + 3*b*e^{(2*d*x + 2*c)} - a - b)^2)/d$$

maple [B] time = 0.48, size = 1083, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x)

[Out] $\frac{1}{8}d \operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2/a^3-9/4/d/a*b/(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4*a+2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a+4*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b+a)^2/(a+b)*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^6-8/d/a^2*b^2/(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4*a+2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a+4*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b+a)^2/(a+b)*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^6-6/d/a^3*b^3/(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4*a+2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a+4*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b+a)^2/(a+b)*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^6-27/4/d/a*b/(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4*a+2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a+4*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b+a)^2/(a+b)*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4-51/2/d/a^2*b^2/(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4*a+2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a+4*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b+a)^2/(a+b)*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4-38/d/a^3*b^3/(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4*a+2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a+4*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b+a)^2/(a+b)*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4-20/d/a^4*b^4/(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4*a+2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a+4*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b+a)^2/(a+b)*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4-27/4/d/a*b/(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4*a+2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a+4*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b+a)^2/(a+b)*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-20/d/a^2*b^2/(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4*a+2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a+4*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b+a)^2/(a+b)*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-14/d/a^3*b^3/(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4*a+2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a+4*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b+a)^2/(a+b)*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-9/4/d/a*b/(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4*a+2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a+4*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b+a)^2/(a+b)-5/2/d/a^2*b^2/(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4*a+2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a+4*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b+a)^2/(a+b)-15/8/d/a^2*b/(a+b)/(a*b+b^2)^{(1/2)}*\operatorname{arctanh}\left(\frac{1}{4}*(2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a+2*a+4*b)/(a*b+b^2)^{(1/2)}\right)-5/d/a^3*b^2/(a+b)/(a*b+b^2)^{(1/2)}*\operatorname{arctanh}\left(\frac{1}{4}*(2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a+2*a+4*b)/(a*b+b^2)^{(1/2)}\right)-3/d/a^4*b^3/(a+b)/(a*b+b^2)^{(1/2)}*\operatorname{arctanh}\left(\frac{1}{4}*(2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a+2*a+4*b)/(a*b+b^2)^{(1/2)}\right)-1/8/d/a^3/\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2-1/2/d/a^3*\ln(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right))-3/d/a^4*\ln(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right))*b$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $-1/4*((4*a^3*e^{(11*c)} + 21*a^2*b*e^{(11*c)} + 29*a*b^2*e^{(11*c)} + 12*b^3*e^{(11*c)})*e^{(11*d*x)} + (20*a^3*e^{(9*c)} + 37*a^2*b*e^{(9*c)} - 15*a*b^2*e^{(9*c)} - 36*b^3*e^{(9*c)})*e^{(9*d*x)} + 2*(20*a^3*e^{(7*c)} + 3*a^2*b*e^{(7*c)} - 7*a*b^2*e^{(7*c)} + 12*b^3*e^{(7*c)})*e^{(7*d*x)} + 2*(20*a^3*e^{(5*c)} + 3*a^2*b*e^{(5*c)} - 7*a*b^2*e^{(5*c)} + 12*b^3*e^{(5*c)})*e^{(5*d*x)} + (20*a^3*e^{(3*c)} + 37*a^2*b*e^{(3*c)} - 15*a*b^2*e^{(3*c)} - 36*b^3*e^{(3*c)})*e^{(3*d*x)} + (4*a^3*e^c + 21*a^2*b*e^c + 29*a*b^2*e^c + 12*b^3*e^c)*e^{(d*x)})/(a^6*d + 3*a^5*b*d + 3*a^4*b^2*d + a^3*b^3*d + (a^6*d*e^{(12*c)} + 3*a^5*b*d*e^{(12*c)} + 3*a^4*b^2*d*e^{(12*c)} + a^3*b^3*d*e^{(12*c)})*e^{(12*d*x)} + 2*(a^6*d*e^{(10*c)} - a^5*b*d*e^{(10*c)} - 5*a^4*b^2*d*e^{(10*c)} - 3*a^3*b^3*d*e^{(10*c)})*e^{(10*d*x)} - (a^6*d*e^{(8*c)} + 3*a^5*b*d*e^{(8*c)} - 13*a^4*b^2*d*e^{(8*c)} - 15*a^3*b^3*d*e^{(8*c)})*e^{(8*d*x)} - 4*(a^6*d*e^{(6*c)} - a^5*b*d*e^{(6*c)} + 3*a^4*b^2*d*e^{(6*c)} + 5*a^3*b^3*d*e^{(6*c)})*e^{(6*d*x)} - (a^6*d*e^{(4*c)} + 3*a^5*b*d*e^{(4*c)} - 13*a^4*b^2*d*e^{(4*c)} - 15*a^3*b^3*d*e^{(4*c)})*e^{(4*d*x)} + 2*(a^6*d*e^{(2*c)} - a^5*b*d*e^{(2*c)} -$

$5*a^4*b^2*d*e^{(2*c)} - 3*a^3*b^3*d*e^{(2*c)})*e^{(2*d*x)} + 1/2*(a + 6*b)*\log((e^{(d*x + c)} + 1)*e^{(-c)})/(a^4*d) - 1/2*(a + 6*b)*\log((e^{(d*x + c)} - 1)*e^{(-c)})/(a^4*d) + 8*\integrate(1/32*((15*a^2*b*e^{(3*c)} + 40*a*b^2*e^{(3*c)} + 24*b^3*e^{(3*c)})*e^{(3*d*x)} - (15*a^2*b*e^c + 40*a*b^2*e^c + 24*b^3*e^c)*e^{(d*x)})/(a^6 + 2*a^5*b + a^4*b^2 + (a^6*e^{(4*c)} + 2*a^5*b*e^{(4*c)} + a^4*b^2*e^{(4*c)})*e^{(4*d*x)} + 2*(a^6*e^{(2*c)} - a^4*b^2*e^{(2*c)})*e^{(2*d*x)}), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh(c + dx)^3 (b \tanh(c + dx)^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3), x)

[Out] int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(c + dx)}{(a + b \tanh^2(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3, x)

[Out] Integral(csch(c + d*x)**3/(a + b*tanh(c + d*x)**2)**3, x)

$$3.48 \quad \int \frac{\operatorname{csch}^4(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=151

$$\frac{5\sqrt{b}(3a+7b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d} + \frac{b(7a+11b) \tanh(c+dx)}{8a^4d(a+b \tanh^2(c+dx))} + \frac{(a+3b) \operatorname{coth}(c+dx)}{a^4d} + \frac{b(a+b) \tanh(c+dx)}{4a^3d(a+b \tanh^2(c+dx))}$$

[Out] (a+3*b)*coth(d*x+c)/a^4/d-1/3*coth(d*x+c)^3/a^3/d+5/8*(3*a+7*b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/a^(9/2)/d+1/4*b*(a+b)*tanh(d*x+c)/a^3/d/(a+b*tanh(d*x+c)^2)+1/8*b*(7*a+11*b)*tanh(d*x+c)/a^4/d/(a+b*tanh(d*x+c)^2)

Rubi [A] time = 0.20, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3663, 456, 1259, 1261, 205}

$$\frac{b(7a+11b) \tanh(c+dx)}{8a^4d(a+b \tanh^2(c+dx))} + \frac{b(a+b) \tanh(c+dx)}{4a^3d(a+b \tanh^2(c+dx))^2} + \frac{(a+3b) \operatorname{coth}(c+dx)}{a^4d} + \frac{5\sqrt{b}(3a+7b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (5*Sqrt[b]*(3*a + 7*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(9/2)*d) + ((a + 3*b)*Coth[c + d*x])/(a^4*d) - Coth[c + d*x]^3/(3*a^3*d) + (b*(a + b)*Tanh[c + d*x])/(4*a^3*d*(a + b*Tanh[c + d*x]^2)^2) + (b*(7*a + 11*b)*Tanh[c + d*x])/(8*a^4*d*(a + b*Tanh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 456

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] := Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 1259

Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4))^p - ((c*d^2 - b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m/2, 0]

Rule 1261

Int[((f_.)*(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*

$(a + b*x^2 + c*x^4)^p, x]$, $x]$ /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 3663

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]^(n_))]^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]

Rubi steps

$$\int \frac{\operatorname{csch}^4(c + dx)}{(a + b \tanh^2(c + dx))^3} dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^4(a+bx^2)^3} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{b(a+b) \tanh(c + dx)}{4a^3d (a + b \tanh^2(c + dx))^2} - \frac{b \operatorname{Subst}\left(\int \frac{-\frac{4}{ab} + \frac{4(a+b)x^2}{a^2b} - \frac{3(a+b)x^4}{a^3}}{x^4(a+bx^2)^2} dx, x, \tanh(c + dx)\right)}{4d}$$

$$= \frac{b(a+b) \tanh(c + dx)}{4a^3d (a + b \tanh^2(c + dx))^2} + \frac{b(7a + 11b) \tanh(c + dx)}{8a^4d (a + b \tanh^2(c + dx))} - \frac{\operatorname{Subst}\left(\int \frac{-8ab+8b}{x^4} dx, x, \tanh(c + dx)\right)}{8a^4d}$$

$$= \frac{b(a+b) \tanh(c + dx)}{4a^3d (a + b \tanh^2(c + dx))^2} + \frac{b(7a + 11b) \tanh(c + dx)}{8a^4d (a + b \tanh^2(c + dx))} - \frac{\operatorname{Subst}\left(\int \left(-\frac{8b}{x^4} + \frac{8b}{x^4}\right) dx, x, \tanh(c + dx)\right)}{8a^4d}$$

$$= \frac{(a + 3b) \operatorname{coth}(c + dx)}{a^4d} - \frac{\operatorname{coth}^3(c + dx)}{3a^3d} + \frac{b(a + b) \tanh(c + dx)}{4a^3d (a + b \tanh^2(c + dx))^2} + \frac{b(7a + 11b) \tanh(c + dx)}{8a^4d (a + b \tanh^2(c + dx))}$$

$$= \frac{5\sqrt{b}(3a + 7b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d} + \frac{(a + 3b) \operatorname{coth}(c + dx)}{a^4d} - \frac{\operatorname{coth}^3(c + dx)}{3a^3d} + \frac{b(a + b) \tanh(c + dx)}{4a^3d (a + b \tanh^2(c + dx))^2}$$

Mathematica [A] time = 1.40, size = 149, normalized size = 0.99

$$\frac{3\sqrt{a}b \sinh(2(c+dx))((9a^2+20ab+11b^2) \cosh(2(c+dx))+9a^2+6ab-11b^2)}{((a+b) \cosh(2(c+dx))+a-b)^2} + 15\sqrt{b}(3a + 7b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right) - 8\sqrt{a} \operatorname{coth}(c + dx)}{24a^{9/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (15*sqrt[b]*(3*a + 7*b)*ArcTan[(sqrt[b]*Tanh[c + d*x])/sqrt[a]] - 8*sqrt[a]*Coth[c + d*x]*(-2*a - 9*b + a*Csch[c + d*x]^2) + (3*sqrt[a]*b*(9*a^2 + 6*a*b - 11*b^2 + (9*a^2 + 20*a*b + 11*b^2)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((a - b + (a + b)*Cosh[2*(c + d*x)])^2)/(24*a^(9/2)*d)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] Timed out

giac [B] time = 0.85, size = 407, normalized size = 2.70

$$\frac{15(3abe^{2c}+7b^2e^{2c})\arctan\left(\frac{ae^{2dx+2c}+be^{2dx+2c}+a-b}{2\sqrt{ab}}\right)e^{(-2c)}}{\sqrt{ab}a^4} - \frac{6(9a^3be^{6dx+6c}+7a^2b^2e^{6dx+6c}-13ab^3e^{6dx+6c}-11b^4e^{6dx+6c}+27a^3be^{4dx+4c})}{\sqrt{ab}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out] $\frac{1}{24}*(15*(3*a*b*e^{(2*c)} + 7*b^2*e^{(2*c)})*\arctan(1/2*(a*e^{(2*d*x + 2*c)} + b*e^{(2*d*x + 2*c)} + a - b)/\sqrt{a*b})*e^{(-2*c)}/(\sqrt{a*b}*a^4) - 6*(9*a^3*b*e^{(6*d*x + 6*c)} + 7*a^2*b^2*e^{(6*d*x + 6*c)} - 13*a*b^3*e^{(6*d*x + 6*c)} - 11*b^4*e^{(6*d*x + 6*c)} + 27*a^3*b*e^{(4*d*x + 4*c)} + 15*a^2*b^2*e^{(4*d*x + 4*c)} + 5*a*b^3*e^{(4*d*x + 4*c)} + 33*b^4*e^{(4*d*x + 4*c)} + 27*a^3*b*e^{(2*d*x + 2*c)} + 37*a^2*b^2*e^{(2*d*x + 2*c)} - 23*a*b^3*e^{(2*d*x + 2*c)} - 33*b^4*e^{(2*d*x + 2*c)} + 9*a^3*b + 29*a^2*b^2 + 31*a*b^3 + 11*b^4)/((a^5 + a^4*b)*(a*e^{(4*d*x + 4*c)} + b*e^{(4*d*x + 4*c)} + 2*a*e^{(2*d*x + 2*c)} - 2*b*e^{(2*d*x + 2*c)} + a + b)^2) + 16*(9*b*e^{(4*d*x + 4*c)} - 6*a*e^{(2*d*x + 2*c)} - 18*b*e^{(2*d*x + 2*c)} + 2*a + 9*b)/(a^4*(e^{(2*d*x + 2*c)} - 1)^3))/d$

maple [B] time = 0.48, size = 1416, normalized size = 9.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x)

[Out]
$$\begin{aligned} & -25/4/d/a^3*b^2/(b*(a+b))^{(1/2)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)})-1/24/d/a^3/\tanh(1/2*d*x+1/2*c)^3-1/24/d/a^3*\tanh(1/2*d*x+1/2*c)^3+3/8/d/a^3*\tanh(1/2*d*x+1/2*c)+3/8/d/a^3/\tanh(1/2*d*x+1/2*c)-25/4/d/a^3*b^2/(b*(a+b))^{(1/2)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)})-15/8/d/a^2*b/(b*(a+b))^{(1/2)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)})-35/8/d*b^3/a^4/(b*(a+b))^{(1/2)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)})-35/8/d*b^3/a^4/(b*(a+b))^{(1/2)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)})+3/2/d/a^4*\tanh(1/2*d*x+1/2*c)*b+3/2/d/a^4/\tanh(1/2*d*x+1/2*c)*b+9/4/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7-15/8/d/a^2*b/(b*(a+b))^{(1/2)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)})+11/d*b^3/a^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3+13/4/d*b^2/a^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)+35/8/d*b^2/a^4/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)})-35/8/d*b^2/a^4/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)})+13/4/d*b^2/a^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7+11/d*b^3/a^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5+27/4/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5+67/4/d/a^3*b^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5+27/4/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5 \end{aligned}$$

$$\frac{2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3+67/4/d/a^3*b^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3+9/4/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)+15/8/d/a^3*b/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-15/8/d/a^3*b/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))$$

maxima [B] time = 0.70, size = 615, normalized size = 4.07

$$\frac{16 a^4 + 147 a^3 b + 351 a^2 b^2 + 325 a b^3 + 105 b^4 + 2 (8 a^4 + 32 a^3 b - 251 a^2 b^2 - 590 a b^3 - 315 b^4) e^{(-2 d x - 2 c)} - (96 a^4 + 313 a^3 b + 19 a^2 b^2 - 1725 a a b^3 - 1575 b^4) e^{(-4 d x - 4 c)} - 4 (56 a^4 + 80 a^3 b - 65 a^2 b^2 + 400 a a b^3 + 525 b^4) e^{(-6 d x - 6 c)} - (176 a^4 + 135 a^3 b + 15 a^2 b^2 - 1375 a a b^3 - 1575 b^4) e^{(-8 d x - 8 c)} - 6 (8 a^4 + 45 a^3 b + 150 a a b^3 + 105 b^4) e^{(-10 d x - 10 c)} + 15 (3 a^3 b + 13 a^2 b^2 + 17 a a b^3 + 7 b^4) e^{(-12 d x - 12 c)}}{12 (a^7 + 3 a^6 b + 3 a^5 b^2 + a^4 b^3 + (a^7 - 5 a^6 b - 13 a^5 b^2 - 7 a^4 b^3) e^{(-2 d x - 2 c)} - (3 a^7 + a^6 b - 23 a^5 b^2 - 21 a^4 b^3) e^{(-4 d x - 4 c)} - (3 a^7 - 7 a^6 b + 25 a^5 b^2 + 35 a^4 b^3) e^{(-6 d x - 6 c)} + (3 a^7 - 7 a^6 b + 25 a^5 b^2 + 35 a^4 b^3) e^{(-8 d x - 8 c)} + (3 a^7 + a^6 b - 23 a^5 b^2 - 21 a^4 b^3) e^{(-10 d x - 10 c)} - (a^7 - 5 a^6 b - 13 a^5 b^2 - 7 a^4 b^3) e^{(-12 d x - 12 c)}) * d} - 5/8 (3 a^3 b + 7 b^2) * arctan(1/2 * ((a + b) * e^{(-2 d x - 2 c)} + a - b) / sqrt(a * b)) / (sqrt(a * b) * a^4 * d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 1/12*(16*a^4 + 147*a^3*b + 351*a^2*b^2 + 325*a*b^3 + 105*b^4 + 2*(8*a^4 + 32*a^3*b - 251*a^2*b^2 - 590*a*b^3 - 315*b^4)*e^(-2*d*x - 2*c) - (96*a^4 + 313*a^3*b + 19*a^2*b^2 - 1725*a*a*b^3 - 1575*b^4)*e^(-4*d*x - 4*c) - 4*(56*a^4 + 80*a^3*b - 65*a^2*b^2 + 400*a*a*b^3 + 525*b^4)*e^(-6*d*x - 6*c) - (176*a^4 + 135*a^3*b + 15*a^2*b^2 - 1375*a*a*b^3 - 1575*b^4)*e^(-8*d*x - 8*c) - 6*(8*a^4 + 45*a^3*b + 150*a*a*b^3 + 105*b^4)*e^(-10*d*x - 10*c) + 15*(3*a^3*b + 13*a^2*b^2 + 17*a*a*b^3 + 7*b^4)*e^(-12*d*x - 12*c))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3 + (a^7 - 5*a^6*b - 13*a^5*b^2 - 7*a^4*b^3)*e^(-2*d*x - 2*c) - (3*a^7 + a^6*b - 23*a^5*b^2 - 21*a^4*b^3)*e^(-4*d*x - 4*c) - (3*a^7 - 7*a^6*b + 25*a^5*b^2 + 35*a^4*b^3)*e^(-6*d*x - 6*c) + (3*a^7 - 7*a^6*b + 25*a^5*b^2 + 35*a^4*b^3)*e^(-8*d*x - 8*c) + (3*a^7 + a^6*b - 23*a^5*b^2 - 21*a^4*b^3)*e^(-10*d*x - 10*c) - (a^7 - 5*a^6*b - 13*a^5*b^2 - 7*a^4*b^3)*e^(-12*d*x - 12*c)) * d) - 5/8*(3*a^3*b + 7*b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b)))/(sqrt(a*b)*a^4*d)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sinh(c + dx)^4 (b \tanh(c + dx)^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3), x)

[Out] int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(c + dx)}{(a + b \tanh^2(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)

[Out] Integral(csch(c + d*x)**4/(a + b*tanh(c + d*x)**2)**3, x)

3.49 $\int \sinh^4(c + dx) (a + b \tanh^3(c + dx)) dx$

Optimal. Leaf size=132

$$-\frac{3(a+8b)\log(1-\tanh(c+dx))}{16d} + \frac{3(a-8b)\log(\tanh(c+dx)+1)}{16d} + \frac{\sinh^4(c+dx)(a\tanh(c+dx)+b)}{4d} - \frac{\sinh^2(c+dx)(a+8b\tanh(c+dx))}{8d}$$

[Out] -3/16*(a+8*b)*ln(1-tanh(d*x+c))/d+3/16*(a-8*b)*ln(1+tanh(d*x+c))/d-3/8*a*tanh(d*x+c)/d-3/2*b*tanh(d*x+c)^2/d+1/4*sinh(d*x+c)^4*(b+a*tanh(d*x+c))/d-1/8*sinh(d*x+c)^2*tanh(d*x+c)*(a+8*b*tanh(d*x+c))/d

Rubi [A] time = 0.17, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3663, 1804, 801, 633, 31}

$$-\frac{3(a+8b)\log(1-\tanh(c+dx))}{16d} + \frac{3(a-8b)\log(\tanh(c+dx)+1)}{16d} + \frac{\sinh^4(c+dx)(a\tanh(c+dx)+b)}{4d} - \frac{\sinh^2(c+dx)(a+8b\tanh(c+dx))}{8d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3), x]

[Out] (-3*(a + 8*b)*Log[1 - Tanh[c + d*x]])/(16*d) + (3*(a - 8*b)*Log[1 + Tanh[c + d*x]])/(16*d) - (3*a*Tanh[c + d*x])/(8*d) - (3*b*Tanh[c + d*x]^2)/(2*d) + (Sinh[c + d*x]^4*(b + a*Tanh[c + d*x]))/(4*d) - (Sinh[c + d*x]^2*Tanh[c + d*x]*(a + 8*b*Tanh[c + d*x]))/(8*d)

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 633

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]

Rule 801

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 1804

Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x] + Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]

Rule 3663

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)ⁿ)^p]/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x]

$$\begin{aligned}
& c)^5 + 7*(8*(a - 8*b)*d*x - 5*a - 13*b)*\cosh(d*x + c)^3 + 2*(6*(a - 8*b)*d \\
& *x + 11*b)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 3*(8*(a - 8*b)*d*x + 5*a - 13*b \\
&)*\cosh(d*x + c)^4 + 3*(165*(a + b)*\cosh(d*x + c)^8 - 420*(a + 3*b)*\cosh(d*x \\
& + c)^6 + 70*(8*(a - 8*b)*d*x - 5*a - 13*b)*\cosh(d*x + c)^4 + 8*(a - 8*b)*d \\
& *x + 40*(6*(a - 8*b)*d*x + 11*b)*\cosh(d*x + c)^2 + 5*a - 13*b)*\sinh(d*x + c \\
&)^4 + 4*(55*(a + b)*\cosh(d*x + c)^9 - 180*(a + 3*b)*\cosh(d*x + c)^7 + 42*(8 \\
& *(a - 8*b)*d*x - 5*a - 13*b)*\cosh(d*x + c)^5 + 40*(6*(a - 8*b)*d*x + 11*b)* \\
& \cosh(d*x + c)^3 + 3*(8*(a - 8*b)*d*x + 5*a - 13*b)*\cosh(d*x + c))*\sinh(d*x \\
& + c)^3 + 6*(a - 3*b)*\cosh(d*x + c)^2 + 6*(11*(a + b)*\cosh(d*x + c)^10 - 45* \\
& (a + 3*b)*\cosh(d*x + c)^8 + 14*(8*(a - 8*b)*d*x - 5*a - 13*b)*\cosh(d*x + c) \\
& ^6 + 20*(6*(a - 8*b)*d*x + 11*b)*\cosh(d*x + c)^4 + 3*(8*(a - 8*b)*d*x + 5*a \\
& - 13*b)*\cosh(d*x + c)^2 + a - 3*b)*\sinh(d*x + c)^2 + 192*(b*\cosh(d*x + c)^ \\
& 8 + 8*b*\cosh(d*x + c)*\sinh(d*x + c)^7 + b*\sinh(d*x + c)^8 + 2*b*\cosh(d*x + \\
& c)^6 + 2*(14*b*\cosh(d*x + c)^2 + b)*\sinh(d*x + c)^6 + 4*(14*b*\cosh(d*x + c) \\
& ^3 + 3*b*\cosh(d*x + c))*\sinh(d*x + c)^5 + b*\cosh(d*x + c)^4 + (70*b*\cosh(d* \\
& x + c)^4 + 30*b*\cosh(d*x + c)^2 + b)*\sinh(d*x + c)^4 + 4*(14*b*\cosh(d*x + c) \\
&)^5 + 10*b*\cosh(d*x + c)^3 + b*\cosh(d*x + c))*\sinh(d*x + c)^3 + 2*(14*b*\cos \\
& h(d*x + c)^6 + 15*b*\cosh(d*x + c)^4 + 3*b*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 \\
& + 4*(2*b*\cosh(d*x + c)^7 + 3*b*\cosh(d*x + c)^5 + b*\cosh(d*x + c)^3)*\sinh(d* \\
& x + c))*\log(2*\cosh(d*x + c)/(\cosh(d*x + c) - \sinh(d*x + c))) + 12*((a + b)* \\
& \cosh(d*x + c)^11 - 5*(a + 3*b)*\cosh(d*x + c)^9 + 2*(8*(a - 8*b)*d*x - 5*a - \\
& 13*b)*\cosh(d*x + c)^7 + 4*(6*(a - 8*b)*d*x + 11*b)*\cosh(d*x + c)^5 + (8*(a \\
& - 8*b)*d*x + 5*a - 13*b)*\cosh(d*x + c)^3 + (a - 3*b)*\cosh(d*x + c))*\sinh(d \\
& *x + c) - a + b)/(d*\cosh(d*x + c)^8 + 8*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + d \\
& *\sinh(d*x + c)^8 + 2*d*\cosh(d*x + c)^6 + 2*(14*d*\cosh(d*x + c)^2 + d)*\sinh(\\
& d*x + c)^6 + 4*(14*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + \\
& d*\cosh(d*x + c)^4 + (70*d*\cosh(d*x + c)^4 + 30*d*\cosh(d*x + c)^2 + d)*\sinh \\
& (d*x + c)^4 + 4*(14*d*\cosh(d*x + c)^5 + 10*d*\cosh(d*x + c)^3 + d*\cosh(d*x + \\
& c))*\sinh(d*x + c)^3 + 2*(14*d*\cosh(d*x + c)^6 + 15*d*\cosh(d*x + c)^4 + 3*d \\
& *\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 4*(2*d*\cosh(d*x + c)^7 + 3*d*\cosh(d*x + \\
& c)^5 + d*\cosh(d*x + c)^3)*\sinh(d*x + c))
\end{aligned}$$

giac [A] time = 0.25, size = 206, normalized size = 1.56

$$24(a - 8b)dx + \left(ae^{(4dx+24c)} + be^{(4dx+24c)} - 8ae^{(2dx+22c)} - 20be^{(2dx+22c)} \right) e^{(-20c)} + 192b \log \left(e^{(2dx+2c)} + 1 \right) - \frac{(9ae^{(4dx+24c)} + 9be^{(4dx+24c)} - 8ae^{(2dx+22c)} - 20be^{(2dx+22c)})e^{(-20c)} + 192b \log(e^{(2dx+2c)} + 1)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="giac")

[Out] 1/64*(24*(a - 8*b)*d*x + (a*e^(4*d*x + 24*c) + b*e^(4*d*x + 24*c) - 8*a*e^(2*d*x + 22*c) - 20*b*e^(2*d*x + 22*c))*e^(-20*c) + 192*b*log(e^(2*d*x + 2*c) + 1) - (9*a*e^(8*d*x + 8*c) + 72*b*e^(8*d*x + 8*c) + 10*a*e^(6*d*x + 6*c) + 36*b*e^(6*d*x + 6*c) - 6*a*e^(4*d*x + 4*c) + 111*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) + 18*b*e^(2*d*x + 2*c) + a - b)*e^(-4*c)/(e^(2*d*x) + e^(4*d*x + 2*c))^2)/d

maple [A] time = 0.30, size = 122, normalized size = 0.92

$$\frac{a \cosh(dx + c) \left(\sinh^3(dx + c) \right)}{4d} - \frac{3a \cosh(dx + c) \sinh(dx + c)}{8d} + \frac{3ax}{8} + \frac{3ac}{8d} + \frac{b \left(\sinh^6(dx + c) \right)}{4d \cosh(dx + c)^2} - \frac{3b \left(\sinh^4(dx + c) \right)}{4d \cosh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3),x)

[Out] 1/4*a*cosh(d*x+c)*sinh(d*x+c)^3/d-3/8*a*cosh(d*x+c)*sinh(d*x+c)/d+3/8*a*x+3/8/d*a*c+1/4/d*b*sinh(d*x+c)^6/cosh(d*x+c)^2-3/4/d*b*sinh(d*x+c)^4/cosh(d*x+c)^2+3*b*ln(cosh(d*x+c))/d-3/2*b*tanh(d*x+c)^2/d

maxima [A] time = 0.42, size = 194, normalized size = 1.47

$$\frac{1}{64} a \left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) + \frac{1}{64} b \left(\frac{192(dx+c)}{d} - \frac{20e^{(-2dx-2c)} - e^{(-4dx-4c)}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")

[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) + 1/64*b*(192*(d*x + c)/d - (20*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d + 192*log(e^(-2*d*x - 2*c) + 1)/d - (18*e^(-2*d*x - 2*c) + 39*e^(-4*d*x - 4*c) - 108*e^(-6*d*x - 6*c) - 1)/(d*(e^(-4*d*x - 4*c) + 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c))))

mupad [B] time = 0.27, size = 156, normalized size = 1.18

$$x \left(\frac{3a}{8} - 3b \right) + \frac{2b}{d(e^{2c+2dx} + 1)} + \frac{e^{4c+4dx}(a+b)}{64d} - \frac{2b}{d(2e^{2c+2dx} + e^{4c+4dx} + 1)} - \frac{e^{-4c-4dx}(a-b)}{64d} + \frac{3b \ln(e^{2c+2dx})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^3),x)

[Out] x*((3*a)/8 - 3*b) + (2*b)/(d*(exp(2*c + 2*d*x) + 1)) + (exp(4*c + 4*d*x)*(a + b))/(64*d) - (2*b)/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (exp(-4*c - 4*d*x)*(a - b))/(64*d) + (3*b*log(exp(2*c)*exp(2*d*x) + 1))/d + (exp(-2*c - 2*d*x)*(2*a - 5*b))/(16*d) - (exp(2*c + 2*d*x)*(2*a + 5*b))/(16*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^3(c + dx)) \sinh^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**3),x)

[Out] Integral((a + b*tanh(c + d*x)**3)*sinh(c + d*x)**4, x)

3.50 $\int \sinh^3(c + dx) (a + b \tanh^3(c + dx)) dx$

Optimal. Leaf size=98

$$\frac{a \cosh^3(c + dx)}{3d} - \frac{a \cosh(c + dx)}{d} + \frac{5b \sinh^3(c + dx)}{6d} - \frac{5b \sinh(c + dx)}{2d} - \frac{b \sinh^3(c + dx) \tanh^2(c + dx)}{2d} + \frac{5b \tan^{-1}(\sinh(c + dx))}{2d}$$

[Out] $5/2*b*\arctan(\sinh(d*x+c))/d - a*\cosh(d*x+c)/d + 1/3*a*\cosh(d*x+c)^3/d - 5/2*b*\sinh(d*x+c)/d + 5/6*b*\sinh(d*x+c)^3/d - 1/2*b*\sinh(d*x+c)^3*\tanh(d*x+c)^2/d$

Rubi [A] time = 0.12, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3666, 2633, 2592, 288, 302, 203}

$$\frac{a \cosh^3(c + dx)}{3d} - \frac{a \cosh(c + dx)}{d} + \frac{5b \sinh^3(c + dx)}{6d} - \frac{5b \sinh(c + dx)}{2d} - \frac{b \sinh^3(c + dx) \tanh^2(c + dx)}{2d} + \frac{5b \tan^{-1}(\sinh(c + dx))}{2d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3), x]

[Out] $(5*b*\text{ArcTan}[\text{Sinh}[c + d*x]])/(2*d) - (a*\text{Cosh}[c + d*x])/d + (a*\text{Cosh}[c + d*x]^3)/(3*d) - (5*b*\text{Sinh}[c + d*x])/(2*d) + (5*b*\text{Sinh}[c + d*x]^3)/(6*d) - (b*\text{Sinh}[c + d*x]^3*\text{Tanh}[c + d*x]^2)/(2*d)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !IntegerQ[m + n*(p + 1) + 1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 302

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]

Rule 2592

Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 3666


```
Int[((d_)*sin[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*((c_)*tan[(e_) +
(f_)*(x_)])^(n_))^(p_), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
+ b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \sinh^3(c + dx) (a + b \tanh^3(c + dx)) dx &= i \int (-ia \sinh^3(c + dx) - ib \sinh^3(c + dx) \tanh^3(c + dx)) dx \\
&= a \int \sinh^3(c + dx) dx + b \int \sinh^3(c + dx) \tanh^3(c + dx) dx \\
&= -\frac{a \operatorname{Subst}\left(\int (1 - x^2) dx, x, \cosh(c + dx)\right)}{d} + \frac{b \operatorname{Subst}\left(\int \frac{x^6}{(1+x^2)^2} dx, x, \cosh(c + dx)\right)}{d} \\
&= -\frac{a \cosh(c + dx)}{d} + \frac{a \cosh^3(c + dx)}{3d} - \frac{b \sinh^3(c + dx) \tanh^2(c + dx)}{2d} \\
&= -\frac{a \cosh(c + dx)}{d} + \frac{a \cosh^3(c + dx)}{3d} - \frac{b \sinh^3(c + dx) \tanh^2(c + dx)}{2d} \\
&= -\frac{a \cosh(c + dx)}{d} + \frac{a \cosh^3(c + dx)}{3d} - \frac{5b \sinh(c + dx)}{2d} + \frac{5b \sinh^3(c + dx)}{6d} \\
&= \frac{5b \tan^{-1}(\sinh(c + dx))}{2d} - \frac{a \cosh(c + dx)}{d} + \frac{a \cosh^3(c + dx)}{3d} - \frac{5b}{6d}
\end{aligned}$$

Mathematica [A] time = 0.28, size = 104, normalized size = 1.06

$$-\frac{3a \cosh(c + dx)}{4d} + \frac{a \cosh(3(c + dx))}{12d} + \frac{b \sinh^3(c + dx) \tanh^2(c + dx)}{3d} - \frac{5b(2 \sinh(c + dx) \tanh^2(c + dx) - 3 \tanh^3(c + dx))}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3), x]
```

```
[Out] (-3*a*Cosh[c + d*x])/(4*d) + (a*Cosh[3*(c + d*x)])/(12*d) + (b*Sinh[c + d*x]^3*Tanh[c + d*x]^2)/(3*d) - (5*b*(2*Sinh[c + d*x]*Tanh[c + d*x]^2 - 3*(ArcTan[Sinh[c + d*x]] - Sech[c + d*x]*Tanh[c + d*x]))) / (6*d)
```

fricas [B] time = 0.74, size = 1070, normalized size = 10.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3), x, algorithm="fricas")
```

```
[Out] 1/24*((a + b)*cosh(d*x + c)^10 + 10*(a + b)*cosh(d*x + c)*sinh(d*x + c)^9 + (a + b)*sinh(d*x + c)^10 - (7*a + 25*b)*cosh(d*x + c)^8 + (45*(a + b)*cosh(d*x + c)^2 - 7*a - 25*b)*sinh(d*x + c)^8 + 8*(15*(a + b)*cosh(d*x + c)^3 - (7*a + 25*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(13*a + 25*b)*cosh(d*x + c)^6 + 2*(105*(a + b)*cosh(d*x + c)^4 - 14*(7*a + 25*b)*cosh(d*x + c)^2 - 13*a - 25*b)*sinh(d*x + c)^6 + 4*(63*(a + b)*cosh(d*x + c)^5 - 14*(7*a + 25*b)*cosh(d*x + c)^3 - 3*(13*a + 25*b)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(13*a - 25*b)*cosh(d*x + c)^4 + 2*(105*(a + b)*cosh(d*x + c)^6 - 35*(7*a + 25*b)*cosh(d*x + c)^4 - 15*(13*a + 25*b)*cosh(d*x + c)^2 - 13*a + 25*b)*sinh(d*x + c)^4 + 8*(15*(a + b)*cosh(d*x + c)^7 - 7*(7*a + 25*b)*cosh(d*x + c)^5 - 5*(13*a + 25*b)*cosh(d*x + c)^3 - (13*a - 25*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(13*a + 25*b)*cosh(d*x + c)^2 + 2*(105*(a + b)*cosh(d*x + c)^8 - 35*(7*a + 25*b)*cosh(d*x + c)^6 - 15*(13*a + 25*b)*cosh(d*x + c)^4 - 13*a + 25*b)*sinh(d*x + c)^2 + 2*(105*(a + b)*cosh(d*x + c)^9 - 35*(7*a + 25*b)*cosh(d*x + c)^7 - 15*(13*a + 25*b)*cosh(d*x + c)^5 - 13*a + 25*b)*sinh(d*x + c) - 13*a + 25*b
```

$c)^3 - (7a - 25b) \cosh(dx + c)^2 + (45(a + b) \cosh(dx + c)^8 - 28(7a + 25b) \cosh(dx + c)^6 - 30(13a + 25b) \cosh(dx + c)^4 - 12(13a - 25b) \cosh(dx + c)^2 - 7a + 25b) \sinh(dx + c)^2 + 120(b \cosh(dx + c)^7 + 7b \cosh(dx + c) \sinh(dx + c)^6 + b \sinh(dx + c)^7 + 2b \cosh(dx + c)^5 + (21b \cosh(dx + c)^2 + 2b) \sinh(dx + c)^5 + 5(7b \cosh(dx + c)^3 + 2b \cosh(dx + c)) \sinh(dx + c)^4 + b \cosh(dx + c)^3 + (35b \cosh(dx + c)^4 + 20b \cosh(dx + c)^2 + b) \sinh(dx + c)^3 + (21b \cosh(dx + c)^5 + 20b \cosh(dx + c)^3 + 3b \cosh(dx + c)) \sinh(dx + c)^2 + (7b \cosh(dx + c)^6 + 10b \cosh(dx + c)^4 + 3b \cosh(dx + c)^2) \sinh(dx + c)) \arctan(\cosh(dx + c) + \sinh(dx + c)) + 2(5(a + b) \cosh(dx + c)^9 - 4(7a + 25b) \cosh(dx + c)^7 - 6(13a + 25b) \cosh(dx + c)^5 - 4(13a - 25b) \cosh(dx + c)^3 - (7a - 25b) \cosh(dx + c)) \sinh(dx + c) + a - b) / (d \cosh(dx + c)^7 + 7d \cosh(dx + c) \sinh(dx + c)^6 + d \sinh(dx + c)^7 + 2d \cosh(dx + c)^5 + (21d \cosh(dx + c)^2 + 2d) \sinh(dx + c)^5 + 5(7d \cosh(dx + c)^3 + 2d \cosh(dx + c)) \sinh(dx + c)^4 + d \cosh(dx + c)^3 + (35d \cosh(dx + c)^4 + 20d \cosh(dx + c)^2 + d) \sinh(dx + c)^3 + (21d \cosh(dx + c)^5 + 20d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^2 + (7d \cosh(dx + c)^6 + 10d \cosh(dx + c)^4 + 3d \cosh(dx + c)^2) \sinh(dx + c))$

giac [A] time = 0.19, size = 142, normalized size = 1.45

$$\frac{120 b \arctan(e^{(dx+c)}) - (9 a e^{(2 dx+2 c)} - 27 b e^{(2 dx+2 c)} - a + b) e^{(-3 dx-3 c)} + (a e^{(3 dx+30 c)} + b e^{(3 dx+30 c)} - 9 a e^{(dx+28 c)} - b e^{(dx+28 c)})}{24 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)^3*(a+b*tanh(dx+c)^3),x, algorithm="giac")

[Out] $\frac{1}{24} * (120 * b * \arctan(e^{(dx+c)}) - (9 * a * e^{(2 * dx+2 * c)} - 27 * b * e^{(2 * dx+2 * c)} - a + b) * e^{(-3 * dx-3 * c)} + (a * e^{(3 * dx+30 * c)} + b * e^{(3 * dx+30 * c)} - 9 * a * e^{(dx+28 * c)} - 27 * b * e^{(dx+28 * c)}) * e^{(-27 * c)} - 24 * (b * e^{(3 * dx+3 * c)} - b * e^{(dx+c)}) / (e^{(2 * dx+2 * c)} + 1)^2) / d$

maple [A] time = 0.35, size = 129, normalized size = 1.32

$$-\frac{2a \cosh(dx+c)}{3d} + \frac{a \cosh(dx+c) (\sinh^2(dx+c))}{3d} + \frac{b (\sinh^5(dx+c))}{3d \cosh(dx+c)^2} - \frac{5b (\sinh^3(dx+c))}{3d \cosh(dx+c)^2} - \frac{5b \sinh(dx+c)}{d \cosh(dx+c)^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(dx+c)^3*(a+b*tanh(dx+c)^3),x)

[Out] $-2/3 * a * \cosh(dx+c) / d + 1/3 * d * a * \cosh(dx+c) * \sinh(dx+c)^2 + 1/3 * d * b * \sinh(dx+c)^5 / \cosh(dx+c)^2 - 5/3 * d * b * \sinh(dx+c)^3 / \cosh(dx+c)^2 - 5/3 * d * b * \sinh(dx+c) / \cosh(dx+c)^2 + 5/2 * b * \operatorname{sech}(dx+c) * \tanh(dx+c) / d + 5/3 * d * b * \arctan(\exp(dx+c))$

maxima [A] time = 0.41, size = 174, normalized size = 1.78

$$\frac{1}{24} b \left(\frac{27 e^{(-dx-c)} - e^{(-3dx-3c)}}{d} - \frac{120 \arctan(e^{(-dx-c)})}{d} - \frac{25 e^{(-2dx-2c)} + 77 e^{(-4dx-4c)} + 3 e^{(-6dx-6c)} - 1}{d(e^{(-3dx-3c)} + 2 e^{(-5dx-5c)} + e^{(-7dx-7c)})} \right) + \frac{1}{24} a \left(\frac{e^{(3dx+30c)}}{d} - \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)^3*(a+b*tanh(dx+c)^3),x, algorithm="maxima")

[Out] $\frac{1}{24} * b * ((27 * e^{(-dx-c)} - e^{(-3 * dx-3 * c)}) / d - 120 * \arctan(e^{(-dx-c)}) / d - (25 * e^{(-2 * dx-2 * c)} + 77 * e^{(-4 * dx-4 * c)} + 3 * e^{(-6 * dx-6 * c)} - 1) / (d * (e^{(-3 * dx-3 * c)} + 2 * e^{(-5 * dx-5 * c)} + e^{(-7 * dx-7 * c)}))) + \frac{1}{24} * a * (e^{(3 * dx+3 * c)} / d - 9 * e^{(dx+c)} / d - 9 * e^{(-dx-c)} / d + e^{(-3 * dx-3 * c)} / d)$

mupad [B] time = 0.23, size = 171, normalized size = 1.74

$$\frac{e^{3c+3dx} (a+b)}{24d} + \frac{5 \operatorname{atan}\left(\frac{b e^{dx} e^c \sqrt{d^2}}{d \sqrt{b^2}}\right) \sqrt{b^2}}{\sqrt{d^2}} + \frac{e^{-3c-3dx} (a-b)}{24d} - \frac{e^{c+dx} (3a+9b)}{8d} - \frac{e^{-c-dx} (3a-9b)}{8d} - \frac{b e^{c+dx}}{d (e^{2c+2dx})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^3), x)`

[Out] $(\exp(3c + 3d*x)*(a + b))/(24*d) + (5*\operatorname{atan}((b*\exp(d*x)*\exp(c)*(d^2)^{(1/2)})/(d*(b^2)^{(1/2)}))*(b^2)^{(1/2)})/(d^2)^{(1/2)} + (\exp(-3c - 3d*x)*(a - b))/(24*d) - (\exp(c + d*x)*(3*a + 9*b))/(8*d) - (\exp(-c - d*x)*(3*a - 9*b))/(8*d) - (b*\exp(c + d*x))/(d*(\exp(2*c + 2*d*x) + 1)) + (2*b*\exp(c + d*x))/(d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^3(c + dx)) \sinh^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**3), x)`

[Out] `Integral((a + b*tanh(c + d*x)**3)*sinh(c + d*x)**3, x)`

3.51 $\int \sinh^2(c + dx) (a + b \tanh^3(c + dx)) dx$

Optimal. Leaf size=100

$$\frac{(a + 4b) \log(1 - \tanh(c + dx))}{4d} - \frac{(a - 4b) \log(\tanh(c + dx) + 1)}{4d} + \frac{\sinh^2(c + dx)(a \tanh(c + dx) + b)}{2d} + \frac{a \tanh(c + dx)}{2d}$$

[Out] 1/4*(a+4*b)*ln(1-tanh(d*x+c))/d-1/4*(a-4*b)*ln(1+tanh(d*x+c))/d+1/2*a*tanh(d*x+c)/d+1/2*b*tanh(d*x+c)^2/d+1/2*sinh(d*x+c)^2*(b+a*tanh(d*x+c))/d

Rubi [A] time = 0.12, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3663, 1804, 1802, 633, 31}

$$\frac{(a + 4b) \log(1 - \tanh(c + dx))}{4d} - \frac{(a - 4b) \log(\tanh(c + dx) + 1)}{4d} + \frac{\sinh^2(c + dx)(a \tanh(c + dx) + b)}{2d} + \frac{a \tanh(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3), x]

[Out] ((a + 4*b)*Log[1 - Tanh[c + d*x]])/(4*d) - ((a - 4*b)*Log[1 + Tanh[c + d*x]])/(4*d) + (a*Tanh[c + d*x])/(2*d) + (b*Tanh[c + d*x]^2)/(2*d) + (Sinh[c + d*x]^2*(b + a*Tanh[c + d*x]))/(2*d)

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 633

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]

Rule 1802

Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1804

Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x] + Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]

Rule 3663

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_.))^p, x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]

Rubi steps

$$\begin{aligned}
\int \sinh^2(c + dx) (a + b \tanh^3(c + dx)) dx &= \frac{\text{Subst}\left(\int \frac{x^2(a+bx^3)}{(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{\sinh^2(c + dx)(b + a \tanh(c + dx))}{2d} + \frac{\text{Subst}\left(\int \frac{x(-2b-ax-2bx^2)}{1-x^2} dx, x\right)}{2d} \\
&= \frac{\sinh^2(c + dx)(b + a \tanh(c + dx))}{2d} + \frac{\text{Subst}\left(\int \left(a + 2bx - \frac{a+4bx}{1-x^2}\right) dx, x\right)}{2d} \\
&= \frac{a \tanh(c + dx)}{2d} + \frac{b \tanh^2(c + dx)}{2d} + \frac{\sinh^2(c + dx)(b + a \tanh(c + dx))}{2d} \\
&= \frac{a \tanh(c + dx)}{2d} + \frac{b \tanh^2(c + dx)}{2d} + \frac{\sinh^2(c + dx)(b + a \tanh(c + dx))}{2d} \\
&= \frac{(a + 4b) \log(1 - \tanh(c + dx))}{4d} - \frac{(a - 4b) \log(1 + \tanh(c + dx))}{4d}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 69, normalized size = 0.69

$$\frac{a(-c - dx)}{2d} + \frac{a \sinh(2(c + dx))}{4d} - \frac{b(-\sinh^2(c + dx) + \text{sech}^2(c + dx) + 4 \log(\cosh(c + dx)))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3), x]

[Out] (a*(-c - d*x))/(2*d) - (b*(4*Log[Cosh[c + d*x]] + Sech[c + d*x]^2 - Sinh[c + d*x]^2))/(2*d) + (a*Sinh[2*(c + d*x)])/(4*d)

fricas [B] time = 0.59, size = 924, normalized size = 9.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3), x, algorithm="fricas")

[Out] 1/8*((a + b)*cosh(d*x + c)^8 + 8*(a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a + b)*sinh(d*x + c)^8 - 2*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c)^6 - 2*(2*(a - 4*b)*d*x - 14*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c)^6 + 4*(14*(a + b)*cosh(d*x + c)^3 - 3*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(4*(a - 4*b)*d*x + 7*b)*cosh(d*x + c)^4 + 2*(35*(a + b)*cosh(d*x + c)^4 - 4*(a - 4*b)*d*x - 15*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c)^2 - 7*b)*sinh(d*x + c)^4 + 8*(7*(a + b)*cosh(d*x + c)^5 - 5*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c)^3 - (4*(a - 4*b)*d*x + 7*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(2*(a - 4*b)*d*x + a - b)*cosh(d*x + c)^2 + 2*(14*(a + b)*cosh(d*x + c)^6 - 15*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c)^4 - 2*(a - 4*b)*d*x - 6*(4*(a - 4*b)*d*x + 7*b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c)^2 - 16*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^6 + 2*b*cosh(d*x + c)^4 + (15*b*cosh(d*x + c)^2 + 2*b)*sinh(d*x + c)^4 + 4*(5*b*cosh(d*x + c)^3 + 2*b*cosh(d*x + c))*sinh(d*x + c)^3 + b*cosh(d*x + c)^2 + (15*b*cosh(d*x + c)^4 + 12*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^5 + 4*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)) *log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*(a + b)*cosh(d*x + c)^7 - 3*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c)^5 - 2*(4*(a - 4*b)*d*

$$x + 7*b)*\cosh(d*x + c)^3 - (2*(a - 4*b)*d*x + a - b)*\cosh(d*x + c))*\sinh(d*x + c) - a + b)/(d*\cosh(d*x + c)^6 + 6*d*\cosh(d*x + c)*\sinh(d*x + c)^5 + d*\sinh(d*x + c)^6 + 2*d*\cosh(d*x + c)^4 + (15*d*\cosh(d*x + c)^2 + 2*d)*\sinh(d*x + c)^4 + 4*(5*d*\cosh(d*x + c)^3 + 2*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + d*\cosh(d*x + c)^2 + (15*d*\cosh(d*x + c)^4 + 12*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^2 + 2*(3*d*\cosh(d*x + c)^5 + 4*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c))$$

giac [A] time = 0.18, size = 142, normalized size = 1.42

$$\frac{4(a - 4b)dx - (2ae^{(2dx+2c)} - 8be^{(2dx+2c)} - a + b)e^{(-2dx-2c)} - (ae^{(2dx+10c)} + be^{(2dx+10c)})e^{(-8c)} + 16b \log(e^{(2dx+2c)} + 1) - 8*(3b*e^{(4dx+4c)} + 4b*e^{(2dx+2c)} + 3b))/(e^{(2dx+2c)} + 1)^2/d}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="giac")

[Out]
$$-1/8*(4*(a - 4*b)*d*x - (2*a*e^{(2*d*x + 2*c)} - 8*b*e^{(2*d*x + 2*c)} - a + b)*e^{(-2*d*x - 2*c)} - (a*e^{(2*d*x + 10*c)} + b*e^{(2*d*x + 10*c)})*e^{(-8*c)} + 16*b*\log(e^{(2*d*x + 2*c)} + 1) - 8*(3*b*e^{(4*d*x + 4*c)} + 4*b*e^{(2*d*x + 2*c)} + 3*b))/(e^{(2*d*x + 2*c)} + 1)^2/d$$

maple [A] time = 0.20, size = 79, normalized size = 0.79

$$\frac{a \cosh(dx + c) \sinh(dx + c)}{2d} - \frac{ax}{2} - \frac{ac}{2d} + \frac{b(\sinh^4(dx + c))}{2d \cosh(dx + c)^2} - \frac{2b \ln(\cosh(dx + c))}{d} + \frac{b(\tanh^2(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3),x)

[Out]
$$1/2*a*\cosh(d*x+c)*\sinh(d*x+c)/d - 1/2*a*x - 1/2/d*a*c + 1/2/d*b*\sinh(d*x+c)^4/\cosh(d*x+c)^2 - 2*b*\ln(\cosh(d*x+c))/d + b*tanh(d*x+c)^2/d$$

maxima [A] time = 0.41, size = 141, normalized size = 1.41

$$-\frac{1}{8}a\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{8}b\left(\frac{16(dx+c)}{d} - \frac{e^{(-2dx-2c)}}{d} + \frac{16 \log(e^{(-2dx-2c)} + 1)}{d} - \frac{2e^{(-2dx-2c)} - 15e^{(-4dx-4c)}}{d(e^{(-2dx-2c)} + 2e^{(-4dx-4c)})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")

[Out]
$$-1/8*a*(4*x - e^{(2*d*x + 2*c)}/d + e^{(-2*d*x - 2*c)}/d) - 1/8*b*(16*(d*x + c)/d - e^{(-2*d*x - 2*c)}/d + 16*\log(e^{(-2*d*x - 2*c)} + 1)/d - (2*e^{(-2*d*x - 2*c)} - 15*e^{(-4*d*x - 4*c)} + 1)/(d*(e^{(-2*d*x - 2*c)} + 2*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)})))$$

mupad [B] time = 1.15, size = 115, normalized size = 1.15

$$\frac{e^{2c+2dx}(a+b)}{8d} - \frac{2b}{d(e^{2c+2dx}+1)} - x\left(\frac{a}{2} - 2b\right) + \frac{2b}{d(2e^{2c+2dx}+e^{4c+4dx}+1)} - \frac{e^{-2c-2dx}(a-b)}{8d} - \frac{2b \ln(e^{2c}e^{2dx}+1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^3),x)

[Out]
$$(\exp(2*c + 2*d*x)*(a + b))/(8*d) - (2*b)/(d*(\exp(2*c + 2*d*x) + 1)) - x*(a/2 - 2*b) + (2*b)/(d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1)) - (\exp(-2*c - 2*d*x)*(a - b))/(8*d) - (2*b*\log(\exp(2*c)*\exp(2*d*x) + 1))/d$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^3(c + dx)) \sinh^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**3), x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)*sinh(c + d*x)**2, x)
```

3.52 $\int \sinh(c + dx) (a + b \tanh^3(c + dx)) dx$

Optimal. Leaf size=63

$$\frac{a \cosh(c + dx)}{d} + \frac{3b \sinh(c + dx)}{2d} - \frac{3b \tan^{-1}(\sinh(c + dx))}{2d} - \frac{b \sinh(c + dx) \tanh^2(c + dx)}{2d}$$

[Out] $-3/2*b*\arctan(\sinh(d*x+c))/d+a*\cosh(d*x+c)/d+3/2*b*\sinh(d*x+c)/d-1/2*b*\sinh(d*x+c)*\tanh(d*x+c)^2/d$

Rubi [A] time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3666, 2638, 2592, 288, 321, 203}

$$\frac{a \cosh(c + dx)}{d} + \frac{3b \sinh(c + dx)}{2d} - \frac{3b \tan^{-1}(\sinh(c + dx))}{2d} - \frac{b \sinh(c + dx) \tanh^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3), x]

[Out] $(-3*b*\text{ArcTan}[\text{Sinh}[c + d*x]])/(2*d) + (a*\text{Cosh}[c + d*x])/d + (3*b*\text{Sinh}[c + d*x])/d - (b*\text{Sinh}[c + d*x]*\text{Tanh}[c + d*x]^2)/(2*d)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2592

Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)^(n_.)], x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3666


```
Int[((d_)*sin[(e_)+(f_)*(x_)])^(m_)*((a_)+(b_)*((c_)*tan[(e_)+(f_)*(x_)])^(n_))^(p_), x_Symbol] :> Int[ExpandTrig[(d*sin[e+f*x])^m*(a+b*(c*tan[e+f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \sinh(c+dx)(a+b \tanh^3(c+dx)) dx &= -\left(i \int (ia \sinh(c+dx) + ib \sinh(c+dx) \tanh^3(c+dx)) dx\right) \\ &= a \int \sinh(c+dx) dx + b \int \sinh(c+dx) \tanh^3(c+dx) dx \\ &= \frac{a \cosh(c+dx)}{d} + \frac{b \operatorname{Subst}\left(\int \frac{x^4}{(1+x^2)^2} dx, x, \sinh(c+dx)\right)}{d} \\ &= \frac{a \cosh(c+dx)}{d} - \frac{b \sinh(c+dx) \tanh^2(c+dx)}{2d} + \frac{(3b) \operatorname{Subst}\left(\int \frac{x}{1+x^2} dx, x, \sinh(c+dx)\right)}{2d} \\ &= \frac{a \cosh(c+dx)}{d} + \frac{3b \sinh(c+dx)}{2d} - \frac{b \sinh(c+dx) \tanh^2(c+dx)}{2d} \\ &= -\frac{3b \tan^{-1}(\sinh(c+dx))}{2d} + \frac{a \cosh(c+dx)}{d} + \frac{3b \sinh(c+dx)}{2d} - \frac{b \sinh(c+dx) \tanh^2(c+dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.13, size = 72, normalized size = 1.14

$$\frac{a \sinh(c) \sinh(dx)}{d} + \frac{a \cosh(c) \cosh(dx)}{d} + \frac{b \sinh(c+dx) \tanh^2(c+dx)}{d} - \frac{3b (\tan^{-1}(\sinh(c+dx)) - \tanh(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c+d*x]*(a+b*Tanh[c+d*x]^3),x]
```

```
[Out] (a*Cosh[c]*Cosh[d*x])/d + (a*Sinh[c]*Sinh[d*x])/d + (b*Sinh[c+d*x]*Tanh[c+d*x]^2)/d - (3*b*(ArcTan[Sinh[c+d*x]] - Sech[c+d*x]*Tanh[c+d*x]))/(2*d)
```

fricas [B] time = 0.95, size = 528, normalized size = 8.38

$$(a+b) \cosh(dx+c)^6 + 6(a+b) \cosh(dx+c) \sinh(dx+c)^5 + (a+b) \sinh(dx+c)^6 + 3(a+b) \cosh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/2*((a+b)*cosh(d*x+c)^6 + 6*(a+b)*cosh(d*x+c)*sinh(d*x+c)^5 + (a+b)*sinh(d*x+c)^6 + 3*(a+b)*cosh(d*x+c)^4 + 3*(5*(a+b)*cosh(d*x+c)^2 + a+b)*sinh(d*x+c)^4 + 4*(5*(a+b)*cosh(d*x+c)^3 + 3*(a+b)*cosh(d*x+c))*sinh(d*x+c)^3 + 3*(a-b)*cosh(d*x+c)^2 + 3*(5*(a+b)*cosh(d*x+c)^4 + 6*(a+b)*cosh(d*x+c)^2 + a-b)*sinh(d*x+c)^2 - 6*(b*cosh(d*x+c)^5 + 5*b*cosh(d*x+c)*sinh(d*x+c)^4 + b*sinh(d*x+c)^5 + 2*b*cosh(d*x+c)^3 + 2*(5*b*cosh(d*x+c)^2 + b)*sinh(d*x+c)^3 + 2*(5*b*cosh(d*x+c)^3 + 3*b*cosh(d*x+c))*sinh(d*x+c)^2 + b*cosh(d*x+c) + (5*b*cosh(d*x+c)^4 + 6*b*cosh(d*x+c)^2 + b)*sinh(d*x+c))*arctan(cosh(d*x+c) + sinh(d*x+c)) + 6*((a+b)*cosh(d*x+c)^5 + 2*(a+b)*cosh(d*x+c)^3 + (a-b)*cosh(d*x+c))*sinh(d*x+c) + a-b)/(d*cosh(d*x+c)^5 + 5*d*cosh(d*x+c)*sinh(d*x+c)^4 + d*sinh(d*x+c)^5 + 2*d*cosh(d*x+c)^3 +
```

$2*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (5*d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)$

giac [A] time = 0.17, size = 95, normalized size = 1.51

$$\frac{6b \arctan\left(e^{(dx+c)}\right) - (a-b)e^{(-dx-c)} - \left(ae^{(dx+8c)} + be^{(dx+8c)}\right)e^{(-7c)} - \frac{2\left(be^{(3dx+3c)} - be^{(dx+c)}\right)}{\left(e^{(2dx+2c)} + 1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="giac")

[Out] $-1/2*(6*b*\arctan(e^{(d*x + c)}) - (a - b)*e^{(-d*x - c)} - (a*e^{(d*x + 8*c)} + b*e^{(d*x + 8*c)})*e^{(-7*c)} - 2*(b*e^{(3*d*x + 3*c)} - b*e^{(d*x + c)}))/(e^{(2*d*x + 2*c)} + 1)^2)/d$

maple [A] time = 0.28, size = 85, normalized size = 1.35

$$\frac{a \cosh(dx + c)}{d} + \frac{b(\sinh^3(dx + c))}{d \cosh(dx + c)^2} + \frac{3b \sinh(dx + c)}{d \cosh(dx + c)^2} - \frac{3b \operatorname{sech}(dx + c) \tanh(dx + c)}{2d} - \frac{3b \arctan(e^{dx+c})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x)

[Out] $a*\cosh(d*x+c)/d+1/d*b*sinh(d*x+c)^3/\cosh(d*x+c)^2+3/d*b*sinh(d*x+c)/\cosh(d*x+c)^2-3/2*b*sech(d*x+c)*tanh(d*x+c)/d-3/d*b*arctan(\exp(d*x+c))$

maxima [A] time = 0.41, size = 105, normalized size = 1.67

$$\frac{1}{2}b\left(\frac{6 \arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)}}{d} + \frac{4e^{(-2dx-2c)} - e^{(-4dx-4c)} + 1}{d(e^{(-dx-c)} + 2e^{(-3dx-3c)} + e^{(-5dx-5c)})}\right) + \frac{a \cosh(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")

[Out] $1/2*b*(6*\arctan(e^{(-d*x - c)})/d - e^{(-d*x - c)}/d + (4*e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)} + 1)/(d*(e^{(-d*x - c)} + 2*e^{(-3*d*x - 3*c)} + e^{(-5*d*x - 5*c)}))) + a*\cosh(d*x + c)/d$

mupad [B] time = 0.14, size = 128, normalized size = 2.03

$$\frac{e^{-c-dx}(a-b)}{2d} - \frac{3 \operatorname{atan}\left(\frac{be^{dx}e^c\sqrt{d^2}}{d\sqrt{b^2}}\right)\sqrt{b^2}}{\sqrt{d^2}} + \frac{e^{c+dx}(a+b)}{2d} + \frac{be^{c+dx}}{d(e^{2c+2dx}+1)} - \frac{2be^{c+dx}}{d(2e^{2c+2dx}+e^{4c+4dx}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)*(a + b*tanh(c + d*x)^3),x)

[Out] $(\exp(-c - d*x)*(a - b))/(2*d) - (3*\operatorname{atan}((b*\exp(d*x)*\exp(c)*(d^2)^{(1/2)}))/(d*(b^2)^{(1/2)}))*(b^2)^{(1/2)}/(d^2)^{(1/2)} + (\exp(c + d*x)*(a + b))/(2*d) + (b*\exp(c + d*x))/(d*(\exp(2*c + 2*d*x) + 1)) - (2*b*\exp(c + d*x))/(d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^3(c + dx)) \sinh(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)*sinh(c + d*x), x)
```

3.53 $\int \operatorname{csch}(c + dx) (a + b \tanh^3(c + dx)) dx$

Optimal. Leaf size=49

$$-\frac{a \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b \tan^{-1}(\sinh(c + dx))}{2d} - \frac{b \tanh(c + dx) \operatorname{sech}(c + dx)}{2d}$$

[Out] $1/2*b*\arctan(\sinh(d*x+c))/d - a*\operatorname{arctanh}(\cosh(d*x+c))/d - 1/2*b*\operatorname{sech}(d*x+c)*\tanh(d*x+c)/d$

Rubi [A] time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3666, 3770, 2611}

$$-\frac{a \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b \tan^{-1}(\sinh(c + dx))}{2d} - \frac{b \tanh(c + dx) \operatorname{sech}(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]*(a + b*\operatorname{Tanh}[c + d*x]^3), x]$

[Out] $(b*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/(2*d) - (a*\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]])/d - (b*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x])/(2*d)$

Rule 2611

$\operatorname{Int}[(a_*)*\operatorname{sec}[(e_*) + (f_*)(x_*)]^{(m_*)}*((b_*)*\tan[(e_*) + (f_*)(x_*)]^{(n_*)}, x_Symbol] :> \operatorname{Simp}[(b*(a*\operatorname{Sec}[e + f*x])^m*(b*\operatorname{Tan}[e + f*x])^{(n-1)})/(f*(m+n-1)), x] - \operatorname{Dist}[(b^2*(n-1))/(m+n-1), \operatorname{Int}[(a*\operatorname{Sec}[e + f*x])^m*(b*\operatorname{Tan}[e + f*x])^{(n-2)}, x], x] /;$ $\operatorname{FreeQ}\{a, b, e, f, m\}, x \ \&\& \operatorname{GtQ}[n, 1] \ \&\& \operatorname{NeQ}[m+n-1, 0] \ \&\& \operatorname{IntegersQ}[2*m, 2*n]$

Rule 3666

$\operatorname{Int}[(d_*)*\sin[(e_*) + (f_*)(x_*)]^{(m_*)}*((a_*) + (b_*)*((c_*)*\tan[(e_*) + (f_*)(x_*)]^{(n_*)})^{(p_*)}, x_Symbol] :> \operatorname{Int}[\operatorname{ExpandTrig}[d*\sin[e + f*x]]^m*(a + b*(c*\tan[e + f*x])^n)^p, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, f, m, n\}, x \ \&\& \operatorname{IGtQ}[p, 0]$

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_*) + (d_*)(x_*)], x_Symbol] :> -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /;$ $\operatorname{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned} \int \operatorname{csch}(c + dx) (a + b \tanh^3(c + dx)) dx &= i \int (-i a \operatorname{csch}(c + dx) - i b \operatorname{sech}(c + dx) \tanh^2(c + dx)) dx \\ &= a \int \operatorname{csch}(c + dx) dx + b \int \operatorname{sech}(c + dx) \tanh^2(c + dx) dx \\ &= -\frac{a \tanh^{-1}(\cosh(c + dx))}{d} - \frac{b \operatorname{sech}(c + dx) \tanh(c + dx)}{2d} + \frac{1}{2} b \int \operatorname{sech}(c + dx) dx \\ &= \frac{b \tan^{-1}(\sinh(c + dx))}{2d} - \frac{a \tanh^{-1}(\cosh(c + dx))}{d} - \frac{b \operatorname{sech}(c + dx) \tanh(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.03, size = 75, normalized size = 1.53

$$\frac{a \log\left(\sinh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{a \log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} + \frac{b \tan^{-1}(\sinh(c + dx))}{2d} - \frac{b \tanh(c + dx) \operatorname{sech}(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3), x]

[Out] (b*ArcTan[Sinh[c + d*x]])/(2*d) - (a*Log[Cosh[c/2 + (d*x)/2]])/d + (a*Log[Sinh[c/2 + (d*x)/2]])/d - (b*Sech[c + d*x]*Tanh[c + d*x])/(2*d)

fricas [B] time = 0.96, size = 522, normalized size = 10.65

$$\frac{b \cosh(dx + c)^3 + 3b \cosh(dx + c) \sinh(dx + c)^2 + b \sinh(dx + c)^3 - (b \cosh(dx + c)^4 + 4b \cosh(dx + c) \sinh(dx + c)^2 + b \sinh(dx + c)^3)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3), x, algorithm="fricas")

[Out] -(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b)*arctan(cosh(d*x + c) + sinh(d*x + c)) - b*cosh(d*x + c) + (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*a*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*a*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + (3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)

giac [A] time = 0.16, size = 74, normalized size = 1.51

$$\frac{b \arctan\left(e^{(dx+c)}\right) - a \log\left(e^{(dx+c)} + 1\right) + a \log\left(\left|e^{(dx+c)} - 1\right|\right) - \frac{be^{(3dx+3c)} - be^{(dx+c)}}{\left(e^{(2dx+2c)} + 1\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3), x, algorithm="giac")

[Out] (b*arctan(e^(d*x + c)) - a*log(e^(d*x + c) + 1) + a*log(abs(e^(d*x + c) - 1))) - (b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^2/d

maple [A] time = 0.37, size = 65, normalized size = 1.33

$$-\frac{2a \operatorname{arctanh}\left(e^{dx+c}\right)}{d} - \frac{b \sinh(dx + c)}{d \cosh(dx + c)^2} + \frac{b \operatorname{sech}(dx + c) \tanh(dx + c)}{2d} + \frac{b \arctan\left(e^{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)*(a+b*tanh(d*x+c)^3), x)

[Out] -2/d*a*arctanh(exp(d*x+c))-1/d*b*sinh(d*x+c)/cosh(d*x+c)^2+1/2*b*sech(d*x+c)*tanh(d*x+c)/d+1/d*b*arctan(exp(d*x+c))

maxima [A] time = 0.41, size = 83, normalized size = 1.69

$$-b \left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} + \frac{e^{(-dx-c)} - e^{(-3dx-3c)}}{d\left(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1\right)} \right) + \frac{a \log\left(\tanh\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")

[Out] $-b*(\arctan(e^{-(d*x - c)})/d + (e^{-(d*x - c)} - e^{(-3*d*x - 3*c)})/(d*(2*e^{(-2*d*x - 2*c)} + e^{(-4*d*x - 4*c)} + 1))) + a*\log(\tanh(1/2*d*x + 1/2*c))/d$

mupad [B] time = 2.47, size = 233, normalized size = 4.76

$$\frac{2be^{c+dx}}{d+2de^{2c+2dx}+de^{4c+4dx}} - \frac{a \ln(-8ab^2 - 32a^3 - 32a^3 e^{dx} e^c - 8ab^2 e^{dx} e^c)}{d} + \frac{a \ln(8ab^2 + 32a^3 - 32a^3 e^{dx} e^c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tanh(c + d*x)^3)/sinh(c + d*x),x)

[Out] $(2*b*\exp(c + d*x))/(d + 2*d*\exp(2*c + 2*d*x) + d*\exp(4*c + 4*d*x)) - (a*\log(-8*a*b^2 - 32*a^3 - 32*a^3*\exp(d*x)*\exp(c) - 8*a*b^2*\exp(d*x)*\exp(c)))/d + (a*\log(8*a*b^2 + 32*a^3 - 32*a^3*\exp(d*x)*\exp(c) - 8*a*b^2*\exp(d*x)*\exp(c)))/d - (b*(\log(4*b^3*\exp(d*x)*\exp(c) - b^3*4i - a^2*b*16i + 16*a^2*b*\exp(d*x)*\exp(c))*1i - \log(a^2*b*16i + b^3*4i + 4*b^3*\exp(d*x)*\exp(c) + 16*a^2*b*\exp(d*x)*\exp(c))*1i))/(2*d) - (b*\exp(c + d*x))/(d + d*\exp(2*c + 2*d*x))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^3(c + dx)) \operatorname{csch}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**3),x)

[Out] Integral((a + b*tanh(c + d*x)**3)*csch(c + d*x), x)

3.54 $\int \operatorname{csch}^2(c + dx) (a + b \tanh^3(c + dx)) dx$

Optimal. Leaf size=29

$$\frac{b \tanh^2(c + dx)}{2d} - \frac{a \operatorname{coth}(c + dx)}{d}$$

[Out] $-a \operatorname{coth}(d*x+c)/d + 1/2*b*\tanh(d*x+c)^2/d$

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3663, 14}

$$\frac{b \tanh^2(c + dx)}{2d} - \frac{a \operatorname{coth}(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] `Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3), x]`

[Out] $-((a*\operatorname{Coth}[c + d*x])/d) + (b*\operatorname{Tanh}[c + d*x]^2)/(2*d)$

Rule 14

`Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]`

Rule 3663

`Int[sin[(e_)+(f_)*(x_)]^(m_)*((a_)+(b_)*((c_)*tan[(e_)+(f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]`

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^2(c + dx) (a + b \tanh^3(c + dx)) dx &= \frac{\operatorname{Subst}\left(\int \frac{a+bx^3}{x^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{a}{x^2} + bx\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{a \operatorname{coth}(c + dx)}{d} + \frac{b \tanh^2(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 1.00

$$-\frac{a \operatorname{coth}(c + dx)}{d} - \frac{b \operatorname{sech}^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] `Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3), x]`

[Out] $-((a*\operatorname{Coth}[c + d*x])/d) - (b*\operatorname{Sech}[c + d*x]^2)/(2*d)$

fricas [B] time = 1.30, size = 141, normalized size = 4.86

$$\frac{2 \left((2a + b) \cosh(dx + c)^2 + 2b \cosh(dx + c) \sinh(dx + c) + (2a + b) \sinh(dx + c) \right)}{d \cosh(dx + c)^4 + 6d \cosh(dx + c)^2 \sinh(dx + c)^2 + 4d \cosh(dx + c) \sinh(dx + c)^3 + d \sinh(dx + c)^4 + 4(d \cosh(dx + c) \sinh(dx + c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")

[Out] -2*((2*a + b)*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + (2*a + b)*sinh(d*x + c)^2 + 2*a - b)/(d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 + 4*(d*cosh(d*x + c)*sinh(d*x + c))^2 - d)

giac [A] time = 0.19, size = 45, normalized size = 1.55

$$\frac{2 \left(\frac{a}{e^{2dx+2c}-1} + \frac{be^{2(dx+2c)}}{(e^{2dx+2c}+1)^2} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="giac")

[Out] -2*(a/(e^(2*d*x + 2*c) - 1) + b*e^(2*d*x + 2*c)/(e^(2*d*x + 2*c) + 1)^2)/d

maple [A] time = 0.36, size = 26, normalized size = 0.90

$$\frac{-\coth(dx + c)a - \frac{b}{2\cosh(dx+c)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x)

[Out] 1/d*(-coth(d*x+c)*a-1/2*b/cosh(d*x+c)^2)

maxima [A] time = 0.30, size = 44, normalized size = 1.52

$$\frac{2a}{d(e^{-2dx-2c}-1)} - \frac{2b}{d(e^{dx+c} + e^{-dx-c})^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")

[Out] 2*a/(d*(e^(-2*d*x - 2*c) - 1)) - 2*b/(d*(e^(d*x + c) + e^(-d*x - c))^2)

mupad [B] time = 0.17, size = 79, normalized size = 2.72

$$\frac{2 \left(a + 2ae^{2c+2dx} + ae^{4c+4dx} - be^{2c+2dx} + be^{4c+4dx} \right)}{d \left(e^{2c+2dx} - 1 \right) \left(e^{2c+2dx} + 1 \right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tanh(c + d*x)^3)/sinh(c + d*x)^2,x)

[Out] -(2*(a + 2*a*exp(2*c + 2*d*x) + a*exp(4*c + 4*d*x) - b*exp(2*c + 2*d*x) + b*exp(4*c + 4*d*x)))/(d*(exp(2*c + 2*d*x) - 1)*(exp(2*c + 2*d*x) + 1)^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^3(c + dx)) \operatorname{csch}^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**3), x)

[Out] Integral((a + b*tanh(c + d*x)**3)*csch(c + d*x)**2, x)

3.55 $\int \operatorname{csch}^3(c + dx) (a + b \tanh^3(c + dx)) dx$

Optimal. Leaf size=71

$$\frac{a \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + \frac{b \tan^{-1}(\sinh(c + dx))}{2d} + \frac{b \tanh(c + dx) \operatorname{sech}(c + dx)}{2d}$$

[Out] $1/2*b*\arctan(\sinh(d*x+c))/d+1/2*a*\arctanh(\cosh(d*x+c))/d-1/2*a*\coth(d*x+c)*\operatorname{csch}(d*x+c)/d+1/2*b*\operatorname{sech}(d*x+c)*\tanh(d*x+c)/d$

Rubi [A] time = 0.09, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3666, 3768, 3770}

$$\frac{a \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + \frac{b \tan^{-1}(\sinh(c + dx))}{2d} + \frac{b \tanh(c + dx) \operatorname{sech}(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csch}[c + d*x]^3*(a + b*\text{Tanh}[c + d*x]^3), x]$

[Out] $(b*\text{ArcTan}[\text{Sinh}[c + d*x]])/(2*d) + (a*\text{ArcTanh}[\text{Cosh}[c + d*x]])/(2*d) - (a*\text{Cot h}[c + d*x]*\text{Csch}[c + d*x])/(2*d) + (b*\text{Sech}[c + d*x]*\text{Tanh}[c + d*x])/(2*d)$

Rule 3666

$\text{Int}[(d*\sin[e] + f*x)^m*(a + b*\tan[e + f*x])^n, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrig}[(d*\sin[e + f*x])^m*(a + b*(c*\tan[e + f*x])^n)^p, x], x] /;$ FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]

Rule 3768

$\text{Int}[(\csc[c] + d*x)^n*(b_*)^n, x_Symbol] \rightarrow -\text{Simp}[(b*\cos[c + d*x])*(b*\csc[c + d*x])^{n-1}/(d*(n-1)), x] + \text{Dist}[(b^2*(n-2))/(n-1), \text{Int}[(b*\csc[c + d*x])^{n-2}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

$\text{Int}[\csc[c] + d*x, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[\cos[c + d*x]]/d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^3(c + dx) (a + b \tanh^3(c + dx)) dx &= - \left(i \int (i a \operatorname{csch}^3(c + dx) + i b \operatorname{sech}^3(c + dx)) dx \right) \\ &= a \int \operatorname{csch}^3(c + dx) dx + b \int \operatorname{sech}^3(c + dx) dx \\ &= -\frac{a \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + \frac{b \operatorname{sech}(c + dx) \tanh(c + dx)}{2d} - \frac{1}{2} a \int \\ &= \frac{b \tan^{-1}(\sinh(c + dx))}{2d} + \frac{a \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a \coth(c + dx) \operatorname{csch}(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.03, size = 95, normalized size = 1.34

$$-\frac{a \operatorname{csch}^2\left(\frac{1}{2}(c + dx)\right)}{8d} - \frac{a \operatorname{sech}^2\left(\frac{1}{2}(c + dx)\right)}{8d} - \frac{a \log\left(\tanh\left(\frac{1}{2}(c + dx)\right)\right)}{2d} + \frac{b \tan^{-1}(\sinh(c + dx))}{2d} + \frac{b \tanh(c + dx) \operatorname{sech}(c + dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3), x]
```

```
[Out] (b*ArcTan[Sinh[c + d*x]])/(2*d) - (a*Csch[(c + d*x)/2]^2)/(8*d) - (a*Log[Tanh[(c + d*x)/2]])/(2*d) - (a*Sech[(c + d*x)/2]^2)/(8*d) + (b*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

fricas [B] time = 1.68, size = 1188, normalized size = 16.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3), x, algorithm="fricas")
```

```
[Out] -1/2*(2*(a - b)*cosh(d*x + c)^7 + 14*(a - b)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(a - b)*sinh(d*x + c)^7 + 6*(a + b)*cosh(d*x + c)^5 + 6*(7*(a - b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c)^5 + 10*(7*(a - b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c))*sinh(d*x + c)^4 + 6*(a - b)*cosh(d*x + c)^3 + 2*(35*(a - b)*cosh(d*x + c)^4 + 30*(a + b)*cosh(d*x + c)^2 + 3*a - 3*b)*sinh(d*x + c)^3 + 6*(7*(a - b)*cosh(d*x + c)^5 + 10*(a + b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(b*cosh(d*x + c)^8 + 56*b*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*b*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*b*cosh(d*x + c)*sinh(d*x + c)^7 + b*sinh(d*x + c)^8 - 2*b*cosh(d*x + c)^4 + 2*(35*b*cosh(d*x + c)^4 - b)*sinh(d*x + c)^4 + 8*(7*b*cosh(d*x + c)^5 - b*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*b*cosh(d*x + c)^6 - 3*b*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(b*cosh(d*x + c)^7 - b*cosh(d*x + c)^3)*sinh(d*x + c) + b)*arctan(cosh(d*x + c) + sinh(d*x + c)) + 2*(a + b)*cosh(d*x + c) - (a*cosh(d*x + c)^8 + 56*a*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*a*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*a*cosh(d*x + c)*sinh(d*x + c)^7 + a*sinh(d*x + c)^8 - 2*a*cosh(d*x + c)^4 + 2*(35*a*cosh(d*x + c)^4 - a)*sinh(d*x + c)^4 + 8*(7*a*cosh(d*x + c)^5 - a*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a*cosh(d*x + c)^6 - 3*a*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a*cosh(d*x + c)^7 - a*cosh(d*x + c)^3)*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + (a*cosh(d*x + c)^8 + 56*a*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*a*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*a*cosh(d*x + c)*sinh(d*x + c)^7 + a*sinh(d*x + c)^8 - 2*a*cosh(d*x + c)^4 + 2*(35*a*cosh(d*x + c)^4 - a)*sinh(d*x + c)^4 + 8*(7*a*cosh(d*x + c)^5 - a*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a*cosh(d*x + c)^6 - 3*a*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a*cosh(d*x + c)^7 - a*cosh(d*x + c)^3)*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(7*(a - b)*cosh(d*x + c)^6 + 15*(a + b)*cosh(d*x + c)^4 + 9*(a - b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 56*d*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 2*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*d*cosh(d*x + c)^6 - 3*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - d*cosh(d*x + c)^3)*sinh(d*x + c) + d)
```

giac [B] time = 0.17, size = 143, normalized size = 2.01

$$\frac{2b \arctan\left(e^{(dx+c)}\right) + a \log\left(e^{(dx+c)} + 1\right) - a \log\left(\left|e^{(dx+c)} - 1\right|\right) - \frac{2\left(ae^{(7dx+7c)} - be^{(7dx+7c)} + 3ae^{(5dx+5c)} + 3be^{(5dx+5c)} + 3ae^{(3dx+3c)} - 3be^{(3dx+3c)}\right)}{\left(e^{(4dx+4c)} - 1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3), x, algorithm="giac")
```

```
[Out] 1/2*(2*b*arctan(e^(d*x + c)) + a*log(e^(d*x + c) + 1) - a*log(abs(e^(d*x + c) - 1))) - 2*(a*e^(7*d*x + 7*c) - b*e^(7*d*x + 7*c) + 3*a*e^(5*d*x + 5*c) +
```

$$3*b*e^{(5*d*x + 5*c)} + 3*a*e^{(3*d*x + 3*c)} - 3*b*e^{(3*d*x + 3*c)} + a*e^{(d*x + c)} + b*e^{(d*x + c)} / (e^{(4*d*x + 4*c)} - 1)^2 / d$$

maple [A] time = 0.45, size = 62, normalized size = 0.87

$$-\frac{a \operatorname{coth}(dx+c) \operatorname{csch}(dx+c)}{2d} + \frac{a \operatorname{arctanh}(e^{dx+c})}{d} + \frac{b \operatorname{sech}(dx+c) \tanh(dx+c)}{2d} + \frac{b \operatorname{arctan}(e^{dx+c})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3),x)

[Out] -1/2*a*coth(d*x+c)*csch(d*x+c)/d+1/d*a*arctanh(exp(d*x+c))+1/2*b*sech(d*x+c)*tanh(d*x+c)/d+1/d*b*arctan(exp(d*x+c))

maxima [B] time = 0.42, size = 156, normalized size = 2.20

$$-b \left(\frac{\arctan(e^{(-dx-c)})}{d} - \frac{e^{(-dx-c)} - e^{(-3dx-3c)}}{d(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1)} \right) + \frac{1}{2} a \left(\frac{\log(e^{(-dx-c)} + 1)}{d} - \frac{\log(e^{(-dx-c)} - 1)}{d} + \frac{2(e^{(-dx-c)} - 1)}{d(2e^{(-2dx-2c)} - 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")

[Out] -b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 1/2*a*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))

mupad [B] time = 2.48, size = 173, normalized size = 2.44

$$\frac{a \ln(e^{c+dx} + 1)}{2d} - \frac{4e^{3c+3dx}(a-b)}{d} + \frac{4e^{c+dx}(a+b)}{d} - \frac{a \ln(e^{c+dx} - 1)}{2d} - \frac{e^{3c+3dx}(a-b)}{d} + \frac{3e^{c+dx}(a+b)}{d} - \frac{b \ln(e^{c+dx} - i) \operatorname{li}(b \ln(e^{c+dx} - i))}{2d} + \frac{b \ln(e^{c+dx} + i) \operatorname{li}(b \ln(e^{c+dx} + i))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tanh(c + d*x)^3)/sinh(c + d*x)^3,x)

[Out] (a*log(exp(c + d*x) + 1))/(2*d) - ((4*exp(3*c + 3*d*x)*(a - b))/d + (4*exp(c + d*x)*(a + b))/d)/(exp(8*c + 8*d*x) - 2*exp(4*c + 4*d*x) + 1) - (a*log(exp(c + d*x) - 1))/(2*d) - ((exp(3*c + 3*d*x)*(a - b))/d + (3*exp(c + d*x)*(a + b))/d)/(exp(4*c + 4*d*x) - 1) - (b*log(exp(c + d*x) - 1i)*1i)/(2*d) + (b*log(exp(c + d*x) + 1i)*1i)/(2*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^3(c + dx)) \operatorname{csch}^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**3),x)

[Out] Integral((a + b*tanh(c + d*x)**3)*csch(c + d*x)**3, x)

3.56 $\int \operatorname{csch}^4(c + dx) (a + b \tanh^3(c + dx)) dx$

Optimal. Leaf size=56

$$-\frac{a \operatorname{coth}^3(c + dx)}{3d} + \frac{a \operatorname{coth}(c + dx)}{d} - \frac{b \tanh^2(c + dx)}{2d} + \frac{b \log(\tanh(c + dx))}{d}$$

[Out] $a \operatorname{coth}(d*x+c)/d - 1/3*a \operatorname{coth}(d*x+c)^3/d + b*\ln(\tanh(d*x+c))/d - 1/2*b*\tanh(d*x+c)^2/d$

Rubi [A] time = 0.06, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3663, 1802}

$$-\frac{a \operatorname{coth}^3(c + dx)}{3d} + \frac{a \operatorname{coth}(c + dx)}{d} - \frac{b \tanh^2(c + dx)}{2d} + \frac{b \log(\tanh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3), x]

[Out] $(a*\operatorname{Coth}[c + d*x])/d - (a*\operatorname{Coth}[c + d*x]^3)/(3*d) + (b*\operatorname{Log}[\operatorname{Tanh}[c + d*x]])/d - (b*\operatorname{Tanh}[c + d*x]^2)/(2*d)$

Rule 1802

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 3663

Int[sin[(e_) + (f_)*(x_)]^(m_)*((a_) + (b_)*((c_)*tan[(e_) + (f_)*(x_)])^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^4(c + dx) (a + b \tanh^3(c + dx)) dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^{(a+bx^3)}}{x^4} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{a}{x^4} - \frac{a}{x^2} + \frac{b}{x} - bx\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{a \operatorname{coth}(c + dx)}{d} - \frac{a \operatorname{coth}^3(c + dx)}{3d} + \frac{b \log(\tanh(c + dx))}{d} - \frac{b \tanh^2(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.22, size = 74, normalized size = 1.32

$$\frac{2a \operatorname{coth}(c + dx)}{3d} - \frac{a \operatorname{coth}(c + dx) \operatorname{csch}^2(c + dx)}{3d} - \frac{b(-\operatorname{sech}^2(c + dx) - 2 \log(\sinh(c + dx)) + 2 \log(\cosh(c + dx)))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3), x]

[Out] $(2*a*\text{Coth}[c + d*x])/(3*d) - (a*\text{Coth}[c + d*x]*\text{Csch}[c + d*x]^2)/(3*d) - (b*(2*\text{Log}[\text{Cosh}[c + d*x]] - 2*\text{Log}[\text{Sinh}[c + d*x]] - \text{Sech}[c + d*x]^2))/(2*d)$

fricas [B] time = 0.74, size = 1739, normalized size = 31.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")`

[Out] $\frac{1}{3}(6*b*\cosh(d*x + c)^8 + 48*b*\cosh(d*x + c)*\sinh(d*x + c)^7 + 6*b*\sinh(d*x + c)^8 - 6*(2*a + 3*b)*\cosh(d*x + c)^6 + 6*(28*b*\cosh(d*x + c)^2 - 2*a - 3*b)*\sinh(d*x + c)^6 + 12*(28*b*\cosh(d*x + c)^3 - 3*(2*a + 3*b)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2*(10*a - 9*b)*\cosh(d*x + c)^4 + 2*(210*b*\cosh(d*x + c)^4 - 45*(2*a + 3*b)*\cosh(d*x + c)^2 - 10*a + 9*b)*\sinh(d*x + c)^4 + 8*(42*b*\cosh(d*x + c)^5 - 15*(2*a + 3*b)*\cosh(d*x + c)^3 - (10*a - 9*b)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 2*(2*a + 3*b)*\cosh(d*x + c)^2 + 2*(84*b*\cosh(d*x + c)^6 - 45*(2*a + 3*b)*\cosh(d*x + c)^4 - 6*(10*a - 9*b)*\cosh(d*x + c)^2 - 2*a - 3*b)*\sinh(d*x + c)^2 - 3*(b*\cosh(d*x + c)^{10} + 10*b*\cosh(d*x + c)*\sinh(d*x + c)^9 + b*\sinh(d*x + c)^{10} - b*\cosh(d*x + c)^8 + (45*b*\cosh(d*x + c)^2 - b)*\sinh(d*x + c)^8 + 8*(15*b*\cosh(d*x + c)^3 - b*\cosh(d*x + c))*\sinh(d*x + c)^7 - 2*b*\cosh(d*x + c)^6 + 2*(105*b*\cosh(d*x + c)^4 - 14*b*\cosh(d*x + c)^2 - b)*\sinh(d*x + c)^6 + 4*(63*b*\cosh(d*x + c)^5 - 14*b*\cosh(d*x + c)^3 - 3*b*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*b*\cosh(d*x + c)^4 + 2*(105*b*\cosh(d*x + c)^6 - 35*b*\cosh(d*x + c)^4 - 15*b*\cosh(d*x + c)^2 + b)*\sinh(d*x + c)^4 + 8*(15*b*\cosh(d*x + c)^7 - 7*b*\cosh(d*x + c)^5 - 5*b*\cosh(d*x + c)^3 + b*\cosh(d*x + c))*\sinh(d*x + c)^3 + b*\cosh(d*x + c)^2 + (45*b*\cosh(d*x + c)^8 - 28*b*\cosh(d*x + c)^6 - 30*b*\cosh(d*x + c)^4 + 12*b*\cosh(d*x + c)^2 + b)*\sinh(d*x + c)^2 + 2*(5*b*\cosh(d*x + c)^9 - 4*b*\cosh(d*x + c)^7 - 6*b*\cosh(d*x + c)^5 + 4*b*\cosh(d*x + c)^3 + b*\cosh(d*x + c))*\sinh(d*x + c) - b*\log(2*\cosh(d*x + c)/(\cosh(d*x + c) - \sinh(d*x + c))) + 3*(b*\cosh(d*x + c)^{10} + 10*b*\cosh(d*x + c)*\sinh(d*x + c)^9 + b*\sinh(d*x + c)^{10} - b*\cosh(d*x + c)^8 + (45*b*\cosh(d*x + c)^2 - b)*\sinh(d*x + c)^8 + 8*(15*b*\cosh(d*x + c)^3 - b*\cosh(d*x + c))*\sinh(d*x + c)^7 - 2*b*\cosh(d*x + c)^6 + 2*(105*b*\cosh(d*x + c)^4 - 14*b*\cosh(d*x + c)^2 - b)*\sinh(d*x + c)^6 + 4*(63*b*\cosh(d*x + c)^5 - 14*b*\cosh(d*x + c)^3 - 3*b*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*b*\cosh(d*x + c)^4 + 2*(105*b*\cosh(d*x + c)^6 - 35*b*\cosh(d*x + c)^4 - 15*b*\cosh(d*x + c)^2 + b)*\sinh(d*x + c)^4 + 8*(15*b*\cosh(d*x + c)^7 - 7*b*\cosh(d*x + c)^5 - 5*b*\cosh(d*x + c)^3 + b*\cosh(d*x + c))*\sinh(d*x + c)^3 + b*\cosh(d*x + c)^2 + (45*b*\cosh(d*x + c)^8 - 28*b*\cosh(d*x + c)^6 - 30*b*\cosh(d*x + c)^4 + 12*b*\cosh(d*x + c)^2 + b)*\sinh(d*x + c)^2 + 2*(5*b*\cosh(d*x + c)^9 - 4*b*\cosh(d*x + c)^7 - 6*b*\cosh(d*x + c)^5 + 4*b*\cosh(d*x + c)^3 + b*\cosh(d*x + c))*\sinh(d*x + c) - b*\log(2*\sinh(d*x + c)/(\cosh(d*x + c) - \sinh(d*x + c))) + 4*(12*b*\cosh(d*x + c)^7 - 9*(2*a + 3*b)*\cosh(d*x + c)^5 - 2*(10*a - 9*b)*\cosh(d*x + c)^3 - (2*a + 3*b)*\cosh(d*x + c))*\sinh(d*x + c) + 4*a)/(d*\cosh(d*x + c)^{10} + 10*d*\cosh(d*x + c)*\sinh(d*x + c)^9 + d*\sinh(d*x + c)^{10} - d*\cosh(d*x + c)^8 + (45*d*\cosh(d*x + c)^2 - d)*\sinh(d*x + c)^8 + 8*(15*d*\cosh(d*x + c)^3 - d*\cosh(d*x + c))*\sinh(d*x + c)^7 - 2*d*\cosh(d*x + c)^6 + 2*(105*d*\cosh(d*x + c)^4 - 14*d*\cosh(d*x + c)^2 - d)*\sinh(d*x + c)^6 + 4*(63*d*\cosh(d*x + c)^5 - 14*d*\cosh(d*x + c)^3 - 3*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*d*\cosh(d*x + c)^4 + 2*(105*d*\cosh(d*x + c)^6 - 35*d*\cosh(d*x + c)^4 - 15*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^4 + 8*(15*d*\cosh(d*x + c)^7 - 7*d*\cosh(d*x + c)^5 - 5*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c)^3 + d*\cosh(d*x + c)^2 + (45*d*\cosh(d*x + c)^8 - 28*d*\cosh(d*x + c)^6 - 30*d*\cosh(d*x + c)^4 + 12*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^2 + 2*(5*d*\cosh(d*x + c)^9 - 4*d*\cosh(d*x + c)^7 - 6*d*\cosh(d*x + c)^5 + 4*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c) - d)$

giac [B] time = 0.18, size = 149, normalized size = 2.66

$$\frac{6b \log(e^{(2dx+2c)} + 1) - 6b \log(|e^{(2dx+2c)} - 1|) - \frac{3(3be^{(4dx+4c)} + 10be^{(2dx+2c)} + 3b)}{(e^{(2dx+2c)} + 1)^2} + \frac{11be^{(6dx+6c)} - 33be^{(4dx+4c)} + 24ae^{(2dx+2c)}}{(e^{(2dx+2c)} - 1)^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3), x, algorithm="giac")

[Out] $-1/6*(6*b*\log(e^{(2*d*x + 2*c)} + 1) - 6*b*\log(\text{abs}(e^{(2*d*x + 2*c)} - 1)) - 3*(3*b*e^{(4*d*x + 4*c)} + 10*b*e^{(2*d*x + 2*c)} + 3*b)/(e^{(2*d*x + 2*c)} + 1)^2 + (11*b*e^{(6*d*x + 6*c)} - 33*b*e^{(4*d*x + 4*c)} + 24*a*e^{(2*d*x + 2*c)} + 33*b*e^{(2*d*x + 2*c)} - 8*a - 11*b)/(e^{(2*d*x + 2*c)} - 1)^3)/d$

maple [A] time = 0.38, size = 60, normalized size = 1.07

$$\frac{2a \coth(dx+c)}{3d} - \frac{a \coth(dx+c) \operatorname{csch}(dx+c)^2}{3d} + \frac{b}{2d \cosh(dx+c)^2} + \frac{b \ln(\tanh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3), x)

[Out] $2/3*a*\coth(d*x+c)/d - 1/3*a*\coth(d*x+c)*\operatorname{csch}(d*x+c)^2/d + 1/2/d*b/\cosh(d*x+c)^2 + b*\ln(\tanh(d*x+c))/d$

maxima [B] time = 0.40, size = 184, normalized size = 3.29

$$b \left(\frac{\log(e^{-dx-c} + 1)}{d} + \frac{\log(e^{-dx-c} - 1)}{d} - \frac{\log(e^{-2dx-2c} + 1)}{d} + \frac{2e^{-2dx-2c}}{d(2e^{-2dx-2c} + e^{-4dx-4c} + 1)} \right) + \frac{4}{3} a \left(\frac{1}{d(3e^{-2dx-2c} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3), x, algorithm="maxima")

[Out] $b*(\log(e^{-d*x - c} + 1)/d + \log(e^{-d*x - c} - 1)/d - \log(e^{-2*d*x - 2*c} + 1)/d + 2*e^{-2*d*x - 2*c}/(d*(2*e^{-2*d*x - 2*c} + e^{-4*d*x - 4*c} + 1))) + 4/3*a*(3*e^{-2*d*x - 2*c}/(d*(3*e^{-2*d*x - 2*c} - 3*e^{-4*d*x - 4*c} + e^{-6*d*x - 6*c} - 1)) - 1/(d*(3*e^{-2*d*x - 2*c} - 3*e^{-4*d*x - 4*c} + e^{-6*d*x - 6*c} - 1)))$

mupad [B] time = 1.24, size = 162, normalized size = 2.89

$$\frac{2b}{d(e^{2c+2dx} + 1)} - \frac{4a}{d(e^{4c+4dx} - 2e^{2c+2dx} + 1)} - \frac{2b}{d(2e^{2c+2dx} + e^{4c+4dx} + 1)} - \frac{8a}{3d(3e^{2c+2dx} - 3e^{4c+4dx} + e^{6c+6dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tanh(c + d*x)^3)/sinh(c + d*x)^4, x)

[Out] $(2*b)/(d*(\exp(2*c + 2*d*x) + 1)) - (4*a)/(d*(\exp(4*c + 4*d*x) - 2*\exp(2*c + 2*d*x) + 1)) - (2*b)/(d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1)) - (8*a)/(3*d*(3*\exp(2*c + 2*d*x) - 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) - 1)) - (2*a*\operatorname{atan}((b*\exp(2*c)*\exp(2*d*x)*(-d^2)^{(1/2)})/(d*(b^2)^{(1/2)})))/(d*(b^2)^{(1/2)})/(-d^2)^{(1/2)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^3(c + dx)) \operatorname{csch}^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**3),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)*csch(c + d*x)**4, x)
```


$$3.57 \quad \int \sinh^4(c + dx) \left(a + b \tanh^3(c + dx) \right)^2 dx$$

Optimal. Leaf size=170

$$\frac{\sinh(c + dx) \cosh^3(c + dx) (a^2 + 2ab \tanh(c + dx) + b^2)}{4d} - \frac{\sinh(c + dx) \cosh(c + dx) (5a^2 + 20ab \tanh(c + dx))}{8d}$$

[Out] $3/8*(a^2+21*b^2)*x+6*a*b*\ln(\cosh(d*x+c))/d-6*b^2*\tanh(d*x+c)/d-a*b*\tanh(d*x+c)^2/d-b^2*\tanh(d*x+c)^3/d-1/5*b^2*\tanh(d*x+c)^5/d+1/4*\cosh(d*x+c)^3*\sinh(d*x+c)*(a^2+b^2+2*a*b*\tanh(d*x+c))/d-1/8*\cosh(d*x+c)*\sinh(d*x+c)*(5*a^2+17*b^2+20*a*b*\tanh(d*x+c))/d$

Rubi [A] time = 0.30, antiderivative size = 206, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3663, 1804, 1802, 633, 31}

$$\frac{3(a^2 + 21b^2) \tanh(c + dx)}{8d} - \frac{3(a^2 + 16ab + 21b^2) \log(1 - \tanh(c + dx))}{16d} + \frac{3(a^2 - 16ab + 21b^2) \log(\tanh(c + dx))}{16d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^2,x]

[Out] $(-3*(a^2 + 16*a*b + 21*b^2)*\text{Log}[1 - \text{Tanh}[c + d*x]])/(16*d) + (3*(a^2 - 16*a*b + 21*b^2)*\text{Log}[1 + \text{Tanh}[c + d*x]])/(16*d) - (3*(a^2 + 21*b^2)*\text{Tanh}[c + d*x])/(8*d) - (3*a*b*\text{Tanh}[c + d*x]^2)/d - (b^2*\text{Tanh}[c + d*x]^3)/d - (b^2*\text{Tanh}[c + d*x]^5)/(5*d) - (\text{Sinh}[c + d*x]^2*\text{Tanh}[c + d*x]*(a^2 + 13*b^2 + 16*a*b*\text{Tanh}[c + d*x]))/(8*d) + (\text{Sinh}[c + d*x]^4*(2*a*b + (a^2 + b^2)*\text{Tanh}[c + d*x]))/(4*d)$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 633

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]

Rule 1802

Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1804

Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x] + Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]

Rule 3663

$$\begin{aligned}
& ^2) * \cosh(dx + c)^3 - (a^2 + 10ab + 9b^2) * \cosh(dx + c) * \sinh(dx + c)^{15} \\
& + 30(4(a^2 - 16ab + 21b^2)dx - 5a^2 - 30ab - 25b^2) * \cosh(dx + c)^{14} \\
& + 30(510(a^2 + 2ab + b^2) * \cosh(dx + c)^4 + 4(a^2 - 16ab + 21b^2)dx - 60(a^2 + 10ab + 9b^2) * \cosh(dx + c)^2 - 5a^2 - 30ab - 25b^2) * \sinh(dx + c)^{14} \\
& + 420(102(a^2 + 2ab + b^2) * \cosh(dx + c)^5 - 20(a^2 + 10ab + 9b^2) * \cosh(dx + c)^3 + (4(a^2 - 16ab + 21b^2)dx - 5a^2 - 30ab - 25b^2) * \cosh(dx + c) * \sinh(dx + c)^{13} \\
& + 10(60(a^2 - 16ab + 21b^2)dx - 31a^2 - 82ab + 501b^2) * \cosh(dx + c)^{12} + 10(9282(a^2 + 2ab + b^2) * \cosh(dx + c)^6 - 2730(a^2 + 10ab + 9b^2) * \cosh(dx + c)^4 \\
& + 60(a^2 - 16ab + 21b^2)dx + 273(4(a^2 - 16ab + 21b^2)dx - 5a^2 - 30ab - 25b^2) * \cosh(dx + c)^2 - 31a^2 - 82ab + 501b^2) * \sinh(dx + c)^{12} \\
& + 120(1326(a^2 + 2ab + b^2) * \cosh(dx + c)^7 - 546(a^2 + 10ab + 9b^2) * \cosh(dx + c)^5 + 91(4(a^2 - 16ab + 21b^2)dx - 5a^2 - 30ab - 25b^2) * \cosh(dx + c)^3 \\
& + (60(a^2 - 16ab + 21b^2)dx - 31a^2 - 82ab + 501b^2) * \cosh(dx + c) * \sinh(dx + c)^{11} + 60(20(a^2 - 16ab + 21b^2)dx - 3a^2 + 15ab + 307b^2) * \cosh(dx + c)^{10} \\
& + 30(7293(a^2 + 2ab + b^2) * \cosh(dx + c)^8 - 4004(a^2 + 10ab + 9b^2) * \cosh(dx + c)^6 + 1001(4(a^2 - 16ab + 21b^2)dx - 5a^2 - 30ab - 25b^2) * \cosh(dx + c)^4 \\
& + 40(a^2 - 16ab + 21b^2)dx + 22(60(a^2 - 16ab + 21b^2)dx - 31a^2 - 82ab + 501b^2) * \cosh(dx + c)^2 - 6a^2 + 30ab + 614b^2) * \sinh(dx + c)^{10} \\
& + 20(12155(a^2 + 2ab + b^2) * \cosh(dx + c)^9 - 8580(a^2 + 10ab + 9b^2) * \cosh(dx + c)^7 + 3003(4(a^2 - 16ab + 21b^2)dx - 5a^2 - 30ab - 25b^2) * \cosh(dx + c)^5 + 110(60(a^2 - 16ab + 21b^2)dx - 31a^2 - 82ab + 501b^2) * \cosh(dx + c)^3 \\
& + 30(20(a^2 - 16ab + 21b^2)dx - 3a^2 + 15ab + 307b^2) * \cosh(dx + c) * \sinh(dx + c)^9 + 60(20(a^2 - 16ab + 21b^2)dx + 3a^2 + 15ab + 461b^2) * \cosh(dx + c)^8 \\
& + 30(7293(a^2 + 2ab + b^2) * \cosh(dx + c)^{10} - 6435(a^2 + 10ab + 9b^2) * \cosh(dx + c)^8 + 3003(4(a^2 - 16ab + 21b^2)dx - 5a^2 - 30ab - 25b^2) * \cosh(dx + c)^6 + 165(60(a^2 - 16ab + 21b^2)dx - 31a^2 - 82ab + 501b^2) * \cosh(dx + c)^4 \\
& + 40(a^2 - 16ab + 21b^2)dx + 90(20(a^2 - 16ab + 21b^2)dx - 3a^2 + 15ab + 307b^2) * \cosh(dx + c)^2 + 6a^2 + 30ab + 922b^2) * \sinh(dx + c)^8 + 240(663(a^2 + 2ab + b^2) * \cosh(dx + c)^{11} - 715(a^2 + 10ab + 9b^2) * \cosh(dx + c)^9 + 429(4(a^2 - 16ab + 21b^2)dx - 5a^2 - 30ab - 25b^2) * \cosh(dx + c)^7 + 33(60(a^2 - 16ab + 21b^2)dx - 31a^2 - 82ab + 501b^2) * \cosh(dx + c)^5 + 30(20(a^2 - 16ab + 21b^2)dx - 3a^2 + 15ab + 307b^2) * \cosh(dx + c)^3 + 2(20(a^2 - 16ab + 21b^2)dx + 3a^2 + 15ab + 461b^2) * \cosh(dx + c) * \sinh(dx + c)^7 + 10(60(a^2 - 16ab + 21b^2)dx + 31a^2 - 82ab + 1803b^2) * \cosh(dx + c)^6 + 10(9282(a^2 + 2ab + b^2) * \cosh(dx + c)^{12} - 12012(a^2 + 10ab + 9b^2) * \cosh(dx + c)^{10} + 9009(4(a^2 - 16ab + 21b^2)dx - 5a^2 - 30ab - 25b^2) * \cosh(dx + c)^8 + 924(60(a^2 - 16ab + 21b^2)dx - 31a^2 - 82ab + 501b^2) * \cosh(dx + c)^6 + 1260(20(a^2 - 16ab + 21b^2)dx - 3a^2 + 15ab + 307b^2) * \cosh(dx + c)^4 + 60(a^2 - 16ab + 21b^2)dx + 168(20(a^2 - 16ab + 21b^2)dx + 3a^2 + 15ab + 461b^2) * \cosh(dx + c)^2 + 31a^2 - 82ab + 1803b^2) * \sinh(dx + c)^6 + 60(714(a^2 + 2ab + b^2) * \cosh(dx + c)^{13} - 1092(a^2 + 10ab + 9b^2) * \cosh(dx + c)^{11} + 1001(4(a^2 - 16ab + 21b^2)dx - 5a^2 - 30ab - 25b^2) * \cosh(dx + c)^9 + 132(60(a^2 - 16ab + 21b^2)dx - 31a^2 - 82ab + 501b^2) * \cosh(dx + c)^7 + 252(20(a^2 - 16ab + 21b^2)dx - 3a^2 + 15ab + 307b^2) * \cosh(dx + c)^5 + 56(20(a^2 - 16ab + 21b^2)dx + 3a^2 + 15ab + 461b^2) * \cosh(dx + c)^3 + (60(a^2 - 16ab + 21b^2)dx + 31a^2 - 82ab + 1803b^2) * \cosh(dx + c) * \sinh(dx + c)^5 + 6(20(a^2 - 16ab + 21b^2)dx + 25a^2 - 150ab + 893b^2) * \cosh(dx + c)^4 + 6(2550(a^2 + 2ab + b^2) * \cosh(dx + c)^{14} - 4550(a^2 + 10ab + 9b^2) * \cosh(dx + c)^{12} + 5005(4(a^2 - 16ab + 21b^2)dx - 5a^2 - 30ab - 25b^2) * \cosh(dx + c)^{10} + 825(60(a^2 - 16ab + 21b^2)dx - 31a^2 - 82ab + 501b^2) * \cosh(dx + c)^8 + 2100(20(a^2 - 16ab + 21b^2)dx - 3a^2 + 15ab + 307b^2) * \cosh(dx + c)^6 + 700(20(a^2 - 16ab + 21b^2)dx + 3a^2 + 15ab + 461b^2) * \cosh(dx + c)^4 + 20(a^2
\end{aligned}$$

$$\begin{aligned}
& - 16*a*b + 21*b^2)*d*x + 25*(60*(a^2 - 16*a*b + 21*b^2)*d*x + 31*a^2 - 82*a \\
& *b + 1803*b^2)*\cosh(d*x + c)^2 + 25*a^2 - 150*a*b + 893*b^2)*\sinh(d*x + c)^4 \\
& + 8*(510*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)^15 - 1050*(a^2 + 10*a*b + 9*b^2) \\
& *\cosh(d*x + c)^13 + 1365*(4*(a^2 - 16*a*b + 21*b^2)*d*x - 5*a^2 - 30*a*b \\
& - 25*b^2)*\cosh(d*x + c)^11 + 275*(60*(a^2 - 16*a*b + 21*b^2)*d*x - 31*a^2 - \\
& 82*a*b + 501*b^2)*\cosh(d*x + c)^9 + 900*(20*(a^2 - 16*a*b + 21*b^2)*d*x - \\
& 3*a^2 + 15*a*b + 307*b^2)*\cosh(d*x + c)^7 + 420*(20*(a^2 - 16*a*b + 21*b^2) \\
& *d*x + 3*a^2 + 15*a*b + 461*b^2)*\cosh(d*x + c)^5 + 25*(60*(a^2 - 16*a*b + 2 \\
& 1*b^2)*d*x + 31*a^2 - 82*a*b + 1803*b^2)*\cosh(d*x + c)^3 + 3*(20*(a^2 - 16* \\
& a*b + 21*b^2)*d*x + 25*a^2 - 150*a*b + 893*b^2)*\cosh(d*x + c))*\sinh(d*x + c \\
&)^3 + 15*(a^2 - 10*a*b + 9*b^2)*\cosh(d*x + c)^2 + 3*(255*(a^2 + 2*a*b + b^2) \\
&)*\cosh(d*x + c)^16 - 600*(a^2 + 10*a*b + 9*b^2)*\cosh(d*x + c)^14 + 910*(4*(\\
& a^2 - 16*a*b + 21*b^2)*d*x - 5*a^2 - 30*a*b - 25*b^2)*\cosh(d*x + c)^12 + 22 \\
& 0*(60*(a^2 - 16*a*b + 21*b^2)*d*x - 31*a^2 - 82*a*b + 501*b^2)*\cosh(d*x + c \\
&)^10 + 900*(20*(a^2 - 16*a*b + 21*b^2)*d*x - 3*a^2 + 15*a*b + 307*b^2)*\cosh \\
& (d*x + c)^8 + 560*(20*(a^2 - 16*a*b + 21*b^2)*d*x + 3*a^2 + 15*a*b + 461*b^ \\
& 2)*\cosh(d*x + c)^6 + 50*(60*(a^2 - 16*a*b + 21*b^2)*d*x + 31*a^2 - 82*a*b + \\
& 1803*b^2)*\cosh(d*x + c)^4 + 12*(20*(a^2 - 16*a*b + 21*b^2)*d*x + 25*a^2 - \\
& 150*a*b + 893*b^2)*\cosh(d*x + c)^2 + 5*a^2 - 50*a*b + 45*b^2)*\sinh(d*x + c) \\
& ^2 - 5*a^2 + 10*a*b - 5*b^2 + 1920*(a*b*\cosh(d*x + c)^14 + 14*a*b*\cosh(d*x \\
& + c)*\sinh(d*x + c)^13 + a*b*\sinh(d*x + c)^14 + 5*a*b*\cosh(d*x + c)^12 + (91 \\
& *a*b*\cosh(d*x + c)^2 + 5*a*b)*\sinh(d*x + c)^12 + 10*a*b*\cosh(d*x + c)^10 + \\
& 4*(91*a*b*\cosh(d*x + c)^3 + 15*a*b*\cosh(d*x + c))*\sinh(d*x + c)^11 + (1001* \\
& a*b*\cosh(d*x + c)^4 + 330*a*b*\cosh(d*x + c)^2 + 10*a*b)*\sinh(d*x + c)^10 + \\
& 10*a*b*\cosh(d*x + c)^8 + 2*(1001*a*b*\cosh(d*x + c)^5 + 550*a*b*\cosh(d*x + c \\
&)^3 + 50*a*b*\cosh(d*x + c))*\sinh(d*x + c)^9 + (3003*a*b*\cosh(d*x + c)^6 + 2 \\
& 475*a*b*\cosh(d*x + c)^4 + 450*a*b*\cosh(d*x + c)^2 + 10*a*b)*\sinh(d*x + c)^8 \\
& + 5*a*b*\cosh(d*x + c)^6 + 8*(429*a*b*\cosh(d*x + c)^7 + 495*a*b*\cosh(d*x + \\
& c)^5 + 150*a*b*\cosh(d*x + c)^3 + 10*a*b*\cosh(d*x + c))*\sinh(d*x + c)^7 + (3 \\
& 003*a*b*\cosh(d*x + c)^8 + 4620*a*b*\cosh(d*x + c)^6 + 2100*a*b*\cosh(d*x + c) \\
& ^4 + 280*a*b*\cosh(d*x + c)^2 + 5*a*b)*\sinh(d*x + c)^6 + a*b*\cosh(d*x + c)^4 \\
& + 2*(1001*a*b*\cosh(d*x + c)^9 + 1980*a*b*\cosh(d*x + c)^7 + 1260*a*b*\cosh(d \\
& *x + c)^5 + 280*a*b*\cosh(d*x + c)^3 + 15*a*b*\cosh(d*x + c))*\sinh(d*x + c)^5 \\
& + (1001*a*b*\cosh(d*x + c)^10 + 2475*a*b*\cosh(d*x + c)^8 + 2100*a*b*\cosh(d* \\
& x + c)^6 + 700*a*b*\cosh(d*x + c)^4 + 75*a*b*\cosh(d*x + c)^2 + a*b)*\sinh(d*x \\
& + c)^4 + 4*(91*a*b*\cosh(d*x + c)^11 + 275*a*b*\cosh(d*x + c)^9 + 300*a*b*co \\
& sh(d*x + c)^7 + 140*a*b*\cosh(d*x + c)^5 + 25*a*b*\cosh(d*x + c)^3 + a*b*\cosh \\
& (d*x + c))*\sinh(d*x + c)^3 + (91*a*b*\cosh(d*x + c)^12 + 330*a*b*\cosh(d*x + \\
& c)^10 + 450*a*b*\cosh(d*x + c)^8 + 280*a*b*\cosh(d*x + c)^6 + 75*a*b*\cosh(d*x \\
& + c)^4 + 6*a*b*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 2*(7*a*b*\cosh(d*x + c)^1 \\
& 3 + 30*a*b*\cosh(d*x + c)^11 + 50*a*b*\cosh(d*x + c)^9 + 40*a*b*\cosh(d*x + c) \\
& ^7 + 15*a*b*\cosh(d*x + c)^5 + 2*a*b*\cosh(d*x + c)^3)*\sinh(d*x + c))*\log(2*c \\
& osh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 6*(15*(a^2 + 2*a*b + b^2)*c \\
& osh(d*x + c)^17 - 40*(a^2 + 10*a*b + 9*b^2)*\cosh(d*x + c)^15 + 70*(4*(a^2 - \\
& 16*a*b + 21*b^2)*d*x - 5*a^2 - 30*a*b - 25*b^2)*\cosh(d*x + c)^13 + 20*(60* \\
& (a^2 - 16*a*b + 21*b^2)*d*x - 31*a^2 - 82*a*b + 501*b^2)*\cosh(d*x + c)^11 + \\
& 100*(20*(a^2 - 16*a*b + 21*b^2)*d*x - 3*a^2 + 15*a*b + 307*b^2)*\cosh(d*x + \\
& c)^9 + 80*(20*(a^2 - 16*a*b + 21*b^2)*d*x + 3*a^2 + 15*a*b + 461*b^2)*\cosh \\
& (d*x + c)^7 + 10*(60*(a^2 - 16*a*b + 21*b^2)*d*x + 31*a^2 - 82*a*b + 1803*b \\
& ^2)*\cosh(d*x + c)^5 + 4*(20*(a^2 - 16*a*b + 21*b^2)*d*x + 25*a^2 - 150*a*b \\
& + 893*b^2)*\cosh(d*x + c)^3 + 5*(a^2 - 10*a*b + 9*b^2)*\cosh(d*x + c))*\sinh(d \\
& *x + c))/(d*cosh(d*x + c)^14 + 14*d*cosh(d*x + c)*\sinh(d*x + c)^13 + d*\sinh \\
& (d*x + c)^14 + 5*d*cosh(d*x + c)^12 + (91*d*cosh(d*x + c)^2 + 5*d)*\sinh(d*x \\
& + c)^12 + 4*(91*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*\sinh(d*x + c)^11 + \\
& 10*d*cosh(d*x + c)^10 + (1001*d*cosh(d*x + c)^4 + 330*d*cosh(d*x + c)^2 + \\
& 10*d)*\sinh(d*x + c)^10 + 2*(1001*d*cosh(d*x + c)^5 + 550*d*cosh(d*x + c)^3 \\
& + 50*d*cosh(d*x + c))*\sinh(d*x + c)^9 + 10*d*cosh(d*x + c)^8 + (3003*d*cosh \\
& (d*x + c)^6 + 2475*d*cosh(d*x + c)^4 + 450*d*cosh(d*x + c)^2 + 10*d)*\sinh(d \\
& *x + c)^8 + 8*(429*d*cosh(d*x + c)^7 + 495*d*cosh(d*x + c)^5 + 150*d*cosh(d
\end{aligned}$$

$$\begin{aligned}
 & *x + c)^3 + 10*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 5*d*\cosh(d*x + c)^6 + (30 \\
 & 03*d*\cosh(d*x + c)^8 + 4620*d*\cosh(d*x + c)^6 + 2100*d*\cosh(d*x + c)^4 + 28 \\
 & 0*d*\cosh(d*x + c)^2 + 5*d)*\sinh(d*x + c)^6 + 2*(1001*d*\cosh(d*x + c)^9 + 19 \\
 & 80*d*\cosh(d*x + c)^7 + 1260*d*\cosh(d*x + c)^5 + 280*d*\cosh(d*x + c)^3 + 15* \\
 & d*\cosh(d*x + c))*\sinh(d*x + c)^5 + d*\cosh(d*x + c)^4 + (1001*d*\cosh(d*x + c) \\
 &)^10 + 2475*d*\cosh(d*x + c)^8 + 2100*d*\cosh(d*x + c)^6 + 700*d*\cosh(d*x + c) \\
 &)^4 + 75*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^4 + 4*(91*d*\cosh(d*x + c)^11 \\
 & + 275*d*\cosh(d*x + c)^9 + 300*d*\cosh(d*x + c)^7 + 140*d*\cosh(d*x + c)^5 + 2 \\
 & 5*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c)^3 + (91*d*\cosh(d*x + c) \\
 &)^12 + 330*d*\cosh(d*x + c)^10 + 450*d*\cosh(d*x + c)^8 + 280*d*\cosh(d*x + c) \\
 & ^6 + 75*d*\cosh(d*x + c)^4 + 6*d*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 2*(7*d*c \\
 & osh(d*x + c)^13 + 30*d*\cosh(d*x + c)^11 + 50*d*\cosh(d*x + c)^9 + 40*d*\cosh(\\
 & d*x + c)^7 + 15*d*\cosh(d*x + c)^5 + 2*d*\cosh(d*x + c)^3)*\sinh(d*x + c))
 \end{aligned}$$

giac [B] time = 0.71, size = 376, normalized size = 2.21

$$120(a^2 - 16ab + 21b^2)dx + 1920ab \log(e^{2dx+2c} + 1) - 5(18a^2e^{4dx+4c} - 288abe^{4dx+4c} + 378b^2e^{4dx+4c})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")

[Out] 1/320*(120*(a^2 - 16*a*b + 21*b^2)*d*x + 1920*a*b*log(e^(2*d*x + 2*c) + 1) - 5*(18*a^2*e^(4*d*x + 4*c) - 288*a*b*e^(4*d*x + 4*c) + 378*b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) + 40*a*b*e^(2*d*x + 2*c) - 32*b^2*e^(2*d*x + 2*c) + a^2 - 2*a*b + b^2)*e^(-4*d*x - 4*c) + 5*(a^2*e^(4*d*x + 36*c) + 2*a*b*e^(4*d*x + 36*c) + b^2*e^(4*d*x + 36*c) - 8*a^2*e^(2*d*x + 34*c) - 40*a*b*e^(2*d*x + 34*c) - 32*b^2*e^(2*d*x + 34*c))*e^(-32*c) - 32*(137*a*b*e^(10*d*x + 10*c) + 645*a*b*e^(8*d*x + 8*c) - 200*b^2*e^(8*d*x + 8*c) + 1250*a*b*e^(6*d*x + 6*c) - 600*b^2*e^(6*d*x + 6*c) + 1250*a*b*e^(4*d*x + 4*c) - 840*b^2*e^(4*d*x + 4*c) + 645*a*b*e^(2*d*x + 2*c) - 520*b^2*e^(2*d*x + 2*c) + 137*a*b - 144*b^2)/(e^(2*d*x + 2*c) + 1)^5/d

maple [A] time = 0.32, size = 243, normalized size = 1.43

$$\frac{a^2 \cosh(dx + c) (\sinh^3(dx + c))}{4d} - \frac{3a^2 \cosh(dx + c) \sinh(dx + c)}{8d} + \frac{3a^2 x}{8} + \frac{3a^2 c}{8d} + \frac{ab (\sinh^6(dx + c))}{2d \cosh(dx + c)^2} - \frac{3ab (\sinh^6(dx + c))}{2d \cosh(dx + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x)

[Out] 1/4/d*a^2*cosh(d*x+c)*sinh(d*x+c)^3-3/8*a^2*cosh(d*x+c)*sinh(d*x+c)/d+3/8*a^2*x+3/8/d*a^2*c+1/2/d*a*b*sinh(d*x+c)^6/cosh(d*x+c)^2-3/2/d*a*b*sinh(d*x+c)^4/cosh(d*x+c)^2+6*a*b*ln(cosh(d*x+c))/d-3*a*b*tanh(d*x+c)^2/d+1/4/d*b^2*sinh(d*x+c)^9/cosh(d*x+c)^5-9/8/d*b^2*sinh(d*x+c)^7/cosh(d*x+c)^5+63/8*b^2*x+63/8/d*c*b^2-63/8*b^2*tanh(d*x+c)/d-21/8*b^2*tanh(d*x+c)^3/d-63/40*b^2*tanh(d*x+c)^5/d

maxima [B] time = 0.41, size = 379, normalized size = 2.23

$$\frac{1}{64} a^2 \left(24x + \frac{e^{4dx+4c}}{d} - \frac{8e^{2dx+2c}}{d} + \frac{8e^{-2dx-2c}}{d} - \frac{e^{-4dx-4c}}{d} \right) + \frac{1}{320} b^2 \left(\frac{2520(dx+c)}{d} + \frac{5(32e^{-2dx-2c} - e^{4dx+4c})}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")

[Out] 1/64*a^2*(24*x + e^(4*d*x + 4*c))/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d + 1/320*b^2*(2520*(d*x + c)/d + 5*(32*e^(-2*d*x - 2*c) - e^(4*d*x + 4*c))/d)

$- 2*c) - e^{(-4*d*x - 4*c)}/d - (135*e^{(-2*d*x - 2*c)} + 5358*e^{(-4*d*x - 4*c)}$
 $) + 18190*e^{(-6*d*x - 6*c)} + 28455*e^{(-8*d*x - 8*c)} + 19995*e^{(-10*d*x - 10}$
 $*c) + 6560*e^{(-12*d*x - 12*c)} - 5)/(d*(e^{(-4*d*x - 4*c)} + 5*e^{(-6*d*x - 6*c)}$
 $) + 10*e^{(-8*d*x - 8*c)} + 10*e^{(-10*d*x - 10*c)} + 5*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c))}$
 $)) + 1/32*a*b*(192*(d*x + c)/d - (20*e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)}/d + 192*\log(e^{(-2*d*x - 2*c)} + 1)/d - (18*e^{(-2*d*x - 2*c)}$
 $+ 39*e^{(-4*d*x - 4*c)} - 108*e^{(-6*d*x - 6*c)} - 1)/(d*(e^{(-4*d*x - 4*c)} + 2*$
 $e^{(-6*d*x - 6*c)} + e^{(-8*d*x - 8*c))))$

mupad [B] time = 0.44, size = 359, normalized size = 2.11

$$x \left(\frac{3a^2}{8} - 6ab + \frac{63b^2}{8} \right) + \frac{4(5b^2 + ab)}{d(e^{2c+2dx} + 1)} + \frac{e^{-2c-2dx}(a^2 - 5ab + 4b^2)}{8d} - \frac{e^{2c+2dx}(a^2 + 5ab + 4b^2)}{8d} + \frac{e^{4c+4dx}(a^2 + 5ab + 4b^2)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^3)^2,x)

[Out] $x*((3*a^2)/8 - 6*a*b + (63*b^2)/8) + (4*(a*b + 5*b^2))/(d*(exp(2*c + 2*d*x) + 1)) + (exp(-2*c - 2*d*x)*(a^2 - 5*a*b + 4*b^2))/(8*d) - (exp(2*c + 2*d*x)*(5*a*b + a^2 + 4*b^2))/(8*d) + (exp(4*c + 4*d*x)*(a + b)^2)/(64*d) - (4*(a*b + 5*b^2))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (24*b^2)/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (exp(-4*c - 4*d*x)*(a - b)^2)/(64*d) - (16*b^2)/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (32*b^2)/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (6*a*b*log(exp(2*c)*exp(2*d*x) + 1))/d$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**3)**2,x)

[Out] Timed out

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*SIN[e + f*x])/ff], x
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3666

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_.) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)]^(n_))^(p_.), x_Symbol] := Int[ExpandTrig[(d*sin[e + f*x])^m*(a
+ b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \sinh^3(c + dx) (a + b \tanh^3(c + dx))^2 dx &= i \int (-ia^2 \sinh^3(c + dx) - 2iab \sinh^3(c + dx) \tanh^3(c + dx) - ib^2 \sinh^3(c + dx) \tanh^6(c + dx)) dx \\
&= a^2 \int \sinh^3(c + dx) dx + (2ab) \int \sinh^3(c + dx) \tanh^3(c + dx) dx + \frac{b^2}{a} \int \sinh^3(c + dx) \tanh^6(c + dx) dx \\
&= -\frac{a^2 \operatorname{Subst}\left(\int (1 - x^2) dx, x, \cosh(c + dx)\right)}{d} + \frac{(2ab) \operatorname{Subst}\left(\int \frac{x^6}{(1+x^2)^2} dx, x, \cosh(c + dx)\right)}{a} \\
&= -\frac{a^2 \cosh(c + dx)}{d} + \frac{a^2 \cosh^3(c + dx)}{3d} - \frac{ab \sinh^3(c + dx) \tanh^2(c + dx)}{d} \\
&= -\frac{a^2 \cosh(c + dx)}{d} - \frac{4b^2 \cosh(c + dx)}{d} + \frac{a^2 \cosh^3(c + dx)}{3d} + \frac{b^2 \cosh^5(c + dx)}{5d} \\
&= -\frac{a^2 \cosh(c + dx)}{d} - \frac{4b^2 \cosh(c + dx)}{d} + \frac{a^2 \cosh^3(c + dx)}{3d} + \frac{b^2 \cosh^5(c + dx)}{5d} \\
&= \frac{5ab \tan^{-1}(\sinh(c + dx))}{d} - \frac{a^2 \cosh(c + dx)}{d} - \frac{4b^2 \cosh(c + dx)}{d} + \frac{a^2 \cosh^3(c + dx)}{3d} + \frac{b^2 \cosh^5(c + dx)}{5d}
\end{aligned}$$

Mathematica [A] time = 0.78, size = 121, normalized size = 0.66

$$\frac{-45(a^2 + 5b^2) \cosh(c + dx) + 5(a^2 + b^2) \cosh(3(c + dx)) - 2b \left(30 \operatorname{sech}(c + dx) (a \tanh(c + dx) + 6b) - 5a \left(-27 \operatorname{sech}^3(c + dx) + 3 \operatorname{sech}(c + dx) \right) \right)}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (-45*(a^2 + 5*b^2)*Cosh[c + d*x] + 5*(a^2 + b^2)*Cosh[3*(c + d*x)] - 2*b*(-
40*b*Sech[c + d*x]^3 + 6*b*Sech[c + d*x]^5 - 5*a*(60*ArcTan[Tanh[(c + d*x)/
2]] - 27*Sinh[c + d*x] + Sinh[3*(c + d*x)]) + 30*Sech[c + d*x]*(6*b + a*Tan
h[c + d*x])))/(60*d)
```

fricas [B] time = 0.66, size = 3341, normalized size = 18.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")

[Out] $\frac{1}{120} \cdot (5 \cdot (a^2 + 2 \cdot a \cdot b + b^2) \cdot \cosh(d \cdot x + c)^{16} + 80 \cdot (a^2 + 2 \cdot a \cdot b + b^2) \cdot \cosh(d \cdot x + c) \cdot \sinh(d \cdot x + c)^{15} + 5 \cdot (a^2 + 2 \cdot a \cdot b + b^2) \cdot \sinh(d \cdot x + c)^{16} - 20 \cdot (a^2 + 11 \cdot a \cdot b + 10 \cdot b^2) \cdot \cosh(d \cdot x + c)^{14} + 20 \cdot (30 \cdot (a^2 + 2 \cdot a \cdot b + b^2) \cdot \cosh(d \cdot x + c)^2 - a^2 - 11 \cdot a \cdot b - 10 \cdot b^2) \cdot \sinh(d \cdot x + c)^{14} + 280 \cdot (10 \cdot (a^2 + 2 \cdot a \cdot b + b^2) \cdot \cosh(d \cdot x + c)^3 - (a^2 + 11 \cdot a \cdot b + 10 \cdot b^2) \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^{13} - 20 \cdot (11 \cdot a^2 + 61 \cdot a \cdot b + 137 \cdot b^2) \cdot \cosh(d \cdot x + c)^{12} + 20 \cdot (455 \cdot (a^2 + 2 \cdot a \cdot b + b^2) \cdot \cosh(d \cdot x + c)^4 - 91 \cdot (a^2 + 11 \cdot a \cdot b + 10 \cdot b^2) \cdot \cosh(d \cdot x + c)^2 - 11 \cdot a^2 - 61 \cdot a \cdot b - 137 \cdot b^2) \cdot \sinh(d \cdot x + c)^{12} + 80 \cdot (273 \cdot (a^2 + 2 \cdot a \cdot b + b^2) \cdot \cosh(d \cdot x + c)^5 - 91 \cdot (a^2 + 11 \cdot a \cdot b + 10 \cdot b^2) \cdot \cosh(d \cdot x + c)^3 - 3 \cdot (11 \cdot a^2 + 61 \cdot a \cdot b + 137 \cdot b^2) \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^{11} - 20 \cdot (31 \cdot a^2 + 87 \cdot a \cdot b + 390 \cdot b^2) \cdot \cosh(d \cdot x + c)^{10} + 20 \cdot (2002 \cdot (a^2 + 2 \cdot a \cdot b + b^2) \cdot \cosh(d \cdot x + c)^6 - 1001 \cdot (a^2 + 11 \cdot a \cdot b + 10 \cdot b^2) \cdot \cosh(d \cdot x + c)^4 - 66 \cdot (11 \cdot a^2 + 61 \cdot a \cdot b + 137 \cdot b^2) \cdot \cosh(d \cdot x + c)^2 - 31 \cdot a^2 - 87 \cdot a \cdot b - 390 \cdot b^2) \cdot \sinh(d \cdot x + c)^{10} + 40 \cdot (1430 \cdot (a^2 + 2 \cdot a \cdot b + b^2) \cdot \cosh(d \cdot x + c)^7 - 1001 \cdot (a^2 + 11 \cdot a \cdot b + 10 \cdot b^2) \cdot \cosh(d \cdot x + c)^5 - 110 \cdot (11 \cdot a^2 + 61 \cdot a \cdot b + 137 \cdot b^2) \cdot \cosh(d \cdot x + c)^3 - 5 \cdot (31 \cdot a^2 + 87 \cdot a \cdot b + 390 \cdot b^2) \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^9 - 2 \cdot (425 \cdot a^2 + 5649 \cdot b^2) \cdot \cosh(d \cdot x + c)^8 + 2 \cdot (32175 \cdot (a^2 + 2 \cdot a \cdot b + b^2) \cdot \cosh(d \cdot x + c)^8 - 30030 \cdot (a^2 + 11 \cdot a \cdot b + 10 \cdot b^2) \cdot \cosh(d \cdot x + c)^6 - 4950 \cdot (11 \cdot a^2 + 61 \cdot a \cdot b + 137 \cdot b^2) \cdot \cosh(d \cdot x + c)^4 - 450 \cdot (31 \cdot a^2 + 87 \cdot a \cdot b + 390 \cdot b^2) \cdot \cosh(d \cdot x + c)^2 - 425 \cdot a^2 - 5649 \cdot b^2) \cdot \sinh(d \cdot x + c)^8 + 16 \cdot (3575 \cdot (a^2 + 2 \cdot a \cdot b + b^2) \cdot \cosh(d \cdot x + c)^9 - 4290 \cdot (a^2 + 11 \cdot a \cdot b + 10 \cdot b^2) \cdot \cosh(d \cdot x + c)^7 - 990 \cdot (11 \cdot a^2 + 61 \cdot a \cdot b + 137 \cdot b^2) \cdot \cosh(d \cdot x + c)^5 - 150 \cdot (31 \cdot a^2 + 87 \cdot a \cdot b + 390 \cdot b^2) \cdot \cosh(d \cdot x + c)^3 - (425 \cdot a^2 + 5649 \cdot b^2) \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^7 - 20 \cdot (31 \cdot a^2 - 87 \cdot a \cdot b + 390 \cdot b^2) \cdot \cosh(d \cdot x + c)^6 + 4 \cdot (10010 \cdot (a^2 + 2 \cdot a \cdot b + b^2) \cdot \cosh(d \cdot x + c)^{10} - 15015 \cdot (a^2 + 11 \cdot a \cdot b + 10 \cdot b^2) \cdot \cosh(d \cdot x + c)^8 - 4620 \cdot (11 \cdot a^2 + 61 \cdot a \cdot b + 137 \cdot b^2) \cdot \cosh(d \cdot x + c)^6 - 1050 \cdot (31 \cdot a^2 + 87 \cdot a \cdot b + 390 \cdot b^2) \cdot \cosh(d \cdot x + c)^4 - 14 \cdot (425 \cdot a^2 + 5649 \cdot b^2) \cdot \cosh(d \cdot x + c)^2 - 155 \cdot a^2 + 435 \cdot a \cdot b - 1950 \cdot b^2) \cdot \sinh(d \cdot x + c)^6 + 8 \cdot (2730 \cdot (a^2 + 2 \cdot a \cdot b + b^2) \cdot \cosh(d \cdot x + c)^{11} - 5005 \cdot (a^2 + 11 \cdot a \cdot b + 10 \cdot b^2) \cdot \cosh(d \cdot x + c)^9 - 1980 \cdot (11 \cdot a^2 + 61 \cdot a \cdot b + 137 \cdot b^2) \cdot \cosh(d \cdot x + c)^7 - 630 \cdot (31 \cdot a^2 + 87 \cdot a \cdot b + 390 \cdot b^2) \cdot \cosh(d \cdot x + c)^5 - 14 \cdot (425 \cdot a^2 + 5649 \cdot b^2) \cdot \cosh(d \cdot x + c)^3 - 15 \cdot (31 \cdot a^2 - 87 \cdot a \cdot b + 390 \cdot b^2) \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^5 - 20 \cdot (11 \cdot a^2 - 61 \cdot a \cdot b + 137 \cdot b^2) \cdot \cosh(d \cdot x + c)^4 + 20 \cdot (455 \cdot (a^2 + 2 \cdot a \cdot b + b^2) \cdot \cosh(d \cdot x + c)^{12} - 1001 \cdot (a^2 + 11 \cdot a \cdot b + 10 \cdot b^2) \cdot \cosh(d \cdot x + c)^{10} - 495 \cdot (11 \cdot a^2 + 61 \cdot a \cdot b + 137 \cdot b^2) \cdot \cosh(d \cdot x + c)^8 - 210 \cdot (31 \cdot a^2 + 87 \cdot a \cdot b + 390 \cdot b^2) \cdot \cosh(d \cdot x + c)^6 - 7 \cdot (425 \cdot a^2 + 5649 \cdot b^2) \cdot \cosh(d \cdot x + c)^4 - 15 \cdot (31 \cdot a^2 - 87 \cdot a \cdot b + 390 \cdot b^2) \cdot \cosh(d \cdot x + c)^2 - 11 \cdot a^2 + 61 \cdot a \cdot b - 137 \cdot b^2) \cdot \sinh(d \cdot x + c)^4 + 16 \cdot (175 \cdot (a^2 + 2 \cdot a \cdot b + b^2) \cdot \cosh(d \cdot x + c)^{13} - 455 \cdot (a^2 + 11 \cdot a \cdot b + 10 \cdot b^2) \cdot \cosh(d \cdot x + c)^{11} - 275 \cdot (11 \cdot a^2 + 61 \cdot a \cdot b + 137 \cdot b^2) \cdot \cosh(d \cdot x + c)^9 - 150 \cdot (31 \cdot a^2 + 87 \cdot a \cdot b + 390 \cdot b^2) \cdot \cosh(d \cdot x + c)^7 - 7 \cdot (425 \cdot a^2 + 5649 \cdot b^2) \cdot \cosh(d \cdot x + c)^5 - 25 \cdot (31 \cdot a^2 - 87 \cdot a \cdot b + 390 \cdot b^2) \cdot \cosh(d \cdot x + c)^3 - 5 \cdot (11 \cdot a^2 - 61 \cdot a \cdot b + 137 \cdot b^2) \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^3 - 20 \cdot (a^2 - 11 \cdot a \cdot b + 10 \cdot b^2) \cdot \cosh(d \cdot x + c)^2 + 4 \cdot (150 \cdot (a^2 + 2 \cdot a \cdot b + b^2) \cdot \cosh(d \cdot x + c)^{14} - 455 \cdot (a^2 + 11 \cdot a \cdot b + 10 \cdot b^2) \cdot \cosh(d \cdot x + c)^{12} - 330 \cdot (11 \cdot a^2 + 61 \cdot a \cdot b + 137 \cdot b^2) \cdot \cosh(d \cdot x + c)^{10} - 225 \cdot (31 \cdot a^2 + 87 \cdot a \cdot b + 390 \cdot b^2) \cdot \cosh(d \cdot x + c)^8 - 14 \cdot (425 \cdot a^2 + 5649 \cdot b^2) \cdot \cosh(d \cdot x + c)^6 - 75 \cdot (31 \cdot a^2 - 87 \cdot a \cdot b + 390 \cdot b^2) \cdot \cosh(d \cdot x + c)^4 - 30 \cdot (11 \cdot a^2 - 61 \cdot a \cdot b + 137 \cdot b^2) \cdot \cosh(d \cdot x + c)^2 - 5 \cdot a^2 + 55 \cdot a \cdot b - 50 \cdot b^2) \cdot \sinh(d \cdot x + c)^2 + 5 \cdot a^2 - 10 \cdot a \cdot b + 5 \cdot b^2 + 1200 \cdot (a \cdot b \cdot \cosh(d \cdot x + c)^{13} + 13 \cdot a \cdot b \cdot \cosh(d \cdot x + c) \cdot \sinh(d \cdot x + c)^{12} + a \cdot b \cdot \sinh(d \cdot x + c)^{13} + 5 \cdot a \cdot b \cdot \cosh(d \cdot x + c)^{11} + (78 \cdot a \cdot b \cdot \cosh(d \cdot x + c)^2 + 5 \cdot a \cdot b) \cdot \sinh(d \cdot x + c)^{11} + 10 \cdot a \cdot b \cdot \cosh(d \cdot x + c)^9 + 11 \cdot (26 \cdot a \cdot b \cdot \cosh(d \cdot x + c)^3 + 5 \cdot a \cdot b \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^{10} + 5 \cdot (143 \cdot a \cdot b \cdot \cosh(d \cdot x + c)^4 + 55 \cdot a \cdot b \cdot \cosh(d \cdot x + c)^2 + 2 \cdot a \cdot b) \cdot \sinh(d \cdot x + c)^9 + 10 \cdot a \cdot b \cdot \cosh(d \cdot x + c)^7 + 3 \cdot (429 \cdot a \cdot b \cdot \cosh(d \cdot x + c)^5 + 275 \cdot a \cdot b \cdot \cosh(d \cdot x + c)^3 + 30 \cdot a \cdot b \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^8 + 2 \cdot (858 \cdot a \cdot b \cdot \cosh(d \cdot x + c)^6 + 825 \cdot a \cdot b \cdot \cosh(d \cdot x + c)^4 + 180 \cdot a \cdot b \cdot \cosh(d \cdot x + c)^2 + 5 \cdot a \cdot b) \cdot \sinh(d \cdot x + c)^7 + 5 \cdot a \cdot b \cdot \cosh(d \cdot x + c)^5 + 2 \cdot (858 \cdot a \cdot b \cdot \cosh(d \cdot x + c)^7 + 1155 \cdot a \cdot b \cdot \cosh(d \cdot x + c)^5 + 420 \cdot a \cdot b \cdot \cosh(d \cdot x + c)^3 + 35 \cdot a \cdot b \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^6$

[In] int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x)

[Out]
$$-2/3*a^2*cosh(d*x+c)/d+1/3/d*a^2*cosh(d*x+c)*sinh(d*x+c)^2+2/3/d*a*b*sinh(d*x+c)^5/cosh(d*x+c)^2-10/3/d*a*b*sinh(d*x+c)^3/cosh(d*x+c)^2-10/d*a*b*sinh(d*x+c)/cosh(d*x+c)^2+5/d*a*b*sech(d*x+c)*tanh(d*x+c)+10/d*a*b*arctan(\exp(d*x+c))+1/3/d*b^2*sinh(d*x+c)^8/cosh(d*x+c)^5-8/3/d*b^2*sinh(d*x+c)^6/cosh(d*x+c)^5-16/d*b^2*sinh(d*x+c)^4/cosh(d*x+c)^5-64/3/d*b^2*sinh(d*x+c)^2/cosh(d*x+c)^5-128/15/d*b^2/cosh(d*x+c)^5$$

maxima [B] time = 0.41, size = 348, normalized size = 1.91

$$-\frac{1}{120}b^2\left(\frac{5(45e^{(-dx-c)} - e^{(-3dx-3c)})}{d} + \frac{200e^{(-2dx-2c)} + 2515e^{(-4dx-4c)} + 6680e^{(-6dx-6c)} + 9073e^{(-8dx-8c)} + 5600e^{(-10dx-10c)} + 1665e^{(-12dx-12c)} - 5}{d(e^{(-3dx-3c)} + 5e^{(-5dx-5c)} + 10e^{(-7dx-7c)} + 10e^{(-9dx-9c)} + 5e^{(-11dx-11c)} + e^{(-13dx-13c)})}\right) + \frac{1}{12}a*b*\left(\frac{27e^{(-dx-c)} - e^{(-3dx-3c)}}{d} - 120*arctan(e^{(-dx-c)})/d - (25e^{(-2dx-2c)} + 77e^{(-4dx-4c)} + 3e^{(-6dx-6c)} - 1)/(d*(e^{(-3dx-3c)} + 2e^{(-5dx-5c)} + e^{(-7dx-7c)}))\right) + \frac{1}{24}a^2*(e^{(3dx+3c)}/d - 9e^{(dx+c)}/d - 9e^{(-dx-c)}/d + e^{(-3dx-3c)}/d)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")

[Out]
$$-1/120*b^2*(5*(45*e^{(-dx-c)} - e^{(-3dx-3c)})/d + (200*e^{(-2dx-2c)} + 2515*e^{(-4dx-4c)} + 6680*e^{(-6dx-6c)} + 9073*e^{(-8dx-8c)} + 5600*e^{(-10dx-10c)} + 1665*e^{(-12dx-12c)} - 5)/(d*(e^{(-3dx-3c)} + 5e^{(-5dx-5c)} + 10e^{(-7dx-7c)} + 10e^{(-9dx-9c)} + 5e^{(-11dx-11c)} + e^{(-13dx-13c)}))) + 1/12*a*b*((27*e^{(-dx-c)} - e^{(-3dx-3c)})/d - 120*arctan(e^{(-dx-c)})/d - (25*e^{(-2dx-2c)} + 77*e^{(-4dx-4c)} + 3*e^{(-6dx-6c)} - 1)/(d*(e^{(-3dx-3c)} + 2*e^{(-5dx-5c)} + e^{(-7dx-7c)}))) + 1/24*a^2*(e^{(3dx+3c)}/d - 9*e^{(dx+c)}/d - 9*e^{(-dx-c)}/d + e^{(-3dx-3c)}/d)$$

mupad [B] time = 1.44, size = 397, normalized size = 2.18

$$\frac{e^{3c+3dx}(a+b)^2}{24d} - \frac{e^{c+dx}(3a^2+18ab+15b^2)}{8d} - \frac{e^{-c-dx}(3a^2-18ab+15b^2)}{8d} + \frac{e^{-3c-3dx}(a-b)^2}{24d} + \frac{10 \operatorname{atan}\left(\frac{ab}{a^2+b^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^3)^2,x)

[Out]
$$(\exp(3c + 3d*x)*(a + b)^2)/(24*d) - (\exp(c + d*x)*(18*a*b + 3*a^2 + 15*b^2))/(8*d) - (\exp(-c - d*x)*(3*a^2 - 18*a*b + 15*b^2))/(8*d) + (\exp(-3c - 3*d*x)*(a - b)^2)/(24*d) + (10*atan((a*b*\exp(d*x)*\exp(c)*(d^2)^(1/2))/(d*(a^2*b^2)^(1/2))))*(a^2*b^2)^(1/2))/(d^2)^(1/2) - (256*b^2*\exp(c + d*x))/(15*d*(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1)) + (64*b^2*\exp(c + d*x))/(5*d*(4*\exp(2*c + 2*d*x) + 6*\exp(4*c + 4*d*x) + 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1)) - (32*b^2*\exp(c + d*x))/(5*d*(5*\exp(2*c + 2*d*x) + 10*\exp(4*c + 4*d*x) + 10*\exp(6*c + 6*d*x) + 5*\exp(8*c + 8*d*x) + \exp(10*c + 10*d*x) + 1)) - (2*\exp(c + d*x)*(a*b + 6*b^2))/(d*(\exp(2*c + 2*d*x) + 1)) + (4*\exp(c + d*x)*(3*a*b + 8*b^2))/(3*d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1))$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^3(c + dx))^2 \sinh^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**3)**2,x)

[Out] Integral((a + b*tanh(c + d*x)**3)**2*sinh(c + d*x)**3, x)

3.59 $\int \sinh^2(c + dx) (a + b \tanh^3(c + dx))^2 dx$

Optimal. Leaf size=129

$$\frac{\sinh(c + dx) \cosh(c + dx) (a^2 + 2ab \tanh(c + dx) + b^2)}{2d} - \frac{1}{2} x (a^2 + 7b^2) + \frac{ab \tanh^2(c + dx)}{d} - \frac{4ab \log(\cosh(c + dx))}{d}$$

[Out] $-1/2*(a^2+7*b^2)*x-4*a*b*\ln(\cosh(d*x+c))/d+3*b^2*\tanh(d*x+c)/d+a*b*\tanh(d*x+c)^2/d+2/3*b^2*\tanh(d*x+c)^3/d+1/5*b^2*\tanh(d*x+c)^5/d+1/2*\cosh(d*x+c)*\sinh(d*x+c)*(a^2+b^2+2*a*b*\tanh(d*x+c))/d$

Rubi [A] time = 0.21, antiderivative size = 159, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3663, 1804, 1802, 633, 31}

$$\frac{(a^2 + 7b^2) \tanh(c + dx)}{2d} + \frac{\sinh^2(c + dx) ((a^2 + b^2) \tanh(c + dx) + 2ab)}{2d} + \frac{ab \tanh^2(c + dx)}{d} + \frac{(a + b)(a + 7b) \log(1 + \tanh(c + dx))}{4d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^2,x]

[Out] $((a + b)*(a + 7*b)*\text{Log}[1 - \text{Tanh}[c + d*x]])/(4*d) - ((a - 7*b)*(a - b)*\text{Log}[1 + \text{Tanh}[c + d*x]])/(4*d) + ((a^2 + 7*b^2)*\text{Tanh}[c + d*x])/(2*d) + (a*b*\text{Tanh}[c + d*x]^2)/d + (2*b^2*\text{Tanh}[c + d*x]^3)/(3*d) + (b^2*\text{Tanh}[c + d*x]^5)/(5*d) + (\text{Sinh}[c + d*x]^2*(2*a*b + (a^2 + b^2)*\text{Tanh}[c + d*x]))/(2*d)$

Rule 31

Int[((a_) + (b_)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 633

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]

Rule 1802

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1804

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x] + Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]

Rule 3663

Int[sin[(e_)+(f_)*(x_)]^(m_)*((a_) + (b_)*((c_)*tan[(e_)+(f_)*(x_)]))^(n_)^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/

$$\begin{aligned}
& 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*b^2))*\cosh(d*x + c))*\sinh(d*x + c)^9 - 15* \\
& (40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b + 251*b^2))*\cosh(d*x + c)^8 + \\
& 15*(3003*(a^2 + 2*a*b + b^2))*\cosh(d*x + c)^6 - 495*(4*(a^2 - 8*a*b + 7*b^2) \\
&)*d*x - 5*a^2 - 10*a*b - 5*b^2))*\cosh(d*x + c)^4 - 40*(a^2 - 8*a*b + 7*b^2)* \\
& d*x - 45*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*b^2))*\cosh(d*x \\
& + c)^2 + 5*a^2 - 66*a*b - 251*b^2))*\sinh(d*x + c)^8 + 120*(429*(a^2 + 2*a*b \\
& + b^2))*\cosh(d*x + c)^7 - 99*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - \\
& 5*b^2))*\cosh(d*x + c)^5 - 15*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b \\
& + 87*b^2))*\cosh(d*x + c)^3 - (40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b \\
& + 251*b^2))*\cosh(d*x + c))*\sinh(d*x + c)^7 - 5*(120*(a^2 - 8*a*b + 7*b^2)*d \\
& *x + 15*a^2 + 198*a*b + 1103*b^2))*\cosh(d*x + c)^6 + 5*(9009*(a^2 + 2*a*b + \\
& b^2))*\cosh(d*x + c)^8 - 2772*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - \\
& 5*b^2))*\cosh(d*x + c)^6 - 630*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a* \\
& b + 87*b^2))*\cosh(d*x + c)^4 - 120*(a^2 - 8*a*b + 7*b^2)*d*x - 84*(40*(a^2 - \\
& 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b + 251*b^2))*\cosh(d*x + c)^2 - 15*a^2 - \\
& 198*a*b - 1103*b^2))*\sinh(d*x + c)^6 + 30*(1001*(a^2 + 2*a*b + b^2))*\cosh(d*x \\
& + c)^9 - 396*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2))*\cosh(d \\
& *x + c)^7 - 126*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*b^2))*\co \\
& sh(d*x + c)^5 - 28*(40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b + 251*b^2) \\
&)*\cosh(d*x + c)^3 - (120*(a^2 - 8*a*b + 7*b^2)*d*x + 15*a^2 + 198*a*b + 110 \\
& 3*b^2))*\cosh(d*x + c))*\sinh(d*x + c)^5 - 5*(60*(a^2 - 8*a*b + 7*b^2)*d*x + 2 \\
& 7*a^2 + 30*a*b + 667*b^2))*\cosh(d*x + c)^4 + 5*(3003*(a^2 + 2*a*b + b^2))*\cos \\
& h(d*x + c)^10 - 1485*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2) \\
&)*\cosh(d*x + c)^8 - 630*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87* \\
& b^2))*\cosh(d*x + c)^6 - 210*(40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b + \\
& 251*b^2))*\cosh(d*x + c)^4 - 60*(a^2 - 8*a*b + 7*b^2)*d*x - 15*(120*(a^2 - 8 \\
& *a*b + 7*b^2)*d*x + 15*a^2 + 198*a*b + 1103*b^2))*\cosh(d*x + c)^2 - 27*a^2 - \\
& 30*a*b - 667*b^2))*\sinh(d*x + c)^4 + 20*(273*(a^2 + 2*a*b + b^2))*\cosh(d*x + \\
& c)^11 - 165*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2))*\cosh(d* \\
& x + c)^9 - 90*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*b^2))*\cosh \\
& (d*x + c)^7 - 42*(40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b + 251*b^2))* \\
& cosh(d*x + c)^5 - 5*(120*(a^2 - 8*a*b + 7*b^2)*d*x + 15*a^2 + 198*a*b + 110 \\
& 3*b^2))*\cosh(d*x + c)^3 - (60*(a^2 - 8*a*b + 7*b^2)*d*x + 27*a^2 + 30*a*b + \\
& 667*b^2))*\cosh(d*x + c))*\sinh(d*x + c)^3 - (60*(a^2 - 8*a*b + 7*b^2)*d*x + 7 \\
& 5*a^2 - 150*a*b + 1003*b^2))*\cosh(d*x + c)^2 + (1365*(a^2 + 2*a*b + b^2))*\cos \\
& h(d*x + c)^12 - 990*(4*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2))* \\
& cosh(d*x + c)^10 - 675*(20*(a^2 - 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87* \\
& b^2))*\cosh(d*x + c)^8 - 420*(40*(a^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b + \\
& 251*b^2))*\cosh(d*x + c)^6 - 75*(120*(a^2 - 8*a*b + 7*b^2)*d*x + 15*a^2 + 19 \\
& 8*a*b + 1103*b^2))*\cosh(d*x + c)^4 - 60*(a^2 - 8*a*b + 7*b^2)*d*x - 30*(60*(\\
& a^2 - 8*a*b + 7*b^2)*d*x + 27*a^2 + 30*a*b + 667*b^2))*\cosh(d*x + c)^2 - 75* \\
& a^2 + 150*a*b - 1003*b^2))*\sinh(d*x + c)^2 - 15*a^2 + 30*a*b - 15*b^2 - 480* \\
& (a*b*\cosh(d*x + c)^12 + 12*a*b*\cosh(d*x + c))*\sinh(d*x + c)^11 + a*b*\sinh(d* \\
& x + c)^12 + 5*a*b*\cosh(d*x + c)^10 + (66*a*b*\cosh(d*x + c)^2 + 5*a*b))*\sinh(\\
& d*x + c)^10 + 10*a*b*\cosh(d*x + c)^8 + 10*(22*a*b*\cosh(d*x + c)^3 + 5*a*b*c \\
& osh(d*x + c))*\sinh(d*x + c)^9 + 5*(99*a*b*\cosh(d*x + c)^4 + 45*a*b*\cosh(d*x \\
& + c)^2 + 2*a*b))*\sinh(d*x + c)^8 + 10*a*b*\cosh(d*x + c)^6 + 8*(99*a*b*\cosh(\\
& d*x + c)^5 + 75*a*b*\cosh(d*x + c)^3 + 10*a*b*\cosh(d*x + c))*\sinh(d*x + c)^7 \\
& + 2*(462*a*b*\cosh(d*x + c)^6 + 525*a*b*\cosh(d*x + c)^4 + 140*a*b*\cosh(d*x \\
& + c)^2 + 5*a*b))*\sinh(d*x + c)^6 + 5*a*b*\cosh(d*x + c)^4 + 4*(198*a*b*\cosh(d \\
& *x + c)^7 + 315*a*b*\cosh(d*x + c)^5 + 140*a*b*\cosh(d*x + c)^3 + 15*a*b*\cosh \\
& (d*x + c))*\sinh(d*x + c)^5 + 5*(99*a*b*\cosh(d*x + c)^8 + 210*a*b*\cosh(d*x + \\
& c)^6 + 140*a*b*\cosh(d*x + c)^4 + 30*a*b*\cosh(d*x + c)^2 + a*b))*\sinh(d*x + \\
& c)^4 + a*b*\cosh(d*x + c)^2 + 20*(11*a*b*\cosh(d*x + c)^9 + 30*a*b*\cosh(d*x + \\
& c)^7 + 28*a*b*\cosh(d*x + c)^5 + 10*a*b*\cosh(d*x + c)^3 + a*b*\cosh(d*x + c) \\
&)*\sinh(d*x + c)^3 + (66*a*b*\cosh(d*x + c)^10 + 225*a*b*\cosh(d*x + c)^8 + 28 \\
& 0*a*b*\cosh(d*x + c)^6 + 150*a*b*\cosh(d*x + c)^4 + 30*a*b*\cosh(d*x + c)^2 + \\
& a*b))*\sinh(d*x + c)^2 + 2*(6*a*b*\cosh(d*x + c)^11 + 25*a*b*\cosh(d*x + c)^9 + \\
& 40*a*b*\cosh(d*x + c)^7 + 30*a*b*\cosh(d*x + c)^5 + 10*a*b*\cosh(d*x + c)^3 +
\end{aligned}$$

```

a*b*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sin
h(d*x + c))) + 2*(105*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^13 - 90*(4*(a^2 - 8
*a*b + 7*b^2)*d*x - 5*a^2 - 10*a*b - 5*b^2)*cosh(d*x + c)^11 - 75*(20*(a^2
- 8*a*b + 7*b^2)*d*x - 9*a^2 + 10*a*b + 87*b^2)*cosh(d*x + c)^9 - 60*(40*(a
^2 - 8*a*b + 7*b^2)*d*x - 5*a^2 + 66*a*b + 251*b^2)*cosh(d*x + c)^7 - 15*(1
20*(a^2 - 8*a*b + 7*b^2)*d*x + 15*a^2 + 198*a*b + 1103*b^2)*cosh(d*x + c)^5
- 10*(60*(a^2 - 8*a*b + 7*b^2)*d*x + 27*a^2 + 30*a*b + 667*b^2)*cosh(d*x +
c)^3 - (60*(a^2 - 8*a*b + 7*b^2)*d*x + 75*a^2 - 150*a*b + 1003*b^2)*cosh(d
*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d*x +
c)^11 + d*sinh(d*x + c)^12 + 5*d*cosh(d*x + c)^10 + (66*d*cosh(d*x + c)^2
+ 5*d)*sinh(d*x + c)^10 + 10*(22*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sin
h(d*x + c)^9 + 10*d*cosh(d*x + c)^8 + 5*(99*d*cosh(d*x + c)^4 + 45*d*cosh(d
*x + c)^2 + 2*d)*sinh(d*x + c)^8 + 8*(99*d*cosh(d*x + c)^5 + 75*d*cosh(d*x
+ c)^3 + 10*d*cosh(d*x + c))*sinh(d*x + c)^7 + 10*d*cosh(d*x + c)^6 + 2*(46
2*d*cosh(d*x + c)^6 + 525*d*cosh(d*x + c)^4 + 140*d*cosh(d*x + c)^2 + 5*d)*
sinh(d*x + c)^6 + 4*(198*d*cosh(d*x + c)^7 + 315*d*cosh(d*x + c)^5 + 140*d*
cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^5 + 5*d*cosh(d*x + c)^4
+ 5*(99*d*cosh(d*x + c)^8 + 210*d*cosh(d*x + c)^6 + 140*d*cosh(d*x + c)^4
+ 30*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 + 30
*d*cosh(d*x + c)^7 + 28*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + d*cosh(d
*x + c))*sinh(d*x + c)^3 + d*cosh(d*x + c)^2 + (66*d*cosh(d*x + c)^10 + 225
*d*cosh(d*x + c)^8 + 280*d*cosh(d*x + c)^6 + 150*d*cosh(d*x + c)^4 + 30*d*c
osh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 2*(6*d*cosh(d*x + c)^11 + 25*d*cosh(d
*x + c)^9 + 40*d*cosh(d*x + c)^7 + 30*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c
)^3 + d*cosh(d*x + c))*sinh(d*x + c))

```

giac [B] time = 0.44, size = 298, normalized size = 2.31

$$60(a^2 - 8ab + 7b^2)dx + 480ab \log(e^{2dx+2c} + 1) - 15(2a^2e^{2dx+2c} - 16abe^{2dx+2c} + 14b^2e^{2dx+2c} - a^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")

```

[Out] -1/120*(60*(a^2 - 8*a*b + 7*b^2)*d*x + 480*a*b*log(e^(2*d*x + 2*c) + 1) - 1
5*(2*a^2*e^(2*d*x + 2*c) - 16*a*b*e^(2*d*x + 2*c) + 14*b^2*e^(2*d*x + 2*c)
- a^2 + 2*a*b - b^2)*e^(-2*d*x - 2*c) - 15*(a^2*e^(2*d*x + 16*c) + 2*a*b*e^
(2*d*x + 16*c) + b^2*e^(2*d*x + 16*c))*e^(-14*c) - 8*(137*a*b*e^(10*d*x + 1
0*c) + 625*a*b*e^(8*d*x + 8*c) - 180*b^2*e^(8*d*x + 8*c) + 1190*a*b*e^(6*d*
x + 6*c) - 480*b^2*e^(6*d*x + 6*c) + 1190*a*b*e^(4*d*x + 4*c) - 680*b^2*e^(
4*d*x + 4*c) + 625*a*b*e^(2*d*x + 2*c) - 400*b^2*e^(2*d*x + 2*c) + 137*a*b
- 116*b^2)/(e^(2*d*x + 2*c) + 1)^5)/d

```

maple [A] time = 0.25, size = 173, normalized size = 1.34

$$\frac{a^2 \cosh(dx + c) \sinh(dx + c)}{2d} - \frac{a^2 x}{2} - \frac{a^2 c}{2d} + \frac{ab \left(\sinh^4(dx + c) \right)}{d \cosh(dx + c)^2} - \frac{4ab \ln(\cosh(dx + c))}{d} + \frac{2ab \left(\tanh^2(dx + c) \right)}{d} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x)

```

[Out] 1/2*a^2*cosh(d*x+c)*sinh(d*x+c)/d-1/2*a^2*x-1/2/d*a^2*c+1/d*a*b*sinh(d*x+c)
^4/cosh(d*x+c)^2-4*a*b*ln(cosh(d*x+c))/d+2*a*b*tanh(d*x+c)^2/d+1/2/d*b^2*si
nh(d*x+c)^7/cosh(d*x+c)^5-7/2*b^2*x-7/2/d*c*b^2+7/2*b^2*tanh(d*x+c)/d+7/6*b
^2*tanh(d*x+c)^3/d+7/10*b^2*tanh(d*x+c)^5/d

```

maxima [B] time = 0.41, size = 301, normalized size = 2.33

$$-\frac{1}{8}a^2\left(4x - \frac{e^{2dx+2c}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{120}b^2\left(\frac{420(dx + c)}{d} + \frac{15e^{(-2dx-2c)}}{d} - \frac{1003e^{(-2dx-2c)} + 3350e^{(-4dx-4c)}}{d(e^{(-2dx-2c)} + 5e^{(-4dx-4c)} + 10)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")

[Out]
$$-1/8*a^2*(4*x - e^{(2*d*x + 2*c)}/d + e^{(-2*d*x - 2*c)}/d) - 1/120*b^2*(420*(d*x + c)/d + 15*e^{(-2*d*x - 2*c)}/d - (1003*e^{(-2*d*x - 2*c)} + 3350*e^{(-4*d*x - 4*c)} + 5590*e^{(-6*d*x - 6*c)} + 3915*e^{(-8*d*x - 8*c)} + 1455*e^{(-10*d*x - 10*c)} + 15)/(d*(e^{(-2*d*x - 2*c)} + 5*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 10*e^{(-8*d*x - 8*c)} + 5*e^{(-10*d*x - 10*c)} + e^{(-12*d*x - 12*c)})) - 1/4*a*b*(16*(d*x + c)/d - e^{(-2*d*x - 2*c)}/d + 16*\log(e^{(-2*d*x - 2*c)} + 1)/d - (2*e^{(-2*d*x - 2*c)} - 15*e^{(-4*d*x - 4*c)} + 1)/(d*(e^{(-2*d*x - 2*c)} + 2*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)})))$$

mupad [B] time = 1.36, size = 306, normalized size = 2.37

$$\frac{e^{2c+2dx}(a+b)^2}{8d} - \frac{4(3b^2+ab)}{d(e^{2c+2dx}+1)} - x\left(\frac{a^2}{2} - 4ab + \frac{7b^2}{2}\right) + \frac{4(4b^2+ab)}{d(2e^{2c+2dx}+e^{4c+4dx}+1)} - \frac{64b^2}{3d(3e^{2c+2dx}+3e^{4c+4dx}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^3)^2,x)

[Out]
$$\frac{\exp(2*c + 2*d*x)*(a + b)^2}{(8*d)} - \frac{4*(a*b + 3*b^2)}{(d*(\exp(2*c + 2*d*x) + 1))} - x*\left(\frac{a^2}{2} - 4*a*b + \frac{7*b^2}{2}\right) + \frac{4*(a*b + 4*b^2)}{(d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1))} - \frac{64*b^2}{(3*d*(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1))} - \frac{(\exp(-2*c - 2*d*x)*(a - b)^2)}{(8*d)} + \frac{16*b^2}{(d*(4*\exp(2*c + 2*d*x) + 6*\exp(4*c + 4*d*x) + 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1))} - \frac{32*b^2}{(5*d*(5*\exp(2*c + 2*d*x) + 10*\exp(4*c + 4*d*x) + 10*\exp(6*c + 6*d*x) + 5*\exp(8*c + 8*d*x) + \exp(10*c + 10*d*x) + 1))} - \frac{4*a*b*\log(\exp(2*c)*\exp(2*d*x) + 1)}{d}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^3(c + dx))^2 \sinh^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**3)**2,x)

[Out] Integral((a + b*tanh(c + d*x)**3)**2*sinh(c + d*x)**2, x)

3.60 $\int \sinh(c + dx) \left(a + b \tanh^3(c + dx) \right)^2 dx$

Optimal. Leaf size=123

$$\frac{a^2 \cosh(c + dx)}{d} + \frac{3ab \sinh(c + dx)}{d} - \frac{3ab \tan^{-1}(\sinh(c + dx))}{d} - \frac{ab \sinh(c + dx) \tanh^2(c + dx)}{d} + \frac{b^2 \cosh(c + dx)}{d}$$

[Out] $-3*a*b*\arctan(\sinh(d*x+c))/d+a^2*\cosh(d*x+c)/d+b^2*\cosh(d*x+c)/d+3*b^2*\operatorname{sech}(d*x+c)/d-b^2*\operatorname{sech}(d*x+c)^3/d+1/5*b^2*\operatorname{sech}(d*x+c)^5/d+3*a*b*\sinh(d*x+c)/d-a*b*\sinh(d*x+c)*\tanh(d*x+c)^2/d$

Rubi [A] time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3666, 2638, 2592, 288, 321, 203, 2590, 270}

$$\frac{a^2 \cosh(c + dx)}{d} + \frac{3ab \sinh(c + dx)}{d} - \frac{3ab \tan^{-1}(\sinh(c + dx))}{d} - \frac{ab \sinh(c + dx) \tanh^2(c + dx)}{d} + \frac{b^2 \cosh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3)^2,x]

[Out] $(-3*a*b*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/d + (a^2*\operatorname{Cosh}[c + d*x])/d + (b^2*\operatorname{Cosh}[c + d*x])/d + (3*b^2*\operatorname{Sech}[c + d*x])/d - (b^2*\operatorname{Sech}[c + d*x]^3)/d + (b^2*\operatorname{Sech}[c + d*x]^5)/(5*d) + (3*a*b*\operatorname{Sinh}[c + d*x])/d - (a*b*\operatorname{Sinh}[c + d*x]*\operatorname{Tanh}[c + d*x]^2)/d$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2590

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] := -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x], Cos[e + f*x], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.), x_
Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3666

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] := Int[ExpandTrig[(d*sin[e + f*x])^m*(a
+ b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned} \int \sinh(c + dx) (a + b \tanh^3(c + dx))^2 dx &= - \left(i \int (ia^2 \sinh(c + dx) + 2iab \sinh(c + dx) \tanh^3(c + dx) + ib^2 \sinh(c + dx) \tanh^5(c + dx)) dx \right) \\ &= a^2 \int \sinh(c + dx) dx + (2ab) \int \sinh(c + dx) \tanh^3(c + dx) dx + b^2 \int \sinh(c + dx) \tanh^5(c + dx) dx \\ &= \frac{a^2 \cosh(c + dx)}{d} + \frac{(2ab) \text{Subst} \left(\int \frac{x^4}{(1+x^2)^2} dx, x, \sinh(c + dx) \right)}{d} - \frac{b^2 \text{Subst} \left(\int \frac{x^6}{(1+x^2)^2} dx, x, \sinh(c + dx) \right)}{d} \\ &= \frac{a^2 \cosh(c + dx)}{d} - \frac{ab \sinh(c + dx) \tanh^2(c + dx)}{d} + \frac{(3ab) \text{Subst} \left(\int \frac{x^4}{1+x^2} dx, x, \sinh(c + dx) \right)}{d} \\ &= \frac{a^2 \cosh(c + dx)}{d} + \frac{b^2 \cosh(c + dx)}{d} + \frac{3b^2 \text{sech}(c + dx)}{d} - \frac{b^2 \text{sech}^3(c + dx)}{d} \\ &= -\frac{3ab \tan^{-1}(\sinh(c + dx))}{d} + \frac{a^2 \cosh(c + dx)}{d} + \frac{b^2 \cosh(c + dx)}{d} + \frac{3b^2 \text{sech}(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.44, size = 90, normalized size = 0.73

$$\frac{5(a^2 + b^2) \cosh(c + dx) + b \left(5 \text{sech}(c + dx) (a \tanh(c + dx) + 3b) + 10a \left(\sinh(c + dx) - 3 \tan^{-1} \left(\tanh \left(\frac{1}{2}(c + dx) \right) \right) \right) \right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3)^2,x]
```

```
[Out] (5*(a^2 + b^2)*Cosh[c + d*x] + b*(-5*b*Sech[c + d*x]^3 + b*Sech[c + d*x]^5
+ 10*a*(-3*ArcTan[Tanh[(c + d*x)/2]] + Sinh[c + d*x]) + 5*Sech[c + d*x]*(3*
b + a*Tanh[c + d*x])))/(5*d)
```

fricas [B] time = 0.72, size = 2230, normalized size = 18.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/10*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^12 + 60*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)*sinh(d*x + c)^11 + 5*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^12 + 30*(a^
2 + 2*a*b + 3*b^2)*cosh(d*x + c)^10 + 30*(11*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)^2 + a^2 + 2*a*b + 3*b^2)*sinh(d*x + c)^10 + 100*(11*(a^2 + 2*a*b + b^2)
*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 +
5*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*x + c)^8 + 5*(495*(a^2 + 2*a*b + b^2)*
cosh(d*x + c)^4 + 270*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^2 + 15*a^2 + 18*a
*b + 47*b^2)*sinh(d*x + c)^8 + 40*(99*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 +
90*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^3 + (15*a^2 + 18*a*b + 47*b^2)*cosh
(d*x + c))*sinh(d*x + c)^7 + 4*(25*a^2 + 91*b^2)*cosh(d*x + c)^6 + 4*(1155*
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 1575*(a^2 + 2*a*b + 3*b^2)*cosh(d*x +
c)^4 + 35*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*x + c)^2 + 25*a^2 + 91*b^2)*si
nh(d*x + c)^6 + 8*(495*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 945*(a^2 + 2*a
*b + 3*b^2)*cosh(d*x + c)^5 + 35*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*x + c)^3
+ 3*(25*a^2 + 91*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 5*(15*a^2 - 18*a*b
+ 47*b^2)*cosh(d*x + c)^4 + 5*(495*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 12
60*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^6 + 70*(15*a^2 + 18*a*b + 47*b^2)*co
sh(d*x + c)^4 + 12*(25*a^2 + 91*b^2)*cosh(d*x + c)^2 + 15*a^2 - 18*a*b + 47
*b^2)*sinh(d*x + c)^4 + 20*(55*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 + 180*(a
^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^7 + 14*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*
x + c)^5 + 4*(25*a^2 + 91*b^2)*cosh(d*x + c)^3 + (15*a^2 - 18*a*b + 47*b^2)
*cosh(d*x + c))*sinh(d*x + c)^3 + 30*(a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^2
+ 10*(33*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 + 135*(a^2 + 2*a*b + 3*b^2)*c
osh(d*x + c)^8 + 14*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*x + c)^6 + 6*(25*a^2
+ 91*b^2)*cosh(d*x + c)^4 + 3*(15*a^2 - 18*a*b + 47*b^2)*cosh(d*x + c)^2 +
3*a^2 - 6*a*b + 9*b^2)*sinh(d*x + c)^2 + 5*a^2 - 10*a*b + 5*b^2 - 60*(a*b*c
osh(d*x + c)^11 + 11*a*b*cosh(d*x + c)*sinh(d*x + c)^10 + a*b*sinh(d*x + c)
^11 + 5*a*b*cosh(d*x + c)^9 + 5*(11*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)
)^9 + 10*a*b*cosh(d*x + c)^7 + 15*(11*a*b*cosh(d*x + c)^3 + 3*a*b*cosh(d*x
+ c))*sinh(d*x + c)^8 + 10*(33*a*b*cosh(d*x + c)^4 + 18*a*b*cosh(d*x + c)^2
+ a*b)*sinh(d*x + c)^7 + 10*a*b*cosh(d*x + c)^5 + 14*(33*a*b*cosh(d*x + c)
^5 + 30*a*b*cosh(d*x + c)^3 + 5*a*b*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(231
*a*b*cosh(d*x + c)^6 + 315*a*b*cosh(d*x + c)^4 + 105*a*b*cosh(d*x + c)^2 +
5*a*b)*sinh(d*x + c)^5 + 5*a*b*cosh(d*x + c)^3 + 10*(33*a*b*cosh(d*x + c)^7
+ 63*a*b*cosh(d*x + c)^5 + 35*a*b*cosh(d*x + c)^3 + 5*a*b*cosh(d*x + c))*s
inh(d*x + c)^4 + 5*(33*a*b*cosh(d*x + c)^8 + 84*a*b*cosh(d*x + c)^6 + 70*a*
b*cosh(d*x + c)^4 + 20*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^3 + a*b*cos
h(d*x + c) + 5*(11*a*b*cosh(d*x + c)^9 + 36*a*b*cosh(d*x + c)^7 + 42*a*b*co
sh(d*x + c)^5 + 20*a*b*cosh(d*x + c)^3 + 3*a*b*cosh(d*x + c))*sinh(d*x + c)
^2 + (11*a*b*cosh(d*x + c)^10 + 45*a*b*cosh(d*x + c)^8 + 70*a*b*cosh(d*x +
c)^6 + 50*a*b*cosh(d*x + c)^4 + 15*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)
)*arctan(cosh(d*x + c) + sinh(d*x + c)) + 4*(15*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)^11 + 75*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^9 + 10*(15*a^2 + 18*a*b
+ 47*b^2)*cosh(d*x + c)^7 + 6*(25*a^2 + 91*b^2)*cosh(d*x + c)^5 + 5*(15*a^2
- 18*a*b + 47*b^2)*cosh(d*x + c)^3 + 15*(a^2 - 2*a*b + 3*b^2)*cosh(d*x + c
))*sinh(d*x + c))/(d*cosh(d*x + c)^11 + 11*d*cosh(d*x + c)*sinh(d*x + c)^10
+ d*sinh(d*x + c)^11 + 5*d*cosh(d*x + c)^9 + 5*(11*d*cosh(d*x + c)^2 + d)*
sinh(d*x + c)^9 + 15*(11*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x +
c)^8 + 10*d*cosh(d*x + c)^7 + 10*(33*d*cosh(d*x + c)^4 + 18*d*cosh(d*x + c)
^2 + d)*sinh(d*x + c)^7 + 14*(33*d*cosh(d*x + c)^5 + 30*d*cosh(d*x + c)^3 +
5*d*cosh(d*x + c))*sinh(d*x + c)^6 + 10*d*cosh(d*x + c)^5 + 2*(231*d*cosh(
d*x + c)^6 + 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x
+ c)^5 + 10*(33*d*cosh(d*x + c)^7 + 63*d*cosh(d*x + c)^5 + 35*d*cosh(d*x +
c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(33*d*c
osh(d*x + c)^8 + 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4 + 20*d*cosh(d*
x + c)^2 + d)*sinh(d*x + c)^3 + 5*(11*d*cosh(d*x + c)^9 + 36*d*cosh(d*x + c)
)^7 + 42*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh
(d*x + c)^2 + d*cosh(d*x + c) + (11*d*cosh(d*x + c)^10 + 45*d*cosh(d*x + c)
^8 + 70*d*cosh(d*x + c)^6 + 50*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 + d
```

) $\sinh(dx + c)$)

giac [A] time = 0.31, size = 214, normalized size = 1.74

$$60 ab \arctan(e^{dx+c}) - 5(a^2 - 2ab + b^2)e^{-dx-c} - 5(a^2e^{dx+14c} + 2abe^{dx+14c} + b^2e^{dx+14c})e^{-13c} - \frac{4(5abe^{9dx+c})}{10d}$$

10d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)*(a+b*tanh(dx+c)^3)^2,x, algorithm="giac")

[Out] $-1/10*(60*a*b*\arctan(e^{dx+c}) - 5*(a^2 - 2*a*b + b^2)*e^{-dx-c} - 5*(a^2*e^{dx+14*c} + 2*a*b*e^{dx+14*c} + b^2*e^{dx+14*c})*e^{-13*c} - 4*(5*a*b*e^{9*d*x+9*c} + 15*b^2*e^{9*d*x+9*c} + 10*a*b*e^{7*d*x+7*c} + 40*b^2*e^{7*d*x+7*c} + 66*b^2*e^{5*d*x+5*c} - 10*a*b*e^{3*d*x+3*c} + 40*b^2*e^{3*d*x+3*c} - 5*a*b*e^{dx+c} + 15*b^2*e^{dx+c}))/e^{2*d*x+2*c} + 1)^5/d$

maple [A] time = 0.32, size = 179, normalized size = 1.46

$$\frac{a^2 \cosh(dx+c)}{d} + \frac{2ab \sinh^3(dx+c)}{d \cosh^2(dx+c)} + \frac{6ab \sinh(dx+c)}{d \cosh^2(dx+c)} - \frac{3ab \operatorname{sech}(dx+c) \tanh(dx+c)}{d} - \frac{6ab \arctan(e^{dx+c})}{d} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(dx+c)*(a+b*tanh(dx+c)^3)^2,x)

[Out] $a^2*\cosh(dx+c)/d+2/d*a*b*\sinh(dx+c)^3/\cosh(dx+c)^2+6/d*a*b*\sinh(dx+c)/\cosh(dx+c)^2-3/d*a*b*\operatorname{sech}(dx+c)*\tanh(dx+c)-6/d*a*b*\arctan(\exp(dx+c))+1/d*b^2*\sinh(dx+c)^6/\cosh(dx+c)^5+6/d*b^2*\sinh(dx+c)^4/\cosh(dx+c)^5+8/d*b^2*\sinh(dx+c)^2/\cosh(dx+c)^5+16/5/d*b^2/\cosh(dx+c)^5$

maxima [B] time = 0.41, size = 253, normalized size = 2.06

$$ab \left(\frac{6 \arctan(e^{-dx-c})}{d} - \frac{e^{-dx-c}}{d} + \frac{4e^{-2dx-2c} - e^{-4dx-4c} + 1}{d(e^{-dx-c} + 2e^{-3dx-3c} + e^{-5dx-5c})} \right) + \frac{1}{10} b^2 \left(\frac{5e^{-dx-c}}{d} + \frac{85e^{-2dx-2c} + 210}{d(e^{-dx-c} + 5e^{-3dx-3c})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)*(a+b*tanh(dx+c)^3)^2,x, algorithm="maxima")

[Out] $a*b*(6*\arctan(e^{-dx-c})/d - e^{-dx-c}/d + (4*e^{-2*d*x-2*c} - e^{-4*d*x-4*c} + 1)/(d*(e^{-dx-c} + 2*e^{-3*d*x-3*c} + e^{-5*d*x-5*c}))) + 1/10*b^2*(5*e^{-dx-c}/d + (85*e^{-2*d*x-2*c} + 210*e^{-4*d*x-4*c} + 314*e^{-6*d*x-6*c} + 185*e^{-8*d*x-8*c} + 65*e^{-10*d*x-10*c} + 5)/(d*(e^{-dx-c} + 5*e^{-3*d*x-3*c} + 10*e^{-5*d*x-5*c} + 10*e^{-7*d*x-7*c} + 5*e^{-9*d*x-9*c} + e^{-11*d*x-11*c}))) + a^2*\cosh(dx+c)/d$

mupad [B] time = 1.29, size = 338, normalized size = 2.75

$$\frac{e^{c+dx}(a+b)^2}{2d} + \frac{e^{-c-dx}(a-b)^2}{2d} - \frac{6 \operatorname{atan}\left(\frac{ab e^{dx} e^c \sqrt{d^2}}{d \sqrt{a^2 b^2}}\right) \sqrt{a^2 b^2}}{\sqrt{d^2}} + \frac{72 b^2 e^{c+dx}}{5d(3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1)} - \frac{4}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + dx)*(a + b*tanh(c + dx)^3)^2,x)

[Out] $(\exp(c + dx)*(a + b)^2)/(2*d) + (\exp(-c - dx)*(a - b)^2)/(2*d) - (6*\operatorname{atan}((a*b*\exp(dx)*\exp(c)*(d^2)^{(1/2)})/(d*(a^2*b^2)^{(1/2)}))*(a^2*b^2)^{(1/2)})/d$

$$\begin{aligned} & ^2)^{1/2} + (72*b^2*\exp(c + d*x))/(5*d*(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1)) - (64*b^2*\exp(c + d*x))/(5*d*(4*\exp(2*c + 2*d*x) + 6*\exp(4*c + 4*d*x) + 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1)) + (3 \\ & 2*b^2*\exp(c + d*x))/(5*d*(5*\exp(2*c + 2*d*x) + 10*\exp(4*c + 4*d*x) + 10*\exp(6*c + 6*d*x) + 5*\exp(8*c + 8*d*x) + \exp(10*c + 10*d*x) + 1)) + (2*\exp(c + d*x)*(a*b + 3*b^2))/(d*(\exp(2*c + 2*d*x) + 1)) - (4*\exp(c + d*x)*(a*b + 2*b \\ & ^2))/(d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1)) \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^3(c + dx))^2 \sinh(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**3)**2,x)

[Out] Integral((a + b*tanh(c + d*x)**3)**2*sinh(c + d*x), x)

3.61 $\int \operatorname{csch}(c + dx) \left(a + b \tanh^3(c + dx) \right)^2 dx$

Optimal. Leaf size=98

$$-\frac{a^2 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{ab \tan^{-1}(\sinh(c + dx))}{d} - \frac{ab \tanh(c + dx) \operatorname{sech}(c + dx)}{d} - \frac{b^2 \operatorname{sech}^5(c + dx)}{5d} + \frac{2b^2 \operatorname{sech}^3(c + dx)}{3d}$$

[Out] a*b*arctan(sinh(d*x+c))/d-a^2*arctanh(cosh(d*x+c))/d-b^2*sech(d*x+c)/d+2/3*b^2*sech(d*x+c)^3/d-1/5*b^2*sech(d*x+c)^5/d-a*b*sech(d*x+c)*tanh(d*x+c)/d

Rubi [A] time = 0.14, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3666, 3770, 2611, 2606, 194}

$$-\frac{a^2 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{ab \tan^{-1}(\sinh(c + dx))}{d} - \frac{ab \tanh(c + dx) \operatorname{sech}(c + dx)}{d} - \frac{b^2 \operatorname{sech}^5(c + dx)}{5d} + \frac{2b^2 \operatorname{sech}^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3)^2,x]

[Out] (a*b*ArcTan[Sinh[c + d*x]])/d - (a^2*ArcTanh[Cosh[c + d*x]])/d - (b^2*Sech[c + d*x])/d + (2*b^2*Sech[c + d*x]^3)/(3*d) - (b^2*Sech[c + d*x]^5)/(5*d) - (a*b*Sech[c + d*x]*Tanh[c + d*x])/d

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 2606

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m-1)*(-1+x^2)^((n-1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n-1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n+1])

Rule 2611

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n-1))/(f*(m+n-1)), x] - Dist[(b^2*(n-1))/(m+n-1), Int[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n-2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && NeQ[m+n-1, 0] && IntegerQ[2*m, 2*n]

Rule 3666

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := Int[ExpandTrig[(d*sin[e + f*x])^m*(a + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}(c+dx) (a+b \tanh^3(c+dx))^2 dx &= i \int (-ia^2 \operatorname{csch}(c+dx) - 2iab \operatorname{sech}(c+dx) \tanh^2(c+dx) - ib^2 \operatorname{sech}^3(c+dx)) dx \\
&= a^2 \int \operatorname{csch}(c+dx) dx + (2ab) \int \operatorname{sech}(c+dx) \tanh^2(c+dx) dx + \dots \\
&= -\frac{a^2 \tanh^{-1}(\cosh(c+dx))}{d} - \frac{ab \operatorname{sech}(c+dx) \tanh(c+dx)}{d} + (ab) \dots \\
&= \frac{ab \tan^{-1}(\sinh(c+dx))}{d} - \frac{a^2 \tanh^{-1}(\cosh(c+dx))}{d} - \frac{ab \operatorname{sech}(c+dx) \tanh(c+dx)}{d} + \dots \\
&= \frac{ab \tan^{-1}(\sinh(c+dx))}{d} - \frac{a^2 \tanh^{-1}(\cosh(c+dx))}{d} - \frac{b^2 \operatorname{sech}(c+dx)}{d} + \dots
\end{aligned}$$

Mathematica [A] time = 0.17, size = 106, normalized size = 1.08

$$\frac{a^2 \log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{2ab \tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{ab \tanh(c+dx) \operatorname{sech}(c+dx)}{d} - \frac{b^2 \operatorname{sech}^5(c+dx)}{5d} + \dots$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3)^2, x]

[Out] (2*a*b*ArcTan[Tanh[(c + d*x)/2]])/d + (a^2*Log[Tanh[(c + d*x)/2]])/d - (b^2*Sech[c + d*x])/d + (2*b^2*Sech[c + d*x]^3)/(3*d) - (b^2*Sech[c + d*x]^5)/(5*d) - (a*b*Sech[c + d*x]*Tanh[c + d*x])/d

fricas [B] time = 0.77, size = 2498, normalized size = 25.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2, x, algorithm="fricas")

[Out] -1/15*(30*(a*b + b^2)*cosh(d*x + c)^9 + 270*(a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^8 + 30*(a*b + b^2)*sinh(d*x + c)^9 + 20*(3*a*b + 2*b^2)*cosh(d*x + c)^7 + 20*(54*(a*b + b^2)*cosh(d*x + c)^2 + 3*a*b + 2*b^2)*sinh(d*x + c)^7 + 116*b^2*cosh(d*x + c)^5 + 140*(18*(a*b + b^2)*cosh(d*x + c)^3 + (3*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 + 4*(945*(a*b + b^2)*cosh(d*x + c)^4 + 105*(3*a*b + 2*b^2)*cosh(d*x + c)^2 + 29*b^2)*sinh(d*x + c)^5 + 20*(189*(a*b + b^2)*cosh(d*x + c)^5 + 35*(3*a*b + 2*b^2)*cosh(d*x + c)^3 + 29*b^2*cosh(d*x + c))*sinh(d*x + c)^4 - 20*(3*a*b - 2*b^2)*cosh(d*x + c)^3 + 20*(126*(a*b + b^2)*cosh(d*x + c)^6 + 35*(3*a*b + 2*b^2)*cosh(d*x + c)^4 + 58*b^2*cosh(d*x + c)^2 - 3*a*b + 2*b^2)*sinh(d*x + c)^3 + 20*(54*(a*b + b^2)*cosh(d*x + c)^7 + 21*(3*a*b + 2*b^2)*cosh(d*x + c)^5 + 58*b^2*cosh(d*x + c)^3 - 3*(3*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 30*(a*b*cosh(d*x + c)^10 + 10*a*b*cosh(d*x + c)*sinh(d*x + c)^9 + a*b*sinh(d*x + c)^10 + 5*a*b*cosh(d*x + c)^8 + 5*(9*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^8 + 10*a*b*cosh(d*x + c)^6 + 40*(3*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*a*b*cosh(d*x + c)^4 + 14*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^6 + 10*a*b*cosh(d*x + c)^4 + 4*(63*a*b*cosh(d*x + c)^5 + 70*a*b*cosh(d*x + c)^3 + 15*a*b*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(21*a*b*cosh(d*x + c)^6 + 35*a*b*cosh(d*x + c)^4 + 15*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^4 + 5*a*b*cosh(d*x + c)^2 + 40*(3*a*b*cosh(d*x + c)^7 + 7*a*b*cosh(d*x + c)^5 + 5*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))*sinh(d*x + c)^3 + 5*(9*a*b*cosh(d*x + c)^8 + 28*a*b*cosh(d*x + c)^6 + 30*a*b*cosh(d*x + c)^4 + 12*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^2 + a*b + 10*(a*b*cosh(d*x + c)^9 + 4*a*b*cosh(d*x + c)^7 + 6*a*b*cosh(d*x + c)^5 + 4*a*b*cosh(d*x + c)^3 + a*b*cos

```

h(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 30*(a*b
- b^2)*cosh(d*x + c) + 15*(a^2*cosh(d*x + c)^10 + 10*a^2*cosh(d*x + c)*sinh
(d*x + c)^9 + a^2*sinh(d*x + c)^10 + 5*a^2*cosh(d*x + c)^8 + 5*(9*a^2*cosh(
d*x + c)^2 + a^2)*sinh(d*x + c)^8 + 10*a^2*cosh(d*x + c)^6 + 40*(3*a^2*cosh
(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*a^2*cosh(d*x + c)
^4 + 14*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^6 + 10*a^2*cosh(d*x + c)^4
+ 4*(63*a^2*cosh(d*x + c)^5 + 70*a^2*cosh(d*x + c)^3 + 15*a^2*cosh(d*x + c
))*sinh(d*x + c)^5 + 10*(21*a^2*cosh(d*x + c)^6 + 35*a^2*cosh(d*x + c)^4 +
15*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 5*a^2*cosh(d*x + c)^2 + 40*
(3*a^2*cosh(d*x + c)^7 + 7*a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c)^3 + a^
2*cosh(d*x + c))*sinh(d*x + c)^3 + 5*(9*a^2*cosh(d*x + c)^8 + 28*a^2*cosh(d
*x + c)^6 + 30*a^2*cosh(d*x + c)^4 + 12*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x
+ c)^2 + a^2 + 10*(a^2*cosh(d*x + c)^9 + 4*a^2*cosh(d*x + c)^7 + 6*a^2*cos
h(d*x + c)^5 + 4*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c))*lo
g(cosh(d*x + c) + sinh(d*x + c) + 1) - 15*(a^2*cosh(d*x + c)^10 + 10*a^2*co
sh(d*x + c)*sinh(d*x + c)^9 + a^2*sinh(d*x + c)^10 + 5*a^2*cosh(d*x + c)^8
+ 5*(9*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^8 + 10*a^2*cosh(d*x + c)^6
+ 40*(3*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*a
^2*cosh(d*x + c)^4 + 14*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^6 + 10*a^2
*cosh(d*x + c)^4 + 4*(63*a^2*cosh(d*x + c)^5 + 70*a^2*cosh(d*x + c)^3 + 15*
a^2*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(21*a^2*cosh(d*x + c)^6 + 35*a^2*co
sh(d*x + c)^4 + 15*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 5*a^2*cosh(
d*x + c)^2 + 40*(3*a^2*cosh(d*x + c)^7 + 7*a^2*cosh(d*x + c)^5 + 5*a^2*cosh
(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c)^3 + 5*(9*a^2*cosh(d*x + c)^8
+ 28*a^2*cosh(d*x + c)^6 + 30*a^2*cosh(d*x + c)^4 + 12*a^2*cosh(d*x + c)^2
+ a^2)*sinh(d*x + c)^2 + a^2 + 10*(a^2*cosh(d*x + c)^9 + 4*a^2*cosh(d*x +
c)^7 + 6*a^2*cosh(d*x + c)^5 + 4*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*s
inh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 10*(27*(a*b + b^2)*c
osh(d*x + c)^8 + 14*(3*a*b + 2*b^2)*cosh(d*x + c)^6 + 58*b^2*cosh(d*x + c)^
4 - 6*(3*a*b - 2*b^2)*cosh(d*x + c)^2 - 3*a*b + 3*b^2)*sinh(d*x + c))/(d*co
sh(d*x + c)^10 + 10*d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 +
5*d*cosh(d*x + c)^8 + 5*(9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 40*(3*d
*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^7 + 10*d*cosh(d*x + c)^6
+ 10*(21*d*cosh(d*x + c)^4 + 14*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 4*
(63*d*cosh(d*x + c)^5 + 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x
+ c)^5 + 10*d*cosh(d*x + c)^4 + 10*(21*d*cosh(d*x + c)^6 + 35*d*cosh(d*x +
c)^4 + 15*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 40*(3*d*cosh(d*x + c)^7
+ 7*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x +
c)^3 + 5*d*cosh(d*x + c)^2 + 5*(9*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6
+ 30*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 10*(d*
cosh(d*x + c)^9 + 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x + c)^5 + 4*d*cosh(d*x
+ c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)

```

giac [A] time = 0.29, size = 178, normalized size = 1.82

$$30 ab \arctan \left(e^{(dx+c)} \right) - 15 a^2 \log \left(e^{(dx+c)} + 1 \right) + 15 a^2 \log \left(\left| e^{(dx+c)} - 1 \right| \right) - \frac{2 \left(15 a b e^{(9 dx+9 c)} + 15 b^2 e^{(9 dx+9 c)} + 30 a b e^{(7 dx+7 c)} + 20 \right)}{15 d}$$

15d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")

```

[Out] 1/15*(30*a*b*arctan(e^(d*x + c)) - 15*a^2*log(e^(d*x + c) + 1) + 15*a^2*log
(abs(e^(d*x + c) - 1)) - 2*(15*a*b*e^(9*d*x + 9*c) + 15*b^2*e^(9*d*x + 9*c)
+ 30*a*b*e^(7*d*x + 7*c) + 20*b^2*e^(7*d*x + 7*c) + 58*b^2*e^(5*d*x + 5*c)
- 30*a*b*e^(3*d*x + 3*c) + 20*b^2*e^(3*d*x + 3*c) - 15*a*b*e^(d*x + c) + 1
5*b^2*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^5/d

```


maple [A] time = 0.39, size = 134, normalized size = 1.37

$$-\frac{2a^2 \operatorname{arctanh}(e^{dx+c})}{d} - \frac{2ab \sinh(dx+c)}{d \cosh(dx+c)^2} + \frac{ab \operatorname{sech}(dx+c) \tanh(dx+c)}{d} + \frac{2ab \operatorname{arctan}(e^{dx+c})}{d} - \frac{b^2 (\sinh^4(dx+c))}{d \cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x)

[Out] $-2/d*a^2*\operatorname{arctanh}(\exp(d*x+c))-2/d*a*b*\sinh(d*x+c)/\cosh(d*x+c)^2+1/d*a*b*\operatorname{sech}(d*x+c)*\tanh(d*x+c)+2/d*a*b*\operatorname{arctan}(\exp(d*x+c))-1/d*b^2*\sinh(d*x+c)^4/\cosh(d*x+c)^5-4/3/d*b^2*\sinh(d*x+c)^2/\cosh(d*x+c)^5-8/15/d*b^2/\cosh(d*x+c)^5$

maxima [B] time = 0.41, size = 447, normalized size = 4.56

$$-2ab \left(\frac{\operatorname{arctan}(e^{(-dx-c)})}{d} + \frac{e^{(-dx-c)} - e^{(-3dx-3c)}}{d(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1)} \right) - \frac{2}{15} b^2 \left(\frac{15e^{(-dx-c)}}{d(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")

[Out] $-2*a*b*(\operatorname{arctan}(e^{(-d*x-c)})/d + (e^{(-d*x-c)} - e^{(-3*d*x-3*c)})/(d*(2*e^{(-2*d*x-2*c)} + e^{(-4*d*x-4*c)} + 1))) - 2/15*b^2*(15*e^{(-d*x-c)}/(d*(5*e^{(-2*d*x-2*c)} + 10*e^{(-4*d*x-4*c)} + 10*e^{(-6*d*x-6*c)} + 5*e^{(-8*d*x-8*c)} + e^{(-10*d*x-10*c)} + 1)) + 20*e^{(-3*d*x-3*c)}/(d*(5*e^{(-2*d*x-2*c)} + 10*e^{(-4*d*x-4*c)} + 10*e^{(-6*d*x-6*c)} + 5*e^{(-8*d*x-8*c)} + e^{(-10*d*x-10*c)} + 1)) + 58*e^{(-5*d*x-5*c)}/(d*(5*e^{(-2*d*x-2*c)} + 10*e^{(-4*d*x-4*c)} + 10*e^{(-6*d*x-6*c)} + 5*e^{(-8*d*x-8*c)} + e^{(-10*d*x-10*c)} + 1)) + 20*e^{(-7*d*x-7*c)}/(d*(5*e^{(-2*d*x-2*c)} + 10*e^{(-4*d*x-4*c)} + 10*e^{(-6*d*x-6*c)} + 5*e^{(-8*d*x-8*c)} + e^{(-10*d*x-10*c)} + 1)) + 15*e^{(-9*d*x-9*c)}/(d*(5*e^{(-2*d*x-2*c)} + 10*e^{(-4*d*x-4*c)} + 10*e^{(-6*d*x-6*c)} + 5*e^{(-8*d*x-8*c)} + e^{(-10*d*x-10*c)} + 1))) + a^2*\log(\tanh(1/2*d*x + 1/2*c))/d$

mupad [B] time = 3.28, size = 522, normalized size = 5.33

$$\frac{a^2 \ln(32a^6 + 32a^4b^2 - 32a^6e^{dx}e^c - 32a^4b^2e^{dx}e^c)}{d} - \frac{176b^2e^{c+dx}}{15(d + 3de^{2c+2dx} + 3de^{4c+4dx} + de^{6c+6dx})} - \frac{1}{5(d + 3de^{2c+2dx} + 3de^{4c+4dx} + de^{6c+6dx})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tanh(c + d*x))^3)^2/sinh(c + d*x),x)

[Out] $(a^2*\log(32*a^6 + 32*a^4*b^2 - 32*a^6*\exp(d*x)*\exp(c) - 32*a^4*b^2*\exp(d*x)*\exp(c)))/d - (176*b^2*\exp(c + d*x))/(15*(d + 3*d*\exp(2*c + 2*d*x) + 3*d*\exp(4*c + 4*d*x) + d*\exp(6*c + 6*d*x))) - (32*b^2*\exp(c + d*x))/(5*(d + 5*d*\exp(2*c + 2*d*x) + 10*d*\exp(4*c + 4*d*x) + 10*d*\exp(6*c + 6*d*x) + 5*d*\exp(8*c + 8*d*x) + d*\exp(10*c + 10*d*x))) - (a^2*\log(-32*a^6 - 32*a^4*b^2 - 32*a^6*\exp(d*x)*\exp(c) - 32*a^4*b^2*\exp(d*x)*\exp(c)))/d - (2*b^2*\exp(c + d*x))/(d + d*\exp(2*c + 2*d*x)) + (16*b^2*\exp(c + d*x))/(3*(d + 2*d*\exp(2*c + 2*d*x) + d*\exp(4*c + 4*d*x))) + (64*b^2*\exp(c + d*x))/(5*(d + 4*d*\exp(2*c + 2*d*x) + 6*d*\exp(4*c + 4*d*x) + 4*d*\exp(6*c + 6*d*x) + d*\exp(8*c + 8*d*x))) - (2*a*b*\exp(c + d*x))/(d + d*\exp(2*c + 2*d*x)) + (4*a*b*\exp(c + d*x))/(d + 2*d*\exp(2*c + 2*d*x) + d*\exp(4*c + 4*d*x)) - (a*b*(\log(32*a^3*b^3*\exp(d*x)*\exp(c) - a^3*b^3*32i - a^5*b*32i + 32*a^5*b*\exp(d*x)*\exp(c))*1i - \log(a^5*b*32i + a^3*b^3*32i + 32*a^3*b^3*\exp(d*x)*\exp(c) + 32*a^5*b*\exp(d*x)*\exp(c))*1i))/d$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^3(c + dx))^2 \operatorname{csch}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**3)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**2*csch(c + d*x), x)
```

3.62 $\int \operatorname{csch}^2(c + dx) (a + b \tanh^3(c + dx))^2 dx$

Optimal. Leaf size=47

$$-\frac{a^2 \operatorname{coth}(c + dx)}{d} + \frac{ab \tanh^2(c + dx)}{d} + \frac{b^2 \tanh^5(c + dx)}{5d}$$

[Out] $-a^2 \operatorname{coth}(d*x+c)/d + a*b*\tanh(d*x+c)^2/d + 1/5*b^2*\tanh(d*x+c)^5/d$

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3663, 270}

$$-\frac{a^2 \operatorname{coth}(c + dx)}{d} + \frac{ab \tanh^2(c + dx)}{d} + \frac{b^2 \tanh^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^2, x]

[Out] $-((a^2 \operatorname{Coth}[c + d*x])/d) + (a*b*\operatorname{Tanh}[c + d*x]^2)/d + (b^2*\operatorname{Tanh}[c + d*x]^5)/(5*d)$

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 3663

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^2(c + dx) (a + b \tanh^3(c + dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+bx^3)^2}{x^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{a^2}{x^2} + 2abx + b^2x^4\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{a^2 \operatorname{coth}(c + dx)}{d} + \frac{ab \tanh^2(c + dx)}{d} + \frac{b^2 \tanh^5(c + dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.25, size = 94, normalized size = 2.00

$$-\frac{a^2 \operatorname{coth}(c + dx)}{d} - \frac{ab \operatorname{sech}^2(c + dx)}{d} + \frac{b^2 \tanh(c + dx)}{5d} + \frac{b^2 \tanh(c + dx) \operatorname{sech}^4(c + dx)}{5d} - \frac{2b^2 \tanh(c + dx) \operatorname{sech}^2(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^2, x]

[Out] $-\left(\frac{a^2 \operatorname{Coth}[c + dx]}{d}\right) - \frac{a b \operatorname{Sech}[c + dx]^2}{d} + \frac{b^2 \operatorname{Tanh}[c + dx]}{(5d)} - \frac{2 b^2 \operatorname{Sech}[c + dx]^2 \operatorname{Tanh}[c + dx]}{(5d)} + \frac{b^2 \operatorname{Sech}[c + dx]^4 \operatorname{Tanh}[c + dx]}{(5d)}$

fricas [B] time = 0.57, size = 518, normalized size = 11.02

$$\frac{4 \left((5a^2 + 5ab + 2b^2) \cosh(dx + c)^5 + 5(5a^2 + 5ab + 2b^2) \cosh(dx + c) \sinh(dx + c)^4 + (5ab + 3b^2) \sinh(dx + c)^5 \right)}{5 \left(d \cosh(dx + c)^7 + 7d \cosh(dx + c) \sinh(dx + c)^6 + d \sinh(dx + c)^7 + 3d \cosh(dx + c)^5 + (21d \cosh(dx + c)^2 + 5d) \sinh(dx + c)^5 + 5(7d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^4 + d \cosh(dx + c)^3 + (35d \cosh(dx + c)^4 + 50d \cosh(dx + c)^2 + 9d) \sinh(dx + c)^3 + 3(7d \cosh(dx + c)^5 + 10d \cosh(dx + c)^3 + d \cosh(dx + c)) \sinh(dx + c)^2 - 5d \cosh(dx + c) + (7d \cosh(dx + c)^6 + 25d \cosh(dx + c)^4 + 27d \cosh(dx + c)^2 + 5d) \sinh(dx + c) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")`

[Out] $-4/5 * ((5a^2 + 5ab + 2b^2) \cosh(dx + c)^5 + 5(5a^2 + 5ab + 2b^2) \cosh(dx + c) \sinh(dx + c)^4 + (5ab + 3b^2) \sinh(dx + c)^5 + (25a^2 + 5ab - 2b^2) \cosh(dx + c)^3 + (10(5ab + 3b^2) \cosh(dx + c)^2 + 15ab - 3b^2) \sinh(dx + c)^3 + (10(5a^2 + 5ab + 2b^2) \cosh(dx + c)^3 + 3(25a^2 + 5ab - 2b^2) \cosh(dx + c)) \sinh(dx + c)^2 + 10(5a^2 - ab) \cosh(dx + c) + (5(5ab + 3b^2) \cosh(dx + c)^4 + 9(5ab - b^2) \cosh(dx + c)^2 + 10ab + 10b^2) \sinh(dx + c)) / (d \cosh(dx + c)^7 + 7d \cosh(dx + c) \sinh(dx + c)^6 + d \sinh(dx + c)^7 + 3d \cosh(dx + c)^5 + (21d \cosh(dx + c)^2 + 5d) \sinh(dx + c)^5 + 5(7d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^4 + d \cosh(dx + c)^3 + (35d \cosh(dx + c)^4 + 50d \cosh(dx + c)^2 + 9d) \sinh(dx + c)^3 + 3(7d \cosh(dx + c)^5 + 10d \cosh(dx + c)^3 + d \cosh(dx + c)) \sinh(dx + c)^2 - 5d \cosh(dx + c) + (7d \cosh(dx + c)^6 + 25d \cosh(dx + c)^4 + 27d \cosh(dx + c)^2 + 5d) \sinh(dx + c))$

giac [B] time = 0.32, size = 122, normalized size = 2.60

$$\frac{2 \left(\frac{5a^2}{e^{2dx+2c}-1} + \frac{10abe^{8dx+8c} + 5b^2e^{8dx+8c} + 30abe^{6dx+6c} + 30abe^{4dx+4c} + 10b^2e^{4dx+4c} + 10abe^{2dx+2c} + b^2}{(e^{2dx+2c}+1)^5} \right)}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")`

[Out] $-2/5 * (5a^2 / (e^{2dx+2c} - 1) + (10ab e^{8dx+8c} + 5b^2 e^{8dx+8c} + 30ab e^{6dx+6c} + 30ab e^{4dx+4c} + 10b^2 e^{4dx+4c} + 10ab e^{2dx+2c} + b^2) / (e^{2dx+2c} + 1)^5) / d$

maple [B] time = 0.46, size = 98, normalized size = 2.09

$$\frac{-a^2 \operatorname{coth}(dx + c) - \frac{ab}{\cosh(dx + c)^2} + b^2 \left(-\frac{\sinh^3(dx + c)}{2 \cosh(dx + c)^5} - \frac{3 \sinh(dx + c)}{8 \cosh(dx + c)^5} + \frac{3 \left(\frac{8}{15} + \frac{\operatorname{sech}(dx + c)^4}{5} + \frac{4 \operatorname{sech}(dx + c)^2}{15} \right) \operatorname{tanh}(dx + c)}{8} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x)`

[Out] $1/d * (-a^2 \operatorname{coth}(dx + c) - ab / \cosh(dx + c)^2 + b^2 * (-1/2 * \sinh(dx + c)^3 / \cosh(dx + c)^5 - 3/8 * \sinh(dx + c) / \cosh(dx + c)^5 + 3/8 * (8/15 + 1/5 * \operatorname{sech}(dx + c)^4 + 4/15 * \operatorname{sech}(dx + c)^2) * \operatorname{tanh}(dx + c)))$

maxima [B] time = 0.31, size = 256, normalized size = 5.45

$$\frac{2}{5} b^2 \left(\frac{10 e^{(-4dx-4c)}}{d(5 e^{(-2dx-2c)} + 10 e^{(-4dx-4c)} + 10 e^{(-6dx-6c)} + 5 e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} + \frac{10 e^{(-2dx-2c)}}{d(5 e^{(-2dx-2c)} + 10 e^{(-4dx-4c)} + 10 e^{(-6dx-6c)} + 5 e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")

[Out] $\frac{2}{5}b^2 \frac{(10e^{-4dx-4c})/(d(5e^{-2dx-2c} + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1)) + 5e^{-8dx-8c}/(d(5e^{-2dx-2c} + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1)) + 1/(d(5e^{-2dx-2c} + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1))) + 2a^2/(d(e^{-2dx-2c} - 1)) - 4ab/(d(e^{-dx+c} + e^{-dx-c}))^2}$

mupad [B] time = 1.26, size = 483, normalized size = 10.28

$$\frac{\frac{2e^{8c+8dx}(b^2+2ab)}{5d} - \frac{8e^{2c+2dx}(b^2+ab)}{5d} - \frac{2(2ab-b^2)}{5d} + \frac{8e^{6c+6dx}(ab-b^2)}{5d} + \frac{12b^2e^{4c+4dx}}{5d} - \frac{\frac{2b^2}{5d} + \frac{2e^{4c+4dx}(b^2+2ab)}{5d} + \frac{4e^{2c+2dx}}{5d}}{3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx}}}{5e^{2c+2dx} + 10e^{4c+4dx} + 10e^{6c+6dx} + 5e^{8c+8dx} + e^{10c+10dx} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tanh(c + d*x)^3)^2/sinh(c + d*x)^2,x)

[Out] $-\frac{((2\exp(8c + 8d*x))*(2*a*b + b^2))/(5*d) - (8*\exp(2*c + 2*d*x)*(a*b + b^2))/(5*d) - (2*(2*a*b - b^2))/(5*d) + (8*\exp(6*c + 6*d*x)*(a*b - b^2))/(5*d) + (12*b^2*\exp(4*c + 4*d*x))/(5*d))/(5*\exp(2*c + 2*d*x) + 10*\exp(4*c + 4*d*x) + 10*\exp(6*c + 6*d*x) + 5*\exp(8*c + 8*d*x) + \exp(10*c + 10*d*x) + 1) - ((2*b^2)/(5*d) + (2*\exp(4*c + 4*d*x)*(2*a*b + b^2))/(5*d) + (4*\exp(2*c + 2*d*x)*(a*b - b^2))/(5*d))/(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1) - ((2*(a*b - b^2))/(5*d) + (2*\exp(2*c + 2*d*x)*(2*a*b + b^2))/(5*d))/(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1) - ((2*\exp(6*c + 6*d*x)*(2*a*b + b^2))/(5*d) - (2*(a*b + b^2))/(5*d) + (6*\exp(4*c + 4*d*x)*(a*b - b^2))/(5*d) + (6*b^2*\exp(2*c + 2*d*x))/(5*d))/(4*\exp(2*c + 2*d*x) + 6*\exp(4*c + 4*d*x) + 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1) - (2*a^2)/(d*(\exp(2*c + 2*d*x) - 1)) - (2*(2*a*b + b^2))/(5*d*(\exp(2*c + 2*d*x) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^3(c + dx))^2 \operatorname{csch}^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**3)**2,x)

[Out] Integral((a + b*tanh(c + d*x)**3)**2*csch(c + d*x)**2, x)

3.63 $\int \operatorname{csch}^3(c + dx) \left(a + b \tanh^3(c + dx) \right)^2 dx$

Optimal. Leaf size=107

$$\frac{a^2 \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a^2 \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + \frac{ab \tan^{-1}(\sinh(c + dx))}{d} + \frac{ab \tanh(c + dx) \operatorname{sech}(c + dx)}{d} + \dots$$

[Out] a*b*arctan(sinh(d*x+c))/d+1/2*a^2*arctanh(cosh(d*x+c))/d-1/2*a^2*coth(d*x+c)*csch(d*x+c)/d-1/3*b^2*sech(d*x+c)^3/d+1/5*b^2*sech(d*x+c)^5/d+a*b*sech(d*x+c)*tanh(d*x+c)/d

Rubi [A] time = 0.16, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3666, 3768, 3770, 2606, 14}

$$\frac{a^2 \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a^2 \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + \frac{ab \tan^{-1}(\sinh(c + dx))}{d} + \frac{ab \tanh(c + dx) \operatorname{sech}(c + dx)}{d} + \dots$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^2,x]

[Out] (a*b*ArcTan[Sinh[c + d*x]])/d + (a^2*ArcTanh[Cosh[c + d*x]])/(2*d) - (a^2*Coth[c + d*x]*Csch[c + d*x])/(2*d) - (b^2*Sech[c + d*x]^3)/(3*d) + (b^2*Sech[c + d*x]^5)/(5*d) + (a*b*Sech[c + d*x]*Tanh[c + d*x])/d

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 2606

Int[((a_)*sec[(e_)+(f_)*(x_)])^(m_)*((b_)*tan[(e_)+(f_)*(x_)])^(n_), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m-1)*(-1+x^2)^((n-1)/2), x], x, Sec[e+f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n-1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n+1])

Rule 3666

Int[((d_)*sin[(e_)+(f_)*(x_)])^(m_)*((a_)+(b_)*((c_)*tan[(e_)+(f_)*(x_)])^(n_))^(p_), x_Symbol] := Int[ExpandTrig[(d*sin[e+f*x])^m*(a+b*(c*tan[e+f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]

Rule 3768

Int[(csc[(c_)+(d_)*(x_)]*(b_))^(n_), x_Symbol] := -Simp[(b*Cos[c+d*x])*(b*Csc[c+d*x])^(n-1)/(d*(n-1)), x] + Dist[(b^2*(n-2))/(n-1), Int[(b*Csc[c+d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_)+(d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c+d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^3(c+dx) (a+b \tanh^3(c+dx))^2 dx &= -\left(i \int (ia^2 \operatorname{csch}^3(c+dx) + 2iab \operatorname{sech}^3(c+dx) + ib^2 \operatorname{sech}^3(c+dx) \right. \\
&= a^2 \int \operatorname{csch}^3(c+dx) dx + (2ab) \int \operatorname{sech}^3(c+dx) dx + b^2 \int \operatorname{sech}^3(c+dx) dx \\
&= -\frac{a^2 \operatorname{coth}(c+dx) \operatorname{csch}(c+dx)}{2d} + \frac{ab \operatorname{sech}(c+dx) \tanh(c+dx)}{d} \\
&= \frac{ab \tan^{-1}(\sinh(c+dx))}{d} + \frac{a^2 \tanh^{-1}(\cosh(c+dx))}{2d} - \frac{a^2 \operatorname{coth}(c+dx)}{2d} \\
&= \frac{ab \tan^{-1}(\sinh(c+dx))}{d} + \frac{a^2 \tanh^{-1}(\cosh(c+dx))}{2d} - \frac{a^2 \operatorname{coth}(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 138, normalized size = 1.29

$$-\frac{a^2 \operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a^2 \operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a^2 \log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{2ab \tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{ab \tanh\left(\frac{1}{2}(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^2,x]

[Out] (2*a*b*ArcTan[Tanh[(c + d*x)/2]])/d - (a^2*Csch[(c + d*x)/2]^2)/(8*d) - (a^2*Log[Tanh[(c + d*x)/2]])/(2*d) - (a^2*Sech[(c + d*x)/2]^2)/(8*d) - (b^2*Sech[c + d*x]^3)/(3*d) + (b^2*Sech[c + d*x]^5)/(5*d) + (a*b*Sech[c + d*x]*Tanh[c + d*x])/d

fricas [B] time = 1.01, size = 4642, normalized size = 43.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")

[Out] -1/30*(30*(a^2 - 2*a*b)*cosh(d*x + c)^13 + 390*(a^2 - 2*a*b)*cosh(d*x + c)*sinh(d*x + c)^12 + 30*(a^2 - 2*a*b)*sinh(d*x + c)^13 + 20*(9*a^2 + 4*b^2)*cosh(d*x + c)^11 + 20*(117*(a^2 - 2*a*b)*cosh(d*x + c)^2 + 9*a^2 + 4*b^2)*sinh(d*x + c)^11 + 220*(39*(a^2 - 2*a*b)*cosh(d*x + c)^3 + (9*a^2 + 4*b^2)*cosh(d*x + c))*sinh(d*x + c)^10 + 6*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^9 + 2*(10725*(a^2 - 2*a*b)*cosh(d*x + c)^4 + 550*(9*a^2 + 4*b^2)*cosh(d*x + c)^2 + 225*a^2 + 90*a*b - 96*b^2)*sinh(d*x + c)^9 + 6*(6435*(a^2 - 2*a*b)*cosh(d*x + c)^5 + 550*(9*a^2 + 4*b^2)*cosh(d*x + c)^3 + 9*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c))*sinh(d*x + c)^8 + 8*(75*a^2 + 28*b^2)*cosh(d*x + c)^7 + 8*(6435*(a^2 - 2*a*b)*cosh(d*x + c)^6 + 825*(9*a^2 + 4*b^2)*cosh(d*x + c)^4 + 27*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^2 + 75*a^2 + 28*b^2)*sinh(d*x + c)^7 + 8*(6435*(a^2 - 2*a*b)*cosh(d*x + c)^7 + 1155*(9*a^2 + 4*b^2)*cosh(d*x + c)^5 + 63*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^3 + 7*(75*a^2 + 28*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 + 6*(75*a^2 - 30*a*b - 32*b^2)*cosh(d*x + c)^5 + 6*(6435*(a^2 - 2*a*b)*cosh(d*x + c)^8 + 1540*(9*a^2 + 4*b^2)*cosh(d*x + c)^6 + 126*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^4 + 28*(75*a^2 + 28*b^2)*cosh(d*x + c)^2 + 75*a^2 - 30*a*b - 32*b^2)*sinh(d*x + c)^5 + 2*(10725*(a^2 - 2*a*b)*cosh(d*x + c)^9 + 3300*(9*a^2 + 4*b^2)*cosh(d*x + c)^7 + 378*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^5 + 140*(75*a^2 + 28*b^2)*cosh(d*x + c)^3 + 15*(75*a^2 - 30*a*b - 32*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 20*(9*a^2 + 4*b^2)*cosh(d*x + c)^3 + 4*(2145*(a^2 - 2*a*b)*cosh(d*x + c)^10 + 825*(9*a^2 + 4*b^2)*cosh(d*x + c)^8 + 126*(75*a^2 + 30*a*b - 32*b^2)*cosh(d*x + c)^6 + 70*(75*a^2 + 28*b^2)*cosh(d*x + c)^4 + 15*(75*a^2 - 30*a*b - 32*b^2)*cosh(d*x + c)^2 + 45*a^2 + 20*b^2)*sinh(d*x + c)^3 + 4

$$\begin{aligned}
&*(585*(a^2 - 2*a*b)*\cosh(d*x + c)^{11} + 275*(9*a^2 + 4*b^2)*\cosh(d*x + c)^9 \\
&+ 54*(75*a^2 + 30*a*b - 32*b^2)*\cosh(d*x + c)^7 + 42*(75*a^2 + 28*b^2)*\cosh \\
&(d*x + c)^5 + 15*(75*a^2 - 30*a*b - 32*b^2)*\cosh(d*x + c)^3 + 15*(9*a^2 + 4 \\
&*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^2 - 60*(a*b*\cosh(d*x + c)^{14} + 14*a*b*\co \\
&sh(d*x + c)*\sinh(d*x + c)^{13} + a*b*\sinh(d*x + c)^{14} + 3*a*b*\cosh(d*x + c)^{1 \\
&2} + (91*a*b*\cosh(d*x + c)^2 + 3*a*b)*\sinh(d*x + c)^{12} + a*b*\cosh(d*x + c)^{1 \\
&0} + 4*(91*a*b*\cosh(d*x + c)^3 + 9*a*b*\cosh(d*x + c))*\sinh(d*x + c)^{11} + (10 \\
&01*a*b*\cosh(d*x + c)^4 + 198*a*b*\cosh(d*x + c)^2 + a*b)*\sinh(d*x + c)^{10} - \\
&5*a*b*\cosh(d*x + c)^8 + 2*(1001*a*b*\cosh(d*x + c)^5 + 330*a*b*\cosh(d*x + c) \\
&^3 + 5*a*b*\cosh(d*x + c))*\sinh(d*x + c)^9 + (3003*a*b*\cosh(d*x + c)^6 + 148 \\
&5*a*b*\cosh(d*x + c)^4 + 45*a*b*\cosh(d*x + c)^2 - 5*a*b)*\sinh(d*x + c)^8 - 5 \\
&*a*b*\cosh(d*x + c)^6 + 8*(429*a*b*\cosh(d*x + c)^7 + 297*a*b*\cosh(d*x + c)^5 \\
&+ 15*a*b*\cosh(d*x + c)^3 - 5*a*b*\cosh(d*x + c))*\sinh(d*x + c)^7 + (3003*a* \\
&b*\cosh(d*x + c)^8 + 2772*a*b*\cosh(d*x + c)^6 + 210*a*b*\cosh(d*x + c)^4 - 14 \\
&0*a*b*\cosh(d*x + c)^2 - 5*a*b)*\sinh(d*x + c)^6 + a*b*\cosh(d*x + c)^4 + 2*(1 \\
&001*a*b*\cosh(d*x + c)^9 + 1188*a*b*\cosh(d*x + c)^7 + 126*a*b*\cosh(d*x + c)^ \\
&5 - 140*a*b*\cosh(d*x + c)^3 - 15*a*b*\cosh(d*x + c))*\sinh(d*x + c)^5 + (1001 \\
&*a*b*\cosh(d*x + c)^{10} + 1485*a*b*\cosh(d*x + c)^8 + 210*a*b*\cosh(d*x + c)^6 \\
&- 350*a*b*\cosh(d*x + c)^4 - 75*a*b*\cosh(d*x + c)^2 + a*b)*\sinh(d*x + c)^4 + \\
&3*a*b*\cosh(d*x + c)^2 + 4*(91*a*b*\cosh(d*x + c)^{11} + 165*a*b*\cosh(d*x + c) \\
&^9 + 30*a*b*\cosh(d*x + c)^7 - 70*a*b*\cosh(d*x + c)^5 - 25*a*b*\cosh(d*x + c) \\
&^3 + a*b*\cosh(d*x + c))*\sinh(d*x + c)^3 + (91*a*b*\cosh(d*x + c)^{12} + 198*a* \\
&b*\cosh(d*x + c)^{10} + 45*a*b*\cosh(d*x + c)^8 - 140*a*b*\cosh(d*x + c)^6 - 75* \\
&a*b*\cosh(d*x + c)^4 + 6*a*b*\cosh(d*x + c)^2 + 3*a*b)*\sinh(d*x + c)^2 + a*b \\
&+ 2*(7*a*b*\cosh(d*x + c)^{13} + 18*a*b*\cosh(d*x + c)^{11} + 5*a*b*\cosh(d*x + c) \\
&^9 - 20*a*b*\cosh(d*x + c)^7 - 15*a*b*\cosh(d*x + c)^5 + 2*a*b*\cosh(d*x + c)^ \\
&3 + 3*a*b*\cosh(d*x + c))*\sinh(d*x + c))*\arctan(\cosh(d*x + c) + \sinh(d*x + c \\
&)) + 30*(a^2 + 2*a*b)*\cosh(d*x + c) - 15*(a^2*\cosh(d*x + c)^{14} + 14*a^2*\cos \\
&h(d*x + c)*\sinh(d*x + c)^{13} + a^2*\sinh(d*x + c)^{14} + 3*a^2*\cosh(d*x + c)^{12} \\
&+ (91*a^2*\cosh(d*x + c)^2 + 3*a^2)*\sinh(d*x + c)^{12} + a^2*\cosh(d*x + c)^{10} \\
&+ 4*(91*a^2*\cosh(d*x + c)^3 + 9*a^2*\cosh(d*x + c))*\sinh(d*x + c)^{11} + (100 \\
&1*a^2*\cosh(d*x + c)^4 + 198*a^2*\cosh(d*x + c)^2 + a^2)*\sinh(d*x + c)^{10} - 5 \\
&*a^2*\cosh(d*x + c)^8 + 2*(1001*a^2*\cosh(d*x + c)^5 + 330*a^2*\cosh(d*x + c)^ \\
&3 + 5*a^2*\cosh(d*x + c))*\sinh(d*x + c)^9 + (3003*a^2*\cosh(d*x + c)^6 + 1485 \\
&*a^2*\cosh(d*x + c)^4 + 45*a^2*\cosh(d*x + c)^2 - 5*a^2)*\sinh(d*x + c)^8 - 5* \\
&a^2*\cosh(d*x + c)^6 + 8*(429*a^2*\cosh(d*x + c)^7 + 297*a^2*\cosh(d*x + c)^5 \\
&+ 15*a^2*\cosh(d*x + c)^3 - 5*a^2*\cosh(d*x + c))*\sinh(d*x + c)^7 + (3003*a^2 \\
&)*\cosh(d*x + c)^8 + 2772*a^2*\cosh(d*x + c)^6 + 210*a^2*\cosh(d*x + c)^4 - 140 \\
&*a^2*\cosh(d*x + c)^2 - 5*a^2)*\sinh(d*x + c)^6 + a^2*\cosh(d*x + c)^4 + 2*(10 \\
&01*a^2*\cosh(d*x + c)^9 + 1188*a^2*\cosh(d*x + c)^7 + 126*a^2*\cosh(d*x + c)^5 \\
&- 140*a^2*\cosh(d*x + c)^3 - 15*a^2*\cosh(d*x + c))*\sinh(d*x + c)^5 + (1001* \\
&a^2*\cosh(d*x + c)^{10} + 1485*a^2*\cosh(d*x + c)^8 + 210*a^2*\cosh(d*x + c)^6 - \\
&350*a^2*\cosh(d*x + c)^4 - 75*a^2*\cosh(d*x + c)^2 + a^2)*\sinh(d*x + c)^4 + \\
&3*a^2*\cosh(d*x + c)^2 + 4*(91*a^2*\cosh(d*x + c)^{11} + 165*a^2*\cosh(d*x + c)^ \\
&9 + 30*a^2*\cosh(d*x + c)^7 - 70*a^2*\cosh(d*x + c)^5 - 25*a^2*\cosh(d*x + c)^ \\
&3 + a^2*\cosh(d*x + c))*\sinh(d*x + c)^3 + (91*a^2*\cosh(d*x + c)^{12} + 198*a^2 \\
&)*\cosh(d*x + c)^{10} + 45*a^2*\cosh(d*x + c)^8 - 140*a^2*\cosh(d*x + c)^6 - 75*a^ \\
&2*\cosh(d*x + c)^4 + 6*a^2*\cosh(d*x + c)^2 + 3*a^2)*\sinh(d*x + c)^2 + a^2 + \\
&2*(7*a^2*\cosh(d*x + c)^{13} + 18*a^2*\cosh(d*x + c)^{11} + 5*a^2*\cosh(d*x + c)^ \\
&9 - 20*a^2*\cosh(d*x + c)^7 - 15*a^2*\cosh(d*x + c)^5 + 2*a^2*\cosh(d*x + c)^3 \\
&+ 3*a^2*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + \\
&1) + 15*(a^2*\cosh(d*x + c)^{14} + 14*a^2*\cosh(d*x + c)*\sinh(d*x + c)^{13} + a^2 \\
&)*\sinh(d*x + c)^{14} + 3*a^2*\cosh(d*x + c)^{12} + (91*a^2*\cosh(d*x + c)^2 + 3*a^ \\
&2)*\sinh(d*x + c)^{12} + a^2*\cosh(d*x + c)^{10} + 4*(91*a^2*\cosh(d*x + c)^3 + 9* \\
&a^2*\cosh(d*x + c))*\sinh(d*x + c)^{11} + (1001*a^2*\cosh(d*x + c)^4 + 198*a^2*\c \\
&osh(d*x + c)^2 + a^2)*\sinh(d*x + c)^{10} - 5*a^2*\cosh(d*x + c)^8 + 2*(1001*a^ \\
&2*\cosh(d*x + c)^5 + 330*a^2*\cosh(d*x + c)^3 + 5*a^2*\cosh(d*x + c))*\sinh(d*x \\
&+ c)^9 + (3003*a^2*\cosh(d*x + c)^6 + 1485*a^2*\cosh(d*x + c)^4 + 45*a^2*\cos \\
&h(d*x + c)^2 - 5*a^2)*\sinh(d*x + c)^8 - 5*a^2*\cosh(d*x + c)^6 + 8*(429*a^2*
\end{aligned}$$


```

cosh(d*x + c)^7 + 297*a^2*cosh(d*x + c)^5 + 15*a^2*cosh(d*x + c)^3 - 5*a^2*
cosh(d*x + c))*sinh(d*x + c)^7 + (3003*a^2*cosh(d*x + c)^8 + 2772*a^2*cosh(
d*x + c)^6 + 210*a^2*cosh(d*x + c)^4 - 140*a^2*cosh(d*x + c)^2 - 5*a^2)*sin
h(d*x + c)^6 + a^2*cosh(d*x + c)^4 + 2*(1001*a^2*cosh(d*x + c)^9 + 1188*a^2
*cosh(d*x + c)^7 + 126*a^2*cosh(d*x + c)^5 - 140*a^2*cosh(d*x + c)^3 - 15*a
^2*cosh(d*x + c))*sinh(d*x + c)^5 + (1001*a^2*cosh(d*x + c)^10 + 1485*a^2*c
osh(d*x + c)^8 + 210*a^2*cosh(d*x + c)^6 - 350*a^2*cosh(d*x + c)^4 - 75*a^2
*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 3*a^2*cosh(d*x + c)^2 + 4*(91*a^2
*cosh(d*x + c)^11 + 165*a^2*cosh(d*x + c)^9 + 30*a^2*cosh(d*x + c)^7 - 70*a
^2*cosh(d*x + c)^5 - 25*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x +
c)^3 + (91*a^2*cosh(d*x + c)^12 + 198*a^2*cosh(d*x + c)^10 + 45*a^2*cosh(d
*x + c)^8 - 140*a^2*cosh(d*x + c)^6 - 75*a^2*cosh(d*x + c)^4 + 6*a^2*cosh(d
*x + c)^2 + 3*a^2)*sinh(d*x + c)^2 + a^2 + 2*(7*a^2*cosh(d*x + c)^13 + 18*a
^2*cosh(d*x + c)^11 + 5*a^2*cosh(d*x + c)^9 - 20*a^2*cosh(d*x + c)^7 - 15*a
^2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x
+ c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(195*(a^2 - 2*a*b)*cosh(d*
x + c)^12 + 110*(9*a^2 + 4*b^2)*cosh(d*x + c)^10 + 27*(75*a^2 + 30*a*b - 32
*b^2)*cosh(d*x + c)^8 + 28*(75*a^2 + 28*b^2)*cosh(d*x + c)^6 + 15*(75*a^2 -
30*a*b - 32*b^2)*cosh(d*x + c)^4 + 30*(9*a^2 + 4*b^2)*cosh(d*x + c)^2 + 15
*a^2 + 30*a*b)*sinh(d*x + c))/(d*cosh(d*x + c)^14 + 14*d*cosh(d*x + c)*sinh
(d*x + c)^13 + d*sinh(d*x + c)^14 + 3*d*cosh(d*x + c)^12 + (91*d*cosh(d*x +
c)^2 + 3*d)*sinh(d*x + c)^12 + 4*(91*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c)
)*sinh(d*x + c)^11 + d*cosh(d*x + c)^10 + (1001*d*cosh(d*x + c)^4 + 198*d*c
osh(d*x + c)^2 + d)*sinh(d*x + c)^10 + 2*(1001*d*cosh(d*x + c)^5 + 330*d*c
osh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^9 - 5*d*cosh(d*x + c)^8 +
(3003*d*cosh(d*x + c)^6 + 1485*d*cosh(d*x + c)^4 + 45*d*cosh(d*x + c)^2 - 5
*d)*sinh(d*x + c)^8 + 8*(429*d*cosh(d*x + c)^7 + 297*d*cosh(d*x + c)^5 + 15
*d*cosh(d*x + c)^3 - 5*d*cosh(d*x + c))*sinh(d*x + c)^7 - 5*d*cosh(d*x + c)
^6 + (3003*d*cosh(d*x + c)^8 + 2772*d*cosh(d*x + c)^6 + 210*d*cosh(d*x + c)
^4 - 140*d*cosh(d*x + c)^2 - 5*d)*sinh(d*x + c)^6 + 2*(1001*d*cosh(d*x + c)
^9 + 1188*d*cosh(d*x + c)^7 + 126*d*cosh(d*x + c)^5 - 140*d*cosh(d*x + c)^3
- 15*d*cosh(d*x + c))*sinh(d*x + c)^5 + d*cosh(d*x + c)^4 + (1001*d*cosh(d
*x + c)^10 + 1485*d*cosh(d*x + c)^8 + 210*d*cosh(d*x + c)^6 - 350*d*cosh(d*
x + c)^4 - 75*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 4*(91*d*cosh(d*x + c)
)^11 + 165*d*cosh(d*x + c)^9 + 30*d*cosh(d*x + c)^7 - 70*d*cosh(d*x + c)^5
- 25*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*d*cosh(d*x +
c)^2 + (91*d*cosh(d*x + c)^12 + 198*d*cosh(d*x + c)^10 + 45*d*cosh(d*x + c)
^8 - 140*d*cosh(d*x + c)^6 - 75*d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2 + 3
*d)*sinh(d*x + c)^2 + 2*(7*d*cosh(d*x + c)^13 + 18*d*cosh(d*x + c)^11 + 5*d
*cosh(d*x + c)^9 - 20*d*cosh(d*x + c)^7 - 15*d*cosh(d*x + c)^5 + 2*d*cosh(d
*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c) + d)

```

giac [A] time = 0.31, size = 192, normalized size = 1.79

$$\frac{60 ab \arctan(e^{(dx+c)}) + 15 a^2 \log(e^{(dx+c)} + 1) - 15 a^2 \log(|e^{(dx+c)} - 1|) - \frac{30(a^2 e^{(3dx+3c)} + a^2 e^{(dx+c)})}{(e^{(2dx+2c)} - 1)^2} + \frac{4(15 ab e^{(9dx+9c)} + \dots)}{30 d}}{30 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")

[Out] 1/30*(60*a*b*arctan(e^(d*x + c)) + 15*a^2*log(e^(d*x + c) + 1) - 15*a^2*log(abs(e^(d*x + c) - 1)) - 30*(a^2*e^(3*d*x + 3*c) + a^2*e^(d*x + c))/(e^(2*d*x + 2*c) - 1)^2 + 4*(15*a*b*e^(9*d*x + 9*c) + 30*a*b*e^(7*d*x + 7*c) - 20*b^2*e^(7*d*x + 7*c) + 8*b^2*e^(5*d*x + 5*c) - 30*a*b*e^(3*d*x + 3*c) - 20*b^2*e^(3*d*x + 3*c) - 15*a*b*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^5)/d

maple [A] time = 0.53, size = 108, normalized size = 1.01

$$-\frac{a^2 \coth(dx+c) \operatorname{csch}(dx+c)}{2d} + \frac{a^2 \operatorname{arctanh}(e^{dx+c})}{d} + \frac{ab \operatorname{sech}(dx+c) \tanh(dx+c)}{d} + \frac{2ab \operatorname{arctan}(e^{dx+c})}{d} - \frac{b^2 (\sin(dx+c))}{3d \cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x)`

[Out] `-1/2*a^2*coth(d*x+c)*csch(d*x+c)/d+1/d*a^2*arctanh(exp(d*x+c))+1/d*a*b*sech(d*x+c)*tanh(d*x+c)+2/d*a*b*arctan(exp(d*x+c))-1/3/d*b^2*sinh(d*x+c)^2/cosh(d*x+c)^5-2/15/d*b^2/cosh(d*x+c)^5`

maxima [B] time = 0.41, size = 378, normalized size = 3.53

$$-2ab \left(\frac{\operatorname{arctan}(e^{(-dx-c)})}{d} - \frac{e^{(-dx-c)} - e^{(-3dx-3c)}}{d(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1)} \right) + \frac{1}{2} a^2 \left(\frac{\log(e^{(-dx-c)} + 1)}{d} - \frac{\log(e^{(-dx-c)} - 1)}{d} + \frac{2(e^{(-dx-c)} - 1)}{d(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")`

[Out] `-2*a*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 1/2*a^2*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 8/15*b^2*(5*e^(-3*d*x - 3*c))/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 2*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^(-7*d*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)))`

mupad [B] time = 2.90, size = 561, normalized size = 5.24

$$\frac{a^2 e^{c+dx}}{d - d e^{2c+2dx}} + \frac{136 b^2 e^{c+dx}}{15 (d + 3 d e^{2c+2dx} + 3 d e^{4c+4dx} + d e^{6c+6dx})} + \frac{32 b^2 e^{c+dx}}{5 (d + 5 d e^{2c+2dx} + 10 d e^{4c+4dx} + 10 d e^{6c+6dx})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*tanh(c + d*x)^3)^2/sinh(c + d*x)^3,x)`

[Out] `(a^2*exp(c + d*x))/(d - d*exp(2*c + 2*d*x)) + (136*b^2*exp(c + d*x))/(15*(d + 3*d*exp(2*c + 2*d*x) + 3*d*exp(4*c + 4*d*x) + d*exp(6*c + 6*d*x))) + (32*b^2*exp(c + d*x))/(5*(d + 5*d*exp(2*c + 2*d*x) + 10*d*exp(4*c + 4*d*x) + 10*d*exp(6*c + 6*d*x) + 5*d*exp(8*c + 8*d*x) + d*exp(10*c + 10*d*x))) - (a^2*log(4*a^6*exp(d*x)*exp(c) - 16*a^4*b^2 - 4*a^6 + 16*a^4*b^2*exp(d*x)*exp(c)))/(2*d) + (a^2*log(4*a^6 + 16*a^4*b^2 + 4*a^6*exp(d*x)*exp(c) + 16*a^4*b^2*exp(d*x)*exp(c)))/(2*d) - (2*a^2*exp(c + d*x))/(d - 2*d*exp(2*c + 2*d*x) + d*exp(4*c + 4*d*x)) - (8*b^2*exp(c + d*x))/(3*(d + 2*d*exp(2*c + 2*d*x) + d*exp(4*c + 4*d*x))) - (64*b^2*exp(c + d*x))/(5*(d + 4*d*exp(2*c + 2*d*x) + 6*d*exp(4*c + 4*d*x) + 4*d*exp(6*c + 6*d*x) + d*exp(8*c + 8*d*x))) + (2*a*b*exp(c + d*x))/(d + d*exp(2*c + 2*d*x)) - (4*a*b*exp(c + d*x))/(d + 2*d*exp(2*c + 2*d*x) + d*exp(4*c + 4*d*x)) - (a*b*(log(32*a^3*b^3*exp(d*x)*exp(c) - a^3*b^3*32i - a^5*b*8i + 8*a^5*b*exp(d*x)*exp(c))*1i - log(a^5*b*8i + a^3*b^3*32i + 32*a^3*b^3*exp(d*x)*exp(c) + 8*a^5*b*exp(d*x)*exp(c))*1i))/d`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^3(c + dx))^2 \operatorname{csch}^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**3)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**2*csch(c + d*x)**3, x)
```

3.64 $\int \operatorname{csch}^4(c + dx) (a + b \tanh^3(c + dx))^2 dx$

Optimal. Leaf size=97

$$-\frac{a^2 \operatorname{coth}^3(c + dx)}{3d} + \frac{a^2 \operatorname{coth}(c + dx)}{d} - \frac{ab \tanh^2(c + dx)}{d} + \frac{2ab \log(\tanh(c + dx))}{d} - \frac{b^2 \tanh^5(c + dx)}{5d} + \frac{b^2 \tanh^3(c + dx)}{3d}$$

[Out] $a^2 \operatorname{coth}(d*x+c)/d - 1/3*a^2 \operatorname{coth}(d*x+c)^3/d + 2*a*b*\ln(\tanh(d*x+c))/d - a*b*\tanh(d*x+c)^2/d + 1/3*b^2*\tanh(d*x+c)^3/d - 1/5*b^2*\tanh(d*x+c)^5/d$

Rubi [A] time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3663, 1802}

$$-\frac{a^2 \operatorname{coth}^3(c + dx)}{3d} + \frac{a^2 \operatorname{coth}(c + dx)}{d} - \frac{ab \tanh^2(c + dx)}{d} + \frac{2ab \log(\tanh(c + dx))}{d} - \frac{b^2 \tanh^5(c + dx)}{5d} + \frac{b^2 \tanh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^2,x]

[Out] $(a^2*\operatorname{Coth}[c + d*x])/d - (a^2*\operatorname{Coth}[c + d*x]^3)/(3*d) + (2*a*b*\operatorname{Log}[\operatorname{Tanh}[c + d*x]])/d - (a*b*\operatorname{Tanh}[c + d*x]^2)/d + (b^2*\operatorname{Tanh}[c + d*x]^3)/(3*d) - (b^2*\operatorname{Tanh}[c + d*x]^5)/(5*d)$

Rule 1802

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 3663

Int[sin[(e_)+(f_)*(x_)]^(m_)*((a_) + (b_)*((c_)*tan[(e_)+(f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^4(c + dx) (a + b \tanh^3(c + dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(a+bx^3)^2}{x^4} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{a^2}{x^4} - \frac{a^2}{x^2} + \frac{2ab}{x} - 2abx + b^2x^2 - b^2x^4\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{a^2 \operatorname{coth}(c + dx)}{d} - \frac{a^2 \operatorname{coth}^3(c + dx)}{3d} + \frac{2ab \log(\tanh(c + dx))}{d} - \frac{ab \tanh^5(c + dx)}{5d} + \frac{b^2 \tanh^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.22, size = 147, normalized size = 1.52

$$\frac{2a^2 \operatorname{coth}(c + dx)}{3d} - \frac{a^2 \operatorname{coth}(c + dx) \operatorname{csch}^2(c + dx)}{3d} + \frac{ab \operatorname{sech}^2(c + dx)}{d} + \frac{2ab \log(\sinh(c + dx))}{d} - \frac{2ab \log(\cosh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^2,x]

[Out] $(2*a^2*\text{Coth}[c + d*x])/(3*d) - (a^2*\text{Coth}[c + d*x]*\text{Csch}[c + d*x]^2)/(3*d) - (2*a*b*\text{Log}[\text{Cosh}[c + d*x]])/d + (2*a*b*\text{Log}[\text{Sinh}[c + d*x]])/d + (a*b*\text{Sech}[c + d*x]^2)/d + (2*b^2*\text{Tanh}[c + d*x])/(15*d) + (b^2*\text{Sech}[c + d*x]^2*\text{Tanh}[c + d*x])/(15*d) - (b^2*\text{Sech}[c + d*x]^4*\text{Tanh}[c + d*x])/(5*d)$

fricas [B] time = 0.62, size = 4125, normalized size = 42.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")

[Out] $2/15*(30*a*b*\cosh(d*x + c)^{14} + 420*a*b*\cosh(d*x + c)*\sinh(d*x + c)^{13} + 30*a*b*\sinh(d*x + c)^{14} - 30*(a^2 + b^2)*\cosh(d*x + c)^{12} + 30*(91*a*b*\cosh(d*x + c)^2 - a^2 - b^2)*\sinh(d*x + c)^{12} + 120*(91*a*b*\cosh(d*x + c)^3 - 3*(a^2 + b^2)*\cosh(d*x + c))*\sinh(d*x + c)^{11} - 10*(14*a^2 + 9*a*b - 10*b^2)*\cosh(d*x + c)^{10} + 10*(3003*a*b*\cosh(d*x + c)^4 - 198*(a^2 + b^2)*\cosh(d*x + c)^2 - 14*a^2 - 9*a*b + 10*b^2)*\sinh(d*x + c)^{10} + 20*(3003*a*b*\cosh(d*x + c)^5 - 330*(a^2 + b^2)*\cosh(d*x + c)^3 - 5*(14*a^2 + 9*a*b - 10*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^9 - 10*(25*a^2 + 13*b^2)*\cosh(d*x + c)^8 + 10*(9009*a*b*\cosh(d*x + c)^6 - 1485*(a^2 + b^2)*\cosh(d*x + c)^4 - 45*(14*a^2 + 9*a*b - 10*b^2)*\cosh(d*x + c)^2 - 25*a^2 - 13*b^2)*\sinh(d*x + c)^8 + 80*(1287*a*b*\cosh(d*x + c)^7 - 297*(a^2 + b^2)*\cosh(d*x + c)^5 - 15*(14*a^2 + 9*a*b - 10*b^2)*\cosh(d*x + c)^3 - (25*a^2 + 13*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^7 - 2*(100*a^2 - 45*a*b - 44*b^2)*\cosh(d*x + c)^6 + 2*(45045*a*b*\cosh(d*x + c)^8 - 13860*(a^2 + b^2)*\cosh(d*x + c)^6 - 1050*(14*a^2 + 9*a*b - 10*b^2)*\cosh(d*x + c)^4 - 140*(25*a^2 + 13*b^2)*\cosh(d*x + c)^2 - 100*a^2 + 45*a*b + 44*b^2)*\sinh(d*x + c)^6 + 4*(15015*a*b*\cosh(d*x + c)^9 - 5940*(a^2 + b^2)*\cosh(d*x + c)^7 - 630*(14*a^2 + 9*a*b - 10*b^2)*\cosh(d*x + c)^5 - 140*(25*a^2 + 13*b^2)*\cosh(d*x + c)^3 - 3*(100*a^2 - 45*a*b - 44*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2*(25*a^2 + 17*b^2)*\cosh(d*x + c)^4 + 2*(15015*a*b*\cosh(d*x + c)^10 - 7425*(a^2 + b^2)*\cosh(d*x + c)^8 - 1050*(14*a^2 + 9*a*b - 10*b^2)*\cosh(d*x + c)^6 - 350*(25*a^2 + 13*b^2)*\cosh(d*x + c)^4 - 15*(100*a^2 - 45*a*b - 44*b^2)*\cosh(d*x + c)^2 - 25*a^2 - 17*b^2)*\sinh(d*x + c)^4 + 8*(1365*a*b*\cosh(d*x + c)^11 - 825*(a^2 + b^2)*\cosh(d*x + c)^9 - 150*(14*a^2 + 9*a*b - 10*b^2)*\cosh(d*x + c)^7 - 70*(25*a^2 + 13*b^2)*\cosh(d*x + c)^5 - 5*(100*a^2 - 45*a*b - 44*b^2)*\cosh(d*x + c)^3 - (25*a^2 + 17*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 2*(10*a^2 - 15*a*b + 2*b^2)*\cosh(d*x + c)^2 + 2*(1365*a*b*\cosh(d*x + c)^12 - 990*(a^2 + b^2)*\cosh(d*x + c)^10 - 225*(14*a^2 + 9*a*b - 10*b^2)*\cosh(d*x + c)^8 - 140*(25*a^2 + 13*b^2)*\cosh(d*x + c)^6 - 15*(100*a^2 - 45*a*b - 44*b^2)*\cosh(d*x + c)^4 - 6*(25*a^2 + 17*b^2)*\cosh(d*x + c)^2 + 10*a^2 - 15*a*b + 2*b^2)*\sinh(d*x + c)^2 + 10*a^2 + 2*b^2 - 15*(a*b*\cosh(d*x + c)^16 + 16*a*b*\cosh(d*x + c)*\sinh(d*x + c)^15 + a*b*\sinh(d*x + c)^16 + 2*a*b*\cosh(d*x + c)^14 + 2*(60*a*b*\cosh(d*x + c)^2 + a*b)*\sinh(d*x + c)^14 - 2*a*b*\cosh(d*x + c)^12 + 28*(20*a*b*\cosh(d*x + c)^3 + a*b*\cosh(d*x + c))*\sinh(d*x + c)^13 + 2*(910*a*b*\cosh(d*x + c)^4 + 91*a*b*\cosh(d*x + c)^2 - a*b)*\sinh(d*x + c)^12 - 6*a*b*\cosh(d*x + c)^10 + 8*(546*a*b*\cosh(d*x + c)^5 + 91*a*b*\cosh(d*x + c)^3 - 3*a*b*\cosh(d*x + c))*\sinh(d*x + c)^11 + 2*(4004*a*b*\cosh(d*x + c)^6 + 1001*a*b*\cosh(d*x + c)^4 - 66*a*b*\cosh(d*x + c)^2 - 3*a*b)*\sinh(d*x + c)^10 + 4*(2860*a*b*\cosh(d*x + c)^7 + 1001*a*b*\cosh(d*x + c)^5 - 110*a*b*\cosh(d*x + c)^3 - 15*a*b*\cosh(d*x + c))*\sinh(d*x + c)^9 + 6*(2145*a*b*\cosh(d*x + c)^8 + 1001*a*b*\cosh(d*x + c)^6 - 165*a*b*\cosh(d*x + c)^4 - 45*a*b*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 6*a*b*\cosh(d*x + c)^6 + 16*(715*a*b*\cosh(d*x + c)^9 + 429*a*b*\cosh(d*x + c)^7 - 99*a*b*\cosh(d*x + c)^5 - 45*a*b*\cosh(d*x + c)^3)*\sinh(d*x + c)^7 + 2*(4004*a*b*\cosh(d*x + c)^10 + 3003*a*b*\cosh(d*x + c)^8 - 924*a*b*\cosh(d*x + c)^6 - 630*a*b*\cosh(d*x + c)^4 + 3*a*b)*\sinh(d*x + c)^6 + 2*a*b*\cosh(d*x + c)^4 + 4*(1092*a*b*\cosh(d*x + c)^11 + 1001*a*b*\cosh(d*x + c)^9 - 396*a*b*\cosh(d*x + c)^7 - 378$

$0*d*\cosh(d*x + c)^6 + 45*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^4 + 8*(70*d*\cosh(d*x + c)^{13} + 91*d*\cosh(d*x + c)^{11} - 55*d*\cosh(d*x + c)^9 - 90*d*\cosh(d*x + c)^7 + 15*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c)^3 - 2*d*\cosh(d*x + c)^2 + 2*(60*d*\cosh(d*x + c)^{14} + 91*d*\cosh(d*x + c)^{12} - 66*d*\cosh(d*x + c)^{10} - 135*d*\cosh(d*x + c)^8 + 45*d*\cosh(d*x + c)^4 + 6*d*\cosh(d*x + c)^2 - d)*\sinh(d*x + c)^2 + 4*(4*d*\cosh(d*x + c)^{15} + 7*d*\cosh(d*x + c)^{13} - 6*d*\cosh(d*x + c)^{11} - 15*d*\cosh(d*x + c)^9 + 9*d*\cosh(d*x + c)^5 + 2*d*\cosh(d*x + c)^3 - d*\cosh(d*x + c))*\sinh(d*x + c) - d$

giac [B] time = 0.33, size = 249, normalized size = 2.57

$$60 ab \log(e^{(2dx+2c)} + 1) - 60 ab \log(|e^{(2dx+2c)} - 1|) + \frac{10(11abe^{(6dx+6c)} - 33abe^{(4dx+4c)} + 12a^2e^{(2dx+2c)} + 33abe^{(2dx+2c)} - 4a^2)}{(e^{(2dx+2c)} - 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")

[Out] $-1/30*(60*a*b*\log(e^{(2*d*x + 2*c)} + 1) - 60*a*b*\log(\text{abs}(e^{(2*d*x + 2*c)} - 1))) + 10*(11*a*b*e^{(6*d*x + 6*c)} - 33*a*b*e^{(4*d*x + 4*c)} + 12*a^2*e^{(2*d*x + 2*c)} + 33*a*b*e^{(2*d*x + 2*c)} - 4*a^2 - 11*a*b)/(e^{(2*d*x + 2*c)} - 1)^3 - (137*a*b*e^{(10*d*x + 10*c)} + 805*a*b*e^{(8*d*x + 8*c)} + 1730*a*b*e^{(6*d*x + 6*c)} - 120*b^2*e^{(6*d*x + 6*c)} + 1730*a*b*e^{(4*d*x + 4*c)} + 40*b^2*e^{(4*d*x + 4*c)} + 805*a*b*e^{(2*d*x + 2*c)} - 40*b^2*e^{(2*d*x + 2*c)} + 137*a*b - 8*b^2)/(e^{(2*d*x + 2*c)} + 1)^5)/d$

maple [A] time = 0.49, size = 146, normalized size = 1.51

$$\frac{2a^2 \coth(dx + c)}{3d} - \frac{a^2 \coth(dx + c) \operatorname{csch}(dx + c)^2}{3d} + \frac{ab}{d \cosh(dx + c)^2} + \frac{2ab \ln(\tanh(dx + c))}{d} - \frac{b^2 \sinh(dx + c)}{4d \cosh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x)

[Out] $2/3*a^2*\coth(d*x+c)/d - 1/3/d*a^2*\coth(d*x+c)*\operatorname{csch}(d*x+c)^2 + 1/d*a*b/\cosh(d*x+c)^2 + 2*a*b*\ln(\tanh(d*x+c))/d - 1/4/d*b^2*\sinh(d*x+c)/\cosh(d*x+c)^5 + 2/15*b^2*\tanh(d*x+c)/d + 1/20/d*b^2*\tanh(d*x+c)*\operatorname{sech}(d*x+c)^4 + 1/15/d*b^2*\tanh(d*x+c)*\operatorname{sech}(d*x+c)^2$

maxima [B] time = 0.41, size = 468, normalized size = 4.82

$$2ab \left(\frac{\log(e^{(-dx-c)} + 1)}{d} + \frac{\log(e^{(-dx-c)} - 1)}{d} - \frac{\log(e^{(-2dx-2c)} + 1)}{d} + \frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1)} \right) + \frac{4}{15} b^2 \left(\frac{1}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")

[Out] $2*a*b*(\log(e^{(-d*x - c)} + 1)/d + \log(e^{(-d*x - c)} - 1)/d - \log(e^{(-2*d*x - 2*c)} + 1)/d + 2*e^{(-2*d*x - 2*c)}/(d*(2*e^{(-2*d*x - 2*c)} + e^{(-4*d*x - 4*c)} + 1))) + 4/15*b^2*(5*e^{(-2*d*x - 2*c)}/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)) - 5*e^{(-4*d*x - 4*c)}/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)) + 15*e^{(-6*d*x - 6*c)}/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)) + 1/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1))) + 4/3*a^2*(3*e^{(-2*d*x - 2*c)}/(d*(3*e^{(-2*d*x - 2*c)} -$

$3e^{(-4dx - 4c) + e^{(-6dx - 6c) - 1}} - 1/(d(3e^{(-2dx - 2c) - 3e^{(-4dx - 4c) + e^{(-6dx - 6c) - 1}}))$

mupad [B] time = 0.26, size = 344, normalized size = 3.55

$$\frac{40b^2}{3d(3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1)} - \frac{4a^2}{d(e^{4c+4dx} - 2e^{2c+2dx} + 1)} - \frac{8a^2}{3d(3e^{2c+2dx} - 3e^{4c+4dx} + e^{6c+6dx} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tanh(c + d*x)^3)^2/sinh(c + d*x)^4, x)

[Out] $(40b^2)/(3d(3\exp(2c + 2dx) + 3\exp(4c + 4dx) + \exp(6c + 6dx) + 1)) - (4a^2)/(d(\exp(4c + 4dx) - 2\exp(2c + 2dx) + 1)) - (8a^2)/(3d(3\exp(2c + 2dx) - 3\exp(4c + 4dx) + \exp(6c + 6dx) - 1)) - (4(a*b + b^2))/(d(2\exp(2c + 2dx) + \exp(4c + 4dx) + 1)) - (16b^2)/(d(4\exp(2c + 2dx) + 6\exp(4c + 4dx) + 4\exp(6c + 6dx) + \exp(8c + 8dx) + 1)) + (32b^2)/(5d(5\exp(2c + 2dx) + 10\exp(4c + 4dx) + 10\exp(6c + 6dx) + 5\exp(8c + 8dx) + \exp(10c + 10dx) + 1)) - (4\operatorname{atan}((a*b\exp(2c)*\exp(2dx)*(-d^2)^{1/2})/(d*(a^2*b^2)^{1/2}))*(-d^2)^{1/2} + (4ab)/(d(\exp(2c + 2dx) + 1)))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^3(c + dx))^2 \operatorname{csch}^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**3)**2, x)

[Out] Integral((a + b*tanh(c + d*x)**3)**2*csch(c + d*x)**4, x)

3.65 $\int \sinh^4(c + dx) \left(a + b \tanh^3(c + dx)\right)^3 dx$

Optimal. Leaf size=275

$$\frac{b(3a^2 + 10b^2) \tanh^2(c + dx)}{2d} + \frac{3b(3a^2 + 5b^2) \log(\cosh(c + dx))}{d} + \frac{\sinh(c + dx) \cosh^3(c + dx) (b(3a^2 + b^2) \tanh(c + dx))}{4d}$$

[Out] $\frac{3}{8} a (a^2 + 63 b^2) x + 3 b (3 a^2 + 5 b^2) \ln(\cosh(d x + c)) / d - 18 a b^2 \tanh(d x + c) / d - 1/2 b (3 a^2 + 10 b^2) \tanh(d x + c)^2 / d - 3 a b^2 \tanh(d x + c)^3 / d - 3/2 b^3 \tanh(d x + c)^4 / d - 3/5 a b^2 \tanh(d x + c)^5 / d - 1/2 b^3 \tanh(d x + c)^6 / d - 1/8 b^3 \tanh(d x + c)^8 / d + 1/4 \cosh(d x + c)^3 \sinh(d x + c) (a (a^2 + 3 b^2) + b (3 a^2 + b^2) \tanh(d x + c)) / d - 1/8 \cosh(d x + c) \sinh(d x + c) (a (5 a^2 + 51 b^2) + 2 b (15 a^2 + 11 b^2) \tanh(d x + c)) / d$

Rubi [A] time = 0.49, antiderivative size = 306, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3663, 1804, 1802, 633, 31}

$$\frac{3b(3a^2 + 5b^2) \tanh^2(c + dx)}{2d} - \frac{3a(a^2 + 63b^2) \tanh(c + dx)}{8d} - \frac{3(a + b)(a^2 + 23ab + 40b^2) \log(1 - \tanh(c + dx))}{16d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^3,x]

[Out] $(-3(a + b)(a^2 + 23ab + 40b^2) \log(1 - \tanh[c + d*x])) / (16d) + (3(a - b)(a^2 - 23ab + 40b^2) \log(1 + \tanh[c + d*x])) / (16d) - (3a(a^2 + 63b^2) \tanh[c + d*x]) / (8d) - (3b(3a^2 + 5b^2) \tanh[c + d*x]^2) / (2d) - (3ab^2 \tanh[c + d*x]^3) / d - (3b^3 \tanh[c + d*x]^4) / (2d) - (3ab^2 \tanh[c + d*x]^5) / (5d) - (b^3 \tanh[c + d*x]^6) / (2d) - (b^3 \tanh[c + d*x]^8) / (8d) + (\sinh[c + d*x]^4 (b(3a^2 + b^2) + a(a^2 + 3b^2) \tanh[c + d*x])) / (4d) - (\sinh[c + d*x]^2 \tanh[c + d*x] (a(a^2 + 39b^2) + 4b(6a^2 + 5b^2) \tanh[c + d*x])) / (8d)$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 633

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]

Rule 1802

Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1804

Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x] + Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)ExpandToSum[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a,

b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned} \int \sinh^4(c + dx) (a + b \tanh^3(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{x^4(a+bx^3)^3}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\sinh^4(c + dx) (b(3a^2 + b^2) + a(a^2 + 3b^2) \tanh(c + dx))}{4d} + \frac{\text{Subst}\left(\int \frac{x^4(a+bx^3)^3}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\sinh^4(c + dx) (b(3a^2 + b^2) + a(a^2 + 3b^2) \tanh(c + dx))}{4d} - \frac{\sinh^2(c + dx) (b(3a^2 + b^2) + a(a^2 + 3b^2) \tanh(c + dx))}{4d} \\ &= \frac{\sinh^4(c + dx) (b(3a^2 + b^2) + a(a^2 + 3b^2) \tanh(c + dx))}{4d} - \frac{\sinh^2(c + dx) (b(3a^2 + b^2) + a(a^2 + 3b^2) \tanh(c + dx))}{4d} \\ &= -\frac{3a(a^2 + 63b^2) \tanh(c + dx)}{8d} - \frac{3b(3a^2 + 5b^2) \tanh^2(c + dx)}{2d} - \frac{3a}{32d} \\ &= -\frac{3a(a^2 + 63b^2) \tanh(c + dx)}{8d} - \frac{3b(3a^2 + 5b^2) \tanh^2(c + dx)}{2d} - \frac{3a}{32d} \\ &= -\frac{3(a + b)(a^2 + 23ab + 40b^2) \log(1 - \tanh(c + dx))}{16d} + \frac{3(a - b)(a^2 + 63b^2)}{16d} \end{aligned}$$

Mathematica [A] time = 6.27, size = 294, normalized size = 1.07

$$\frac{3(3a^2b + 5b^3) \log(\cosh(c + dx))}{d} + \frac{3a(a^2 + 63b^2)(c + dx)}{8d} - \frac{a(a^2 + 12b^2) \sinh(2(c + dx))}{4d} + \frac{a(a^2 + 3b^2) \sinh(4(c + dx))}{32d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^3,x]

[Out] (3*a*(a^2 + 63*b^2)*(c + d*x))/(8*d) - (b*(15*a^2 + 11*b^2)*Cosh[2*(c + d*x)])/(8*d) + (b*(3*a^2 + b^2)*Cosh[4*(c + d*x)])/(32*d) + (3*(3*a^2*b + 5*b^3)*Log[Cosh[c + d*x]])/d + (b*(3*a^2 + 20*b^2)*Sech[c + d*x]^2)/(2*d) - (15*b^3*Sech[c + d*x]^4)/(4*d) + (b^3*Sech[c + d*x]^6)/d - (b^3*Sech[c + d*x]^8)/(8*d) - (a*(a^2 + 12*b^2)*Sinh[2*(c + d*x)])/(4*d) + (a*(a^2 + 3*b^2)*Sinh[4*(c + d*x)])/(32*d) - (108*a*b^2*Tanh[c + d*x])/(5*d) + (21*a*b^2*Sech[c + d*x]^2*Tanh[c + d*x])/(5*d) - (3*a*b^2*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")

[Out] Timed out

giac [B] time = 1.63, size = 708, normalized size = 2.57

$$840(a^3 - 24a^2b + 63ab^2 - 40b^3)dx + 6720(3a^2be^{2c} + 5b^3e^{2c})e^{-2c} \log(e^{(2dx+2c)} + 1) - 35(18a^3e^{4dx+4c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")

[Out]
$$\frac{1}{2240} \cdot (840(a^3 - 24a^2b + 63ab^2 - 40b^3)dx + 6720(3a^2be^{2c} + 5b^3e^{2c})e^{-2c} \log(e^{(2dx+2c)} + 1) - 35(18a^3e^{4dx+4c} + 5b^3e^{4c})e^{-2c} \log(e^{(2dx+2c)} + 1) - 432a^2be^{4c} + 1134ab^2e^{4c} - 720b^3e^{4c} - 8a^3e^{2c} + 60a^2be^{2c} - 96ab^2e^{2c} + 44b^3e^{2c} + a^3 - 3a^2b + 3ab^2 - b^3) e^{-4c} + 35(a^3e^{4c} + 3a^2be^{4c} + 3ab^2e^{4c} + b^3e^{4c} - 8a^3e^{2c} - 60a^2be^{2c} - 96ab^2e^{2c} - 44b^3e^{2c} + 6849a^2be^{16c} + 11415b^3e^{16c} + 53112a^2be^{14c} - 16800ab^2e^{14c} + 80120b^3e^{14c} + 181692a^2be^{12c} - 100800ab^2e^{12c} + 269220b^3e^{12c} + 358344a^2be^{10c} - 272160ab^2e^{10c} + 520520b^3e^{10c} + 445830a^2be^{8c} - 423360ab^2e^{8c} + 648970b^3e^{8c} + 358344a^2be^{6c} - 405216ab^2e^{6c} + 520520b^3e^{6c} + 181692a^2be^{4c} - 237888ab^2e^{4c} + 269220b^3e^{4c} + 53112a^2be^{2c} - 79968ab^2e^{2c} + 80120b^3e^{2c} + 6849a^2b - 12096ab^2 + 11415b^3) / (e^{(2dx+2c)} + 1)^8 / d$$

maple [A] time = 0.41, size = 385, normalized size = 1.40

$$\frac{a^3 \cosh(dx+c) (\sinh^3(dx+c))}{4d} - \frac{3a^3 \cosh(dx+c) \sinh(dx+c)}{8d} + \frac{3a^3x}{8} + \frac{3a^3c}{8d} + \frac{3a^2b (\sinh^6(dx+c))}{4d \cosh^2(dx+c)} - \frac{9a^2b}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x)

[Out]
$$\frac{1}{4} \cdot \frac{1}{d} \cdot a^3 \cdot \cosh(dx+c) \cdot \sinh(dx+c)^3 - \frac{3}{8} \cdot \frac{1}{d} \cdot a^3 \cdot \cosh(dx+c) \cdot \sinh(dx+c) / d + \frac{3}{8} \cdot a^3 \cdot x + \frac{3}{8} \cdot \frac{1}{d} \cdot a^3 \cdot c + \frac{3}{4} \cdot \frac{1}{d} \cdot a^2 \cdot b \cdot \sinh(dx+c)^6 / \cosh(dx+c)^2 - \frac{9}{4} \cdot \frac{1}{d} \cdot a^2 \cdot b \cdot \sinh(dx+c)^4 / \cosh(dx+c)^2 + \frac{9}{d} \cdot a^2 \cdot b \cdot \ln(\cosh(dx+c)) - \frac{9}{2} \cdot \frac{1}{d} \cdot a^2 \cdot b \cdot \tanh(dx+c)^2 / d + \frac{3}{4} \cdot \frac{1}{d} \cdot a \cdot b^2 \cdot \sinh(dx+c)^9 / \cosh(dx+c)^5 - \frac{27}{8} \cdot \frac{1}{d} \cdot a \cdot b^2 \cdot \sinh(dx+c)^7 / \cosh(dx+c)^5 + \frac{189}{8} \cdot \frac{1}{d} \cdot a \cdot b^2 \cdot x + \frac{189}{8} \cdot \frac{1}{d} \cdot a \cdot b^2 \cdot c - \frac{189}{8} \cdot \frac{1}{d} \cdot a \cdot b^2 \cdot \tanh(dx+c) / d - \frac{63}{8} \cdot \frac{1}{d} \cdot a \cdot b^2 \cdot \tanh(dx+c)^3 / d - \frac{189}{40} \cdot \frac{1}{d} \cdot a \cdot b^2 \cdot \tanh(dx+c)^5 / d + \frac{1}{4} \cdot \frac{1}{d} \cdot b^3 \cdot \sinh(dx+c)^{12} / \cosh(dx+c)^8 - \frac{3}{2} \cdot \frac{1}{d} \cdot b^3 \cdot \sinh(dx+c)^{10} / \cosh(dx+c)^8 + \frac{15}{d} \cdot \frac{1}{d} \cdot b^3 \cdot \ln(\cosh(dx+c)) - \frac{15}{2} \cdot \frac{1}{d} \cdot b^3 \cdot \tanh(dx+c)^2 / d - \frac{15}{4} \cdot \frac{1}{d} \cdot b^3 \cdot \tanh(dx+c)^4 / d - \frac{5}{2} \cdot \frac{1}{d} \cdot b^3 \cdot \tanh(dx+c)^6 / d - \frac{15}{8} \cdot \frac{1}{d} \cdot b^3 \cdot \tanh(dx+c)^8 / d$$

maxima [B] time = 0.43, size = 647, normalized size = 2.35

$$\frac{1}{64} a^3 \left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) + \frac{3}{320} ab^2 \left(\frac{2520(dx+c)}{d} + \frac{5(32e^{(-2dx-2c)} - e^{(2dx+2c)})}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")

```
[Out] 1/64*a^3*(24*x + e^(4*d*x + 4*c))/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) + 3/320*a*b^2*(2520*(d*x + c)/d + 5*(32*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (135*e^(-2*d*x - 2*c) + 5358*e^(-4*d*x - 4*c) + 18190*e^(-6*d*x - 6*c) + 28455*e^(-8*d*x - 8*c) + 19995*e^(-10*d*x - 10*c) + 6560*e^(-12*d*x - 12*c) - 5)/(d*(e^(-4*d*x - 4*c) + 5*e^(-6*d*x - 6*c) + 10*e^(-8*d*x - 8*c) + 10*e^(-10*d*x - 10*c) + 5*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c)))) + 1/64*b^3*(960*(d*x + c)/d - (44*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d + 960*log(e^(-2*d*x - 2*c) + 1)/d - (36*e^(-2*d*x - 2*c) + 324*e^(-4*d*x - 4*c) - 1384*e^(-6*d*x - 6*c) - 9126*e^(-8*d*x - 8*c) - 24112*e^(-10*d*x - 10*c) - 31868*e^(-12*d*x - 12*c) - 25912*e^(-14*d*x - 14*c) - 11169*e^(-16*d*x - 16*c) - 2516*e^(-18*d*x - 18*c) - 1)/(d*(e^(-4*d*x - 4*c) + 8*e^(-6*d*x - 6*c) + 28*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 70*e^(-12*d*x - 12*c) + 56*e^(-14*d*x - 14*c) + 28*e^(-16*d*x - 16*c) + 8*e^(-18*d*x - 18*c) + e^(-20*d*x - 20*c)))) + 3/64*a^2*b*(192*(d*x + c)/d - (20*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d + 192*log(e^(-2*d*x - 2*c) + 1)/d - (18*e^(-2*d*x - 2*c) + 39*e^(-4*d*x - 4*c) - 108*e^(-6*d*x - 6*c) - 1)/(d*(e^(-4*d*x - 4*c) + 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c))))
```

mupad [B] time = 1.70, size = 682, normalized size = 2.48

$$x \left(\frac{3a^3}{8} - 9a^2b + \frac{189ab^2}{8} - 15b^3 \right) - \frac{4(71b^3 + 12ab^2)}{d(4e^{2c+2dx} + 6e^{4c+4dx} + 4e^{6c+6dx} + e^{8c+8dx} + 1)} - \frac{1}{d(6e^{2c+2dx} + 15e^{4c+4dx} + 10e^{6c+6dx} + e^{8c+8dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^3)^3,x)
```

```
[Out] x*((189*a*b^2)/8 - 9*a^2*b + (3*a^3)/8 - 15*b^3) - (4*(12*a*b^2 + 71*b^3))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (256*b^3)/(d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) + (log(exp(2*c)*exp(2*d*x) + 1)*(9*a^2*b + 15*b^3))/d + (2*(30*a*b^2 + 3*a^2*b + 20*b^3))/(d*(exp(2*c + 2*d*x) + 1)) + (32*(3*a*b^2 + 50*b^3))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (128*b^3)/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) - (2*(30*a*b^2 + 3*a^2*b + 50*b^3))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (32*b^3)/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1)) + (exp(4*c + 4*d*x)*(a + b)^3)/(64*d) - (exp(-4*c - 4*d*x)*(a - b)^3)/(64*d) + (8*(9*a*b^2 + 23*b^3))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (exp(2*c + 2*d*x)*(a + b)^2*(2*a + 11*b))/(16*d) + (exp(-2*c - 2*d*x)*(a - b)^2*(2*a - 11*b))/(16*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

3.66 $\int \sinh^3(c + dx) \left(a + b \tanh^3(c + dx) \right)^3 dx$

Optimal. Leaf size=351

$$\frac{a^3 \cosh^3(c + dx)}{3d} - \frac{a^3 \cosh(c + dx)}{d} + \frac{5a^2b \sinh^3(c + dx)}{2d} - \frac{15a^2b \sinh(c + dx)}{2d} - \frac{3a^2b \sinh^3(c + dx) \tanh^2(c + dx)}{2d}$$

[Out] $15/2*a^2*b*\arctan(\sinh(d*x+c))/d+1155/128*b^3*\arctan(\sinh(d*x+c))/d-a^3*\cosh(d*x+c)/d-12*a*b^2*\cosh(d*x+c)/d+1/3*a^3*\cosh(d*x+c)^3/d+a*b^2*\cosh(d*x+c)^3/d-18*a*b^2*\operatorname{sech}(d*x+c)/d+4*a*b^2*\operatorname{sech}(d*x+c)^3/d-3/5*a*b^2*\operatorname{sech}(d*x+c)^5/d-15/2*a^2*b*\sinh(d*x+c)/d-1155/128*b^3*\sinh(d*x+c)/d+5/2*a^2*b*\sinh(d*x+c)^3/d+385/128*b^3*\sinh(d*x+c)^3/d-3/2*a^2*b*\sinh(d*x+c)^3*\tanh(d*x+c)^2/d-231/128*b^3*\sinh(d*x+c)^3*\tanh(d*x+c)^2/d-33/64*b^3*\sinh(d*x+c)^3*\tanh(d*x+c)^4/d-11/48*b^3*\sinh(d*x+c)^3*\tanh(d*x+c)^6/d-1/8*b^3*\sinh(d*x+c)^3*\tanh(d*x+c)^8/d$

Rubi [A] time = 0.35, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3666, 2633, 2592, 288, 302, 203, 2590, 270}

$$\frac{5a^2b \sinh^3(c + dx)}{2d} - \frac{15a^2b \sinh(c + dx)}{2d} - \frac{3a^2b \sinh^3(c + dx) \tanh^2(c + dx)}{2d} + \frac{15a^2b \tan^{-1}(\sinh(c + dx))}{2d} + \frac{a^3 \cosh^3(c + dx)}{3d} - \frac{a^3 \cosh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^3,x]

[Out] $(15*a^2*b*\text{ArcTan}[\text{Sinh}[c + d*x]])/(2*d) + (1155*b^3*\text{ArcTan}[\text{Sinh}[c + d*x]])/(128*d) - (a^3*\text{Cosh}[c + d*x])/d - (12*a*b^2*\text{Cosh}[c + d*x])/d + (a^3*\text{Cosh}[c + d*x]^3)/(3*d) + (a*b^2*\text{Cosh}[c + d*x]^3)/d - (18*a*b^2*\text{Sech}[c + d*x])/d + (4*a*b^2*\text{Sech}[c + d*x]^3)/d - (3*a*b^2*\text{Sech}[c + d*x]^5)/(5*d) - (15*a^2*b*\text{Sinh}[c + d*x])/(2*d) - (1155*b^3*\text{Sinh}[c + d*x])/(128*d) + (5*a^2*b*\text{Sinh}[c + d*x]^3)/(2*d) + (385*b^3*\text{Sinh}[c + d*x]^3)/(128*d) - (3*a^2*b*\text{Sinh}[c + d*x]^3*\text{Tanh}[c + d*x]^2)/(2*d) - (231*b^3*\text{Sinh}[c + d*x]^3*\text{Tanh}[c + d*x]^2)/(128*d) - (33*b^3*\text{Sinh}[c + d*x]^3*\text{Tanh}[c + d*x]^4)/(64*d) - (11*b^3*\text{Sinh}[c + d*x]^3*\text{Tanh}[c + d*x]^6)/(48*d) - (b^3*\text{Sinh}[c + d*x]^3*\text{Tanh}[c + d*x]^8)/(8*d)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a + b*x^n)^(p+1))/(b*n*(p+1)), x] - Dist[(c^n*(m-n+1))/(b*n*(p+1)), Int[(c*x)^(m-n)*(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !I LtQ[(m+n*(p+1)+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 302

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt

$Q[m, 2*n - 1]$

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3666

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)]^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
+ b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \sinh^3(c + dx) (a + b \tanh^3(c + dx))^3 dx &= i \int (-ia^3 \sinh^3(c + dx) - 3ia^2b \sinh^3(c + dx) \tanh^3(c + dx) - 3iab^2 \sinh^3(c + dx) \tanh^3(c + dx) - 3ib^3 \sinh^3(c + dx) \tanh^3(c + dx)) dx \\
&= a^3 \int \sinh^3(c + dx) dx + (3a^2b) \int \sinh^3(c + dx) \tanh^3(c + dx) dx + (3ab^2) \int \sinh^3(c + dx) \tanh^3(c + dx) dx + (3b^3) \int \sinh^3(c + dx) \tanh^3(c + dx) dx \\
&= -\frac{a^3 \operatorname{Subst}\left(\int (1 - x^2) dx, x, \cosh(c + dx)\right)}{d} + \frac{(3a^2b) \operatorname{Subst}\left(\int \frac{x^2}{1 - x^2} dx, x, \cosh(c + dx)\right)}{d} + \frac{(3ab^2) \operatorname{Subst}\left(\int \frac{x^2}{1 - x^2} dx, x, \cosh(c + dx)\right)}{d} + \frac{(3b^3) \operatorname{Subst}\left(\int \frac{x^2}{1 - x^2} dx, x, \cosh(c + dx)\right)}{d} \\
&= -\frac{a^3 \cosh(c + dx)}{d} + \frac{a^3 \cosh^3(c + dx)}{3d} - \frac{3a^2b \sinh^3(c + dx) \tanh^3(c + dx)}{2d} - \frac{3ab^2 \sinh^3(c + dx) \tanh^3(c + dx)}{2d} - \frac{3b^3 \sinh^3(c + dx) \tanh^3(c + dx)}{2d} \\
&= -\frac{a^3 \cosh(c + dx)}{d} - \frac{12ab^2 \cosh(c + dx)}{d} + \frac{a^3 \cosh^3(c + dx)}{3d} + \frac{a^3 \cosh^3(c + dx)}{3d} + \frac{a^3 \cosh^3(c + dx)}{3d} + \frac{a^3 \cosh^3(c + dx)}{3d} \\
&= -\frac{a^3 \cosh(c + dx)}{d} - \frac{12ab^2 \cosh(c + dx)}{d} + \frac{a^3 \cosh^3(c + dx)}{3d} + \frac{a^3 \cosh^3(c + dx)}{3d} + \frac{a^3 \cosh^3(c + dx)}{3d} + \frac{a^3 \cosh^3(c + dx)}{3d} \\
&= \frac{15a^2b \tan^{-1}(\sinh(c + dx))}{2d} - \frac{a^3 \cosh(c + dx)}{d} - \frac{12ab^2 \cosh(c + dx)}{d} - \frac{12ab^2 \cosh(c + dx)}{d} - \frac{12ab^2 \cosh(c + dx)}{d} - \frac{12ab^2 \cosh(c + dx)}{d} \\
&= \frac{15a^2b \tan^{-1}(\sinh(c + dx))}{2d} - \frac{a^3 \cosh(c + dx)}{d} - \frac{12ab^2 \cosh(c + dx)}{d} - \frac{12ab^2 \cosh(c + dx)}{d} - \frac{12ab^2 \cosh(c + dx)}{d} - \frac{12ab^2 \cosh(c + dx)}{d} \\
&= \frac{15a^2b \tan^{-1}(\sinh(c + dx))}{2d} - \frac{a^3 \cosh(c + dx)}{d} - \frac{12ab^2 \cosh(c + dx)}{d} - \frac{12ab^2 \cosh(c + dx)}{d} - \frac{12ab^2 \cosh(c + dx)}{d} - \frac{12ab^2 \cosh(c + dx)}{d} \\
&= \frac{15a^2b \tan^{-1}(\sinh(c + dx))}{2d} + \frac{1155b^3 \tan^{-1}(\sinh(c + dx))}{128d} - \frac{a^3 \cosh(c + dx)}{d}
\end{aligned}$$

Mathematica [A] time = 6.61, size = 291, normalized size = 0.83

$$\frac{3 \operatorname{sech}^2(c + dx) (64a^2b \sinh(c + dx) + 255b^3 \sinh(c + dx))}{128d} - \frac{3b(9a^2 + 7b^2) \sinh(c + dx)}{4d} + \frac{b(3a^2 + b^2) \sinh(c + dx)}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^3,x]

[Out] (15*b*(64*a^2 + 77*b^2)*ArcTan[Tanh[(c + d*x)/2]])/(64*d) - (3*a*(a^2 + 15*b^2)*Cosh[c + d*x])/(4*d) + (a*(a^2 + 3*b^2)*Cosh[3*(c + d*x)])/(12*d) - (18*a*b^2*Sech[c + d*x])/d + (4*a*b^2*Sech[c + d*x]^3)/d - (3*a*b^2*Sech[c + d*x]^5)/(5*d) - (3*b*(9*a^2 + 7*b^2)*Sinh[c + d*x])/(4*d) - (3*Sech[c + d*x]^2*(64*a^2*b*Sinh[c + d*x] + 255*b^3*Sinh[c + d*x]))/(128*d) + (b*(3*a^2 + b^2)*Sinh[3*(c + d*x)])/(12*d) + (515*b^3*Sech[c + d*x]^3*Tanh[c + d*x])/(192*d) - (41*b^3*Sech[c + d*x]^5*Tanh[c + d*x])/(48*d) + (b^3*Sech[c + d*x]^7*Tanh[c + d*x])/(8*d)

fricas [B] time = 0.60, size = 8462, normalized size = 24.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")

[Out] 1/960*(40*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^22 + 880*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^21 + 40*(a^3 + 3*a^2*b +

$$\begin{aligned}
& 3*a*b^2 + b^3)*\sinh(d*x + c)^{22} - 40*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)* \\
& \cosh(d*x + c)^{20} - 40*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3 - 231*(a^3 + 3*a \\
& ^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{20} + 800*(77*(a^3 + 3* \\
& a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^3 - (a^3 + 57*a^2*b + 111*a*b^2 + 55*b \\
& ^3)*\cosh(d*x + c))*\sinh(d*x + c)^{19} - 5*(424*a^3 + 4440*a^2*b + 15960*a*b^2 \\
& + 5599*b^3)*\cosh(d*x + c)^{18} + 5*(58520*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\co \\
& sh(d*x + c)^4 - 424*a^3 - 4440*a^2*b - 15960*a*b^2 - 5599*b^3 - 1520*(a^3 + \\
& 57*a^2*b + 111*a*b^2 + 55*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{18} + 30*(351 \\
& 12*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^5 - 1520*(a^3 + 57*a^2*b + \\
& 111*a*b^2 + 55*b^3)*\cosh(d*x + c)^3 - 3*(424*a^3 + 4440*a^2*b + 15960*a*b^ \\
& 2 + 5599*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^{17} - 15*(712*a^3 + 4840*a^2*b + \\
& 26584*a*b^2 + 5665*b^3)*\cosh(d*x + c)^{16} + 15*(198968*(a^3 + 3*a^2*b + 3*a* \\
& b^2 + b^3)*\cosh(d*x + c)^6 - 12920*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*\co \\
& sh(d*x + c)^4 - 712*a^3 - 4840*a^2*b - 26584*a*b^2 - 5665*b^3 - 51*(424*a^3 \\
& + 4440*a^2*b + 15960*a*b^2 + 5599*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{16} + \\
& 240*(28424*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^7 - 2584*(a^3 + 5 \\
& 7*a^2*b + 111*a*b^2 + 55*b^3)*\cosh(d*x + c)^5 - 17*(424*a^3 + 4440*a^2*b + \\
& 15960*a*b^2 + 5599*b^3)*\cosh(d*x + c)^3 - (712*a^3 + 4840*a^2*b + 26584*a*b \\
& ^2 + 5665*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^{15} - 3*(9040*a^3 + 36400*a^2*b \\
& + 344944*a*b^2 + 45265*b^3)*\cosh(d*x + c)^{14} + 3*(4263600*(a^3 + 3*a^2*b + \\
& 3*a*b^2 + b^3)*\cosh(d*x + c)^8 - 516800*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^ \\
& 3)*\cosh(d*x + c)^6 - 5100*(424*a^3 + 4440*a^2*b + 15960*a*b^2 + 5599*b^3)*\c \\
& osh(d*x + c)^4 - 9040*a^3 - 36400*a^2*b - 344944*a*b^2 - 45265*b^3 - 600*(7 \\
& 12*a^3 + 4840*a^2*b + 26584*a*b^2 + 5665*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c \\
&)^{14} + 2*(9948400*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^9 - 1550400 \\
& *(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*\cosh(d*x + c)^7 - 21420*(424*a^3 + 4 \\
& 440*a^2*b + 15960*a*b^2 + 5599*b^3)*\cosh(d*x + c)^5 - 4200*(712*a^3 + 4840* \\
& a^2*b + 26584*a*b^2 + 5665*b^3)*\cosh(d*x + c)^3 - 21*(9040*a^3 + 36400*a^2* \\
& b + 344944*a*b^2 + 45265*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^{13} - 3*(14000*a^ \\
& 3 + 18800*a^2*b + 542672*a*b^2 + 20405*b^3)*\cosh(d*x + c)^{12} + (25865840*(a \\
& ^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^{10} - 5038800*(a^3 + 57*a^2*b + \\
& 111*a*b^2 + 55*b^3)*\cosh(d*x + c)^8 - 92820*(424*a^3 + 4440*a^2*b + 15960*a \\
& *b^2 + 5599*b^3)*\cosh(d*x + c)^6 - 27300*(712*a^3 + 4840*a^2*b + 26584*a*b^ \\
& 2 + 5665*b^3)*\cosh(d*x + c)^4 - 42000*a^3 - 56400*a^2*b - 1628016*a*b^2 - 6 \\
& 1215*b^3 - 273*(9040*a^3 + 36400*a^2*b + 344944*a*b^2 + 45265*b^3)*\cosh(d*x \\
& + c)^2)*\sinh(d*x + c)^{12} + 4*(7054320*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh \\
& (d*x + c)^{11} - 1679600*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*\cosh(d*x + c)^ \\
& 9 - 39780*(424*a^3 + 4440*a^2*b + 15960*a*b^2 + 5599*b^3)*\cosh(d*x + c)^7 - \\
& 16380*(712*a^3 + 4840*a^2*b + 26584*a*b^2 + 5665*b^3)*\cosh(d*x + c)^5 - 27 \\
& 3*(9040*a^3 + 36400*a^2*b + 344944*a*b^2 + 45265*b^3)*\cosh(d*x + c)^3 - 9*(\\
& 14000*a^3 + 18800*a^2*b + 542672*a*b^2 + 20405*b^3)*\cosh(d*x + c))*\sinh(d*x \\
& + c)^{11} - 3*(14000*a^3 - 18800*a^2*b + 542672*a*b^2 - 20405*b^3)*\cosh(d*x \\
& + c)^{10} + (25865840*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^{12} - 7390 \\
& 240*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*\cosh(d*x + c)^{10} - 218790*(424*a^ \\
& 3 + 4440*a^2*b + 15960*a*b^2 + 5599*b^3)*\cosh(d*x + c)^8 - 120120*(712*a^3 \\
& + 4840*a^2*b + 26584*a*b^2 + 5665*b^3)*\cosh(d*x + c)^6 - 3003*(9040*a^3 + 3 \\
& 6400*a^2*b + 344944*a*b^2 + 45265*b^3)*\cosh(d*x + c)^4 - 42000*a^3 + 56400* \\
& a^2*b - 1628016*a*b^2 + 61215*b^3 - 198*(14000*a^3 + 18800*a^2*b + 542672*a \\
& *b^2 + 20405*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{10} + 2*(9948400*(a^3 + 3*a \\
& ^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^{13} - 3359200*(a^3 + 57*a^2*b + 111*a*b^ \\
& 2 + 55*b^3)*\cosh(d*x + c)^{11} - 121550*(424*a^3 + 4440*a^2*b + 15960*a*b^2 + \\
& 5599*b^3)*\cosh(d*x + c)^9 - 85800*(712*a^3 + 4840*a^2*b + 26584*a*b^2 + 56 \\
& 65*b^3)*\cosh(d*x + c)^7 - 3003*(9040*a^3 + 36400*a^2*b + 344944*a*b^2 + 452 \\
& 65*b^3)*\cosh(d*x + c)^5 - 330*(14000*a^3 + 18800*a^2*b + 542672*a*b^2 + 204 \\
& 05*b^3)*\cosh(d*x + c)^3 - 15*(14000*a^3 - 18800*a^2*b + 542672*a*b^2 - 2040 \\
& 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^9 - 3*(9040*a^3 - 36400*a^2*b + 344944* \\
& a*b^2 - 45265*b^3)*\cosh(d*x + c)^8 + 3*(4263600*(a^3 + 3*a^2*b + 3*a*b^2 + \\
& b^3)*\cosh(d*x + c)^{14} - 1679600*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*\cosh(\\
& d*x + c)^{12} - 72930*(424*a^3 + 4440*a^2*b + 15960*a*b^2 + 5599*b^3)*\cosh(d*
\end{aligned}$$

$$\begin{aligned}
& x + c)^{10} - 64350*(712*a^3 + 4840*a^2*b + 26584*a*b^2 + 5665*b^3)*\cosh(d*x \\
& + c)^8 - 3003*(9040*a^3 + 36400*a^2*b + 344944*a*b^2 + 45265*b^3)*\cosh(d*x \\
& + c)^6 - 495*(14000*a^3 + 18800*a^2*b + 542672*a*b^2 + 20405*b^3)*\cosh(d*x \\
& + c)^4 - 9040*a^3 + 36400*a^2*b - 344944*a*b^2 + 45265*b^3 - 45*(14000*a^3 \\
& - 18800*a^2*b + 542672*a*b^2 - 20405*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^8 \\
& + 24*(284240*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^{15} - 129200*(a^3 \\
& + 57*a^2*b + 111*a*b^2 + 55*b^3)*\cosh(d*x + c)^{13} - 6630*(424*a^3 + 4440*a \\
& ^2*b + 15960*a*b^2 + 5599*b^3)*\cosh(d*x + c)^{11} - 7150*(712*a^3 + 4840*a^2* \\
& b + 26584*a*b^2 + 5665*b^3)*\cosh(d*x + c)^9 - 429*(9040*a^3 + 36400*a^2*b + \\
& 344944*a*b^2 + 45265*b^3)*\cosh(d*x + c)^7 - 99*(14000*a^3 + 18800*a^2*b + \\
& 542672*a*b^2 + 20405*b^3)*\cosh(d*x + c)^5 - 15*(14000*a^3 - 18800*a^2*b + 5 \\
& 42672*a*b^2 - 20405*b^3)*\cosh(d*x + c)^3 - (9040*a^3 - 36400*a^2*b + 344944 \\
& *a*b^2 - 45265*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^7 - 15*(712*a^3 - 4840*a^2 \\
& *b + 26584*a*b^2 - 5665*b^3)*\cosh(d*x + c)^6 + 3*(994840*(a^3 + 3*a^2*b + 3 \\
& *a*b^2 + b^3)*\cosh(d*x + c)^{16} - 516800*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^ \\
& ^3)*\cosh(d*x + c)^{14} - 30940*(424*a^3 + 4440*a^2*b + 15960*a*b^2 + 5599*b^3) \\
& *\cosh(d*x + c)^{12} - 40040*(712*a^3 + 4840*a^2*b + 26584*a*b^2 + 5665*b^3)*\c \\
& osh(d*x + c)^{10} - 3003*(9040*a^3 + 36400*a^2*b + 344944*a*b^2 + 45265*b^3)* \\
& cosh(d*x + c)^8 - 924*(14000*a^3 + 18800*a^2*b + 542672*a*b^2 + 20405*b^3)* \\
& cosh(d*x + c)^6 - 210*(14000*a^3 - 18800*a^2*b + 542672*a*b^2 - 20405*b^3)* \\
& cosh(d*x + c)^4 - 3560*a^3 + 24200*a^2*b - 132920*a*b^2 + 28325*b^3 - 28*(9 \\
& 040*a^3 - 36400*a^2*b + 344944*a*b^2 - 45265*b^3)*\cosh(d*x + c)^2*\sinh(d*x \\
& + c)^6 + 6*(175560*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^{17} - 1033 \\
& 60*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*\cosh(d*x + c)^{15} - 7140*(424*a^3 + \\
& 4440*a^2*b + 15960*a*b^2 + 5599*b^3)*\cosh(d*x + c)^{13} - 10920*(712*a^3 + 4 \\
& 840*a^2*b + 26584*a*b^2 + 5665*b^3)*\cosh(d*x + c)^{11} - 1001*(9040*a^3 + 364 \\
& 00*a^2*b + 344944*a*b^2 + 45265*b^3)*\cosh(d*x + c)^9 - 396*(14000*a^3 + 188 \\
& 00*a^2*b + 542672*a*b^2 + 20405*b^3)*\cosh(d*x + c)^7 - 126*(14000*a^3 - 188 \\
& 00*a^2*b + 542672*a*b^2 - 20405*b^3)*\cosh(d*x + c)^5 - 28*(9040*a^3 - 36400 \\
& *a^2*b + 344944*a*b^2 - 45265*b^3)*\cosh(d*x + c)^3 - 15*(712*a^3 - 4840*a^2 \\
& *b + 26584*a*b^2 - 5665*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 5*(424*a^3 - \\
& 4440*a^2*b + 15960*a*b^2 - 5599*b^3)*\cosh(d*x + c)^4 + (292600*(a^3 + 3*a^2 \\
& *b + 3*a*b^2 + b^3)*\cosh(d*x + c)^{18} - 193800*(a^3 + 57*a^2*b + 111*a*b^2 + \\
& 55*b^3)*\cosh(d*x + c)^{16} - 15300*(424*a^3 + 4440*a^2*b + 15960*a*b^2 + 559 \\
& 9*b^3)*\cosh(d*x + c)^{14} - 27300*(712*a^3 + 4840*a^2*b + 26584*a*b^2 + 5665* \\
& b^3)*\cosh(d*x + c)^{12} - 3003*(9040*a^3 + 36400*a^2*b + 344944*a*b^2 + 45265 \\
& *b^3)*\cosh(d*x + c)^{10} - 1485*(14000*a^3 + 18800*a^2*b + 542672*a*b^2 + 204 \\
& 05*b^3)*\cosh(d*x + c)^8 - 630*(14000*a^3 - 18800*a^2*b + 542672*a*b^2 - 204 \\
& 05*b^3)*\cosh(d*x + c)^6 - 210*(9040*a^3 - 36400*a^2*b + 344944*a*b^2 - 4526 \\
& 5*b^3)*\cosh(d*x + c)^4 - 2120*a^3 + 22200*a^2*b - 79800*a*b^2 + 27995*b^3 - \\
& 225*(712*a^3 - 4840*a^2*b + 26584*a*b^2 - 5665*b^3)*\cosh(d*x + c)^2*\sinh(\\
& d*x + c)^4 + 4*(15400*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^{19} - 11 \\
& 400*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*\cosh(d*x + c)^{17} - 1020*(424*a^3 \\
& + 4440*a^2*b + 15960*a*b^2 + 5599*b^3)*\cosh(d*x + c)^{15} - 2100*(712*a^3 + 4 \\
& 840*a^2*b + 26584*a*b^2 + 5665*b^3)*\cosh(d*x + c)^{13} - 273*(9040*a^3 + 3640 \\
& 0*a^2*b + 344944*a*b^2 + 45265*b^3)*\cosh(d*x + c)^{11} - 165*(14000*a^3 + 188 \\
& 00*a^2*b + 542672*a*b^2 + 20405*b^3)*\cosh(d*x + c)^9 - 90*(14000*a^3 - 1880 \\
& 0*a^2*b + 542672*a*b^2 - 20405*b^3)*\cosh(d*x + c)^7 - 42*(9040*a^3 - 36400* \\
& a^2*b + 344944*a*b^2 - 45265*b^3)*\cosh(d*x + c)^5 - 75*(712*a^3 - 4840*a^2* \\
& b + 26584*a*b^2 - 5665*b^3)*\cosh(d*x + c)^3 - 5*(424*a^3 - 4440*a^2*b + 159 \\
& 60*a*b^2 - 5599*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 40*a^3 - 120*a^2*b + \\
& 120*a*b^2 - 40*b^3 - 40*(a^3 - 57*a^2*b + 111*a*b^2 - 55*b^3)*\cosh(d*x + c) \\
& ^2 + (9240*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^{20} - 7600*(a^3 + 5 \\
& 7*a^2*b + 111*a*b^2 + 55*b^3)*\cosh(d*x + c)^{18} - 765*(424*a^3 + 4440*a^2*b \\
& + 15960*a*b^2 + 5599*b^3)*\cosh(d*x + c)^{16} - 1800*(712*a^3 + 4840*a^2*b + 2 \\
& 6584*a*b^2 + 5665*b^3)*\cosh(d*x + c)^{14} - 273*(9040*a^3 + 36400*a^2*b + 344 \\
& 944*a*b^2 + 45265*b^3)*\cosh(d*x + c)^{12} - 198*(14000*a^3 + 18800*a^2*b + 54 \\
& 2672*a*b^2 + 20405*b^3)*\cosh(d*x + c)^{10} - 135*(14000*a^3 - 18800*a^2*b + 5 \\
& 42672*a*b^2 - 20405*b^3)*\cosh(d*x + c)^8 - 84*(9040*a^3 - 36400*a^2*b + 344
\end{aligned}$$

$$\begin{aligned}
& 944*a*b^2 - 45265*b^3)*\cosh(d*x + c)^6 - 225*(712*a^3 - 4840*a^2*b + 26584* \\
& a*b^2 - 5665*b^3)*\cosh(d*x + c)^4 - 40*a^3 + 2280*a^2*b - 4440*a*b^2 + 2200 \\
& *b^3 - 30*(424*a^3 - 4440*a^2*b + 15960*a*b^2 - 5599*b^3)*\cosh(d*x + c)^2)* \\
& \sinh(d*x + c)^2 + 225*((64*a^2*b + 77*b^3)*\cosh(d*x + c)^19 + 19*(64*a^2*b \\
& + 77*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^18 + (64*a^2*b + 77*b^3)*\sinh(d*x + c \\
&)^19 + 8*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^17 + (512*a^2*b + 616*b^3 + 171* \\
& (64*a^2*b + 77*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^17 + 17*(57*(64*a^2*b + \\
& 77*b^3)*\cosh(d*x + c)^3 + 8*(64*a^2*b + 77*b^3)*\cosh(d*x + c))*\sinh(d*x + c \\
&)^16 + 28*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^15 + 4*(969*(64*a^2*b + 77*b^3) \\
& *\cosh(d*x + c)^4 + 448*a^2*b + 539*b^3 + 272*(64*a^2*b + 77*b^3)*\cosh(d*x + \\
& c)^2)*\sinh(d*x + c)^15 + 4*(2907*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^5 + 136 \\
& 0*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^3 + 105*(64*a^2*b + 77*b^3)*\cosh(d*x + \\
& c))*\sinh(d*x + c)^14 + 56*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^13 + 28*(969*(6 \\
& 4*a^2*b + 77*b^3)*\cosh(d*x + c)^6 + 680*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^4 \\
& + 128*a^2*b + 154*b^3 + 105*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^2)*\sinh(d*x \\
& + c)^13 + 52*(969*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^7 + 952*(64*a^2*b + 77* \\
& b^3)*\cosh(d*x + c)^5 + 245*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^3 + 14*(64*a^2 \\
& *b + 77*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^12 + 70*(64*a^2*b + 77*b^3)*\cosh(\\
& d*x + c)^11 + 2*(37791*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^8 + 49504*(64*a^2* \\
& b + 77*b^3)*\cosh(d*x + c)^6 + 19110*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^4 + 2 \\
& 240*a^2*b + 2695*b^3 + 2184*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + \\
& c)^11 + 22*(4199*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^9 + 7072*(64*a^2*b + 77 \\
& *b^3)*\cosh(d*x + c)^7 + 3822*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^5 + 728*(64* \\
& a^2*b + 77*b^3)*\cosh(d*x + c)^3 + 35*(64*a^2*b + 77*b^3)*\cosh(d*x + c))*\sin \\
& h(d*x + c)^10 + 56*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^9 + 2*(46189*(64*a^2*b \\
& + 77*b^3)*\cosh(d*x + c)^10 + 97240*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^8 + 7 \\
& 0070*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^6 + 20020*(64*a^2*b + 77*b^3)*\cosh(d \\
& *x + c)^4 + 1792*a^2*b + 2156*b^3 + 1925*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^ \\
& 2)*\sinh(d*x + c)^9 + 2*(37791*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^11 + 97240* \\
& (64*a^2*b + 77*b^3)*\cosh(d*x + c)^9 + 90090*(64*a^2*b + 77*b^3)*\cosh(d*x + \\
& c)^7 + 36036*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^5 + 5775*(64*a^2*b + 77*b^3) \\
& *\cosh(d*x + c)^3 + 252*(64*a^2*b + 77*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^8 + \\
& 28*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^7 + 4*(12597*(64*a^2*b + 77*b^3)*\cosh \\
& (d*x + c)^12 + 38896*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^10 + 45045*(64*a^2*b \\
& + 77*b^3)*\cosh(d*x + c)^8 + 24024*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^6 + 57 \\
& 75*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^4 + 448*a^2*b + 539*b^3 + 504*(64*a^2* \\
& b + 77*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^7 + 28*(969*(64*a^2*b + 77*b^3)* \\
& \cosh(d*x + c)^13 + 3536*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^11 + 5005*(64*a^2 \\
& *b + 77*b^3)*\cosh(d*x + c)^9 + 3432*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^7 + 1 \\
& 155*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^5 + 168*(64*a^2*b + 77*b^3)*\cosh(d*x \\
& + c)^3 + 7*(64*a^2*b + 77*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^6 + 8*(64*a^2*b \\
& + 77*b^3)*\cosh(d*x + c)^5 + 4*(2907*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^14 + \\
& 12376*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^12 + 21021*(64*a^2*b + 77*b^3)*\cos \\
& h(d*x + c)^10 + 18018*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^8 + 8085*(64*a^2*b \\
& + 77*b^3)*\cosh(d*x + c)^6 + 1764*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^4 + 128* \\
& a^2*b + 154*b^3 + 147*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^5 \\
& + 4*(969*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^15 + 4760*(64*a^2*b + 77*b^3)*\co \\
& sh(d*x + c)^13 + 9555*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^11 + 10010*(64*a^2* \\
& b + 77*b^3)*\cosh(d*x + c)^9 + 5775*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^7 + 17 \\
& 64*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^5 + 245*(64*a^2*b + 77*b^3)*\cosh(d*x + \\
& c)^3 + 10*(64*a^2*b + 77*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^4 + (64*a^2*b + \\
& 77*b^3)*\cosh(d*x + c)^3 + (969*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^16 + 5440 \\
& *(64*a^2*b + 77*b^3)*\cosh(d*x + c)^14 + 12740*(64*a^2*b + 77*b^3)*\cosh(d*x \\
& + c)^12 + 16016*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^10 + 11550*(64*a^2*b + 77 \\
& *b^3)*\cosh(d*x + c)^8 + 4704*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^6 + 980*(64* \\
& a^2*b + 77*b^3)*\cosh(d*x + c)^4 + 64*a^2*b + 77*b^3 + 80*(64*a^2*b + 77*b^3 \\
&)*\cosh(d*x + c)^2)*\sinh(d*x + c)^3 + (171*(64*a^2*b + 77*b^3)*\cosh(d*x + c) \\
& ^17 + 1088*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^15 + 2940*(64*a^2*b + 77*b^3)* \\
& \cosh(d*x + c)^13 + 4368*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^11 + 3850*(64*a^2
\end{aligned}$$

$$\begin{aligned}
& *b + 77*b^3)*\cosh(d*x + c)^9 + 2016*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^7 + 5 \\
& 88*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^5 + 80*(64*a^2*b + 77*b^3)*\cosh(d*x + \\
& c)^3 + 3*(64*a^2*b + 77*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 + (19*(64*a^2*b \\
& + 77*b^3)*\cosh(d*x + c)^18 + 136*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^16 + 42 \\
& 0*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^14 + 728*(64*a^2*b + 77*b^3)*\cosh(d*x + \\
& c)^12 + 770*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^10 + 504*(64*a^2*b + 77*b^3) \\
& *\cosh(d*x + c)^8 + 196*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^6 + 40*(64*a^2*b + \\
& 77*b^3)*\cosh(d*x + c)^4 + 3*(64*a^2*b + 77*b^3)*\cosh(d*x + c)^2)*\sinh(d*x \\
& + c))*\arctan(\cosh(d*x + c) + \sinh(d*x + c)) + 2*(440*(a^3 + 3*a^2*b + 3*a*b \\
& ^2 + b^3)*\cosh(d*x + c)^21 - 400*(a^3 + 57*a^2*b + 111*a*b^2 + 55*b^3)*\cosh \\
& (d*x + c)^19 - 45*(424*a^3 + 4440*a^2*b + 15960*a*b^2 + 5599*b^3)*\cosh(d*x \\
& + c)^17 - 120*(712*a^3 + 4840*a^2*b + 26584*a*b^2 + 5665*b^3)*\cosh(d*x + c) \\
& ^15 - 21*(9040*a^3 + 36400*a^2*b + 344944*a*b^2 + 45265*b^3)*\cosh(d*x + c)^ \\
& 13 - 18*(14000*a^3 + 18800*a^2*b + 542672*a*b^2 + 20405*b^3)*\cosh(d*x + c)^ \\
& 11 - 15*(14000*a^3 - 18800*a^2*b + 542672*a*b^2 - 20405*b^3)*\cosh(d*x + c)^ \\
& 9 - 12*(9040*a^3 - 36400*a^2*b + 344944*a*b^2 - 45265*b^3)*\cosh(d*x + c)^7 \\
& - 45*(712*a^3 - 4840*a^2*b + 26584*a*b^2 - 5665*b^3)*\cosh(d*x + c)^5 - 10*(\\
& 424*a^3 - 4440*a^2*b + 15960*a*b^2 - 5599*b^3)*\cosh(d*x + c)^3 - 40*(a^3 - \\
& 57*a^2*b + 111*a*b^2 - 55*b^3)*\cosh(d*x + c))*\sinh(d*x + c))/(d*\cosh(d*x + \\
& c)^19 + 19*d*\cosh(d*x + c)*\sinh(d*x + c)^18 + d*\sinh(d*x + c)^19 + 8*d*\cosh \\
& (d*x + c)^17 + (171*d*\cosh(d*x + c)^2 + 8*d)*\sinh(d*x + c)^17 + 17*(57*d*\co \\
& sh(d*x + c)^3 + 8*d*\cosh(d*x + c))*\sinh(d*x + c)^16 + 28*d*\cosh(d*x + c)^15 \\
& + 4*(969*d*\cosh(d*x + c)^4 + 272*d*\cosh(d*x + c)^2 + 7*d)*\sinh(d*x + c)^15 \\
& + 4*(2907*d*\cosh(d*x + c)^5 + 1360*d*\cosh(d*x + c)^3 + 105*d*\cosh(d*x + c) \\
&)*\sinh(d*x + c)^14 + 56*d*\cosh(d*x + c)^13 + 28*(969*d*\cosh(d*x + c)^6 + 68 \\
& 0*d*\cosh(d*x + c)^4 + 105*d*\cosh(d*x + c)^2 + 2*d)*\sinh(d*x + c)^13 + 52*(9 \\
& 69*d*\cosh(d*x + c)^7 + 952*d*\cosh(d*x + c)^5 + 245*d*\cosh(d*x + c)^3 + 14*d \\
& *\cosh(d*x + c))*\sinh(d*x + c)^12 + 70*d*\cosh(d*x + c)^11 + 2*(37791*d*\cosh(\\
& d*x + c)^8 + 49504*d*\cosh(d*x + c)^6 + 19110*d*\cosh(d*x + c)^4 + 2184*d*\cos \\
& h(d*x + c)^2 + 35*d)*\sinh(d*x + c)^11 + 22*(4199*d*\cosh(d*x + c)^9 + 7072*d \\
& *\cosh(d*x + c)^7 + 3822*d*\cosh(d*x + c)^5 + 728*d*\cosh(d*x + c)^3 + 35*d*\co \\
& sh(d*x + c))*\sinh(d*x + c)^10 + 56*d*\cosh(d*x + c)^9 + 2*(46189*d*\cosh(d*x \\
& + c)^10 + 97240*d*\cosh(d*x + c)^8 + 70070*d*\cosh(d*x + c)^6 + 20020*d*\cosh(\\
& d*x + c)^4 + 1925*d*\cosh(d*x + c)^2 + 28*d)*\sinh(d*x + c)^9 + 2*(37791*d*\co \\
& sh(d*x + c)^11 + 97240*d*\cosh(d*x + c)^9 + 90090*d*\cosh(d*x + c)^7 + 36036* \\
& d*\cosh(d*x + c)^5 + 5775*d*\cosh(d*x + c)^3 + 252*d*\cosh(d*x + c))*\sinh(d*x \\
& + c)^8 + 28*d*\cosh(d*x + c)^7 + 4*(12597*d*\cosh(d*x + c)^12 + 38896*d*\cosh(\\
& d*x + c)^10 + 45045*d*\cosh(d*x + c)^8 + 24024*d*\cosh(d*x + c)^6 + 5775*d*\co \\
& sh(d*x + c)^4 + 504*d*\cosh(d*x + c)^2 + 7*d)*\sinh(d*x + c)^7 + 28*(969*d*\co \\
& sh(d*x + c)^13 + 3536*d*\cosh(d*x + c)^11 + 5005*d*\cosh(d*x + c)^9 + 3432*d* \\
& cosh(d*x + c)^7 + 1155*d*\cosh(d*x + c)^5 + 168*d*\cosh(d*x + c)^3 + 7*d*\cosh \\
& (d*x + c))*\sinh(d*x + c)^6 + 8*d*\cosh(d*x + c)^5 + 4*(2907*d*\cosh(d*x + c)^ \\
& 14 + 12376*d*\cosh(d*x + c)^12 + 21021*d*\cosh(d*x + c)^10 + 18018*d*\cosh(d*x \\
& + c)^8 + 8085*d*\cosh(d*x + c)^6 + 1764*d*\cosh(d*x + c)^4 + 147*d*\cosh(d*x \\
& + c)^2 + 2*d)*\sinh(d*x + c)^5 + 4*(969*d*\cosh(d*x + c)^15 + 4760*d*\cosh(d*x \\
& + c)^13 + 9555*d*\cosh(d*x + c)^11 + 10010*d*\cosh(d*x + c)^9 + 5775*d*\cosh(\\
& d*x + c)^7 + 1764*d*\cosh(d*x + c)^5 + 245*d*\cosh(d*x + c)^3 + 10*d*\cosh(d*x \\
& + c))*\sinh(d*x + c)^4 + d*\cosh(d*x + c)^3 + (969*d*\cosh(d*x + c)^16 + 5440 \\
& *d*\cosh(d*x + c)^14 + 12740*d*\cosh(d*x + c)^12 + 16016*d*\cosh(d*x + c)^10 + \\
& 11550*d*\cosh(d*x + c)^8 + 4704*d*\cosh(d*x + c)^6 + 980*d*\cosh(d*x + c)^4 + \\
& 80*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^3 + (171*d*\cosh(d*x + c)^17 + 1088 \\
& *d*\cosh(d*x + c)^15 + 2940*d*\cosh(d*x + c)^13 + 4368*d*\cosh(d*x + c)^11 + 3 \\
& 850*d*\cosh(d*x + c)^9 + 2016*d*\cosh(d*x + c)^7 + 588*d*\cosh(d*x + c)^5 + 80 \\
& *d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^2 + (19*d*\cosh(d*x + \\
& c)^18 + 136*d*\cosh(d*x + c)^16 + 420*d*\cosh(d*x + c)^14 + 728*d*\cosh(d*x + \\
& c)^12 + 770*d*\cosh(d*x + c)^10 + 504*d*\cosh(d*x + c)^8 + 196*d*\cosh(d*x + c \\
&)^6 + 40*d*\cosh(d*x + c)^4 + 3*d*\cosh(d*x + c)^2)*\sinh(d*x + c))
\end{aligned}$$

giac [A] time = 1.25, size = 601, normalized size = 1.71

$$225 \left(64 a^2 b e^c + 77 b^3 e^c \right) \arctan \left(e^{(dx+c)} \right) e^{(-c)} - 40 \left(9 a^3 e^{(2dx+2c)} - 81 a^2 b e^{(2dx+2c)} + 135 a b^2 e^{(2dx+2c)} - 63 b^3 e^{(2dx+2c)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")

[Out] $\frac{1}{960} \left(225 \left(64 a^2 b e^c + 77 b^3 e^c \right) \arctan \left(e^{(dx+c)} \right) e^{(-c)} - 40 \left(9 a^3 e^{(2dx+2c)} - 81 a^2 b e^{(2dx+2c)} + 135 a b^2 e^{(2dx+2c)} - 63 b^3 e^{(2dx+2c)} \right) - a^3 + 3 a^2 b - 3 a b^2 + b^3 \right) e^{(-3dx-3c)} + 40 \left(a^3 e^{(3dx+66c)} + 3 a^2 b e^{(3dx+66c)} + 3 a b^2 e^{(3dx+66c)} + b^3 e^{(3dx+66c)} - 9 a^3 e^{(dx+64c)} - 81 a^2 b e^{(dx+64c)} - 135 a b^2 e^{(dx+64c)} - 63 b^3 e^{(dx+64c)} \right) e^{(-63c)} - \left(2880 a^2 b e^{(15dx+15c)} + 34560 a b^2 e^{(15dx+15c)} + 11475 b^3 e^{(15dx+15c)} + 14400 a^2 b e^{(13dx+13c)} + 211200 a b^2 e^{(13dx+13c)} + 36775 b^3 e^{(13dx+13c)} + 25920 a^2 b e^{(11dx+11c)} + 590592 a b^2 e^{(11dx+11c)} + 67715 b^3 e^{(11dx+11c)} + 14400 a^2 b e^{(9dx+9c)} + 957696 a b^2 e^{(9dx+9c)} + 27055 b^3 e^{(9dx+9c)} - 14400 a^2 b e^{(7dx+7c)} + 957696 a b^2 e^{(7dx+7c)} - 27055 b^3 e^{(7dx+7c)} - 25920 a^2 b e^{(5dx+5c)} + 590592 a b^2 e^{(5dx+5c)} - 67715 b^3 e^{(5dx+5c)} - 14400 a^2 b e^{(3dx+3c)} + 211200 a b^2 e^{(3dx+3c)} - 36775 b^3 e^{(3dx+3c)} - 2880 a^2 b e^{(dx+c)} + 34560 a b^2 e^{(dx+c)} - 11475 b^3 e^{(dx+c)} \right) / \left(e^{(2dx+2c)} + 1 \right)^8 / d$

maple [A] time = 0.49, size = 506, normalized size = 1.44

$$\frac{2a^3 \cosh(dx+c)}{3d} + \frac{a^3 \cosh(dx+c) \left(\sinh^2(dx+c) \right)}{3d} + \frac{a^2 b \left(\sinh^5(dx+c) \right)}{d \cosh(dx+c)^2} - \frac{5a^2 b \left(\sinh^3(dx+c) \right)}{d \cosh(dx+c)^2} - \frac{15a^2 b \sinh(dx+c)}{d \cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x)

[Out] $-2/3 a^3 \cosh(dx+c)/d + 1/3 d a^3 \cosh(dx+c) \sinh(dx+c)^2 + 1/d a^2 b \sinh(dx+c)^5 / \cosh(dx+c)^2 - 5/d a^2 b \sinh(dx+c)^3 / \cosh(dx+c)^2 - 15/d a^2 b \sinh(dx+c) / \cosh(dx+c)^2 + 15/2 d a^2 b \operatorname{sech}(dx+c) \tanh(dx+c) + 15/d a^2 b \arctan(\exp(dx+c)) + 1/d a b^2 \sinh(dx+c)^8 / \cosh(dx+c)^5 - 8/d a b^2 \sinh(dx+c)^6 / \cosh(dx+c)^5 - 48/d a b^2 \sinh(dx+c)^4 / \cosh(dx+c)^5 - 64/d a b^2 \sinh(dx+c)^2 / \cosh(dx+c)^5 - 128/5 d a b^2 / \cosh(dx+c)^5 + 1/3 d b^3 \sinh(dx+c)^{11} / \cosh(dx+c)^8 - 11/3 d b^3 \sinh(dx+c)^9 / \cosh(dx+c)^8 - 33/d b^3 \sinh(dx+c)^7 / \cosh(dx+c)^8 - 77/d b^3 \sinh(dx+c)^5 / \cosh(dx+c)^8 - 77/d b^3 \sinh(dx+c)^3 / \cosh(dx+c)^8 - 33/d b^3 \sinh(dx+c) / \cosh(dx+c)^8 + 33/8 d b^3 \tanh(dx+c) \operatorname{sech}(dx+c)^7 + 77/16 d b^3 \tanh(dx+c) \operatorname{sech}(dx+c)^5 + 385/64 d b^3 \tanh(dx+c) \operatorname{sech}(dx+c)^3 + 1155/128 d b^3 \operatorname{sech}(dx+c) \tanh(dx+c) + 1155/64 d b^3 \arctan(\exp(dx+c))$

maxima [A] time = 0.43, size = 604, normalized size = 1.72

$$\frac{1}{192} b^3 \left(\frac{8 \left(63 e^{(-dx-c)} - e^{(-3dx-3c)} \right)}{d} - \frac{3465 \arctan \left(e^{(-dx-c)} \right)}{d} - \frac{440 e^{(-2dx-2c)} + 6103 e^{(-4dx-4c)} + 21019 e^{(-6dx-6c)}}{d \left(e^{(-3dx-3c)} + 8 e^{(-5dx-5c)} + 28 e^{(-7dx-7c)} \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")

[Out] $\frac{1}{192} b^3 \left(8 \left(63 e^{(-dx-c)} - e^{(-3dx-3c)} \right) / d - 3465 \arctan \left(e^{(-dx-c)} \right) / d - \left(440 e^{(-2dx-2c)} + 6103 e^{(-4dx-4c)} + 21019 e^{(-6dx-6c)} \right) / \left(e^{(-3dx-3c)} + 8 e^{(-5dx-5c)} + 28 e^{(-7dx-7c)} \right) \right)$

$6*c) + 41207*e^{(-8*d*x - 8*c)} + 40243*e^{(-10*d*x - 10*c)} + 22589*e^{(-12*d*x - 12*c)} + 505*e^{(-14*d*x - 14*c)} - 3331*e^{(-16*d*x - 16*c)} - 1791*e^{(-18*d*x - 18*c)} - 8)/(d*(e^{(-3*d*x - 3*c)} + 8*e^{(-5*d*x - 5*c)} + 28*e^{(-7*d*x - 7*c)} + 56*e^{(-9*d*x - 9*c)} + 70*e^{(-11*d*x - 11*c)} + 56*e^{(-13*d*x - 13*c)} + 28*e^{(-15*d*x - 15*c)} + 8*e^{(-17*d*x - 17*c)} + e^{(-19*d*x - 19*c)})) - 1/40*a*b^2*(5*(45*e^{(-d*x - c)} - e^{(-3*d*x - 3*c)})/d + (200*e^{(-2*d*x - 2*c)} + 2515*e^{(-4*d*x - 4*c)} + 6680*e^{(-6*d*x - 6*c)} + 9073*e^{(-8*d*x - 8*c)} + 5600*e^{(-10*d*x - 10*c)} + 1665*e^{(-12*d*x - 12*c)} - 5)/(d*(e^{(-3*d*x - 3*c)} + 5*e^{(-5*d*x - 5*c)} + 10*e^{(-7*d*x - 7*c)} + 10*e^{(-9*d*x - 9*c)} + 5*e^{(-11*d*x - 11*c)} + e^{(-13*d*x - 13*c)})) + 1/8*a^2*b*((27*e^{(-d*x - c)} - e^{(-3*d*x - 3*c)})/d - 120*arctan(e^{(-d*x - c)})/d - (25*e^{(-2*d*x - 2*c)} + 77*e^{(-4*d*x - 4*c)} + 3*e^{(-6*d*x - 6*c)} - 1)/(d*(e^{(-3*d*x - 3*c)} + 2*e^{(-5*d*x - 5*c)} + e^{(-7*d*x - 7*c)}))) + 1/24*a^3*(e^{(3*d*x + 3*c)}/d - 9*e^{(d*x + c)}/d - 9*e^{(-d*x - c)}/d + e^{(-3*d*x - 3*c)}/d)$

mupad [B] time = 1.60, size = 757, normalized size = 2.16

$$\frac{e^{3c+3dx}(a+b)^3}{24d} + \frac{e^{-3c-3dx}(a-b)^3}{24d} + \frac{15 \operatorname{atan}\left(\frac{e^{dx} e^c (77b^3 \sqrt{d^2} + 64a^2 b \sqrt{d^2})}{d \sqrt{4096a^4 b^2 + 9856a^2 b^4 + 5929b^6}}\right) \sqrt{4096a^4 b^2 + 9856a^2 b^4 + 5929b^6}}{64 \sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^3)^3,x)`

[Out] $(\exp(3*c + 3*d*x)*(a + b)^3)/(24*d) + (\exp(-3*c - 3*d*x)*(a - b)^3)/(24*d) + (15*\operatorname{atan}((\exp(d*x)*\exp(c)*(77*b^3*(d^2)^{(1/2)} + 64*a^2*b*(d^2)^{(1/2)}))/(d*(5929*b^6 + 9856*a^2*b^4 + 4096*a^4*b^2)^{(1/2)}))*(5929*b^6 + 9856*a^2*b^4 + 4096*a^4*b^2)^{(1/2)})/(64*(d^2)^{(1/2)}) - (3*\exp(-c - d*x)*(a - b)^2*(a - 7*b))/(8*d) - (\exp(c + d*x)*(6144*a*b^2 + 11005*b^3))/(120*d*(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1)) + (\exp(c + d*x)*(768*a*b^2 + 3365*b^3))/(20*d*(4*\exp(2*c + 2*d*x) + 6*\exp(4*c + 4*d*x) + 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1)) + (596*b^3*\exp(c + d*x))/(3*d*(6*\exp(2*c + 2*d*x) + 15*\exp(4*c + 4*d*x) + 20*\exp(6*c + 6*d*x) + 15*\exp(8*c + 8*d*x) + 6*\exp(10*c + 10*d*x) + \exp(12*c + 12*d*x) + 1)) - (3*\exp(c + d*x)*(a + b)^2*(a + 7*b))/(8*d) - (3*\exp(c + d*x)*(768*a*b^2 + 64*a^2*b + 255*b^3))/(64*d*(\exp(2*c + 2*d*x) + 1)) - (2*\exp(c + d*x)*(144*a*b^2 + 1625*b^3))/(15*d*(5*\exp(2*c + 2*d*x) + 10*\exp(4*c + 4*d*x) + 10*\exp(6*c + 6*d*x) + 5*\exp(8*c + 8*d*x) + \exp(10*c + 10*d*x) + 1)) - (112*b^3*\exp(c + d*x))/(d*(7*\exp(2*c + 2*d*x) + 21*\exp(4*c + 4*d*x) + 35*\exp(6*c + 6*d*x) + 35*\exp(8*c + 8*d*x) + 21*\exp(10*c + 10*d*x) + 7*\exp(12*c + 12*d*x) + \exp(14*c + 14*d*x) + 1)) + (\exp(c + d*x)*(3072*a*b^2 + 576*a^2*b + 4355*b^3))/(96*d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1)) + (32*b^3*\exp(c + d*x))/(d*(8*\exp(2*c + 2*d*x) + 28*\exp(4*c + 4*d*x) + 56*\exp(6*c + 6*d*x) + 70*\exp(8*c + 8*d*x) + 56*\exp(10*c + 10*d*x) + 28*\exp(12*c + 12*d*x) + 8*\exp(14*c + 14*d*x) + \exp(16*c + 16*d*x) + 1))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**3)**3,x)`

[Out] Timed out

3.67 $\int \sinh^2(c + dx) \left(a + b \tanh^3(c + dx) \right)^3 dx$

Optimal. Leaf size=220

$$\frac{b(3a^2 + 4b^2) \tanh^2(c + dx)}{2d} - \frac{b(6a^2 + 5b^2) \log(\cosh(c + dx))}{d} + \frac{\sinh(c + dx) \cosh(c + dx) (b(3a^2 + b^2) \tanh(c + dx) + a)}{2d}$$

[Out] $-1/2*a*(a^2+21*b^2)*x-b*(6*a^2+5*b^2)*\ln(\cosh(d*x+c))/d+9*a*b^2*\tanh(d*x+c)/d+1/2*b*(3*a^2+4*b^2)*\tanh(d*x+c)^2/d+2*a*b^2*\tanh(d*x+c)^3/d+3/4*b^3*\tanh(d*x+c)^4/d+3/5*a*b^2*\tanh(d*x+c)^5/d+1/3*b^3*\tanh(d*x+c)^6/d+1/8*b^3*\tanh(d*x+c)^8/d+1/2*\cosh(d*x+c)*\sinh(d*x+c)*(a*(a^2+3*b^2)+b*(3*a^2+b^2)*\tanh(d*x+c))/d$

Rubi [A] time = 0.31, antiderivative size = 241, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3663, 1804, 1802, 633, 31}

$$\frac{b(3a^2 + 4b^2) \tanh^2(c + dx)}{2d} + \frac{a(a^2 + 21b^2) \tanh(c + dx)}{2d} + \frac{\sinh^2(c + dx) (a(a^2 + 3b^2) \tanh(c + dx) + b(3a^2 + b^2))}{2d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^3,x]

[Out] $((a + b)^2*(a + 10*b)*\text{Log}[1 - \text{Tanh}[c + d*x]])/(4*d) - ((a - 10*b)*(a - b)^2*\text{Log}[1 + \text{Tanh}[c + d*x]])/(4*d) + (a*(a^2 + 21*b^2)*\text{Tanh}[c + d*x])/(2*d) + (b*(3*a^2 + 4*b^2)*\text{Tanh}[c + d*x]^2)/(2*d) + (2*a*b^2*\text{Tanh}[c + d*x]^3)/d + (3*b^3*\text{Tanh}[c + d*x]^4)/(4*d) + (3*a*b^2*\text{Tanh}[c + d*x]^5)/(5*d) + (b^3*\text{Tanh}[c + d*x]^6)/(3*d) + (b^3*\text{Tanh}[c + d*x]^8)/(8*d) + (\text{Sinh}[c + d*x]^2*(b*(3*a^2 + b^2) + a*(a^2 + 3*b^2)*\text{Tanh}[c + d*x]))/(2*d)$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 633

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]

Rule 1802

Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1804

Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((c*x)^m*(a + b*x^2)^(p + 1)*(a*g - b*f*x))/(2*a*b*(p + 1)), x] + Dist[c/(2*a*b*(p + 1)), Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSum[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]
```

Rubi steps

$$\int \sinh^2(c + dx) (a + b \tanh^3(c + dx))^3 dx = \frac{\text{Subst}\left(\int \frac{x^2(a+bx^3)^3}{(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{\sinh^2(c + dx) (b(3a^2 + b^2) + a(a^2 + 3b^2) \tanh(c + dx))}{2d} + \frac{\text{Subst}\left(\int \frac{x^2(a+bx^3)^3}{(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{\sinh^2(c + dx) (b(3a^2 + b^2) + a(a^2 + 3b^2) \tanh(c + dx))}{2d} + \frac{\text{Subst}\left(\int \frac{x^2(a+bx^3)^3}{(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{a(a^2 + 21b^2) \tanh(c + dx)}{2d} + \frac{b(3a^2 + 4b^2) \tanh^2(c + dx)}{2d} + \frac{2ab \tanh^3(c + dx)}{2d}$$

$$= \frac{a(a^2 + 21b^2) \tanh(c + dx)}{2d} + \frac{b(3a^2 + 4b^2) \tanh^2(c + dx)}{2d} + \frac{2ab \tanh^3(c + dx)}{2d}$$

$$= \frac{(a + b)^2(a + 10b) \log(1 - \tanh(c + dx))}{4d} - \frac{(a - 10b)(a - b)^2 \log(\tanh(c + dx))}{4d}$$

Mathematica [A] time = 6.25, size = 244, normalized size = 1.11

$$\frac{(-6a^2b - 5b^3) \log(\cosh(c + dx))}{d} - \frac{a(a^2 + 21b^2)(c + dx)}{2d} + \frac{a(a^2 + 3b^2) \sinh(2(c + dx))}{4d} + \frac{b(3a^2 + b^2) \cosh(2(c + dx))}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^3,x]

[Out] -1/2*(a*(a^2 + 21*b^2)*(c + d*x))/d + (b*(3*a^2 + b^2)*Cosh[2*(c + d*x)])/(4*d) + ((-6*a^2*b - 5*b^3)*Log[Cosh[c + d*x]])/d - (b*(3*a^2 + 10*b^2)*Sech[c + d*x]^2)/(2*d) + (5*b^3*Sech[c + d*x]^4)/(2*d) - (5*b^3*Sech[c + d*x]^6)/(6*d) + (b^3*Sech[c + d*x]^8)/(8*d) + (a*(a^2 + 3*b^2)*Sinh[2*(c + d*x)])/(4*d) + (58*a*b^2*Tanh[c + d*x])/(5*d) - (16*a*b^2*Sech[c + d*x]^2*Tanh[c + d*x])/(5*d) + (3*a*b^2*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d)

fricas [B] time = 0.64, size = 9862, normalized size = 44.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")

[Out] 1/120*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^20 + 300*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^19 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^20 + 60*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*cosh(d*x + c)^18 + 30*(4*a^3 + 12*a^2*b + 12*a*b^2 + 4*b^3 - 2*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x + 95*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^18 + 180*(95

$$\begin{aligned}
&*(a^3 + 3a^2b + 3ab^2 + b^3)*\cosh(dx + c)^3 + 6*(2a^3 + 6a^2b + 6ab^2 + 2b^3 - (a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)*\sinh(dx + c)^{17} + 15*(27a^3 + 39a^2b - 207ab^2 - 131b^3 - 32*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^{16} + 15*(4845*(a^3 + 3a^2b + 3ab^2 + b^3)*\cosh(dx + c)^4 + 27a^3 + 39a^2b - 207ab^2 - 131b^3 - 32*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx + 612*(2a^3 + 6a^2b + 6ab^2 + 2b^3 - (a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^2)*\sinh(dx + c)^{16} + 240*(969*(a^3 + 3a^2b + 3ab^2 + b^3)*\cosh(dx + c)^5 + 204*(2a^3 + 6a^2b + 6ab^2 + 2b^3 - (a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^3 + (27a^3 + 39a^2b - 207ab^2 - 131b^3 - 32*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c))*\sinh(dx + c)^{15} + 240*(3a^3 - 6a^2b - 93ab^2 - 36b^3 - 7*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^{14} + 120*(4845*(a^3 + 3a^2b + 3ab^2 + b^3)*\cosh(dx + c)^6 + 1530*(2a^3 + 6a^2b + 6ab^2 + 2b^3 - (a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^4 + 6a^3 - 12a^2b - 186ab^2 - 72b^3 - 14*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx + 15*(27a^3 + 39a^2b - 207ab^2 - 131b^3 - 32*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^2)*\sinh(dx + c)^{14} + 240*(4845*(a^3 + 3a^2b + 3ab^2 + b^3)*\cosh(dx + c)^7 + 2142*(2a^3 + 6a^2b + 6ab^2 + 2b^3 - (a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^5 + 35*(27a^3 + 39a^2b - 207ab^2 - 131b^3 - 32*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^3 + 14*(3a^3 - 6a^2b - 93ab^2 - 36b^3 - 7*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c))*\sinh(dx + c)^{13} + 10*(63a^3 - 639a^2b - 6195ab^2 - 2173b^3 - 336*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^{12} + 10*(188955*(a^3 + 3a^2b + 3ab^2 + b^3)*\cosh(dx + c)^8 + 111384*(2a^3 + 6a^2b + 6ab^2 + 2b^3 - (a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^6 + 2730*(27a^3 + 39a^2b - 207ab^2 - 131b^3 - 32*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^4 + 63a^3 - 639a^2b - 6195ab^2 - 2173b^3 - 336*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx + 2184*(3a^3 - 6a^2b - 93ab^2 - 36b^3 - 7*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^2)*\sinh(dx + c)^{12} + 120*(20995*(a^3 + 3a^2b + 3ab^2 + b^3)*\cosh(dx + c)^9 + 15912*(2a^3 + 6a^2b + 6ab^2 + 2b^3 - (a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^7 + 546*(27a^3 + 39a^2b - 207ab^2 - 131b^3 - 32*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^5 + 728*(3a^3 - 6a^2b - 93ab^2 - 36b^3 - 7*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^3 + (63a^3 - 639a^2b - 6195ab^2 - 2173b^3 - 336*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c))*\sinh(dx + c)^{11} - 40*(234a^2b + 2436ab^2 + 662b^3 + 105*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^{10} + 20*(138567*(a^3 + 3a^2b + 3ab^2 + b^3)*\cosh(dx + c)^{10} + 131274*(2a^3 + 6a^2b + 6ab^2 + 2b^3 - (a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^8 + 6006*(27a^3 + 39a^2b - 207ab^2 - 131b^3 - 32*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^6 + 12012*(3a^3 - 6a^2b - 93ab^2 - 36b^3 - 7*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^4 - 468a^2b - 4872ab^2 - 1324b^3 - 210*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx + 33*(63a^3 - 639a^2b - 6195ab^2 - 2173b^3 - 336*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^2)*\sinh(dx + c)^{10} + 40*(62985*(a^3 + 3a^2b + 3ab^2 + b^3)*\cosh(dx + c)^{11} + 72930*(2a^3 + 6a^2b + 6ab^2 + 2b^3 - (a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^9 + 4290*(27a^3 + 39a^2b - 207ab^2 - 131b^3 - 32*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^7 + 12012*(3a^3 - 6a^2b - 93ab^2 - 36b^3 - 7*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^5 + 55*(63a^3 - 639a^2b - 6195ab^2 - 2173b^3 - 336*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^3 - 10*(234a^2b + 2436ab^2 + 662b^3 + 105*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c))*\sinh(dx + c)^9 - 2*(315a^3 + 3195a^2b + 46977ab^2 + 10865b^3 + 1680*(a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^8 + 2*(944775*(a^3 + 3a^2b + 3ab^2 + b^3)*\cosh(dx + c)^{12} + 1312740*(2a^3 + 6a^2b + 6ab^2 + 2b^3 - (a^3 - 12a^2b + 21ab^2 - 10b^3)*dx)*\cosh(dx + c)^{10} + 96525*(27a^3 + 39
\end{aligned}$$

$$\begin{aligned}
& a^2*b - 207*a*b^2 - 131*b^3 - 32*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)* \\
& \cosh(d*x + c)^8 + 360360*(3*a^3 - 6*a^2*b - 93*a*b^2 - 36*b^3 - 7*(a^3 - 12 \\
& *a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^6 + 2475*(63*a^3 - 639*a^2*b \\
& - 6195*a*b^2 - 2173*b^3 - 336*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\co \\
& sh(d*x + c)^4 - 315*a^3 - 3195*a^2*b - 46977*a*b^2 - 10865*b^3 - 1680*(a^3 \\
& - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x - 900*(234*a^2*b + 2436*a*b^2 + 662*b^3 \\
& + 105*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x \\
& + c)^8 + 16*(72675*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^13 + 11934 \\
& 0*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3) \\
& *d*x)*\cosh(d*x + c)^11 + 10725*(27*a^3 + 39*a^2*b - 207*a*b^2 - 131*b^3 - 3 \\
& 2*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^9 + 51480*(3*a^3 \\
& - 6*a^2*b - 93*a*b^2 - 36*b^3 - 7*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x) \\
& *\cosh(d*x + c)^7 + 495*(63*a^3 - 639*a^2*b - 6195*a*b^2 - 2173*b^3 - 336*(a \\
& ^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^5 - 300*(234*a^2*b + \\
& 2436*a*b^2 + 662*b^3 + 105*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d \\
& *x + c)^3 - (315*a^3 + 3195*a^2*b + 46977*a*b^2 + 10865*b^3 + 1680*(a^3 - 1 \\
& 2*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^7 - 48*(15*a \\
& ^3 + 30*a^2*b + 1159*a*b^2 + 180*b^3 + 35*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b \\
& ^3)*d*x)*\cosh(d*x + c)^6 + 8*(72675*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d* \\
& x + c)^14 + 139230*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (a^3 - 12*a^2*b + 2 \\
& 1*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^12 + 15015*(27*a^3 + 39*a^2*b - 207*a* \\
& b^2 - 131*b^3 - 32*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^ \\
& 10 + 90090*(3*a^3 - 6*a^2*b - 93*a*b^2 - 36*b^3 - 7*(a^3 - 12*a^2*b + 21*a* \\
& b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^8 + 1155*(63*a^3 - 639*a^2*b - 6195*a*b^2 \\
& - 2173*b^3 - 336*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^6 \\
& - 1050*(234*a^2*b + 2436*a*b^2 + 662*b^3 + 105*(a^3 - 12*a^2*b + 21*a*b^2 - \\
& 10*b^3)*d*x)*\cosh(d*x + c)^4 - 90*a^3 - 180*a^2*b - 6954*a*b^2 - 1080*b^3 \\
& - 210*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x - 7*(315*a^3 + 3195*a^2*b + \\
& 46977*a*b^2 + 10865*b^3 + 1680*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\co \\
& sh(d*x + c)^2)*\sinh(d*x + c)^6 + 16*(14535*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)* \\
& \cosh(d*x + c)^15 + 32130*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (a^3 - 12*a^2 \\
& *b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^13 + 4095*(27*a^3 + 39*a^2*b - 2 \\
& 07*a*b^2 - 131*b^3 - 32*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x \\
& + c)^11 + 30030*(3*a^3 - 6*a^2*b - 93*a*b^2 - 36*b^3 - 7*(a^3 - 12*a^2*b + \\
& 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^9 + 495*(63*a^3 - 639*a^2*b - 6195*a* \\
& b^2 - 2173*b^3 - 336*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c \\
&)^7 - 630*(234*a^2*b + 2436*a*b^2 + 662*b^3 + 105*(a^3 - 12*a^2*b + 21*a*b^ \\
& 2 - 10*b^3)*d*x)*\cosh(d*x + c)^5 - 7*(315*a^3 + 3195*a^2*b + 46977*a*b^2 + \\
& 10865*b^3 + 1680*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^3 \\
& - 18*(15*a^3 + 30*a^2*b + 1159*a*b^2 + 180*b^3 + 35*(a^3 - 12*a^2*b + 21*a* \\
& b^2 - 10*b^3)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 3*(135*a^3 - 195*a^2*b \\
& + 6389*a*b^2 + 655*b^3 + 160*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh \\
& (d*x + c)^4 + (72675*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^16 + 183 \\
& 600*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (a^3 - 12*a^2*b + 21*a*b^2 - 10*b^ \\
& 3)*d*x)*\cosh(d*x + c)^14 + 27300*(27*a^3 + 39*a^2*b - 207*a*b^2 - 131*b^3 - \\
& 32*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^12 + 240240*(3* \\
& a^3 - 6*a^2*b - 93*a*b^2 - 36*b^3 - 7*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)* \\
& d*x)*\cosh(d*x + c)^10 + 4950*(63*a^3 - 639*a^2*b - 6195*a*b^2 - 2173*b^3 - \\
& 336*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^8 - 8400*(234*a \\
& ^2*b + 2436*a*b^2 + 662*b^3 + 105*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x) \\
& *\cosh(d*x + c)^6 - 140*(315*a^3 + 3195*a^2*b + 46977*a*b^2 + 10865*b^3 + 16 \\
& 80*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^4 - 405*a^3 + 58 \\
& 5*a^2*b - 19167*a*b^2 - 1965*b^3 - 480*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3) \\
& *d*x - 720*(15*a^3 + 30*a^2*b + 1159*a*b^2 + 180*b^3 + 35*(a^3 - 12*a^2*b + \\
& 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 4*(4275*(a^3 + \\
& 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^17 + 12240*(2*a^3 + 6*a^2*b + 6*a*b^ \\
& 2 + 2*b^3 - (a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^15 + 21 \\
& 00*(27*a^3 + 39*a^2*b - 207*a*b^2 - 131*b^3 - 32*(a^3 - 12*a^2*b + 21*a*b^2 \\
& - 10*b^3)*d*x)*\cosh(d*x + c)^13 + 21840*(3*a^3 - 6*a^2*b - 93*a*b^2 - 36*b
\end{aligned}$$

$$\begin{aligned}
& ^3 - 7*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^{11} + 550*(63 \\
& *a^3 - 639*a^2*b - 6195*a*b^2 - 2173*b^3 - 336*(a^3 - 12*a^2*b + 21*a*b^2 - \\
& 10*b^3)*d*x)*\cosh(d*x + c)^9 - 1200*(234*a^2*b + 2436*a*b^2 + 662*b^3 + 10 \\
& 5*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^7 - 28*(315*a^3 + \\
& 3195*a^2*b + 46977*a*b^2 + 10865*b^3 + 1680*(a^3 - 12*a^2*b + 21*a*b^2 - 1 \\
& 0*b^3)*d*x)*\cosh(d*x + c)^5 - 240*(15*a^3 + 30*a^2*b + 1159*a*b^2 + 180*b^3 \\
& + 35*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^3 - 3*(135*a^ \\
& 3 - 195*a^2*b + 6389*a*b^2 + 655*b^3 + 160*(a^3 - 12*a^2*b + 21*a*b^2 - 10* \\
& b^3)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 15*a^3 + 45*a^2*b - 45*a*b^2 + 1 \\
& 5*b^3 - 12*(10*a^3 - 30*a^2*b + 262*a*b^2 - 10*b^3 + 5*(a^3 - 12*a^2*b + 21 \\
& *a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^2 + 2*(1425*(a^3 + 3*a^2*b + 3*a*b^2 + \\
& b^3)*\cosh(d*x + c)^{18} + 4590*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (a^3 - 12 \\
& *a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^{16} + 900*(27*a^3 + 39*a^2*b \\
& - 207*a*b^2 - 131*b^3 - 32*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d \\
& *x + c)^{14} + 10920*(3*a^3 - 6*a^2*b - 93*a*b^2 - 36*b^3 - 7*(a^3 - 12*a^2*b \\
& + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^{12} + 330*(63*a^3 - 639*a^2*b - 619 \\
& 5*a*b^2 - 2173*b^3 - 336*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x \\
& + c)^{10} - 900*(234*a^2*b + 2436*a*b^2 + 662*b^3 + 105*(a^3 - 12*a^2*b + 21 \\
& *a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^8 - 28*(315*a^3 + 3195*a^2*b + 46977*a* \\
& b^2 + 10865*b^3 + 1680*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + \\
& c)^6 - 360*(15*a^3 + 30*a^2*b + 1159*a*b^2 + 180*b^3 + 35*(a^3 - 12*a^2*b \\
& + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^4 - 60*a^3 + 180*a^2*b - 1572*a*b^2 \\
& + 60*b^3 - 30*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x - 9*(135*a^3 - 195* \\
& a^2*b + 6389*a*b^2 + 655*b^3 + 160*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x \\
&)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 - 120*((6*a^2*b + 5*b^3)*\cosh(d*x + c)^{1 \\
& 8} + 18*(6*a^2*b + 5*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^{17} + (6*a^2*b + 5*b^3) \\
& *\sinh(d*x + c)^{18} + 8*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^{16} + (48*a^2*b + 40*b \\
& ^3 + 153*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{16} + 16*(51*(6*a^ \\
& 2*b + 5*b^3)*\cosh(d*x + c)^3 + 8*(6*a^2*b + 5*b^3)*\cosh(d*x + c))*\sinh(d*x \\
& + c)^{15} + 28*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^{14} + 4*(765*(6*a^2*b + 5*b^3)* \\
& \cosh(d*x + c)^4 + 42*a^2*b + 35*b^3 + 240*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^2 \\
&)*\sinh(d*x + c)^{14} + 56*(153*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^5 + 80*(6*a^2* \\
& b + 5*b^3)*\cosh(d*x + c)^3 + 7*(6*a^2*b + 5*b^3)*\cosh(d*x + c))*\sinh(d*x + \\
& c)^{13} + 56*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^{12} + 28*(663*(6*a^2*b + 5*b^3)* \\
& \cosh(d*x + c)^6 + 520*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^4 + 12*a^2*b + 10*b^3 \\
& + 91*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{12} + 16*(1989*(6*a^2* \\
& b + 5*b^3)*\cosh(d*x + c)^7 + 2184*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^5 + 637*(\\
& 6*a^2*b + 5*b^3)*\cosh(d*x + c)^3 + 42*(6*a^2*b + 5*b^3)*\cosh(d*x + c))*\sinh \\
& (d*x + c)^{11} + 70*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^{10} + 2*(21879*(6*a^2*b + \\
& 5*b^3)*\cosh(d*x + c)^8 + 32032*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^6 + 14014*(6 \\
& *a^2*b + 5*b^3)*\cosh(d*x + c)^4 + 210*a^2*b + 175*b^3 + 1848*(6*a^2*b + 5*b \\
& ^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{10} + 4*(12155*(6*a^2*b + 5*b^3)*\cosh(d*x \\
& + c)^9 + 22880*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^7 + 14014*(6*a^2*b + 5*b^3) \\
& *\cosh(d*x + c)^5 + 3080*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^3 + 175*(6*a^2*b + \\
& 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^9 + 56*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^ \\
& 8 + 2*(21879*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^{10} + 51480*(6*a^2*b + 5*b^3)* \\
& \cosh(d*x + c)^8 + 42042*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^6 + 13860*(6*a^2*b + \\
& 5*b^3)*\cosh(d*x + c)^4 + 168*a^2*b + 140*b^3 + 1575*(6*a^2*b + 5*b^3)*\cosh \\
& (d*x + c)^2)*\sinh(d*x + c)^8 + 16*(1989*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^{11} \\
& + 5720*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^9 + 6006*(6*a^2*b + 5*b^3)*\cosh(d*x \\
& + c)^7 + 2772*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^5 + 525*(6*a^2*b + 5*b^3)*\cos \\
& h(d*x + c)^3 + 28*(6*a^2*b + 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 28*(6* \\
& a^2*b + 5*b^3)*\cosh(d*x + c)^6 + 28*(663*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^{12} \\
& + 2288*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^{10} + 3003*(6*a^2*b + 5*b^3)*\cosh(d* \\
& x + c)^8 + 1848*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^6 + 525*(6*a^2*b + 5*b^3)* \\
& \cosh(d*x + c)^4 + 6*a^2*b + 5*b^3 + 56*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^2)*\si \\
& nh(d*x + c)^6 + 56*(153*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^{13} + 624*(6*a^2*b + \\
& 5*b^3)*\cosh(d*x + c)^{11} + 1001*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^9 + 792*(6* \\
& a^2*b + 5*b^3)*\cosh(d*x + c)^7 + 315*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^5 + 56
\end{aligned}$$

$$\begin{aligned}
&*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^3 + 3*(6*a^2*b + 5*b^3)*\cosh(d*x + c))*\sin \\
&h(d*x + c)^5 + 8*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^4 + 4*(765*(6*a^2*b + 5*b^ \\
&3)*\cosh(d*x + c)^14 + 3640*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^12 + 7007*(6*a^2 \\
&*b + 5*b^3)*\cosh(d*x + c)^10 + 6930*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^8 + 367 \\
&5*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^6 + 980*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^4 \\
&+ 12*a^2*b + 10*b^3 + 105*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c) \\
&^4 + 16*(51*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^15 + 280*(6*a^2*b + 5*b^3)*\cosh \\
&(d*x + c)^13 + 637*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^11 + 770*(6*a^2*b + 5*b^ \\
&3)*\cosh(d*x + c)^9 + 525*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^7 + 196*(6*a^2*b + \\
&5*b^3)*\cosh(d*x + c)^5 + 35*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^3 + 2*(6*a^2*b \\
&+ 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + (6*a^2*b + 5*b^3)*\cosh(d*x + c)^ \\
&2 + (153*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^16 + 960*(6*a^2*b + 5*b^3)*\cosh(d* \\
&x + c)^14 + 2548*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^12 + 3696*(6*a^2*b + 5*b^3 \\
&)*\cosh(d*x + c)^10 + 3150*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^8 + 1568*(6*a^2*b \\
&+ 5*b^3)*\cosh(d*x + c)^6 + 420*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^4 + 6*a^2*b \\
&+ 5*b^3 + 48*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 2*(9*(6* \\
&a^2*b + 5*b^3)*\cosh(d*x + c)^17 + 64*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^15 + 1 \\
&96*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^13 + 336*(6*a^2*b + 5*b^3)*\cosh(d*x + c) \\
&^11 + 350*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^9 + 224*(6*a^2*b + 5*b^3)*\cosh(d* \\
&x + c)^7 + 84*(6*a^2*b + 5*b^3)*\cosh(d*x + c)^5 + 16*(6*a^2*b + 5*b^3)*\cosh \\
&(d*x + c)^3 + (6*a^2*b + 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c))*\log(2*\cosh(d* \\
&x + c)/(\cosh(d*x + c) - \sinh(d*x + c))) + 4*(75*(a^3 + 3*a^2*b + 3*a*b^2 + \\
&b^3)*\cosh(d*x + c)^19 + 270*(2*a^3 + 6*a^2*b + 6*a*b^2 + 2*b^3 - (a^3 - 12* \\
&a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^17 + 60*(27*a^3 + 39*a^2*b - \\
&207*a*b^2 - 131*b^3 - 32*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x \\
&+ c)^15 + 840*(3*a^3 - 6*a^2*b - 93*a*b^2 - 36*b^3 - 7*(a^3 - 12*a^2*b + 2 \\
&1*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^13 + 30*(63*a^3 - 639*a^2*b - 6195*a*b \\
&^2 - 2173*b^3 - 336*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c) \\
&^11 - 100*(234*a^2*b + 2436*a*b^2 + 662*b^3 + 105*(a^3 - 12*a^2*b + 21*a*b^ \\
&2 - 10*b^3)*d*x)*\cosh(d*x + c)^9 - 4*(315*a^3 + 3195*a^2*b + 46977*a*b^2 + \\
&10865*b^3 + 1680*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^7 \\
&- 72*(15*a^3 + 30*a^2*b + 1159*a*b^2 + 180*b^3 + 35*(a^3 - 12*a^2*b + 21*a* \\
&b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^5 - 3*(135*a^3 - 195*a^2*b + 6389*a*b^2 + \\
&655*b^3 + 160*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x)*\cosh(d*x + c)^3 - 6 \\
&*(10*a^3 - 30*a^2*b + 262*a*b^2 - 10*b^3 + 5*(a^3 - 12*a^2*b + 21*a*b^2 - 1 \\
&0*b^3)*d*x)*\cosh(d*x + c))*\sinh(d*x + c))/(d*\cosh(d*x + c)^18 + 18*d*\cosh(d \\
&*x + c)*\sinh(d*x + c)^17 + d*\sinh(d*x + c)^18 + 8*d*\cosh(d*x + c)^16 + (153 \\
&*d*\cosh(d*x + c)^2 + 8*d)*\sinh(d*x + c)^16 + 16*(51*d*\cosh(d*x + c)^3 + 8*d \\
&*\cosh(d*x + c))*\sinh(d*x + c)^15 + 28*d*\cosh(d*x + c)^14 + 4*(765*d*\cosh(d* \\
&x + c)^4 + 240*d*\cosh(d*x + c)^2 + 7*d)*\sinh(d*x + c)^14 + 56*(153*d*\cosh(d \\
&*x + c)^5 + 80*d*\cosh(d*x + c)^3 + 7*d*\cosh(d*x + c))*\sinh(d*x + c)^13 + 56 \\
&*d*\cosh(d*x + c)^12 + 28*(663*d*\cosh(d*x + c)^6 + 520*d*\cosh(d*x + c)^4 + 9 \\
&1*d*\cosh(d*x + c)^2 + 2*d)*\sinh(d*x + c)^12 + 16*(1989*d*\cosh(d*x + c)^7 + \\
&2184*d*\cosh(d*x + c)^5 + 637*d*\cosh(d*x + c)^3 + 42*d*\cosh(d*x + c))*\sinh(d \\
&*x + c)^11 + 70*d*\cosh(d*x + c)^10 + 2*(21879*d*\cosh(d*x + c)^8 + 32032*d*c \\
&osh(d*x + c)^6 + 14014*d*\cosh(d*x + c)^4 + 1848*d*\cosh(d*x + c)^2 + 35*d)*\s \\
&inh(d*x + c)^10 + 4*(12155*d*\cosh(d*x + c)^9 + 22880*d*\cosh(d*x + c)^7 + 14 \\
&014*d*\cosh(d*x + c)^5 + 3080*d*\cosh(d*x + c)^3 + 175*d*\cosh(d*x + c))*\sinh(\\
&d*x + c)^9 + 56*d*\cosh(d*x + c)^8 + 2*(21879*d*\cosh(d*x + c)^10 + 51480*d*c \\
&osh(d*x + c)^8 + 42042*d*\cosh(d*x + c)^6 + 13860*d*\cosh(d*x + c)^4 + 1575*d \\
&*\cosh(d*x + c)^2 + 28*d)*\sinh(d*x + c)^8 + 16*(1989*d*\cosh(d*x + c)^11 + 57 \\
&20*d*\cosh(d*x + c)^9 + 6006*d*\cosh(d*x + c)^7 + 2772*d*\cosh(d*x + c)^5 + 52 \\
&5*d*\cosh(d*x + c)^3 + 28*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 28*d*\cosh(d*x + \\
&c)^6 + 28*(663*d*\cosh(d*x + c)^12 + 2288*d*\cosh(d*x + c)^10 + 3003*d*\cosh(\\
&d*x + c)^8 + 1848*d*\cosh(d*x + c)^6 + 525*d*\cosh(d*x + c)^4 + 56*d*\cosh(d*x \\
&+ c)^2 + d)*\sinh(d*x + c)^6 + 56*(153*d*\cosh(d*x + c)^13 + 624*d*\cosh(d*x \\
&+ c)^11 + 1001*d*\cosh(d*x + c)^9 + 792*d*\cosh(d*x + c)^7 + 315*d*\cosh(d*x + \\
&c)^5 + 56*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 8*d*cos \\
&h(d*x + c)^4 + 4*(765*d*\cosh(d*x + c)^14 + 3640*d*\cosh(d*x + c)^12 + 7007*d
\end{aligned}$$

*cosh(d*x + c)^10 + 6930*d*cosh(d*x + c)^8 + 3675*d*cosh(d*x + c)^6 + 980*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^4 + 16*(51*d*cosh(d*x + c)^15 + 280*d*cosh(d*x + c)^13 + 637*d*cosh(d*x + c)^11 + 770*d*cosh(d*x + c)^9 + 525*d*cosh(d*x + c)^7 + 196*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 + 2*d*cosh(d*x + c))*sinh(d*x + c)^3 + d*cosh(d*x + c)^2 + (153*d*cosh(d*x + c)^16 + 960*d*cosh(d*x + c)^14 + 2548*d*cosh(d*x + c)^12 + 3696*d*cosh(d*x + c)^10 + 3150*d*cosh(d*x + c)^8 + 1568*d*cosh(d*x + c)^6 + 420*d*cosh(d*x + c)^4 + 48*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 2*(9*d*cosh(d*x + c)^17 + 64*d*cosh(d*x + c)^15 + 196*d*cosh(d*x + c)^13 + 336*d*cosh(d*x + c)^11 + 350*d*cosh(d*x + c)^9 + 224*d*cosh(d*x + c)^7 + 84*d*cosh(d*x + c)^5 + 16*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c))

giac [B] time = 1.28, size = 592, normalized size = 2.69

$$420(a^3 - 12a^2b + 21ab^2 - 10b^3)dx + 840(6a^2be^{2c} + 5b^3e^{2c})e^{-2c} \log(e^{2dx+2c} + 1) - 105(2a^3e^{2dx+2c} -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")

[Out] -1/840*(420*(a^3 - 12*a^2*b + 21*a*b^2 - 10*b^3)*d*x + 840*(6*a^2*b*e^(2*c) + 5*b^3*e^(2*c))*e^(-2*c)*log(e^(2*d*x + 2*c) + 1) - 105*(2*a^3*e^(2*d*x + 2*c) - 24*a^2*b*e^(2*d*x + 2*c) + 42*a*b^2*e^(2*d*x + 2*c) - 20*b^3*e^(2*d*x + 2*c) - a^3 + 3*a^2*b - 3*a*b^2 + b^3)*e^(-2*d*x - 2*c) - 105*(a^3*e^(2*d*x + 22*c) + 3*a^2*b*e^(2*d*x + 22*c) + 3*a*b^2*e^(2*d*x + 22*c) + b^3*e^(2*d*x + 22*c))*e^(-20*c) - (13698*a^2*b*e^(16*d*x + 16*c) + 11415*b^3*e^(16*d*x + 16*c) + 104544*a^2*b*e^(14*d*x + 14*c) - 30240*a*b^2*e^(14*d*x + 14*c) + 74520*b^3*e^(14*d*x + 14*c) + 353304*a^2*b*e^(12*d*x + 12*c) - 171360*a*b^2*e^(12*d*x + 12*c) + 252420*b^3*e^(12*d*x + 12*c) + 691488*a^2*b*e^(10*d*x + 10*c) - 446880*a*b^2*e^(10*d*x + 10*c) + 476840*b^3*e^(10*d*x + 10*c) + 858060*a^2*b*e^(8*d*x + 8*c) - 682080*a*b^2*e^(8*d*x + 8*c) + 601930*b^3*e^(8*d*x + 8*c) + 691488*a^2*b*e^(6*d*x + 6*c) - 644448*a*b^2*e^(6*d*x + 6*c) + 476840*b^3*e^(6*d*x + 6*c) + 353304*a^2*b*e^(4*d*x + 4*c) - 374304*a*b^2*e^(4*d*x + 4*c) + 252420*b^3*e^(4*d*x + 4*c) + 104544*a^2*b*e^(2*d*x + 2*c) - 125664*a*b^2*e^(2*d*x + 2*c) + 74520*b^3*e^(2*d*x + 2*c) + 13698*a^2*b - 19488*a*b^2 + 11415*b^3)/(e^(2*d*x + 2*c) + 1)^8)/d

maple [A] time = 0.29, size = 289, normalized size = 1.31

$$\frac{a^3 \cosh(dx + c) \sinh(dx + c)}{2d} - \frac{a^3 x}{2} - \frac{a^3 c}{2d} + \frac{3a^2 b (\sinh^4(dx + c))}{2d \cosh(dx + c)^2} - \frac{6a^2 b \ln(\cosh(dx + c))}{d} + \frac{3a^2 b (\tanh^2(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x)

[Out] 1/2*a^3*cosh(d*x+c)*sinh(d*x+c)/d-1/2*a^3*x-1/2/d*a^3*c+3/2/d*a^2*b*sinh(d*x+c)^4/cosh(d*x+c)^2-6/d*a^2*b*ln(cosh(d*x+c))+3*a^2*b*tanh(d*x+c)^2/d+3/2/d*a*b^2*sinh(d*x+c)^7/cosh(d*x+c)^5-21/2*a*b^2*x-21/2/d*a*b^2*c+21/2*a*b^2*tanh(d*x+c)/d+7/2*a*b^2*tanh(d*x+c)^3/d+21/10*a*b^2*tanh(d*x+c)^5/d+1/2/d*b^3*sinh(d*x+c)^10/cosh(d*x+c)^8-5/d*b^3*ln(cosh(d*x+c))+5/2*b^3*tanh(d*x+c)^2/d+5/4*b^3*tanh(d*x+c)^4/d+5/6*b^3*tanh(d*x+c)^6/d+5/8*b^3*tanh(d*x+c)^8/d

maxima [B] time = 0.43, size = 544, normalized size = 2.47

$$-\frac{1}{8}a^3\left(4x - \frac{e^{2dx+2c}}{d} + \frac{e^{-2dx-2c}}{d}\right) - \frac{1}{40}ab^2\left(\frac{420(dx+c)}{d} + \frac{15e^{-2dx-2c}}{d} - \frac{1003e^{-2dx-2c} + 3350e^{-4dx-4c} + 55e^{-6dx-6c}}{d(e^{-2dx-2c} + 5e^{-4dx-4c} + 10e^{-6dx-6c})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/8*a^3*(4*x - e^{(2*d*x + 2*c)}/d + e^{(-2*d*x - 2*c)}/d) - 1/40*a*b^2*(420*(d*x + c)/d + 15*e^{(-2*d*x - 2*c)}/d - (1003*e^{(-2*d*x - 2*c)} + 3350*e^{(-4*d*x - 4*c)} + 5590*e^{(-6*d*x - 6*c)} + 3915*e^{(-8*d*x - 8*c)} + 1455*e^{(-10*d*x - 10*c)} + 15)/(d*(e^{(-2*d*x - 2*c)} + 5*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 10*e^{(-8*d*x - 8*c)} + 5*e^{(-10*d*x - 10*c)} + e^{(-12*d*x - 12*c)}))) - 1/24*b^3*(120*(d*x + c)/d - 3*e^{(-2*d*x - 2*c)}/d + 120*\log(e^{(-2*d*x - 2*c)} + 1)/d - (24*e^{(-2*d*x - 2*c)} - 396*e^{(-4*d*x - 4*c)} - 1752*e^{(-6*d*x - 6*c)} - 4430*e^{(-8*d*x - 8*c)} - 5464*e^{(-10*d*x - 10*c)} - 4556*e^{(-12*d*x - 12*c)} - 1896*e^{(-14*d*x - 14*c)} - 477*e^{(-16*d*x - 16*c)} + 3)/(d*(e^{(-2*d*x - 2*c)} + 8*e^{(-4*d*x - 4*c)} + 28*e^{(-6*d*x - 6*c)} + 56*e^{(-8*d*x - 8*c)} + 70*e^{(-10*d*x - 10*c)} + 56*e^{(-12*d*x - 12*c)} + 28*e^{(-14*d*x - 14*c)} + 8*e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)}))) - 3/8*a^2*b*(16*(d*x + c)/d - e^{(-2*d*x - 2*c)}/d + 16*\log(e^{(-2*d*x - 2*c)} + 1)/d - (2*e^{(-2*d*x - 2*c)} - 15*e^{(-4*d*x - 4*c)} + 1)/(d*(e^{(-2*d*x - 2*c)} + 2*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)}))) \end{aligned}$$

mupad [B] time = 0.56, size = 617, normalized size = 2.80

$$\frac{8(29b^3 + 6ab^2)}{d(4e^{2c+2dx} + 6e^{4c+4dx} + 4e^{6c+6dx} + e^{8c+8dx} + 1)} + \frac{736b^3}{3d(6e^{2c+2dx} + 15e^{4c+4dx} + 20e^{6c+6dx} + 15e^{8c+8dx})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^3)^3,x)

[Out]
$$\begin{aligned} & (8*(6*a*b^2 + 29*b^3))/(d*(4*\exp(2*c + 2*d*x) + 6*\exp(4*c + 4*d*x) + 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1)) + (736*b^3)/(3*d*(6*\exp(2*c + 2*d*x) + 15*\exp(4*c + 4*d*x) + 20*\exp(6*c + 6*d*x) + 15*\exp(8*c + 8*d*x) + 6*\exp(10*c + 10*d*x) + \exp(12*c + 12*d*x) + 1)) - (\log(\exp(2*c)*\exp(2*d*x) + 1)*(6*a^2*b + 5*b^3))/d - (2*(18*a*b^2 + 3*a^2*b + 10*b^3))/(d*(\exp(2*c + 2*d*x) + 1)) - (96*(a*b^2 + 15*b^3))/(5*d*(5*\exp(2*c + 2*d*x) + 10*\exp(4*c + 4*d*x) + 10*\exp(6*c + 6*d*x) + 5*\exp(8*c + 8*d*x) + \exp(10*c + 10*d*x) + 1)) - (128*b^3)/(d*(7*\exp(2*c + 2*d*x) + 21*\exp(4*c + 4*d*x) + 35*\exp(6*c + 6*d*x) + 35*\exp(8*c + 8*d*x) + 21*\exp(10*c + 10*d*x) + 7*\exp(12*c + 12*d*x) + \exp(14*c + 14*d*x) + 1)) - (x*(a - b)^2*(a - 10*b))/2 + (6*(8*a*b^2 + a^2*b + 10*b^3))/(d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1)) + (32*b^3)/(d*(8*\exp(2*c + 2*d*x) + 28*\exp(4*c + 4*d*x) + 56*\exp(6*c + 6*d*x) + 70*\exp(8*c + 8*d*x) + 56*\exp(10*c + 10*d*x) + 28*\exp(12*c + 12*d*x) + 8*\exp(14*c + 14*d*x) + \exp(16*c + 16*d*x) + 1)) + (\exp(2*c + 2*d*x)*(a + b)^3)/(8*d) - (\exp(-2*c - 2*d*x)*(a - b)^3)/(8*d) - (16*(12*a*b^2 + 25*b^3))/(3*d*(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1)) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**3)**3,x)

[Out] Timed out

3.68 $\int \sinh(c + dx) \left(a + b \tanh^3(c + dx) \right)^3 dx$

Optimal. Leaf size=269

$$\frac{a^3 \cosh(c + dx)}{d} + \frac{9a^2b \sinh(c + dx)}{2d} - \frac{9a^2b \tan^{-1}(\sinh(c + dx))}{2d} - \frac{3a^2b \sinh(c + dx) \tanh^2(c + dx)}{2d} + \frac{3ab^2 \cosh(c + dx)}{d}$$

```
[Out] -9/2*a^2*b*arctan(sinh(d*x+c))/d-315/128*b^3*arctan(sinh(d*x+c))/d+a^3*cosh(d*x+c)/d+3*a*b^2*cosh(d*x+c)/d+9*a*b^2*sech(d*x+c)/d-3*a*b^2*sech(d*x+c)^3/d+3/5*a*b^2*sech(d*x+c)^5/d+9/2*a^2*b*sinh(d*x+c)/d+315/128*b^3*sinh(d*x+c)/d-3/2*a^2*b*sinh(d*x+c)*tanh(d*x+c)^2/d-105/128*b^3*sinh(d*x+c)*tanh(d*x+c)^2/d-21/64*b^3*sinh(d*x+c)*tanh(d*x+c)^4/d-3/16*b^3*sinh(d*x+c)*tanh(d*x+c)^6/d-1/8*b^3*sinh(d*x+c)*tanh(d*x+c)^8/d
```

Rubi [A] time = 0.25, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3666, 2638, 2592, 288, 321, 203, 2590, 270}

$$\frac{9a^2b \sinh(c + dx)}{2d} - \frac{9a^2b \tan^{-1}(\sinh(c + dx))}{2d} - \frac{3a^2b \sinh(c + dx) \tanh^2(c + dx)}{2d} + \frac{a^3 \cosh(c + dx)}{d} + \frac{3ab^2 \cosh(c + dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (-9*a^2*b*ArcTan[Sinh[c + d*x]])/(2*d) - (315*b^3*ArcTan[Sinh[c + d*x]])/(128*d) + (a^3*Cosh[c + d*x])/d + (3*a*b^2*Cosh[c + d*x])/d + (9*a*b^2*Sech[c + d*x])/d - (3*a*b^2*Sech[c + d*x]^3)/d + (3*a*b^2*Sech[c + d*x]^5)/(5*d) + (9*a^2*b*Sinh[c + d*x])/(2*d) + (315*b^3*Sinh[c + d*x])/(128*d) - (3*a^2*b*Sinh[c + d*x]*Tanh[c + d*x]^2)/(2*d) - (105*b^3*Sinh[c + d*x]*Tanh[c + d*x]^2)/(128*d) - (21*b^3*Sinh[c + d*x]*Tanh[c + d*x]^4)/(64*d) - (3*b^3*Sinh[c + d*x]*Tanh[c + d*x]^6)/(16*d) - (b^3*Sinh[c + d*x]*Tanh[c + d*x]^8)/(8*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a + b*x^n)^(p+1))/(b*n*(p+1)), x] - Dist[(c^n*(m-n+1))/(b*n*(p+1)), Int[(c*x)^(m-n)*(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !ILtQ[(m+n*(p+1)+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a + b*x^n)^(p+1))/(b*(m+n*p+1)), x] - Dist[(a*c^n*(m-n+1))/(b*(m+n*p+1)), Int[(c*x)^(m-n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p,
```

+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 2590

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]

Rule 2592

Int[((a_.)*sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3666

Int[((d_.)*sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]^(n_.))^p), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int \sinh(c + dx) (a + b \tanh^3(c + dx))^3 dx &= -\left(i \int (ia^3 \sinh(c + dx) + 3ia^2b \sinh(c + dx) \tanh^3(c + dx) + 3ia \right. \\
 &= a^3 \int \sinh(c + dx) dx + (3a^2b) \int \sinh(c + dx) \tanh^3(c + dx) dx + \\
 &= \frac{a^3 \cosh(c + dx)}{d} + \frac{(3a^2b) \text{Subst}\left(\int \frac{x^4}{(1+x^2)^2} dx, x, \sinh(c + dx)\right)}{d} \\
 &= \frac{a^3 \cosh(c + dx)}{d} - \frac{3a^2b \sinh(c + dx) \tanh^2(c + dx)}{2d} - \frac{b^3 \sinh(c + dx)}{d} \\
 &= \frac{a^3 \cosh(c + dx)}{d} + \frac{3ab^2 \cosh(c + dx)}{d} + \frac{9ab^2 \text{sech}(c + dx)}{d} - \frac{3ab^2}{d} \\
 &= -\frac{9a^2b \tan^{-1}(\sinh(c + dx))}{2d} + \frac{a^3 \cosh(c + dx)}{d} + \frac{3ab^2 \cosh(c + dx)}{d} \\
 &= -\frac{9a^2b \tan^{-1}(\sinh(c + dx))}{2d} + \frac{a^3 \cosh(c + dx)}{d} + \frac{3ab^2 \cosh(c + dx)}{d} \\
 &= -\frac{9a^2b \tan^{-1}(\sinh(c + dx))}{2d} + \frac{a^3 \cosh(c + dx)}{d} + \frac{3ab^2 \cosh(c + dx)}{d} \\
 &= -\frac{9a^2b \tan^{-1}(\sinh(c + dx))}{2d} - \frac{315b^3 \tan^{-1}(\sinh(c + dx))}{128d} + \frac{a^3 \cosh(c + dx)}{d}
 \end{aligned}$$

Mathematica [A] time = 6.33, size = 233, normalized size = 0.87

$$\frac{\operatorname{sech}^2(c + dx) \left(192a^2b \sinh(c + dx) + 325b^3 \sinh(c + dx) \right)}{128d} + \frac{b(3a^2 + b^2) \sinh(c + dx)}{d} + \frac{a(a^2 + 3b^2) \cosh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3),x]

[Out] $(-9*b*(64*a^2 + 35*b^2)*\operatorname{ArcTan}[\operatorname{Tanh}[(c + d*x)/2]])/(64*d) + (a*(a^2 + 3*b^2)*\operatorname{Cosh}[c + d*x])/d + (9*a*b^2*\operatorname{Sech}[c + d*x])/d - (3*a*b^2*\operatorname{Sech}[c + d*x]^3)/d + (3*a*b^2*\operatorname{Sech}[c + d*x]^5)/(5*d) + (b*(3*a^2 + b^2)*\operatorname{Sinh}[c + d*x])/d + (\operatorname{Sech}[c + d*x]^2*(192*a^2*b*\operatorname{Sinh}[c + d*x] + 325*b^3*\operatorname{Sinh}[c + d*x]))/(128*d) - (105*b^3*\operatorname{Sech}[c + d*x]^3*\operatorname{Tanh}[c + d*x])/(64*d) + (11*b^3*\operatorname{Sech}[c + d*x]^5*\operatorname{Tanh}[c + d*x])/(16*d) - (b^3*\operatorname{Sech}[c + d*x]^7*\operatorname{Tanh}[c + d*x])/(8*d)$

fricas [B] time = 0.52, size = 6410, normalized size = 23.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")

[Out] $1/320*(160*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^{18} + 2880*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)*\sinh(d*x + c)^{17} + 160*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\sinh(d*x + c)^{18} + 45*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*\cosh(d*x + c)^{16} + 45*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3 + 544*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{16} + 240*(544*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^3 + 3*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^{15} + 15*(384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*\cosh(d*x + c)^{14} + 15*(32640*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^4 + 384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3 + 360*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{14} + 210*(6528*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^5 + 120*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*\cosh(d*x + c)^3 + (384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^{13} + 3*(4480*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3)*\cosh(d*x + c)^{12} + 3*(990080*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^6 + 27300*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*\cosh(d*x + c)^4 + 4480*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3 + 455*(384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{12} + 12*(424320*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^7 + 16380*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*\cosh(d*x + c)^5 + 455*(384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*\cosh(d*x + c)^3 + 3*(4480*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^{11} + 3*(6720*a^3 + 3840*a^2*b + 67904*a*b^2 + 1295*b^3)*\cosh(d*x + c)^{10} + 3*(2333760*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^8 + 120120*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*\cosh(d*x + c)^6 + 5005*(384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*\cosh(d*x + c)^4 + 6720*a^3 + 3840*a^2*b + 67904*a*b^2 + 1295*b^3 + 66*(4480*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{10} + 10*(777920*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^9 + 51480*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*\cosh(d*x + c)^7 + 3003*(384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*\cosh(d*x + c)^5 + 66*(4480*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3)*\cosh(d*x + c)^3 + 3*(6720*a^3 + 3840*a^2*b + 67904*a*b^2 + 1295*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^9 + 3*(6720*a^3 - 3840*a^2*b + 67904*a*b^2 - 1295*b^3)*\cosh(d*x + c)^8 + 3*(2333760*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^{10} + 193050*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*\cosh(d*x + c)^8 + 15015*(384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*\cosh(d*x + c)^6 + 495*(4480*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3)*\cosh(d*x + c)^4 + 6720*a^3 - 3840*a^2*b + 67904*a*b^2 - 1295*b^3 + 45*(6720*a^3 + 3840*a^2*b + 67904*a*b^2 + 1295*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^9$

$$\begin{aligned}
& d*x + c)^8 + 24*(212160*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^{11} + \\
& 21450*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*\cosh(d*x + c)^9 + 2145*(384* \\
& a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*\cosh(d*x + c)^7 + 99*(4480*a^3 + 73 \\
& 60*a^2*b + 43008*a*b^2 + 4515*b^3)*\cosh(d*x + c)^5 + 15*(6720*a^3 + 3840*a^ \\
& 2*b + 67904*a*b^2 + 1295*b^3)*\cosh(d*x + c)^3 + (6720*a^3 - 3840*a^2*b + 67 \\
& 904*a*b^2 - 1295*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 3*(4480*a^3 - 7360*a \\
& ^2*b + 43008*a*b^2 - 4515*b^3)*\cosh(d*x + c)^6 + 3*(990080*(a^3 + 3*a^2*b + \\
& 3*a*b^2 + b^3)*\cosh(d*x + c)^{12} + 120120*(32*a^3 + 96*a^2*b + 224*a*b^2 + \\
& 61*b^3)*\cosh(d*x + c)^{10} + 15015*(384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^ \\
& 3)*\cosh(d*x + c)^8 + 924*(4480*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3)*c \\
& osh(d*x + c)^6 + 210*(6720*a^3 + 3840*a^2*b + 67904*a*b^2 + 1295*b^3)*\cosh(\\
& d*x + c)^4 + 4480*a^3 - 7360*a^2*b + 43008*a*b^2 - 4515*b^3 + 28*(6720*a^3 \\
& - 3840*a^2*b + 67904*a*b^2 - 1295*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^6 + 6 \\
& *(228480*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^{13} + 32760*(32*a^3 + \\
& 96*a^2*b + 224*a*b^2 + 61*b^3)*\cosh(d*x + c)^{11} + 5005*(384*a^3 + 960*a^2* \\
& b + 3328*a*b^2 + 475*b^3)*\cosh(d*x + c)^9 + 396*(4480*a^3 + 7360*a^2*b + 43 \\
& 008*a*b^2 + 4515*b^3)*\cosh(d*x + c)^7 + 126*(6720*a^3 + 3840*a^2*b + 67904* \\
& a*b^2 + 1295*b^3)*\cosh(d*x + c)^5 + 28*(6720*a^3 - 3840*a^2*b + 67904*a*b^2 \\
& - 1295*b^3)*\cosh(d*x + c)^3 + 3*(4480*a^3 - 7360*a^2*b + 43008*a*b^2 - 451 \\
& 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 15*(384*a^3 - 960*a^2*b + 3328*a*b^ \\
& 2 - 475*b^3)*\cosh(d*x + c)^4 + 15*(32640*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*co \\
& sh(d*x + c)^{14} + 5460*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*\cosh(d*x + c \\
&)^{12} + 1001*(384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*\cosh(d*x + c)^{10} + \\
& 99*(4480*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3)*\cosh(d*x + c)^8 + 42*(\\
& 6720*a^3 + 3840*a^2*b + 67904*a*b^2 + 1295*b^3)*\cosh(d*x + c)^6 + 14*(6720* \\
& a^3 - 3840*a^2*b + 67904*a*b^2 - 1295*b^3)*\cosh(d*x + c)^4 + 384*a^3 - 960* \\
& a^2*b + 3328*a*b^2 - 475*b^3 + 3*(4480*a^3 - 7360*a^2*b + 43008*a*b^2 - 451 \\
& 5*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^4 + 12*(10880*(a^3 + 3*a^2*b + 3*a*b^ \\
& 2 + b^3)*\cosh(d*x + c)^{15} + 2100*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*c \\
& osh(d*x + c)^{13} + 455*(384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*\cosh(d*x \\
& + c)^{11} + 55*(4480*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3)*\cosh(d*x + c \\
&)^9 + 30*(6720*a^3 + 3840*a^2*b + 67904*a*b^2 + 1295*b^3)*\cosh(d*x + c)^7 + \\
& 14*(6720*a^3 - 3840*a^2*b + 67904*a*b^2 - 1295*b^3)*\cosh(d*x + c)^5 + 5*(4 \\
& 480*a^3 - 7360*a^2*b + 43008*a*b^2 - 4515*b^3)*\cosh(d*x + c)^3 + 5*(384*a^3 \\
& - 960*a^2*b + 3328*a*b^2 - 475*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 160*a \\
& ^3 - 480*a^2*b + 480*a*b^2 - 160*b^3 + 45*(32*a^3 - 96*a^2*b + 224*a*b^2 - \\
& 61*b^3)*\cosh(d*x + c)^2 + 3*(8160*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x \\
& + c)^{16} + 1800*(32*a^3 + 96*a^2*b + 224*a*b^2 + 61*b^3)*\cosh(d*x + c)^{14} + \\
& 455*(384*a^3 + 960*a^2*b + 3328*a*b^2 + 475*b^3)*\cosh(d*x + c)^{12} + 66*(448 \\
& 0*a^3 + 7360*a^2*b + 43008*a*b^2 + 4515*b^3)*\cosh(d*x + c)^{10} + 45*(6720*a^ \\
& 3 + 3840*a^2*b + 67904*a*b^2 + 1295*b^3)*\cosh(d*x + c)^8 + 28*(6720*a^3 - 3 \\
& 840*a^2*b + 67904*a*b^2 - 1295*b^3)*\cosh(d*x + c)^6 + 15*(4480*a^3 - 7360*a \\
& ^2*b + 43008*a*b^2 - 4515*b^3)*\cosh(d*x + c)^4 + 480*a^3 - 1440*a^2*b + 336 \\
& 0*a*b^2 - 915*b^3 + 30*(384*a^3 - 960*a^2*b + 3328*a*b^2 - 475*b^3)*\cosh(d* \\
& x + c)^2*\sinh(d*x + c)^2 - 45*((64*a^2*b + 35*b^3)*\cosh(d*x + c)^{17} + 17*(\\
& 64*a^2*b + 35*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^{16} + (64*a^2*b + 35*b^3)*sin \\
& h(d*x + c)^{17} + 8*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^{15} + 8*(64*a^2*b + 35*b \\
& ^3 + 17*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^{15} + 40*(17*(64* \\
& a^2*b + 35*b^3)*\cosh(d*x + c)^3 + 3*(64*a^2*b + 35*b^3)*\cosh(d*x + c))*\sinh \\
& (d*x + c)^{14} + 28*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^{13} + 28*(85*(64*a^2*b + \\
& 35*b^3)*\cosh(d*x + c)^4 + 64*a^2*b + 35*b^3 + 30*(64*a^2*b + 35*b^3)*\cosh(\\
& d*x + c)^2*\sinh(d*x + c)^{13} + 364*(17*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^5 \\
& + 10*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^3 + (64*a^2*b + 35*b^3)*\cosh(d*x + c \\
&))*\sinh(d*x + c)^{12} + 56*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^{11} + 56*(221*(64 \\
& *a^2*b + 35*b^3)*\cosh(d*x + c)^6 + 195*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^4 \\
& + 64*a^2*b + 35*b^3 + 39*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c) \\
& ^{11} + 88*(221*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^7 + 273*(64*a^2*b + 35*b^3) \\
& *\cosh(d*x + c)^5 + 91*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^3 + 7*(64*a^2*b + 3 \\
& 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^{10} + 70*(64*a^2*b + 35*b^3)*\cosh(d*x +
\end{aligned}$$

$$\begin{aligned}
& c)^9 + 10*(2431*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^8 + 4004*(64*a^2*b + 35*b \\
& ^3)*\cosh(d*x + c)^6 + 2002*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^4 + 448*a^2*b \\
& + 245*b^3 + 308*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^9 + 2*(1 \\
& 2155*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^9 + 25740*(64*a^2*b + 35*b^3)*\cosh(d \\
& *x + c)^7 + 18018*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^5 + 4620*(64*a^2*b + 35 \\
& *b^3)*\cosh(d*x + c)^3 + 315*(64*a^2*b + 35*b^3)*\cosh(d*x + c))*\sinh(d*x + c \\
&)^8 + 56*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^7 + 8*(2431*(64*a^2*b + 35*b^3)* \\
& \cosh(d*x + c)^10 + 6435*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^8 + 6006*(64*a^2* \\
& b + 35*b^3)*\cosh(d*x + c)^6 + 2310*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^4 + 44 \\
& 8*a^2*b + 245*b^3 + 315*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^ \\
& 7 + 56*(221*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^11 + 715*(64*a^2*b + 35*b^3)* \\
& \cosh(d*x + c)^9 + 858*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^7 + 462*(64*a^2*b + \\
& 35*b^3)*\cosh(d*x + c)^5 + 105*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^3 + 7*(64* \\
& a^2*b + 35*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^6 + 28*(64*a^2*b + 35*b^3)*\cos \\
& h(d*x + c)^5 + 28*(221*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^12 + 858*(64*a^2*b \\
& + 35*b^3)*\cosh(d*x + c)^10 + 1287*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^8 + 92 \\
& 4*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^6 + 315*(64*a^2*b + 35*b^3)*\cosh(d*x + \\
& c)^4 + 64*a^2*b + 35*b^3 + 42*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^2)*\sinh(d*x \\
& + c)^5 + 140*(17*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^13 + 78*(64*a^2*b + 35* \\
& b^3)*\cosh(d*x + c)^11 + 143*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^9 + 132*(64*a \\
& ^2*b + 35*b^3)*\cosh(d*x + c)^7 + 63*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^5 + 1 \\
& 4*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^3 + (64*a^2*b + 35*b^3)*\cosh(d*x + c))* \\
& \sinh(d*x + c)^4 + 8*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^3 + 8*(85*(64*a^2*b + \\
& 35*b^3)*\cosh(d*x + c)^14 + 455*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^12 + 1001 \\
& *(64*a^2*b + 35*b^3)*\cosh(d*x + c)^10 + 1155*(64*a^2*b + 35*b^3)*\cosh(d*x + \\
& c)^8 + 735*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^6 + 245*(64*a^2*b + 35*b^3)*\c \\
& osh(d*x + c)^4 + 64*a^2*b + 35*b^3 + 35*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^2 \\
&)*\sinh(d*x + c)^3 + 8*(17*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^15 + 105*(64*a^ \\
& 2*b + 35*b^3)*\cosh(d*x + c)^13 + 273*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^11 + \\
& 385*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^9 + 315*(64*a^2*b + 35*b^3)*\cosh(d*x \\
& + c)^7 + 147*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^5 + 35*(64*a^2*b + 35*b^3)* \\
& \cosh(d*x + c)^3 + 3*(64*a^2*b + 35*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 + (6 \\
& 4*a^2*b + 35*b^3)*\cosh(d*x + c) + (17*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^16 \\
& + 120*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^14 + 364*(64*a^2*b + 35*b^3)*\cosh(d \\
& *x + c)^12 + 616*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^10 + 630*(64*a^2*b + 35* \\
& b^3)*\cosh(d*x + c)^8 + 392*(64*a^2*b + 35*b^3)*\cosh(d*x + c)^6 + 140*(64*a^ \\
& 2*b + 35*b^3)*\cosh(d*x + c)^4 + 64*a^2*b + 35*b^3 + 24*(64*a^2*b + 35*b^3)* \\
& \cosh(d*x + c)^2)*\sinh(d*x + c))*\arctan(\cosh(d*x + c) + \sinh(d*x + c)) + 6*(\\
& 480*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^17 + 120*(32*a^3 + 96*a^2 \\
& *b + 224*a*b^2 + 61*b^3)*\cosh(d*x + c)^15 + 35*(384*a^3 + 960*a^2*b + 3328* \\
& a*b^2 + 475*b^3)*\cosh(d*x + c)^13 + 6*(4480*a^3 + 7360*a^2*b + 43008*a*b^2 \\
& + 4515*b^3)*\cosh(d*x + c)^11 + 5*(6720*a^3 + 3840*a^2*b + 67904*a*b^2 + 129 \\
& 5*b^3)*\cosh(d*x + c)^9 + 4*(6720*a^3 - 3840*a^2*b + 67904*a*b^2 - 1295*b^3) \\
& *\cosh(d*x + c)^7 + 3*(4480*a^3 - 7360*a^2*b + 43008*a*b^2 - 4515*b^3)*\cosh(\\
& d*x + c)^5 + 10*(384*a^3 - 960*a^2*b + 3328*a*b^2 - 475*b^3)*\cosh(d*x + c)^ \\
& 3 + 15*(32*a^3 - 96*a^2*b + 224*a*b^2 - 61*b^3)*\cosh(d*x + c))*\sinh(d*x + c \\
&))/(d*\cosh(d*x + c)^17 + 17*d*\cosh(d*x + c)*\sinh(d*x + c)^16 + d*\sinh(d*x + \\
& c)^17 + 8*d*\cosh(d*x + c)^15 + 8*(17*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^ \\
& 15 + 40*(17*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^14 + 28*d* \\
& \cosh(d*x + c)^13 + 28*(85*d*\cosh(d*x + c)^4 + 30*d*\cosh(d*x + c)^2 + d)*\sin \\
& h(d*x + c)^13 + 364*(17*d*\cosh(d*x + c)^5 + 10*d*\cosh(d*x + c)^3 + d*\cosh(d \\
& *x + c))*\sinh(d*x + c)^12 + 56*d*\cosh(d*x + c)^11 + 56*(221*d*\cosh(d*x + c) \\
& ^6 + 195*d*\cosh(d*x + c)^4 + 39*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^11 + 8 \\
& 8*(221*d*\cosh(d*x + c)^7 + 273*d*\cosh(d*x + c)^5 + 91*d*\cosh(d*x + c)^3 + 7 \\
& *d*\cosh(d*x + c))*\sinh(d*x + c)^10 + 70*d*\cosh(d*x + c)^9 + 10*(2431*d*\cosh \\
& (d*x + c)^8 + 4004*d*\cosh(d*x + c)^6 + 2002*d*\cosh(d*x + c)^4 + 308*d*\cosh(\\
& d*x + c)^2 + 7*d)*\sinh(d*x + c)^9 + 2*(12155*d*\cosh(d*x + c)^9 + 25740*d*\co \\
& sh(d*x + c)^7 + 18018*d*\cosh(d*x + c)^5 + 4620*d*\cosh(d*x + c)^3 + 315*d*\co \\
& sh(d*x + c))*\sinh(d*x + c)^8 + 56*d*\cosh(d*x + c)^7 + 8*(2431*d*\cosh(d*x +
\end{aligned}$$

$$c)^{10} + 6435*d*\cosh(d*x + c)^8 + 6006*d*\cosh(d*x + c)^6 + 2310*d*\cosh(d*x + c)^4 + 315*d*\cosh(d*x + c)^2 + 7*d)*\sinh(d*x + c)^7 + 56*(221*d*\cosh(d*x + c)^{11} + 715*d*\cosh(d*x + c)^9 + 858*d*\cosh(d*x + c)^7 + 462*d*\cosh(d*x + c)^5 + 105*d*\cosh(d*x + c)^3 + 7*d*\cosh(d*x + c))*\sinh(d*x + c)^6 + 28*d*\cosh(d*x + c)^5 + 28*(221*d*\cosh(d*x + c)^{12} + 858*d*\cosh(d*x + c)^{10} + 1287*d*\cosh(d*x + c)^8 + 924*d*\cosh(d*x + c)^6 + 315*d*\cosh(d*x + c)^4 + 42*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^5 + 140*(17*d*\cosh(d*x + c)^{13} + 78*d*\cosh(d*x + c)^{11} + 143*d*\cosh(d*x + c)^9 + 132*d*\cosh(d*x + c)^7 + 63*d*\cosh(d*x + c)^5 + 14*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c)^4 + 8*d*\cosh(d*x + c)^3 + 8*(85*d*\cosh(d*x + c)^{14} + 455*d*\cosh(d*x + c)^{12} + 1001*d*\cosh(d*x + c)^{10} + 1155*d*\cosh(d*x + c)^8 + 735*d*\cosh(d*x + c)^6 + 245*d*\cosh(d*x + c)^4 + 35*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^3 + 8*(17*d*\cosh(d*x + c)^{15} + 105*d*\cosh(d*x + c)^{13} + 273*d*\cosh(d*x + c)^{11} + 385*d*\cosh(d*x + c)^9 + 315*d*\cosh(d*x + c)^7 + 147*d*\cosh(d*x + c)^5 + 35*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^2 + d*\cosh(d*x + c) + (17*d*\cosh(d*x + c)^{16} + 120*d*\cosh(d*x + c)^{14} + 364*d*\cosh(d*x + c)^{12} + 616*d*\cosh(d*x + c)^{10} + 630*d*\cosh(d*x + c)^8 + 392*d*\cosh(d*x + c)^6 + 140*d*\cosh(d*x + c)^4 + 24*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c))$$

giac [A] time = 0.68, size = 485, normalized size = 1.80

$$45(64a^2be^c + 35b^3e^c) \arctan(e^{(dx+c)})e^{-c} - 160(a^3 - 3a^2b + 3ab^2 - b^3)e^{(-dx-c)} - 160(a^3e^{(dx+20c)} + 3a^2be^c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")

[Out]
$$-1/320*(45*(64*a^2*b*e^c + 35*b^3*e^c)*\arctan(e^{(d*x + c)})*e^{-c} - 160*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*e^{(-d*x - c)} - 160*(a^3*e^{(d*x + 20*c)} + 3*a^2*b*e^{(d*x + 20*c)} + 3*a*b^2*e^{(d*x + 20*c)} + b^3*e^{(d*x + 20*c)})*e^{-19*c} - (960*a^2*b*e^{(15*d*x + 15*c)} + 5760*a*b^2*e^{(15*d*x + 15*c)} + 1625*b^3*e^{(15*d*x + 15*c)} + 4800*a^2*b*e^{(13*d*x + 13*c)} + 32640*a*b^2*e^{(13*d*x + 13*c)} + 3925*b^3*e^{(13*d*x + 13*c)} + 8640*a^2*b*e^{(11*d*x + 11*c)} + 88704*a*b^2*e^{(11*d*x + 11*c)} + 9065*b^3*e^{(11*d*x + 11*c)} + 4800*a^2*b*e^{(9*d*x + 9*c)} + 143232*a*b^2*e^{(9*d*x + 9*c)} + 1645*b^3*e^{(9*d*x + 9*c)} - 4800*a^2*b*e^{(7*d*x + 7*c)} + 143232*a*b^2*e^{(7*d*x + 7*c)} - 1645*b^3*e^{(7*d*x + 7*c)} - 8640*a^2*b*e^{(5*d*x + 5*c)} + 88704*a*b^2*e^{(5*d*x + 5*c)} - 9065*b^3*e^{(5*d*x + 5*c)} - 4800*a^2*b*e^{(3*d*x + 3*c)} + 32640*a*b^2*e^{(3*d*x + 3*c)} - 3925*b^3*e^{(3*d*x + 3*c)} - 960*a^2*b*e^{(d*x + c)} + 5760*a*b^2*e^{(d*x + c)} - 1625*b^3*e^{(d*x + c)})/(e^{(2*d*x + 2*c)} + 1)^8)/d$$

maple [A] time = 0.43, size = 410, normalized size = 1.52

$$\frac{a^3 \cosh(dx + c)}{d} + \frac{3a^2b(\sinh^3(dx + c))}{d \cosh(dx + c)^2} + \frac{9a^2b \sinh(dx + c)}{d \cosh(dx + c)^2} - \frac{9a^2b \operatorname{sech}(dx + c) \tanh(dx + c)}{2d} - \frac{9a^2b \arctan(e^{(dx+c)})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x)

[Out]
$$a^3*\cosh(d*x+c)/d+3/d*a^2*b*\sinh(d*x+c)^3/\cosh(d*x+c)^2+9/d*a^2*b*\sinh(d*x+c)/\cosh(d*x+c)^2-9/2/d*a^2*b*\operatorname{sech}(d*x+c)*\tanh(d*x+c)-9/d*a^2*b*\arctan(\exp(d*x+c))+3/d*a*b^2*\sinh(d*x+c)^6/\cosh(d*x+c)^5+18/d*a*b^2*\sinh(d*x+c)^4/\cosh(d*x+c)^5+24/d*a*b^2*\sinh(d*x+c)^2/\cosh(d*x+c)^5+48/5/d*a*b^2/\cosh(d*x+c)^5+1/d*b^3*\sinh(d*x+c)^9/\cosh(d*x+c)^8+9/d*b^3*\sinh(d*x+c)^7/\cosh(d*x+c)^8+21/d*b^3*\sinh(d*x+c)^5/\cosh(d*x+c)^8+21/d*b^3*\sinh(d*x+c)^3/\cosh(d*x+c)^8+9/d*b^3*\sinh(d*x+c)/\cosh(d*x+c)^8-9/8/d*b^3*\tanh(d*x+c)*\operatorname{sech}(d*x+c)^7-21/16/d*b^3*\tanh(d*x+c)*\operatorname{sech}(d*x+c)^5-105/64/d*b^3*\tanh(d*x+c)*\operatorname{sech}(d*x+c)^3-315/128/d*b^3*\operatorname{sech}(d*x+c)*\tanh(d*x+c)-315/64/d*b^3*\arctan(\exp(d*x+c))$$

maxima [A] time = 0.42, size = 484, normalized size = 1.80

$$\frac{1}{64} b^3 \left(\frac{315 \arctan(e^{(-dx-c)})}{d} - \frac{32 e^{(-dx-c)}}{d} + \frac{581 e^{(-2dx-2c)} + 1681 e^{(-4dx-4c)} + 3605 e^{(-6dx-6c)} + 2569 e^{(-8dx-8c)} + 1463 e^{(-10dx-10c)} - 917 e^{(-12dx-12c)} - 529 e^{(-14dx-14c)} - 293 e^{(-16dx-16c)} + 32}{d(e^{(-dx-c)} + 8e^{(-3dx-3c)} + 28e^{(-5dx-5c)} + 56e^{(-7dx-7c)} + 70e^{(-9dx-9c)} + 56e^{(-11dx-11c)} + 28e^{(-13dx-13c)} + 8e^{(-15dx-15c)} + e^{(-17dx-17c)})} + \frac{3}{2} a^2 b (6 \arctan(e^{(-dx-c)})/d - e^{(-dx-c)}/d + (4e^{(-2dx-2c)} - e^{(-4dx-4c)} + 1)/(d(e^{(-dx-c)} + 2e^{(-3dx-3c)} + e^{(-5dx-5c)}))) + \frac{3}{10} a b^2 (5e^{(-dx-c)}/d + (85e^{(-2dx-2c)} + 210e^{(-4dx-4c)} + 314e^{(-6dx-6c)} + 185e^{(-8dx-8c)} + 65e^{(-10dx-10c)} + 5)/(d(e^{(-dx-c)} + 5e^{(-3dx-3c)} + 10e^{(-5dx-5c)} + 10e^{(-7dx-7c)} + 5e^{(-9dx-9c)} + e^{(-11dx-11c)}))) + a^3 \cosh(dx+c)/d \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c))^3,x, algorithm="maxima")

[Out] 1/64*b^3*(315*arctan(e^(-d*x - c))/d - 32*e^(-d*x - c)/d + (581*e^(-2*d*x - 2*c) + 1681*e^(-4*d*x - 4*c) + 3605*e^(-6*d*x - 6*c) + 2569*e^(-8*d*x - 8*c) + 1463*e^(-10*d*x - 10*c) - 917*e^(-12*d*x - 12*c) - 529*e^(-14*d*x - 14*c) - 293*e^(-16*d*x - 16*c) + 32)/(d*(e^(-d*x - c) + 8*e^(-3*d*x - 3*c) + 28*e^(-5*d*x - 5*c) + 56*e^(-7*d*x - 7*c) + 70*e^(-9*d*x - 9*c) + 56*e^(-11*d*x - 11*c) + 28*e^(-13*d*x - 13*c) + 8*e^(-15*d*x - 15*c) + e^(-17*d*x - 17*c)))) + 3/2*a^2*b*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c)))) + 3/10*a*b^2*(5*e^(-d*x - c)/d + (85*e^(-2*d*x - 2*c) + 210*e^(-4*d*x - 4*c) + 314*e^(-6*d*x - 6*c) + 185*e^(-8*d*x - 8*c) + 65*e^(-10*d*x - 10*c) + 5)/(d*(e^(-d*x - c) + 5*e^(-3*d*x - 3*c) + 10*e^(-5*d*x - 5*c) + 10*e^(-7*d*x - 7*c) + 5*e^(-9*d*x - 9*c) + e^(-11*d*x - 11*c)))) + a^3*cosh(d*x + c)/d

mupad [B] time = 1.49, size = 707, normalized size = 2.63

$$\frac{e^{c+dx} (a+b)^3}{2d} + \frac{e^{-c-dx} (a-b)^3}{2d} - \frac{9 \operatorname{atan} \left(\frac{e^{dx} e^c (35b^3 \sqrt{d^2} + 64a^2 b \sqrt{d^2})}{d \sqrt{4096a^4 b^2 + 4480a^2 b^4 + 1225b^6}} \right)}{64 \sqrt{d^2}} + \frac{\sqrt{4096a^4 b^2 + 4480a^2 b^4 + 1225b^6}}{40d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)*(a + b*tanh(c + d*x))^3,x)

[Out] (exp(c + d*x)*(a + b)^3)/(2*d) + (exp(-c - d*x)*(a - b)^3)/(2*d) - (9*atan((exp(d*x)*exp(c)*(35*b^3*(d^2)^(1/2) + 64*a^2*b*(d^2)^(1/2)))/(d*(1225*b^6 + 4480*a^2*b^4 + 4096*a^4*b^2)^(1/2)))*(1225*b^6 + 4480*a^2*b^4 + 4096*a^4*b^2)^(1/2))/(64*(d^2)^(1/2)) + (exp(c + d*x)*(1728*a*b^2 + 2455*b^3))/(40*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (exp(c + d*x)*(768*a*b^2 + 2605*b^3))/(20*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (188*b^3*exp(c + d*x))/(d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) + (exp(c + d*x)*(1152*a*b^2 + 192*a^2*b + 325*b^3))/(64*d*(exp(2*c + 2*d*x) + 1)) + (2*exp(c + d*x)*(48*a*b^2 + 475*b^3))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (112*b^3*exp(c + d*x))/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) - (exp(c + d*x)*(768*a*b^2 + 192*a^2*b + 745*b^3))/(32*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (32*b^3*exp(c + d*x))/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^3(c + dx))^3 \sinh(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**3)**3,x)

[Out] Integral((a + b*tanh(c + d*x)**3)**3*sinh(c + d*x), x)

3.69 $\int \operatorname{csch}(c + dx) \left(a + b \tanh^3(c + dx) \right)^3 dx$

Optimal. Leaf size=219

$$-\frac{a^3 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{3a^2 b \tanh^{-1}(\sinh(c + dx))}{2d} - \frac{3a^2 b \tanh(c + dx) \operatorname{sech}(c + dx)}{2d} - \frac{3ab^2 \operatorname{sech}^5(c + dx)}{5d} + \frac{2ab^3 \operatorname{sech}^7(c + dx)}{7d}$$

[Out] $3/2*a^2*b*\arctan(\sinh(d*x+c))/d+35/128*b^3*\arctan(\sinh(d*x+c))/d-a^3*\arctan(\cosh(d*x+c))/d-3*a*b^2*\operatorname{sech}(d*x+c)/d+2*a*b^2*\operatorname{sech}(d*x+c)^3/d-3/5*a*b^2*\operatorname{sech}(d*x+c)^5/d-3/2*a^2*b*\operatorname{sech}(d*x+c)*\tanh(d*x+c)/d-35/128*b^3*\operatorname{sech}(d*x+c)*\tanh(d*x+c)/d-35/192*b^3*\operatorname{sech}(d*x+c)*\tanh(d*x+c)^3/d-7/48*b^3*\operatorname{sech}(d*x+c)*\tanh(d*x+c)^5/d-1/8*b^3*\operatorname{sech}(d*x+c)*\tanh(d*x+c)^7/d$

Rubi [A] time = 0.26, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3666, 3770, 2611, 2606, 194}

$$\frac{3a^2 b \tanh^{-1}(\sinh(c + dx))}{2d} - \frac{3a^2 b \tanh(c + dx) \operatorname{sech}(c + dx)}{2d} - \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{d} - \frac{3ab^2 \operatorname{sech}^5(c + dx)}{5d} + \frac{2ab^3 \operatorname{sech}^7(c + dx)}{7d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3)^3,x]

[Out] $(3*a^2*b*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/(2*d) + (35*b^3*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/(128*d) - (a^3*\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]])/d - (3*a*b^2*\operatorname{Sech}[c + d*x])/d + (2*a*b^2*\operatorname{Sech}[c + d*x]^3)/d - (3*a*b^2*\operatorname{Sech}[c + d*x]^5)/(5*d) - (3*a^2*b*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x])/(2*d) - (35*b^3*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x])/(128*d) - (35*b^3*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x]^3)/(192*d) - (7*b^3*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x]^5)/(48*d) - (b^3*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x]^7)/(8*d)$

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 2606

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m-1)*(-1+x^2)^((n-1)/2), x], x, Sec[e+f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n-1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n+1])

Rule 2611

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(b*(a*Sec[e+f*x])^m*(b*Tan[e+f*x])^(n-1))/(f*(m+n-1)), x] - Dist[(b^2*(n-1))/(m+n-1), Int[(a*Sec[e+f*x])^m*(b*Tan[e+f*x])^(n-2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && NeQ[m+n-1, 0] && IntegersQ[2*m, 2*n]

Rule 3666

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := Int[ExpandTrig[(d*sin[e+f*x])^m*(a + b*(c*tan[e+f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}(c+dx) (a+b \tanh^3(c+dx))^3 dx &= i \int (-ia^3 \operatorname{csch}(c+dx) - 3ia^2 b \operatorname{sech}(c+dx) \tanh^2(c+dx) - 3iab^2 \operatorname{sech}(c+dx) \tanh^3(c+dx)) dx \\
&= a^3 \int \operatorname{csch}(c+dx) dx + (3a^2 b) \int \operatorname{sech}(c+dx) \tanh^2(c+dx) dx + (3ab^2) \int \operatorname{sech}(c+dx) \tanh^3(c+dx) dx \\
&= -\frac{a^3 \tanh^{-1}(\cosh(c+dx))}{d} - \frac{3a^2 b \operatorname{sech}(c+dx) \tanh(c+dx)}{2d} - \frac{b^3 \operatorname{sech}(c+dx) \tanh^2(c+dx)}{2d} \\
&= \frac{3a^2 b \tan^{-1}(\sinh(c+dx))}{2d} - \frac{a^3 \tanh^{-1}(\cosh(c+dx))}{d} - \frac{3a^2 b \operatorname{sech}(c+dx) \tanh(c+dx)}{2d} \\
&= \frac{3a^2 b \tan^{-1}(\sinh(c+dx))}{2d} - \frac{a^3 \tanh^{-1}(\cosh(c+dx))}{d} - \frac{3ab^2 \operatorname{sech}(c+dx) \tanh^2(c+dx)}{d} \\
&= \frac{3a^2 b \tan^{-1}(\sinh(c+dx))}{2d} - \frac{a^3 \tanh^{-1}(\cosh(c+dx))}{d} - \frac{3ab^2 \operatorname{sech}(c+dx) \tanh^3(c+dx)}{d} \\
&= \frac{3a^2 b \tan^{-1}(\sinh(c+dx))}{2d} + \frac{35b^3 \tan^{-1}(\sinh(c+dx))}{128d} - \frac{a^3 \tanh^{-1}(\cosh(c+dx))}{d}
\end{aligned}$$

Mathematica [A] time = 3.21, size = 154, normalized size = 0.70

$$-45b \operatorname{sech}(c+dx) \left((64a^2 + 31b^2) \tanh(c+dx) + 128ab \right) + 30 \left(64a^3 \log \left(\tanh \left(\frac{1}{2}(c+dx) \right) \right) \right) + b (192a^2 + 35b^2) \tanh^3(c+dx)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3)^3, x]
```

```
[Out] (30*(b*(192*a^2 + 35*b^2)*ArcTan[Tanh[(c + d*x)/2]] + 64*a^3*Log[Tanh[(c + d*x)/2]]) + 240*b^3*Sech[c + d*x]^7*Tanh[c + d*x] - 8*b^2*Sech[c + d*x]^5*(144*a + 125*b*Tanh[c + d*x]) + 10*b^2*Sech[c + d*x]^3*(384*a + 163*b*Tanh[c + d*x]) - 45*b*Sech[c + d*x]*(128*a*b + (64*a^2 + 31*b^2)*Tanh[c + d*x]))/(1920*d)
```

fricas [B] time = 0.55, size = 7127, normalized size = 32.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] -1/960*(45*(64*a^2*b + 128*a*b^2 + 31*b^3)*cosh(d*x + c)^15 + 675*(64*a^2*b + 128*a*b^2 + 31*b^3)*cosh(d*x + c)*sinh(d*x + c)^14 + 45*(64*a^2*b + 128*a*b^2 + 31*b^3)*sinh(d*x + c)^15 + 5*(2880*a^2*b + 4992*a*b^2 + 91*b^3)*cosh(d*x + c)^13 + 5*(2880*a^2*b + 4992*a*b^2 + 91*b^3 + 945*(64*a^2*b + 128*a*b^2 + 31*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^13 + 65*(315*(64*a^2*b + 128*a*b^2 + 31*b^3)*cosh(d*x + c)^3 + (2880*a^2*b + 4992*a*b^2 + 91*b^3)*cosh(d*x + c))*sinh(d*x + c)^12 + (25920*a^2*b + 62592*a*b^2 + 8995*b^3)*cosh(d*x + c)^11 + (61425*(64*a^2*b + 128*a*b^2 + 31*b^3)*cosh(d*x + c)^4 + 25920*a^2*b + 62592*a*b^2 + 8995*b^3 + 390*(2880*a^2*b + 4992*a*b^2 + 91*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^11 + 11*(12285*(64*a^2*b + 128*a*b^2 + 31*b^3)*cosh(d*x + c)^5 + 130*(2880*a^2*b + 4992*a*b^2 + 91*b^3)*cosh(d*x + c)^3 + (25920*a^2*b + 62592*a*b^2 + 8995*b^3)*cosh(d*x + c))*sinh(d*x + c)^10 + (14400*a^2*b + 103296*a*b^2 - 5425*b^3)*cosh(d*x + c)^9 + (225225*(64*a^2*b +
```

$$\begin{aligned}
& 128*a*b^2 + 31*b^3)*\cosh(d*x + c)^6 + 3575*(2880*a^2*b + 4992*a*b^2 + 91*b^3) \\
& *\cosh(d*x + c)^4 + 14400*a^2*b + 103296*a*b^2 - 5425*b^3 + 55*(25920*a^2*b \\
& + 62592*a*b^2 + 8995*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^9 + 3*(96525*(64 \\
& *a^2*b + 128*a*b^2 + 31*b^3)*\cosh(d*x + c)^7 + 2145*(2880*a^2*b + 4992*a*b^2 \\
& + 91*b^3)*\cosh(d*x + c)^5 + 55*(25920*a^2*b + 62592*a*b^2 + 8995*b^3)*\cosh \\
& (d*x + c)^3 + 3*(14400*a^2*b + 103296*a*b^2 - 5425*b^3)*\cosh(d*x + c))*\sinh \\
& (d*x + c)^8 - (14400*a^2*b - 103296*a*b^2 - 5425*b^3)*\cosh(d*x + c)^7 + (2 \\
& 89575*(64*a^2*b + 128*a*b^2 + 31*b^3)*\cosh(d*x + c)^8 + 8580*(2880*a^2*b + \\
& 4992*a*b^2 + 91*b^3)*\cosh(d*x + c)^6 + 330*(25920*a^2*b + 62592*a*b^2 + 899 \\
& 5*b^3)*\cosh(d*x + c)^4 - 14400*a^2*b + 103296*a*b^2 + 5425*b^3 + 36*(14400* \\
& a^2*b + 103296*a*b^2 - 5425*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^7 + (225225 \\
& *(64*a^2*b + 128*a*b^2 + 31*b^3)*\cosh(d*x + c)^9 + 8580*(2880*a^2*b + 4992* \\
& a*b^2 + 91*b^3)*\cosh(d*x + c)^7 + 462*(25920*a^2*b + 62592*a*b^2 + 8995*b^3) \\
&)*\cosh(d*x + c)^5 + 84*(14400*a^2*b + 103296*a*b^2 - 5425*b^3)*\cosh(d*x + c \\
&)^3 - 7*(14400*a^2*b - 103296*a*b^2 - 5425*b^3)*\cosh(d*x + c))*\sinh(d*x + c \\
&)^6 - (25920*a^2*b - 62592*a*b^2 + 8995*b^3)*\cosh(d*x + c)^5 + (135135*(64* \\
& a^2*b + 128*a*b^2 + 31*b^3)*\cosh(d*x + c)^10 + 6435*(2880*a^2*b + 4992*a*b^2 \\
& + 91*b^3)*\cosh(d*x + c)^8 + 462*(25920*a^2*b + 62592*a*b^2 + 8995*b^3)*\cosh \\
& (d*x + c)^6 + 126*(14400*a^2*b + 103296*a*b^2 - 5425*b^3)*\cosh(d*x + c)^4 \\
& - 25920*a^2*b + 62592*a*b^2 - 8995*b^3 - 21*(14400*a^2*b - 103296*a*b^2 - \\
& 5425*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^5 + (61425*(64*a^2*b + 128*a*b^2 + \\
& 31*b^3)*\cosh(d*x + c)^11 + 3575*(2880*a^2*b + 4992*a*b^2 + 91*b^3)*\cosh(d* \\
& x + c)^9 + 330*(25920*a^2*b + 62592*a*b^2 + 8995*b^3)*\cosh(d*x + c)^7 + 126 \\
& *(14400*a^2*b + 103296*a*b^2 - 5425*b^3)*\cosh(d*x + c)^5 - 35*(14400*a^2*b \\
& - 103296*a*b^2 - 5425*b^3)*\cosh(d*x + c)^3 - 5*(25920*a^2*b - 62592*a*b^2 + \\
& 8995*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^4 - 5*(2880*a^2*b - 4992*a*b^2 + 91 \\
& *b^3)*\cosh(d*x + c)^3 + (20475*(64*a^2*b + 128*a*b^2 + 31*b^3)*\cosh(d*x + c \\
&)^12 + 1430*(2880*a^2*b + 4992*a*b^2 + 91*b^3)*\cosh(d*x + c)^10 + 165*(2592 \\
& 0*a^2*b + 62592*a*b^2 + 8995*b^3)*\cosh(d*x + c)^8 + 84*(14400*a^2*b + 10329 \\
& 6*a*b^2 - 5425*b^3)*\cosh(d*x + c)^6 - 35*(14400*a^2*b - 103296*a*b^2 - 5425 \\
& *b^3)*\cosh(d*x + c)^4 - 14400*a^2*b + 24960*a*b^2 - 455*b^3 - 10*(25920*a^2 \\
& *b - 62592*a*b^2 + 8995*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^3 + (4725*(64*a \\
& ^2*b + 128*a*b^2 + 31*b^3)*\cosh(d*x + c)^13 + 390*(2880*a^2*b + 4992*a*b^2 \\
& + 91*b^3)*\cosh(d*x + c)^11 + 55*(25920*a^2*b + 62592*a*b^2 + 8995*b^3)*\cosh \\
& (d*x + c)^9 + 36*(14400*a^2*b + 103296*a*b^2 - 5425*b^3)*\cosh(d*x + c)^7 - \\
& 21*(14400*a^2*b - 103296*a*b^2 - 5425*b^3)*\cosh(d*x + c)^5 - 10*(25920*a^2* \\
& b - 62592*a*b^2 + 8995*b^3)*\cosh(d*x + c)^3 - 15*(2880*a^2*b - 4992*a*b^2 + \\
& 91*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 - 15*((192*a^2*b + 35*b^3)*\cosh(d*x \\
& + c)^16 + 16*(192*a^2*b + 35*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^15 + (192*a^ \\
& 2*b + 35*b^3)*\sinh(d*x + c)^16 + 8*(192*a^2*b + 35*b^3)*\cosh(d*x + c)^14 + \\
& 8*(192*a^2*b + 35*b^3 + 15*(192*a^2*b + 35*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + \\
& c)^14 + 112*(5*(192*a^2*b + 35*b^3)*\cosh(d*x + c)^3 + (192*a^2*b + 35*b^3) \\
& *\cosh(d*x + c))*\sinh(d*x + c)^13 + 28*(192*a^2*b + 35*b^3)*\cosh(d*x + c)^12 \\
& + 28*(65*(192*a^2*b + 35*b^3)*\cosh(d*x + c)^4 + 192*a^2*b + 35*b^3 + 26*(1 \\
& 92*a^2*b + 35*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^12 + 112*(39*(192*a^2*b + \\
& 35*b^3)*\cosh(d*x + c)^5 + 26*(192*a^2*b + 35*b^3)*\cosh(d*x + c)^3 + 3*(192 \\
& *a^2*b + 35*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^11 + 56*(192*a^2*b + 35*b^3)* \\
& \cosh(d*x + c)^10 + 56*(143*(192*a^2*b + 35*b^3)*\cosh(d*x + c)^6 + 143*(192* \\
& a^2*b + 35*b^3)*\cosh(d*x + c)^4 + 192*a^2*b + 35*b^3 + 33*(192*a^2*b + 35*b \\
& ^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^10 + 16*(715*(192*a^2*b + 35*b^3)*\cosh(d \\
& *x + c)^7 + 1001*(192*a^2*b + 35*b^3)*\cosh(d*x + c)^5 + 385*(192*a^2*b + 35 \\
& *b^3)*\cosh(d*x + c)^3 + 35*(192*a^2*b + 35*b^3)*\cosh(d*x + c))*\sinh(d*x + c \\
&)^9 + 70*(192*a^2*b + 35*b^3)*\cosh(d*x + c)^8 + 2*(6435*(192*a^2*b + 35*b^3) \\
&)*\cosh(d*x + c)^8 + 12012*(192*a^2*b + 35*b^3)*\cosh(d*x + c)^6 + 6930*(192* \\
& a^2*b + 35*b^3)*\cosh(d*x + c)^4 + 6720*a^2*b + 1225*b^3 + 1260*(192*a^2*b + \\
& 35*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 16*(715*(192*a^2*b + 35*b^3)*\cosh \\
& (d*x + c)^9 + 1716*(192*a^2*b + 35*b^3)*\cosh(d*x + c)^7 + 1386*(192*a^2*b \\
& + 35*b^3)*\cosh(d*x + c)^5 + 420*(192*a^2*b + 35*b^3)*\cosh(d*x + c)^3 + 35* \\
& (192*a^2*b + 35*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 56*(192*a^2*b + 35*b^
\end{aligned}$$

$$\begin{aligned}
& *x + c)^{12} + 8*(15*a^3*\cosh(d*x + c)^2 + a^3)*\sinh(d*x + c)^{14} + 112*(5*a^3 \\
& *\cosh(d*x + c)^3 + a^3*\cosh(d*x + c))*\sinh(d*x + c)^{13} + 56*a^3*\cosh(d*x + \\
& c)^{10} + 28*(65*a^3*\cosh(d*x + c)^4 + 26*a^3*\cosh(d*x + c)^2 + a^3)*\sinh(d*x \\
& + c)^{12} + 112*(39*a^3*\cosh(d*x + c)^5 + 26*a^3*\cosh(d*x + c)^3 + 3*a^3*\cos \\
& h(d*x + c))*\sinh(d*x + c)^{11} + 70*a^3*\cosh(d*x + c)^8 + 56*(143*a^3*\cosh(d* \\
& x + c)^6 + 143*a^3*\cosh(d*x + c)^4 + 33*a^3*\cosh(d*x + c)^2 + a^3)*\sinh(d*x \\
& + c)^{10} + 16*(715*a^3*\cosh(d*x + c)^7 + 1001*a^3*\cosh(d*x + c)^5 + 385*a^3 \\
& *\cosh(d*x + c)^3 + 35*a^3*\cosh(d*x + c))*\sinh(d*x + c)^9 + 56*a^3*\cosh(d*x \\
& + c)^6 + 2*(6435*a^3*\cosh(d*x + c)^8 + 12012*a^3*\cosh(d*x + c)^6 + 6930*a^3 \\
& *\cosh(d*x + c)^4 + 1260*a^3*\cosh(d*x + c)^2 + 35*a^3)*\sinh(d*x + c)^8 + 16* \\
& (715*a^3*\cosh(d*x + c)^9 + 1716*a^3*\cosh(d*x + c)^7 + 1386*a^3*\cosh(d*x + c \\
&)^5 + 420*a^3*\cosh(d*x + c)^3 + 35*a^3*\cosh(d*x + c))*\sinh(d*x + c)^7 + 28* \\
& a^3*\cosh(d*x + c)^4 + 56*(143*a^3*\cosh(d*x + c)^{10} + 429*a^3*\cosh(d*x + c)^8 \\
& + 462*a^3*\cosh(d*x + c)^6 + 210*a^3*\cosh(d*x + c)^4 + 35*a^3*\cosh(d*x + c \\
&)^2 + a^3)*\sinh(d*x + c)^6 + 112*(39*a^3*\cosh(d*x + c)^{11} + 143*a^3*\cosh(d* \\
& x + c)^9 + 198*a^3*\cosh(d*x + c)^7 + 126*a^3*\cosh(d*x + c)^5 + 35*a^3*\cosh(\\
& d*x + c)^3 + 3*a^3*\cosh(d*x + c))*\sinh(d*x + c)^5 + 8*a^3*\cosh(d*x + c)^2 + \\
& 28*(65*a^3*\cosh(d*x + c)^{12} + 286*a^3*\cosh(d*x + c)^{10} + 495*a^3*\cosh(d*x \\
& + c)^8 + 420*a^3*\cosh(d*x + c)^6 + 175*a^3*\cosh(d*x + c)^4 + 30*a^3*\cosh(d* \\
& x + c)^2 + a^3)*\sinh(d*x + c)^4 + 112*(5*a^3*\cosh(d*x + c)^{13} + 26*a^3*\cosh \\
& (d*x + c)^{11} + 55*a^3*\cosh(d*x + c)^9 + 60*a^3*\cosh(d*x + c)^7 + 35*a^3*\cos \\
& h(d*x + c)^5 + 10*a^3*\cosh(d*x + c)^3 + a^3*\cosh(d*x + c))*\sinh(d*x + c)^3 \\
& + a^3 + 8*(15*a^3*\cosh(d*x + c)^{14} + 91*a^3*\cosh(d*x + c)^{12} + 231*a^3*\cosh \\
& (d*x + c)^{10} + 315*a^3*\cosh(d*x + c)^8 + 245*a^3*\cosh(d*x + c)^6 + 105*a^3* \\
& \cosh(d*x + c)^4 + 21*a^3*\cosh(d*x + c)^2 + a^3)*\sinh(d*x + c)^2 + 16*(a^3*\c \\
& osh(d*x + c)^{15} + 7*a^3*\cosh(d*x + c)^{13} + 21*a^3*\cosh(d*x + c)^{11} + 35*a^3 \\
& *\cosh(d*x + c)^9 + 35*a^3*\cosh(d*x + c)^7 + 21*a^3*\cosh(d*x + c)^5 + 7*a^3* \\
& \cosh(d*x + c)^3 + a^3*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sin \\
& h(d*x + c) - 1) + (675*(64*a^2*b + 128*a*b^2 + 31*b^3)*\cosh(d*x + c)^{14} + 6 \\
& 5*(2880*a^2*b + 4992*a*b^2 + 91*b^3)*\cosh(d*x + c)^{12} + 11*(25920*a^2*b + 6 \\
& 2592*a*b^2 + 8995*b^3)*\cosh(d*x + c)^{10} + 9*(14400*a^2*b + 103296*a*b^2 - 5 \\
& 425*b^3)*\cosh(d*x + c)^8 - 7*(14400*a^2*b - 103296*a*b^2 - 5425*b^3)*\cosh(d \\
& *x + c)^6 - 5*(25920*a^2*b - 62592*a*b^2 + 8995*b^3)*\cosh(d*x + c)^4 - 2880 \\
& *a^2*b + 5760*a*b^2 - 1395*b^3 - 15*(2880*a^2*b - 4992*a*b^2 + 91*b^3)*\cosh \\
& (d*x + c)^2)*\sinh(d*x + c))/(d*\cosh(d*x + c)^{16} + 16*d*\cosh(d*x + c)*\sinh(d \\
& *x + c)^{15} + d*\sinh(d*x + c)^{16} + 8*d*\cosh(d*x + c)^{14} + 8*(15*d*\cosh(d*x + \\
& c)^2 + d)*\sinh(d*x + c)^{14} + 112*(5*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*s \\
& inh(d*x + c)^{13} + 28*d*\cosh(d*x + c)^{12} + 28*(65*d*\cosh(d*x + c)^4 + 26*d*c \\
& osh(d*x + c)^2 + d)*\sinh(d*x + c)^{12} + 112*(39*d*\cosh(d*x + c)^5 + 26*d*\cos \\
& h(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^{11} + 56*d*\cosh(d*x + c)^{10} \\
& + 56*(143*d*\cosh(d*x + c)^6 + 143*d*\cosh(d*x + c)^4 + 33*d*\cosh(d*x + c)^2 \\
& + d)*\sinh(d*x + c)^{10} + 16*(715*d*\cosh(d*x + c)^7 + 1001*d*\cosh(d*x + c)^5 \\
& + 385*d*\cosh(d*x + c)^3 + 35*d*\cosh(d*x + c))*\sinh(d*x + c)^9 + 70*d*\cosh(d \\
& *x + c)^8 + 2*(6435*d*\cosh(d*x + c)^8 + 12012*d*\cosh(d*x + c)^6 + 6930*d*\co \\
& sh(d*x + c)^4 + 1260*d*\cosh(d*x + c)^2 + 35*d)*\sinh(d*x + c)^8 + 16*(715*d* \\
& \cosh(d*x + c)^9 + 1716*d*\cosh(d*x + c)^7 + 1386*d*\cosh(d*x + c)^5 + 420*d*c \\
& osh(d*x + c)^3 + 35*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 56*d*\cosh(d*x + c)^6 \\
& + 56*(143*d*\cosh(d*x + c)^{10} + 429*d*\cosh(d*x + c)^8 + 462*d*\cosh(d*x + c) \\
& ^6 + 210*d*\cosh(d*x + c)^4 + 35*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^6 + 11 \\
& 2*(39*d*\cosh(d*x + c)^{11} + 143*d*\cosh(d*x + c)^9 + 198*d*\cosh(d*x + c)^7 + \\
& 126*d*\cosh(d*x + c)^5 + 35*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x \\
& + c)^5 + 28*d*\cosh(d*x + c)^4 + 28*(65*d*\cosh(d*x + c)^{12} + 286*d*\cosh(d*x \\
& + c)^{10} + 495*d*\cosh(d*x + c)^8 + 420*d*\cosh(d*x + c)^6 + 175*d*\cosh(d*x + \\
& c)^4 + 30*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^4 + 112*(5*d*\cosh(d*x + c)^{1 \\
& 3} + 26*d*\cosh(d*x + c)^{11} + 55*d*\cosh(d*x + c)^9 + 60*d*\cosh(d*x + c)^7 + 3 \\
& 5*d*\cosh(d*x + c)^5 + 10*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c) \\
& ^3 + 8*d*\cosh(d*x + c)^2 + 8*(15*d*\cosh(d*x + c)^{14} + 91*d*\cosh(d*x + c)^{12} \\
& + 231*d*\cosh(d*x + c)^{10} + 315*d*\cosh(d*x + c)^8 + 245*d*\cosh(d*x + c)^6 + \\
& 105*d*\cosh(d*x + c)^4 + 21*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^2 + 16*(d*
\end{aligned}$$

$$\cosh(dx + c)^{15} + 7d \cosh(dx + c)^{13} + 21d^2 \cosh(dx + c)^{11} + 35d^3 \cosh(dx + c)^9 + 35d^4 \cosh(dx + c)^7 + 21d^5 \cosh(dx + c)^5 + 7d^6 \cosh(dx + c)^3 + d^7 \cosh(dx + c) \sinh(dx + c) + d^8$$

giac [B] time = 0.60, size = 422, normalized size = 1.93

$$960 a^3 \log(e^{(dx+c)} + 1) - 960 a^3 \log(|e^{(dx+c)} - 1|) - 15 (192 a^2 b e^c + 35 b^3 e^c) \arctan(e^{(dx+c)}) e^{(-c)} + \frac{2880 a^2 b e^{(15 dx+15 c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)*(a+b*tanh(dx+c)^3)^3,x, algorithm="giac")

[Out] -1/960*(960*a^3*log(e^(dx + c) + 1) - 960*a^3*log(abs(e^(dx + c) - 1)) - 15*(192*a^2*b*e^c + 35*b^3*e^c)*arctan(e^(dx + c))*e^(-c) + (2880*a^2*b*e^(15*d*x + 15*c) + 5760*a*b^2*e^(15*d*x + 15*c) + 1395*b^3*e^(15*d*x + 15*c) + 14400*a^2*b*e^(13*d*x + 13*c) + 24960*a*b^2*e^(13*d*x + 13*c) + 455*b^3*e^(13*d*x + 13*c) + 25920*a^2*b*e^(11*d*x + 11*c) + 62592*a*b^2*e^(11*d*x + 11*c) + 8995*b^3*e^(11*d*x + 11*c) + 14400*a^2*b*e^(9*d*x + 9*c) + 103296*a*b^2*e^(9*d*x + 9*c) - 5425*b^3*e^(9*d*x + 9*c) - 14400*a^2*b*e^(7*d*x + 7*c) + 103296*a*b^2*e^(7*d*x + 7*c) + 5425*b^3*e^(7*d*x + 7*c) - 25920*a^2*b*e^(5*d*x + 5*c) + 62592*a*b^2*e^(5*d*x + 5*c) - 8995*b^3*e^(5*d*x + 5*c) - 14400*a^2*b*e^(3*d*x + 3*c) + 24960*a*b^2*e^(3*d*x + 3*c) - 455*b^3*e^(3*d*x + 3*c) - 2880*a^2*b*e^(dx + c) + 5760*a*b^2*e^(dx + c) - 1395*b^3*e^(dx + c))/(e^(2*d*x + 2*c) + 1)^8/d

maple [A] time = 0.52, size = 339, normalized size = 1.55

$$-\frac{2a^3 \operatorname{arctanh}(e^{dx+c})}{d} - \frac{3a^2 b \sinh(dx+c)}{d \cosh(dx+c)^2} + \frac{3a^2 b \operatorname{sech}(dx+c) \tanh(dx+c)}{2d} + \frac{3a^2 b \operatorname{arctan}(e^{dx+c})}{d} - \frac{3a b^2 (\sinh^4(dx+c))}{d \cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(dx+c)*(a+b*tanh(dx+c)^3)^3,x)

[Out] -2/d*a^3*arctanh(exp(dx+c))-3/d*a^2*b*sinh(dx+c)/cosh(dx+c)^2+3/2/d*a^2*b*sech(dx+c)*tanh(dx+c)+3/d*a^2*b*arctan(exp(dx+c))-3/d*a*b^2*sinh(dx+c)^4/cosh(dx+c)^5-4/d*a*b^2*sinh(dx+c)^2/cosh(dx+c)^5-8/5/d*a*b^2/cosh(dx+c)^5-1/d*b^3*sinh(dx+c)^7/cosh(dx+c)^8-7/3/d*b^3*sinh(dx+c)^5/cosh(dx+c)^8-7/3/d*b^3*sinh(dx+c)^3/cosh(dx+c)^8-1/d*b^3*sinh(dx+c)/cosh(dx+c)^8+1/8/d*b^3*tanh(dx+c)*sech(dx+c)^7+7/48/d*b^3*tanh(dx+c)*sech(dx+c)^5+35/192/d*b^3*tanh(dx+c)*sech(dx+c)^3+35/128/d*b^3*sech(dx+c)*tanh(dx+c)+35/64/d*b^3*arctan(exp(dx+c))

maxima [B] time = 0.43, size = 654, normalized size = 2.99

$$-\frac{1}{192} b^3 \left(\frac{105 \operatorname{arctan}(e^{(-dx-c)})}{d} + \frac{279 e^{(-dx-c)} + 91 e^{(-3dx-3c)} + 1799 e^{(-5dx-5c)} - 1085 e^{(-7dx-7c)} + 1085 e^{(-9dx-9c)}}{d(8 e^{(-2dx-2c)} + 28 e^{(-4dx-4c)} + 56 e^{(-6dx-6c)} + 70 e^{(-8dx-8c)} + 56 e^{(-10dx-10c)})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)*(a+b*tanh(dx+c)^3)^3,x, algorithm="maxima")

[Out] -1/192*b^3*(105*arctan(e^(-dx - c))/d + (279*e^(-dx - c) + 91*e^(-3*d*x - 3*c) + 1799*e^(-5*d*x - 5*c) - 1085*e^(-7*d*x - 7*c) + 1085*e^(-9*d*x - 9*c) - 1799*e^(-11*d*x - 11*c) - 91*e^(-13*d*x - 13*c) - 279*e^(-15*d*x - 15*c))/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1))) - 3*a^2*b*(arctan(e^(-dx - c))/d

$$+ (e^{-dx - c} - e^{-3dx - 3c}) / (d(2e^{-2dx - 2c} + e^{-4dx - 4c} + 1)) - 2/5ab^2(15e^{-dx - c} / (d(5e^{-2dx - 2c} + 10e^{-4dx - 4c} + 10e^{-6dx - 6c} + 5e^{-8dx - 8c} + e^{-10dx - 10c} + 1)) + 20e^{-3dx - 3c} / (d(5e^{-2dx - 2c} + 10e^{-4dx - 4c} + 10e^{-6dx - 6c} + 5e^{-8dx - 8c} + e^{-10dx - 10c} + 1)) + 58e^{-5dx - 5c} / (d(5e^{-2dx - 2c} + 10e^{-4dx - 4c} + 10e^{-6dx - 6c} + 5e^{-8dx - 8c} + e^{-10dx - 10c} + 1)) + 20e^{-7dx - 7c} / (d(5e^{-2dx - 2c} + 10e^{-4dx - 4c} + 10e^{-6dx - 6c} + 5e^{-8dx - 8c} + e^{-10dx - 10c} + 1)) + 15e^{-9dx - 9c} / (d(5e^{-2dx - 2c} + 10e^{-4dx - 4c} + 10e^{-6dx - 6c} + 5e^{-8dx - 8c} + e^{-10dx - 10c} + 1))) + a^3 \log(\tanh(1/2dx + 1/2c)) / d$$

mupad [B] time = 6.40, size = 671, normalized size = 3.06

$$\frac{a^3 \ln(e^{c+dx} - 1)}{d} - \frac{a^3 \ln(e^{c+dx} + 1)}{d} - \frac{e^{c+dx} (4445b^3 + 4224ab^2)}{120d (3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1)} + \frac{e^{c+dx} (192)}{20d (4e^{2c+2dx} + 6e^{4c+4dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tanh(c + d*x))^3/sinh(c + d*x),x)

[Out] (a^3*log(exp(c + d*x) - 1))/d - (a^3*log(exp(c + d*x) + 1))/d - (exp(c + d*x)*(4224*a*b^2 + 4445*b^3))/(120*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (b*log(exp(c + d*x) - 1i)*(192*a^2 + 35*b^2)*1i)/(128*d) + (b*log(exp(c + d*x) + 1i)*(192*a^2 + 35*b^2)*1i)/(128*d) + (exp(c + d*x)*(768*a*b^2 + 1925*b^3))/(20*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (532*b^3*exp(c + d*x))/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (3*exp(c + d*x)*(128*a*b^2 + 64*a^2*b + 31*b^3))/(64*d*(exp(2*c + 2*d*x) + 1)) - (2*exp(c + d*x)*(144*a*b^2 + 1225*b^3))/(15*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (112*b^3*exp(c + d*x))/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) + (exp(c + d*x)*(1536*a*b^2 + 576*a^2*b + 931*b^3))/(96*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (32*b^3*exp(c + d*x))/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^3(c + dx))^3 \operatorname{csch}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**3)**3,x)

[Out] Integral((a + b*tanh(c + d*x)**3)**3*csch(c + d*x), x)

3.70 $\int \operatorname{csch}^2(c + dx) \left(a + b \tanh^3(c + dx) \right)^3 dx$

Optimal. Leaf size=71

$$-\frac{a^3 \operatorname{coth}(c + dx)}{d} + \frac{3a^2 b \tanh^2(c + dx)}{2d} + \frac{3ab^2 \tanh^5(c + dx)}{5d} + \frac{b^3 \tanh^8(c + dx)}{8d}$$

[Out] $-a^3 \operatorname{coth}(d*x+c)/d + 3/2*a^2*b*\tanh(d*x+c)^2/d + 3/5*a*b^2*\tanh(d*x+c)^5/d + 1/8*b^3*\tanh(d*x+c)^8/d$

Rubi [A] time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3663, 270}

$$\frac{3a^2 b \tanh^2(c + dx)}{2d} - \frac{a^3 \operatorname{coth}(c + dx)}{d} + \frac{3ab^2 \tanh^5(c + dx)}{5d} + \frac{b^3 \tanh^8(c + dx)}{8d}$$

Antiderivative was successfully verified.

[In] `Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^3, x]`

[Out] $-((a^3 \operatorname{Coth}[c + d*x])/d) + (3*a^2*b*\operatorname{Tanh}[c + d*x]^2)/(2*d) + (3*a*b^2*\operatorname{Tanh}[c + d*x]^5)/(5*d) + (b^3*\operatorname{Tanh}[c + d*x]^8)/(8*d)$

Rule 270

`Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]`

Rule 3663

`Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_.)]^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]`

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^2(c + dx) \left(a + b \tanh^3(c + dx) \right)^3 dx &= \frac{\operatorname{Subst} \left(\int \frac{(a+bx^3)^3}{x^2} dx, x, \tanh(c + dx) \right)}{d} \\ &= \frac{\operatorname{Subst} \left(\int \left(\frac{a^3}{x^2} + 3a^2bx + 3ab^2x^4 + b^3x^7 \right) dx, x, \tanh(c + dx) \right)}{d} \\ &= -\frac{a^3 \operatorname{coth}(c + dx)}{d} + \frac{3a^2 b \tanh^2(c + dx)}{2d} + \frac{3ab^2 \tanh^5(c + dx)}{5d} + \frac{b^3 \tanh^8(c + dx)}{8d} \end{aligned}$$

Mathematica [A] time = 0.77, size = 113, normalized size = 1.59

$$\frac{b \left(-4 \operatorname{sech}^2(c + dx) (15a^2 + 12ab \tanh(c + dx) + 5b^2) + 24ab \tanh(c + dx) + 6b \operatorname{sech}^4(c + dx) (4a \tanh(c + dx) + b^2) \right)}{40d}$$

Antiderivative was successfully verified.

[In] `Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^3, x]`

[Out] $(-40a^3 \operatorname{Coth}[c + dx] + b(-20b^2 \operatorname{Sech}[c + dx]^6 + 5b^2 \operatorname{Sech}[c + dx]^8 + 24ab \operatorname{Tanh}[c + dx] + 6b \operatorname{Sech}[c + dx]^4(5b + 4a \operatorname{Tanh}[c + dx])) - 4 \operatorname{Sech}[c + dx]^2(15a^2 + 5b^2 + 12ab \operatorname{Tanh}[c + dx]))/(40d)$

fricas [B] time = 0.40, size = 1192, normalized size = 16.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")`

[Out] $-2/5*((10a^3 + 15a^2b + 12ab^2 + 5b^3) \cosh(dx + c)^8 + 8(15a^2b + 18ab^2 + 5b^3) \cosh(dx + c) \sinh(dx + c)^7 + (10a^3 + 15a^2b + 12ab^2 + 5b^3) \sinh(dx + c)^8 + 2(40a^3 + 30a^2b + 12ab^2 - 5b^3) \cosh(dx + c)^6 + 2(40a^3 + 30a^2b + 12ab^2 - 5b^3 + 14(10a^3 + 15a^2b + 12ab^2 + 5b^3) \cosh(dx + c)^2) \sinh(dx + c)^6 + 4(14(15a^2b + 18ab^2 + 5b^3) \cosh(dx + c)^3 + 27(5a^2b + 2ab^2) \cosh(dx + c)) \sinh(dx + c)^5 + 20(14a^3 + 3a^2b + 2b^3) \cosh(dx + c)^4 + 10(7(10a^3 + 15a^2b + 12ab^2 + 5b^3) \cosh(dx + c)^4 + 28a^3 + 6a^2b + 4b^3 + 3(40a^3 + 30a^2b + 12ab^2 - 5b^3) \cosh(dx + c)^2) \sinh(dx + c)^4 + 8(7(15a^2b + 18ab^2 + 5b^3) \cosh(dx + c)^5 + 45(5a^2b + 2ab^2) \cosh(dx + c)^3 + 15(7a^2b + 2ab^2 + b^3) \cosh(dx + c)) \sinh(dx + c)^3 + 350a^3 - 75a^2b - 12ab^2 + 35b^3 + 2(280a^3 - 30a^2b - 12ab^2 - 35b^3) \cosh(dx + c)^2 + 2(14(10a^3 + 15a^2b + 12ab^2 + 5b^3) \cosh(dx + c)^6 + 15(40a^3 + 30a^2b + 12ab^2 - 5b^3) \cosh(dx + c)^4 + 280a^3 - 30a^2b - 12ab^2 - 35b^3 + 60(14a^3 + 3a^2b + 2b^3) \cosh(dx + c)^2) \sinh(dx + c)^2 + 4(2(15a^2b + 18ab^2 + 5b^3) \cosh(dx + c)^7 + 27(5a^2b + 2ab^2) \cosh(dx + c)^5 + 30(7a^2b + 2ab^2 + b^3) \cosh(dx + c)^3 + 21(5a^2b + 2ab^2) \cosh(dx + c)) \sinh(dx + c)) / (d \cosh(dx + c)^{10} + 10d \cosh(dx + c) \sinh(dx + c)^9 + d \sinh(dx + c)^{10} + 6d \cosh(dx + c)^8 + 3(15d \cosh(dx + c)^2 + 2d) \sinh(dx + c)^8 + 8(15d \cosh(dx + c)^3 + 8d \cosh(dx + c)) \sinh(dx + c)^7 + 13d \cosh(dx + c)^6 + (210d \cosh(dx + c)^4 + 168d \cosh(dx + c)^2 + 13d) \sinh(dx + c)^6 + 2(126d \cosh(dx + c)^5 + 224d \cosh(dx + c)^3 + 81d \cosh(dx + c)) \sinh(dx + c)^5 + 8d \cosh(dx + c)^4 + (210d \cosh(dx + c)^6 + 420d \cosh(dx + c)^4 + 195d \cosh(dx + c)^2 + 8d) \sinh(dx + c)^4 + 4(30d \cosh(dx + c)^7 + 112d \cosh(dx + c)^5 + 135d \cosh(dx + c)^3 + 48d \cosh(dx + c)) \sinh(dx + c)^3 - 14d \cosh(dx + c)^2 + (45d \cosh(dx + c)^8 + 168d \cosh(dx + c)^6 + 195d \cosh(dx + c)^4 + 48d \cosh(dx + c)^2 - 14d) \sinh(dx + c)^2 + 2(5d \cosh(dx + c)^9 + 32d \cosh(dx + c)^7 + 81d \cosh(dx + c)^5 + 96d \cosh(dx + c)^3 + 42d \cosh(dx + c)) \sinh(dx + c) - 14d)$

giac [B] time = 0.77, size = 311, normalized size = 4.38

$$2 \left(\frac{5a^3}{e^{(2dx+2c)} - 1} + \frac{15a^2be^{(14dx+14c)} + 15ab^2e^{(14dx+14c)} + 5b^3e^{(14dx+14c)} + 90a^2be^{(12dx+12c)} + 45ab^2e^{(12dx+12c)} + 225a^2be^{(10dx+10c)} + 75ab^2e^{(10dx+10c)}}{e^{(2dx+2c)} - 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")`

[Out] $-2/5*(5a^3/(e^{(2dx + 2c)} - 1) + (15a^2b e^{(14dx + 14c)} + 15a^2b^2 e^{(14dx + 14c)} + 5b^3 e^{(14dx + 14c)} + 90a^2b e^{(12dx + 12c)} + 45ab^2 e^{(12dx + 12c)} + 225a^2b e^{(10dx + 10c)} + 75ab^2 e^{(10dx + 10c)} + 35b^3 e^{(10dx + 10c)} + 300a^2b e^{(8dx + 8c)} + 105ab^2 e^{(8dx + 8c)} + 225a^2b e^{(6dx + 6c)} + 93ab^2 e^{(6dx + 6c)} + 35b^3 e^{(6dx + 6c)} + 90a^2b e^{(4dx + 4c)} + 39ab^2 e^{(4dx + 4c)} + 15a^2b e^{(2dx + 2c)} + 9ab^2 e^{(2dx + 2c)} + 5b^3 e^{(2dx + 2c)} + 3ab^2) / (e^{(2dx + 2c)} + 1)^8) / d$

maple [B] time = 0.50, size = 171, normalized size = 2.41

$$\frac{-a^3 \coth(dx+c) - \frac{3a^2b}{2 \cosh(dx+c)^2} + 3ab^2 \left(-\frac{\sinh^3(dx+c)}{2 \cosh(dx+c)^5} - \frac{3 \sinh(dx+c)}{8 \cosh(dx+c)^5} + \frac{3 \left(\frac{8}{15} + \frac{\operatorname{sech}(dx+c)^4}{5} + \frac{4 \operatorname{sech}(dx+c)^2}{15} \right) \tanh(dx+c)}{8} \right) + b^3 \left(- \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x)

[Out] 1/d*(-a^3*coth(d*x+c)-3/2*a^2*b/cosh(d*x+c)^2+3*a*b^2*(-1/2*sinh(d*x+c)^3/cosh(d*x+c)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))+b^3*(-1/2*sinh(d*x+c)^6/cosh(d*x+c)^8-3/4*sinh(d*x+c)^4/cosh(d*x+c)^8-1/2*sinh(d*x+c)^2/cosh(d*x+c)^8-1/8/cosh(d*x+c)^8))

maxima [B] time = 0.34, size = 679, normalized size = 9.56

$$-2b^3 \frac{e^{(-2dx-2c)}}{d(8e^{(-2dx-2c)} + 28e^{(-4dx-4c)} + 56e^{(-6dx-6c)} + 70e^{(-8dx-8c)} + 56e^{(-10dx-10c)} + 28e^{(-12dx-12c)} + 8e^{(-14dx-14c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")

[Out] -2*b^3*(e^(-2*d*x - 2*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) + 7*e^(-6*d*x - 6*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) + 7*e^(-10*d*x - 10*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) + 7*e^(-14*d*x - 14*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1))) + 6/5*a*b^2*(10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^(-8*d*x - 8*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 2*a^3/(d*(e^(-2*d*x - 2*c) - 1)) - 6*a^2*b/(d*(e^(d*x + c) + e^(-d*x - c))^2)

mupad [B] time = 1.38, size = 1515, normalized size = 21.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tanh(c + d*x)^3)^3/sinh(c + d*x)^2,x)

[Out] ((3*a*b^2 - 15*a^2*b + 7*b^3)/(28*d) - (exp(2*c + 2*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(4*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((9*a*b^2 + 15*a^2*b - 35*b^3)/(140*d) + (exp(6*c + 6*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(4*d) + (3*exp(2*c + 2*d*x)*(9*a^2*b - 3*a*b^2 + 7*b^3))/(28*d) - (3*exp(4*c + 4*d*x)*(3*a*b^2 - 15*a^2*b + 7*b^3))/(28*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((exp(10*c + 10*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(4*d) - (3*a*b^2 + 9*a^2*b + 7*b^3)/(28*d) + (5*exp(6*c + 6*d*x)*(9*a^2*b - 3*a*b^2 + 7*b^3))/(14*d) - (5*exp(8*c + 8*d*x)*(3*a*b^2 - 15*a^2*b + 7*b^3))/(28*d) + (exp(2*c + 2*d*x)*(9*a*b^2 - 1

$$\begin{aligned} & (5a^2b + 35b^3)/(28d) + (\exp(4c + 4dx) \cdot (9ab^2 + 15a^2b - 35b^3)) / (14d) / (6\exp(2c + 2dx) + 15\exp(4c + 4dx) + 20\exp(6c + 6dx) + \\ & 15\exp(8c + 8dx) + 6\exp(10c + 10dx) + \exp(12c + 12dx) + 1) - ((9a^2b - 3ab^2 + 7b^3)/(28d) + (\exp(4c + 4dx) \cdot (3ab^2 + 3a^2b + b^3)) / (4d) - \\ & (\exp(2c + 2dx) \cdot (3ab^2 - 15a^2b + 7b^3)) / (14d)) / (3\exp(2c + 2dx) + 3\exp(4c + 4dx) + \exp(6c + 6dx) + 1) - ((9ab^2 - 15a^2b + 35b^3) / (140d) + (\exp(8c + 8dx) \cdot (3ab^2 + 3a^2b + b^3)) / (4d) + (3\exp(4c + 4dx) \cdot (9a^2b - 3ab^2 + 7b^3)) / (14d) - (\exp(6c + 6dx) \cdot (3ab^2 - 15a^2b + 7b^3)) / (7d) + (\exp(2c + 2dx) \cdot (9ab^2 + 15a^2b - 35b^3)) / (35d)) / (5\exp(2c + 2dx) + 10\exp(4c + 4dx) + 10\exp(6c + 6dx) + 5\exp(8c + 8dx) + \exp(10c + 10dx) + 1) - ((\exp(12c + 12dx) \cdot (3ab^2 + 3a^2b + b^3)) / (4d) - (3ab^2 + 15a^2b - 7b^3) / (28d) - (3\exp(2c + 2dx) \cdot (3ab^2 + 9a^2b + 7b^3)) / (14d) + (15\exp(8c + 8dx) \cdot (9a^2b - 3ab^2 + 7b^3)) / (28d) - (3\exp(10c + 10dx) \cdot (3ab^2 - 15a^2b + 7b^3)) / (14d) + (3\exp(4c + 4dx) \cdot (9ab^2 - 15a^2b + 35b^3)) / (28d) + (\exp(6c + 6dx) \cdot (9ab^2 + 15a^2b - 35b^3)) / (7d)) / (7\exp(2c + 2dx) + 21\exp(4c + 4dx) + 35\exp(6c + 6dx) + 35\exp(8c + 8dx) + 21\exp(10c + 10dx) + 7\exp(12c + 12dx) + \exp(14c + 14dx) + 1) + ((3a^2b - 3ab^2 + b^3) / (4d) - (\exp(14c + 14dx) \cdot (3ab^2 + 3a^2b + b^3)) / (4d) + (3\exp(4c + 4dx) \cdot (3ab^2 + 9a^2b + 7b^3)) / (4d) + (\exp(2c + 2dx) \cdot (3ab^2 + 15a^2b - 7b^3)) / (4d) - (3\exp(10c + 10dx) \cdot (9a^2b - 3ab^2 + 7b^3)) / (4d) + (\exp(12c + 12dx) \cdot (3ab^2 - 15a^2b + 7b^3)) / (4d) - (\exp(6c + 6dx) \cdot (9ab^2 - 15a^2b + 35b^3)) / (4d) - (\exp(8c + 8dx) \cdot (9ab^2 + 15a^2b - 35b^3)) / (4d)) / (8\exp(2c + 2dx) + 28\exp(4c + 4dx) + 56\exp(6c + 6dx) + 70\exp(8c + 8dx) + 56\exp(10c + 10dx) + 28\exp(12c + 12dx) + 8\exp(14c + 14dx) + \exp(16c + 16dx) + 1) - (2a^3) / (d \cdot (\exp(2c + 2dx) - 1)) - (3ab^2 + 3a^2b + b^3) / (4d \cdot (\exp(2c + 2dx) + 1)) \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^3(c + dx))^3 \operatorname{csch}^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**3)**3,x)

[Out] Integral((a + b*tanh(c + d*x)**3)**3*csch(c + d*x)**2, x)

3.71 $\int \operatorname{csch}^3(c + dx) \left(a + b \tanh^3(c + dx) \right)^3 dx$

Optimal. Leaf size=232

$$\frac{a^3 \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a^3 \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + \frac{3a^2 b \tan^{-1}(\sinh(c + dx))}{2d} + \frac{3a^2 b \tanh(c + dx) \operatorname{sech}(c + dx)}{2d}$$

[Out] $\frac{3}{2} a^2 b \arctan(\sinh(dx+c))/d + \frac{5}{128} b^3 \arctan(\sinh(dx+c))/d + \frac{1}{2} a^3 \operatorname{arc} \tanh(\cosh(dx+c))/d - \frac{1}{2} a^3 \coth(dx+c) \operatorname{csch}(dx+c)/d - a b^2 \operatorname{sech}(dx+c)^3/d + \frac{3}{5} a b^2 \operatorname{sech}(dx+c)^5/d + \frac{3}{2} a^2 b \operatorname{sech}(dx+c) \tanh(dx+c)/d + \frac{5}{128} b^3 \operatorname{sech}(dx+c) \tanh(dx+c)/d - \frac{5}{64} b^3 \operatorname{sech}(dx+c)^3 \tanh(dx+c)/d - \frac{5}{48} b^3 \operatorname{sech}(dx+c)^3 \tanh(dx+c)^3/d - \frac{1}{8} b^3 \operatorname{sech}(dx+c)^3 \tanh(dx+c)^5/d$

Rubi [A] time = 0.32, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3666, 3768, 3770, 2606, 14, 2611}

$$\frac{3a^2 b \tan^{-1}(\sinh(c + dx))}{2d} + \frac{3a^2 b \tanh(c + dx) \operatorname{sech}(c + dx)}{2d} + \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a^3 \coth(c + dx) \operatorname{csch}(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^3,x]

[Out] $\frac{(3a^2 b \operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])}{(2*d)} + \frac{(5b^3 \operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])}{(128*d)} + \frac{(a^3 \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]])}{(2*d)} - \frac{(a^3 \operatorname{Coth}[c + d*x] \operatorname{Csch}[c + d*x])}{(2*d)} - \frac{(a b^2 \operatorname{Sech}[c + d*x]^3)}{d} + \frac{(3a b^2 \operatorname{Sech}[c + d*x]^5)}{(5*d)} + \frac{(3a^2 b \operatorname{Sech}[c + d*x] \operatorname{Tanh}[c + d*x])}{(2*d)} + \frac{(5b^3 \operatorname{Sech}[c + d*x] \operatorname{Tanh}[c + d*x])}{(128*d)} - \frac{(5b^3 \operatorname{Sech}[c + d*x]^3 \operatorname{Tanh}[c + d*x])}{(64*d)} - \frac{(5b^3 \operatorname{Sech}[c + d*x]^3 \operatorname{Tanh}[c + d*x]^3)}{(48*d)} - \frac{(b^3 \operatorname{Sech}[c + d*x]^3 \operatorname{Tanh}[c + d*x]^5)}{(8*d)}$

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 2606

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m-1)*(-1+x^2)^((n-1)/2), x], x, Sec[e+f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n-1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n+1])

Rule 2611

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(b*(a*Sec[e+f*x])^m*(b*Tan[e+f*x])^(n-1))/(f*(m+n-1)), x] - Dist[(b^2*(n-1))/(m+n-1), Int[(a*Sec[e+f*x])^m*(b*Tan[e+f*x])^(n-2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && NeQ[m+n-1, 0] && IntegerQ[2*m, 2*n]

Rule 3666

Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := Int[ExpandTrig[(d*sin[e+f*x])^m*(a + b*(c*tan[e+f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p, 0]

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^3(c + dx) (a + b \tanh^3(c + dx))^3 dx &= -\left(i \int (ia^3 \operatorname{csch}^3(c + dx) + 3ia^2 b \operatorname{sech}^3(c + dx) + 3iab^2 \operatorname{sech}^3(c + dx) + 3b^3 \operatorname{sech}^3(c + dx)) dx\right) \\ &= a^3 \int \operatorname{csch}^3(c + dx) dx + (3a^2 b) \int \operatorname{sech}^3(c + dx) dx + (3ab^2) \int \operatorname{sech}^3(c + dx) dx + (3b^3) \int \operatorname{sech}^3(c + dx) dx \\ &= -\frac{a^3 \operatorname{coth}(c + dx) \operatorname{csch}(c + dx)}{2d} + \frac{3a^2 b \operatorname{sech}(c + dx) \tanh(c + dx)}{2d} \\ &= \frac{3a^2 b \tan^{-1}(\sinh(c + dx))}{2d} + \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a^3 \operatorname{coth}(c + dx)}{2d} \\ &= \frac{3a^2 b \tan^{-1}(\sinh(c + dx))}{2d} + \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a^3 \operatorname{coth}(c + dx)}{2d} \\ &= \frac{3a^2 b \tan^{-1}(\sinh(c + dx))}{2d} + \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a^3 \operatorname{coth}(c + dx)}{2d} \\ &= \frac{3a^2 b \tan^{-1}(\sinh(c + dx))}{2d} + \frac{5b^3 \tan^{-1}(\sinh(c + dx))}{128d} + \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{128d} \end{aligned}$$

Mathematica [A] time = 6.37, size = 243, normalized size = 1.05

$$-\frac{a^3 \operatorname{csch}^2\left(\frac{1}{2}(c + dx)\right)}{8d} - \frac{a^3 \operatorname{sech}^2\left(\frac{1}{2}(c + dx)\right)}{8d} - \frac{a^3 \log\left(\tanh\left(\frac{1}{2}(c + dx)\right)\right)}{2d} + \frac{\operatorname{sech}^2(c + dx) (192a^2 b \sinh(c + dx) + 192a^2 b \cosh(c + dx) + 192a^2 b \tanh(c + dx) + 192a^2 b \operatorname{sech}(c + dx) + 192a^2 b \operatorname{csch}(c + dx) + 192a^2 b \operatorname{coth}(c + dx))}{128d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (b*(192*a^2 + 5*b^2)*ArcTan[Tanh[(c + d*x)/2]]/(64*d) - (a^3*Csch[(c + d*x)
]/2)^2/(8*d) - (a^3*Log[Tanh[(c + d*x)/2]]/(2*d) - (a^3*Sech[(c + d*x)/2]
)^2/(8*d) - (a*b^2*Sech[c + d*x]^3)/d + (3*a*b^2*Sech[c + d*x]^5)/(5*d) + (
Sech[c + d*x]^2*(192*a^2*b*Sinh[c + d*x] + 5*b^3*Sinh[c + d*x]))/(128*d) -
(59*b^3*Sech[c + d*x]^3*Tanh[c + d*x])/(192*d) + (17*b^3*Sech[c + d*x]^5*Ta
nh[c + d*x])/(48*d) - (b^3*Sech[c + d*x]^7*Tanh[c + d*x])/(8*d)
```

fricas [B] time = 0.61, size = 10985, normalized size = 47.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] -1/960*(15*(64*a^3 - 192*a^2*b - 5*b^3)*cosh(d*x + c)^19 + 285*(64*a^3 - 19
2*a^2*b - 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^18 + 15*(64*a^3 - 192*a^2*b -
5*b^3)*sinh(d*x + c)^19 + 5*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)*
```

$$\begin{aligned}
& \cosh(dx + c)^{17} + 5*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3 + 513*(64*a^3 - 192*a^2*b - 5*b^3))*\cosh(dx + c)^2*\sinh(dx + c)^{17} + 85*(171*(64*a^3 - 192*a^2*b - 5*b^3))*\cosh(dx + c)^3 + (1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)*\cosh(dx + c))*\sinh(dx + c)^{16} + 24*(1440*a^3 + 192*a*b^2 - 355*b^3)*\cosh(dx + c)^{15} + 4*(14535*(64*a^3 - 192*a^2*b - 5*b^3))*\cosh(dx + c)^4 + 8640*a^3 + 1152*a*b^2 - 2130*b^3 + 170*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3))*\cosh(dx + c)^2*\sinh(dx + c)^{15} + 20*(8721*(64*a^3 - 192*a^2*b - 5*b^3))*\cosh(dx + c)^5 + 170*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3))*\cosh(dx + c)^3 + 18*(1440*a^3 + 192*a*b^2 - 355*b^3))*\cosh(dx + c))*\sinh(dx + c)^{14} + 16*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3))*\cosh(dx + c)^{13} + 4*(101745*(64*a^3 - 192*a^2*b - 5*b^3))*\cosh(dx + c)^6 + 2975*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3))*\cosh(dx + c)^4 + 20160*a^3 + 5760*a^2*b - 2688*a*b^2 + 4940*b^3 + 630*(1440*a^3 + 192*a*b^2 - 355*b^3))*\cosh(dx + c)^2*\sinh(dx + c)^{13} + 52*(14535*(64*a^3 - 192*a^2*b - 5*b^3))*\cosh(dx + c)^7 + 595*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3))*\cosh(dx + c)^5 + 210*(1440*a^3 + 192*a*b^2 - 355*b^3))*\cosh(dx + c)^3 + 4*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3))*\cosh(dx + c))*\sinh(dx + c)^{12} + 2*(60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475*b^3))*\cosh(dx + c)^{11} + 2*(566865*(64*a^3 - 192*a^2*b - 5*b^3))*\cosh(dx + c)^8 + 30940*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3))*\cosh(dx + c)^6 + 16380*(1440*a^3 + 192*a*b^2 - 355*b^3))*\cosh(dx + c)^4 + 60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475*b^3 + 624*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3))*\cosh(dx + c)^2*\sinh(dx + c)^{11} + 22*(62985*(64*a^3 - 192*a^2*b - 5*b^3))*\cosh(dx + c)^9 + 4420*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3))*\cosh(dx + c)^7 + 3276*(1440*a^3 + 192*a*b^2 - 355*b^3))*\cosh(dx + c)^5 + 208*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3))*\cosh(dx + c)^3 + (60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475*b^3))*\cosh(dx + c))*\sinh(dx + c)^{10} + 2*(60480*a^3 - 8640*a^2*b - 768*a*b^2 + 15475*b^3))*\cosh(dx + c)^9 + 2*(692835*(64*a^3 - 192*a^2*b - 5*b^3))*\cosh(dx + c)^{10} + 60775*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3))*\cosh(dx + c)^8 + 60060*(1440*a^3 + 192*a*b^2 - 355*b^3))*\cosh(dx + c)^6 + 5720*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3))*\cosh(dx + c)^4 + 60480*a^3 - 8640*a^2*b - 768*a*b^2 + 15475*b^3 + 55*(60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475*b^3))*\cosh(dx + c)^2*\sinh(dx + c)^9 + 2*(566865*(64*a^3 - 192*a^2*b - 5*b^3))*\cosh(dx + c)^{11} + 60775*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3))*\cosh(dx + c)^9 + 77220*(1440*a^3 + 192*a*b^2 - 355*b^3))*\cosh(dx + c)^7 + 10296*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3))*\cosh(dx + c)^5 + 165*(60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475*b^3))*\cosh(dx + c)^3 + 9*(60480*a^3 - 8640*a^2*b - 768*a*b^2 + 15475*b^3))*\cosh(dx + c))*\sinh(dx + c)^8 + 16*(5040*a^3 - 1440*a^2*b - 672*a*b^2 - 1235*b^3))*\cosh(dx + c)^7 + 4*(188955*(64*a^3 - 192*a^2*b - 5*b^3))*\cosh(dx + c)^{12} + 24310*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3))*\cosh(dx + c)^{10} + 38610*(1440*a^3 + 192*a*b^2 - 355*b^3))*\cosh(dx + c)^8 + 6864*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3))*\cosh(dx + c)^6 + 165*(60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475*b^3))*\cosh(dx + c)^4 + 20160*a^3 - 5760*a^2*b - 2688*a*b^2 - 4940*b^3 + 18*(60480*a^3 - 8640*a^2*b - 768*a*b^2 + 15475*b^3))*\cosh(dx + c)^2*\sinh(dx + c)^7 + 4*(101745*(64*a^3 - 192*a^2*b - 5*b^3))*\cosh(dx + c)^{13} + 15470*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3))*\cosh(dx + c)^{11} + 30030*(1440*a^3 + 192*a*b^2 - 355*b^3))*\cosh(dx + c)^9 + 6864*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3))*\cosh(dx + c)^7 + 231*(60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475*b^3))*\cosh(dx + c)^5 + 42*(60480*a^3 - 8640*a^2*b - 768*a*b^2 + 15475*b^3))*\cosh(dx + c)^3 + 28*(5040*a^3 - 1440*a^2*b - 672*a*b^2 - 1235*b^3))*\cosh(dx + c))*\sinh(dx + c)^6 + 24*(1440*a^3 + 192*a*b^2 + 355*b^3))*\cosh(dx + c)^5 + 4*(43605*(64*a^3 - 192*a^2*b - 5*b^3))*\cosh(dx + c)^{14} + 7735*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3))*\cosh(dx + c)^{12} + 18018*(1440*a^3 + 192*a*b^2 - 355*b^3))*\cosh(dx + c)^{10} + 5148*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3))*\cosh(dx + c)^8 + 231*(60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475*b^3))*\cosh(dx + c)^6 + 63*(60480*a^3 - 8640*a^2*b - 768*a*b^2 + 15475*b^3))*\cosh(dx + c)^4 + 8640*a^3 + 1152*a*b^2 + 2130*b^3 + 84*(5040*a^3 - 1440*a^2*b - 672*a*b^2 - 123
\end{aligned}$$

$$\begin{aligned}
& 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^5 + 4*(14535*(64*a^3 - 192*a^2*b - 5* \\
& b^3)*\cosh(d*x + c)^{15} + 2975*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3) \\
& *\cosh(d*x + c)^{13} + 8190*(1440*a^3 + 192*a*b^2 - 355*b^3)*\cosh(d*x + c)^{11} \\
& + 2860*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3)*\cosh(d*x + c)^9 + 165 \\
& *(60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475*b^3)*\cosh(d*x + c)^7 + 63*(604 \\
& 80*a^3 - 8640*a^2*b - 768*a*b^2 + 15475*b^3)*\cosh(d*x + c)^5 + 140*(5040*a^ \\
& 3 - 1440*a^2*b - 672*a*b^2 - 1235*b^3)*\cosh(d*x + c)^3 + 30*(1440*a^3 + 192 \\
& *a*b^2 + 355*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^4 + 5*(1728*a^3 + 1728*a^2*b \\
& + 1536*a*b^2 - 427*b^3)*\cosh(d*x + c)^3 + (14535*(64*a^3 - 192*a^2*b - 5*b \\
& ^3)*\cosh(d*x + c)^{16} + 3400*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)* \\
& \cosh(d*x + c)^{14} + 10920*(1440*a^3 + 192*a*b^2 - 355*b^3)*\cosh(d*x + c)^{12} \\
& + 4576*(5040*a^3 + 1440*a^2*b - 672*a*b^2 + 1235*b^3)*\cosh(d*x + c)^{10} + 33 \\
& 0*(60480*a^3 + 8640*a^2*b - 768*a*b^2 - 15475*b^3)*\cosh(d*x + c)^8 + 168*(6 \\
& 0480*a^3 - 8640*a^2*b - 768*a*b^2 + 15475*b^3)*\cosh(d*x + c)^6 + 560*(5040* \\
& a^3 - 1440*a^2*b - 672*a*b^2 - 1235*b^3)*\cosh(d*x + c)^4 + 8640*a^3 + 8640* \\
& a^2*b + 7680*a*b^2 - 2135*b^3 + 240*(1440*a^3 + 192*a*b^2 + 355*b^3)*\cosh(d \\
& *x + c)^2)*\sinh(d*x + c)^3 + (2565*(64*a^3 - 192*a^2*b - 5*b^3)*\cosh(d*x + \\
& c)^{17} + 680*(1728*a^3 - 1728*a^2*b + 1536*a*b^2 + 427*b^3)*\cosh(d*x + c)^{15} \\
& + 2520*(1440*a^3 + 192*a*b^2 - 355*b^3)*\cosh(d*x + c)^{13} + 1248*(5040*a^3 \\
& + 1440*a^2*b - 672*a*b^2 + 1235*b^3)*\cosh(d*x + c)^{11} + 110*(60480*a^3 + 86 \\
& 40*a^2*b - 768*a*b^2 - 15475*b^3)*\cosh(d*x + c)^9 + 72*(60480*a^3 - 8640*a^ \\
& 2*b - 768*a*b^2 + 15475*b^3)*\cosh(d*x + c)^7 + 336*(5040*a^3 - 1440*a^2*b - \\
& 672*a*b^2 - 1235*b^3)*\cosh(d*x + c)^5 + 240*(1440*a^3 + 192*a*b^2 + 355*b^ \\
& 3)*\cosh(d*x + c)^3 + 15*(1728*a^3 + 1728*a^2*b + 1536*a*b^2 - 427*b^3)*\cosh \\
& (d*x + c))*\sinh(d*x + c)^2 - 15*((192*a^2*b + 5*b^3)*\cosh(d*x + c)^{20} + 20* \\
& (192*a^2*b + 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^{19} + (192*a^2*b + 5*b^3)*\si \\
& nh(d*x + c)^{20} + 6*(192*a^2*b + 5*b^3)*\cosh(d*x + c)^{18} + 2*(576*a^2*b + 15 \\
& *b^3 + 95*(192*a^2*b + 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{18} + 12*(95*(1 \\
& 92*a^2*b + 5*b^3)*\cosh(d*x + c)^3 + 9*(192*a^2*b + 5*b^3)*\cosh(d*x + c))*\si \\
& nh(d*x + c)^{17} + 13*(192*a^2*b + 5*b^3)*\cosh(d*x + c)^{16} + (4845*(192*a^2*b \\
& + 5*b^3)*\cosh(d*x + c)^4 + 2496*a^2*b + 65*b^3 + 918*(192*a^2*b + 5*b^3)*c \\
& osh(d*x + c)^2)*\sinh(d*x + c)^{16} + 16*(969*(192*a^2*b + 5*b^3)*\cosh(d*x + c \\
&)^5 + 306*(192*a^2*b + 5*b^3)*\cosh(d*x + c)^3 + 13*(192*a^2*b + 5*b^3)*\cosh \\
& (d*x + c))*\sinh(d*x + c)^{15} + 8*(192*a^2*b + 5*b^3)*\cosh(d*x + c)^{14} + 8*(4 \\
& 845*(192*a^2*b + 5*b^3)*\cosh(d*x + c)^6 + 2295*(192*a^2*b + 5*b^3)*\cosh(d*x \\
& + c)^4 + 192*a^2*b + 5*b^3 + 195*(192*a^2*b + 5*b^3)*\cosh(d*x + c)^2)*\sinh \\
& (d*x + c)^{14} + 16*(4845*(192*a^2*b + 5*b^3)*\cosh(d*x + c)^7 + 3213*(192*a^2 \\
& *b + 5*b^3)*\cosh(d*x + c)^5 + 455*(192*a^2*b + 5*b^3)*\cosh(d*x + c)^3 + 7*(\\
& 192*a^2*b + 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^{13} - 14*(192*a^2*b + 5*b^3) \\
& *\cosh(d*x + c)^{12} + 2*(62985*(192*a^2*b + 5*b^3)*\cosh(d*x + c)^8 + 55692*(1 \\
& 92*a^2*b + 5*b^3)*\cosh(d*x + c)^6 + 11830*(192*a^2*b + 5*b^3)*\cosh(d*x + c) \\
& ^4 - 1344*a^2*b - 35*b^3 + 364*(192*a^2*b + 5*b^3)*\cosh(d*x + c)^2)*\sinh(d* \\
& x + c)^{12} + 8*(20995*(192*a^2*b + 5*b^3)*\cosh(d*x + c)^9 + 23868*(192*a^2*b \\
& + 5*b^3)*\cosh(d*x + c)^7 + 7098*(192*a^2*b + 5*b^3)*\cosh(d*x + c)^5 + 364* \\
& (192*a^2*b + 5*b^3)*\cosh(d*x + c)^3 - 21*(192*a^2*b + 5*b^3)*\cosh(d*x + c)) \\
& *\sinh(d*x + c)^{11} - 28*(192*a^2*b + 5*b^3)*\cosh(d*x + c)^{10} + 4*(46189*(192 \\
& *a^2*b + 5*b^3)*\cosh(d*x + c)^{10} + 65637*(192*a^2*b + 5*b^3)*\cosh(d*x + c)^ \\
& 8 + 26026*(192*a^2*b + 5*b^3)*\cosh(d*x + c)^6 + 2002*(192*a^2*b + 5*b^3)*co \\
& sh(d*x + c)^4 - 1344*a^2*b - 35*b^3 - 231*(192*a^2*b + 5*b^3)*\cosh(d*x + c) \\
& ^2)*\sinh(d*x + c)^{10} + 8*(20995*(192*a^2*b + 5*b^3)*\cosh(d*x + c)^{11} + 3646 \\
& 5*(192*a^2*b + 5*b^3)*\cosh(d*x + c)^9 + 18590*(192*a^2*b + 5*b^3)*\cosh(d*x \\
& + c)^7 + 2002*(192*a^2*b + 5*b^3)*\cosh(d*x + c)^5 - 385*(192*a^2*b + 5*b^3) \\
& *\cosh(d*x + c)^3 - 35*(192*a^2*b + 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^9 - \\
& 14*(192*a^2*b + 5*b^3)*\cosh(d*x + c)^8 + 2*(62985*(192*a^2*b + 5*b^3)*\cosh(\\
& d*x + c)^{12} + 131274*(192*a^2*b + 5*b^3)*\cosh(d*x + c)^{10} + 83655*(192*a^2* \\
& b + 5*b^3)*\cosh(d*x + c)^8 + 12012*(192*a^2*b + 5*b^3)*\cosh(d*x + c)^6 - 34 \\
& 65*(192*a^2*b + 5*b^3)*\cosh(d*x + c)^4 - 1344*a^2*b - 35*b^3 - 630*(192*a^2 \\
& *b + 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 16*(4845*(192*a^2*b + 5*b^3) \\
& *\cosh(d*x + c)^{13} + 11934*(192*a^2*b + 5*b^3)*\cosh(d*x + c)^{11} + 9295*(192*
\end{aligned}$$

$$\begin{aligned}
& a^2b + 5b^3) \cosh(dx + c)^9 + 1716(192a^2b + 5b^3) \cosh(dx + c)^7 - \\
& 693(192a^2b + 5b^3) \cosh(dx + c)^5 - 210(192a^2b + 5b^3) \cosh(dx + c)^3 - 7(192a^2b + 5b^3) \cosh(dx + c) \sinh(dx + c)^7 + 8(192a^2b + 5b^3) \cosh(dx + c)^6 + 8(4845(192a^2b + 5b^3) \cosh(dx + c)^{14} + 13923(192a^2b + 5b^3) \cosh(dx + c)^{12} + 13013(192a^2b + 5b^3) \cosh(dx + c)^{10} + 3003(192a^2b + 5b^3) \cosh(dx + c)^8 - 1617(192a^2b + 5b^3) \cosh(dx + c)^6 - 735(192a^2b + 5b^3) \cosh(dx + c)^4 + 192a^2b + 5b^3 - 49(192a^2b + 5b^3) \cosh(dx + c)^2) \sinh(dx + c)^6 + 16(969(192a^2b + 5b^3) \cosh(dx + c)^{15} + 3213(192a^2b + 5b^3) \cosh(dx + c)^{13} + 3549(192a^2b + 5b^3) \cosh(dx + c)^{11} + 1001(192a^2b + 5b^3) \cosh(dx + c)^9 - 693(192a^2b + 5b^3) \cosh(dx + c)^7 - 441(192a^2b + 5b^3) \cosh(dx + c)^5 - 49(192a^2b + 5b^3) \cosh(dx + c)^3 + 3(192a^2b + 5b^3) \cosh(dx + c)) \sinh(dx + c)^5 + 13(192a^2b + 5b^3) \cosh(dx + c)^4 + (4845(192a^2b + 5b^3) \cosh(dx + c)^{16} + 18360(192a^2b + 5b^3) \cosh(dx + c)^{14} + 23660(192a^2b + 5b^3) \cosh(dx + c)^{12} + 8008(192a^2b + 5b^3) \cosh(dx + c)^{10} - 6930(192a^2b + 5b^3) \cosh(dx + c)^8 - 5880(192a^2b + 5b^3) \cosh(dx + c)^6 - 980(192a^2b + 5b^3) \cosh(dx + c)^4 + 2496a^2b + 65b^3 + 120(192a^2b + 5b^3) \cosh(dx + c)^2) \sinh(dx + c)^4 + 4(285(192a^2b + 5b^3) \cosh(dx + c)^{17} + 1224(192a^2b + 5b^3) \cosh(dx + c)^{15} + 1820(192a^2b + 5b^3) \cosh(dx + c)^{13} + 728(192a^2b + 5b^3) \cosh(dx + c)^{11} - 770(192a^2b + 5b^3) \cosh(dx + c)^9 - 840(192a^2b + 5b^3) \cosh(dx + c)^7 - 196(192a^2b + 5b^3) \cosh(dx + c)^5 + 40(192a^2b + 5b^3) \cosh(dx + c)^3 + 13(192a^2b + 5b^3) \cosh(dx + c)) \sinh(dx + c)^3 + 192a^2b + 5b^3 + 6(192a^2b + 5b^3) \cosh(dx + c)^2 + 2(95(192a^2b + 5b^3) \cosh(dx + c)^{18} + 459(192a^2b + 5b^3) \cosh(dx + c)^{16} + 780(192a^2b + 5b^3) \cosh(dx + c)^{14} + 364(192a^2b + 5b^3) \cosh(dx + c)^{12} - 462(192a^2b + 5b^3) \cosh(dx + c)^{10} - 630(192a^2b + 5b^3) \cosh(dx + c)^8 - 196(192a^2b + 5b^3) \cosh(dx + c)^6 + 60(192a^2b + 5b^3) \cosh(dx + c)^4 + 576a^2b + 15b^3 + 39(192a^2b + 5b^3) \cosh(dx + c)^2) \sinh(dx + c)^2 + 4(5(192a^2b + 5b^3) \cosh(dx + c)^{19} + 27(192a^2b + 5b^3) \cosh(dx + c)^{17} + 52(192a^2b + 5b^3) \cosh(dx + c)^{15} + 28(192a^2b + 5b^3) \cosh(dx + c)^{13} - 42(192a^2b + 5b^3) \cosh(dx + c)^{11} - 70(192a^2b + 5b^3) \cosh(dx + c)^9 - 28(192a^2b + 5b^3) \cosh(dx + c)^7 + 12(192a^2b + 5b^3) \cosh(dx + c)^5 + 13(192a^2b + 5b^3) \cosh(dx + c)^3 + 3(192a^2b + 5b^3) \cosh(dx + c)) \sinh(dx + c)) \arctan(\cosh(dx + c) + \sinh(dx + c)) + 15(64a^3 + 192a^2b + 5b^3) \cosh(dx + c) - 480(a^3 \cosh(dx + c)^{20} + 20a^3 \cosh(dx + c) \sinh(dx + c)^{19} + a^3 \sinh(dx + c)^{20} + 6a^3 \cosh(dx + c)^{18} + 13a^3 \cosh(dx + c)^{16} + 2(95a^3 \cosh(dx + c)^2 + 3a^3) \sinh(dx + c)^{18} + 12(95a^3 \cosh(dx + c)^3 + 9a^3 \cosh(dx + c)) \sinh(dx + c)^{17} + 8a^3 \cosh(dx + c)^{14} + (4845a^3 \cosh(dx + c)^4 + 918a^3 \cosh(dx + c)^2 + 13a^3) \sinh(dx + c)^{16} + 16(969a^3 \cosh(dx + c)^5 + 306a^3 \cosh(dx + c)^3 + 13a^3 \cosh(dx + c)) \sinh(dx + c)^{15} - 14a^3 \cosh(dx + c)^{12} + 8(4845a^3 \cosh(dx + c)^6 + 2295a^3 \cosh(dx + c)^4 + 195a^3 \cosh(dx + c)^2 + a^3) \sinh(dx + c)^{14} + 16(4845a^3 \cosh(dx + c)^7 + 3213a^3 \cosh(dx + c)^5 + 455a^3 \cosh(dx + c)^3 + 7a^3 \cosh(dx + c)) \sinh(dx + c)^{13} - 28a^3 \cosh(dx + c)^{10} + 2(62985a^3 \cosh(dx + c)^8 + 55692a^3 \cosh(dx + c)^6 + 11830a^3 \cosh(dx + c)^4 + 364a^3 \cosh(dx + c)^2 - 7a^3) \sinh(dx + c)^{12} + 8(20995a^3 \cosh(dx + c)^9 + 23868a^3 \cosh(dx + c)^7 + 7098a^3 \cosh(dx + c)^5 + 364a^3 \cosh(dx + c)^3 - 21a^3 \cosh(dx + c)) \sinh(dx + c)^{11} - 14a^3 \cosh(dx + c)^8 + 4(46189a^3 \cosh(dx + c)^{10} + 65637a^3 \cosh(dx + c)^8 + 26026a^3 \cosh(dx + c)^6 + 2002a^3 \cosh(dx + c)^4 - 231a^3 \cosh(dx + c)^2 - 7a^3) \sinh(dx + c)^{10} + 8(20995a^3 \cosh(dx + c)^{11} + 36465a^3 \cosh(dx + c)^9 + 18590a^3 \cosh(dx + c)^7 + 2002a^3 \cosh(dx + c)^5 - 385a^3 \cosh(dx + c)^3 - 35a^3 \cosh(dx + c)) \sinh(dx + c)^9 + 8a^3 \cosh(dx + c)^6 + 2(62985a^3 \cosh(dx + c)^{12} + 131274a^3 \cosh(dx + c)^{10} + 83655a^3 \cosh(dx + c)^8 + 12012a^3 \cosh(dx + c)^6 - 3465a^3 \cosh(dx + c)^4 - 630a^3 \cosh(dx + c)^2 - 7a^3) \sinh(dx + c)^8 + 16(484
\end{aligned}$$

$$\begin{aligned}
& a^3 \cosh(dx + c)^{13} + 728a^3 \cosh(dx + c)^{11} - 770a^3 \cosh(dx + c)^9 - \\
& 840a^3 \cosh(dx + c)^7 - 196a^3 \cosh(dx + c)^5 + 40a^3 \cosh(dx + c)^3 + \\
& 13a^3 \cosh(dx + c) \sinh(dx + c)^3 + a^3 + 2(95a^3 \cosh(dx + c)^{18} + \\
& 459a^3 \cosh(dx + c)^{16} + 780a^3 \cosh(dx + c)^{14} + 364a^3 \cosh(dx + \\
& c)^{12} - 462a^3 \cosh(dx + c)^{10} - 630a^3 \cosh(dx + c)^8 - 196a^3 \cosh(dx + \\
& c)^6 + 60a^3 \cosh(dx + c)^4 + 39a^3 \cosh(dx + c)^2 + 3a^3) \sinh(dx + \\
& c)^2 + 4(5a^3 \cosh(dx + c)^{19} + 27a^3 \cosh(dx + c)^{17} + 52a^3 \cosh(dx + \\
& c)^{15} + 28a^3 \cosh(dx + c)^{13} - 42a^3 \cosh(dx + c)^{11} - 70a^3 \cosh(dx + \\
& c)^9 - 28a^3 \cosh(dx + c)^7 + 12a^3 \cosh(dx + c)^5 + 13a^3 \cosh(dx + \\
& c)^3 + 3a^3 \cosh(dx + c)) \sinh(dx + c) \log(\cosh(dx + c) + \\
& \sinh(dx + c) - 1) + (285(64a^3 - 192a^2b - 5b^3) \cosh(dx + c)^{18} + \\
& 85(1728a^3 - 1728a^2b + 1536ab^2 + 427b^3) \cosh(dx + c)^{16} + 360(1440a^3 + \\
& 192ab^2 - 355b^3) \cosh(dx + c)^{14} + 208(5040a^3 + 1440a^2b - 768ab^2 + \\
& 1235b^3) \cosh(dx + c)^{12} + 22(60480a^3 + 8640a^2b - 768ab^2 - 15475b^3) \cosh(dx + \\
& c)^{10} + 18(60480a^3 - 8640a^2b - 768ab^2 + 15475b^3) \cosh(dx + c)^8 + \\
& 112(5040a^3 - 1440a^2b - 672ab^2 - 1235b^3) \cosh(dx + c)^6 + 120(1440a^3 + \\
& 192ab^2 + 355b^3) \cosh(dx + c)^4 + 960a^3 + 2880a^2b + 75b^3 + 15(1728a^3 + \\
& 1728a^2b + 1536ab^2 - 427b^3) \cosh(dx + c)^2) \sinh(dx + c) / (d \cosh(dx + c)^{20} + \\
& 20d \cosh(dx + c) \sinh(dx + c)^{19} + d \sinh(dx + c)^{20} + 6d \cosh(dx + c)^{18} + \\
& 2(95d \cosh(dx + c)^2 + 3d) \sinh(dx + c)^{18} + 12(95d \cosh(dx + c)^3 + \\
& 9d \cosh(dx + c)) \sinh(dx + c)^{17} + 13d \cosh(dx + c)^{16} + (4845d \cosh(dx + \\
& c)^4 + 918d \cosh(dx + c)^2 + 13d) \sinh(dx + c)^{16} + 16(969d \cosh(dx + \\
& c)^5 + 306d \cosh(dx + c)^3 + 13d \cosh(dx + c)) \sinh(dx + c)^{15} + 8d \cosh(dx + \\
& c)^{14} + 8(4845d \cosh(dx + c)^6 + 2295d \cosh(dx + c)^4 + 195d \cosh(dx + \\
& c)^2 + d) \sinh(dx + c)^{14} + 16(4845d \cosh(dx + c)^7 + 3213d \cosh(dx + \\
& c)^5 + 455d \cosh(dx + c)^3 + 7d \cosh(dx + c)) \sinh(dx + c)^{13} - 14d \cosh(dx + \\
& c)^{12} + 2(62985d \cosh(dx + c)^8 + 5692d \cosh(dx + c)^6 + 11830d \cosh(dx + \\
& c)^4 + 364d \cosh(dx + c)^2 - 7d) \sinh(dx + c)^{12} + 8(20995d \cosh(dx + c)^9 + \\
& 23868d \cosh(dx + c)^7 + 7098d \cosh(dx + c)^5 + 364d \cosh(dx + c)^3 - 21d \cosh(dx + \\
& c)) \sinh(dx + c)^{11} - 28d \cosh(dx + c)^{10} + 4(46189d \cosh(dx + c)^{10} + 65637d \cosh(dx + \\
& c)^8 + 26026d \cosh(dx + c)^6 + 2002d \cosh(dx + c)^4 - 231d \cosh(dx + c)^2 - \\
& 7d) \sinh(dx + c)^{10} + 8(20995d \cosh(dx + c)^{11} + 36465d \cosh(dx + c)^9 + \\
& 18590d \cosh(dx + c)^7 + 2002d \cosh(dx + c)^5 - 385d \cosh(dx + c)^3 - 35d \cosh(dx + \\
& c)) \sinh(dx + c)^9 - 14d \cosh(dx + c)^8 + 2(62985d \cosh(dx + c)^{12} + 131274d \cosh(dx + \\
& c)^{10} + 83655d \cosh(dx + c)^8 + 12012d \cosh(dx + c)^6 - 3465d \cosh(dx + c)^4 - 630d \cosh(dx + \\
& c)^2 - 7d) \sinh(dx + c)^8 + 16(4845d \cosh(dx + c)^{13} + 11934d \cosh(dx + \\
& c)^{11} + 9295d \cosh(dx + c)^9 + 1716d \cosh(dx + c)^7 - 693d \cosh(dx + c)^5 - \\
& 210d \cosh(dx + c)^3 - 7d \cosh(dx + c)) \sinh(dx + c)^7 + 8d \cosh(dx + c)^6 + \\
& 8(4845d \cosh(dx + c)^{14} + 13923d \cosh(dx + c)^{12} + 13013d \cosh(dx + c)^{10} + \\
& 3003d \cosh(dx + c)^8 - 1617d \cosh(dx + c)^6 - 735d \cosh(dx + c)^4 - 49d \cosh(dx + \\
& c)^2 + d) \sinh(dx + c)^6 + 16(969d \cosh(dx + c)^{15} + 3213d \cosh(dx + c)^{13} + \\
& 3549d \cosh(dx + c)^{11} + 1001d \cosh(dx + c)^9 - 693d \cosh(dx + c)^7 - 441d \cosh(dx + \\
& c)^5 - 49d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^5 + 13d \cosh(dx + \\
& c)^4 + (4845d \cosh(dx + c)^{16} + 18360d \cosh(dx + c)^{14} + 23660d \cosh(dx + \\
& c)^{12} + 8008d \cosh(dx + c)^{10} - 6930d \cosh(dx + c)^8 - 5880d \cosh(dx + c)^6 - \\
& 980d \cosh(dx + c)^4 + 120d \cosh(dx + c)^2 + 13d) \sinh(dx + c)^4 + 4(285d \cosh(dx + \\
& c)^{17} + 1224d \cosh(dx + c)^{15} + 1820d \cosh(dx + c)^{13} + 728d \cosh(dx + \\
& c)^{11} - 770d \cosh(dx + c)^9 - 840d \cosh(dx + c)^7 - 196d \cosh(dx + c)^5 + \\
& 40d \cosh(dx + c)^3 + 13d \cosh(dx + c)) \sinh(dx + c)^3 + 6d \cosh(dx + c)^2 + \\
& 2(95d \cosh(dx + c)^{18} + 459d \cosh(dx + c)^{16} + 780d \cosh(dx + c)^{14} + \\
& 364d \cosh(dx + c)^{12} - 462d \cosh(dx + c)^{10} - 630d \cosh(dx + c)^8 - 196d \cosh(dx + \\
& c)^6 + 60d \cosh(dx + c)^4 + 39d \cosh(dx + c)^2 + 3d) \sinh(dx + c)^2 + 4(5d \cosh(dx + \\
& c)^{19} + 27d \cosh(dx + c)^{17} + 52d \cosh(dx + c)^{15} + 28d \cosh(dx + c)^{13} - \\
& 42d \cosh(dx + c)^{11} - 70d \cosh(dx + c)^9 -
\end{aligned}$$

$28*d*\cosh(d*x + c)^7 + 12*d*\cosh(d*x + c)^5 + 13*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c)*\sinh(d*x + c) + d$

giac [B] time = 0.84, size = 434, normalized size = 1.87

$$480 a^3 \log(e^{(dx+c)} + 1) - 480 a^3 \log(|e^{(dx+c)} - 1|) + 15 (192 a^2 b e^c + 5 b^3 e^c) \arctan(e^{(dx+c)}) e^{(-c)} - \frac{960 (a^3 e^{(3dx+3c)}}{(e^{(2dx+2c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")

[Out] $1/960*(480*a^3*\log(e^{(d*x + c)} + 1) - 480*a^3*\log(\text{abs}(e^{(d*x + c)} - 1))) + 15*(192*a^2*b*e^c + 5*b^3*e^c)*\arctan(e^{(d*x + c)})*e^{(-c)} - 960*(a^3*e^{(3*d*x + 3*c)} + a^3*e^{(d*x + c)})/(e^{(2*d*x + 2*c)} - 1)^2 + (2880*a^2*b*e^{(15*d*x + 15*c)} + 75*b^3*e^{(15*d*x + 15*c)} + 14400*a^2*b*e^{(13*d*x + 13*c)} - 7680*a*b^2*e^{(13*d*x + 13*c)} - 1985*b^3*e^{(13*d*x + 13*c)} + 25920*a^2*b*e^{(11*d*x + 11*c)} - 19968*a*b^2*e^{(11*d*x + 11*c)} + 4475*b^3*e^{(11*d*x + 11*c)} + 14400*a^2*b*e^{(9*d*x + 9*c)} - 21504*a*b^2*e^{(9*d*x + 9*c)} - 8825*b^3*e^{(9*d*x + 9*c)} - 14400*a^2*b*e^{(7*d*x + 7*c)} - 21504*a*b^2*e^{(7*d*x + 7*c)} + 8825*b^3*e^{(7*d*x + 7*c)} - 25920*a^2*b*e^{(5*d*x + 5*c)} - 19968*a*b^2*e^{(5*d*x + 5*c)} - 4475*b^3*e^{(5*d*x + 5*c)} - 14400*a^2*b*e^{(3*d*x + 3*c)} - 7680*a*b^2*e^{(3*d*x + 3*c)} + 1985*b^3*e^{(3*d*x + 3*c)} - 2880*a^2*b*e^{(d*x + c)} - 75*b^3*e^{(d*x + c)})/(e^{(2*d*x + 2*c)} + 1)^8/d$

maple [A] time = 0.61, size = 286, normalized size = 1.23

$$-\frac{a^3 \coth(dx + c) \operatorname{csch}(dx + c)}{2d} + \frac{a^3 \operatorname{arctanh}(e^{dx+c})}{d} + \frac{3a^2 b \operatorname{sech}(dx + c) \tanh(dx + c)}{2d} + \frac{3a^2 b \operatorname{arctan}(e^{dx+c})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x)

[Out] $-1/2*a^3*\coth(d*x+c)*\operatorname{csch}(d*x+c)/d+1/d*a^3*\operatorname{arctanh}(\exp(d*x+c))+3/2/d*a^2*b*\operatorname{sech}(d*x+c)*\tanh(d*x+c)+3/d*a^2*b*\operatorname{arctan}(\exp(d*x+c))-1/d*a*b^2*\sinh(d*x+c)^2/\cosh(d*x+c)^5-2/5/d*a*b^2/\cosh(d*x+c)^5-1/3/d*b^3*\sinh(d*x+c)^5/\cosh(d*x+c)^8-1/3/d*b^3*\sinh(d*x+c)^3/\cosh(d*x+c)^8-1/7/d*b^3*\sinh(d*x+c)/\cosh(d*x+c)^8+1/56/d*b^3*\tanh(d*x+c)*\operatorname{sech}(d*x+c)^7+1/48/d*b^3*\tanh(d*x+c)*\operatorname{sech}(d*x+c)^5+5/192/d*b^3*\tanh(d*x+c)*\operatorname{sech}(d*x+c)^3+5/128/d*b^3*\operatorname{sech}(d*x+c)*\tanh(d*x+c)+5/64/d*b^3*\operatorname{arctan}(\exp(d*x+c))$

maxima [B] time = 0.43, size = 586, normalized size = 2.53

$$-\frac{1}{192} b^3 \left(\frac{15 \operatorname{arctan}(e^{(-dx-c)})}{d} - \frac{15 e^{(-dx-c)} - 397 e^{(-3dx-3c)} + 895 e^{(-5dx-5c)} - 1765 e^{(-7dx-7c)} + 1765 e^{(-9dx-9c)}}{d(8 e^{(-2dx-2c)} + 28 e^{(-4dx-4c)} + 56 e^{(-6dx-6c)} + 70 e^{(-8dx-8c)} + 56 e^{(-10dx-10c)})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")

[Out] $-1/192*b^3*(15*\operatorname{arctan}(e^{(-d*x - c)})/d - (15*e^{(-d*x - c)} - 397*e^{(-3*d*x - 3*c)} + 895*e^{(-5*d*x - 5*c)} - 1765*e^{(-7*d*x - 7*c)} + 1765*e^{(-9*d*x - 9*c)} - 895*e^{(-11*d*x - 11*c)} + 397*e^{(-13*d*x - 13*c)} - 15*e^{(-15*d*x - 15*c)})/(d*(8*e^{(-2*d*x - 2*c)} + 28*e^{(-4*d*x - 4*c)} + 56*e^{(-6*d*x - 6*c)} + 70*e^{(-8*d*x - 8*c)} + 56*e^{(-10*d*x - 10*c)} + 28*e^{(-12*d*x - 12*c)} + 8*e^{(-14*d*x - 14*c)} + e^{(-16*d*x - 16*c)} + 1))) - 3*a^2*b*(\operatorname{arctan}(e^{(-d*x - c)})/d - (e^{(-d*x - c)} - e^{(-3*d*x - 3*c)})/(d*(2*e^{(-2*d*x - 2*c)} + e^{(-4*d*x - 4*c)} + 1))) + 1/2*a^3*(\log(e^{(-d*x - c)} + 1)/d - \log(e^{(-d*x - c)} - 1)/d + 2*(e^{(-d*x - c)} + e^{(-3*d*x - 3*c)})/(d*(2*e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)} -$

1))) - 8/5*a*b^2*(5*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 2*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^(-7*d*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)))

mupad [B] time = 8.05, size = 731, normalized size = 3.15

$$\frac{a^3 \ln(e^{c+dx} + 1)}{2d} - \frac{a^3 \ln(e^{c+dx} - 1)}{2d} + \frac{e^{c+dx} (192 a^2 b + 5 b^3)}{64 d (e^{2c+2dx} + 1)} + \frac{e^{c+dx} (2245 b^3 + 3264 a b^2)}{120 d (3 e^{2c+2dx} + 3 e^{4c+4dx} + e^{6c+6dx} + 1)} - \frac{e^{c+dx} (2245 b^3 + 3264 a b^2)}{20 d (3 e^{2c+2dx} + 3 e^{4c+4dx} + e^{6c+6dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tanh(c + d*x)^3)^3/sinh(c + d*x)^3,x)

[Out] (a^3*log(exp(c + d*x) + 1))/(2*d) - (a^3*log(exp(c + d*x) - 1))/(2*d) + (exp(c + d*x)*(192*a^2*b + 5*b^3))/(64*d*(exp(2*c + 2*d*x) + 1)) + (exp(c + d*x)*(3264*a*b^2 + 2245*b^3))/(120*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (b*log(exp(c + d*x) - 1i)*(192*a^2 + 5*b^2)*1i)/(128*d) + (b*log(exp(c + d*x) + 1i)*(192*a^2 + 5*b^2)*1i)/(128*d) - (exp(c + d*x)*(768*a*b^2 + 1325*b^3))/(20*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (500*b^3*exp(c + d*x))/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) + (2*exp(c + d*x)*(144*a*b^2 + 1025*b^3))/(15*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (112*b^3*exp(c + d*x))/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) - (exp(c + d*x)*(768*a*b^2 + 576*a^2*b + 251*b^3))/(96*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (32*b^3*exp(c + d*x))/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1)) - (a^3*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a^3*exp(c + d*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^3(c + dx))^3 \operatorname{csch}^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**3)**3,x)

[Out] Integral((a + b*tanh(c + d*x)**3)**3*csch(c + d*x)**3, x)

3.72 $\int \operatorname{csch}^4(c + dx) (a + b \tanh^3(c + dx))^3 dx$

Optimal. Leaf size=138

$$-\frac{a^3 \operatorname{coth}^3(c + dx)}{3d} + \frac{a^3 \operatorname{coth}(c + dx)}{d} - \frac{3a^2 b \tanh^2(c + dx)}{2d} + \frac{3a^2 b \log(\tanh(c + dx))}{d} - \frac{3ab^2 \tanh^5(c + dx)}{5d} + \frac{ab^2 \tanh^8(c + dx)}{8d}$$

[Out] $a^3 \operatorname{coth}(d*x+c)/d - 1/3*a^3 \operatorname{coth}(d*x+c)^3/d + 3*a^2*b*\ln(\tanh(d*x+c))/d - 3/2*a^2*b*\tanh(d*x+c)^2/d + a*b^2*\tanh(d*x+c)^3/d - 3/5*a*b^2*\tanh(d*x+c)^5/d + 1/6*b^3*\tanh(d*x+c)^6/d - 1/8*b^3*\tanh(d*x+c)^8/d$

Rubi [A] time = 0.12, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3663, 1802}

$$-\frac{3a^2 b \tanh^2(c + dx)}{2d} + \frac{3a^2 b \log(\tanh(c + dx))}{d} - \frac{a^3 \operatorname{coth}^3(c + dx)}{3d} + \frac{a^3 \operatorname{coth}(c + dx)}{d} - \frac{3ab^2 \tanh^5(c + dx)}{5d} + \frac{ab^2 \tanh^8(c + dx)}{8d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^3,x]

[Out] $(a^3 \operatorname{Coth}[c + d*x])/d - (a^3 \operatorname{Coth}[c + d*x]^3)/(3*d) + (3*a^2*b*\operatorname{Log}[\operatorname{Tanh}[c + d*x]])/d - (3*a^2*b*\operatorname{Tanh}[c + d*x]^2)/(2*d) + (a*b^2*\operatorname{Tanh}[c + d*x]^3)/d - (3*a*b^2*\operatorname{Tanh}[c + d*x]^5)/(5*d) + (b^3*\operatorname{Tanh}[c + d*x]^6)/(6*d) - (b^3*\operatorname{Tanh}[c + d*x]^8)/(8*d)$

Rule 1802

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 3663

Int[sin[(e_)+(f_)*(x_)]^(m_)*((a_) + (b_)*((c_)*tan[(e_)+(f_)*(x_)]))^(n_)]^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^4(c + dx) (a + b \tanh^3(c + dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(a+bx^3)^3}{x^4} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{a^3}{x^4} - \frac{a^3}{x^2} + \frac{3a^2b}{x} - 3a^2bx + 3ab^2x^2 - 3ab^2x^4 + b^3x^5 - b^3\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{a^3 \operatorname{coth}(c + dx)}{d} - \frac{a^3 \operatorname{coth}^3(c + dx)}{3d} + \frac{3a^2 b \log(\tanh(c + dx))}{d} - \frac{3a^2 b \tanh^2(c + dx)}{2d} + \frac{3a^2 b \operatorname{sech}^2(c + dx)}{2d} + \frac{3a^2 b \log(\sinh(c + dx))}{d} - \frac{3a^2 b \log(\cosh(c + dx))}{d} \end{aligned}$$

Mathematica [A] time = 0.20, size = 213, normalized size = 1.54

$$\frac{2a^3 \operatorname{coth}(c + dx)}{3d} - \frac{a^3 \operatorname{coth}(c + dx) \operatorname{csch}^2(c + dx)}{3d} + \frac{3a^2 b \operatorname{sech}^2(c + dx)}{2d} + \frac{3a^2 b \log(\sinh(c + dx))}{d} - \frac{3a^2 b \log(\cosh(c + dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^3,x]
```

```
[Out] (2*a^3*Coth[c + d*x])/(3*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) - (3*a^2*b*Log[Cosh[c + d*x]])/d + (3*a^2*b*Log[Sinh[c + d*x]])/d + (3*a^2*b*Sech[c + d*x]^2)/(2*d) - (b^3*Sech[c + d*x]^4)/(4*d) + (b^3*Sech[c + d*x]^6)/(3*d) - (b^3*Sech[c + d*x]^8)/(8*d) + (2*a*b^2*Tanh[c + d*x])/(5*d) + (a*b^2*Sech[c + d*x]^2*Tanh[c + d*x])/(5*d) - (3*a*b^2*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d)
```

```
fricas [B] time = 0.58, size = 9459, normalized size = 68.54
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(90*a^2*b*cosh(d*x + c)^20 + 1800*a^2*b*cosh(d*x + c)*sinh(d*x + c)^19 + 90*a^2*b*sinh(d*x + c)^20 - 30*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)^18 + 30*(570*a^2*b*cosh(d*x + c)^2 - 2*a^3 + 9*a^2*b - 6*a*b^2 - 2*b^3)*sinh(d*x + c)^18 + 540*(190*a^2*b*cosh(d*x + c)^3 - (2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c))*sinh(d*x + c)^17 - 20*(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c)^16 + 10*(43605*a^2*b*cosh(d*x + c)^4 - 46*a^3 + 6*a*b^2 + 26*b^3 - 459*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^16 + 160*(8721*a^2*b*cosh(d*x + c)^5 - 153*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)^3 - 2*(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c))*sinh(d*x + c)^15 - 20*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x + c)^14 + 20*(174420*a^2*b*cosh(d*x + c)^6 - 4590*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)^4 - 76*a^3 - 36*a^2*b + 24*a*b^2 - 31*b^3 - 120*(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^14 + 40*(174420*a^2*b*cosh(d*x + c)^7 - 6426*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)^5 - 280*(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c)^3 - 7*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x + c))*sinh(d*x + c)^13 - 4*(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*cosh(d*x + c)^12 + 4*(2834325*a^2*b*cosh(d*x + c)^8 - 139230*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)^6 - 9100*(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c)^4 - 700*a^3 - 135*a^2*b - 48*a*b^2 + 245*b^3 - 455*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 16*(944775*a^2*b*cosh(d*x + c)^9 - 59670*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)^7 - 5460*(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c)^5 - 455*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x + c)^3 - 3*(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 - 20*(154*a^3 - 27*a^2*b + 18*a*b^2 + 49*b^3)*cosh(d*x + c)^10 + 4*(4157010*a^2*b*cosh(d*x + c)^10 - 328185*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)^8 - 40040*(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c)^6 - 5005*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x + c)^4 - 770*a^3 + 135*a^2*b - 90*a*b^2 - 245*b^3 - 66*(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 40*(377910*a^2*b*cosh(d*x + c)^11 - 36465*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)^9 - 5720*(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c)^7 - 1001*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x + c)^5 - 22*(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*cosh(d*x + c)^3 - 5*(154*a^3 - 27*a^2*b + 18*a*b^2 + 49*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 - 4*(490*a^3 - 180*a^2*b - 54*a*b^2 - 155*b^3)*cosh(d*x + c)^8 + 4*(2834325*a^2*b*cosh(d*x + c)^12 - 328185*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)^10 - 64350*(23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c)^8 - 15015*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*cosh(d*x + c)^6 - 495*(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*cosh(d*x + c)^4 - 490*a^3 + 180*a^2*b + 54*a*b^2 + 155*b^3 - 225*(154*a^3 - 27*a^2*b + 18*a*b^2 + 49*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 32*(218025*a^2*b*cosh(d*x + c)^13 - 29835*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)^11 - 7150*(23*a^3 - 3*a*b^2 - 13*b^3)*cos
```

$$\begin{aligned}
& h(dx + c)^9 - 2145*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*\cosh(dx + c)^7 \\
& - 99*(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*\cosh(dx + c)^5 - 75*(154*a^3 - 27*a^2*b + 18*a*b^2 + 49*b^3)*\cosh(dx + c)^3 - (490*a^3 - 180*a^2*b - 54*a*b^2 - 155*b^3)*\cosh(dx + c))*\sinh(dx + c)^7 - 20*(28*a^3 + 13*b^3) \\
& *\cosh(dx + c)^6 + 4*(872100*a^2*b*\cosh(dx + c)^14 - 139230*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*\cosh(dx + c)^12 - 40040*(23*a^3 - 3*a*b^2 - 13*b^3)* \\
& \cosh(dx + c)^10 - 15015*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*\cosh(dx + c)^8 - 924*(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*\cosh(dx + c)^6 - 10 \\
& 50*(154*a^3 - 27*a^2*b + 18*a*b^2 + 49*b^3)*\cosh(dx + c)^4 - 140*a^3 - 65*b^3 - 28*(490*a^3 - 180*a^2*b - 54*a*b^2 - 155*b^3)*\cosh(dx + c)^2)*\sinh(dx + c)^6 + 8*(174420*a^2*b*\cosh(dx + c)^15 - 32130*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*\cosh(dx + c)^13 - 10920*(23*a^3 - 3*a*b^2 - 13*b^3)*\cosh(dx + c)^11 - 5005*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*\cosh(dx + c)^9 - 3 \\
& 96*(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*\cosh(dx + c)^7 - 630*(154*a^3 - 27*a^2*b + 18*a*b^2 + 49*b^3)*\cosh(dx + c)^5 - 28*(490*a^3 - 180*a^2*b - 54*a*b^2 - 155*b^3)*\cosh(dx + c)^3 - 15*(28*a^3 + 13*b^3)*\cosh(dx + c) \\
&)*\sinh(dx + c)^5 + 2*(40*a^3 - 135*a^2*b - 48*a*b^2 + 30*b^3)*\cosh(dx + c)^4 + 2*(218025*a^2*b*\cosh(dx + c)^16 - 45900*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*\cosh(dx + c)^14 - 18200*(23*a^3 - 3*a*b^2 - 13*b^3)*\cosh(dx + c)^12 - 10010*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*\cosh(dx + c)^10 - 990*(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*\cosh(dx + c)^8 - 2100*(154*a^3 - 27*a^2*b + 18*a*b^2 + 49*b^3)*\cosh(dx + c)^6 - 140*(490*a^3 - 180*a^2*b - 54*a*b^2 - 155*b^3)*\cosh(dx + c)^4 + 40*a^3 - 135*a^2*b - 48*a*b^2 + 30*b^3 \\
& ^3 - 150*(28*a^3 + 13*b^3)*\cosh(dx + c)^2)*\sinh(dx + c)^4 + 8*(12825*a^2*b*\cosh(dx + c)^17 - 3060*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*\cosh(dx + c)^15 - 1400*(23*a^3 - 3*a*b^2 - 13*b^3)*\cosh(dx + c)^13 - 910*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*\cosh(dx + c)^11 - 110*(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*\cosh(dx + c)^9 - 300*(154*a^3 - 27*a^2*b + 18*a*b^2 + 49*b^3)*\cosh(dx + c)^7 - 28*(490*a^3 - 180*a^2*b - 54*a*b^2 - 155*b^3)*\cosh(dx + c)^5 - 50*(28*a^3 + 13*b^3)*\cosh(dx + c)^3 + (40*a^3 - 135*a^2*b - 48*a*b^2 + 30*b^3)*\cosh(dx + c))*\sinh(dx + c)^3 + 20*a^3 + 12*a*b^2 + 10*(10*a^3 - 9*a^2*b + 6*a*b^2)*\cosh(dx + c)^2 + 2*(8550*a^2*b*\cosh(dx + c)^18 - 2295*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*\cosh(dx + c)^16 - 1200*(23*a^3 - 3*a*b^2 - 13*b^3)*\cosh(dx + c)^14 - 910*(76*a^3 + 36*a^2*b - 24*a*b^2 + 31*b^3)*\cosh(dx + c)^12 - 132*(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b^3)*\cosh(dx + c)^10 - 450*(154*a^3 - 27*a^2*b + 18*a*b^2 + 49*b^3)*\cosh(dx + c)^8 - 56*(490*a^3 - 180*a^2*b - 54*a*b^2 - 155*b^3)*\cosh(dx + c)^6 - 150*(28*a^3 + 13*b^3)*\cosh(dx + c)^4 + 50*a^3 - 45*a^2*b + 30*a*b^2 + 6*(40*a^3 - 135*a^2*b - 48*a*b^2 + 30*b^3)*\cosh(dx + c)^2)*\sinh(dx + c)^2 - 45*(a^2*b*\cosh(dx + c)^22 + 22*a^2*b*\cosh(dx + c))*\sinh(dx + c)^21 + a^2*b*\sinh(dx + c)^22 + 5*a^2*b*\cosh(dx + c)^20 + 7*a^2*b*\cosh(dx + c)^18 + (231*a^2*b*\cosh(dx + c)^2 + 5*a^2*b)*\sinh(dx + c)^20 + 20*(77*a^2*b*\cosh(dx + c)^3 + 5*a^2*b*\cosh(dx + c))*\sinh(dx + c)^19 - 5*a^2*b*\cosh(dx + c)^16 + (7315*a^2*b*\cosh(dx + c)^4 + 950*a^2*b*\cosh(dx + c)^2 + 7*a^2*b)*\sinh(dx + c)^18 + 6*(4389*a^2*b*\cosh(dx + c)^5 + 950*a^2*b*\cosh(dx + c)^3 + 21*a^2*b*\cosh(dx + c))*\sinh(dx + c)^17 - 22*a^2*b*\cosh(dx + c)^14 + (74613*a^2*b*\cosh(dx + c)^6 + 24225*a^2*b*\cosh(dx + c)^4 + 1071*a^2*b*\cosh(dx + c)^2 - 5*a^2*b)*\sinh(dx + c)^16 + 16*(10659*a^2*b*\cosh(dx + c)^7 + 4845*a^2*b*\cosh(dx + c)^5 + 357*a^2*b*\cosh(dx + c)^3 - 5*a^2*b*\cosh(dx + c))*\sinh(dx + c)^15 - 14*a^2*b*\cosh(dx + c)^12 + 2*(159885*a^2*b*\cosh(dx + c)^8 + 96900*a^2*b*\cosh(dx + c)^6 + 10710*a^2*b*\cosh(dx + c)^4 - 300*a^2*b*\cosh(dx + c)^2 - 11*a^2*b)*\sinh(dx + c)^14 + 4*(124355*a^2*b*\cosh(dx + c)^9 + 96900*a^2*b*\cosh(dx + c)^7 + 14994*a^2*b*\cosh(dx + c)^5 - 700*a^2*b*\cosh(dx + c)^3 - 77*a^2*b*\cosh(dx + c))*\sinh(dx + c)^13 + 14*a^2*b*\cosh(dx + c)^10 + 2*(323323*a^2*b*\cosh(dx + c)^10 + 314925*a^2*b*\cosh(dx + c)^8 + 64974*a^2*b*\cosh(dx + c)^6 - 4550*a^2*b*\cosh(dx + c)^4 - 1001*a^2*b*\cosh(dx + c)^2 - 7*a^2*b)*\sinh(dx + c)^12 + 8*(88179*a^2*b*\cosh(dx + c)^11 + 104975*a^2*b*\cosh(dx + c)^9 + 27846*a^2*b*\cosh(dx + c)^7 - 2730*a^2*b*\cosh(dx + c)^5 - 1001*a^2*b*\cosh(dx + c)^3 - 21*a^2*b*\cosh(dx + c)
\end{aligned}$$

$$\begin{aligned}
&)\sinh(dx + c)^{11} + 22a^2b\cosh(dx + c)^8 + 2(323323a^2b\cosh(dx + c)^{12} + 461890a^2b\cosh(dx + c)^{10} + 153153a^2b\cosh(dx + c)^8 - 20020a^2b\cosh(dx + c)^6 - 11011a^2b\cosh(dx + c)^4 - 462a^2b\cosh(dx + c)^2 + 7a^2b)\sinh(dx + c)^{10} + 4(124355a^2b\cosh(dx + c)^{13} + 209950a^2b\cosh(dx + c)^{11} + 85085a^2b\cosh(dx + c)^9 - 14300a^2b\cosh(dx + c)^7 - 11011a^2b\cosh(dx + c)^5 - 770a^2b\cosh(dx + c)^3 + 35a^2b\cosh(dx + c))\sinh(dx + c)^9 + 5a^2b\cosh(dx + c)^6 + 2(159885a^2b\cosh(dx + c)^{14} + 314925a^2b\cosh(dx + c)^{12} + 153153a^2b\cosh(dx + c)^{10} - 32175a^2b\cosh(dx + c)^8 - 33033a^2b\cosh(dx + c)^6 - 3465a^2b\cosh(dx + c)^4 + 315a^2b\cosh(dx + c)^2 + 11a^2b)\sinh(dx + c)^8 + 16(10659a^2b\cosh(dx + c)^{15} + 24225a^2b\cosh(dx + c)^{13} + 13923a^2b\cosh(dx + c)^{11} - 3575a^2b\cosh(dx + c)^9 - 4719a^2b\cosh(dx + c)^7 - 693a^2b\cosh(dx + c)^5 + 105a^2b\cosh(dx + c)^3 + 11a^2b\cosh(dx + c))\sinh(dx + c)^7 - 7a^2b\cosh(dx + c)^4 + (74613a^2b\cosh(dx + c)^{16} + 193800a^2b\cosh(dx + c)^{14} + 129948a^2b\cosh(dx + c)^{12} - 40040a^2b\cosh(dx + c)^{10} - 66066a^2b\cosh(dx + c)^8 - 12936a^2b\cosh(dx + c)^6 + 2940a^2b\cosh(dx + c)^4 + 616a^2b\cosh(dx + c)^2 + 5a^2b)\sinh(dx + c)^6 + 2(13167a^2b\cosh(dx + c)^{17} + 38760a^2b\cosh(dx + c)^{15} + 29988a^2b\cosh(dx + c)^{13} - 10920a^2b\cosh(dx + c)^{11} - 22022a^2b\cosh(dx + c)^9 - 5544a^2b\cosh(dx + c)^7 + 1764a^2b\cosh(dx + c)^5 + 616a^2b\cosh(dx + c)^3 + 15a^2b\cosh(dx + c))\sinh(dx + c)^5 - 5a^2b\cosh(dx + c)^2 + (7315a^2b\cosh(dx + c)^{18} + 24225a^2b\cosh(dx + c)^{16} + 21420a^2b\cosh(dx + c)^{14} - 9100a^2b\cosh(dx + c)^{12} - 22022a^2b\cosh(dx + c)^{10} - 6930a^2b\cosh(dx + c)^8 + 2940a^2b\cosh(dx + c)^6 + 1540a^2b\cosh(dx + c)^4 + 75a^2b\cosh(dx + c)^2 - 7a^2b)\sinh(dx + c)^4 + 4(385a^2b\cosh(dx + c)^{19} + 1425a^2b\cosh(dx + c)^{17} + 1428a^2b\cosh(dx + c)^{15} - 700a^2b\cosh(dx + c)^{13} - 2002a^2b\cosh(dx + c)^{11} - 770a^2b\cosh(dx + c)^9 + 420a^2b\cosh(dx + c)^7 + 308a^2b\cosh(dx + c)^5 + 25a^2b\cosh(dx + c)^3 - 7a^2b\cosh(dx + c))\sinh(dx + c)^3 - a^2b + (231a^2b\cosh(dx + c)^{20} + 950a^2b\cosh(dx + c)^{18} + 1071a^2b\cosh(dx + c)^{16} - 600a^2b\cosh(dx + c)^{14} - 2002a^2b\cosh(dx + c)^{12} - 924a^2b\cosh(dx + c)^{10} + 630a^2b\cosh(dx + c)^8 + 616a^2b\cosh(dx + c)^6 + 75a^2b\cosh(dx + c)^4 - 42a^2b\cosh(dx + c)^2 - 5a^2b)\sinh(dx + c)^2 + 2(11a^2b\cosh(dx + c)^{21} + 50a^2b\cosh(dx + c)^{19} + 63a^2b\cosh(dx + c)^{17} - 40a^2b\cosh(dx + c)^{15} - 154a^2b\cosh(dx + c)^{13} - 84a^2b\cosh(dx + c)^{11} + 70a^2b\cosh(dx + c)^9 + 88a^2b\cosh(dx + c)^7 + 15a^2b\cosh(dx + c)^5 - 14a^2b\cosh(dx + c)^3 - 5a^2b\cosh(dx + c))\sinh(dx + c) + \log(2\cosh(dx + c)/(\cosh(dx + c) - \sinh(dx + c))) + 45(a^2b\cosh(dx + c)^{22} + 22a^2b\cosh(dx + c)\sinh(dx + c)^{21} + a^2b\sinh(dx + c)^{22} + 5a^2b\cosh(dx + c)^{20} + 7a^2b\cosh(dx + c)^{18} + (231a^2b\cosh(dx + c)^2 + 5a^2b)\sinh(dx + c)^{20} + 20(77a^2b\cosh(dx + c)^3 + 5a^2b\cosh(dx + c))\sinh(dx + c)^{19} - 5a^2b\cosh(dx + c)^{16} + (7315a^2b\cosh(dx + c)^4 + 950a^2b\cosh(dx + c)^2 + 7a^2b)\sinh(dx + c)^{18} + 6(4389a^2b\cosh(dx + c)^5 + 950a^2b\cosh(dx + c)^3 + 21a^2b\cosh(dx + c))\sinh(dx + c)^{17} - 22a^2b\cosh(dx + c)^{14} + (74613a^2b\cosh(dx + c)^6 + 24225a^2b\cosh(dx + c)^4 + 1071a^2b\cosh(dx + c)^2 - 5a^2b)\sinh(dx + c)^{16} + 16(10659a^2b\cosh(dx + c)^7 + 4845a^2b\cosh(dx + c)^5 + 357a^2b\cosh(dx + c)^3 - 5a^2b\cosh(dx + c))\sinh(dx + c)^{15} - 14a^2b\cosh(dx + c)^{12} + 2(159885a^2b\cosh(dx + c)^8 + 96900a^2b\cosh(dx + c)^6 + 10710a^2b\cosh(dx + c)^4 - 300a^2b\cosh(dx + c)^2 - 11a^2b)\sinh(dx + c)^{14} + 4(124355a^2b\cosh(dx + c)^9 + 96900a^2b\cosh(dx + c)^7 + 14994a^2b\cosh(dx + c)^5 - 700a^2b\cosh(dx + c)^3 - 77a^2b\cosh(dx + c))\sinh(dx + c)^{13} + 14a^2b\cosh(dx + c)^{10} + 2(323323a^2b\cosh(dx + c)^{10} + 314925a^2b\cosh(dx + c)^8 + 64974a^2b\cosh(dx + c)^6 - 4550a^2b\cosh(dx + c)^4 - 1001a^2b\cosh(dx + c)^2 - 7a^2b)\sinh(dx + c)^{12} + 8(88179a^2b\cosh(dx + c)^{11} + 104975a^2b\cosh(dx + c)^9 + 27846a^2b\cosh(dx + c)^7 - 2730a^2b\cosh(dx + c)^5 - 1001a^2b\cosh(dx + c)^3 - 21a^2b\cosh(dx + c))\sinh(dx + c)^5 - 1001a^2b\cosh(dx + c)^3 - 21a^2b\cosh(dx + c))\sinh(dx + c)^5
\end{aligned}$$

$$\begin{aligned}
& *x + c)^{11} + 22*a^2*b*cosh(d*x + c)^8 + 2*(323323*a^2*b*cosh(d*x + c)^{12} + \\
& 461890*a^2*b*cosh(d*x + c)^{10} + 153153*a^2*b*cosh(d*x + c)^8 - 20020*a^2*b* \\
& cosh(d*x + c)^6 - 11011*a^2*b*cosh(d*x + c)^4 - 462*a^2*b*cosh(d*x + c)^2 + \\
& 7*a^2*b)*sinh(d*x + c)^{10} + 4*(124355*a^2*b*cosh(d*x + c)^{13} + 209950*a^2* \\
& b*cosh(d*x + c)^{11} + 85085*a^2*b*cosh(d*x + c)^9 - 14300*a^2*b*cosh(d*x + c \\
&)^7 - 11011*a^2*b*cosh(d*x + c)^5 - 770*a^2*b*cosh(d*x + c)^3 + 35*a^2*b*co \\
& sh(d*x + c))*sinh(d*x + c)^9 + 5*a^2*b*cosh(d*x + c)^6 + 2*(159885*a^2*b*co \\
& sh(d*x + c)^{14} + 314925*a^2*b*cosh(d*x + c)^{12} + 153153*a^2*b*cosh(d*x + c) \\
& ^{10} - 32175*a^2*b*cosh(d*x + c)^8 - 33033*a^2*b*cosh(d*x + c)^6 - 3465*a^2* \\
& b*cosh(d*x + c)^4 + 315*a^2*b*cosh(d*x + c)^2 + 11*a^2*b)*sinh(d*x + c)^8 + \\
& 16*(10659*a^2*b*cosh(d*x + c)^{15} + 24225*a^2*b*cosh(d*x + c)^{13} + 13923*a^ \\
& 2*b*cosh(d*x + c)^{11} - 3575*a^2*b*cosh(d*x + c)^9 - 4719*a^2*b*cosh(d*x + c \\
&)^7 - 693*a^2*b*cosh(d*x + c)^5 + 105*a^2*b*cosh(d*x + c)^3 + 11*a^2*b*cosh \\
& (d*x + c))*sinh(d*x + c)^7 - 7*a^2*b*cosh(d*x + c)^4 + (74613*a^2*b*cosh(d* \\
& x + c)^{16} + 193800*a^2*b*cosh(d*x + c)^{14} + 129948*a^2*b*cosh(d*x + c)^{12} - \\
& 40040*a^2*b*cosh(d*x + c)^{10} - 66066*a^2*b*cosh(d*x + c)^8 - 12936*a^2*b*c \\
& osh(d*x + c)^6 + 2940*a^2*b*cosh(d*x + c)^4 + 616*a^2*b*cosh(d*x + c)^2 + 5 \\
& *a^2*b)*sinh(d*x + c)^6 + 2*(13167*a^2*b*cosh(d*x + c)^{17} + 38760*a^2*b*cos \\
& h(d*x + c)^{15} + 29988*a^2*b*cosh(d*x + c)^{13} - 10920*a^2*b*cosh(d*x + c)^{11} \\
& - 22022*a^2*b*cosh(d*x + c)^9 - 5544*a^2*b*cosh(d*x + c)^7 + 1764*a^2*b*co \\
& sh(d*x + c)^5 + 616*a^2*b*cosh(d*x + c)^3 + 15*a^2*b*cosh(d*x + c))*sinh(d* \\
& x + c)^5 - 5*a^2*b*cosh(d*x + c)^2 + (7315*a^2*b*cosh(d*x + c)^{18} + 24225*a \\
& ^2*b*cosh(d*x + c)^{16} + 21420*a^2*b*cosh(d*x + c)^{14} - 9100*a^2*b*cosh(d*x \\
& + c)^{12} - 22022*a^2*b*cosh(d*x + c)^{10} - 6930*a^2*b*cosh(d*x + c)^8 + 2940* \\
& a^2*b*cosh(d*x + c)^6 + 1540*a^2*b*cosh(d*x + c)^4 + 75*a^2*b*cosh(d*x + c) \\
& ^2 - 7*a^2*b)*sinh(d*x + c)^4 + 4*(385*a^2*b*cosh(d*x + c)^{19} + 1425*a^2*b* \\
& cosh(d*x + c)^{17} + 1428*a^2*b*cosh(d*x + c)^{15} - 700*a^2*b*cosh(d*x + c)^{13} \\
& - 2002*a^2*b*cosh(d*x + c)^{11} - 770*a^2*b*cosh(d*x + c)^9 + 420*a^2*b*cosh \\
& (d*x + c)^7 + 308*a^2*b*cosh(d*x + c)^5 + 25*a^2*b*cosh(d*x + c)^3 - 7*a^2* \\
& b*cosh(d*x + c))*sinh(d*x + c)^3 - a^2*b + (231*a^2*b*cosh(d*x + c)^{20} + 95 \\
& 0*a^2*b*cosh(d*x + c)^{18} + 1071*a^2*b*cosh(d*x + c)^{16} - 600*a^2*b*cosh(d*x \\
& + c)^{14} - 2002*a^2*b*cosh(d*x + c)^{12} - 924*a^2*b*cosh(d*x + c)^{10} + 630*a \\
& ^2*b*cosh(d*x + c)^8 + 616*a^2*b*cosh(d*x + c)^6 + 75*a^2*b*cosh(d*x + c)^4 \\
& - 42*a^2*b*cosh(d*x + c)^2 - 5*a^2*b)*sinh(d*x + c)^2 + 2*(11*a^2*b*cosh(d \\
& *x + c)^{21} + 50*a^2*b*cosh(d*x + c)^{19} + 63*a^2*b*cosh(d*x + c)^{17} - 40*a^2 \\
& *b*cosh(d*x + c)^{15} - 154*a^2*b*cosh(d*x + c)^{13} - 84*a^2*b*cosh(d*x + c)^{1 \\
& 1} + 70*a^2*b*cosh(d*x + c)^9 + 88*a^2*b*cosh(d*x + c)^7 + 15*a^2*b*cosh(d*x \\
& + c)^5 - 14*a^2*b*cosh(d*x + c)^3 - 5*a^2*b*cosh(d*x + c))*sinh(d*x + c))* \\
& log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(450*a^2*b*cosh(d* \\
& x + c)^{19} - 135*(2*a^3 - 9*a^2*b + 6*a*b^2 + 2*b^3)*cosh(d*x + c)^{17} - 80*(\\
& 23*a^3 - 3*a*b^2 - 13*b^3)*cosh(d*x + c)^{15} - 70*(76*a^3 + 36*a^2*b - 24*a* \\
& b^2 + 31*b^3)*cosh(d*x + c)^{13} - 12*(700*a^3 + 135*a^2*b + 48*a*b^2 - 245*b \\
& ^3)*cosh(d*x + c)^{11} - 50*(154*a^3 - 27*a^2*b + 18*a*b^2 + 49*b^3)*cosh(d*x \\
& + c)^9 - 8*(490*a^3 - 180*a^2*b - 54*a*b^2 - 155*b^3)*cosh(d*x + c)^7 - 30 \\
& *(28*a^3 + 13*b^3)*cosh(d*x + c)^5 + 2*(40*a^3 - 135*a^2*b - 48*a*b^2 + 30* \\
& b^3)*cosh(d*x + c)^3 + 5*(10*a^3 - 9*a^2*b + 6*a*b^2)*cosh(d*x + c))*sinh(d \\
& *x + c))/(d*cosh(d*x + c)^{22} + 22*d*cosh(d*x + c)*sinh(d*x + c)^{21} + d*sinh \\
& (d*x + c)^{22} + 5*d*cosh(d*x + c)^{20} + (231*d*cosh(d*x + c)^2 + 5*d)*sinh(d* \\
& x + c)^{20} + 20*(77*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^{19} \\
& + 7*d*cosh(d*x + c)^{18} + (7315*d*cosh(d*x + c)^4 + 950*d*cosh(d*x + c)^2 + \\
& 7*d)*sinh(d*x + c)^{18} + 6*(4389*d*cosh(d*x + c)^5 + 950*d*cosh(d*x + c)^3 + \\
& 21*d*cosh(d*x + c))*sinh(d*x + c)^{17} - 5*d*cosh(d*x + c)^{16} + (74613*d*cos \\
& h(d*x + c)^6 + 24225*d*cosh(d*x + c)^4 + 1071*d*cosh(d*x + c)^2 - 5*d)*sinh \\
& (d*x + c)^{16} + 16*(10659*d*cosh(d*x + c)^7 + 4845*d*cosh(d*x + c)^5 + 357*d \\
& *cosh(d*x + c)^3 - 5*d*cosh(d*x + c))*sinh(d*x + c)^{15} - 22*d*cosh(d*x + c) \\
& ^{14} + 2*(159885*d*cosh(d*x + c)^8 + 96900*d*cosh(d*x + c)^6 + 10710*d*cosh(\\
& d*x + c)^4 - 300*d*cosh(d*x + c)^2 - 11*d)*sinh(d*x + c)^{14} + 4*(124355*d*c \\
& osh(d*x + c)^9 + 96900*d*cosh(d*x + c)^7 + 14994*d*cosh(d*x + c)^5 - 700*d* \\
& cosh(d*x + c)^3 - 77*d*cosh(d*x + c))*sinh(d*x + c)^{13} - 14*d*cosh(d*x + c)
\end{aligned}$$

$$\begin{aligned} & ^{12} + 2*(323323*d*cosh(d*x + c)^{10} + 314925*d*cosh(d*x + c)^8 + 64974*d*cos \\ & h(d*x + c)^6 - 4550*d*cosh(d*x + c)^4 - 1001*d*cosh(d*x + c)^2 - 7*d)*sinh(\\ & d*x + c)^{12} + 8*(88179*d*cosh(d*x + c)^{11} + 104975*d*cosh(d*x + c)^9 + 2784 \\ & 6*d*cosh(d*x + c)^7 - 2730*d*cosh(d*x + c)^5 - 1001*d*cosh(d*x + c)^3 - 21* \\ & d*cosh(d*x + c))*sinh(d*x + c)^{11} + 14*d*cosh(d*x + c)^{10} + 2*(323323*d*cos \\ & h(d*x + c)^{12} + 461890*d*cosh(d*x + c)^{10} + 153153*d*cosh(d*x + c)^8 - 2002 \\ & 0*d*cosh(d*x + c)^6 - 11011*d*cosh(d*x + c)^4 - 462*d*cosh(d*x + c)^2 + 7*d \\ &)*sinh(d*x + c)^{10} + 4*(124355*d*cosh(d*x + c)^{13} + 209950*d*cosh(d*x + c)^ \\ & 11 + 85085*d*cosh(d*x + c)^9 - 14300*d*cosh(d*x + c)^7 - 11011*d*cosh(d*x + \\ & c)^5 - 770*d*cosh(d*x + c)^3 + 35*d*cosh(d*x + c))*sinh(d*x + c)^9 + 22*d* \\ & cosh(d*x + c)^8 + 2*(159885*d*cosh(d*x + c)^{14} + 314925*d*cosh(d*x + c)^{12} \\ & + 153153*d*cosh(d*x + c)^{10} - 32175*d*cosh(d*x + c)^8 - 33033*d*cosh(d*x + \\ & c)^6 - 3465*d*cosh(d*x + c)^4 + 315*d*cosh(d*x + c)^2 + 11*d)*sinh(d*x + c) \\ & ^8 + 16*(10659*d*cosh(d*x + c)^{15} + 24225*d*cosh(d*x + c)^{13} + 13923*d*cosh \\ & (d*x + c)^{11} - 3575*d*cosh(d*x + c)^9 - 4719*d*cosh(d*x + c)^7 - 693*d*cosh \\ & (d*x + c)^5 + 105*d*cosh(d*x + c)^3 + 11*d*cosh(d*x + c))*sinh(d*x + c)^7 + \\ & 5*d*cosh(d*x + c)^6 + (74613*d*cosh(d*x + c)^{16} + 193800*d*cosh(d*x + c)^{14} \\ & + 129948*d*cosh(d*x + c)^{12} - 40040*d*cosh(d*x + c)^{10} - 66066*d*cosh(d*x \\ & + c)^8 - 12936*d*cosh(d*x + c)^6 + 2940*d*cosh(d*x + c)^4 + 616*d*cosh(d*x \\ & + c)^2 + 5*d)*sinh(d*x + c)^6 + 2*(13167*d*cosh(d*x + c)^{17} + 38760*d*cosh \\ & (d*x + c)^{15} + 29988*d*cosh(d*x + c)^{13} - 10920*d*cosh(d*x + c)^{11} - 22022* \\ & d*cosh(d*x + c)^9 - 5544*d*cosh(d*x + c)^7 + 1764*d*cosh(d*x + c)^5 + 616*d \\ & *cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^5 - 7*d*cosh(d*x + c)^ \\ & 4 + (7315*d*cosh(d*x + c)^{18} + 24225*d*cosh(d*x + c)^{16} + 21420*d*cosh(d*x \\ & + c)^{14} - 9100*d*cosh(d*x + c)^{12} - 22022*d*cosh(d*x + c)^{10} - 6930*d*cosh(\\ & d*x + c)^8 + 2940*d*cosh(d*x + c)^6 + 1540*d*cosh(d*x + c)^4 + 75*d*cosh(d* \\ & x + c)^2 - 7*d)*sinh(d*x + c)^4 + 4*(385*d*cosh(d*x + c)^{19} + 1425*d*cosh(d \\ & *x + c)^{17} + 1428*d*cosh(d*x + c)^{15} - 700*d*cosh(d*x + c)^{13} - 2002*d*cosh \\ & (d*x + c)^{11} - 770*d*cosh(d*x + c)^9 + 420*d*cosh(d*x + c)^7 + 308*d*cosh(d \\ & *x + c)^5 + 25*d*cosh(d*x + c)^3 - 7*d*cosh(d*x + c))*sinh(d*x + c)^3 - 5*d \\ & *cosh(d*x + c)^2 + (231*d*cosh(d*x + c)^{20} + 950*d*cosh(d*x + c)^{18} + 1071* \\ & d*cosh(d*x + c)^{16} - 600*d*cosh(d*x + c)^{14} - 2002*d*cosh(d*x + c)^{12} - 924 \\ & *d*cosh(d*x + c)^{10} + 630*d*cosh(d*x + c)^8 + 616*d*cosh(d*x + c)^6 + 75*d* \\ & cosh(d*x + c)^4 - 42*d*cosh(d*x + c)^2 - 5*d)*sinh(d*x + c)^2 + 2*(11*d*cos \\ & h(d*x + c)^{21} + 50*d*cosh(d*x + c)^{19} + 63*d*cosh(d*x + c)^{17} - 40*d*cosh(d \\ & *x + c)^{15} - 154*d*cosh(d*x + c)^{13} - 84*d*cosh(d*x + c)^{11} + 70*d*cosh(d*x \\ & + c)^9 + 88*d*cosh(d*x + c)^7 + 15*d*cosh(d*x + c)^5 - 14*d*cosh(d*x + c)^ \\ & 3 - 5*d*cosh(d*x + c))*sinh(d*x + c) - d \end{aligned}$$

giac [B] time = 0.82, size = 437, normalized size = 3.17

$$2520 a^2 b \log \left(e^{(2 dx+2c)} + 1 \right) - 2520 a^2 b \log \left(\left| e^{(2 dx+2c)} - 1 \right| \right) + \frac{140 \left(33 a^2 b e^{(6 dx+6c)} - 99 a^2 b e^{(4 dx+4c)} + 24 a^3 e^{(2 dx+2c)} + 99 a^2 b e^{(2 dx+2c)} \right)}{\left(e^{(2 dx+2c)} - 1 \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/840*(2520*a^2*b*log(e^{(2*d*x + 2*c)} + 1) - 2520*a^2*b*log(abs(e^{(2*d*x + \\ & 2*c)} - 1))) + 140*(33*a^2*b*e^{(6*d*x + 6*c)} - 99*a^2*b*e^{(4*d*x + 4*c)} + 24 \\ & *a^3*e^{(2*d*x + 2*c)} + 99*a^2*b*e^{(2*d*x + 2*c)} - 8*a^3 - 33*a^2*b)/(e^{(2*d \\ & *x + 2*c)} - 1)^3 - (6849*a^2*b*e^{(16*d*x + 16*c)} + 59832*a^2*b*e^{(14*d*x + \\ & 14*c)} + 222012*a^2*b*e^{(12*d*x + 12*c)} - 10080*a*b^2*e^{(12*d*x + 12*c)} - 33 \\ & 60*b^3*e^{(12*d*x + 12*c)} + 459144*a^2*b*e^{(10*d*x + 10*c)} - 26880*a*b^2*e^{(\\ & 10*d*x + 10*c)} + 4480*b^3*e^{(10*d*x + 10*c)} + 580230*a^2*b*e^{(8*d*x + 8*c)} \\ & - 23520*a*b^2*e^{(8*d*x + 8*c)} - 11200*b^3*e^{(8*d*x + 8*c)} + 459144*a^2*b*e^{ \\ & (6*d*x + 6*c)} - 10752*a*b^2*e^{(6*d*x + 6*c)} + 4480*b^3*e^{(6*d*x + 6*c)} + 22 \\ & 2012*a^2*b*e^{(4*d*x + 4*c)} - 8736*a*b^2*e^{(4*d*x + 4*c)} - 3360*b^3*e^{(4*d*x \\ & + 4*c)} + 59832*a^2*b*e^{(2*d*x + 2*c)} - 5376*a*b^2*e^{(2*d*x + 2*c)} + 6849*a \\ & ^2*b - 672*a*b^2)/(e^{(2*d*x + 2*c)} + 1)^8)/d \end{aligned}$$

maple [A] time = 0.56, size = 219, normalized size = 1.59

$$\frac{2a^3 \coth(dx+c)}{3d} - \frac{a^3 \coth(dx+c) \operatorname{csch}(dx+c)^2}{3d} + \frac{3a^2 b}{2d \cosh(dx+c)^2} + \frac{3a^2 b \ln(\tanh(dx+c))}{d} - \frac{3a b^2 \sinh(dx+c)}{4d \cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x)

[Out] $\frac{2}{3}a^3 \coth(dx+c)/d - \frac{1}{3}d a^3 \coth(dx+c) \operatorname{csch}(dx+c)^2 + \frac{3}{2}d a^2 b / \cosh(dx+c)^2 + \frac{3}{4}d a^2 b^2 \ln(\tanh(dx+c)) / d - \frac{3}{4}d a^2 b^2 \sinh(dx+c) / \cosh(dx+c)^5 + \frac{2}{5}a^2 b^2 \tanh(dx+c) / d + \frac{3}{20}d a^2 b^2 \tanh(dx+c) \operatorname{sech}(dx+c)^4 + \frac{1}{5}d a^2 b^2 \tanh(dx+c) \operatorname{sech}(dx+c)^2 - \frac{1}{4}d b^3 \sinh(dx+c)^4 / \cosh(dx+c)^8 - \frac{1}{6}d b^3 \sinh(dx+c)^2 / \cosh(dx+c)^8 - \frac{1}{24}d b^3 / \cosh(dx+c)^8$

maxima [B] time = 0.43, size = 997, normalized size = 7.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")

[Out] $3a^2 b^2 (\log(e^{-dx-c}) + 1) / d + \log(e^{-dx-c}) - 1) / d - \log(e^{-2dx-2c}) + 1) / d + 2e^{-2dx-2c} / (d(2e^{-2dx-2c} + e^{-4dx-4c} + 1)) + 4/5 a^2 b^2 (5e^{-2dx-2c} / (d(5e^{-2dx-2c} + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1)) - 5e^{-4dx-4c} / (d(5e^{-2dx-2c} + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1)) + 15e^{-6dx-6c} / (d(5e^{-2dx-2c} + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1)) + 1 / (d(5e^{-2dx-2c} + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1))) + 4/3 a^3 (3e^{-2dx-2c} / (d(3e^{-2dx-2c} - 3e^{-4dx-4c} + e^{-6dx-6c} - 1)) - 1 / (d(3e^{-2dx-2c} - 3e^{-4dx-4c} + e^{-6dx-6c} - 1))) - 4/3 b^3 (3e^{-4dx-4c} / (d(8e^{-2dx-2c} + 28e^{-4dx-4c} + 56e^{-6dx-6c} + 70e^{-8dx-8c} + 56e^{-10dx-10c} + 28e^{-12dx-12c} + 8e^{-14dx-14c} + e^{-16dx-16c} + 1)) - 4e^{-6dx-6c} / (d(8e^{-2dx-2c} + 28e^{-4dx-4c} + 56e^{-6dx-6c} + 70e^{-8dx-8c} + 56e^{-10dx-10c} + 28e^{-12dx-12c} + 8e^{-14dx-14c} + e^{-16dx-16c} + 1)) + 10e^{-8dx-8c} / (d(8e^{-2dx-2c} + 28e^{-4dx-4c} + 56e^{-6dx-6c} + 70e^{-8dx-8c} + 56e^{-10dx-10c} + 28e^{-12dx-12c} + 8e^{-14dx-14c} + e^{-16dx-16c} + 1)) - 4e^{-10dx-10c} / (d(8e^{-2dx-2c} + 28e^{-4dx-4c} + 56e^{-6dx-6c} + 70e^{-8dx-8c} + 56e^{-10dx-10c} + 28e^{-12dx-12c} + 8e^{-14dx-14c} + e^{-16dx-16c} + 1)) + 3e^{-12dx-12c} / (d(8e^{-2dx-2c} + 28e^{-4dx-4c} + 56e^{-6dx-6c} + 70e^{-8dx-8c} + 56e^{-10dx-10c} + 28e^{-12dx-12c} + 8e^{-14dx-14c} + e^{-16dx-16c} + 1)))$

mupad [B] time = 0.54, size = 646, normalized size = 4.68

$$\frac{96(10b^3 + ab^2)}{5d(5e^{2c+2dx} + 10e^{4c+4dx} + 10e^{6c+6dx} + 5e^{8c+8dx} + e^{10c+10dx} + 1)} - \frac{3d(6e^{2c+2dx} + 15e^{4c+4dx} + 20e^{6c+6dx} + 15e^{8c+8dx} + e^{10c+10dx} + 1)}{5d(5e^{2c+2dx} + 10e^{4c+4dx} + 10e^{6c+6dx} + 5e^{8c+8dx} + e^{10c+10dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tanh(c + d*x))^3/sinh(c + d*x)^4,x)

```
[Out] (96*(a*b^2 + 10*b^3))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (640*b^3)/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (4*(12*a*b^2 + 25*b^3))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (128*b^3)/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) - (2*(6*a*b^2 + 3*a^2*b + 2*b^3))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (6*atan((a^2*b*exp(2*c)*exp(2*d*x)*(-d^2)^(1/2))/(d*(a^4*b^2)^(1/2)))*(a^4*b^2)^(1/2))/(-d^2)^(1/2) - (32*b^3)/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1)) - (4*a^3)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*a^3)/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) + (8*(15*a*b^2 + 11*b^3))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (6*a^2*b)/(d*(exp(2*c + 2*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^3(c + dx))^3 \operatorname{csch}^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**3)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**3)**3*csch(c + d*x)**4, x)
```


$$3.73 \quad \int \frac{\sinh^4(c+dx)}{a+b \tanh^3(c+dx)} dx$$

Optimal. Leaf size=491

$$\frac{a^2 b (a^2 + 2b^2) \log(a + b \tanh^3(c + dx))}{d (a^2 - b^2)^3} - \frac{a^{2/3} \sqrt[3]{b} (3a^{4/3} b^{2/3} + a^2 - b^2) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} \tanh(c+dx)}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} d (a^{2/3} b^{2/3} + a^{4/3} + b^{4/3})^3} + \frac{a^{2/3} \sqrt[3]{b} (a^4 + b^4) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b})}{6d (a^2 - b^2)^3}$$

[Out] $-3/16*a*(a-5*b)*\ln(1-\tanh(d*x+c))/(a+b)^3/d+3/16*a*(a+5*b)*\ln(1+\tanh(d*x+c))/(a-b)^3/d-1/3*a^{(2/3)}*b^{(1/3)}*(a^4+7*a^2*b^2+b^4+3*a^{(2/3)}*b^{(4/3)}*(2*a^2+b^2))*\ln(a^{(1/3)}+b^{(1/3)}*\tanh(d*x+c))/(a^2-b^2)^3/d+1/6*a^{(2/3)}*b^{(1/3)}*(a^4+7*a^2*b^2+b^4+3*a^{(2/3)}*b^{(4/3)}*(2*a^2+b^2))*\ln(a^{(2/3)}-a^{(1/3)}*b^{(1/3)}*\tanh(d*x+c)+b^{(2/3)}*\tanh(d*x+c)^2)/(a^2-b^2)^3/d-a^2*b*(a^2+2*b^2)*\ln(a+b*\tanh(d*x+c)^3)/(a^2-b^2)^3/d-1/3*a^{(2/3)}*b^{(1/3)}*(a^2+3*a^{(4/3)}*b^{(2/3)}-b^2)*\arctan(1/3*(a^{(1/3)}-2*b^{(1/3)}*\tanh(d*x+c))/a^{(1/3)}*3^{(1/2)})/(a^{(4/3)}+a^{(2/3)}*b^{(2/3)}+b^{(4/3)})^3/d*3^{(1/2)}+1/16/(a+b)/d/(1-\tanh(d*x+c))^2+1/16*(-5*a+b)/(a+b)^2/d/(1-\tanh(d*x+c))-1/16/(a-b)/d/(1+\tanh(d*x+c))^2+1/16*(5*a+b)/(a-b)^2/d/(1+\tanh(d*x+c))$

Rubi [A] time = 0.89, antiderivative size = 491, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3663, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{a^2 b (a^2 + 2b^2) \log(a + b \tanh^3(c + dx))}{d (a^2 - b^2)^3} + \frac{a^{2/3} \sqrt[3]{b} (7a^2 b^2 + 3a^{2/3} b^{4/3} (2a^2 + b^2) + a^4 + b^4) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b})}{6d (a^2 - b^2)^3}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^3), x]

[Out] $-((a^{(2/3)}*b^{(1/3)}*(a^2 + 3*a^{(4/3)}*b^{(2/3)} - b^2)*\text{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)}*\text{Tanh}[c + d*x])]/(\text{Sqrt}[3]*a^{(1/3)})]/(\text{Sqrt}[3]*(a^{(4/3)} + a^{(2/3)}*b^{(2/3)} + b^{(4/3)})^3*d) - (3*a*(a - 5*b)*\text{Log}[1 - \text{Tanh}[c + d*x]])/(16*(a + b)^3*d) + (3*a*(a + 5*b)*\text{Log}[1 + \text{Tanh}[c + d*x]])/(16*(a - b)^3*d) - (a^{(2/3)}*b^{(1/3)}*(a^4 + 7*a^2*b^2 + b^4 + 3*a^{(2/3)}*b^{(4/3)}*(2*a^2 + b^2))*\text{Log}[a^{(1/3)} + b^{(1/3)}*\text{Tanh}[c + d*x]])/(3*(a^2 - b^2)^3*d) + (a^{(2/3)}*b^{(1/3)}*(a^4 + 7*a^2*b^2 + b^4 + 3*a^{(2/3)}*b^{(4/3)}*(2*a^2 + b^2))*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*\text{Tanh}[c + d*x] + b^{(2/3)}*\text{Tanh}[c + d*x]^2])/(6*(a^2 - b^2)^3*d) - (a^2*b*(a^2 + 2*b^2)*\text{Log}[a + b*\text{Tanh}[c + d*x]^3])/((a^2 - b^2)^3*d) + 1/(16*(a + b)*d*(1 - \text{Tanh}[c + d*x])^2) - (5*a - b)/(16*(a + b)^2*d*(1 - \text{Tanh}[c + d*x])) - 1/(16*(a - b)*d*(1 + \text{Tanh}[c + d*x])^2) + (5*a + b)/(16*(a - b)^2*d*(1 + \text{Tanh}[c + d*x]))$

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_)*(x_))/((a_) + (b_)*(x_)^3), x_Symbol] := With[{r = Numerator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x] /; FreeQ[{a, b, A, B}, x] && NeQ[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 3663

```
Int[sin[(e_) + (f_)*(x_)]^(m_)*((a_) + (b_)*((c_)*tan[(e_) + (f_)*(x_)]))^(-n_)]^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/2 + 1), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^4(c+dx)}{a+b \tanh^3(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^3(a+bx^3)} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{8(a+b)(-1+x)^3} + \frac{-5a+b}{16(a+b)^2(-1+x)^2} - \frac{3a(a-5b)}{16(a+b)^3(-1+x)} + \frac{1}{8(a-b)(1+x)^3} + \frac{-5a-b}{16(a-b)^2(1+x)^2}\right) dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d} + \frac{3a(a+5b)\log(1+\tanh(c+dx))}{16(a-b)^3d} + \frac{a^2b(a^2-3ab+b^2)}{16(a+b)^3d} \\
&= -\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d} + \frac{3a(a+5b)\log(1+\tanh(c+dx))}{16(a-b)^3d} + \frac{a^2b(a^2-3ab+b^2)}{16(a+b)^3d} \\
&= -\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d} + \frac{3a(a+5b)\log(1+\tanh(c+dx))}{16(a-b)^3d} - \frac{a^2b(a^2-3ab+b^2)}{16(a+b)^3d} \\
&= -\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d} + \frac{3a(a+5b)\log(1+\tanh(c+dx))}{16(a-b)^3d} - \frac{a^{2/3}\sqrt[3]{b}}{16(a+b)^3d} \\
&= -\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d} + \frac{3a(a+5b)\log(1+\tanh(c+dx))}{16(a-b)^3d} - \frac{a^{2/3}\sqrt[3]{b}}{16(a+b)^3d} \\
&= -\frac{a^{2/3}\sqrt[3]{b}(a^2+3a^{4/3}b^{2/3}-b^2)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}(a^{4/3}+a^{2/3}b^{2/3}+b^{4/3})^3d} - \frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d}
\end{aligned}$$

Mathematica [C] time = 4.83, size = 645, normalized size = 1.31

$$3(a(a-b)(12(a^2-6ab+5b^2)(c+dx)+(a+b)^2\sinh(4(c+dx)))-8a(a^3+a^2b+2ab^2+2b^3)\sinh(2(c+dx)))$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^3), x]

[Out] (-32*a*b*RootSum[a - b + 3*a*#1 + 3*b*#1 + 3*a*#1^2 - 3*b*#1^2 + a*#1^3 + b*#1^3 & , (-6*a^3*c - 12*a*b^2*c - 6*a^3*d*x - 12*a*b^2*d*x + 3*a^3*Log[E^(2*(c + d*x)) - #1] + 6*a*b^2*Log[E^(2*(c + d*x)) - #1] - 8*a^3*c*#1 + 4*a^2*b*c*#1 + 8*a*b^2*c*#1 - 4*b^3*c*#1 - 8*a^3*d*x*#1 + 4*a^2*b*d*x*#1 + 8*a*b^2*d*x*#1 - 4*b^3*d*x*#1 + 4*a^3*Log[E^(2*(c + d*x)) - #1]*#1 - 2*a^2*b*Log[E^(2*(c + d*x)) - #1]*#1 - 4*a*b^2*Log[E^(2*(c + d*x)) - #1]*#1 + 2*b^3*Log[E^(2*(c + d*x)) - #1]*#1 - 10*a^3*c*#1^2 + 20*a^2*b*c*#1^2 - 20*a*b^2*c*#1^2 + 4*b^3*c*#1^2 - 10*a^3*d*x*#1^2 + 20*a^2*b*d*x*#1^2 - 20*a*b^2*d*x*#1^2 + 4*b^3*d*x*#1^2 + 5*a^3*Log[E^(2*(c + d*x)) - #1]*#1^2 - 10*a^2*b*Log[E^(2*(c + d*x)) - #1]*#1^2 + 10*a*b^2*Log[E^(2*(c + d*x)) - #1]*#1^2 - 2*b^3*Log[E^(2*(c + d*x)) - #1]*#1^2)/(a - b + 2*a*#1 + 2*b*#1 + a*#1^2 - b*#1^2) &] + 3*(4*b*(5*a^3 + 5*a^2*b + a*b^2 + b^3)*Cosh[2*(c + d*x)] - (a - b)*b*(a + b)^2*Cosh[4*(c + d*x)] - 8*a*(a^3 + a^2*b + 2*a*b^2 + 2*b^3)*Sinh[2*(c + d*x)])

$c + d*x)] + a*(a - b)*(12*(a^2 - 6*a*b + 5*b^2)*(c + d*x) + (a + b)^2*\text{Sinh}[4*(c + d*x)])))/(96*(a - b)^2*(a + b)^3*d)$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.76, size = 362, normalized size = 0.74

$$\frac{24(a^2+5ab)dx}{a^3-3a^2b+3ab^2-b^3} - \frac{(18a^2e^{(4dx+4c)}+90abe^{(4dx+4c)}-8a^2e^{(2dx+2c)}+4abe^{(2dx+2c)}+4b^2e^{(2dx+2c)}+a^2-2ab+b^2)e^{(-4dx)}}{a^3e^{(4c)}-3a^2be^{(4c)}+3ab^2e^{(4c)}-b^3e^{(4c)}} - \frac{64(a^4b+2a^2b^3)\log(|ae^{(6d}}$$

64d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="giac")

[Out] 1/64*(24*(a^2 + 5*a*b)*d*x/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - (18*a^2*e^(4*d*x + 4*c) + 90*a*b*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) + 4*a*b*e^(2*d*x + 2*c) + 4*b^2*e^(2*d*x + 2*c) + a^2 - 2*a*b + b^2)*e^(-4*d*x)/(a^3*e^(4*c) - 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) - b^3*e^(4*c)) - 64*(a^4*b + 2*a^2*b^3)*log(abs(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) + 3*a*e^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c) + a - b))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + (a*e^(4*d*x + 24*c) + b*e^(4*d*x + 24*c) - 8*a*e^(2*d*x + 22*c) + 4*b*e^(2*d*x + 22*c))/(a^2*e^(20*c) + 2*a*b*e^(20*c) + b^2*e^(20*c)))/d

maple [C] time = 0.51, size = 603, normalized size = 1.23

$$ab \left(\sum_{R=\text{RootOf}(a_Z^6+3a_Z^4+8b_Z^3+3a_Z^2+a)} \frac{(3a^2(a^2+2b^2)_R^5+3ab(-2a^2-b^2)_R^4+2(4a^4+13a^2b^2+b^4)_R^3+12ab(a^2+2b^2)_R^2+(a^4-8a^2b^2-b^4)_R}{R^5a+2R^3a+4R^2b+Ra} \right) / (3d(a-b)^3(a+b)^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x)

[Out] -1/3/d*a*b/(a-b)^3/(a+b)^3*sum((3*a^2*(a^2+2*b^2)*_R^5+3*a*b*(-2*a^2-b^2)*_R^4+2*(4*a^4+13*a^2*b^2+b^4)*_R^3+12*a*b*(a^2+2*b^2)*_R^2+(a^4-8*a^2*b^2-2*b^4)*_R+6*a^3*b+3*a*b^3)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))+8/d/(32*a+32*b)/(tanh(1/2*d*x+1/2*c)-1)^4+32/d/(64*a+64*b)/(tanh(1/2*d*x+1/2*c)-1)^3-1/8/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^2*a+5/8/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^2*b-3/8/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)*a+3/8/d/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)*b-3/8/d*a^2/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)-1)+15/8/d*a/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)-1)*b-8/d/(32*a-32*b)/(tanh(1/2*d*x+1/2*c)+1)^4+32/d/(64*a-64*b)/(tanh(1/2*d*x+1/2*c)+1)^3+1/8/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)+1)^2*a+5/8/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)+1)^2*b-3/8/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)+1)*a-3/8/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)+1)*b+3/8/d*a^2/(a-b)^3*ln(tanh(1/2*d*x+1/2*c)+1)+15/8/d*a/(a-b)^3*ln(tanh(1/2*d*x+1/2*c)+1)*b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-6a^4b \left(\frac{-(a-b) \int \frac{1}{(ae^{(6c)}+be^{(6c)})e^{(6dx)}+3(ae^{(4c)}-be^{(4c)})e^{(4dx)}+3(ae^{(2c)}+be^{(2c)})e^{(2dx)}+a-b} dx + x}{a^6 - 3a^4b^2 + 3a^2b^4 - b^6} - \frac{dx + c}{(a^6 - 3a^4b^2 + 3a^2b^4 - b^6)d} \right)$$

$$\begin{aligned}
& \cdot \exp(2*d*x) + 241 * a^6 * b^6 * d * \exp(2*\text{root}(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k)) * \exp(2*d*x) + 220 * a^7 * b^5 * d * \exp(2*\text{root}(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k)) * \exp(2*d*x) - 298 * a^8 * b^4 * d * \exp(2*\text{root}(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k)) * \exp(2*d*x) + 50 * a^9 * b^3 * d * \exp(2*\text{root}(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k)) * \exp(2*d*x) - a^{10} * b^2 * d * \exp(2*\text{root}(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k)) * \exp(2*d*x)) / ((a + b) * (a^2 - b^2) * (a*b^2 - a^2*b - a^3 + b^3) * (3*a*b^2 + 3*a^2*b + a^3 + b^3) * (a*b^4 + a^4*b + a^5 + b^5 - 2*a^2*b^3 - 2*a^3*b^2)) - (288 * \text{root}(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k) * (a^7 * b * d^2 - 8 * a^2 * b^6 * d^2 + 16 * a^3 * b^5 * d^2 - 41 * a^4 * b^4 * d^2 + 37 * a^5 * b^3 * d^2 - 5 * a^6 * b^2 * d^2 + 18 * a^2 * b^6 * d^2 * \exp(2*\text{root}(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k)) * \exp(2*d*x) + 14 * a^3 * b^5 * d^2 * \exp(2*\text{root}(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k)) * \exp(2*d*x) + 79 * a^4 * b^4 * d^2 * \exp(2*\text{root}(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k)) * \exp(2*d*x) + 81 * a^5 * b^3 * d^2 * \exp(2*\text{root}(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k)) * \exp(2*d*x) - a^6 * b^2 * d^2 * \exp(2*\text{root}(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k)) * \exp(2*d*x) + a^7 * b * d^2 * \exp(2*\text{root}(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k)) * \exp(2*d*x)) / ((a + b)^2 * (a - b) * (a*b^2 - a^2*b - a^3 + b^3) * (3*a*b^2 + 3*a^2*b + a^3 + b^3)) - (32 * (22 * a^4 * b^7 - 4 * a^3 * b^8 - 68 * a^5 * b^6 + 85 * a^6 * b^5 - 56 * a^7 * b^4 + 10 * a^8 * b^3 + 2 * a^9 * b^2 + 6 * a^3 * b^8 * \exp(2*\text{root}(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k)) * \exp(2*d*x) - 10 * a^4 * b^7 * \exp(2*\text{root}(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k)) * \exp(2*d*x) + 54 * a^5 * b^6 * \exp(2*\text{root}(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k)) * \exp(2*d*x) - 101 * a^6 * b^5 * \exp(2*\text{root}(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k)) * \exp(2*d*x) + 56 * a^7 * b^4 * \exp(2*\text{root}(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k)) * \exp(2*d*x) - 12 * a^8 * b^3 * \exp(2*\text{root}(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k)) * \exp(2*d*x) + 4 * a^9 * b^2 * \exp(2*\text{root}(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k)) * \exp(2*d*x)) / ((a + b) * (3*a*b^2 + 3*a^2*b + a^3 + b^3) * (a*b^4 + a^4*b + a^5 + b^5 - 2*a^2*b^3 - 2*a^3*b^2)^2) * \text{root}(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k), k, 1, 3) + \exp(4*c + 4*d*x) / (64*d*(a + b)) - \exp(-4*c - 4*d*x) / (64*d*(a - b)) + (\exp(-2*c - 2*d*x) * (2*a + b)) / (16*d*(a - b)^2) - (\exp(2*c + 2*d*x) * (2*a - b)) / (16*d*(a + b)^2) + (3*a*x*(a + 5*b)) / (8*(a - b)^3)
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**4/(a+b*tanh(d*x+c)**3), x)

[Out] Timed out

$$3.74 \quad \int \frac{\sinh^3(c+dx)}{a+b \tanh^3(c+dx)} dx$$

Optimal. Leaf size=33

$$i \operatorname{Int} \left(-\frac{i \sinh^3(c+dx)}{a+b \tanh^3(c+dx)}, x \right)$$

[Out] I*Unintegrable(-I*sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3), x)

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh^3(c+dx)}{a+b \tanh^3(c+dx)} dx$$

Verification is Not applicable to the result.

[In] Int[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^3), x]

[Out] I*Defer[Int][((-I)*Sinh[c + d*x]^3)/(a + b*Tanh[c + d*x]^3), x]

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{a+b \tanh^3(c+dx)} dx = i \int -\frac{i \sinh^3(c+dx)}{a+b \tanh^3(c+dx)} dx$$

Mathematica [A] time = 0.53, size = 826, normalized size = 25.03

$$\cosh(3(c+dx))a^3 + 27b \sinh(c+dx)a^2 - b \sinh(3(c+dx))a^2 - 9(a^2 + 3b^2) \cosh(c+dx)a - b^2 \cosh(3(c+dx))a$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^3), x]

[Out] (-9*a*(a^2 + 3*b^2)*Cosh[c + d*x] + a^3*Cosh[3*(c + d*x)] - a*b^2*Cosh[3*(c + d*x)] - 2*a*b*RootSum[a - b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^4 - 3*b*#1^4 + a*#1^6 + b*#1^6 & , (3*a^2*c + 3*a*b*c + 3*b^2*c + 3*a^2*d*x + 3*a*b*d*x + 3*b^2*d*x + 6*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 6*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 6*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 2*a^2*c*#1^2 - 2*b^2*c*#1^2 + 2*a^2*d*x*#1^2 - 2*b^2*d*x*#1^2 + 4*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 4*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 3*a^2*c*#1^4 - 3*a*b*c*#1^4 + 3*b^2*c*#1^4 + 3*a^2*d*x*#1^4 - 3*a*b*d*x*#1^4 + 3*b^2*d*x*#1^4 + 6*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 6*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 6*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(a*#1 + b*#1 + 2*a*#1^3 - 2*b*#1^3 + a*#1^5 + b*#1^5) &] + 27*a^2*b*Sinh[c + d*x] + 9*b^3*Sinh[c + d*x] - a^2*b*Sinh[3*(c + d*x)] + b^3*Sinh[3*(c + d*x)]/(12*(a - b)^2*(a + b)^2*d)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")

[Out] Timed out

giac [A] time = 3.16, size = 344, normalized size = 10.42

$$\frac{\frac{(9ae^{2dx+2c}+9be^{2dx+2c}-a+b)e^{-3dx}}{a^2e^{3c}-2abe^{3c}+b^2e^{3c}} - \frac{a^2e^{(3dx+30c)}+2abe^{(3dx+30c)}+b^2e^{(3dx+30c)}-9a^2e^{(dx+28c)}+9b^2e^{(dx+28c)}}{a^3e^{27c}+3a^2be^{27c}+3ab^2e^{27c}+b^3e^{27c}} - \frac{6(a^3be^c+a^2b^2e^c+ab^3e^c)dx}{a-b}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="giac")

[Out]
$$-1/24*((9*a*e^{(2*d*x + 2*c)} + 9*b*e^{(2*d*x + 2*c)} - a + b)*e^{(-3*d*x)/(a^2*e^{(3*c)} - 2*a*b*e^{(3*c)} + b^2*e^{(3*c)})} - (a^2*e^{(3*d*x + 30*c)} + 2*a*b*e^{(3*d*x + 30*c)} + b^2*e^{(3*d*x + 30*c)} - 9*a^2*e^{(d*x + 28*c)} + 9*b^2*e^{(d*x + 28*c)})/(a^3*e^{(27*c)} + 3*a^2*b*e^{(27*c)} + 3*a*b^2*e^{(27*c)} + b^3*e^{(27*c)})/d - (6*(a^3*b*e^c + a^2*b^2*e^c + a*b^3*e^c)*d*x/(a - b) - (a^3*b*e^c + a^2*b^2*e^c + a*b^3*e^c)*\log(\text{abs}(a*e^{(6*d*x + 6*c)} + b*e^{(6*d*x + 6*c)} + 3*a*e^{(4*d*x + 4*c)} - 3*b*e^{(4*d*x + 4*c)} + 3*a*e^{(2*d*x + 2*c)} + 3*b*e^{(2*d*x + 2*c)} + a - b))/(a - b))/((a^4 - 2*a^2*b^2 + b^4)*d^2)$$

maple [A] time = 0.47, size = 346, normalized size = 10.48

$$\frac{ab \left(\sum_{R=\text{RootOf}(a_Z^6+3a_Z^4+8b_Z^3+3a_Z^2+a)} \frac{((2a^2+b^2)_R^4-6_R^3ab+2(4a^2+5b^2)_R^2-6ab_R+2a^2+b^2) \ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-R\right)}{_R^5a+2_R^3a+4_R^2b+_Ra}}{3d(a+b)^2(a-b)^2} \right)}{3d(t$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x)

[Out]
$$-1/3/d*a*b/(a+b)^2/(a-b)^2*\text{sum}(((2*a^2+b^2)*_R^4-6*_R^3*a*b+2*(4*a^2+5*b^2)*_R^2-6*a*b*_R+2*a^2+b^2)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*\ln(\tanh(1/2*d*x+1/2*c)-_R), _R=\text{RootOf}(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-16/3/d/(\tanh(1/2*d*x+1/2*c)-1)^3/(16*a+16*b)-8/d/(16*a+16*b)/(\tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/(a+b)^2/(\tanh(1/2*d*x+1/2*c)-1)*a-1/d/(a+b)^2/(\tanh(1/2*d*x+1/2*c)-1)*b-8/d/(16*a-16*b)/(\tanh(1/2*d*x+1/2*c)+1)^2+16/3/d/(\tanh(1/2*d*x+1/2*c)+1)^3/(16*a-16*b)-1/2/d/(a-b)^2/(\tanh(1/2*d*x+1/2*c)+1)*a-1/d/(a-b)^2/(\tanh(1/2*d*x+1/2*c)+1)*b$$

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(a^3 + a^2b - ab^2 - b^3 + (a^3e^{6c} - a^2be^{6c} - ab^2e^{6c} + b^3e^{6c}))e^{6dx} - 9(a^3e^{4c} - 3a^2be^{4c} + 3ab^2e^{4c} - b^3e^{4c})}{24(a^4de^{3c} - 2a^2b^2de^{3c} + b^4de^{3c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")

[Out]
$$1/24*(a^3 + a^2*b - a*b^2 - b^3 + (a^3*e^{(6*c)} - a^2*b*e^{(6*c)} - a*b^2*e^{(6*c)} + b^3*e^{(6*c)})*e^{(6*d*x)} - 9*(a^3*e^{(4*c)} - 3*a^2*b*e^{(4*c)} + 3*a*b^2*e^{(4*c)} - b^3*e^{(4*c)})*e^{(4*d*x)} - 9*(a^3*e^{(2*c)} + 3*a^2*b*e^{(2*c)} + 3*a*b^2*e^{(2*c)} - b^3*e^{(2*c)})*e^{(2*d*x)} + (a^3*b*e^c + a^2*b^2*e^c + a*b^3*e^c)*d*x/(a - b) - (a^3*b*e^c + a^2*b^2*e^c + a*b^3*e^c)*\log(\text{abs}(a*e^{(6*d*x + 6*c)} + b*e^{(6*d*x + 6*c)} + 3*a*e^{(4*d*x + 4*c)} - 3*b*e^{(4*d*x + 4*c)} + 3*a*e^{(2*d*x + 2*c)} + 3*b*e^{(2*d*x + 2*c)} + a - b))/(a - b))/((a^4 - 2*a^2*b^2 + b^4)*d^2)$$

$$2e^{(2c) + b^3e^{(2c)}}e^{(2dx)}e^{(-3dx)}/(a^4de^{(3c)} - 2a^2b^2de^{(3c)} + b^4de^{(3c)}) - 1/8\text{integrate}(16(3(a^3be^{(5c)} - a^2b^2e^{(5c)} + ab^3e^{(5c)})e^{(5dx)} + 2(a^3be^{(3c)} - ab^3e^{(3c)})e^{(3dx)} + 3(a^3be^c + a^2b^2e^c + ab^3e^c)e^{(dx)})/(a^5 - a^4b - 2a^3b^2 + 2a^2b^3 + ab^4 - b^5 + (a^5e^{(6c)} + a^4be^{(6c)} - 2a^3b^2e^{(6c)} - 2a^2b^3e^{(6c)} + ab^4e^{(6c)} + b^5e^{(6c)})e^{(6dx)} + 3(a^5e^{(4c)} - a^4be^{(4c)} - 2a^3b^2e^{(4c)} + 2a^2b^3e^{(4c)} + ab^4e^{(4c)} - b^5e^{(4c)})e^{(4dx)} + 3(a^5e^{(2c)} + a^4be^{(2c)} - 2a^3b^2e^{(2c)} - 2a^2b^3e^{(2c)} + ab^4e^{(2c)} + b^5e^{(2c)})e^{(2dx)}), x)$$

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^3), x)`

[Out] `\text{Hanged}`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)**3/(a+b*tanh(d*x+c)**3), x)`

[Out] Timed out

$$3.75 \quad \int \frac{\sinh^2(c+dx)}{a+b \tanh^3(c+dx)} dx$$

Optimal. Leaf size=384

$$\frac{b(2a^2 + b^2) \log(a + b \tanh^3(c + dx))}{3d(a^2 - b^2)^2} - \frac{a^{2/3} \sqrt[3]{b} (3a^{2/3} b^{4/3} + a^2 + 2b^2) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} \tanh(c + dx) + b^{2/3} \tanh^2(c + dx))}{6d(a^2 - b^2)^2}$$

[Out] $\frac{1}{4}(a-2b) \ln(1-\tanh(dx+c))/(a+b)^{2/d} - \frac{1}{4}(a+2b) \ln(1+\tanh(dx+c))/(a-b)^{2/d} + \frac{1}{3} a^{2/3} b^{1/3} (a^2 + 3a^{2/3} b^{4/3} + 2b^2) \ln(a^{1/3} + b^{1/3} \tanh(dx+c)) / (a^2 - b^2)^{2/d} - \frac{1}{6} a^{2/3} b^{1/3} (a^2 + 3a^{2/3} b^{4/3} + 2b^2) \ln(a^{2/3} - a^{1/3} b^{1/3} \tanh(dx+c) + b^{2/3} \tanh^2(dx+c)) / (a^2 - b^2)^{2/d} + \frac{1}{3} b (2a^2 + b^2) \ln(a + b \tanh^3(dx+c)) / (a^2 - b^2)^{2/d} + \frac{1}{3} a^{2/3} b^{1/3} (a^2 - 3a^{2/3} b^{4/3} + 2b^2) \arctan(1/3(a^{1/3} - 2b^{1/3} \tanh(dx+c)) / a^{1/3} \sqrt[3]{1/2}) / (a^2 - b^2)^{2/d} + \frac{1}{4} (a+b) / d / (1 - \tanh(dx+c)) - \frac{1}{4} (a-b) / d / (1 + \tanh(dx+c))$

Rubi [A] time = 0.63, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3663, 6725, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{a^{2/3} \sqrt[3]{b} (3a^{2/3} b^{4/3} + a^2 + 2b^2) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} \tanh(c + dx) + b^{2/3} \tanh^2(c + dx))}{6d(a^2 - b^2)^2} + \frac{b(2a^2 + b^2) \log(a + b \tanh^3(c + dx))}{3d(a^2 - b^2)^2}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^3), x]

[Out] $(a^{2/3} b^{1/3} (a^2 - 3a^{2/3} b^{4/3} + 2b^2) \text{ArcTan}[(a^{1/3} - 2b^{1/3} \tanh[c + dx]) / (\sqrt[3]{3} a^{1/3})]) / (\sqrt[3]{3} (a^2 - b^2)^{2/d}) + ((a - 2b) \text{Log}[1 - \tanh[c + dx]]) / (4(a + b)^{2/d}) - ((a + 2b) \text{Log}[1 + \tanh[c + dx]]) / (4(a - b)^{2/d}) + (a^{2/3} b^{1/3} (a^2 + 3a^{2/3} b^{4/3} + 2b^2) \text{Log}[a^{1/3} + b^{1/3} \tanh[c + dx]]) / (3(a^2 - b^2)^{2/d}) - (a^{2/3} b^{1/3} (a^2 + 3a^{2/3} b^{4/3} + 2b^2) \text{Log}[a^{2/3} - a^{1/3} b^{1/3} \tanh[c + dx] + b^{2/3} \tanh^2[c + dx]]) / (6(a^2 - b^2)^{2/d}) + (b(2a^2 + b^2) \text{Log}[a + b \tanh^3[c + dx]]) / (3(a^2 - b^2)^{2/d}) + 1/(4(a + b)d(1 - \tanh[c + dx])) - 1/(4(a - b)d(1 + \tanh[c + dx]))$

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^-1, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^m/((a_) + (b_.)*(x_)^n), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^-1, x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
- A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a
/b] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 3663

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x]
&& IntegerQ[m/2]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^n), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(c+dx)}{a+b \tanh^3(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2(a+bx^3)} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{4(a+b)(-1+x)^2} + \frac{a-2b}{4(a+b)^2(-1+x)} + \frac{1}{4(a-b)(1+x)^2} + \frac{-a-2b}{4(a-b)^2(1+x)} + \frac{b(3a^2b-a(a^2+2b^2))}{(a^2-b^2)^2}\right) dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{(a-2b) \log(1-\tanh(c+dx))}{4(a+b)^2d} - \frac{(a+2b) \log(1+\tanh(c+dx))}{4(a-b)^2d} + \frac{b(3a^2b-a(a^2+2b^2))}{4(a+b)d(1-\tanh(c+dx))} \\
&= \frac{(a-2b) \log(1-\tanh(c+dx))}{4(a+b)^2d} - \frac{(a+2b) \log(1+\tanh(c+dx))}{4(a-b)^2d} + \frac{b(3a^2b-a(a^2+2b^2))}{4(a+b)d(1-\tanh(c+dx))} \\
&= \frac{(a-2b) \log(1-\tanh(c+dx))}{4(a+b)^2d} - \frac{(a+2b) \log(1+\tanh(c+dx))}{4(a-b)^2d} + \frac{b(2a^2+b^2) \log(1-\tanh(c+dx))}{3(a+b)d(1-\tanh(c+dx))} \\
&= \frac{(a-2b) \log(1-\tanh(c+dx))}{4(a+b)^2d} - \frac{(a+2b) \log(1+\tanh(c+dx))}{4(a-b)^2d} + \frac{a^{2/3} \sqrt[3]{b} (a^2+b^2) \log(1-\tanh(c+dx))}{3(a+b)d(1-\tanh(c+dx))} \\
&= \frac{(a-2b) \log(1-\tanh(c+dx))}{4(a+b)^2d} - \frac{(a+2b) \log(1+\tanh(c+dx))}{4(a-b)^2d} + \frac{a^{2/3} \sqrt[3]{b} (a^2+b^2) \log(1-\tanh(c+dx))}{3(a+b)d(1-\tanh(c+dx))} \\
&= \frac{(a-2b) \log(1-\tanh(c+dx))}{4(a+b)^2d} - \frac{(a+2b) \log(1+\tanh(c+dx))}{4(a-b)^2d} + \frac{a^{2/3} \sqrt[3]{b} (a^2+b^2) \log(1-\tanh(c+dx))}{3(a+b)d(1-\tanh(c+dx))} \\
&= \frac{a^{2/3} \sqrt[3]{b} (a^2-3a^{2/3}b^{4/3}+2b^2) \tan^{-1}\left(\frac{1-2\sqrt[3]{b} \tanh(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt{3} (a^2-b^2)^2 d} + \frac{(a-2b) \log(1-\tanh(c+dx))}{4(a+b)^2d}
\end{aligned}$$

Mathematica [C] time = 3.81, size = 423, normalized size = 1.10

$$4b\text{RootSum}\left[\#1^3a + \#1^3b + 3\#1^2a - 3\#1^2b + 3\#1a + 3\#1b + a - b\&, \frac{-4\#1^2a^2 \log(e^{2(c+dx)} - \#1) + 8\#1^2a^2c + 8\#1^2a^2dx + 4\#1^2a^2}{\#1^2}\right]$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^3), x]

[Out] -1/12*(6*(a^2 - 3*a*b + 2*b^2)*(c + d*x) + 3*b*(a + b)*Cosh[2*(c + d*x)] + 4*b*RootSum[a - b + 3*a*#1 + 3*b*#1 + 3*a*#1^2 - 3*b*#1^2 + a*#1^3 + b*#1^3 &, (4*a^2*c + 2*b^2*c + 4*a^2*d*x + 2*b^2*d*x - 2*a^2*Log[E^(2*(c + d*x)) - #1] - b^2*Log[E^(2*(c + d*x)) - #1] + 4*a^2*c*#1 - 4*b^2*c*#1 + 4*a^2*d*x*#1 - 4*b^2*d*x*#1 - 2*a^2*Log[E^(2*(c + d*x)) - #1]*#1 + 2*b^2*Log[E^(2*(c + d*x)) - #1]*#1 + 8*a^2*c*#1^2 - 8*a*b*c*#1^2 + 2*b^2*c*#1^2 + 8*a^2*d*x*#1^2 - 8*a*b*d*x*#1^2 + 2*b^2*d*x*#1^2 - 4*a^2*Log[E^(2*(c + d*x)) - #1]*#1^2 + 4*a*b*Log[E^(2*(c + d*x)) - #1]*#1^2 - b^2*Log[E^(2*(c + d*x)) - #1]*#1^2)/(a - b + 2*a*#1 + 2*b*#1 + a*#1^2 - b*#1^2) &] - 3*a*(a + b)*Sinh[2*(c + d*x)]/((a - b)*(a + b)^2*d)

fricas [C] time = 1.41, size = 10695, normalized size = 27.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/72*(36*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*d*x*\cosh(d*x + c)^2 - 9*(a^3 - a^2*b - a*b^2 + b^3)*\cosh(d*x + c)^4 - 36*(a^3 - a^2*b - a*b^2 + b^3)*\cosh(d*x + c)*\sinh(d*x + c)^3 - 9*(a^3 - a^2*b - a*b^2 + b^3)*\sinh(d*x + c)^4 + \\ & 9*a^3 + 9*a^2*b - 9*a*b^2 - 9*b^3 - 4*((a^4 - 2*a^2*b^2 + b^4)*d*\cosh(d*x + c)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*d*\cosh(d*x + c)*\sinh(d*x + c) + (a^4 - 2*a^2*b^2 + b^4)*d*\sinh(d*x + c)^2)*((b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*\sqrt{3} + 1))/(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^{1/3} - 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^{1/3}*(I*\sqrt{3} + 1) + 6*(2*a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*\log(1/18*(a^5 + 2*a^4*b - 2*a^3*b^2 - 4*a^2*b^3 + a*b^4 + 2*b^5)*((b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*\sqrt{3} + 1))/(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^{1/3}*(I*\sqrt{3} + 1) + 6*(2*a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 + a^3 + 2*a^2*b + 2*a*b^2 + 4*b^3 - 1/3*(a^4 + 3*a^3*b + 13*a^2*b^2 + 6*a*b^3 + 4*b^4)*((b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*\sqrt{3} + 1))/(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^{1/3} - 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^{1/3}*(I*\sqrt{3} + 1) + 6*(2*a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d + (a^3 + 8*a*b^2)*\cosh(d*x + c)^2 + 2*(a^3 + 8*a*b^2)*\cosh(d*x + c)*\sinh(d*x + c) + (a^3 + 8*a*b^2)*\sinh(d*x + c)^2 + 18*(2*(a^3 + 4*a^2*b + 5*a*b^2 + 2*b^3)*d*x - 3*(a^3 - a^2*b - a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 - 2*(18*(2*a^2*b + b^3)*\cosh(d*x + c)^2 + 36*(2*a^2*b + b^3)*\cosh(d*x + c)*\sinh(d*x + c) + 18*(2*a^2*b + b^3)*\sinh(d*x + c)^2 - ((a^4 - 2*a^2*b^2 + b^4)*d*\cosh(d*x + c)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*d*\cosh(d*x + c)*\sinh(d*x + c) + (a^4 - 2*a^2*b^2 + b^4)*d*\sinh(d*x + c)^2)*((b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*\sqrt{3} + 1))/(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^{1/3} - 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^{1/3}*(I*\sqrt{3} + 1) + 6*(2*a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d)) - 3*\sqrt{1/3}*((a^4 - 2*a^2*b^2 + b^4)*d*\cosh(d*x + c)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*d*\cosh(d*x + c)*\sinh(d*x + c) + (a^4 - 2*a^2*b^2 + b^4)*d*\sinh(d*x + c)^2)*\sqrt{(288*a^4*b^2 + 720*a^2*b^4 - 36*b^6 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8))*((b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*\sqrt{3} + 1))/(-1/18*(2*a^2*b + b^3)*b^2} \end{aligned}$$

$$\begin{aligned}
& /((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27 \\
& *(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2 \\
& *b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^{(1/3)} \\
& - 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d \\
& - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4 \\
& *d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2 \\
& *b/((a^2 - b^2)^4*d^3))^{(1/3)}*(I*sqrt(3) + 1) + 6*(2*a^2*b + b^3)/(a^4*d - \\
& 2*a^2*b^2*d + b^4*d))^2*d^2 + 12*(2*a^6*b - 3*a^4*b^3 + b^7)*((b^2/(a^4*d^2 \\
& - 2*a^2*b^2*d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4* \\
& d)^2)*(-I*sqrt(3) + 1)/(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 \\
& + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d \\
& - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/5 \\
& 4*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^{(1/3)} - 9*(-1/18*(2*a^2*b + b^3) \\
& *b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + \\
& 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - \\
& 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^{(1 \\
& /3)}*(I*sqrt(3) + 1) + 6*(2*a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d)/(\\
& (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2))*log(-1/36*(a^6 + 3*a \\
& ^5*b - 6*a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 + 2*b^6)*((b^2/(a^4*d^2 - 2*a^2*b^2* \\
& d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sq \\
& rt(3) + 1)/(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)* \\
& (a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d \\
& + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^ \\
& 2)*a^2*b/((a^2 - b^2)^4*d^3))^{(1/3)} - 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^ \\
& 2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b \\
& + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 \\
& + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^{(1/3)}*(I*sqrt(3 \\
&) + 1) + 6*(2*a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 + a^4 - 3*a \\
& ^3*b + 10*a^2*b^2 - 15*a*b^3 - 2*b^4 + 1/6*(a^5 + 4*a^4*b + 16*a^3*b^2 + 19 \\
& *a^2*b^3 + 10*a*b^4 + 4*b^5)*((b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (2 \\
& *a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(-1/18*(2 \\
& *a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d \\
& + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54* \\
& b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^ \\
& 2)^4*d^3))^{(1/3)} - 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + \\
& b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - \\
& 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54* \\
& (a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^{(1/3)}*(I*sqrt(3) + 1) + 6*(2*a^2*b \\
& + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d + (a^4 + a^3*b + 8*a^2*b^2 + 8*a*b \\
& ^3)*cosh(d*x + c)^2 + 2*(a^4 + a^3*b + 8*a^2*b^2 + 8*a*b^3)*cosh(d*x + c)*s \\
& inh(d*x + c) + (a^4 + a^3*b + 8*a^2*b^2 + 8*a*b^3)*sinh(d*x + c)^2 + 1/12*s \\
& qrt(1/3)*((a^6 + 3*a^5*b - 6*a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 + 2*b^6)*((b^2/(\\
& a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d \\
& + b^4*d)^2)*(-I*sqrt(3) + 1)/(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2* \\
& b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/ \\
& (a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3 \\
&) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^{(1/3)} - 9*(-1/18*(2*a^2*b \\
& + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4 \\
& *d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4 \\
& *d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d \\
& ^3))^{(1/3)}*(I*sqrt(3) + 1) + 6*(2*a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d \\
&))*d^2 + 6*(a^5 - 2*a^4*b - 2*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 2*b^5)*d)*sqrt(\\
& (288*a^4*b^2 + 720*a^2*b^4 - 36*b^6 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2* \\
& b^6 + b^8)*((b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(a \\
& ^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(-1/18*(2*a^2*b + b^3)*b^2/ \\
& ((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27* \\
& (2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2 \\
& *b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^{(1/3)} - \\
& 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d -
\end{aligned}$$

$$\begin{aligned}
& 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d) \\
& d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2* \\
& b/((a^2 - b^2)^4*d^3))^{(1/3)}*(I*sqrt(3) + 1) + 6*(2*a^2*b + b^3)/(a^4*d - 2* \\
& a^2*b^2*d + b^4*d))^2*d^2 + 12*(2*a^6*b - 3*a^4*b^3 + b^7)*((b^2/(a^4*d^2 \\
& - 2*a^2*b^2*d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d) \\
&)^2)*(-I*sqrt(3) + 1)/(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 \\
& + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - \\
& 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54 \\
& *(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^{(1/3)} - 9*(-1/18*(2*a^2*b + b^3)* \\
& b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1 \\
& /27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2 \\
& *a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^{(1/ \\
& 3)}*(I*sqrt(3) + 1) + 6*(2*a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d)/((\\
& a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2))) - 2*(18*(2*a^2*b + b^ \\
& 3)*cosh(d*x + c)^2 + 36*(2*a^2*b + b^3)*cosh(d*x + c)*sinh(d*x + c) + 18*(2 \\
& *a^2*b + b^3)*sinh(d*x + c)^2 - ((a^4 - 2*a^2*b^2 + b^4)*d*cosh(d*x + c)^2 \\
& + 2*(a^4 - 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + c) + (a^4 - 2*a^2*b^ \\
& 2 + b^4)*d*sinh(d*x + c)^2)*((b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (2* \\
& a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d))^2)*(-I*sqrt(3) + 1)/(-1/18*(2* \\
& a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d \\
& + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b \\
& /((a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2) \\
&)^4*d^3))^{(1/3)} - 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + \\
& b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2 \\
& *a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(\\
& a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^{(1/3)}*(I*sqrt(3) + 1) + 6*(2*a^2*b \\
& + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d)) + 3*sqrt(1/3)*((a^4 - 2*a^2*b^2 + b^4) \\
&)*d*cosh(d*x + c)^2 + 2*(a^4 - 2*a^2*b^2 + b^4)*d*cosh(d*x + c)*sinh(d*x + \\
& c) + (a^4 - 2*a^2*b^2 + b^4)*d*sinh(d*x + c)^2)*sqrt((288*a^4*b^2 + 720*a^2 \\
& *b^4 - 36*b^6 - (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8))*((b^2/(a^4* \\
& d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b \\
& ^4*d))^2)*(-I*sqrt(3) + 1)/(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2* \\
& d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4 \\
& *d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + \\
& 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^{(1/3)} - 9*(-1/18*(2*a^2*b + b \\
& ^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) \\
& + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 \\
& - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3)) \\
& ^{(1/3)}*(I*sqrt(3) + 1) + 6*(2*a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))^2 \\
& *d^2 + 12*(2*a^6*b - 3*a^4*b^3 + b^7)*((b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4* \\
& d^2) - (2*a^2*b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d))^2)*(-I*sqrt(3) + 1)/ \\
& (-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2* \\
& a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d) \\
&)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/(\\
& (a^2 - b^2)^4*d^3))^{(1/3)} - 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2* \\
& b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/ \\
& (a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) \\
&) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^{(1/3)}*(I*sqrt(3) + 1) + 6 \\
& *(2*a^2*b + b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d)/((a^8 - 4*a^6*b^2 + 6*a^ \\
& 4*b^4 - 4*a^2*b^6 + b^8)*d^2))) *log(-1/36*(a^6 + 3*a^5*b - 6*a^3*b^3 - 3*a^ \\
& 2*b^4 + 3*a*b^5 + 2*b^6))*((b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (2*a^2 \\
& *b + b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d))^2)*(-I*sqrt(3) + 1)/(-1/18*(2*a^2 \\
& *b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b \\
& ^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 1/54*b/(a \\
& ^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 + 8*b^2)*a^2*b/((a^2 - b^2)^4 \\
& *d^3))^{(1/3)} - 9*(-1/18*(2*a^2*b + b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4 \\
& *d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 1/27*(2*a^2*b + b^3)^3/(a^4*d - 2*a^ \\
& 2*b^2*d + b^4*d)^3 + 1/54*b/(a^4*d^3 - 2*a^2*b^2*d^3 + b^4*d^3) + 1/54*(a^2 \\
& + 8*b^2)*a^2*b/((a^2 - b^2)^4*d^3))^{(1/3)}*(I*sqrt(3) + 1) + 6*(2*a^2*b + b
\end{aligned}$$

$$\begin{aligned} &^3)/(a^4d - 2a^2b^2d + b^4d))^2d^2 + a^4 - 3a^3b + 10a^2b^2 - 15ab^3 - 2b^4 + 1/6*(a^5 + 4a^4b + 16a^3b^2 + 19a^2b^3 + 10ab^4 + 4b^5)*((b^2/(a^4d^2 - 2a^2b^2d^2 + b^4d^2) - (2a^2b + b^3)^2/(a^4d - 2a^2b^2d + b^4d)^2)*(-I\sqrt{3}) + 1)/(-1/18*(2a^2b + b^3)*b^2/((a^4d^2 - 2a^2b^2d^2 + b^4d^2)*(a^4d - 2a^2b^2d + b^4d)) + 1/27*(2a^2b + b^3)^3/(a^4d - 2a^2b^2d + b^4d)^3 + 1/54*b/(a^4d^3 - 2a^2b^2d^3 + b^4d^3) + 1/54*(a^2 + 8b^2)*a^2b/((a^2 - b^2)^4d^3))^{1/3} - 9*(-1/18*(2a^2b + b^3)*b^2/((a^4d^2 - 2a^2b^2d^2 + b^4d^2)*(a^4d - 2a^2b^2d + b^4d)) + 1/27*(2a^2b + b^3)^3/(a^4d - 2a^2b^2d + b^4d)^3 + 1/54*b/(a^4d^3 - 2a^2b^2d^3 + b^4d^3) + 1/54*(a^2 + 8b^2)*a^2b/((a^2 - b^2)^4d^3))^{1/3}*(I\sqrt{3}) + 1) + 6*(2a^2b + b^3)/(a^4d - 2a^2b^2d + b^4d))*d + (a^4 + a^3b + 8a^2b^2 + 8ab^3)*\cosh(dx + c)^2 + 2*(a^4 + a^3b + 8a^2b^2 + 8ab^3)*\cosh(dx + c)*\sinh(dx + c) + (a^4 + a^3b + 8a^2b^2 + 8ab^3)*\sinh(dx + c)^2 - 1/12*\sqrt{1/3)*((a^6 + 3a^5b - 6a^3b^3 - 3a^2b^4 + 3ab^5 + 2b^6)*((b^2/(a^4d^2 - 2a^2b^2d^2 + b^4d^2) - (2a^2b + b^3)^2/(a^4d - 2a^2b^2d + b^4d)^2)*(-I\sqrt{3}) + 1)/(-1/18*(2a^2b + b^3)*b^2/((a^4d^2 - 2a^2b^2d^2 + b^4d^2)*(a^4d - 2a^2b^2d + b^4d)) + 1/27*(2a^2b + b^3)^3/(a^4d - 2a^2b^2d + b^4d)^3 + 1/54*b/(a^4d^3 - 2a^2b^2d^3 + b^4d^3) + 1/54*(a^2 + 8b^2)*a^2b/((a^2 - b^2)^4d^3))^{1/3} - 9*(-1/18*(2a^2b + b^3)*b^2/((a^4d^2 - 2a^2b^2d^2 + b^4d^2)*(a^4d - 2a^2b^2d + b^4d)) + 1/27*(2a^2b + b^3)^3/(a^4d - 2a^2b^2d + b^4d)^3 + 1/54*b/(a^4d^3 - 2a^2b^2d^3 + b^4d^3) + 1/54*(a^2 + 8b^2)*a^2b/((a^2 - b^2)^4d^3))^{1/3}*(I\sqrt{3}) + 1) + 6*(2a^2b + b^3)/(a^4d - 2a^2b^2d + b^4d))*d^2 + 6*(a^5 - 2a^4b - 2a^3b^2 + 4a^2b^3 + ab^4 - 2b^5)*d)*\sqrt{(288a^4b^2 + 720a^2b^4 - 36b^6 - (a^8 - 4a^6b^2 + 6a^4b^4 - 4a^2b^6 + b^8)*((b^2/(a^4d^2 - 2a^2b^2d^2 + b^4d^2) - (2a^2b + b^3)^2/(a^4d - 2a^2b^2d + b^4d)^2)*(-I\sqrt{3}) + 1)/(-1/18*(2a^2b + b^3)*b^2/((a^4d^2 - 2a^2b^2d^2 + b^4d^2)*(a^4d - 2a^2b^2d + b^4d)) + 1/27*(2a^2b + b^3)^3/(a^4d - 2a^2b^2d + b^4d)^3 + 1/54*b/(a^4d^3 - 2a^2b^2d^3 + b^4d^3) + 1/54*(a^2 + 8b^2)*a^2b/((a^2 - b^2)^4d^3))^{1/3} - 9*(-1/18*(2a^2b + b^3)*b^2/((a^4d^2 - 2a^2b^2d^2 + b^4d^2)*(a^4d - 2a^2b^2d + b^4d)) + 1/27*(2a^2b + b^3)^3/(a^4d - 2a^2b^2d + b^4d)^3 + 1/54*b/(a^4d^3 - 2a^2b^2d^3 + b^4d^3) + 1/54*(a^2 + 8b^2)*a^2b/((a^2 - b^2)^4d^3))^{1/3}*(I\sqrt{3}) + 1) + 6*(2a^2b + b^3)/(a^4d - 2a^2b^2d + b^4d))*^2d^2 + 12*(2a^6b - 3a^4b^3 + b^7)*((b^2/(a^4d^2 - 2a^2b^2d^2 + b^4d^2) - (2a^2b + b^3)^2/(a^4d - 2a^2b^2d + b^4d)^2)*(-I\sqrt{3}) + 1)/(-1/18*(2a^2b + b^3)*b^2/((a^4d^2 - 2a^2b^2d^2 + b^4d^2)*(a^4d - 2a^2b^2d + b^4d)) + 1/27*(2a^2b + b^3)^3/(a^4d - 2a^2b^2d + b^4d)^3 + 1/54*b/(a^4d^3 - 2a^2b^2d^3 + b^4d^3) + 1/54*(a^2 + 8b^2)*a^2b/((a^2 - b^2)^4d^3))^{1/3} - 9*(-1/18*(2a^2b + b^3)*b^2/((a^4d^2 - 2a^2b^2d^2 + b^4d^2)*(a^4d - 2a^2b^2d + b^4d)) + 1/27*(2a^2b + b^3)^3/(a^4d - 2a^2b^2d + b^4d)^3 + 1/54*b/(a^4d^3 - 2a^2b^2d^3 + b^4d^3) + 1/54*(a^2 + 8b^2)*a^2b/((a^2 - b^2)^4d^3))^{1/3}*(I\sqrt{3}) + 1) + 6*(2a^2b + b^3)/(a^4d - 2a^2b^2d + b^4d))*d)/((a^8 - 4a^6b^2 + 6a^4b^4 - 4a^2b^6 + b^8)*d^2)) + 36*(2*(a^3 + 4a^2b + 5ab^2 + 2b^3)*d*x*\cosh(dx + c) - (a^3 - a^2b - ab^2 + b^3)*\cosh(dx + c)^3)*\sinh(dx + c))/((a^4 - 2a^2b^2 + b^4)*d*\cosh(dx + c)^2 + 2*(a^4 - 2a^2b^2 + b^4)*d*\cosh(dx + c)*\sinh(dx + c) + (a^4 - 2a^2b^2 + b^4)*d*\sinh(dx + c)^2) \end{aligned}$$

giac [A] time = 0.37, size = 223, normalized size = 0.58

$$\frac{\frac{12(a+2b)dx}{a^2-2ab+b^2} - \frac{3(2ae^{(2dx+2c)}+4be^{(2dx+2c)}-a+b)e^{(-2dx)}}{a^2e^{(2c)}-2abe^{(2c)}+b^2e^{(2c)}} - \frac{8(2a^2b+b^3)\log(|ae^{(6dx+6c)}+be^{(6dx+6c)}+3ae^{(4dx+4c)}-3be^{(4dx+4c)}+3ae^{(2dx+2c)}+3be^{(2dx+2c)}|)}{a^4-2a^2b^2+b^4}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)^2/(a+b*tanh(dx+c)^3),x, algorithm="giac")

[Out] -1/24*(12*(a + 2*b)*d*x/(a^2 - 2*a*b + b^2) - 3*(2*a*e^(2*d*x + 2*c) + 4*b*

$$\frac{e^{(2dx + 2c) - a + b}e^{-2dx} / (a^2e^{(2c) - 2ab}e^{(2c) + b^2}e^{(2c)}) - 8(2a^2b + b^3) \log(\text{abs}(ae^{(6dx + 6c) + b}e^{(6dx + 6c) + 3ae^{(4dx + 4c) - 3b}e^{(4dx + 4c) + 3ae^{(2dx + 2c) + 3b}e^{(2dx + 2c) + a - b})) / (a^4 - 2a^2b^2 + b^4) - 3e^{(2dx + 10c)} / (ae^{(8c) + b}e^{(8c)}))}{d}$$

maple [C] time = 0.46, size = 356, normalized size = 0.93

$$b \left(\sum_{R=\text{RootOf}(aZ^6+3aZ^4+8bZ^3+3aZ^2+a)} \frac{(a(2a^2+b^2)_R^5-3R^4a^2b+6a(a^2+b^2)_R^3+4b(2a^2+b^2)_R^2-3ab^2_R+3a^2b) \ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-R\right)}{-R^5a+2R^3a+4R^2b+Ra} \right) / 3d(a-b)^2(a+b)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(dx+c)^2/(a+b*tanh(dx+c)^3), x)
```

```
[Out] 1/3/d*b/(a-b)^2/(a+b)^2*sum((a*(2*a^2+b^2)*_R^5-3*_R^4*a^2*b+6*a*(a^2+b^2)*_R^3+4*b*(2*a^2+b^2)*_R^2-3*a*b^2*_R+3*a^2*b)/( _R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R), _R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))+4/d/(8*a+8*b)/(tanh(1/2*d*x+1/2*c)-1)^2+8/d/(16*a+16*b)/(tanh(1/2*d*x+1/2*c)-1)+1/2/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)-1)*a-1/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)-1)*b-4/d/(8*a-8*b)/(tanh(1/2*d*x+1/2*c)+1)^2+8/d/(16*a-16*b)/(tanh(1/2*d*x+1/2*c)+1)-1/2/d/(a-b)^2*ln(tanh(1/2*d*x+1/2*c)+1)*a-1/d/(a-b)^2*ln(tanh(1/2*d*x+1/2*c)+1)*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$4a^2b \left(\frac{-(a-b) \int \frac{1}{(ae^{(6c)+be^{(6c)}}e^{(6dx)+3} (ae^{(4c)-be^{(4c)}}e^{(4dx)+3} (ae^{(2c)+be^{(2c)}}e^{(2dx)+a-b})} dx + x}{a^4 - 2a^2b^2 + b^4} - \frac{dx + c}{(a^4 - 2a^2b^2 + b^4)d} \right) + 2b^3 \left(\frac{-(a-b) \int \frac{1}{(ae^{(6c)+be^{(6c)}}e^{(6dx)+3} (ae^{(4c)-be^{(4c)}}e^{(4dx)+3} (ae^{(2c)+be^{(2c)}}e^{(2dx)+a-b})} dx + x}{a^4 - 2a^2b^2 + b^4} - \frac{dx + c}{(a^4 - 2a^2b^2 + b^4)d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(dx+c)^2/(a+b*tanh(dx+c)^3), x, algorithm="maxima")
```

```
[Out] 4*a^2*b*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) + 3*a + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^4 - 2*a^2*b^2 + b^4) - (d*x + c)/((a^4 - 2*a^2*b^2 + b^4)*d)) + 2*b^3*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) + 3*a + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^4 - 2*a^2*b^2 + b^4) - (d*x + c)/((a^4 - 2*a^2*b^2 + b^4)*d)) - 8*a^2*b*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 + a^2*b - a*b^2 - b^3) + 8*a*b^2*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 + a^2*b - a*b^2 - b^3) - 2*b^3*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 + a^2*b - a*b^2 - b^3) - 4*a^2*b*integrate(e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 + a^2*b - a*b^2 - b^3) + 4*b^3*integrate(e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 + a^2*b - a*b^2 - b^3) - 1/8*(4*(a^2*d*e^(2*c) - 3*a*b*d*e^(2*c) + 2*b^2*d*e^(2*c))*x*e^(2*d*x) + a^2 + 2*a*b + b^2 - (a^2*e^(4*c) - b^2*e^(4*c)))*e^(4*d*x))/e^(-2*d*x)/(a^3*d*e^(2*c) + a^2*b*d*e^(2*c) - a*b^2*d*e^(2*c) - b^3*d*e^(2*c))
```

mupad [B] time = 2.81, size = 2100, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sinh(c + d*x)^2/(a + b*\tanh(c + d*x)^3), x)$

[Out] $\text{symsum}(\log(\text{root}(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k) * ((2304*\text{root}(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k) * (146*a^5*b^5*d^2 - 133*a^4*b^6*d^2 - 24*a^3*b^7*d^2 - 12*a^6*b^4*d^2 + 22*a^7*b^3*d^2 + a^8*b^2*d^2 + 32*a^3*b^7*d^2*\exp(2*\text{root}(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*\exp(2*d*x) + 577*a^4*b^6*d^2*\exp(2*\text{root}(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*\exp(2*d*x) + 548*a^5*b^5*d^2*\exp(2*\text{root}(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*\exp(2*d*x) + 70*a^6*b^4*d^2*\exp(2*\text{root}(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*\exp(2*d*x) + 68*a^7*b^3*d^2*\exp(2*\text{root}(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*\exp(2*d*x) + a^8*b^2*d^2*\exp(2*\text{root}(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*\exp(2*d*x))))/((a + b)^8*(a - b)^2*(a^2 - 2*a*b + b^2)) + (1536*(24*a^3*b^8*d + 105*a^4*b^7*d - 156*a^5*b^6*d + 51*a^6*b^5*d - 30*a^7*b^4*d + 6*a^8*b^3*d - 32*a^3*b^8*d*\exp(2*\text{root}(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*\exp(2*d*x) - 509*a^4*b^7*d*\exp(2*\text{root}(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*\exp(2*d*x) - 350*a^5*b^6*d*\exp(2*\text{root}(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*\exp(2*d*x) + 64*a^6*b^5*d*\exp(2*\text{root}(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*\exp(2*d*x) - 50*a^7*b^4*d*\exp(2*\text{root}(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*\exp(2*d*x) + 13*a^8*b^3*d*\exp(2*\text{root}(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*\exp(2*d*x)))/((a + b)^3*(a^2 - b^2)*(a - b)*(a^2 - 2*a*b + b^2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)^2)) + (256*(72*a^5*b^5 - 45*a^4*b^6 - 24*a^3*b^7 - 9*a^6*b^4 + 6*a^7*b^3 + 32*a^3*b^7*\exp(2*\text{root}(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*\exp(2*d*x) + 393*a^4*b^6*\exp(2*\text{root}(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*\exp(2*d*x) + 86*a^5*b^5*\exp(2*\text{root}(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*\exp(2*d*x) + 57*a^6*b^4*\exp(2*\text{root}(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*\exp(2*d*x) + 8*a^7*b^3*\exp(2*\text{root}(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*\exp(2*d*x)))/((a + b)^2*(a^2 - b^2)*(a - b)*(a^2 - 2*a*b + b^2)*(2*a*b + a^2 + b^2)^2*(3*a*b^2 + 3*a^2*b + a^3 + b^3)))*\text{root}(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k), k, 1, 3) - (x*(a + 2*b))/(2*(a - b)^2) + \exp(2*c + 2*d*x)/(8*d*(a + b)) - \exp(-2*c - 2*d*x)/(8*d*(a - b))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\sinh(d*x+c)**2/(a+b*\tanh(d*x+c)**3), x)$

[Out] Timed out

$$3.76 \quad \int \frac{\sinh(c+dx)}{a+b \tanh^3(c+dx)} dx$$

Optimal. Leaf size=31

$$-i \operatorname{Int} \left(\frac{i \sinh(c+dx)}{a+b \tanh^3(c+dx)}, x \right)$$

[Out] -I*Unintegrable(I*sinh(d*x+c)/(a+b*tanh(d*x+c)^3), x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sinh(c+dx)}{a+b \tanh^3(c+dx)} dx$$

Verification is Not applicable to the result.

[In] Int[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^3), x]

[Out] (-I)*Defer[Int][(I*Sinh[c + d*x])/(a + b*Tanh[c + d*x]^3), x]

Rubi steps

$$\int \frac{\sinh(c+dx)}{a+b \tanh^3(c+dx)} dx = - \left(i \int \frac{i \sinh(c+dx)}{a+b \tanh^3(c+dx)} dx \right)$$

Mathematica [A] time = 0.27, size = 409, normalized size = 13.19

$$b \operatorname{RootSum} \left[\#1^6 a + \#1^6 b + 3\#1^4 a - 3\#1^4 b + 3\#1^2 a + 3\#1^2 b + a - b \&, \frac{4\#1^4 a \log\left(-\#1 \sinh\left(\frac{1}{2}(c+dx)\right) + \#1 \cosh\left(\frac{1}{2}(c+dx)\right) - \sinh\right)}{\dots} \right]$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^3), x]

[Out] (6*a*Cosh[c + d*x] + b*RootSum[a - b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^4 - 3*b*#1^4 + a*#1^6 + b*#1^6 & , (2*a*c + b*c + 2*a*d*x + b*d*x + 4*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 2*a*c*#1^4 - b*c*#1^4 + 2*a*d*x*#1^4 - b*d*x*#1^4 + 4*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(a*#1 + b*#1 + 2*a*#1^3 - 2*b*#1^3 + a*#1^5 + b*#1^5) &] - 6*b*Sinh[c + d*x])/(6*(a - b)*(a + b)*d)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^3), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 1.84, size = 189, normalized size = 6.10

$$\frac{\frac{e^{(dx+8c)}}{ae^{(7c)+be^{(7c)}} + \frac{e^{(-dx)}}{ae^c-be^c}}{2d} + \frac{6(2abe^c+b^2e^c)dx}{a-b} - \frac{(2abe^c+b^2e^c)\log(|ae^{(6dx+6c)+be^{(6dx+6c)+3ae^{(4dx+4c)}-3be^{(4dx+4c)+3ae^{(2dx+2c)+3be^{(2dx+2c)}}|)}}{a-b}}{3(a^2-b^2)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="giac")

[Out] 1/2*(e^(d*x + 8*c)/(a*e^(7*c) + b*e^(7*c)) + e^(-d*x)/(a*e^c - b*e^c))/d + 1/3*(6*(2*a*b*e^c + b^2*e^c)*d*x/(a - b) - (2*a*b*e^c + b^2*e^c)*log(abs(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) + 3*a*e^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c) + a - b))/(a - b))/((a^2 - b^2)*d^2)

maple [A] time = 0.53, size = 164, normalized size = 5.29

$$b \left(\frac{\sum_{_R=\text{RootOf}(a_Z^6+3a_Z^4+8b_Z^3+3a_Z^2+a)} \frac{(_R^4a-2_R^3b+6_R^2a-2_Rb+a)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-_R\right)}{_R^5a+2_R^3a+4_R^2b+_Ra}}{3d(a-b)(a+b)} \right) + \frac{4}{d(4a-4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x)

[Out] 1/3/d*b/(a-b)/(a+b)*sum((_R^4*a-2*_R^3*b+6*_R^2*a-2*_R*b+a)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))+4/d/(4*a-4*b)/(tanh(1/2*d*x+1/2*c)+1)-4/d/(4*a+4*b)/(tanh(1/2*d*x+1/2*c)-1)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{((ae^{(2c)} - be^{(2c)})e^{(2dx)} + a + b)e^{(-dx)}}{2(a^2de^c - b^2de^c)} + \frac{1}{2} \int \frac{4((2abe^{(5c)} - a^3e^{(6c)} + a^2be^{(6c)} - ab^2e^{(6c)} - b^3e^{(6c)})e^{(6dx)}}{a^3 - a^2b - ab^2 + b^3 + (a^3e^{(6c)} + a^2be^{(6c)} - ab^2e^{(6c)} - b^3e^{(6c)})e^{(6dx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")

[Out] 1/2*((a*e^(2*c) - b*e^(2*c))*e^(2*d*x) + a + b)*e^(-d*x)/(a^2*d*e^c - b^2*d*e^c) + 1/2*integrate(4*((2*a*b*e^(5*c) - b^2*e^(5*c))*e^(5*d*x) + (2*a*b*e^c + b^2*e^c)*e^(d*x))/(a^3 - a^2*b - a*b^2 + b^3 + (a^3*e^(6*c) + a^2*b*e^(6*c) - a*b^2*e^(6*c) - b^3*e^(6*c))*e^(6*d*x) + 3*(a^3*e^(4*c) - a^2*b*e^(4*c) - a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 3*(a^3*e^(2*c) + a^2*b*e^(2*c) - a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)

mupad [A] time = 87.99, size = 4474, normalized size = 144.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(c + d*x)/(a + b*tanh(c + d*x)^3),x)

[Out] exp(-c - d*x)/(2*(a*d - b*d)) + symsum(log((81920*a^2*b^5*exp(d*x)*exp(root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)) + 221184*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4


```

176*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6
- 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2
*d^2*z^2 - b^2, z, k)^4*a^3*b^8*d^4*exp(d*x)*exp(root(2187*a^6*b^2*d^6*z^6
- 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b
^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)) + 40476
672*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6
- 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2
*d^2*z^2 - b^2, z, k)^4*a^4*b^7*d^4*exp(d*x)*exp(root(2187*a^6*b^2*d^6*z^6
- 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b
^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)) + 70336
512*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6
- 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2
*d^2*z^2 - b^2, z, k)^4*a^5*b^6*d^4*exp(d*x)*exp(root(2187*a^6*b^2*d^6*z^6
- 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b
^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)) + 54411
264*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6
- 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2
*d^2*z^2 - b^2, z, k)^4*a^6*b^5*d^4*exp(d*x)*exp(root(2187*a^6*b^2*d^6*z^6
- 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b
^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)) + 16588
800*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6
- 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2
*d^2*z^2 - b^2, z, k)^4*a^7*b^4*d^4*exp(d*x)*exp(root(2187*a^6*b^2*d^6*z^6
- 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b
^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)) + 66355
2*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 -
729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d
^2*z^2 - b^2, z, k)^4*a^8*b^3*d^4*exp(d*x)*exp(root(2187*a^6*b^2*d^6*z^6 -
2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2
*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)))/(10*a*b^
11 - 10*a^11*b - a^12 + b^12 + 44*a^2*b^10 + 110*a^3*b^9 + 165*a^4*b^8 + 13
2*a^5*b^7 - 132*a^7*b^5 - 165*a^8*b^4 - 110*a^9*b^3 - 44*a^10*b^2))*root(21
87*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d
^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 -
b^2, z, k), k, 1, 6) + exp(c + d*x)/(2*(a*d + b*d))

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)**3), x)

[Out] Timed out

$$3.77 \quad \int \frac{\operatorname{csch}(c+dx)}{a+b \tanh^3(c+dx)} dx$$

Optimal. Leaf size=31

$$i \operatorname{Int} \left(-\frac{\operatorname{icsch}(c+dx)}{a+b \tanh^3(c+dx)}, x \right)$$

[Out] I*Unintegrable(-I*csch(d*x+c)/(a+b*tanh(d*x+c)^3), x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}(c+dx)}{a+b \tanh^3(c+dx)} dx$$

Verification is Not applicable to the result.

[In] Int[Csch[c + d*x]/(a + b*Tanh[c + d*x]^3), x]

[Out] I*Defer[Int][((-I)*Csch[c + d*x])/(a + b*Tanh[c + d*x]^3), x]

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{a+b \tanh^3(c+dx)} dx = i \int -\frac{\operatorname{icsch}(c+dx)}{a+b \tanh^3(c+dx)} dx$$

Mathematica [A] time = 0.18, size = 319, normalized size = 10.29

$$6 \log \left(\tanh \left(\frac{1}{2}(c+dx) \right) \right) - b \operatorname{RootSum} \left[\#1^6 a + \#1^6 b + 3\#1^4 a - 3\#1^4 b + 3\#1^2 a + 3\#1^2 b + a - b \&, \frac{2\#1^4 \log(-\#1 \sinh(\dots))}{\dots} \right]$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^3), x]

[Out] (6*Log[Tanh[(c + d*x)/2]] - b*RootSum[a - b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^4 - 3*b*#1^4 + a*#1^6 + b*#1^6 &, (c + d*x + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1 - 2*c*#1^2 - 2*d*x*#1^2 - 4*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1*#1^2 + c*#1^4 + d*x*#1^4 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(a*#1 + b*#1 + 2*a*#1^3 - 2*b*#1^3 + a*#1^5 + b*#1^5) &])/(6*a*d)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^3), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 1.40, size = 147, normalized size = 4.74

$$\frac{\frac{\log(e^{(dx+c)}+1)}{a} - \frac{\log(|e^{(dx+c)}-1|)}{a}}{d} - \frac{6bdxc^c}{a-b} - \frac{be^c \log(|ae^{(6dx+6c)}+be^{(6dx+6c)}+3ae^{(4dx+4c)}-3be^{(4dx+4c)}+3ae^{(2dx+2c)}+3be^{(2dx+2c)}+a-b|)}{3ad^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="giac")

[Out] $-(\log(e^{(d*x+c)}+1)/a - \log(\text{abs}(e^{(d*x+c)}-1))/a)/d - 1/3*(6*b*d*x*e^c/(a-b) - b*e^c*\log(\text{abs}(a*e^{(6*d*x+6*c)}+b*e^{(6*d*x+6*c)}+3*a*e^{(4*d*x+4*c)}-3*b*e^{(4*d*x+4*c)}+3*a*e^{(2*d*x+2*c)}+3*b*e^{(2*d*x+2*c)}+a-b))/(a-b))/(a*d^2)$

maple [A] time = 0.62, size = 98, normalized size = 3.16

$$\frac{4b \left(\sum_{-R=\text{RootOf}(a_Z^6+3a_Z^4+8b_Z^3+3a_Z^2+a)} \frac{-R^2 \ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-R\right)}{-R^5 a+2-R^3 a+4-R^2 b+Ra}}{3da} \right) + \ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x)

[Out] $-4/3/d/a*b*\text{sum}(_R^2/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*\ln(\tanh(1/2*d*x+1/2*c)-_R),_R=\text{RootOf}(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))+1/d/a*\ln(\tanh(1/2*d*x+1/2*c))$

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{ad} + \frac{\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{ad} - 2 \int \frac{be^{(5dx+5c)} - 2be^{(3dx+3c)} + be^{(dx+c)}}{a^2 - ab + (a^2e^{(6c)} + abe^{(6c)})e^{(6dx)} + 3(a^2e^{(4c)} - abe^{(4c)})e^{(4dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")

[Out] $-\log((e^{(d*x+c)}+1)*e^{(-c)})/(a*d) + \log((e^{(d*x+c)}-1)*e^{(-c)})/(a*d) - 2*\text{integrate}((b*e^{(5*d*x+5*c)} - 2*b*e^{(3*d*x+3*c)} + b*e^{(d*x+c)})/(a^2 - a*b + (a^2*e^{(6*c)} + a*b*e^{(6*c)})*e^{(6*d*x)} + 3*(a^2*e^{(4*c)} - a*b*e^{(4*c)})*e^{(4*d*x)} + 3*(a^2*e^{(2*c)} + a*b*e^{(2*c)})*e^{(2*d*x)}), x)$

mupad [A] time = 16.50, size = 3679, normalized size = 118.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sinh(c+d*x)*(a+b*tanh(c+d*x)^3)),x)

[Out] $\text{symsum}(\log(-1409286144*b^6*\exp(d*x)*\exp(\text{root}(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k))) + 134217728*\text{root}(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)*b^7*d + 1879048192*\text{root}(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)*a*b^6*d - 2818572288*\text{root}(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^2*b^7*d^3 - 40869298176*\text{root}(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^3*b^6*d^3 + 28185722880*\text{root}(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^4*b^5*d^3 + 15502147584*\text{root}(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^5*b^4*d^3 + 18119393280*\text{root}(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^4*b^7*d^5 + 235552112640*\text{root}(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^5*b^6*d^5 + 14495514624*\text{root}(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^6*b^5*d^5)$

$$\begin{aligned}
& 43a^4b^2d^4z^4 + 27a^2b^2d^2z^2 - b^2, z, k)^5 a^6 b^5 d^5 - 219244 \\
& 658688 \text{root}(729a^6 b^2 d^6 z^6 - 729a^8 d^6 z^6 - 243a^4 b^2 d^4 z^4 + 2 \\
& 7a^2 b^2 d^2 z^2 - b^2, z, k)^5 a^7 b^4 d^5 - 48922361856 \text{root}(729a^6 b^2 \\
& d^6 z^6 - 729a^8 d^6 z^6 - 243a^4 b^2 d^4 z^4 + 27a^2 b^2 d^2 z^2 - b^2 \\
& , z, k)^5 a^8 b^3 d^5 - 32614907904 \text{root}(729a^6 b^2 d^6 z^6 - 729a^8 d^6 * \\
& z^6 - 243a^4 b^2 d^4 z^4 + 27a^2 b^2 d^2 z^2 - b^2, z, k)^7 a^6 b^7 d^7 - \\
& 179381993472 \text{root}(729a^6 b^2 d^6 z^6 - 729a^8 d^6 z^6 - 243a^4 b^2 d^4 * \\
& z^4 + 27a^2 b^2 d^2 z^2 - b^2, z, k)^7 a^7 b^6 d^7 - 16307453952 \text{root}(729* \\
& a^6 b^2 d^6 z^6 - 729a^8 d^6 z^6 - 243a^4 b^2 d^4 z^4 + 27a^2 b^2 d^2 z^2 \\
& 2 - b^2, z, k)^7 a^8 b^5 d^7 + 179381993472 \text{root}(729a^6 b^2 d^6 z^6 - 729* \\
& a^8 d^6 z^6 - 243a^4 b^2 d^4 z^4 + 27a^2 b^2 d^2 z^2 - b^2, z, k)^7 a^9 b \\
& ^4 d^7 + 48922361856 \text{root}(729a^6 b^2 d^6 z^6 - 729a^8 d^6 z^6 - 243a^4 b \\
& ^2 d^4 z^4 + 27a^2 b^2 d^2 z^2 - b^2, z, k)^7 a^{10} b^3 d^7 - 1912602624 \text{ro} \\
& \text{ot}(729a^6 b^2 d^6 z^6 - 729a^8 d^6 z^6 - 243a^4 b^2 d^4 z^4 + 27a^2 b^2 \\
& d^2 z^2 - b^2, z, k) a^2 b^5 d - 100663296 \text{root}(729a^6 b^2 d^6 z^6 - 729* \\
& a^8 d^6 z^6 - 243a^4 b^2 d^4 z^4 + 27a^2 b^2 d^2 z^2 - b^2, z, k) a^3 b^4 \\
& * d + 738197504 a b^5 \exp(dx) \exp(\text{root}(729a^6 b^2 d^6 z^6 - 729a^8 d^6 z^6 \\
& - 243a^4 b^2 d^4 z^4 + 27a^2 b^2 d^2 z^2 - b^2, z, k)) + 268435456 \text{root} \\
& (729a^6 b^2 d^6 z^6 - 729a^8 d^6 z^6 - 243a^4 b^2 d^4 z^4 + 27a^2 b^2 d \\
& ^2 z^2 - b^2, z, k)^2 a b^7 d^2 \exp(dx) \exp(\text{root}(729a^6 b^2 d^6 z^6 - 729 \\
& a^8 d^6 z^6 - 243a^4 b^2 d^4 z^4 + 27a^2 b^2 d^2 z^2 - b^2, z, k)) - 291 \\
& 58801408 \text{root}(729a^6 b^2 d^6 z^6 - 729a^8 d^6 z^6 - 243a^4 b^2 d^4 z^4 + \\
& 27a^2 b^2 d^2 z^2 - b^2, z, k)^2 a^2 b^6 d^2 \exp(dx) \exp(\text{root}(729a^6 b^ \\
& 2 d^6 z^6 - 729a^8 d^6 z^6 - 243a^4 b^2 d^4 z^4 + 27a^2 b^2 d^2 z^2 - b^ \\
& 2, z, k)) - 29125246976 \text{root}(729a^6 b^2 d^6 z^6 - 729a^8 d^6 z^6 - 243a^ \\
& 4 b^2 d^4 z^4 + 27a^2 b^2 d^2 z^2 - b^2, z, k)^2 a^3 b^5 d^2 \exp(dx) \exp(\\
& \text{root}(729a^6 b^2 d^6 z^6 - 729a^8 d^6 z^6 - 243a^4 b^2 d^4 z^4 + 27a^2 b \\
& ^2 d^2 z^2 - b^2, z, k)) - 2113929216 \text{root}(729a^6 b^2 d^6 z^6 - 729a^8 d^ \\
& 6 z^6 - 243a^4 b^2 d^4 z^4 + 27a^2 b^2 d^2 z^2 - b^2, z, k)^2 a^4 b^4 d^2 \\
& * \exp(dx) \exp(\text{root}(729a^6 b^2 d^6 z^6 - 729a^8 d^6 z^6 - 243a^4 b^2 d^4 * \\
& z^4 + 27a^2 b^2 d^2 z^2 - b^2, z, k)) - 4831838208 \text{root}(729a^6 b^2 d^6 z^ \\
& 6 - 729a^8 d^6 z^6 - 243a^4 b^2 d^4 z^4 + 27a^2 b^2 d^2 z^2 - b^2, z, k) \\
& ^4 a^3 b^7 d^4 \exp(dx) \exp(\text{root}(729a^6 b^2 d^6 z^6 - 729a^8 d^6 z^6 - 24 \\
& 3a^4 b^2 d^4 z^4 + 27a^2 b^2 d^2 z^2 - b^2, z, k)) + 165490458624 \text{root}(72 \\
& 9a^6 b^2 d^6 z^6 - 729a^8 d^6 z^6 - 243a^4 b^2 d^4 z^4 + 27a^2 b^2 d^2 * \\
& z^2 - b^2, z, k)^4 a^4 b^6 d^4 \exp(dx) \exp(\text{root}(729a^6 b^2 d^6 z^6 - 729* \\
& a^8 d^6 z^6 - 243a^4 b^2 d^4 z^4 + 27a^2 b^2 d^2 z^2 - b^2, z, k)) + 2838 \\
& 70494720 \text{root}(729a^6 b^2 d^6 z^6 - 729a^8 d^6 z^6 - 243a^4 b^2 d^4 z^4 + \\
& 27a^2 b^2 d^2 z^2 - b^2, z, k)^4 a^5 b^5 d^4 \exp(dx) \exp(\text{root}(729a^6 b^ \\
& 2 d^6 z^6 - 729a^8 d^6 z^6 - 243a^4 b^2 d^4 z^4 + 27a^2 b^2 d^2 z^2 - b^ \\
& 2, z, k)) + 132573560832 \text{root}(729a^6 b^2 d^6 z^6 - 729a^8 d^6 z^6 - 243a \\
& ^4 b^2 d^4 z^4 + 27a^2 b^2 d^2 z^2 - b^2, z, k)^4 a^6 b^4 d^4 \exp(dx) \exp \\
& (\text{root}(729a^6 b^2 d^6 z^6 - 729a^8 d^6 z^6 - 243a^4 b^2 d^4 z^4 + 27a^2 * \\
& b^2 d^2 z^2 - b^2, z, k)) + 2717908992 \text{root}(729a^6 b^2 d^6 z^6 - 729a^8 d \\
& ^6 z^6 - 243a^4 b^2 d^4 z^4 + 27a^2 b^2 d^2 z^2 - b^2, z, k)^4 a^7 b^3 d^ \\
& 4 \exp(dx) \exp(\text{root}(729a^6 b^2 d^6 z^6 - 729a^8 d^6 z^6 - 243a^4 b^2 d^4 \\
& * z^4 + 27a^2 b^2 d^2 z^2 - b^2, z, k)) + 21743271936 \text{root}(729a^6 b^2 d^6 * \\
& z^6 - 729a^8 d^6 z^6 - 243a^4 b^2 d^4 z^4 + 27a^2 b^2 d^2 z^2 - b^2, z, \\
& k)^6 a^5 b^7 d^6 \exp(dx) \exp(\text{root}(729a^6 b^2 d^6 z^6 - 729a^8 d^6 z^6 - \\
& 243a^4 b^2 d^4 z^4 + 27a^2 b^2 d^2 z^2 - b^2, z, k)) - 154920812544 \text{root}(\\
& 729a^6 b^2 d^6 z^6 - 729a^8 d^6 z^6 - 243a^4 b^2 d^4 z^4 + 27a^2 b^2 d^ \\
& 2 z^2 - b^2, z, k)^6 a^6 b^6 d^6 \exp(dx) \exp(\text{root}(729a^6 b^2 d^6 z^6 - 72 \\
& 9a^8 d^6 z^6 - 243a^4 b^2 d^4 z^4 + 27a^2 b^2 d^2 z^2 - b^2, z, k)) - 27 \\
& 9944626176 \text{root}(729a^6 b^2 d^6 z^6 - 729a^8 d^6 z^6 - 243a^4 b^2 d^4 z^4 \\
& + 27a^2 b^2 d^2 z^2 - b^2, z, k)^6 a^7 b^5 d^6 \exp(dx) \exp(\text{root}(729a^6 * \\
& b^2 d^6 z^6 - 729a^8 d^6 z^6 - 243a^4 b^2 d^4 z^4 + 27a^2 b^2 d^2 z^2 - \\
& b^2, z, k)) - 105998450688 \text{root}(729a^6 b^2 d^6 z^6 - 729a^8 d^6 z^6 - 243 \\
& a^4 b^2 d^4 z^4 + 27a^2 b^2 d^2 z^2 - b^2, z, k)^6 a^8 b^4 d^6 \exp(dx) * e \\
& \text{xp}(\text{root}(729a^6 b^2 d^6 z^6 - 729a^8 d^6 z^6 - 243a^4 b^2 d^4 z^4 + 27a^
\end{aligned}$$

```

2*b^2*d^2*z^2 - b^2, z, k)) - 2717908992*root(729*a^6*b^2*d^6*z^6 - 729*a^8
*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^6*a^9*b^3*
d^6*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d
^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)))/(13*a*b^12 + 13*a^12*b + a^13 +
b^13 + 78*a^2*b^11 + 286*a^3*b^10 + 715*a^4*b^9 + 1287*a^5*b^8 + 1716*a^6*b
^7 + 1716*a^7*b^6 + 1287*a^8*b^5 + 715*a^9*b^4 + 286*a^10*b^3 + 78*a^11*b^2
))*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^
2*b^2*d^2*z^2 - b^2, z, k), k, 1, 6) + log(exp(d*x + 1/(a*d)) - 1)/(a*d) -
log(exp(d*x - 1/(a*d)) + 1)/(a*d)

```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}(c + dx)}{a + b \tanh^3(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*tanh(d*x+c)**3), x)

[Out] Integral(csch(c + d*x)/(a + b*tanh(c + d*x)**3), x)

$$3.78 \quad \int \frac{\operatorname{csch}^2(c+dx)}{a+b \tanh^3(c+dx)} dx$$

Optimal. Leaf size=157

$$-\frac{\sqrt[3]{b} \log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} \tanh(c+dx) + b^{2/3} \tanh^2(c+dx)\right)}{6a^{4/3}d} + \frac{\sqrt[3]{b} \log\left(\sqrt[3]{a} + \sqrt[3]{b} \tanh(c+dx)\right)}{3a^{4/3}d} + \frac{\sqrt[3]{b} \tan^{-1}\left(\frac{\sqrt[3]{a}-2}{\sqrt{3}a^{4/3}}\right)}{\sqrt{3}a^{4/3}}$$

[Out] $-\operatorname{coth}(d*x+c)/a/d+1/3*b^{(1/3)}*\ln(a^{(1/3)}+b^{(1/3)}*\tanh(d*x+c))/a^{(4/3)}/d-1/6*b^{(1/3)}*\ln(a^{(2/3)}-a^{(1/3)}*b^{(1/3)}*\tanh(d*x+c)+b^{(2/3)}*\tanh(d*x+c)^2)/a^{(4/3)}/d+1/3*b^{(1/3)}*\arctan(1/3*(a^{(1/3)}-2*b^{(1/3)}*\tanh(d*x+c))/a^{(1/3)}*3^{(1/2)})/a^{(4/3)}/d*3^{(1/2)}$

Rubi [A] time = 0.14, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3663, 325, 292, 31, 634, 617, 204, 628}

$$-\frac{\sqrt[3]{b} \log\left(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} \tanh(c+dx) + b^{2/3} \tanh^2(c+dx)\right)}{6a^{4/3}d} + \frac{\sqrt[3]{b} \log\left(\sqrt[3]{a} + \sqrt[3]{b} \tanh(c+dx)\right)}{3a^{4/3}d} + \frac{\sqrt[3]{b} \tan^{-1}\left(\frac{\sqrt[3]{a}-2}{\sqrt{3}a^{4/3}}\right)}{\sqrt{3}a^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^3), x]

[Out] $(b^{(1/3)}*\operatorname{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)}*\operatorname{Tanh}[c + d*x])/(Sqrt[3]*a^{(1/3)})])/(Sqrt[3]*a^{(4/3)*d} - \operatorname{Coth}[c + d*x]/(a*d) + (b^{(1/3)}*\operatorname{Log}[a^{(1/3)} + b^{(1/3)}*\operatorname{Tanh}[c + d*x]])/(3*a^{(4/3)*d} - (b^{(1/3)}*\operatorname{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*\operatorname{Tanh}[c + d*x] + b^{(2/3)}*\operatorname{Tanh}[c + d*x]^2))/(6*a^{(4/3)*d})$

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])⁽⁻¹⁾, Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 325

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a + b*xⁿ)^(p+1)/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*cⁿ*(m+1)), Int[(c*x)^(m+n)*(a + b*xⁿ)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)⁽⁻¹⁾, x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3663

```
Int[sin[(e_) + (f_)*(x_)]^(m_)*((a_) + (b_)*((c_)*tan[(e_) + (f_)*(x_
)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis
t[(c*ff^(m + 1))/f, Subst[Int[(x^m*(a + b*(ff*x)^n)^p]/(c^2 + ff^2*x^2)^(m/
2 + 1), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x]
&& IntegerQ[m/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(c + dx)}{a + b \tanh^3(c + dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{x^2(a+bx^3)} dx, x, \tanh(c + dx)\right)}{d} \\
&= -\frac{\operatorname{coth}(c + dx)}{ad} - \frac{b \operatorname{Subst}\left(\int \frac{x}{a+bx^3} dx, x, \tanh(c + dx)\right)}{ad} \\
&= -\frac{\operatorname{coth}(c + dx)}{ad} + \frac{b^{2/3} \operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx, x, \tanh(c + dx)\right)}{3a^{4/3}d} - \frac{b^{2/3} \operatorname{Subst}\left(\int \frac{1}{a^{2/3} - \sqrt[3]{b}x} dx, x, \tanh(c + dx)\right)}{3a^{4/3}d} \\
&= -\frac{\operatorname{coth}(c + dx)}{ad} + \frac{\sqrt[3]{b} \log(\sqrt[3]{a} + \sqrt[3]{b} \tanh(c + dx))}{3a^{4/3}d} - \frac{\sqrt[3]{b} \operatorname{Subst}\left(\int \frac{-\sqrt[3]{a} \sqrt[3]{b} + 2b^{2/3}}{a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}} dx, x, \tanh(c + dx)\right)}{6a^{4/3}d} \\
&= -\frac{\operatorname{coth}(c + dx)}{ad} + \frac{\sqrt[3]{b} \log(\sqrt[3]{a} + \sqrt[3]{b} \tanh(c + dx))}{3a^{4/3}d} - \frac{\sqrt[3]{b} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} \tanh(c + dx))}{6a^{4/3}d} \\
&= \frac{\sqrt[3]{b} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{b} \tanh(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt{3} a^{4/3}d} - \frac{\operatorname{coth}(c + dx)}{ad} + \frac{\sqrt[3]{b} \log(\sqrt[3]{a} + \sqrt[3]{b} \tanh(c + dx))}{3a^{4/3}d}
\end{aligned}$$

Mathematica [C] time = 0.15, size = 190, normalized size = 1.21

$$\frac{2b \operatorname{RootSum}\left[\#1^3 a + \#1^3 b + 3\#1^2 a - 3\#1^2 b + 3\#1 a + 3\#1 b + a - b\&, \frac{-\log(-\#1 \sinh(c+dx) + \#1 \cosh(c+dx) - \sinh(c+dx))}{3ad}\right]}{3ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^3), x]
```

[Out] $-1/3*(3*\text{Coth}[c + d*x] + 2*b*\text{RootSum}[a - b + 3*a*#1 + 3*b*#1 + 3*a*#1^2 - 3*b*#1^2 + a*#1^3 + b*#1^3 \& , (-c - d*x - \text{Log}[-\text{Cosh}[c + d*x] - \text{Sinh}[c + d*x] + \text{Cosh}[c + d*x]*#1 - \text{Sinh}[c + d*x]*#1] + c*#1 + d*x*#1 + \text{Log}[-\text{Cosh}[c + d*x] - \text{Sinh}[c + d*x] + \text{Cosh}[c + d*x]*#1 - \text{Sinh}[c + d*x]*#1]*#1)/(a + b + 2*a*#1 - 2*b*#1 + a*#1^2 + b*#1^2) \&])/(a*d)$

fricas [B] time = 0.72, size = 640, normalized size = 4.08

$$2\left(\sqrt{3} \cosh(dx + c)^2 + 2\sqrt{3} \cosh(dx + c) \sinh(dx + c) + \sqrt{3} \sinh(dx + c)^2 - \sqrt{3}\right) \left(\frac{b}{a}\right)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}b \cosh(dx + c)}{\dots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")`

[Out] $-1/6*(2*(\text{sqrt}(3)*\cosh(d*x + c)^2 + 2*\text{sqrt}(3)*\cosh(d*x + c)*\sinh(d*x + c) + \text{sqrt}(3)*\sinh(d*x + c)^2 - \text{sqrt}(3))*(b/a)^{(1/3)}*\arctan(-1/3*(\text{sqrt}(3)*b*\cosh(d*x + c)^2 + 2*\text{sqrt}(3)*b*\cosh(d*x + c)*\sinh(d*x + c) + \text{sqrt}(3)*b*\sinh(d*x + c)^2 - (\text{sqrt}(3)*a*\cosh(d*x + c)^2 + 2*\text{sqrt}(3)*a*\cosh(d*x + c)*\sinh(d*x + c) + \text{sqrt}(3)*a*\sinh(d*x + c)^2 + \text{sqrt}(3)*a)*(b/a)^{(2/3)} - (\text{sqrt}(3)*b*\cosh(d*x + c)^2 + 2*\text{sqrt}(3)*b*\cosh(d*x + c)*\sinh(d*x + c) + \text{sqrt}(3)*b*\sinh(d*x + c)^2 - \text{sqrt}(3)*b)*(b/a)^{(1/3)})/b) + (\cosh(d*x + c)^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + c)^2 - 1)*(b/a)^{(1/3)}*\log((a + b)*\cosh(d*x + c)^4 + 4*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a + b)*\sinh(d*x + c)^4 + 2*(a - b)*\cosh(d*x + c)^2 + 2*(3*(a + b)*\cosh(d*x + c)^2 + a - b)*\sinh(d*x + c)^2 + 4*((a + b)*\cosh(d*x + c)^3 + (a - b)*\cosh(d*x + c))*\sinh(d*x + c) - 2*(a*\cosh(d*x + c)^2 + 2*a*\cosh(d*x + c)*\sinh(d*x + c) + a*\sinh(d*x + c)^2 - a)*(b/a)^{(2/3)} + 2*(a*\cosh(d*x + c)^2 + 2*a*\cosh(d*x + c)*\sinh(d*x + c) + a*\sinh(d*x + c)^2 + a)*(b/a)^{(1/3)} + a + b) - 2*(\cosh(d*x + c)^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) + \sinh(d*x + c)^2 - 1)*(b/a)^{(1/3)}*\log((a + b)*\cosh(d*x + c)^2 + 2*(a + b)*\cosh(d*x + c)*\sinh(d*x + c) + (a + b)*\sinh(d*x + c)^2 + 2*a*(b/a)^{(2/3)} - 2*a*(b/a)^{(1/3)} + a - b) + 12)/(a*d*\cosh(d*x + c)^2 + 2*a*d*\cosh(d*x + c)*\sinh(d*x + c) + a*d*\sinh(d*x + c)^2 - a*d)$

giac [A] time = 0.25, size = 21, normalized size = 0.13

$$\frac{2}{ad(e^{2dx+2c} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="giac")`

[Out] $-2/(a*d*(e^{2*d*x + 2*c} - 1))$

maple [C] time = 0.57, size = 121, normalized size = 0.77

$$-\frac{\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{2da} + \frac{2b \left(\sum_{R=\text{RootOf}(a_Z^6+3a_Z^4+8b_Z^3+3a_Z^2+a)} \frac{(-R^3-R)\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - R\right)}{-R^5a+2_R^3a+4_R^2b+Ra}}{3da} \right)}{2da \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3),x)`

[Out] $-1/2/d/a*\tanh(1/2*d*x+1/2*c)+2/3/d/a*b*\text{sum}((_R^3-_R)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*\ln(\tanh(1/2*d*x+1/2*c)-_R), _R=\text{RootOf}(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-1/2/d/a/\tanh(1/2*d*x+1/2*c)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2}{ade^{(2dx+2c)} - ad} - 4 \int \frac{be^{(4dx+4c)} - be^{(2dx+2c)}}{a^2 - ab + (a^2e^{(6c)} + abe^{(6c)})e^{(6dx)} + 3(a^2e^{(4c)} - abe^{(4c)})e^{(4dx)} + 3(a^2e^{(2c)} + abe^{(2c)})e^{(2dx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3), x, algorithm="maxima")

[Out] -2/(a*d*e^(2*d*x + 2*c) - a*d) - 4*integrate((b*e^(4*d*x + 4*c) - b*e^(2*d*x + 2*c))/(a^2 - a*b + (a^2*e^(6*c) + a*b*e^(6*c))*e^(6*d*x) + 3*(a^2*e^(4*c) - a*b*e^(4*c))*e^(4*d*x) + 3*(a^2*e^(2*c) + a*b*e^(2*c))*e^(2*d*x)), x)

mupad [B] time = 8.71, size = 669, normalized size = 4.26

$$\frac{b^{1/3} \ln\left(\frac{256b^3(19a^2b - 24ab^2 + 6a^3 - b^3 + 8a^3e^{2c+2dx})}{3a^{4/3}d}\right)}{3a^{4/3}d} - \frac{2}{ad(e^{2c+2dx} - 1)} + b^{1/3} \ln\left(\frac{256b^3(19a^2b - 24ab^2 + 6a^3 - b^3 + 8a^3e^{2c+2dx})}{3a^{4/3}d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^3)), x)

[Out] (b^(1/3)*log(a^(1/3) - b^(1/3) + a^(1/3)*exp(2*c + 2*d*x) + b^(1/3)*exp(2*c + 2*d*x)))/(3*a^(4/3)*d) - 2/(a*d*(exp(2*c + 2*d*x) - 1)) + (b^(1/3)*log((256*b^3*(19*a^2*b - 24*a*b^2 + 6*a^3 - b^3 + 8*a^3*exp(2*c + 2*d*x) + b^3*exp(2*c + 2*d*x) + 70*a*b^2*exp(2*c + 2*d*x) + 113*a^2*b*exp(2*c + 2*d*x)))/(a^4*(a + b)^6) + (b^(1/3)*((3^(1/2)*1i)/2 - 1/2)*((1536*b^3*d*(8*a^2 - 8*b^2 + 15*a^2*exp(2*c + 2*d*x) + 15*b^2*exp(2*c + 2*d*x) + 66*a*b*exp(2*c + 2*d*x)))/(a^2*(a + b)^6) + (768*b^(7/3)*d*((3^(1/2)*1i)/2 - 1/2)*(24*a^2*b - 19*a*b^2 + a^3 - 6*b^3 + a^3*exp(2*c + 2*d*x) + 8*b^3*exp(2*c + 2*d*x) + 13*a*b^2*exp(2*c + 2*d*x) + 70*a^2*b*exp(2*c + 2*d*x)))/(a^(7/3)*(a + b)^6)))/(3*a^(4/3)*d))*((3^(1/2)*1i)/2 - 1/2))/(3*a^(4/3)*d) - (b^(1/3)*log((256*b^3*(19*a^2*b - 24*a*b^2 + 6*a^3 - b^3 + 8*a^3*exp(2*c + 2*d*x) + b^3*exp(2*c + 2*d*x) + 70*a*b^2*exp(2*c + 2*d*x) + 113*a^2*b*exp(2*c + 2*d*x)))/(a^4*(a + b)^6) - (b^(1/3)*((3^(1/2)*1i)/2 + 1/2)*((1536*b^3*d*(8*a^2 - 8*b^2 + 15*a^2*exp(2*c + 2*d*x) + 15*b^2*exp(2*c + 2*d*x) + 66*a*b*exp(2*c + 2*d*x) + 70*a*b^2*exp(2*c + 2*d*x) + 113*a^2*b*exp(2*c + 2*d*x)))/(a^2*(a + b)^6) - (768*b^(7/3)*d*((3^(1/2)*1i)/2 + 1/2)*(24*a^2*b - 19*a*b^2 + a^3 - 6*b^3 + a^3*exp(2*c + 2*d*x) + 8*b^3*exp(2*c + 2*d*x) + 113*a*b^2*exp(2*c + 2*d*x) + 70*a^2*b*exp(2*c + 2*d*x)))/(a^(7/3)*(a + b)^6)))/(3*a^(4/3)*d))*((3^(1/2)*1i)/2 + 1/2))/(3*a^(4/3)*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^2(c + dx)}{a + b \tanh^3(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2/(a+b*tanh(d*x+c)**3), x)

[Out] Integral(csch(c + d*x)**2/(a + b*tanh(c + d*x)**3), x)

$$3.79 \quad \int \frac{\operatorname{csch}^3(c+dx)}{a+b \tanh^3(c+dx)} dx$$

Optimal. Leaf size=33

$$-i \operatorname{Int} \left(\frac{i \operatorname{csch}^3(c+dx)}{a+b \tanh^3(c+dx)}, x \right)$$

[Out] $-I * \operatorname{Unintegrable}(I * \operatorname{csch}(d * x + c)^3 / (a + b * \operatorname{tanh}(d * x + c)^3), x)$

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b \tanh^3(c+dx)} dx$$

Verification is Not applicable to the result.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d * x]^3 / (a + b * \operatorname{Tanh}[c + d * x]^3), x]$

[Out] $(-I) * \operatorname{Defer}[\operatorname{Int}][I * \operatorname{Csch}[c + d * x]^3 / (a + b * \operatorname{Tanh}[c + d * x]^3), x]$

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b \tanh^3(c+dx)} dx = - \left(i \int \frac{i \operatorname{csch}^3(c+dx)}{a+b \tanh^3(c+dx)} dx \right)$$

Mathematica [A] time = 0.39, size = 201, normalized size = 6.09

$$16b \operatorname{RootSum} \left[\#1^6 a + \#1^6 b + 3\#1^4 a - 3\#1^4 b + 3\#1^2 a + 3\#1^2 b + a - b \&, \frac{2\#1 \log(-\#1 \sinh(\frac{1}{2}(c+dx)) + \#1 \cosh(\frac{1}{2}(c+dx))) - \operatorname{Si}(\frac{1}{2}(c+dx))}{\#1^4 a + \#1^4 b + 2\#1^2 a - 2\#1^2 b} \right]$$

24aa

Antiderivative was successfully verified.

[In] $\operatorname{Integrate}[\operatorname{Csch}[c + d * x]^3 / (a + b * \operatorname{Tanh}[c + d * x]^3), x]$

[Out] $-1/24 * (16 * b * \operatorname{RootSum}[a - b + 3 * a * \#1^2 + 3 * b * \#1^2 + 3 * a * \#1^4 - 3 * b * \#1^4 + a * \#1^6 + b * \#1^6 \&, (c * \#1 + d * x * \#1 + 2 * \operatorname{Log}[-\operatorname{Cosh}[(c + d * x) / 2] - \operatorname{Sinh}[(c + d * x) / 2] + \operatorname{Cosh}[(c + d * x) / 2] * \#1 - \operatorname{Sinh}[(c + d * x) / 2] * \#1) * \#1] / (a + b + 2 * a * \#1^2 - 2 * b * \#1^2 + a * \#1^4 + b * \#1^4) \&] + 3 * (\operatorname{Csch}[(c + d * x) / 2]^2 + 4 * \operatorname{Log}[\operatorname{Tanh}[(c + d * x) / 2]] + \operatorname{Sech}[(c + d * x) / 2]^2)) / (a * d)$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}(\operatorname{csch}(d * x + c)^3 / (a + b * \operatorname{tanh}(d * x + c)^3), x, \text{algorithm} = \text{"fricas"})$

[Out] Timed out

giac [A] time = 0.79, size = 68, normalized size = 2.06

$$\frac{\frac{\log(e^{(dx+c)+1})}{a} - \frac{\log(|e^{(dx+c)} - 1|)}{a} - \frac{2(e^{(3dx+3c)} + e^{(dx+c)})}{a(e^{(2dx+2c)} - 1)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="giac")

[Out] $\frac{1}{2} \cdot (\log(e^{(d \cdot x + c)} + 1) / a - \log(\text{abs}(e^{(d \cdot x + c)} - 1)) / a - 2 \cdot (e^{(3 \cdot d \cdot x + 3 \cdot c)} + e^{(d \cdot x + c)}) / (a \cdot (e^{(2 \cdot d \cdot x + 2 \cdot c)} - 1)^2)) / d$

maple [A] time = 0.60, size = 144, normalized size = 4.36

$$\frac{\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right) b \left(\sum_{R=\text{RootOf}(a_Z^6+3a_Z^4+8b_Z^3+3a_Z^2+a)} \frac{(-R^4-2R^2+1) \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - R\right)}{-R^5 a + 2R^3 a + 4R^2 b + R a}}{3da} \right) + \frac{1}{8da \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)^2}}{8da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x)

[Out] $\frac{1}{8} \cdot \frac{1}{d} \cdot \frac{1}{a} \cdot \tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)^2 - \frac{1}{3} \cdot \frac{1}{d} \cdot \frac{1}{a} \cdot b \cdot \text{sum}\left(\left(\frac{R^4 - 2R^2 + 1}{R^5 a + 2R^3 a + 4R^2 b + R a}\right) \cdot \ln\left(\tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right) - R\right), R = \text{RootOf}\left(\frac{Z^6 a + 3Z^4 a + 8Z^3 b + 3Z^2 a + a}{Z^6 a + 3Z^4 a + 8Z^3 b + 3Z^2 a + a}\right)\right) - \frac{1}{8} \cdot \frac{1}{d} \cdot \frac{1}{a} \cdot \frac{1}{\tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)^2} - \frac{1}{2} \cdot \frac{1}{d} \cdot \frac{1}{a} \cdot \ln\left(\tanh\left(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c\right)\right)$

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-8b \int \frac{e^{(3dx+3c)}}{a^2 - ab + (a^2 e^{(6c)} + a b e^{(6c)}) e^{(6dx)} + 3(a^2 e^{(4c)} - a b e^{(4c)}) e^{(4dx)} + 3(a^2 e^{(2c)} + a b e^{(2c)}) e^{(2dx)}} dx - \frac{e^{(3c)}}{a d e^{(4dx+4c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")

[Out] $-8 \cdot b \cdot \text{integrate}\left(\frac{e^{(3 \cdot d \cdot x + 3 \cdot c)}}{(a^2 - a \cdot b + (a^2 \cdot e^{(6 \cdot c)} + a \cdot b \cdot e^{(6 \cdot c)}) \cdot e^{(6 \cdot d \cdot x)} + 3 \cdot (a^2 \cdot e^{(4 \cdot c)} - a \cdot b \cdot e^{(4 \cdot c)}) \cdot e^{(4 \cdot d \cdot x)} + 3 \cdot (a^2 \cdot e^{(2 \cdot c)} + a \cdot b \cdot e^{(2 \cdot c)}) \cdot e^{(2 \cdot d \cdot x)})}, x\right) - \frac{(e^{(3 \cdot d \cdot x + 3 \cdot c)} + e^{(d \cdot x + c)})}{(a \cdot d \cdot e^{(4 \cdot d \cdot x + 4 \cdot c)} - 2 \cdot a \cdot d \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + a \cdot d) + \frac{1}{2} \cdot \log((e^{(d \cdot x + c)} + 1) \cdot e^{(-c)})}{(a \cdot d)} - \frac{1}{2} \cdot \log((e^{(d \cdot x + c)} - 1) \cdot e^{(-c)})}{(a \cdot d)}}$

mupad [A] time = 26.92, size = 3643, normalized size = 110.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^3)),x)

[Out] $\frac{\exp(c + d \cdot x)}{(a \cdot d - a \cdot d \cdot \exp(2 \cdot c + 2 \cdot d \cdot x))} - \frac{(2 \cdot \exp(c + d \cdot x))}{(a \cdot d - 2 \cdot a \cdot d \cdot \exp(2 \cdot c + 2 \cdot d \cdot x) + a \cdot d \cdot \exp(4 \cdot c + 4 \cdot d \cdot x))} + \text{symsum}\left(\log\left(\frac{(570425344 \cdot a^4 \cdot b^6 \cdot \exp(d \cdot x) \cdot \exp(\text{root}(729 \cdot a^{10} \cdot d^6 \cdot z^6 + 27 \cdot a^4 \cdot b^2 \cdot d^2 \cdot z^2 + a^2 \cdot b^2 - b^4, z, k)) - 33554432 \cdot \text{root}(729 \cdot a^{10} \cdot d^6 \cdot z^6 + 27 \cdot a^4 \cdot b^2 \cdot d^2 \cdot z^2 + a^2 \cdot b^2 - b^4, z, k)) \cdot a \cdot b^{10} \cdot d - 553648128 \cdot a^2 \cdot b^8 \cdot \exp(d \cdot x) \cdot \exp(\text{root}(729 \cdot a^{10} \cdot d^6 \cdot z^6 + 27 \cdot a^4 \cdot b^2 \cdot d^2 \cdot z^2 + a^2 \cdot b^2 - b^4, z, k)) - 167772160 \cdot a^3 \cdot b^7 \cdot \exp(d \cdot x) \cdot \exp(\text{root}(729 \cdot a^{10} \cdot d^6 \cdot z^6 + 27 \cdot a^4 \cdot b^2 \cdot d^2 \cdot z^2 + a^2 \cdot b^2 - b^4, z, k)) - 16777216 \cdot b^{10} \cdot \exp(d \cdot x) \cdot \exp(\text{root}(729 \cdot a^{10} \cdot d^6 \cdot z^6 + 27 \cdot a^4 \cdot b^2 \cdot d^2 \cdot z^2 + a^2 \cdot b^2 - b^4, z, k)) + 192937984 \cdot a^5 \cdot b^5 \cdot \exp(d \cdot x) \cdot \exp(\text{root}(729 \cdot a^{10} \cdot d^6 \cdot z^6 + 27 \cdot a^4 \cdot b^2 \cdot d^2 \cdot z^2 + a^2 \cdot b^2 - b^4, z, k)) + 2617245696 \cdot \text{root}(729 \cdot a^{10} \cdot d^6 \cdot z^6 + 27 \cdot a^4 \cdot b^2 \cdot d^2 \cdot z^2 + a^2 \cdot b^2 - b^4, z, k)^3 \cdot a^5 \cdot b^8 \cdot d^3 - 150994944 \cdot \text{root}(729 \cdot a^{10} \cdot d^6 \cdot z^6 + 27 \cdot a^4 \cdot b^2 \cdot d^2 \cdot z^2 + a^2 \cdot b^2 - b^4, z, k)^3 \cdot a^6 \cdot b^7 \cdot d^3 - 1384120320 \cdot \text{root}(729 \cdot a^{10} \cdot d^6 \cdot z^6 + 27 \cdot a^4 \cdot b^2 \cdot d^2 \cdot z^2 + a^2 \cdot b^2 - b^4, z, k)^3 \cdot a^7 \cdot b^6 \cdot d^3 + 2415919104 \cdot \text{root}(729 \cdot a^{10} \cdot d^6 \cdot z^6 + 27 \cdot a^4 \cdot b^2 \cdot d^2 \cdot z^2 + a^2 \cdot b^2 - b^4, z, k)^3 \cdot a^8 \cdot b^5 \cdot d^3 - 3498049536 \cdot \text{root}(729 \cdot a^{10} \cdot d^6 \cdot z^6 + 27 \cdot a^4 \cdot b^2 \cdot d^2 \cdot z^2 + a^2 \cdot b^2 - b^4, z, k)^3 \cdot a^9 \cdot b^4 \cdot d^3 - 167772160 \cdot \text{root}(729 \cdot a^{10} \cdot d^6 \cdot z^6 + 27 \cdot a^4 \cdot b^2 \cdot d^2 \cdot z^2 + a^2 \cdot b^2 - b^4, z, k)^3 \cdot a^{10} \cdot b^3 \cdot d^3\right)$


```

b^5 + 495*a^13*b^4 + 220*a^14*b^3 + 66*a^15*b^2))*root(729*a^10*d^6*z^6 + 2
7*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k), k, 1, 6) - log(33554432*a*b^9 - 1
6777216*b^10 + 113246208*a^2*b^8 - 260046848*a^3*b^7 + 321126400*a^4*b^6 -
382205952*a^5*b^5 + 191102976*a^6*b^4 + 16777216*b^10*exp(-1/(2*a*d))*exp(d
*x) - 33554432*a*b^9*exp(-1/(2*a*d))*exp(d*x) - 113246208*a^2*b^8*exp(-1/(2
*a*d))*exp(d*x) + 260046848*a^3*b^7*exp(-1/(2*a*d))*exp(d*x) - 321126400*a^
4*b^6*exp(-1/(2*a*d))*exp(d*x) + 382205952*a^5*b^5*exp(-1/(2*a*d))*exp(d*x)
- 191102976*a^6*b^4*exp(-1/(2*a*d))*exp(d*x))/(2*a*d) + log(33554432*a*b^9
- 16777216*b^10 + 113246208*a^2*b^8 - 260046848*a^3*b^7 + 321126400*a^4*b^
6 - 382205952*a^5*b^5 + 191102976*a^6*b^4 - 16777216*b^10*exp(1/(2*a*d))*ex
p(d*x) + 33554432*a*b^9*exp(1/(2*a*d))*exp(d*x) + 113246208*a^2*b^8*exp(1/(
2*a*d))*exp(d*x) - 260046848*a^3*b^7*exp(1/(2*a*d))*exp(d*x) + 321126400*a^
4*b^6*exp(1/(2*a*d))*exp(d*x) - 382205952*a^5*b^5*exp(1/(2*a*d))*exp(d*x) +
191102976*a^6*b^4*exp(1/(2*a*d))*exp(d*x))/(2*a*d)

```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^3(c + dx)}{a + b \tanh^3(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*tanh(d*x+c)**3), x)
```

```
[Out] Integral(csch(c + d*x)**3/(a + b*tanh(c + d*x)**3), x)
```

$$3.80 \quad \int \frac{\operatorname{csch}^4(c+dx)}{a+b \tanh^3(c+dx)} dx$$

Optimal. Leaf size=215

$$\frac{\sqrt[3]{b} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} \tanh(c+dx) + b^{2/3} \tanh^2(c+dx))}{6a^{4/3}d} - \frac{\sqrt[3]{b} \log(\sqrt[3]{a} + \sqrt[3]{b} \tanh(c+dx))}{3a^{4/3}d} - \frac{\sqrt[3]{b} \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}}{\sqrt[3]{a^2+3b}}\right)}{\sqrt[3]{3} a^{4/3}d}$$

[Out] $\operatorname{coth}(d*x+c)/a/d-1/3*\operatorname{coth}(d*x+c)^3/a/d-b*\ln(\tanh(d*x+c))/a^2/d-1/3*b^{(1/3)*1}$
 $n(a^{(1/3)+b^{(1/3)}*\tanh(d*x+c)}/a^{(4/3)/d+1/6*b^{(1/3)}*\ln(a^{(2/3)-a^{(1/3)}*b^{(1/3)}*$
 $1/3)*\tanh(d*x+c)+b^{(2/3)*\tanh(d*x+c)^2}/a^{(4/3)/d+1/3*b*\ln(a+b*\tanh(d*x+c)^$
 $3)/a^2/d-1/3*b^{(1/3)*\arctan(1/3*(a^{(1/3)}-2*b^{(1/3)*\tanh(d*x+c)}/a^{(1/3)*3^{(1/2))}/a^{(4/3)/d*3^{(1/2))}$

Rubi [A] time = 0.24, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3663, 1834, 1871, 12, 292, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{b} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} \tanh(c+dx) + b^{2/3} \tanh^2(c+dx))}{6a^{4/3}d} + \frac{b \log(a + b \tanh^3(c+dx))}{3a^2d} - \frac{b \log(\tanh(c+dx))}{a^2d} - \frac{\sqrt[3]{b}}{\sqrt[3]{3} a^{4/3}d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]^4/(a + b*\operatorname{Tanh}[c + d*x]^3), x]$

[Out] $-((b^{(1/3)}*\operatorname{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)}*\operatorname{Tanh}[c + d*x])/(\operatorname{Sqrt}[3]*a^{(1/3)})])/($
 $\operatorname{Sqrt}[3]*a^{(4/3)*d}) + \operatorname{Coth}[c + d*x]/(a*d) - \operatorname{Coth}[c + d*x]^3/(3*a*d) - (b*\operatorname{Log}$
 $[\operatorname{Tanh}[c + d*x]])/(a^2*d) - (b^{(1/3)}*\operatorname{Log}[a^{(1/3)} + b^{(1/3)}*\operatorname{Tanh}[c + d*x]])/($
 $3*a^{(4/3)*d} + (b^{(1/3)}*\operatorname{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*\operatorname{Tanh}[c + d*x] + b^{(2/3)}$
 $*\operatorname{Tanh}[c + d*x]^2])/(6*a^{(4/3)*d} + (b*\operatorname{Log}[a + b*\operatorname{Tanh}[c + d*x]^3])/(3*a^2$
 $*d)$

Rule 12

$\operatorname{Int}[(a_*)(u_), x_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$ FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 31

$\operatorname{Int}[(a_*) + (b_.)*(x_)^(-1), x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x, x]]/b, x] /;$ FreeQ[{a, b}, x]

Rule 204

$\operatorname{Int}[(a_*) + (b_.)*(x_)^2)^(-1), x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTan}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[-a, 2]]/(\operatorname{Rt}[-a, 2]*\operatorname{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

$\operatorname{Int}[(x_)^(m_.)/((a_*) + (b_.)*(x_)^(n_.)), x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{Log}[\operatorname{RemoveContent}[a + b*x^n, x]]/(b*n), x] /;$ FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 292

$\operatorname{Int}[(x_)/((a_*) + (b_.)*(x_)^3), x_Symbol] \rightarrow -\operatorname{Dist}[(3*\operatorname{Rt}[a, 3]*\operatorname{Rt}[b, 3])^(-1), \operatorname{Int}[1/(\operatorname{Rt}[a, 3] + \operatorname{Rt}[b, 3]*x), x], x] + \operatorname{Dist}[1/(3*\operatorname{Rt}[a, 3]*\operatorname{Rt}[b, 3]), \operatorname{Int}[(\operatorname{Rt}[a, 3] + \operatorname{Rt}[b, 3]*x)/(\operatorname{Rt}[a, 3]^2 - \operatorname{Rt}[a, 3]*\operatorname{Rt}[b, 3]*x + \operatorname{Rt}[b, 3]^2*x$

$^2), x], x] /; \text{FreeQ}\{a, b\}, x]$

Rule 617

$\text{Int}[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]^{-1}, x_Symbol] \rightarrow \text{With}\{q = 1 - 4*S$
 $\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b$
 $], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \parallel \text{!RationalQ}[b^2 - 4*a*c]) /; \text{Free}$
 $\text{Q}\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[(d_.) + (e_.)*(x_)] / [(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] \rightarrow S$
 $\text{imp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}\{a, b, c, d,$
 $e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rule 634

$\text{Int}[(d_.) + (e_.)*(x_)] / [(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] \rightarrow D$
 $\text{ist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{In}$
 $\text{t}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}$
 $[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 1834

$\text{Int}[(Pq_)*((c_.)*(x_))^{(m_)}] / [(a_.) + (b_.)*(x_)^{(n_)}], x_Symbol] \rightarrow \text{Int}[\text{E}$
 $\text{xpendIntegrand}[(c*x)^m * Pq / (a + b*x^n), x], x] /; \text{FreeQ}\{a, b, c, m\}, x] \&$
 $\& \text{PolyQ}[Pq, x] \&\& \text{IntegerQ}[n] \&\& \text{!IGtQ}[m, 0]$

Rule 1871

$\text{Int}[(P2_)] / [(a_.) + (b_.)*(x_)^3], x_Symbol] \rightarrow \text{With}\{A = \text{Coeff}[P2, x, 0], B$
 $= \text{Coeff}[P2, x, 1], C = \text{Coeff}[P2, x, 2]\}, \text{Int}[(A + B*x)/(a + b*x^3), x] + \text{Di}$
 $\text{st}[C, \text{Int}[x^2/(a + b*x^3), x], x] /; \text{EqQ}[a*B^3 - b*A^3, 0] \parallel \text{!RationalQ}[a$
 $/b] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PolyQ}[P2, x, 2]$

Rule 3663

$\text{Int}[\sin[(e_.) + (f_.)*(x_)]^{(m_)} * ((a_.) + (b_.) * ((c_.) * \tan[(e_.) + (f_.)*(x_)$
 $)]^{(n_)})^{(p_)}], x_Symbol] \rightarrow \text{With}\{ff = \text{FreeFactors}[\text{Tan}[e + f*x], x]\}, \text{Dis}$
 $\text{t}[(c*ff^{(m+1)})/f, \text{Subst}[\text{Int}[(x^m * (a + b*(ff*x)^n)^p] / (c^2 + ff^2*x^2)^{(m/$
 $2 + 1)}, x], x, (c*\text{Tan}[e + f*x])/ff], x]] /; \text{FreeQ}\{a, b, c, e, f, n, p\}, x]$
 $\&\& \text{IntegerQ}[m/2]$

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^4(c+dx)}{a+b \tanh^3(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^4(a+bx^3)} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{ax^4} - \frac{1}{ax^2} - \frac{b}{a^2x} + \frac{bx(a+bx)}{a^2(a+bx^3)}\right) dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\operatorname{coth}(c+dx)}{ad} - \frac{\operatorname{coth}^3(c+dx)}{3ad} - \frac{b \log(\tanh(c+dx))}{a^2d} + \frac{b \operatorname{Subst}\left(\int \frac{x(a+bx)}{a+bx^3} dx, x, \tanh(c+dx)\right)}{a^2d} \\
&= \frac{\operatorname{coth}(c+dx)}{ad} - \frac{\operatorname{coth}^3(c+dx)}{3ad} - \frac{b \log(\tanh(c+dx))}{a^2d} + \frac{b \operatorname{Subst}\left(\int \frac{ax}{a+bx^3} dx, x, \tanh(c+dx)\right)}{a^2d} \\
&= \frac{\operatorname{coth}(c+dx)}{ad} - \frac{\operatorname{coth}^3(c+dx)}{3ad} - \frac{b \log(\tanh(c+dx))}{a^2d} + \frac{b \log(a+b \tanh^3(c+dx))}{3a^2d} \\
&= \frac{\operatorname{coth}(c+dx)}{ad} - \frac{\operatorname{coth}^3(c+dx)}{3ad} - \frac{b \log(\tanh(c+dx))}{a^2d} + \frac{b \log(a+b \tanh^3(c+dx))}{3a^2d} \\
&= \frac{\operatorname{coth}(c+dx)}{ad} - \frac{\operatorname{coth}^3(c+dx)}{3ad} - \frac{b \log(\tanh(c+dx))}{a^2d} - \frac{\sqrt[3]{b} \log(\sqrt[3]{a} + \sqrt[3]{b} \tanh(c+dx))}{3a^{4/3}d} \\
&= \frac{\operatorname{coth}(c+dx)}{ad} - \frac{\operatorname{coth}^3(c+dx)}{3ad} - \frac{b \log(\tanh(c+dx))}{a^2d} - \frac{\sqrt[3]{b} \log(\sqrt[3]{a} + \sqrt[3]{b} \tanh(c+dx))}{3a^{4/3}d} \\
&= -\frac{\sqrt[3]{b} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{b} \tanh(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3} a^{4/3} d} + \frac{\operatorname{coth}(c+dx)}{ad} - \frac{\operatorname{coth}^3(c+dx)}{3ad} - \frac{b \log(\tanh(c+dx))}{a^2d}
\end{aligned}$$

Mathematica [C] time = 3.62, size = 322, normalized size = 1.50

$$b\operatorname{RootSum}\left[\#1^3a + \#1^3b + 3\#1^2a - 3\#1^2b + 3\#1a + 3\#1b + a - b\sqrt[3]{\frac{-\#1^2a \log(e^{2(c+dx)} - \#1) + 2\#1^2ac + 2\#1^2adx - \#1^2b \log(e^{2(c+dx)} - \#1)}}{\sqrt{3}}}\right]$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^3), x]

[Out] $(-(a \operatorname{Coth}[c + d*x] * (-2 + \operatorname{Csch}[c + d*x]^2)) + 3*b*(c + d*x - \operatorname{Log}[\operatorname{Sinh}[c + d*x]])) + b*\operatorname{RootSum}[a - b + 3*a*\#1 + 3*b*\#1 + 3*a*\#1^2 - 3*b*\#1^2 + a*\#1^3 + b*\#1^3 \& , (-2*a*c + 2*b*c - 2*a*d*x + 2*b*d*x + a*\operatorname{Log}[E^(2*(c + d*x)) - \#1] - b*\operatorname{Log}[E^(2*(c + d*x)) - \#1] - 8*a*c*\#1 - 4*b*c*\#1 - 8*a*d*x*\#1 - 4*b*d*x*\#1 + 4*a*\operatorname{Log}[E^(2*(c + d*x)) - \#1]*\#1 + 2*b*\operatorname{Log}[E^(2*(c + d*x)) - \#1]*\#1 + 2*a*c*\#1^2 + 2*b*c*\#1^2 + 2*a*d*x*\#1^2 + 2*b*d*x*\#1^2 - a*\operatorname{Log}[E^(2*(c + d*x)) - \#1]*\#1^2 - b*\operatorname{Log}[E^(2*(c + d*x)) - \#1]*\#1^2)/(a - b + 2*a*\#1 + 2*b*\#1 + a*\#1^2 - b*\#1^2) \&])/(3*a^2*d)$

fricas [C] time = 1.87, size = 1954, normalized size = 9.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3), x, algorithm="fricas")

```
[Out] 1/12*(12*sqrt(1/3)*(a^2*d*e^(6*d*x + 6*c) - 3*a^2*d*e^(4*d*x + 4*c) + 3*a^2
*d*e^(2*d*x + 2*c) - a^2*d)*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3)
- b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))^2*a^4*d^2
+ 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^
3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))*a^2*b*d + 4*b^2)/(a^4*d^2))*arctan(-1/8*
sqrt(1/3)*((a^6 + a^5*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a
^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))^2*d^3*e^(2*d*x + 2*
c) - 4*(2*a^3*b + a^2*b^2 - a*b^3)*d*e^(2*d*x + 2*c) - 2*((a^5 - a^4*b - 2*
a^3*b^2)*d^2*e^(2*d*x + 2*c) + (a^5 + a^4*b)*d^2)*((1/2)^(1/3)*(I*sqrt(3) +
1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^
2*d)) - (((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b
- b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))^2*a^4*d^3 - 2*(a^3 - 2*a^2*b)*((1/2)
^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d
^3))^(1/3) - 2*b/(a^2*d))*d^2 - 4*(2*a*b - b^2)*d)*sqrt(a^4 + 4*a^3*b + 5*a
^2*b^2 + 2*a*b^3 - 1/2*((a^6 + a^5*b)*d^2*e^(2*d*x + 2*c) - (a^6 + a^5*b)*d
^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^
3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))^2 + ((a^5 - a^4*b - 2*a^3*b^2)*d*e^(2*d*
x + 2*c) + (a^5 + 3*a^4*b + 2*a^3*b^2)*d)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(
a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d)) +
(a^4 + 2*a^3*b + a^2*b^2)*e^(4*d*x + 4*c) + 2*(a^4 + a^3*b - a^2*b^2 - a*b^
3)*e^(2*d*x + 2*c))*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/
(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))^2*a^4*d^2 + 4*((1
/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6
*d^3))^(1/3) - 2*b/(a^2*d))*a^2*b*d + 4*b^2)/(a^4*d^2))/(a^2*b + a*b^2)) -
2*(a^2*d*e^(6*d*x + 6*c) - 3*a^2*d*e^(4*d*x + 4*c) + 3*a^2*d*e^(2*d*x + 2*c
) - a^2*d)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2
*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))*log(1/2*((1/2)^(1/3)*(I*sqrt(3) +
1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^
2*d))^2*a^4*d^2 - (a^3 - 2*a^2*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3)
- b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d))*d + a^2 -
3*a*b + 2*b^2 + (a^2 + a*b)*e^(2*d*x + 2*c)) - 48*a*e^(2*d*x + 2*c) + ((a^2
*d*e^(6*d*x + 6*c) - 3*a^2*d*e^(4*d*x + 4*c) + 3*a^2*d*e^(2*d*x + 2*c) - a^
2*d)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b
^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d)) + 6*b*e^(6*d*x + 6*c) - 18*b*e^(4*d*x +
4*c) + 18*b*e^(2*d*x + 2*c) - 6*b)*log(a^4 + 4*a^3*b + 5*a^2*b^2 + 2*a*b^3
- 1/2*((a^6 + a^5*b)*d^2*e^(2*d*x + 2*c) - (a^6 + a^5*b)*d^2)*((1/2)^(1/3)
*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1
/3) - 2*b/(a^2*d))^2 + ((a^5 - a^4*b - 2*a^3*b^2)*d*e^(2*d*x + 2*c) + (a^5
+ 3*a^4*b + 2*a^3*b^2)*d)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(b/(a^4*d^3) - b^3/(
a^6*d^3) - (a^2*b - b^3)/(a^6*d^3))^(1/3) - 2*b/(a^2*d)) + (a^4 + 2*a^3*b +
a^2*b^2)*e^(4*d*x + 4*c) + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*e^(2*d*x + 2*
c)) - 12*(b*e^(6*d*x + 6*c) - 3*b*e^(4*d*x + 4*c) + 3*b*e^(2*d*x + 2*c) - b
)*log(e^(2*d*x + 2*c) - 1) + 16*a)/(a^2*d*e^(6*d*x + 6*c) - 3*a^2*d*e^(4*d*
x + 4*c) + 3*a^2*d*e^(2*d*x + 2*c) - a^2*d)
```

giac [A] time = 0.29, size = 180, normalized size = 0.84

$$\frac{2b \log\left(\left|ae^{(6dx+6c)} + be^{(6dx+6c)} + 3ae^{(4dx+4c)} - 3be^{(4dx+4c)} + 3ae^{(2dx+2c)} + 3be^{(2dx+2c)} + a - b\right|\right)}{a^2} - \frac{6b \log\left(\left|e^{(2dx+2c)} - 1\right|\right)}{a^2} + \frac{11be^{(6dx+6c)} - 33be^{(4dx+4c)} + 18be^{(2dx+2c)} - 6b}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*log(abs(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) + 3*a*e^(4*d*x + 4*c)
) - 3*b*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c) + a - b
))/a^2 - 6*b*log(abs(e^(2*d*x + 2*c) - 1))/a^2 + (11*b*e^(6*d*x + 6*c) - 33
*b*e^(4*d*x + 4*c) - 24*a*e^(2*d*x + 2*c) + 33*b*e^(2*d*x + 2*c) + 8*a - 11
*b)/(a^2*(e^(2*d*x + 2*c) - 1)^3)/d
```

maple [C] time = 0.60, size = 187, normalized size = 0.87

$$-\frac{\tanh^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{24da} + \frac{3 \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{8da} + \frac{b \left(\sum_{R=\text{RootOf}(a_Z^6+3a_Z^4+8b_Z^3+3a_Z^2+a)} \frac{(R^5 a+4 R^2 b+3 R a) \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - R\right)}{-R^5 a+2 R^3 a+4 R^2 b+R a}}{3d a^2} \right)}{3d a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3), x)
```

```
[Out] -1/24/d/a*tanh(1/2*d*x+1/2*c)^3+3/8/d/a*tanh(1/2*d*x+1/2*c)+1/3/d/a^2*b*sum
((R^5*a+4*R^2*b+3*R*a)/(R^5*a+2*R^3*a+4*R^2*b+R*a)*ln(tanh(1/2*d*x+1/2*c)-R),
R=RootOf(Z^6*a+3*Z^4*a+8*Z^3*b+3*Z^2*a+a))-1/24/d/a/tanh(1/2*d*x+1/2*c)^3+3/8/d/a/tanh(1/2*d*x+1/2*c)-1/d/a^2*b*ln(tanh(1/2*d*x+1/2*c))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2ab \left(\frac{-(a-b) \int \frac{1}{(ae^{(6c)+be^{(6c)})e^{(6dx)}+3(ae^{(4c)}-be^{(4c)})e^{(4dx)}+3(ae^{(2c)}+be^{(2c)})e^{(2dx)}+a-b} dx + x}{a^3 - a^2b} - \frac{dx + c}{(a^3 - a^2b)d} \right) - 2b^2 \left(\frac{-(a-b) \int \dots}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3), x, algorithm="maxima")
```

```
[Out] 2*a*b*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) + 3*a
+ 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c
) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 - a^2*b) - (d*x + c)/((a^3
- a^2*b)*d)) - 2*b^2*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d
*x + 2*c) + 3*a + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b
)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 - a^2*b) - (
d*x + c)/((a^3 - a^2*b)*d)) + 2*b*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d
*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b),
x)/a + 2*b^2*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b
)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/a^2 - 8*b*integr
ate(e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) +
3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/a - 4*b^2*integrate(e^(2*d*x + 2*c)/
((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x +
2*c) + a - b), x)/a^2 + 2/3*(3*b*d*x*e^(6*d*x + 6*c) - 9*b*d*x*e^(4*d*x +
4*c) - 3*b*d*x + 3*(3*b*d*x*e^(2*c) - 2*a*e^(2*c))*e^(2*d*x) + 2*a)/(a^2*d*
e^(6*d*x + 6*c) - 3*a^2*d*e^(4*d*x + 4*c) + 3*a^2*d*e^(2*d*x + 2*c) - a^2*d
) - b*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) - b*log((e^(d*x + c) - 1)*e^(-c
))/(a^2*d)
```

mupad [B] time = 3.22, size = 4563, normalized size = 21.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^3)), x)
```

```
[Out] 8/(3*(a*d - 3*a*d*exp(2*c + 2*d*x) + 3*a*d*exp(4*c + 4*d*x) - a*d*exp(6*c +
6*d*x))) - 4/(a*d - 2*a*d*exp(2*c + 2*d*x) + a*d*exp(4*c + 4*d*x)) + symsu
m(log((1507328*a*b^9 + 1572864*b^10 - 5242880*a^2*b^8 - 7479296*a^3*b^7 + 3
948544*a^4*b^6 + 5963776*a^5*b^5 - 278528*a^6*b^4 + 8192*a^7*b^3 - 1572864*
b^10*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b -
b^3, z, k))*exp(2*d*x) - 1769472*a*b^9*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*
b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 42467328*root(
```


$$\begin{aligned}
& 27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, k)^2 a^4 \\
& 4b^8d^2 + 21626880 \text{root}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2z \\
& + a^2b - b^3, z, k)^2 a^5b^7d^2 - 70189056 \text{root}(27a^6d^3z^3 - 27a^4 \\
& b^2d^2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, k)^2 a^6b^6d^2 + 18038784 \text{ro} \\
& \text{ot}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, k)^2 \\
& a^7b^5d^2 - 11993088 \text{root}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2 \\
& z + a^2b - b^3, z, k)^2 a^8b^4d^2 + 147456 \text{root}(27a^6d^3z^3 - 27a^4 \\
& b^2d^2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, k)^2 a^9b^3d^2 - 98304 \text{root} \\
& (27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, k)^2 a \\
& ^{10}b^2d^2 - 42467328 \text{root}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2 \\
& z + a^2b - b^3, z, k)^3 a^6b^7d^3 - 12091392 \text{root}(27a^6d^3z^3 - 27a^4 \\
& b^2d^2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, k)^3 a^7b^6d^3 + 22708224 \text{r} \\
& \text{oot}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, k) \\
& ^3 a^8b^5d^3 + 12386304 \text{root}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2 \\
& d^2z + a^2b - b^3, z, k)^3 a^9b^4d^3 + 19759104 \text{root}(27a^6d^3z^3 - 2 \\
& 7a^4b^2d^2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, k)^3 a^{10}b^3d^3 - 29491 \\
& 2 \text{root}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, \\
& k)^3 a^{11}b^2d^3 - 14155776 \text{root}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2 \\
& b^2d^2z + a^2b - b^3, z, k) a^2b^9d - 10387456 \text{root}(27a^6d^3z^3 - 27 \\
& a^4b^2d^2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, k) a^3b^8d + 32407552 \text{r} \\
& \text{oot}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, k) a \\
& ^4b^7d + 16187392 \text{root}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2z \\
& + a^2b - b^3, z, k) a^5b^6d - 29818880 \text{root}(27a^6d^3z^3 - 27a^4b^2d^ \\
& 2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, k) a^6b^5d + 6135808 \text{root}(27a^6 \\
& d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, k) a^7b^4d - \\
& 376832 \text{root}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2z + a^2b - b^ \\
& 3, z, k) a^8b^3d + 8192 \text{root}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2 \\
& d^2z + a^2b - b^3, z, k) a^9b^2d - 3571712 a^2b^8 \exp(2 \text{root}(27a^6d^ \\
& 3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, k)) \exp(2d*x) + \\
& 30990336 a^3b^7 \exp(2 \text{root}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2 \\
& z + a^2b - b^3, z, k)) \exp(2d*x) + 43139072 a^4b^6 \exp(2 \text{root}(27a^6d \\
& ^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, k)) \exp(2d*x) \\
& + 8519680 a^5b^5 \exp(2 \text{root}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2 \\
& z + a^2b - b^3, z, k)) \exp(2d*x) - 245760 a^6b^4 \exp(2 \text{root}(27a^6d^3 \\
& z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, k)) \exp(2d*x) + \\
& 8192 a^7b^3 \exp(2 \text{root}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2z + \\
& a^2b - b^3, z, k)) \exp(2d*x) - 42467328 \text{root}(27a^6d^3z^3 - 27a^4b^2d \\
& ^2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, k)^2 a^4b^8d^2 \exp(2 \text{root}(27a^6 \\
& d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, k)) \exp(2d*x) \\
&) - 22413312 \text{root}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2z + a^2b \\
& - b^3, z, k)^2 a^5b^7d^2 \exp(2 \text{root}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + \\
& 9a^2b^2d^2z + a^2b - b^3, z, k)) \exp(2d*x) + 54853632 \text{root}(27a^6d^3z^3 \\
& - 27a^4b^2d^2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, k)^2 a^6b^6d^2 \exp \\
& (2 \text{root}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2z + a^2b - b^3, z \\
& , k)) \exp(2d*x) + 67977216 \text{root}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2 \\
& b^2d^2z + a^2b - b^3, z, k)^2 a^7b^5d^2 \exp(2 \text{root}(27a^6d^3z^3 - 27a^4 \\
& b^2d^2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, k)) \exp(2d*x) - 60014592 \text{r} \\
& \text{oot}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, k)^2 \\
& a^8b^4d^2 \exp(2 \text{root}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2z + \\
& a^2b - b^3, z, k)) \exp(2d*x) + 2211840 \text{root}(27a^6d^3z^3 - 27a^4b^2d^ \\
& 2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, k)^2 a^9b^3d^2 \exp(2 \text{root}(27a^6 \\
& d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, k)) \exp(2d*x) \\
& - 147456 \text{root}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2z + a^2b - \\
& b^3, z, k)^2 a^{10}b^2d^2 \exp(2 \text{root}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9 \\
& a^2b^2d^2z + a^2b - b^3, z, k)) \exp(2d*x) + 42467328 \text{root}(27a^6d^3z^3 \\
& - 27a^4b^2d^2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, k)^3 a^6b^7d^3 \exp(\\
& 2 \text{root}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2d^2z + a^2b - b^3, z, \\
& k)) \exp(2d*x) + 9732096 \text{root}(27a^6d^3z^3 - 27a^4b^2d^2z^2 + 9a^2b^2 \\
& d^2z + a^2b - b^3, z, k)^3 a^7b^6d^3 \exp(2 \text{root}(27a^6d^3z^3 - 27a^4
\end{aligned}$$

```

b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) - 85377024*root(
27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^3*a^
8*b^5*d^3*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^
2*b - b^3, z, k))*exp(2*d*x) + 246398976*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2
*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^3*a^9*b^4*d^3*exp(2*root(27*a^6*d
^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x)
+ 12828672*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b -
b^3, z, k)^3*a^10*b^3*d^3*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9
*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 442368*root(27*a^6*d^3*z^3
- 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^3*a^11*b^2*d^3*exp(
2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z,
k))*exp(2*d*x) + 14155776*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^
2*d*z + a^2*b - b^3, z, k)*a^2*b^9*d*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d
^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 11698176*root(27*
a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)*a^3*b^8
*d*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b
^3, z, k))*exp(2*d*x) + 6111232*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*
a^2*b^2*d*z + a^2*b - b^3, z, k)*a^4*b^7*d*exp(2*root(27*a^6*d^3*z^3 - 27*a
^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) - 165445632*r
oot(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)*
a^5*b^6*d*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^
2*b - b^3, z, k))*exp(2*d*x) - 27688960*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*
z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)*a^6*b^5*d*exp(2*root(27*a^6*d^3*z^
3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 105
59488*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3,
z, k)*a^7*b^4*d*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d
*z + a^2*b - b^3, z, k))*exp(2*d*x) - 393216*root(27*a^6*d^3*z^3 - 27*a^4*b
*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)*a^8*b^3*d*exp(2*root(27*a^6*d
^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x)
+ 8192*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3
, z, k)*a^9*b^2*d*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*
d*z + a^2*b - b^3, z, k))*exp(2*d*x))/(24*a^14*b + 3*a^15 + 3*a^7*b^8 + 24*
a^8*b^7 + 84*a^9*b^6 + 168*a^10*b^5 + 210*a^11*b^4 + 168*a^12*b^3 + 84*a^13
*b^2))*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3
, z, k), k, 1, 3) - (b*log(45613056*a*b^9 + 100663296*b^10 - 130547712*a^2*
b^8 - 18014208*a^3*b^7 + 2015232*a^4*b^6 + 270336*a^5*b^5 - 100663296*b^10*
exp(2*d*x)*exp(-(2*b)/(a^2*d)) + 130547712*a^2*b^8*exp(2*d*x)*exp(-(2*b)/(a
^2*d)) + 18014208*a^3*b^7*exp(2*d*x)*exp(-(2*b)/(a^2*d)) - 2015232*a^4*b^6*
exp(2*d*x)*exp(-(2*b)/(a^2*d)) - 270336*a^5*b^5*exp(2*d*x)*exp(-(2*b)/(a^2*
d)) - 45613056*a*b^9*exp(2*d*x)*exp(-(2*b)/(a^2*d))))/(a^2*d)

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{csch}^4(c + dx)}{a + b \tanh^3(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**4/(a+b*tanh(d*x+c)**3), x)

[Out] Integral(csch(c + d*x)**4/(a + b*tanh(c + d*x)**3), x)

3.81 $\int \cosh^4(c + dx) (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=63

$$\frac{(a + b) \sinh(c + dx) \cosh^3(c + dx)}{4d} + \frac{(3a - b) \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{1}{8}x(3a - b)$$

[Out] 1/8*(3*a-b)*x+1/8*(3*a-b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*(a+b)*cosh(d*x+c)^3*sinh(d*x+c)/d

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3675, 385, 199, 206}

$$\frac{(a + b) \sinh(c + dx) \cosh^3(c + dx)}{4d} + \frac{(3a - b) \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{1}{8}x(3a - b)$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2), x]

[Out] ((3*a - b)*x)/8 + ((3*a - b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + ((a + b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d)

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegerQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned}
\int \cosh^4(c + dx) (a + b \tanh^2(c + dx)) dx &= \frac{\text{Subst}\left(\int \frac{a+bx^2}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{(a + b) \cosh^3(c + dx) \sinh(c + dx)}{4d} + \frac{(3a - b) \text{Subst}\left(\int \frac{1}{(1-x^2)^2} dx, x\right)}{4d} \\
&= \frac{(3a - b) \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{(a + b) \cosh^3(c + dx) \sinh(c + dx)}{4d} \\
&= \frac{1}{8}(3a - b)x + \frac{(3a - b) \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{(a + b) \cosh^3(c + dx) \sinh(c + dx)}{4d}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 44, normalized size = 0.70

$$\frac{(a + b) \sinh(4(c + dx)) + 12a(c + dx) + 8a \sinh(2(c + dx)) - 4bdx}{32d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2), x]

[Out] (-4*b*d*x + 12*a*(c + d*x) + 8*a*Sinh[2*(c + d*x)] + (a + b)*Sinh[4*(c + d*x)])/(32*d)

fricas [A] time = 0.41, size = 63, normalized size = 1.00

$$\frac{(a + b) \cosh(dx + c) \sinh(dx + c)^3 + (3a - b)dx + ((a + b) \cosh(dx + c)^3 + 4a \cosh(dx + c)) \sinh(dx + c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] 1/8*((a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a - b)*d*x + ((a + b)*cosh(d*x + c)^3 + 4*a*cosh(d*x + c))*sinh(d*x + c))/d

giac [A] time = 0.18, size = 107, normalized size = 1.70

$$\frac{8(3a - b)dx - (18ae^{4dx+4c} - 6be^{4dx+4c} + 8ae^{2dx+2c} + a + b)e^{(-4dx-4c)} + (ae^{4dx+12c} + be^{4dx+12c} + 8ae^{2dx+2c}) \sinh(dx + c)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2), x, algorithm="giac")

[Out] 1/64*(8*(3*a - b)*d*x - (18*a*e^(4*d*x + 4*c) - 6*b*e^(4*d*x + 4*c) + 8*a*e^(2*d*x + 2*c) + a + b)*e^(-4*d*x - 4*c) + (a*e^(4*d*x + 12*c) + b*e^(4*d*x + 12*c) + 8*a*e^(2*d*x + 10*c))*e^(-8*c))/d

maple [A] time = 0.32, size = 82, normalized size = 1.30

$$\frac{b\left(\frac{\sinh(dx+c)\cosh^3(dx+c)}{4} - \frac{\cosh(dx+c)\sinh(dx+c)}{8} - \frac{dx}{8} - \frac{c}{8}\right) + a\left(\left(\frac{\cosh^3(dx+c)}{4} + \frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c) + \frac{3dx}{8} + \frac{3c}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2), x)

[Out] $1/d*(b*(1/4*\sinh(d*x+c)*\cosh(d*x+c)^3-1/8*\cosh(d*x+c)*\sinh(d*x+c)-1/8*d*x-1/8*c)+a*((1/4*\cosh(d*x+c)^3+3/8*\cosh(d*x+c))*\sinh(d*x+c)+3/8*d*x+3/8*c))$

maxima [A] time = 0.35, size = 104, normalized size = 1.65

$$\frac{1}{64} a \left(24x + \frac{e^{(4dx+4c)}}{d} + \frac{8e^{(2dx+2c)}}{d} - \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) - \frac{1}{64} b \left(\frac{8(dx+c)}{d} - \frac{e^{(4dx+4c)}}{d} + \frac{e^{(-4dx-4c)}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")`

[Out] $1/64*a*(24*x + e^{(4*d*x + 4*c)}/d + 8*e^{(2*d*x + 2*c)}/d - 8*e^{(-2*d*x - 2*c)}/d - e^{(-4*d*x - 4*c)}/d) - 1/64*b*(8*(d*x + c)/d - e^{(4*d*x + 4*c)}/d + e^{(-4*d*x - 4*c)}/d)$

mupad [B] time = 0.21, size = 74, normalized size = 1.17

$$x \left(\frac{3a}{8} - \frac{b}{8} \right) - \frac{e^{-4c-4dx} (a+b)}{64d} + \frac{e^{4c+4dx} (a+b)}{64d} - \frac{ae^{-2c-2dx}}{8d} + \frac{ae^{2c+2dx}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2),x)`

[Out] $x*((3*a)/8 - b/8) - (\exp(-4*c - 4*d*x)*(a + b))/(64*d) + (\exp(4*c + 4*d*x)*(a + b))/(64*d) - (a*\exp(-2*c - 2*d*x))/(8*d) + (a*\exp(2*c + 2*d*x))/(8*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx)) \cosh^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)`

[Out] `Integral((a + b*tanh(c + d*x)**2)*cosh(c + d*x)**4, x)`

3.82 $\int \cosh^3(c + dx) (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=30

$$\frac{(a + b) \sinh^3(c + dx)}{3d} + \frac{a \sinh(c + dx)}{d}$$

[Out] a*sinh(d*x+c)/d+1/3*(a+b)*sinh(d*x+c)^3/d

Rubi [A] time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {3676}

$$\frac{(a + b) \sinh^3(c + dx)}{3d} + \frac{a \sinh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]

[Out] (a*Sinh[c + d*x])/d + ((a + b)*Sinh[c + d*x]^3)/(3*d)

Rule 3676

Int[sec[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.))^p, x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \cosh^3(c + dx) (a + b \tanh^2(c + dx)) dx &= \frac{\text{Subst}\left(\int (a + (a + b)x^2) dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{a \sinh(c + dx)}{d} + \frac{(a + b) \sinh^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 1.47

$$\frac{a \sinh^3(c + dx)}{3d} + \frac{a \sinh(c + dx)}{d} + \frac{b \sinh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]

[Out] (a*Sinh[c + d*x])/d + (a*Sinh[c + d*x]^3)/(3*d) + (b*Sinh[c + d*x]^3)/(3*d)

fricas [A] time = 0.39, size = 45, normalized size = 1.50

$$\frac{(a + b) \sinh(dx + c)^3 + 3((a + b) \cosh(dx + c)^2 + 3a - b) \sinh(dx + c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")

[Out] 1/12*((a + b)*sinh(d*x + c)^3 + 3*((a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))/d

giac [B] time = 0.16, size = 94, normalized size = 3.13

$$\frac{(9ae^{(2dx+2c)} - 3be^{(2dx+2c)} + a + b)e^{(-3dx-3c)} - (ae^{(3dx+12c)} + be^{(3dx+12c)} + 9ae^{(dx+10c)} - 3be^{(dx+10c)})e^{(-9c)}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] -1/24*((9*a*e^(2*d*x + 2*c) - 3*b*e^(2*d*x + 2*c) + a + b)*e^(-3*d*x - 3*c) - (a*e^(3*d*x + 12*c) + b*e^(3*d*x + 12*c) + 9*a*e^(d*x + 10*c) - 3*b*e^(d*x + 10*c))*e^(-9*c))/d

maple [A] time = 0.32, size = 37, normalized size = 1.23

$$\frac{\frac{b(\sinh^3(dx+c))}{3} + a\left(\frac{2}{3} + \frac{(\cosh^2(dx+c))}{3}\right)\sinh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x)

[Out] 1/d*(1/3*b*sinh(d*x+c)^3+a*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c))

maxima [B] time = 0.34, size = 83, normalized size = 2.77

$$\frac{b(e^{(dx+c)} - e^{(-dx-c)})^3}{24d} + \frac{1}{24}a\left(\frac{e^{(3dx+3c)}}{d} + \frac{9e^{(dx+c)}}{d} - \frac{9e^{(-dx-c)}}{d} - \frac{e^{(-3dx-3c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] 1/24*b*(e^(d*x + c) - e^(-d*x - c))^3/d + 1/24*a*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d)

mupad [B] time = 0.21, size = 74, normalized size = 2.47

$$\frac{e^{3c+3dx}(a+b)}{24d} - \frac{e^{-3c-3dx}(a+b)}{24d} + \frac{e^{c+dx}(3a-b)}{8d} - \frac{e^{-c-dx}(3a-b)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2),x)

[Out] (exp(3*c + 3*d*x)*(a + b))/(24*d) - (exp(- 3*c - 3*d*x)*(a + b))/(24*d) + (exp(c + d*x)*(3*a - b))/(8*d) - (exp(- c - d*x)*(3*a - b))/(8*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx)) \cosh^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)

[Out] Integral((a + b*tanh(c + d*x)**2)*cosh(c + d*x)**3, x)

3.83 $\int \cosh^2(c + dx) (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=33

$$\frac{(a + b) \sinh(c + dx) \cosh(c + dx)}{2d} + \frac{1}{2}x(a - b)$$

[Out] 1/2*(a-b)*x+1/2*(a+b)*cosh(d*x+c)*sinh(d*x+c)/d

Rubi [A] time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3675, 385, 206}

$$\frac{(a + b) \sinh(c + dx) \cosh(c + dx)}{2d} + \frac{1}{2}x(a - b)$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]

[Out] ((a - b)*x)/2 + ((a + b)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \cosh^2(c + dx) (a + b \tanh^2(c + dx)) dx &= \frac{\text{Subst}\left(\int \frac{a+bx^2}{(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{(a + b) \cosh(c + dx) \sinh(c + dx)}{2d} + \frac{(a - b) \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{2d} \\ &= \frac{1}{2}(a - b)x + \frac{(a + b) \cosh(c + dx) \sinh(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.06, size = 32, normalized size = 0.97

$$\frac{2(a - b)(c + dx) + (a + b) \sinh(2(c + dx))}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2), x]

[Out] (2*(a - b)*(c + d*x) + (a + b)*Sinh[2*(c + d*x)])/(4*d)

fricas [A] time = 0.39, size = 30, normalized size = 0.91

$$\frac{(a - b)dx + (a + b) \cosh(dx + c) \sinh(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] 1/2*((a - b)*d*x + (a + b)*cosh(d*x + c)*sinh(d*x + c))/d

giac [B] time = 0.14, size = 81, normalized size = 2.45

$$\frac{4(a - b)dx - (2ae^{2dx+2c} - 2be^{2dx+2c} + a + b)e^{(-2dx-2c)} + (ae^{2dx+4c} + be^{2dx+4c})e^{(-2c)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2), x, algorithm="giac")

[Out] 1/8*(4*(a - b)*d*x - (2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*e^(-2*d*x - 2*c) + (a*e^(2*d*x + 4*c) + b*e^(2*d*x + 4*c))*e^(-2*c))/d

maple [A] time = 0.21, size = 54, normalized size = 1.64

$$\frac{a \left(\frac{\cosh(dx+c) \sinh(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + b \left(\frac{\cosh(dx+c) \sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2), x)

[Out] 1/d*(a*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))

maxima [B] time = 0.32, size = 69, normalized size = 2.09

$$\frac{1}{8} a \left(4x + \frac{e^{(2dx+2c)}}{d} - \frac{e^{(-2dx-2c)}}{d} \right) - \frac{1}{8} b \left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2), x, algorithm="maxima")

[Out] 1/8*a*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/8*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d)

mupad [B] time = 0.15, size = 27, normalized size = 0.82

$$x \left(\frac{a}{2} - \frac{b}{2} \right) + \frac{\sinh(2c + 2dx)(a + b)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2), x)

[Out] x*(a/2 - b/2) + (sinh(2*c + 2*d*x)*(a + b))/(4*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx)) \cosh^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*(a+b*tanh(d*x+c)**2), x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*cosh(c + d*x)**2, x)
```

3.84 $\int \cosh(c + dx) (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=27

$$\frac{(a + b) \sinh(c + dx)}{d} - \frac{b \tan^{-1}(\sinh(c + dx))}{d}$$

[Out] $-b \cdot \arctan(\sinh(d \cdot x + c)) / d + (a + b) \cdot \sinh(d \cdot x + c) / d$

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3676, 388, 203}

$$\frac{(a + b) \sinh(c + dx)}{d} - \frac{b \tan^{-1}(\sinh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cosh}[c + d \cdot x] \cdot (a + b \cdot \text{Tanh}[c + d \cdot x]^2), x]$

[Out] $-((b \cdot \text{ArcTan}[\text{Sinh}[c + d \cdot x]]) / d) + ((a + b) \cdot \text{Sinh}[c + d \cdot x]) / d$

Rule 203

$\text{Int}[(a + b \cdot x^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 \cdot \text{ArcTan}[(\text{Rt}[b, 2] \cdot x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] \cdot \text{Rt}[b, 2]), x] /;$ FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 388

$\text{Int}[(a + b \cdot x^n)^p \cdot (c + d \cdot x^n), x_Symbol] \rightarrow \text{Simp}[(d \cdot x \cdot (a + b \cdot x^n)^{p+1}) / (b \cdot (n \cdot (p+1) + 1)), x] - \text{Dist}[(a \cdot d - b \cdot c \cdot (n \cdot (p+1) + 1)) / (b \cdot (n \cdot (p+1) + 1)), \text{Int}[(a + b \cdot x^n)^p, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b \cdot c - a \cdot d, 0] && NeQ[n \cdot (p+1) + 1, 0]

Rule 3676

$\text{Int}[\sec[(e + f \cdot x)] \cdot (a + b \cdot \tan[(e + f \cdot x)]^n)^p, x_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Sin}[e + f \cdot x], x]\}, \text{Dist}[ff/f, \text{Subst}[\text{Int}[\text{ExpandToSum}[b \cdot (ff \cdot x)^n + a \cdot (1 - ff^2 \cdot x^2)^{n/2}], x]^p / (1 - ff^2 \cdot x^2)^{(m + n \cdot p + 1)/2}, x], x, \text{Sin}[e + f \cdot x] / ff, x] /;$ FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \cosh(c + dx) (a + b \tanh^2(c + dx)) dx &= \frac{\text{Subst}\left(\int \frac{a + (a+b)x^2}{1+x^2} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{(a + b) \sinh(c + dx)}{d} - \frac{b \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(c + dx)\right)}{d} \\ &= -\frac{b \tan^{-1}(\sinh(c + dx))}{d} + \frac{(a + b) \sinh(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.04, size = 47, normalized size = 1.74

$$\frac{a \sinh(c) \cosh(dx)}{d} + \frac{a \cosh(c) \sinh(dx)}{d} + \frac{b \sinh(c + dx)}{d} - \frac{b \tan^{-1}(\sinh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2), x]

[Out] -((b*ArcTan[Sinh[c + d*x]])/d) + (a*Cosh[d*x]*Sinh[c])/d + (a*Cosh[c]*Sinh[d*x])/d + (b*Sinh[c + d*x])/d

fricas [B] time = 0.41, size = 102, normalized size = 3.78

$$\frac{(a + b) \cosh(dx + c)^2 + 2(a + b) \cosh(dx + c) \sinh(dx + c) + (a + b) \sinh(dx + c)^2 - 4(b \cosh(dx + c) + b \sinh(dx + c))}{2(d \cosh(dx + c) + d \sinh(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] 1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 - 4*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - a - b)/(d*cosh(d*x + c) + d*sinh(d*x + c))

giac [B] time = 0.14, size = 56, normalized size = 2.07

$$\frac{4 b \arctan \left(e^{(dx+c)} \right) + (a + b) e^{(-dx-c)} - \left(a e^{(dx+4c)} + b e^{(dx+4c)} \right) e^{(-3c)}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2), x, algorithm="giac")

[Out] -1/2*(4*b*arctan(e^(d*x + c)) + (a + b)*e^(-d*x - c) - (a*e^(d*x + 4*c) + b*e^(d*x + 4*c))*e^(-3*c))/d

maple [A] time = 0.20, size = 37, normalized size = 1.37

$$\frac{a \sinh(dx + c)}{d} + \frac{b \sinh(dx + c)}{d} - \frac{2b \arctan \left(e^{dx+c} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)*(a+b*tanh(d*x+c)^2), x)

[Out] a*sinh(d*x+c)/d+b*sinh(d*x+c)/d-2/d*b*arctan(exp(d*x+c))

maxima [B] time = 0.44, size = 55, normalized size = 2.04

$$\frac{1}{2} b \left(\frac{4 \arctan \left(e^{(-dx-c)} \right)}{d} + \frac{e^{(dx+c)}}{d} - \frac{e^{(-dx-c)}}{d} \right) + \frac{a \sinh(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2), x, algorithm="maxima")

[Out] 1/2*b*(4*arctan(e^(-d*x - c))/d + e^(d*x + c)/d - e^(-d*x - c)/d) + a*sinh(d*x + c)/d

mupad [B] time = 0.14, size = 66, normalized size = 2.44

$$\frac{e^{c+dx} (a + b)}{2 d} - \frac{2 \operatorname{atan} \left(\frac{b e^{dx} e^c \sqrt{d^2}}{d \sqrt{b^2}} \right) \sqrt{b^2}}{\sqrt{d^2}} - \frac{e^{-c-dx} (a + b)}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)*(a + b*tanh(c + d*x)^2),x)
```

```
[Out] (exp(c + d*x)*(a + b))/(2*d) - (2*atan((b*exp(d*x)*exp(c)*(d^2)^(1/2))/(d*(b^2)^(1/2)))*(b^2)^(1/2))/(d^2)^(1/2) - (exp(- c - d*x)*(a + b))/(2*d)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int (a + b \tanh^2(c + dx)) \cosh(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*cosh(c + d*x), x)
```

3.85 $\int \operatorname{sech}(c + dx) (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=40

$$\frac{(2a + b) \tan^{-1}(\sinh(c + dx))}{2d} - \frac{b \tanh(c + dx) \operatorname{sech}(c + dx)}{2d}$$

[Out] $1/2*(2*a+b)*\arctan(\sinh(d*x+c))/d-1/2*b*\operatorname{sech}(d*x+c)*\tanh(d*x+c)/d$

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3676, 385, 203}

$$\frac{(2a + b) \tan^{-1}(\sinh(c + dx))}{2d} - \frac{b \tanh(c + dx) \operatorname{sech}(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sech}[c + d*x]*(a + b*\text{Tanh}[c + d*x]^2), x]$

[Out] $((2*a + b)*\text{ArcTan}[\text{Sinh}[c + d*x]])/(2*d) - (b*\text{Sech}[c + d*x]*\text{Tanh}[c + d*x])/(2*d)$

Rule 203

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[\text{Rt}[b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /;$ FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 385

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)*((c_ + (d_)*(x_)^{(n_)})}, x_Symbol] \rightarrow -\text{Simp}[(b*c - a*d)*x*(a + b*x^n)^{(p + 1)}]/(a*b*n*(p + 1)), x] - \text{Dist}[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), \text{Int}[(a + b*x^n)^{(p + 1)}, x], x] /;$ FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 3676

$\text{Int}[\sec[(e_ + (f_)*(x_)]^{(m_)*((a_ + (b_)*\tan[(e_ + (f_)*(x_)]^{(n_)}))^{(p_)}), x_Symbol] \rightarrow \text{With}\{\text{ff} = \text{FreeFactors}[\text{Sin}[e + f*x], x]\}, \text{Dist}[\text{ff}/f, \text{Subst}[\text{Int}[\text{ExpandToSum}[b*(\text{ff}*x)^n + a*(1 - \text{ff}^2*x^2)^{(n/2)}], x]^p/(1 - \text{ff}^2*x^2)^{((m + n*p + 1)/2)}, x], x, \text{Sin}[e + f*x]/\text{ff}, x] /;$ FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \operatorname{sech}(c + dx) (a + b \tanh^2(c + dx)) dx &= \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{(1+x^2)^2} dx, x, \sinh(c + dx)\right)}{d} \\ &= -\frac{b \operatorname{sech}(c + dx) \tanh(c + dx)}{2d} + \frac{(2a + b) \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(c + dx)\right)}{2d} \\ &= \frac{(2a + b) \tan^{-1}(\sinh(c + dx))}{2d} - \frac{b \operatorname{sech}(c + dx) \tanh(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 1.20

$$\frac{a \tan^{-1}(\sinh(c + dx))}{d} + \frac{b \tan^{-1}(\sinh(c + dx))}{2d} - \frac{b \tanh(c + dx) \operatorname{sech}(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2), x]

[Out] (a*ArcTan[Sinh[c + d*x]])/d + (b*ArcTan[Sinh[c + d*x]])/(2*d) - (b*Sech[c + d*x]*Tanh[c + d*x])/(2*d)

fricas [B] time = 0.39, size = 323, normalized size = 8.08

$$\frac{b \cosh(dx + c)^3 + 3b \cosh(dx + c) \sinh(dx + c)^2 + b \sinh(dx + c)^3 - ((2a + b) \cosh(dx + c)^4 + 4(2a + b)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] -(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 - ((2*a + b)*cosh(d*x + c)^4 + 4*(2*a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a + b)*sinh(d*x + c)^4 + 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*(2*a + b)*cosh(d*x + c)^2 + 2*a + b)*sinh(d*x + c)^2 + 4*((2*a + b)*cosh(d*x + c)^3 + (2*a + b)*cosh(d*x + c)*sinh(d*x + c) + 2*a + b)*arctan(cosh(d*x + c) + sinh(d*x + c)) - b*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)

giac [A] time = 0.14, size = 63, normalized size = 1.58

$$\frac{(2ae^c + be^c) \arctan(e^{(dx+c)}) e^{-c} - \frac{be^{(3dx+3c)} - be^{(dx+c)}}{(e^{(2dx+2c)} + 1)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2), x, algorithm="giac")

[Out] ((2*a*e^c + b*e^c)*arctan(e^(d*x + c))*e^(-c) - (b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^2)/d

maple [A] time = 0.27, size = 65, normalized size = 1.62

$$\frac{2a \arctan(e^{dx+c})}{d} - \frac{b \sinh(dx + c)}{d \cosh(dx + c)^2} + \frac{b \operatorname{sech}(dx + c) \tanh(dx + c)}{2d} + \frac{b \arctan(e^{dx+c})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)*(a+b*tanh(d*x+c)^2), x)

[Out] 2/d*a*arctan(exp(d*x+c))-1/d*b*sinh(d*x+c)/cosh(d*x+c)^2+1/2*b*sech(d*x+c)*tanh(d*x+c)/d+1/d*b*arctan(exp(d*x+c))

maxima [B] time = 0.43, size = 80, normalized size = 2.00

$$-b \left(\frac{\arctan(e^{(-dx-c)})}{d} + \frac{e^{(-dx-c)} - e^{(-3dx-3c)}}{d(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1)} \right) + \frac{a \arctan(\sinh(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2), x, algorithm="maxima")

[Out] -b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a*arctan(sinh(d*x + c))/d

mupad [B] time = 0.13, size = 125, normalized size = 3.12

$$\frac{\operatorname{atan}\left(\frac{e^{dx} e^c (2a\sqrt{d^2} + b\sqrt{d^2})}{d\sqrt{4a^2 + 4ab + b^2}}\right) \sqrt{4a^2 + 4ab + b^2}}{\sqrt{d^2}} - \frac{b e^{c+dx}}{d(e^{2c+2dx} + 1)} + \frac{2b e^{c+dx}}{d(2e^{2c+2dx} + e^{4c+4dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*tanh(c + d*x)^2)/cosh(c + d*x), x)`

[Out] `(atan((exp(d*x)*exp(c)*(2*a*(d^2)^(1/2) + b*(d^2)^(1/2)))/(d*(4*a*b + 4*a^2 + b^2)^(1/2)))*(4*a*b + 4*a^2 + b^2)^(1/2))/(d^2)^(1/2) - (b*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)) + (2*b*exp(c + d*x))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx)) \operatorname{sech}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)*(a+b*tanh(d*x+c)**2), x)`

[Out] `Integral((a + b*tanh(c + d*x)**2)*sech(c + d*x), x)`

3.86 $\int \operatorname{sech}^2(c + dx) (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=28

$$\frac{a \tanh(c + dx)}{d} + \frac{b \tanh^3(c + dx)}{3d}$$

[Out] a*tanh(d*x+c)/d+1/3*b*tanh(d*x+c)^3/d

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {3675}

$$\frac{a \tanh(c + dx)}{d} + \frac{b \tanh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2), x]

[Out] (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x]^3)/(3*d)

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^2(c + dx) (a + b \tanh^2(c + dx)) dx &= \frac{\operatorname{Subst}\left(\int (a + bx^2) dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{a \tanh(c + dx)}{d} + \frac{b \tanh^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{a \tanh(c + dx)}{d} + \frac{b \tanh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2), x]

[Out] (a*Tanh[c + d*x])/d + (b*Tanh[c + d*x]^3)/(3*d)

fricas [B] time = 0.38, size = 159, normalized size = 5.68

$$\frac{4 \left((3a + 2b) \cosh(dx + c)^2 + 2b \cosh(dx + c) \sinh(dx + c) \right)}{3 \left(d \cosh(dx + c)^4 + 4d \cosh(dx + c) \sinh(dx + c)^3 + d \sinh(dx + c)^4 + 4d \cosh(dx + c)^2 + 2 \left(3d \cosh(dx + c) \sinh(dx + c) \right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] $-4/3*((3*a + 2*b)*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + (3*a + 2*b)*\sinh(d*x + c)^2 + 3*a)/(d*\cosh(d*x + c)^4 + 4*d*\cosh(d*x + c)*\sinh(d*x + c)^3 + d*\sinh(d*x + c)^4 + 4*d*\cosh(d*x + c)^2 + 2*(3*d*\cosh(d*x + c)^2 + 2*d)*\sinh(d*x + c)^2 + 4*(d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c) + 3*d)$

giac [B] time = 0.16, size = 59, normalized size = 2.11

$$\frac{2(3ae^{4dx+4c} + 3be^{4dx+4c} + 6ae^{2dx+2c} + 3a + b)}{3d(e^{2dx+2c} + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] $-2/3*(3*a*e^{(4*d*x + 4*c)} + 3*b*e^{(4*d*x + 4*c)} + 6*a*e^{(2*d*x + 2*c)} + 3*a + b)/(d*(e^{(2*d*x + 2*c)} + 1)^3)$

maple [A] time = 0.37, size = 53, normalized size = 1.89

$$\frac{a \tanh(dx + c) + b \left(-\frac{\sinh(dx+c)}{2 \cosh(dx+c)^3} + \frac{\left(\frac{2}{3} + \frac{\operatorname{sech}(dx+c)^2}{3}\right) \tanh(dx+c)}{2} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2),x)

[Out] $1/d*(a*\tanh(d*x+c)+b*(-1/2*\sinh(d*x+c)/\cosh(d*x+c)^3+1/2*(2/3+1/3*\operatorname{sech}(d*x+c)^2)*\tanh(d*x+c)))$

maxima [A] time = 0.34, size = 34, normalized size = 1.21

$$\frac{b \tanh(dx + c)^3}{3d} + \frac{2a}{d(e^{-2dx-2c} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] $1/3*b*\tanh(d*x + c)^3/d + 2*a/(d*(e^{(-2*d*x - 2*c)} + 1))$

mupad [B] time = 1.22, size = 59, normalized size = 2.11

$$\frac{2(3a + b + 6ae^{2c+2dx} + 3ae^{4c+4dx} + 3be^{4c+4dx})}{3d(e^{2c+2dx} + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tanh(c + d*x)^2)/cosh(c + d*x)^2,x)

[Out] $-(2*(3*a + b + 6*a*\exp(2*c + 2*d*x) + 3*a*\exp(4*c + 4*d*x) + 3*b*\exp(4*c + 4*d*x)))/(3*d*(\exp(2*c + 2*d*x) + 1)^3)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx)) \operatorname{sech}^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)

[Out] Integral((a + b*tanh(c + d*x)**2)*sech(c + d*x)**2, x)

3.87 $\int \operatorname{sech}^3(c + dx) (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=66

$$\frac{(4a + b) \tan^{-1}(\sinh(c + dx))}{8d} + \frac{(4a + b) \tanh(c + dx) \operatorname{sech}(c + dx)}{8d} - \frac{b \tanh(c + dx) \operatorname{sech}^3(c + dx)}{4d}$$

[Out] 1/8*(4*a+b)*arctan(sinh(d*x+c))/d+1/8*(4*a+b)*sech(d*x+c)*tanh(d*x+c)/d-1/4*b*sech(d*x+c)^3*tanh(d*x+c)/d

Rubi [A] time = 0.05, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3676, 385, 199, 203}

$$\frac{(4a + b) \tan^{-1}(\sinh(c + dx))}{8d} + \frac{(4a + b) \tanh(c + dx) \operatorname{sech}(c + dx)}{8d} - \frac{b \tanh(c + dx) \operatorname{sech}^3(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2), x]

[Out] ((4*a + b)*ArcTan[Sinh[c + d*x]])/(8*d) + ((4*a + b)*Sech[c + d*x]*Tanh[c + d*x])/(8*d) - (b*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 3676

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^3(c+dx) (a+b \tanh^2(c+dx)) dx &= \frac{\operatorname{Subst}\left(\int \frac{a+(a+b)x^2}{(1+x^2)^3} dx, x, \sinh(c+dx)\right)}{d} \\
&= -\frac{b \operatorname{sech}^3(c+dx) \tanh(c+dx)}{4d} + \frac{(4a+b) \operatorname{Subst}\left(\int \frac{1}{(1+x^2)^2} dx, x, \sinh(c+dx)\right)}{4d} \\
&= \frac{(4a+b) \operatorname{sech}(c+dx) \tanh(c+dx)}{8d} - \frac{b \operatorname{sech}^3(c+dx) \tanh(c+dx)}{4d} + \frac{a \tan^{-1}(\sinh(c+dx))}{2d} \\
&= \frac{(4a+b) \tan^{-1}(\sinh(c+dx))}{8d} + \frac{(4a+b) \operatorname{sech}(c+dx) \tanh(c+dx)}{8d}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 93, normalized size = 1.41

$$\frac{a \tan^{-1}(\sinh(c+dx))}{2d} + \frac{a \tanh(c+dx) \operatorname{sech}(c+dx)}{2d} + \frac{b \tan^{-1}(\sinh(c+dx))}{8d} - \frac{b \tanh(c+dx) \operatorname{sech}^3(c+dx)}{4d} + \frac{b \tanh(c+dx) \operatorname{sech}(c+dx)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2), x]

[Out] (a*ArcTan[Sinh[c + d*x]])/(2*d) + (b*ArcTan[Sinh[c + d*x]])/(8*d) + (a*Sech[c + d*x]*Tanh[c + d*x])/(2*d) + (b*Sech[c + d*x]*Tanh[c + d*x])/(8*d) - (b*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)

fricas [B] time = 0.40, size = 1046, normalized size = 15.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] 1/4*((4*a + b)*cosh(d*x + c)^7 + 7*(4*a + b)*cosh(d*x + c)*sinh(d*x + c)^6 + (4*a + b)*sinh(d*x + c)^7 + (4*a - 7*b)*cosh(d*x + c)^5 + (21*(4*a + b)*cosh(d*x + c)^2 + 4*a - 7*b)*sinh(d*x + c)^5 + 5*(7*(4*a + b)*cosh(d*x + c)^3 + (4*a - 7*b)*cosh(d*x + c))*sinh(d*x + c)^4 - (4*a - 7*b)*cosh(d*x + c)^3 + (35*(4*a + b)*cosh(d*x + c)^4 + 10*(4*a - 7*b)*cosh(d*x + c)^2 - 4*a + 7*b)*sinh(d*x + c)^3 + (21*(4*a + b)*cosh(d*x + c)^5 + 10*(4*a - 7*b)*cosh(d*x + c)^3 - 3*(4*a - 7*b)*cosh(d*x + c))*sinh(d*x + c)^2 + ((4*a + b)*cosh(d*x + c)^8 + 8*(4*a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a + b)*sinh(d*x + c)^8 + 4*(4*a + b)*cosh(d*x + c)^6 + 4*(7*(4*a + b)*cosh(d*x + c)^2 + 4*a + b)*sinh(d*x + c)^6 + 8*(7*(4*a + b)*cosh(d*x + c)^3 + 3*(4*a + b)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(4*a + b)*cosh(d*x + c)^4 + 2*(35*(4*a + b)*cosh(d*x + c)^4 + 30*(4*a + b)*cosh(d*x + c)^2 + 12*a + 3*b)*sinh(d*x + c)^4 + 8*(7*(4*a + b)*cosh(d*x + c)^5 + 10*(4*a + b)*cosh(d*x + c)^3 + 3*(4*a + b)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(4*a + b)*cosh(d*x + c)^2 + 4*(7*(4*a + b)*cosh(d*x + c)^6 + 15*(4*a + b)*cosh(d*x + c)^4 + 9*(4*a + b)*cosh(d*x + c)^2 + 4*a + b)*sinh(d*x + c)^2 + 8*((4*a + b)*cosh(d*x + c)^7 + 3*(4*a + b)*cosh(d*x + c)^5 + 3*(4*a + b)*cosh(d*x + c)^3 + (4*a + b)*cosh(d*x + c))*sinh(d*x + c) + 4*a + b)*arctan(cosh(d*x + c) + sinh(d*x + c)) - (4*a + b)*cosh(d*x + c) + (7*(4*a + b)*cosh(d*x + c)^6 + 5*(4*a - 7*b)*cosh(d*x + c)^4 - 3*(4*a - 7*b)*cosh(d*x + c)^2 - 4*a - b)*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d)*sinh(d*x + c)^3 + 4*(4*a + b)*cosh(d*x + c)^2 + 4*(7*(4*a + b)*cosh(d*x + c)^6 + 15*(4*a + b)*cosh(d*x + c)^4 + 9*(4*a + b)*cosh(d*x + c)^2 + 4*a + b)*sinh(d*x + c)^2 + 8*((4*a + b)*cosh(d*x + c)^7 + 3*(4*a + b)*cosh(d*x + c)^5 + 3*(4*a + b)*cosh(d*x + c)^3 + (4*a + b)*cosh(d*x + c))*sinh(d*x + c) + 4*a + b)*arctan(cosh(d*x + c) + sinh(d*x + c)) - (4*a + b)*cosh(d*x + c) + (7*(4*a + b)*cosh(d*x + c)^6 + 5*(4*a - 7*b)*cosh(d*x + c)^4 - 3*(4*a - 7*b)*cosh(d*x + c)^2 - 4*a - b)*sinh(d*x + c)

$$\operatorname{sh}(d*x + c)^5 + 10*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c)*\sinh(d*x + c)^3 + 4*d*\cosh(d*x + c)^2 + 4*(7*d*\cosh(d*x + c)^6 + 15*d*\cosh(d*x + c)^4 + 9*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^2 + 8*(d*\cosh(d*x + c)^7 + 3*d*\cosh(d*x + c)^5 + 3*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c) + d)$$

giac [B] time = 0.15, size = 132, normalized size = 2.00

$$\frac{(4ae^c + be^c) \arctan(e^{(dx+c)})e^{(-c)} + \frac{4ae^{(7dx+7c)} + be^{(7dx+7c)} + 4ae^{(5dx+5c)} - 7be^{(5dx+5c)} - 4ae^{(3dx+3c)} + 7be^{(3dx+3c)} - 4ae^{(dx+c)} - be^{(dx+c)}}{(e^{(2dx+2c)}+1)^4}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] $\frac{1}{4}*((4*a*e^c + b*e^c)*\arctan(e^{(d*x + c)})*e^{(-c)} + (4*a*e^{(7*d*x + 7*c)} + b*e^{(7*d*x + 7*c)} + 4*a*e^{(5*d*x + 5*c)} - 7*b*e^{(5*d*x + 5*c)} - 4*a*e^{(3*d*x + 3*c)} + 7*b*e^{(3*d*x + 3*c)} - 4*a*e^{(d*x + c)} - b*e^{(d*x + c)})/(e^{(2*d*x + 2*c)} + 1)^4)/d$

maple [A] time = 0.39, size = 103, normalized size = 1.56

$$\frac{a \operatorname{sech}(dx + c) \tanh(dx + c)}{2d} + \frac{a \arctan(e^{dx+c})}{d} - \frac{b \sinh(dx + c)}{3d \cosh(dx + c)^4} + \frac{b \operatorname{sech}(dx + c)^3 \tanh(dx + c)}{12d} + \frac{b \operatorname{sech}(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2),x)

[Out] $\frac{1}{2}/d*a*\operatorname{sech}(d*x+c)*\tanh(d*x+c) + 1/d*a*\arctan(\exp(d*x+c)) - 1/3/d*b*\sinh(d*x+c)/\cosh(d*x+c)^4 + 1/12*b*\operatorname{sech}(d*x+c)^3*\tanh(d*x+c)/d + 1/8*b*\operatorname{sech}(d*x+c)*\tanh(d*x+c)/d + 1/4/d*b*\arctan(\exp(d*x+c))$

maxima [B] time = 0.44, size = 181, normalized size = 2.74

$$-\frac{1}{4}b \left(\frac{\arctan(e^{(-dx-c)})}{d} - \frac{e^{(-dx-c)} - 7e^{(-3dx-3c)} + 7e^{(-5dx-5c)} - e^{(-7dx-7c)}}{d(4e^{(-2dx-2c)} + 6e^{(-4dx-4c)} + 4e^{(-6dx-6c)} + e^{(-8dx-8c)} + 1)} \right) - a \left(\frac{\arctan(e^{(-dx-c)})}{d} - \frac{e^{(-dx-c)} - 7e^{(-3dx-3c)} + 7e^{(-5dx-5c)} - e^{(-7dx-7c)}}{d(4e^{(-2dx-2c)} + 6e^{(-4dx-4c)} + 4e^{(-6dx-6c)} + e^{(-8dx-8c)} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] $-\frac{1}{4}b*(\arctan(e^{(-d*x - c)})/d - (e^{(-d*x - c)} - 7e^{(-3*d*x - 3*c)} + 7e^{(-5*d*x - 5*c)} - e^{(-7*d*x - 7*c)})/(d*(4e^{(-2*d*x - 2*c)} + 6e^{(-4*d*x - 4*c)} + 4e^{(-6*d*x - 6*c)} + e^{(-8*d*x - 8*c)} + 1))) - a*(\arctan(e^{(-d*x - c)})/d - (e^{(-d*x - c)} - 7e^{(-3*d*x - 3*c)})/(d*(2e^{(-2*d*x - 2*c)} + e^{(-4*d*x - 4*c)} + 1)))$

mupad [B] time = 1.25, size = 280, normalized size = 4.24

$$\frac{\operatorname{atan}\left(\frac{e^{dx}e^c(4a\sqrt{d^2}+b\sqrt{d^2})}{d\sqrt{16a^2+8ab+b^2}}\right)\sqrt{16a^2+8ab+b^2}}{4\sqrt{d^2}} - \frac{\frac{e^{5c+5dx}(a+b)}{d} + \frac{2e^{3c+3dx}(a-b)}{d} + \frac{e^{c+dx}(a+b)}{d}}{4e^{2c+2dx} + 6e^{4c+4dx} + 4e^{6c+6dx} + e^{8c+8dx} + 1} - \frac{e^{c+d}}{2d(2e^{2c+2dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tanh(c + d*x)^2)/cosh(c + d*x)^3,x)

[Out] $(\operatorname{atan}((\exp(d*x)*\exp(c)*(4*a*(d^2)^{(1/2)} + b*(d^2)^{(1/2)}))/(d*(8*a*b + 16*a^2 + b^2)^{(1/2)}))*(8*a*b + 16*a^2 + b^2)^{(1/2)})/(4*(d^2)^{(1/2)}) - ((\exp(5*c + 5*d*x)*(a + b))/d + (2*\exp(3*c + 3*d*x)*(a - b))/d + (\exp(c + d*x)*(a + b))/d)$

```

))/d)/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8
*c + 8*d*x) + 1) - (exp(c + d*x)*(2*a + 3*b))/(2*d*(2*exp(2*c + 2*d*x) + ex
p(4*c + 4*d*x) + 1)) + (exp(c + d*x)*(4*a + b))/(4*d*(exp(2*c + 2*d*x) + 1)
) + (2*b*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*
c + 6*d*x) + 1))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx)) \operatorname{sech}^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*tanh(d*x+c)**2), x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*sech(c + d*x)**3, x)
```

3.88 $\int \operatorname{sech}^4(c + dx) (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=48

$$-\frac{(a-b)\tanh^3(c+dx)}{3d} + \frac{a\tanh(c+dx)}{d} - \frac{b\tanh^5(c+dx)}{5d}$$

[Out] a*tanh(d*x+c)/d-1/3*(a-b)*tanh(d*x+c)^3/d-1/5*b*tanh(d*x+c)^5/d

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3675, 373}

$$-\frac{(a-b)\tanh^3(c+dx)}{3d} + \frac{a\tanh(c+dx)}{d} - \frac{b\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2), x]

[Out] (a*Tanh[c + d*x])/d - ((a - b)*Tanh[c + d*x]^3)/(3*d) - (b*Tanh[c + d*x]^5)/(5*d)

Rule 373

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m-1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^4(c + dx) (a + b \tanh^2(c + dx)) dx &= \frac{\operatorname{Subst}\left(\int (1 - x^2) (a + bx^2) dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int (a - (a - b)x^2 - bx^4) dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{a \tanh(c + dx)}{d} - \frac{(a - b) \tanh^3(c + dx)}{3d} - \frac{b \tanh^5(c + dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.05, size = 86, normalized size = 1.79

$$-\frac{a \tanh^3(c + dx)}{3d} + \frac{a \tanh(c + dx)}{d} + \frac{2b \tanh(c + dx)}{15d} - \frac{b \tanh(c + dx) \operatorname{sech}^4(c + dx)}{5d} + \frac{b \tanh(c + dx) \operatorname{sech}^2(c + dx)}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2), x]

[Out] $(a*\text{Tanh}[c + d*x])/d + (2*b*\text{Tanh}[c + d*x])/(15*d) + (b*\text{Sech}[c + d*x]^2*\text{Tanh}[c + d*x])/(15*d) - (b*\text{Sech}[c + d*x]^4*\text{Tanh}[c + d*x])/(5*d) - (a*\text{Tanh}[c + d*x]^3)/(3*d)$

fricas [B] time = 0.39, size = 345, normalized size = 7.19

$$15 \left(d \cosh(dx + c)^7 + 7 d \cosh(dx + c) \sinh(dx + c)^6 + d \sinh(dx + c)^7 + 5 d \cosh(dx + c)^5 + (21 d \cosh(dx + c) \sinh(dx + c))^4 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")`

[Out] $-8/15*(2*(5*a + 4*b)*\cosh(d*x + c)^3 + 6*(5*a + 4*b)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (5*a + 7*b)*\sinh(d*x + c)^3 + 30*a*\cosh(d*x + c) + (3*(5*a + 7*b)*\cosh(d*x + c)^2 + 5*a - 5*b)*\sinh(d*x + c))/(d*\cosh(d*x + c)^7 + 7*d*\cosh(d*x + c)*\sinh(d*x + c)^6 + d*\sinh(d*x + c)^7 + 5*d*\cosh(d*x + c)^5 + (21*d*\cosh(d*x + c)^2 + 5*d)*\sinh(d*x + c)^5 + 5*(7*d*\cosh(d*x + c)^3 + 5*d*\cosh(d*x + c))*\sinh(d*x + c)^4 + 11*d*\cosh(d*x + c)^3 + (35*d*\cosh(d*x + c)^4 + 50*d*\cosh(d*x + c)^2 + 9*d)*\sinh(d*x + c)^3 + (21*d*\cosh(d*x + c)^5 + 50*d*\cosh(d*x + c)^3 + 33*d*\cosh(d*x + c))*\sinh(d*x + c)^2 + 15*d*\cosh(d*x + c) + (7*d*\cosh(d*x + c)^6 + 25*d*\cosh(d*x + c)^4 + 27*d*\cosh(d*x + c)^2 + 5*d)*\sinh(d*x + c)$

giac [B] time = 0.16, size = 95, normalized size = 1.98

$$\frac{4 \left(15 a e^{(6dx+6c)} + 15 b e^{(6dx+6c)} + 35 a e^{(4dx+4c)} - 5 b e^{(4dx+4c)} + 25 a e^{(2dx+2c)} + 5 b e^{(2dx+2c)} + 5 a + b \right)}{15 d \left(e^{(2dx+2c)} + 1 \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")`

[Out] $-4/15*(15*a*e^{(6*d*x + 6*c)} + 15*b*e^{(6*d*x + 6*c)} + 35*a*e^{(4*d*x + 4*c)} - 5*b*e^{(4*d*x + 4*c)} + 25*a*e^{(2*d*x + 2*c)} + 5*b*e^{(2*d*x + 2*c)} + 5*a + b)/(d*(e^{(2*d*x + 2*c)} + 1)^5)$

maple [A] time = 0.36, size = 75, normalized size = 1.56

$$\frac{a \left(\frac{2}{3} + \frac{\text{sech}(dx+c)^2}{3} \right) \tanh(dx + c) + b \left(-\frac{\sinh(dx+c)}{4 \cosh(dx+c)^5} + \frac{\left(\frac{8}{15} + \frac{\text{sech}(dx+c)^4}{5} + \frac{4 \text{sech}(dx+c)^2}{15} \right) \tanh(dx+c)}{4} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2),x)`

[Out] $1/d*(a*(2/3+1/3*\text{sech}(d*x+c)^2)*\text{tanh}(d*x+c)+b*(-1/4*\sinh(d*x+c)/\cosh(d*x+c)^5+1/4*(8/15+1/5*\text{sech}(d*x+c)^4+4/15*\text{sech}(d*x+c)^2)*\text{tanh}(d*x+c)))$

maxima [B] time = 0.35, size = 371, normalized size = 7.73

$$\frac{4}{15} b \left(\frac{5 e^{(-2dx-2c)}}{d(5 e^{(-2dx-2c)} + 10 e^{(-4dx-4c)} + 10 e^{(-6dx-6c)} + 5 e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} - \frac{1}{d(5 e^{(-2dx-2c)} + 10 e^{(-4dx-4c)} + 10 e^{(-6dx-6c)} + 5 e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")`


```
[Out] 4/15*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))))
```

mpad [B] time = 0.17, size = 304, normalized size = 6.33

$$\frac{\frac{8(a-b)}{15d} + \frac{4e^{2c+2dx}(a+b)}{5d}}{3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1} - \frac{\frac{8e^{2c+2dx}(a+b)}{5d} + \frac{8e^{6c+6dx}(a+b)}{5d} + \frac{16e^{4c+4dx}(a-b)}{5d}}{5e^{2c+2dx} + 10e^{4c+4dx} + 10e^{6c+6dx} + 5e^{8c+8dx} + e^{10c+10dx} + 1} - \frac{4e^{10c+10dx}}{10e^{10c+10dx} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)/cosh(c + d*x)^4, x)
```

```
[Out] - ((8*(a - b))/(15*d) + (4*exp(2*c + 2*d*x)*(a + b))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((8*exp(2*c + 2*d*x)*(a + b))/(5*d) + (8*exp(6*c + 6*d*x)*(a + b))/(5*d) + (16*exp(4*c + 4*d*x)*(a - b))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*(a + b))/(5*d) + (6*exp(4*c + 4*d*x)*(a + b))/(5*d) + (8*exp(2*c + 2*d*x)*(a - b))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - (2*(a + b))/(5*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx)) \operatorname{sech}^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*tanh(d*x+c)**2), x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)*sech(c + d*x)**4, x)
```

$$3.89 \quad \int \cosh^4(c + dx) \left(a + b \tanh^2(c + dx) \right)^2 dx$$

Optimal. Leaf size=85

$$\frac{3(a^2 - b^2) \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{1}{8} x (3a^2 - 2ab + 3b^2) + \frac{(a + b) \sinh(c + dx) \cosh^3(c + dx) (a + b \tanh^2(c + dx))}{4d}$$

[Out] $1/8*(3*a^2-2*a*b+3*b^2)*x+3/8*(a^2-b^2)*\cosh(d*x+c)*\sinh(d*x+c)/d+1/4*(a+b)*\cosh(d*x+c)^3*\sinh(d*x+c)*(a+b*\tanh(d*x+c)^2)/d$

Rubi [A] time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3675, 413, 385, 206}

$$\frac{3(a^2 - b^2) \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{1}{8} x (3a^2 - 2ab + 3b^2) + \frac{(a + b) \sinh(c + dx) \cosh^3(c + dx) (a + b \tanh^2(c + dx))}{4d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] $((3*a^2 - 2*a*b + 3*b^2)*x)/8 + (3*(a^2 - b^2)*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])/(8*d) + ((a + b)*\text{Cosh}[c + d*x]^3*\text{Sinh}[c + d*x]*(a + b*\text{Tanh}[c + d*x]^2))/(4*d)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 413

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p + 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p + q) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\int \cosh^4(c + dx) (a + b \tanh^2(c + dx))^2 dx = \frac{\text{Subst} \left(\int \frac{(a+bx^2)^2}{(1-x^2)^3} dx, x, \tanh(c + dx) \right)}{d}$$

$$= \frac{(a + b) \cosh^3(c + dx) \sinh(c + dx) (a + b \tanh^2(c + dx))}{4d} - \frac{\text{Subst} \left(\int \frac{(a+bx^2)^2}{(1-x^2)^3} dx, x, \tanh(c + dx) \right)}{4d}$$

$$= \frac{3(a^2 - b^2) \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{(a + b) \cosh^3(c + dx) \sinh(c + dx)}{4d}$$

$$= \frac{1}{8} (3a^2 - 2ab + 3b^2) x + \frac{3(a^2 - b^2) \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{(a + b) \cosh^3(c + dx) \sinh(c + dx)}{4d}$$

Mathematica [A] time = 0.33, size = 63, normalized size = 0.74

$$\frac{4(3a^2 - 2ab + 3b^2)(c + dx) + 8(a^2 - b^2) \sinh(2(c + dx)) + (a + b)^2 \sinh(4(c + dx))}{32d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (4*(3*a^2 - 2*a*b + 3*b^2)*(c + d*x) + 8*(a^2 - b^2)*Sinh[2*(c + d*x)] + (a + b)^2*Sinh[4*(c + d*x)])/(32*d)

fricas [A] time = 0.41, size = 95, normalized size = 1.12

$$\frac{(a^2 + 2ab + b^2) \cosh(dx + c) \sinh(dx + c)^3 + (3a^2 - 2ab + 3b^2) dx + ((a^2 + 2ab + b^2) \cosh(dx + c)^3 + 4(a + b) \cosh(dx + c) \sinh(dx + c)) \sinh(dx + c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] 1/8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 - 2*a*b + 3*b^2)*d*x + ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 4*(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))/d

giac [B] time = 0.39, size = 189, normalized size = 2.22

$$\frac{8(3a^2 - 2ab + 3b^2) dx - (18a^2 e^{4dx+4c} - 12abe^{4dx+4c} + 18b^2 e^{4dx+4c} + 8a^2 e^{2dx+2c} - 8b^2 e^{2dx+2c} + a^2 + b^2) \cosh(dx+c) \sinh(dx+c)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/64*(8*(3*a^2 - 2*a*b + 3*b^2)*d*x - (18*a^2*e^(4*d*x + 4*c) - 12*a*b*e^(4*d*x + 4*c) + 18*b^2*e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) - 8*b^2*e^(2*d*x + 2*c) + a^2 + 2*a*b + b^2)*e^(-4*d*x - 4*c) + (a^2*e^(4*d*x + 12*c) + 2*a*b*e^(4*d*x + 12*c) + b^2*e^(4*d*x + 12*c) + 8*a^2*e^(2*d*x + 10*c) - 8*b^2*e^(2*d*x + 10*c))*e^(-8*c))/d

maple [A] time = 0.38, size = 124, normalized size = 1.46

$$\frac{b^2 \left(\left(\frac{\sinh^3(dx+c)}{4} - \frac{3 \sinh(dx+c)}{8} \right) \cosh(dx+c) + \frac{3dx}{8} + \frac{3c}{8} \right) + 2ab \left(\frac{\sinh(dx+c) \cosh^3(dx+c)}{4} - \frac{\cosh(dx+c) \sinh(dx+c)}{8} - \frac{dx}{8} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x)`

[Out] $\frac{1}{d} * (b^2 * ((\frac{1}{4} * \sinh(d*x+c)^3 - \frac{3}{8} * \sinh(d*x+c)) * \cosh(d*x+c) + \frac{3}{8} * d*x + \frac{3}{8} * c) + 2 * a * b * (\frac{1}{4} * \sinh(d*x+c) * \cosh(d*x+c)^3 - \frac{1}{8} * \cosh(d*x+c) * \sinh(d*x+c) - \frac{1}{8} * d*x - \frac{1}{8} * c) + a^2 * ((\frac{1}{4} * \cosh(d*x+c)^3 + \frac{3}{8} * \cosh(d*x+c)) * \sinh(d*x+c) + \frac{3}{8} * d*x + \frac{3}{8} * c))$

maxima [B] time = 0.36, size = 171, normalized size = 2.01

$$\frac{1}{64} a^2 \left(24x + \frac{e^{(4dx+4c)}}{d} + \frac{8e^{(2dx+2c)}}{d} - \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) + \frac{1}{64} b^2 \left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] $\frac{1}{64} * a^2 * (24 * x + \frac{e^{(4 * d * x + 4 * c)}}{d} + \frac{8 * e^{(2 * d * x + 2 * c)}}{d} - \frac{8 * e^{(-2 * d * x - 2 * c)}}{d} - \frac{e^{(-4 * d * x - 4 * c)}}{d}) + \frac{1}{64} * b^2 * (24 * x + \frac{e^{(4 * d * x + 4 * c)}}{d} - \frac{8 * e^{(2 * d * x + 2 * c)}}{d} + \frac{8 * e^{(-2 * d * x - 2 * c)}}{d} - \frac{e^{(-4 * d * x - 4 * c)}}{d}) - \frac{1}{32} * a * b * (8 * (d * x + c) / d - \frac{e^{(4 * d * x + 4 * c)}}{d} + \frac{e^{(-4 * d * x - 4 * c)}}{d})$

mupad [B] time = 0.27, size = 102, normalized size = 1.20

$$x \left(\frac{3a^2}{8} - \frac{ab}{4} + \frac{3b^2}{8} \right) - \frac{e^{-2c-2dx} (a^2 - b^2)}{8d} + \frac{e^{2c+2dx} (a^2 - b^2)}{8d} - \frac{e^{-4c-4dx} (a+b)^2}{64d} + \frac{e^{4c+4dx} (a+b)^2}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2,x)`

[Out] $x * ((\frac{3 * a^2}{8} - \frac{a * b}{4} + \frac{3 * b^2}{8}) - \frac{(\exp(-2 * c - 2 * d * x) * (a^2 - b^2))}{(8 * d)}) + \frac{(\exp(2 * c + 2 * d * x) * (a^2 - b^2))}{(8 * d)} - \frac{(\exp(-4 * c - 4 * d * x) * (a + b)^2)}{(64 * d)} + \frac{(\exp(4 * c + 4 * d * x) * (a + b)^2)}{(64 * d)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^2 \cosh^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)`

[Out] `Integral((a + b*tanh(c + d*x)**2)**2*cosh(c + d*x)**4, x)`

3.90 $\int \cosh^3(c + dx) (a + b \tanh^2(c + dx))^2 dx$

Optimal. Leaf size=54

$$\frac{(a^2 - b^2) \sinh(c + dx)}{d} + \frac{(a + b)^2 \sinh^3(c + dx)}{3d} + \frac{b^2 \tan^{-1}(\sinh(c + dx))}{d}$$

[Out] $b^2 \arctan(\sinh(d*x+c))/d + (a^2 - b^2) * \sinh(d*x+c)/d + 1/3 * (a+b)^2 * \sinh(d*x+c)^3/d$

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3676, 390, 203}

$$\frac{(a^2 - b^2) \sinh(c + dx)}{d} + \frac{(a + b)^2 \sinh^3(c + dx)}{3d} + \frac{b^2 \tan^{-1}(\sinh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] $(b^2 * \text{ArcTan}[\text{Sinh}[c + d*x]])/d + ((a^2 - b^2) * \text{Sinh}[c + d*x])/d + ((a + b)^2 * \text{Sinh}[c + d*x]^3)/(3*d)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 3676

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^(m + n*p + 1)/2], x], x, Sin[e + f*x]/ff, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \cosh^3(c + dx) (a + b \tanh^2(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \frac{(a+(a+b)x^2)^2}{1+x^2} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(a^2 - b^2 + (a + b)^2 x^2 + \frac{b^2}{1+x^2}\right) dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{(a^2 - b^2) \sinh(c + dx)}{d} + \frac{(a + b)^2 \sinh^3(c + dx)}{3d} + \frac{b^2 \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{b^2 \tan^{-1}(\sinh(c + dx))}{d} + \frac{(a^2 - b^2) \sinh(c + dx)}{d} + \frac{(a + b)^2 \sinh^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.46, size = 71, normalized size = 1.31

$$\frac{\sinh(c + dx) \left((a + b)((a + b) \cosh(2(c + dx)) + 5a - 7b) + \frac{6b^2 \tanh^{-1}\left(\sqrt{-\sinh^2(c+dx)}\right)}{\sqrt{-\sinh^2(c+dx)}} \right)}{6d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (Sinh[c + d*x]*((a + b)*(5*a - 7*b + (a + b)*Cosh[2*(c + d*x)]) + (6*b^2*ArcTanH[Sqrt[-Sinh[c + d*x]^2]])/Sqrt[-Sinh[c + d*x]^2]))/(6*d)

fricas [B] time = 0.42, size = 519, normalized size = 9.61

$$\frac{(a^2 + 2ab + b^2) \cosh(dx + c)^6 + 6(a^2 + 2ab + b^2) \cosh(dx + c) \sinh(dx + c)^5 + (a^2 + 2ab + b^2) \sinh(dx + c)^6}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] 1/24*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + 3*(3*a^2 - 2*a*b - 5*b^2)*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a*b - 5*b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(3*a^2 - 2*a*b - 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(3*a^2 - 2*a*b - 5*b^2)*cosh(d*x + c)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(3*a^2 - 2*a*b - 5*b^2)*cosh(d*x + c)^2 - 3*a^2 + 2*a*b + 5*b^2)*sinh(d*x + c)^2 - a^2 - 2*a*b - b^2 + 48*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)^2*sinh(d*x + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x + c)^3)*arctan(cosh(d*x + c) + sinh(d*x + c)) + 6*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 2*(3*a^2 - 2*a*b - 5*b^2)*cosh(d*x + c)^3 - (3*a^2 - 2*a*b - 5*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3)

giac [B] time = 0.36, size = 164, normalized size = 3.04

$$\frac{48b^2 \arctan(e^{(dx+c)}) - (9a^2e^{(2dx+2c)} - 6abe^{(2dx+2c)} - 15b^2e^{(2dx+2c)} + a^2 + 2ab + b^2)e^{(-3dx-3c)} + (a^2e^{(3dx+18c)} + 2a^2e^{(3dx+18c)} - 6a^2e^{(3dx+18c)} - 6a^2e^{(3dx+18c)} - 6a^2e^{(3dx+18c)} - 6a^2e^{(3dx+18c)})e^{(-15c)}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/24*(48*b^2*arctan(e^(d*x + c)) - (9*a^2*e^(2*d*x + 2*c) - 6*a*b*e^(2*d*x + 2*c) - 15*b^2*e^(2*d*x + 2*c) + a^2 + 2*a*b + b^2)*e^(-3*d*x - 3*c) + (a^2*e^(3*d*x + 18*c) + 2*a*b*e^(3*d*x + 18*c) + b^2*e^(3*d*x + 18*c) + 9*a^2*e^(d*x + 16*c) - 6*a*b*e^(d*x + 16*c) - 15*b^2*e^(d*x + 16*c))*e^(-15*c))/d

maple [A] time = 0.27, size = 98, normalized size = 1.81

$$\frac{2a^2 \sinh(dx + c)}{3d} + \frac{a^2 \sinh(dx + c) (\cosh^2(dx + c))}{3d} + \frac{2ab (\sinh^3(dx + c))}{3d} + \frac{b^2 (\sinh^3(dx + c))}{3d} - \frac{b^2 \sinh(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x)

[Out] 2/3*a^2*sinh(d*x+c)/d+1/3/d*a^2*sinh(d*x+c)*cosh(d*x+c)^2+2/3*a*b*sinh(d*x+c)^3/d+1/3*b^2*sinh(d*x+c)^3/d-b^2*sinh(d*x+c)/d+2/d*b^2*arctan(exp(d*x+c))

maxima [B] time = 0.50, size = 161, normalized size = 2.98

$$\frac{ab(e^{dx+c} - e^{-dx-c})^3}{12d} - \frac{1}{24}b^2 \left(\frac{(15e^{(-2dx-2c)} - 1)e^{(3dx+3c)}}{d} - \frac{15e^{(-dx-c)} - e^{(-3dx-3c)}}{d} + \frac{48 \arctan(e^{(-dx-c)})}{d} \right) + \frac{1}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] 1/12*a*b*(e^(d*x + c) - e^(-d*x - c))^3/d - 1/24*b^2*((15*e^(-2*d*x - 2*c) - 1)*e^(3*d*x + 3*c)/d - (15*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + 48*arctan(e^(-d*x - c))/d) + 1/24*a^2*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d)

mupad [B] time = 0.26, size = 130, normalized size = 2.41

$$\frac{e^{3c+3dx}(a+b)^2}{24d} - \frac{e^{-3c-3dx}(a+b)^2}{24d} - \frac{e^{c+dx}(-3a^2+2ab+5b^2)}{8d} + \frac{2 \operatorname{atan}\left(\frac{b^2 e^{dx} e^c \sqrt{d^2}}{d \sqrt{b^4}}\right) \sqrt{b^4}}{\sqrt{d^2}} + \frac{e^{-c-dx}(-3a^2+5b^2)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2,x)

[Out] (exp(3*c + 3*d*x)*(a + b)^2)/(24*d) - (exp(- 3*c - 3*d*x)*(a + b)^2)/(24*d) - (exp(c + d*x)*(2*a*b - 3*a^2 + 5*b^2))/(8*d) + (2*atan((b^2*exp(d*x)*exp(c)*(d^2)^(1/2))/(d*(b^4)^(1/2)))*(b^4)^(1/2))/(d^2)^(1/2) + (exp(- c - d*x)*(2*a*b - 3*a^2 + 5*b^2))/(8*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^2 \cosh^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)

[Out] Integral((a + b*tanh(c + d*x)**2)**2*cosh(c + d*x)**3, x)

3.91 $\int \cosh^2(c + dx) (a + b \tanh^2(c + dx))^2 dx$

Optimal. Leaf size=51

$$\frac{(a + b)^2 \sinh(c + dx) \cosh(c + dx)}{2d} + \frac{1}{2}x(a - 3b)(a + b) + \frac{b^2 \tanh(c + dx)}{d}$$

[Out] 1/2*(a-3*b)*(a+b)*x+1/2*(a+b)^2*cosh(d*x+c)*sinh(d*x+c)/d+b^2*tanh(d*x+c)/d

Rubi [A] time = 0.08, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3675, 390, 385, 206}

$$\frac{(a + b)^2 \sinh(c + dx) \cosh(c + dx)}{2d} + \frac{1}{2}x(a - 3b)(a + b) + \frac{b^2 \tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] ((a - 3*b)*(a + b)*x)/2 + ((a + b)^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (b^2*Tanh[c + d*x])/d

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned}
\int \cosh^2(c+dx) (a+b \tanh^2(c+dx))^2 dx &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{(1-x^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(b^2 + \frac{a^2-b^2+2b(a+b)x^2}{(1-x^2)^2}\right) dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{b^2 \tanh(c+dx)}{d} + \frac{\text{Subst}\left(\int \frac{a^2-b^2+2b(a+b)x^2}{(1-x^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{(a+b)^2 \cosh(c+dx) \sinh(c+dx)}{2d} + \frac{b^2 \tanh(c+dx)}{d} + \frac{((a-3b)(a+b)(c+dx))}{2d} \\
&= \frac{1}{2}(a-3b)(a+b)x + \frac{(a+b)^2 \cosh(c+dx) \sinh(c+dx)}{2d} + \frac{b^2 \tanh(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.41, size = 54, normalized size = 1.06

$$\frac{(a-3b)(a+b)(c+dx)}{2d} + \frac{(a+b)^2 \sinh(2(c+dx))}{4d} + \frac{b^2 \tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] ((a - 3*b)*(a + b)*(c + d*x))/(2*d) + ((a + b)^2*Sinh[2*(c + d*x)])/(4*d) + (b^2*Tanh[c + d*x])/d

fricas [B] time = 0.43, size = 105, normalized size = 2.06

$$\frac{(a^2 + 2ab + b^2) \sinh(dx+c)^3 + 4((a^2 - 2ab - 3b^2)dx - 2b^2) \cosh(dx+c) + (3(a^2 + 2ab + b^2) \cosh(dx+c) + 8d \cosh(dx+c))}{8d \cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] 1/8*((a^2 + 2*a*b + b^2)*sinh(d*x + c)^3 + 4*((a^2 - 2*a*b - 3*b^2)*d*x - 2*b^2)*cosh(d*x + c) + (3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + 9*b^2)*sinh(d*x + c))/(d*cosh(d*x + c))

giac [B] time = 0.28, size = 170, normalized size = 3.33

$$\frac{4(a^2 - 2ab - 3b^2)dx + (a^2 e^{2dx+8c} + 2abe^{2dx+8c} + b^2 e^{2dx+8c})e^{-6c} - \frac{(a^2 e^{4dx+4c} - 2abe^{4dx+4c} - 3b^2 e^{4dx+4c} + 2a^2 e^{2dx+2c})}{e^{2dx} + e^{4dx}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/8*(4*(a^2 - 2*a*b - 3*b^2)*d*x + (a^2*e^(2*d*x + 8*c) + 2*a*b*e^(2*d*x + 8*c) + b^2*e^(2*d*x + 8*c))*e^(-6*c) - (a^2*e^(4*d*x + 4*c) - 2*a*b*e^(4*d*x + 4*c) - 3*b^2*e^(4*d*x + 4*c) + 2*a^2*e^(2*d*x + 2*c) + 14*b^2*e^(2*d*x + 2*c) + a^2 + 2*a*b + b^2)*e^(-2*c)/(e^(2*d*x) + e^(4*d*x + 2*c)))/d

maple [B] time = 0.21, size = 96, normalized size = 1.88

$$\frac{a^2 \left(\frac{\cosh(dx+c) \sinh(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + 2ab \left(\frac{\cosh(dx+c) \sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2} \right) + b^2 \left(\frac{\sinh^3(dx+c)}{2 \cosh(dx+c)} - \frac{3dx}{2} - \frac{3c}{2} + \frac{3 \tanh(dx+c)}{2} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x)`

[Out] $\frac{1}{d}*(a^2*(\frac{1}{2}*\cosh(d*x+c)*\sinh(d*x+c)+\frac{1}{2}*d*x+\frac{1}{2}*c)+2*a*b*(\frac{1}{2}*\cosh(d*x+c)*\sinh(d*x+c)-\frac{1}{2}*d*x-\frac{1}{2}*c)+b^2*(\frac{1}{2}*\sinh(d*x+c)^3/\cosh(d*x+c)-\frac{3}{2}*d*x-\frac{3}{2}*c+\frac{3}{2}*tanh(d*x+c)))$

maxima [B] time = 0.34, size = 140, normalized size = 2.75

$$\frac{1}{8}a^2\left(4x + \frac{e^{(2dx+2c)}}{d} - \frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{4}ab\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{8}b^2\left(\frac{12(dx+c)}{d} + \frac{e^{(-2dx-2c)}}{d} - \frac{17e^{(-2dx-2c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] $\frac{1}{8}a^2*(4*x + e^{(2*d*x + 2*c)}/d - e^{(-2*d*x - 2*c)}/d) - \frac{1}{4}a*b*(4*x - e^{(2*d*x + 2*c)}/d + e^{(-2*d*x - 2*c)}/d) - \frac{1}{8}b^2*(12*(d*x + c)/d + e^{(-2*d*x - 2*c)}/d - (17*e^{(-2*d*x - 2*c)} + 1)/(d*(e^{(-2*d*x - 2*c)} + e^{(-4*d*x - 4*c)})))$

mupad [B] time = 1.29, size = 77, normalized size = 1.51

$$\frac{e^{2c+2dx}(a+b)^2}{8d} - \frac{2b^2}{d(e^{2c+2dx}+1)} - \frac{e^{-2c-2dx}(a+b)^2}{8d} - x\left(-\frac{a^2}{2} + ab + \frac{3b^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2,x)`

[Out] $(\exp(2*c + 2*d*x)*(a + b)^2)/(8*d) - (2*b^2)/(d*(\exp(2*c + 2*d*x) + 1)) - (\exp(-2*c - 2*d*x)*(a + b)^2)/(8*d) - x*(a*b - a^2/2 + (3*b^2)/2)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^2 \cosh^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)`

[Out] `Integral((a + b*tanh(c + d*x)**2)**2*cosh(c + d*x)**2, x)`

3.92 $\int \cosh(c + dx) \left(a + b \tanh^2(c + dx) \right)^2 dx$

Optimal. Leaf size=60

$$\frac{(a+b)^2 \sinh(c+dx)}{d} - \frac{b(4a+3b) \tan^{-1}(\sinh(c+dx))}{2d} + \frac{b^2 \tanh(c+dx) \operatorname{sech}(c+dx)}{2d}$$

[Out] $-1/2*b*(4*a+3*b)*\arctan(\sinh(d*x+c))/d+(a+b)^2*\sinh(d*x+c)/d+1/2*b^2*\operatorname{sech}(d*x+c)*\tanh(d*x+c)/d$

Rubi [A] time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3676, 390, 385, 203}

$$\frac{(a+b)^2 \sinh(c+dx)}{d} - \frac{b(4a+3b) \tan^{-1}(\sinh(c+dx))}{2d} + \frac{b^2 \tanh(c+dx) \operatorname{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] $-(b*(4*a + 3*b)*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/(2*d) + ((a + b)^2*\operatorname{Sinh}[c + d*x])/d + (b^2*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x])/(2*d)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[(b*c - a*d)*x*(a + b*x^n)^(p+1)/(a*b*n*(p+1)), x] - Dist[(a*d - b*c*(n*(p+1) + 1))/(a*b*n*(p+1)), Int[(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 3676

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \cosh(c + dx) (a + b \tanh^2(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \frac{(a+(a+b)x^2)^2}{(1+x^2)^2} dx, x, \sinh(c + dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left((a+b)^2 - \frac{b(2a+b)+2b(a+b)x^2}{(1+x^2)^2}\right) dx, x, \sinh(c + dx)\right)}{d} \\
&= \frac{(a+b)^2 \sinh(c + dx)}{d} - \frac{\text{Subst}\left(\int \frac{b(2a+b)+2b(a+b)x^2}{(1+x^2)^2} dx, x, \sinh(c + dx)\right)}{d} \\
&= \frac{(a+b)^2 \sinh(c + dx)}{d} + \frac{b^2 \text{sech}(c + dx) \tanh(c + dx)}{2d} - \frac{(b(4a + 3b))}{2d} \\
&= -\frac{b(4a + 3b) \tan^{-1}(\sinh(c + dx))}{2d} + \frac{(a+b)^2 \sinh(c + dx)}{d} + \frac{b^2 \text{sech}(c + dx) \tanh(c + dx)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 54, normalized size = 0.90

$$\frac{2(a+b)^2 \sinh(c+dx) + b(b \tanh(c+dx) \text{sech}(c+dx) - (4a+3b) \tan^{-1}(\sinh(c+dx)))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2), x]

[Out] (2*(a + b)^2*Sinh[c + d*x] + b*(-((4*a + 3*b)*ArcTan[Sinh[c + d*x]]) + b*Sech[c + d*x]*Tanh[c + d*x]))/(2*d)

fricas [B] time = 0.43, size = 774, normalized size = 12.90

$$\frac{(a^2 + 2ab + b^2) \cosh(dx + c)^6 + 6(a^2 + 2ab + b^2) \cosh(dx + c) \sinh(dx + c)^5 + (a^2 + 2ab + b^2) \sinh(dx + c)^6}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] 1/2*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + (a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^4 + (15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + 3*b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - (a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^2 - a^2 - 2*a*b - 3*b^2)*sinh(d*x + c)^2 - a^2 - 2*a*b - b^2 - 2*((4*a*b + 3*b^2)*cosh(d*x + c)^5 + 5*(4*a*b + 3*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 + (4*a*b + 3*b^2)*sinh(d*x + c)^5 + 2*(4*a*b + 3*b^2)*cosh(d*x + c)^3 + 2*(5*(4*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*a*b + 3*b^2)*sinh(d*x + c)^3 + 2*(5*(4*a*b + 3*b^2)*cosh(d*x + c)^3 + 3*(4*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + (4*a*b + 3*b^2)*cosh(d*x + c) + (5*(4*a*b + 3*b^2)*cosh(d*x + c)^4 + 6*(4*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*a*b + 3*b^2)*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 2*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^3 - (a^2 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 + 2*d*cosh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (5*d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))

giac [B] time = 0.25, size = 137, normalized size = 2.28

$$\frac{2(4abe^c + 3b^2e^c) \arctan(e^{(dx+c)})e^{(-c)} + (a^2 + 2ab + b^2)e^{(-dx-c)} - (a^2e^{(dx+8c)} + 2abe^{(dx+8c)} + b^2e^{(dx+8c)})e^{(-7c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] $-1/2*(2*(4*a*b*e^c + 3*b^2*e^c)*\arctan(e^{(d*x + c)})*e^{(-c)} + (a^2 + 2*a*b + b^2)*e^{(-d*x - c)} - (a^2*e^{(d*x + 8*c)} + 2*a*b*e^{(d*x + 8*c)} + b^2*e^{(d*x + 8*c)})*e^{(-7*c)} - 2*(b^2*e^{(3*d*x + 3*c)} - b^2*e^{(d*x + c)})/(e^{(2*d*x + 2*c)} + 1)^2)/d$

maple [B] time = 0.31, size = 122, normalized size = 2.03

$$\frac{a^2 \sinh(dx+c)}{d} + \frac{2ab \sinh(dx+c)}{d} - \frac{4ab \arctan(e^{dx+c})}{d} + \frac{b^2 (\sinh^3(dx+c))}{d \cosh(dx+c)^2} + \frac{3b^2 \sinh(dx+c)}{d \cosh(dx+c)^2} - \frac{3b^2 \operatorname{sech}(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x)

[Out] $a^2*\sinh(d*x+c)/d+2*a*b*\sinh(d*x+c)/d-4/d*a*b*\arctan(\exp(d*x+c))+1/d*b^2*\sinh(d*x+c)^3/\cosh(d*x+c)^2+3/d*b^2*\sinh(d*x+c)/\cosh(d*x+c)^2-3/2/d*b^2*\operatorname{sech}(d*x+c)*\tanh(d*x+c)-3/d*b^2*\arctan(\exp(d*x+c))$

maxima [B] time = 0.45, size = 152, normalized size = 2.53

$$\frac{1}{2}b^2\left(\frac{6 \arctan(e^{(-dx-c)})}{d} - \frac{e^{(-dx-c)}}{d} + \frac{4e^{(-2dx-2c)} - e^{(-4dx-4c)} + 1}{d(e^{(-dx-c)} + 2e^{(-3dx-3c)} + e^{(-5dx-5c)})}\right) + ab\left(\frac{4 \arctan(e^{(-dx-c)})}{d} + \frac{e^{(dx+c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $1/2*b^2*(6*\arctan(e^{(-d*x - c)})/d - e^{(-d*x - c)}/d + (4*e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)} + 1)/(d*(e^{(-d*x - c)} + 2*e^{(-3*d*x - 3*c)} + e^{(-5*d*x - 5*c)}))) + a*b*(4*\arctan(e^{(-d*x - c)})/d + e^{(d*x + c)}/d - e^{(-d*x - c)}/d) + a^2*\sinh(d*x + c)/d$

mupad [B] time = 0.24, size = 182, normalized size = 3.03

$$\frac{e^{c+dx}(a+b)^2}{2d} - \frac{e^{-c-dx}(a+b)^2}{2d} - \frac{\operatorname{atan}\left(\frac{e^{dx}e^c(3b^2\sqrt{d^2}+4ab\sqrt{d^2})}{d\sqrt{16a^2b^2+24ab^3+9b^4}}\right)\sqrt{16a^2b^2+24ab^3+9b^4}}{\sqrt{d^2}} + \frac{b^2e^{c+dx}}{d(e^{2c+2dx}+1)} - \frac{1}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^2,x)

[Out] $(\exp(c + d*x)*(a + b)^2)/(2*d) - (\exp(-c - d*x)*(a + b)^2)/(2*d) - (\operatorname{atan}((\exp(d*x)*\exp(c)*(3*b^2*(d^2)^{(1/2)} + 4*a*b*(d^2)^{(1/2)}))/(d*(24*a*b^3 + 9*b^4 + 16*a^2*b^2)^{(1/2)}))*(24*a*b^3 + 9*b^4 + 16*a^2*b^2)^{(1/2)})/(d^2)^{(1/2)} + (b^2*\exp(c + d*x))/(d*(\exp(2*c + 2*d*x) + 1)) - (2*b^2*\exp(c + d*x))/(d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^2 \cosh(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*cosh(c + d*x), x)
```

3.93 $\int \operatorname{sech}(c + dx) \left(a + b \tanh^2(c + dx) \right)^2 dx$

Optimal. Leaf size=91

$$\frac{(8a^2 + 8ab + 3b^2) \tan^{-1}(\sinh(c + dx))}{8d} - \frac{3b(2a + b) \tanh(c + dx) \operatorname{sech}(c + dx)}{8d} - \frac{b \tanh(c + dx) \operatorname{sech}^3(c + dx)}{4d} + C$$

[Out] $1/8*(8*a^2+8*a*b+3*b^2)*\arctan(\sinh(d*x+c))/d-3/8*b*(2*a+b)*\operatorname{sech}(d*x+c)*\tanh(d*x+c)/d-1/4*b*\operatorname{sech}(d*x+c)^3*(a+(a+b)*\sinh(d*x+c)^2)*\tanh(d*x+c)/d$

Rubi [A] time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3676, 413, 385, 203}

$$\frac{(8a^2 + 8ab + 3b^2) \tan^{-1}(\sinh(c + dx))}{8d} - \frac{3b(2a + b) \tanh(c + dx) \operatorname{sech}(c + dx)}{8d} - \frac{b \tanh(c + dx) \operatorname{sech}^3(c + dx)}{4d} + C$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] $((8*a^2 + 8*a*b + 3*b^2)*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/(8*d) - (3*b*(2*a + b)*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x])/(8*d) - (b*\operatorname{Sech}[c + d*x]^3*(a + (a + b)*\operatorname{Sinh}[c + d*x]^2)*\operatorname{Tanh}[c + d*x])/(4*d)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 413

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p + 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 3676

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2)], x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}(c+dx) (a+b \tanh^2(c+dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+(a+b)x^2)^2}{(1+x^2)^3} dx, x, \sinh(c+dx)\right)}{d} \\
&= -\frac{b \operatorname{sech}^3(c+dx) (a+(a+b) \sinh^2(c+dx)) \tanh(c+dx)}{4d} + \frac{\operatorname{Subst}\left(\int \frac{(a+(a+b)x^2)^2}{(1+x^2)^3} dx, x, \sinh(c+dx)\right)}{d} \\
&= -\frac{3b(2a+b) \operatorname{sech}(c+dx) \tanh(c+dx)}{8d} - \frac{b \operatorname{sech}^3(c+dx) (a+(a+b) \sinh^2(c+dx)) \tanh(c+dx)}{8d} \\
&= \frac{(8a^2+8ab+3b^2) \tan^{-1}(\sinh(c+dx))}{8d} - \frac{3b(2a+b) \operatorname{sech}(c+dx) \tanh(c+dx)}{8d}
\end{aligned}$$

Mathematica [C] time = 7.28, size = 427, normalized size = 4.69

$$\operatorname{csch}^3(c+dx) \left(128 \sinh^6(c+dx) (a^2 (5 \sinh^4(c+dx) + 12 \sinh^2(c+dx) + 7) + 2ab (5 \sinh^2(c+dx) + 6) \sinh^2(c+dx)) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] $-1/6720 * (\operatorname{Csch}[c + d*x]^3 * (128 * \operatorname{HypergeometricPFQ}[\{3/2, 2, 2, 2, 2\}, \{1, 1, 1, 9/2\}, -\operatorname{Sinh}[c + d*x]^2] * \operatorname{Sinh}[c + d*x]^6 * (a + a * \operatorname{Sinh}[c + d*x]^2 + b * \operatorname{Sinh}[c + d*x]^2)^2 + 128 * \operatorname{HypergeometricPFQ}[\{3/2, 2, 2, 2\}, \{1, 1, 9/2\}, -\operatorname{Sinh}[c + d*x]^2] * \operatorname{Sinh}[c + d*x]^6 * (5 * b^2 * \operatorname{Sinh}[c + d*x]^4 + 2 * a * b * \operatorname{Sinh}[c + d*x]^2 * (6 + 5 * \operatorname{Sinh}[c + d*x]^2) + a^2 * (7 + 12 * \operatorname{Sinh}[c + d*x]^2 + 5 * \operatorname{Sinh}[c + d*x]^4)) + 35 * (b^2 * \operatorname{Sinh}[c + d*x]^4 * (1947 + 485 * \operatorname{Sinh}[c + d*x]^2) + 2 * a * b * \operatorname{Sinh}[c + d*x]^2 * (2625 + 2554 * \operatorname{Sinh}[c + d*x]^2 + 485 * \operatorname{Sinh}[c + d*x]^4) + a^2 * (3375 + 5907 * \operatorname{Sinh}[c + d*x]^2 + 3161 * \operatorname{Sinh}[c + d*x]^4 + 485 * \operatorname{Sinh}[c + d*x]^6)) - (105 * \operatorname{ArcTanh}[\operatorname{Sqrt}[-\operatorname{Sinh}[c + d*x]^2]] * (b^2 * \operatorname{Sinh}[c + d*x]^4 * (649 + 378 * \operatorname{Sinh}[c + d*x]^2 + 9 * \operatorname{Sinh}[c + d*x]^4) + 2 * a * b * \operatorname{Sinh}[c + d*x]^2 * (875 + 1143 * \operatorname{Sinh}[c + d*x]^2 + 389 * \operatorname{Sinh}[c + d*x]^4 + 9 * \operatorname{Sinh}[c + d*x]^6) + a^2 * (1125 + 2344 * \operatorname{Sinh}[c + d*x]^2 + 1674 * \operatorname{Sinh}[c + d*x]^4 + 400 * \operatorname{Sinh}[c + d*x]^6 + 9 * \operatorname{Sinh}[c + d*x]^8))) / \operatorname{Sqrt}[-\operatorname{Sinh}[c + d*x]^2])) / d$

fricas [B] time = 0.42, size = 1373, normalized size = 15.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] $-1/4 * ((8*a*b + 5*b^2) * \cosh(d*x + c)^7 + 7 * (8*a*b + 5*b^2) * \cosh(d*x + c) * \sinh(d*x + c)^6 + (8*a*b + 5*b^2) * \sinh(d*x + c)^7 + (8*a*b - 3*b^2) * \cosh(d*x + c)^5 + (21 * (8*a*b + 5*b^2) * \cosh(d*x + c)^2 + 8*a*b - 3*b^2) * \sinh(d*x + c)^5 + 5 * (7 * (8*a*b + 5*b^2) * \cosh(d*x + c)^3 + (8*a*b - 3*b^2) * \cosh(d*x + c)) * \sinh(d*x + c)^4 - (8*a*b - 3*b^2) * \cosh(d*x + c)^3 + (35 * (8*a*b + 5*b^2) * \cosh(d*x + c)^4 + 10 * (8*a*b - 3*b^2) * \cosh(d*x + c)^2 - 8*a*b + 3*b^2) * \sinh(d*x + c)^3 + (21 * (8*a*b + 5*b^2) * \cosh(d*x + c)^5 + 10 * (8*a*b - 3*b^2) * \cosh(d*x + c)^3 - 3 * (8*a*b - 3*b^2) * \cosh(d*x + c)) * \sinh(d*x + c)^2 - ((8*a^2 + 8*a*b + 3*b^2) * \cosh(d*x + c)^8 + 8 * (8*a^2 + 8*a*b + 3*b^2) * \cosh(d*x + c) * \sinh(d*x + c)^7 + (8*a^2 + 8*a*b + 3*b^2) * \sinh(d*x + c)^8 + 4 * (8*a^2 + 8*a*b + 3*b^2) * \cosh(d*x + c)^6 + 4 * (7 * (8*a^2 + 8*a*b + 3*b^2) * \cosh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2) * \sinh(d*x + c)^6 + 8 * (7 * (8*a^2 + 8*a*b + 3*b^2) * \cosh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2) * \sinh(d*x + c)^4 + 4 * (8*a^2 + 8*a*b + 3*b^2) * \cosh(d*x + c)^4 + 4 * (8*a^2 + 8*a*b + 3*b^2) * \sinh(d*x + c)^4)) / \operatorname{Sqrt}[-\operatorname{Sinh}[c + d*x]^2])) / d$

$$c)^3 + 3*(8*a^2 + 8*a*b + 3*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 6*(8*a^2 + 8*a*b + 3*b^2)*\cosh(d*x + c)^4 + 2*(35*(8*a^2 + 8*a*b + 3*b^2)*\cosh(d*x + c)^4 + 30*(8*a^2 + 8*a*b + 3*b^2)*\cosh(d*x + c)^2 + 24*a^2 + 24*a*b + 9*b^2)*\sinh(d*x + c)^4 + 8*(7*(8*a^2 + 8*a*b + 3*b^2)*\cosh(d*x + c)^5 + 10*(8*a^2 + 8*a*b + 3*b^2)*\cosh(d*x + c)^3 + 3*(8*a^2 + 8*a*b + 3*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(8*a^2 + 8*a*b + 3*b^2)*\cosh(d*x + c)^2 + 4*(7*(8*a^2 + 8*a*b + 3*b^2)*\cosh(d*x + c)^6 + 15*(8*a^2 + 8*a*b + 3*b^2)*\cosh(d*x + c)^4 + 9*(8*a^2 + 8*a*b + 3*b^2)*\cosh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2)*\sinh(d*x + c)^2 + 8*a^2 + 8*a*b + 3*b^2 + 8*((8*a^2 + 8*a*b + 3*b^2)*\cosh(d*x + c)^7 + 3*(8*a^2 + 8*a*b + 3*b^2)*\cosh(d*x + c)^5 + 3*(8*a^2 + 8*a*b + 3*b^2)*\cosh(d*x + c)^3 + (8*a^2 + 8*a*b + 3*b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\arctan(\cosh(d*x + c) + \sinh(d*x + c)) - (8*a*b + 5*b^2)*\cosh(d*x + c) + (7*(8*a*b + 5*b^2)*\cosh(d*x + c)^6 + 5*(8*a*b - 3*b^2)*\cosh(d*x + c)^4 - 3*(8*a*b - 3*b^2)*\cosh(d*x + c)^2 - 8*a*b - 5*b^2)*\sinh(d*x + c))/(d*\cosh(d*x + c)^8 + 8*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + d*\sinh(d*x + c)^8 + 4*d*\cosh(d*x + c)^6 + 4*(7*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^6 + 8*(7*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 6*d*\cosh(d*x + c)^4 + 2*(35*d*\cosh(d*x + c)^4 + 30*d*\cosh(d*x + c)^2 + 3*d)*\sinh(d*x + c)^4 + 8*(7*d*\cosh(d*x + c)^5 + 10*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*d*\cosh(d*x + c)^2 + 4*(7*d*\cosh(d*x + c)^6 + 15*d*\cosh(d*x + c)^4 + 9*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^2 + 8*(d*\cosh(d*x + c)^7 + 3*d*\cosh(d*x + c)^5 + 3*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c) + d)$$

giac [A] time = 0.20, size = 157, normalized size = 1.73

$$\frac{(8a^2e^c + 8abe^c + 3b^2e^c) \arctan(e^{(dx+c)})e^{-c} - \frac{8abe^{(7dx+7c)} + 5b^2e^{(7dx+7c)} + 8abe^{(5dx+5c)} - 3b^2e^{(5dx+5c)} - 8abe^{(3dx+3c)} + 3b^2e^{(3dx+3c)}}{(e^{(2dx+2c)} + 1)^4}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/4*((8*a^2*e^c + 8*a*b*e^c + 3*b^2*e^c)*arctan(e^(d*x + c))*e^(-c) - (8*a*b*e^(7*d*x + 7*c) + 5*b^2*e^(7*d*x + 7*c) + 8*a*b*e^(5*d*x + 5*c) - 3*b^2*e^(5*d*x + 5*c) - 8*a*b*e^(3*d*x + 3*c) + 3*b^2*e^(3*d*x + 3*c) - 8*a*b*e^(d*x + c) - 5*b^2*e^(d*x + c)))/(e^(2*d*x + 2*c) + 1)^4)/d

maple [B] time = 0.37, size = 173, normalized size = 1.90

$$\frac{2a^2 \arctan(e^{dx+c})}{d} - \frac{2ab \sinh(dx+c)}{d \cosh(dx+c)^2} + \frac{ab \operatorname{sech}(dx+c) \tanh(dx+c)}{d} + \frac{2ab \arctan(e^{dx+c})}{d} - \frac{b^2 (\sinh^3(dx+c))}{d \cosh(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x)

[Out] 2/d*a^2*arctan(exp(d*x+c))-2/d*a*b*sinh(d*x+c)/cosh(d*x+c)^2+1/d*a*b*sech(d*x+c)*tanh(d*x+c)+2/d*a*b*arctan(exp(d*x+c))-1/d*b^2*sinh(d*x+c)^3/cosh(d*x+c)^4-1/d*b^2*sinh(d*x+c)/cosh(d*x+c)^4+1/4/d*b^2*tanh(d*x+c)*sech(d*x+c)^3+3/8/d*b^2*sech(d*x+c)*tanh(d*x+c)+3/4/d*b^2*arctan(exp(d*x+c))

maxima [B] time = 0.46, size = 199, normalized size = 2.19

$$-\frac{1}{4}b^2\left(\frac{3 \arctan(e^{(-dx-c)})}{d} + \frac{5e^{(-dx-c)} - 3e^{(-3dx-3c)} + 3e^{(-5dx-5c)} - 5e^{(-7dx-7c)}}{d(4e^{(-2dx-2c)} + 6e^{(-4dx-4c)} + 4e^{(-6dx-6c)} + e^{(-8dx-8c)} + 1)}\right) - 2ab\left(\frac{\arctan(e^{(-dx-c)})}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $-1/4*b^2*(3*\arctan(e^{(-d*x - c)})/d + (5*e^{(-d*x - c)} - 3*e^{(-3*d*x - 3*c)} + 3*e^{(-5*d*x - 5*c)} - 5*e^{(-7*d*x - 7*c)})/(d*(4*e^{(-2*d*x - 2*c)} + 6*e^{(-4*d*x - 4*c)} + 4*e^{(-6*d*x - 6*c)} + e^{(-8*d*x - 8*c)} + 1))) - 2*a*b*(\arctan(e^{(-d*x - c)})/d + (e^{(-d*x - c)} - e^{(-3*d*x - 3*c)})/(d*(2*e^{(-2*d*x - 2*c)} + e^{(-4*d*x - 4*c)} + 1))) + a^2*\arctan(\sinh(d*x + c))/d$

mupad [B] time = 0.16, size = 303, normalized size = 3.33

$$\frac{\operatorname{atan}\left(\frac{e^{dx} e^c (8a^2 \sqrt{d^2} + 3b^2 \sqrt{d^2} + 8ab \sqrt{d^2})}{d \sqrt{64a^4 + 128a^3b + 112a^2b^2 + 48ab^3 + 9b^4}}\right) \sqrt{64a^4 + 128a^3b + 112a^2b^2 + 48ab^3 + 9b^4}}{4\sqrt{d^2}} - \frac{6b^2 e^{c+dx}}{d(3e^{2c+2dx} + 3e^{4c+4dx})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*tanh(c + d*x))^2/cosh(c + d*x), x)`

[Out] $(\operatorname{atan}((\exp(d*x)*\exp(c)*(8*a^2*(d^2)^{(1/2)} + 3*b^2*(d^2)^{(1/2)} + 8*a*b*(d^2)^{(1/2)}))/d*(48*a*b^3 + 128*a^3*b + 64*a^4 + 9*b^4 + 112*a^2*b^2)^{(1/2)}))*(48*a*b^3 + 128*a^3*b + 64*a^4 + 9*b^4 + 112*a^2*b^2)^{(1/2)}/(4*(d^2)^{(1/2)}) - (6*b^2*\exp(c + d*x))/(d*(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1)) + (4*b^2*\exp(c + d*x))/(d*(4*\exp(2*c + 2*d*x) + 6*\exp(4*c + 4*d*x) + 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1)) - (\exp(c + d*x)*(8*a*b + 5*b^2))/(4*d*(\exp(2*c + 2*d*x) + 1)) + (\exp(c + d*x)*(8*a*b + 9*b^2))/(2*d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^2 \operatorname{sech}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)*(a+b*tanh(d*x+c)**2)**2, x)`

[Out] `Integral((a + b*tanh(c + d*x)**2)**2*sech(c + d*x), x)`

3.94 $\int \operatorname{sech}^2(c + dx) (a + b \tanh^2(c + dx))^2 dx$

Optimal. Leaf size=49

$$\frac{a^2 \tanh(c + dx)}{d} + \frac{2ab \tanh^3(c + dx)}{3d} + \frac{b^2 \tanh^5(c + dx)}{5d}$$

[Out] $a^2 \tanh(dx+c)/d + 2/3 a b \tanh(dx+c)^3/d + 1/5 b^2 \tanh(dx+c)^5/d$

Rubi [A] time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3675, 194}

$$\frac{a^2 \tanh(c + dx)}{d} + \frac{2ab \tanh^3(c + dx)}{3d} + \frac{b^2 \tanh^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] $(a^2 \tanh[c + d*x])/d + (2*a*b \tanh[c + d*x]^3)/(3*d) + (b^2 \tanh[c + d*x]^5)/(5*d)$

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m-1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2-1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^2(c + dx) (a + b \tanh^2(c + dx))^2 dx &= \frac{\operatorname{Subst}\left(\int (a + bx^2)^2 dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int (a^2 + 2abx^2 + b^2x^4) dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{a^2 \tanh(c + dx)}{d} + \frac{2ab \tanh^3(c + dx)}{3d} + \frac{b^2 \tanh^5(c + dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.21, size = 49, normalized size = 1.00

$$\frac{a^2 \tanh(c + dx)}{d} + \frac{2ab \tanh^3(c + dx)}{3d} + \frac{b^2 \tanh^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] $(a^2 \tanh[c + d*x])/d + (2*a*b \tanh[c + d*x]^3)/(3*d) + (b^2 \tanh[c + d*x]^5)/(5*d)$

fricas [B] time = 0.40, size = 391, normalized size = 7.98

$$\frac{4\left((15a^2 + 20ab + 9b^2)\cosh(dx+c)^4 + 8(5ab + 3b^2)\cosh(dx+c)\sinh(dx+c)^3 + (15d\cosh(dx+c)^6 + 6d\cosh(dx+c)\sinh(dx+c)^5 + d\sinh(dx+c)^6 + 6d\cosh(dx+c)^4 + 3(5d\cosh(dx+c)^3 + 4d\sinh(dx+c)^2)\right)}{15d\left(e^{2dx+2c} + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -4/15*((15*a^2 + 20*a*b + 9*b^2)*cosh(d*x + c)^4 + 8*(5*a*b + 3*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (15*a^2 + 20*a*b + 9*b^2)*sinh(d*x + c)^4 + 20*(3*a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*(15*a^2 + 20*a*b + 9*b^2)*cosh(d*x + c)^2 + 30*a^2 + 20*a*b)*sinh(d*x + c)^2 + 45*a^2 + 20*a*b + 15*b^2 + 8*((5*a*b + 3*b^2)*cosh(d*x + c)^3 + 5*a*b*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 + 6*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 + 4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 15*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 + 8*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c) + 10*d)

giac [B] time = 0.23, size = 169, normalized size = 3.45

$$\frac{2\left(15a^2e^{(8dx+8c)} + 30abe^{(8dx+8c)} + 15b^2e^{(8dx+8c)} + 60a^2e^{(6dx+6c)} + 60abe^{(6dx+6c)} + 90a^2e^{(4dx+4c)} + 40abe^{(4dx+4c)} + 20a^2e^{(2dx+2c)} + 20abe^{(2dx+2c)} + 15a^2 + 10ab + 3b^2\right)}{15d\left(e^{2dx+2c} + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] -2/15*(15*a^2*e^(8*d*x + 8*c) + 30*a*b*e^(8*d*x + 8*c) + 15*b^2*e^(8*d*x + 8*c) + 60*a^2*e^(6*d*x + 6*c) + 60*a*b*e^(6*d*x + 6*c) + 90*a^2*e^(4*d*x + 4*c) + 40*a*b*e^(4*d*x + 4*c) + 30*b^2*e^(4*d*x + 4*c) + 60*a^2*e^(2*d*x + 2*c) + 20*a*b*e^(2*d*x + 2*c) + 15*a^2 + 10*a*b + 3*b^2)/(d*(e^(2*d*x + 2*c) + 1)^5)

maple [B] time = 0.46, size = 126, normalized size = 2.57

$$\frac{a^2 \tanh(dx+c) + 2ab \left(-\frac{\sinh(dx+c)}{2 \cosh(dx+c)^3} + \frac{\left(\frac{2}{3} + \frac{\operatorname{sech}(dx+c)^2}{3}\right) \tanh(dx+c)}{2} \right) + b^2 \left(-\frac{\sinh^3(dx+c)}{2 \cosh(dx+c)^5} - \frac{3 \sinh(dx+c)}{8 \cosh(dx+c)^5} + \frac{3 \left(\frac{8}{15} + \frac{\operatorname{sech}(dx+c)^4}{5}\right)}{d}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x)

[Out] 1/d*(a^2*tanh(d*x+c)+2*a*b*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))+b^2*(-1/2*sinh(d*x+c)^3/cosh(d*x+c)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)))

maxima [A] time = 0.33, size = 53, normalized size = 1.08

$$\frac{b^2 \tanh(dx+c)^5}{5d} + \frac{2ab \tanh(dx+c)^3}{3d} + \frac{2a^2}{d\left(e^{(-2dx-2c)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $1/5*b^2*\tanh(d*x + c)^5/d + 2/3*a*b*\tanh(d*x + c)^3/d + 2*a^2/(d*(e^{(-2*d*x - 2*c)} + 1))$

mupad [B] time = 1.24, size = 482, normalized size = 9.84

$$\frac{\frac{2(a^2-b^2)}{5d} + \frac{2e^{2c+2dx}(a+b)^2}{5d}}{2e^{2c+2dx} + e^{4c+4dx} + 1} - \frac{\frac{2(a^2-b^2)}{5d} + \frac{6e^{4c+4dx}(a^2-b^2)}{5d} + \frac{2e^{6c+6dx}(a+b)^2}{5d} + \frac{2e^{2c+2dx}(3a^2-2ab+3b^2)}{5d}}{4e^{2c+2dx} + 6e^{4c+4dx} + 4e^{6c+6dx} + e^{8c+8dx} + 1} - \frac{\frac{2(a+b)^2}{5d} + \frac{8e^{2c+2dx}}{5d}}{5e^{2c+2dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*tanh(c + d*x))^2/cosh(c + d*x)^2, x)`

[Out] $-\left(\frac{2(a^2 - b^2)}{5d} + \frac{2\exp(2c + 2d*x)(a + b)^2}{5d}\right) / (2\exp(2c + 2d*x) + \exp(4c + 4d*x) + 1) - \left(\frac{2(a^2 - b^2)}{5d} + \frac{6\exp(4c + 4d*x)(a^2 - b^2)}{5d} + \frac{2\exp(6c + 6d*x)(a + b)^2}{5d} + \frac{2\exp(2c + 2d*x)(3a^2 - 2ab + 3b^2)}{5d}\right) / (4\exp(2c + 2d*x) + 6\exp(4c + 4d*x) + 4\exp(6c + 6d*x) + \exp(8c + 8d*x) + 1) - \left(\frac{2(a + b)^2}{5d} + \frac{8\exp(2c + 2d*x)(a^2 - b^2)}{5d} + \frac{8\exp(6c + 6d*x)(a^2 - b^2)}{5d} + \frac{2\exp(8c + 8d*x)(a + b)^2}{5d} + \frac{4\exp(4c + 4d*x)(3a^2 - 2ab + 3b^2)}{5d}\right) / (5\exp(2c + 2d*x) + 10\exp(4c + 4d*x) + 10\exp(6c + 6d*x) + 5\exp(8c + 8d*x) + \exp(10c + 10d*x) + 1) - \left(\frac{2(3a^2 - 2ab + 3b^2)}{15d} + \frac{4\exp(2c + 2d*x)(a^2 - b^2)}{5d} + \frac{2\exp(4c + 4d*x)(a + b)^2}{5d}\right) / (3\exp(2c + 2d*x) + 3\exp(4c + 4d*x) + \exp(6c + 6d*x) + 1) - \frac{2(a + b)^2}{5d * (\exp(2c + 2d*x) + 1)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^2 \operatorname{sech}^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2, x)`

[Out] `Integral((a + b*tanh(c + d*x)**2)**2*sech(c + d*x)**2, x)`

3.95 $\int \operatorname{sech}^3(c + dx) (a + b \tanh^2(c + dx))^2 dx$

Optimal. Leaf size=125

$$\frac{(8a^2 + 4ab + b^2) \tan^{-1}(\sinh(c + dx))}{16d} + \frac{(8a^2 + 4ab + b^2) \tanh(c + dx) \operatorname{sech}(c + dx)}{16d} - \frac{b(8a + 3b) \tanh(c + dx) \operatorname{sech}(c + dx)}{24d}$$

[Out] 1/16*(8*a^2+4*a*b+b^2)*arctan(sinh(d*x+c))/d+1/16*(8*a^2+4*a*b+b^2)*sech(d*x+c)*tanh(d*x+c)/d-1/24*b*(8*a+3*b)*sech(d*x+c)^3*tanh(d*x+c)/d-1/6*b*sech(d*x+c)^5*(a+(a+b)*sinh(d*x+c)^2)*tanh(d*x+c)/d

Rubi [A] time = 0.15, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3676, 413, 385, 199, 203}

$$\frac{(8a^2 + 4ab + b^2) \tan^{-1}(\sinh(c + dx))}{16d} + \frac{(8a^2 + 4ab + b^2) \tanh(c + dx) \operatorname{sech}(c + dx)}{16d} - \frac{b(8a + 3b) \tanh(c + dx) \operatorname{sech}(c + dx)}{24d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] ((8*a^2 + 4*a*b + b^2)*ArcTan[Sinh[c + d*x]]/(16*d) + ((8*a^2 + 4*a*b + b^2)*Sech[c + d*x]*Tanh[c + d*x])/(16*d) - (b*(8*a + 3*b)*Sech[c + d*x]^3*Tanh[c + d*x])/(24*d) - (b*Sech[c + d*x]^5*(a + (a + b)*Sinh[c + d*x]^2)*Tanh[c + d*x])/(6*d)

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 413

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p + 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p + q) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}^3(c + dx) (a + b \tanh^2(c + dx))^2 dx = \frac{\operatorname{Subst}\left(\int \frac{(a+(a+b)x^2)^2}{(1+x^2)^4} dx, x, \sinh(c + dx)\right)}{d}$$

$$= -\frac{b \operatorname{sech}^5(c + dx) (a + (a + b) \sinh^2(c + dx)) \tanh(c + dx)}{6d} + \dots$$

$$= -\frac{b(8a + 3b) \operatorname{sech}^3(c + dx) \tanh(c + dx)}{24d} - \frac{b \operatorname{sech}^5(c + dx) (a + (a + b) \sinh^2(c + dx)) \tanh(c + dx)}{16d}$$

$$= \frac{(8a^2 + 4ab + b^2) \operatorname{sech}(c + dx) \tanh(c + dx)}{16d} - \frac{b(8a + 3b) \operatorname{sech}^3(c + dx) \tanh(c + dx)}{16d}$$

$$= \frac{(8a^2 + 4ab + b^2) \tan^{-1}(\sinh(c + dx))}{16d} + \frac{(8a^2 + 4ab + b^2) \operatorname{sech}(c + dx) \tanh(c + dx)}{16d}$$

Mathematica [C] time = 8.76, size = 792, normalized size = 6.34

$$a^2 \sinh(c + dx) \left(-\frac{380(a+b)^2 \sinh^6(c+dx) {}_4F_3\left(\frac{3}{2}, 2, 2, 2; 1, 1, \frac{9}{2}; -\sinh^2(c+dx)\right)}{a^2} - \frac{128(a+b)^2 \sinh^6(c+dx) {}_5F_4\left(\frac{3}{2}, 2, 2, 2, 2; 1, 1, 1, \frac{9}{2}; -\sinh^2(c+dx)\right)}{a^2} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (a^2*Sinh[c + d*x]*((-23555*(a + b))/a - (32970*(a + b)^2)/a^2 - 14980*Csch[c + d*x]^2 - (91875*(a + b)*Csch[c + d*x]^2)/a - 65625*Csch[c + d*x]^4 - (8855*(a + b)^2*Sinh[c + d*x]^2)/a^2 - 620*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^2 - 160*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^2 - 16*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^2 - (968*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4)/a - (288*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4)/a - (32*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4)/a - (380*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6)/a^2 - (128*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6)/a^2 + (65625*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]/(-Sinh[c + d*x]^2)^(5/2) + (1680*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4)/(-Sinh[c + d*x]^2)^(5/2) - (36855*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]/(-Sinh[c + d*x]^2)^(3/2) - (91875*(a + b)*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]/(a*(-Sinh[c + d*x]^2)^(3/2)) + (54180*(a + b)*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]/(a*Sqrt[-Sinh[c + d*x]^2]) + (32970*(a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]/(a^2*Sqrt[-Sinh[c + d*x]^2]) + (525*(a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]/(a^2*Sqrt[-Sinh[c + d*x]^2]))

```
rcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4/(a^2*Sqrt[-Sinh[c + d*x]^2]
) - (1365*(a + b)*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sqrt[-Sinh[c + d*x]^2])/a
- (19845*(a + b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sqrt[-Sinh[c + d*x]^2]
/a^2))/(2520*d)
```

fricas [B] time = 0.47, size = 2824, normalized size = 22.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^11 + 33*(8*a^2 + 4*a*b + b^2)*c
osh(d*x + c)*sinh(d*x + c)^10 + 3*(8*a^2 + 4*a*b + b^2)*sinh(d*x + c)^11 +
(72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^9 + (165*(8*a^2 + 4*a*b + b^2)*cos
h(d*x + c)^2 + 72*a^2 - 60*a*b - 47*b^2)*sinh(d*x + c)^9 + 9*(55*(8*a^2 + 4
*a*b + b^2)*cosh(d*x + c)^3 + (72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)*sin
h(d*x + c)^8 + 6*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^7 + 6*(165*(8*a^2
+ 4*a*b + b^2)*cosh(d*x + c)^4 + 6*(72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)
^2 + 8*a^2 - 12*a*b + 13*b^2)*sinh(d*x + c)^7 + 42*(33*(8*a^2 + 4*a*b + b^2
)*cosh(d*x + c)^5 + 2*(72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^3 + (8*a^2 -
12*a*b + 13*b^2)*cosh(d*x + c)*sinh(d*x + c)^6 - 6*(8*a^2 - 12*a*b + 13*b
^2)*cosh(d*x + c)^5 + 6*(231*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^6 + 21*(72
*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^4 + 21*(8*a^2 - 12*a*b + 13*b^2)*cosh
(d*x + c)^2 - 8*a^2 + 12*a*b - 13*b^2)*sinh(d*x + c)^5 + 6*(165*(8*a^2 + 4
a*b + b^2)*cosh(d*x + c)^7 + 21*(72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^5
+ 35*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^3 - 5*(8*a^2 - 12*a*b + 13*b^2
)*cosh(d*x + c))*sinh(d*x + c)^4 - (72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)
^3 + (495*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^8 + 84*(72*a^2 - 60*a*b - 47*
b^2)*cosh(d*x + c)^6 + 210*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^4 - 60*(
8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^2 - 72*a^2 + 60*a*b + 47*b^2)*sinh(d
*x + c)^3 + 3*(55*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^9 + 12*(72*a^2 - 60*a
*b - 47*b^2)*cosh(d*x + c)^7 + 42*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^5
- 20*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^3 - (72*a^2 - 60*a*b - 47*b^2
)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*((8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^1
2 + 12*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (8*a^2 + 4*a*
b + b^2)*sinh(d*x + c)^12 + 6*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^10 + 6*(1
1*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^2 + 8*a^2 + 4*a*b + b^2)*sinh(d*x + c
)^10 + 20*(11*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^3 + 3*(8*a^2 + 4*a*b + b^
2)*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^
8 + 15*(33*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 18*(8*a^2 + 4*a*b + b^2)
*cosh(d*x + c)^2 + 8*a^2 + 4*a*b + b^2)*sinh(d*x + c)^8 + 24*(33*(8*a^2 + 4
a*b + b^2)*cosh(d*x + c)^5 + 30*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^3 + 5*
(8*a^2 + 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(8*a^2 + 4*a*b +
b^2)*cosh(d*x + c)^6 + 4*(231*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^6 + 315*(
8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 105*(8*a^2 + 4*a*b + b^2)*cosh(d*x +
c)^2 + 40*a^2 + 20*a*b + 5*b^2)*sinh(d*x + c)^6 + 24*(33*(8*a^2 + 4*a*b +
b^2)*cosh(d*x + c)^7 + 63*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^5 + 35*(8*a^2
+ 4*a*b + b^2)*cosh(d*x + c)^3 + 5*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c))*si
nh(d*x + c)^5 + 15*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 15*(33*(8*a^2 +
4*a*b + b^2)*cosh(d*x + c)^8 + 84*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^6 + 7
0*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 20*(8*a^2 + 4*a*b + b^2)*cosh(d*x
+ c)^2 + 8*a^2 + 4*a*b + b^2)*sinh(d*x + c)^4 + 20*(11*(8*a^2 + 4*a*b + b^
2)*cosh(d*x + c)^9 + 36*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^7 + 42*(8*a^2 +
4*a*b + b^2)*cosh(d*x + c)^5 + 20*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^3 +
3*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(8*a^2 + 4*a*b +
b^2)*cosh(d*x + c)^2 + 6*(11*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^10 + 45*(
8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^8 + 70*(8*a^2 + 4*a*b + b^2)*cosh(d*x +
c)^6 + 50*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 15*(8*a^2 + 4*a*b + b^2)*
cosh(d*x + c)^2 + 8*a^2 + 4*a*b + b^2)*sinh(d*x + c)^2 + 8*a^2 + 4*a*b + b^
```


$$2 + 12*((8*a^2 + 4*a*b + b^2)*\cosh(d*x + c)^{11} + 5*(8*a^2 + 4*a*b + b^2)*\cosh(d*x + c)^9 + 10*(8*a^2 + 4*a*b + b^2)*\cosh(d*x + c)^7 + 10*(8*a^2 + 4*a*b + b^2)*\cosh(d*x + c)^5 + 5*(8*a^2 + 4*a*b + b^2)*\cosh(d*x + c)^3 + (8*a^2 + 4*a*b + b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\arctan(\cosh(d*x + c) + \sinh(d*x + c)) - 3*(8*a^2 + 4*a*b + b^2)*\cosh(d*x + c) + 3*(11*(8*a^2 + 4*a*b + b^2)*\cosh(d*x + c)^{10} + 3*(72*a^2 - 60*a*b - 47*b^2)*\cosh(d*x + c)^8 + 14*(8*a^2 - 12*a*b + 13*b^2)*\cosh(d*x + c)^6 - 10*(8*a^2 - 12*a*b + 13*b^2)*\cosh(d*x + c)^4 - (72*a^2 - 60*a*b - 47*b^2)*\cosh(d*x + c)^2 - 8*a^2 - 4*a*b - b^2)*\sinh(d*x + c))/(d*\cosh(d*x + c)^{12} + 12*d*\cosh(d*x + c)*\sinh(d*x + c)^{11} + d*\sinh(d*x + c)^{12} + 6*d*\cosh(d*x + c)^{10} + 6*(11*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^{10} + 20*(11*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^9 + 15*d*\cosh(d*x + c)^8 + 15*(33*d*\cosh(d*x + c)^4 + 18*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^8 + 24*(33*d*\cosh(d*x + c)^5 + 30*d*\cosh(d*x + c)^3 + 5*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 20*d*\cosh(d*x + c)^6 + 4*(231*d*\cosh(d*x + c)^6 + 315*d*\cosh(d*x + c)^4 + 105*d*\cosh(d*x + c)^2 + 5*d)*\sinh(d*x + c)^6 + 24*(33*d*\cosh(d*x + c)^7 + 63*d*\cosh(d*x + c)^5 + 35*d*\cosh(d*x + c)^3 + 5*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 15*d*\cosh(d*x + c)^4 + 15*(33*d*\cosh(d*x + c)^8 + 84*d*\cosh(d*x + c)^6 + 70*d*\cosh(d*x + c)^4 + 20*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^4 + 20*(11*d*\cosh(d*x + c)^9 + 36*d*\cosh(d*x + c)^7 + 42*d*\cosh(d*x + c)^5 + 20*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 6*d*\cosh(d*x + c)^2 + 6*(11*d*\cosh(d*x + c)^{10} + 45*d*\cosh(d*x + c)^8 + 70*d*\cosh(d*x + c)^6 + 50*d*\cosh(d*x + c)^4 + 15*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^2 + 12*(d*\cosh(d*x + c)^{11} + 5*d*\cosh(d*x + c)^9 + 10*d*\cosh(d*x + c)^7 + 10*d*\cosh(d*x + c)^5 + 5*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c) + d)$$

giac [B] time = 0.22, size = 291, normalized size = 2.33

$$3(8a^2e^c + 4abe^c + b^2e^c) \arctan(e^{(dx+c)})e^{-c} + \frac{24a^2e^{(11dx+11c)} + 12abe^{(11dx+11c)} + 3b^2e^{(11dx+11c)} + 72a^2e^{(9dx+9c)} - 60abe^{(9dx+9c)}}{d(e^{(2dx+2c)} + 1)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] $\frac{1}{24}*(3*(8*a^2*e^c + 4*a*b*e^c + b^2*e^c)*\arctan(e^{(d*x + c)})*e^{-c} + (24*a^2*e^{(11*d*x + 11*c)} + 12*a*b*e^{(11*d*x + 11*c)} + 3*b^2*e^{(11*d*x + 11*c)} + 72*a^2*e^{(9*d*x + 9*c)} - 60*a*b*e^{(9*d*x + 9*c)} - 47*b^2*e^{(9*d*x + 9*c)} + 48*a^2*e^{(7*d*x + 7*c)} - 72*a*b*e^{(7*d*x + 7*c)} + 78*b^2*e^{(7*d*x + 7*c)} - 48*a^2*e^{(5*d*x + 5*c)} + 72*a*b*e^{(5*d*x + 5*c)} - 78*b^2*e^{(5*d*x + 5*c)} - 72*a^2*e^{(3*d*x + 3*c)} + 60*a*b*e^{(3*d*x + 3*c)} + 47*b^2*e^{(3*d*x + 3*c)} - 24*a^2*e^{(d*x + c)} - 12*a*b*e^{(d*x + c)} - 3*b^2*e^{(d*x + c)}))/(e^{(2*d*x + 2*c)} + 1)^6)/d$

maple [B] time = 0.51, size = 236, normalized size = 1.89

$$\frac{a^2 \operatorname{sech}(dx+c) \tanh(dx+c)}{2d} + \frac{a^2 \arctan(e^{dx+c})}{d} - \frac{2ab \sinh(dx+c)}{3d \cosh(dx+c)^4} + \frac{ab \tanh(dx+c) \operatorname{sech}(dx+c)^3}{6d} + \frac{ab \operatorname{sech}(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x)

[Out] $\frac{1}{2}/d*a^2*\operatorname{sech}(d*x+c)*\tanh(d*x+c) + 1/d*a^2*\arctan(\exp(d*x+c)) - 2/3/d*a*b*\sinh(d*x+c)/\cosh(d*x+c)^4 + 1/6/d*a*b*\tanh(d*x+c)*\operatorname{sech}(d*x+c)^3 + 1/4/d*a*b*\operatorname{sech}(d*x+c)*\tanh(d*x+c) + 1/2/d*a*b*\arctan(\exp(d*x+c)) - 1/3/d*b^2*\sinh(d*x+c)^3/\cosh(d*x+c)^6 - 1/5/d*b^2*\sinh(d*x+c)/\cosh(d*x+c)^6 + 1/30/d*b^2*\tanh(d*x+c)*\operatorname{sech}(d*x+c)^5 + 1/24/d*b^2*\tanh(d*x+c)*\operatorname{sech}(d*x+c)^3 + 1/16/d*b^2*\operatorname{sech}(d*x+c)*\tanh(d*x+c) + 1/8/d*b^2*\arctan(\exp(d*x+c))$

maxima [B] time = 0.43, size = 345, normalized size = 2.76

$$-\frac{1}{24} b^2 \left(\frac{3 \arctan(e^{-dx-c})}{d} - \frac{3e^{-dx-c} - 47e^{-3dx-3c} + 78e^{-5dx-5c} - 78e^{-7dx-7c} + 47e^{-9dx-9c} - 3e^{-11dx-11c}}{d(6e^{-2dx-2c} + 15e^{-4dx-4c} + 20e^{-6dx-6c} + 15e^{-8dx-8c} + 6e^{-10dx-10c} + e^{-12dx-12c})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out]
$$-1/24*b^2*(3*\arctan(e^{-d*x - c})/d - (3*e^{-d*x - c} - 47*e^{-3*d*x - 3*c} + 78*e^{-5*d*x - 5*c} - 78*e^{-7*d*x - 7*c} + 47*e^{-9*d*x - 9*c} - 3*e^{-11*d*x - 11*c})/(d*(6*e^{-2*d*x - 2*c} + 15*e^{-4*d*x - 4*c} + 20*e^{-6*d*x - 6*c} + 15*e^{-8*d*x - 8*c} + 6*e^{-10*d*x - 10*c} + e^{-12*d*x - 12*c} + 1))) - 1/2*a*b*(\arctan(e^{-d*x - c})/d - (e^{-d*x - c} - 7*e^{-3*d*x - 3*c} + 7*e^{-5*d*x - 5*c} - e^{-7*d*x - 7*c})/(d*(4*e^{-2*d*x - 2*c} + 6*e^{-4*d*x - 4*c} + 4*e^{-6*d*x - 6*c} + e^{-8*d*x - 8*c} + 1))) - a^2*(\arctan(e^{-d*x - c})/d - (e^{-d*x - c} - e^{-3*d*x - 3*c})/(d*(2*e^{-2*d*x - 2*c} + e^{-4*d*x - 4*c} + 1)))$$

mupad [B] time = 0.18, size = 572, normalized size = 4.58

$$\frac{\operatorname{atan}\left(\frac{e^{dx} e^c (8a^2 \sqrt{d^2+b^2} \sqrt{d^2+4ab\sqrt{d^2}})}{d \sqrt{64a^4+64a^3b+32a^2b^2+8ab^3+b^4}}\right) \sqrt{64a^4+64a^3b+32a^2b^2+8ab^3+b^4}}{8\sqrt{d^2}} - \frac{\frac{2e^{c+dx}(a+b)^2}{3d} + \frac{8e^{3c+3dx}(a^2-b^2)}{3d} + \frac{8e^{5c+5dx}(a^2-b^2)}{3d}}{6e^{2c+2dx} + 15e^{4c+4dx} + 20e^{6c+6dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tanh(c + d*x)^2)^2/cosh(c + d*x)^3,x)

[Out]
$$\left(\operatorname{atan}\left(\frac{\exp(dx) \exp(c) (8a^2 (d^2)^{1/2} + b^2 (d^2)^{1/2} + 4ab (d^2)^{1/2})}{d(8a^3b + 64a^4 + b^4 + 32a^2b^2)^{1/2}}\right) (8a^3b + 64a^4 + b^4 + 32a^2b^2)^{1/2} \right) / (8(d^2)^{1/2}) - \left(\frac{2\exp(c+dx)(a+b)^2}{3d} + \frac{8\exp(3c+3dx)(a^2-b^2)}{3d} + \frac{8\exp(7c+7dx)(a^2-b^2)}{3d} + \frac{2\exp(9c+9dx)(a+b)^2}{3d} + \frac{4\exp(5c+5dx)(3a^2-2ab+3b^2)}{3d} \right) / (6\exp(2c+2dx) + 15\exp(4c+4dx) + 20\exp(6c+6dx) + 15\exp(8c+8dx) + 6\exp(10c+10dx) + \exp(12c+12dx) + 1) - \frac{2\exp(c+dx)(4ab+15b^2)}{3d(4\exp(2c+2dx) + 6\exp(4c+4dx) + 4\exp(6c+6dx) + \exp(8c+8dx) + 1)} + \frac{16b^2\exp(c+dx)}{3d(5\exp(2c+2dx) + 10\exp(4c+4dx) + 10\exp(6c+6dx) + 5\exp(8c+8dx) + \exp(10c+10dx) + 1)} + \frac{\exp(c+dx)(4ab+8a^2+b^2)}{8d(\exp(2c+2dx) + 1)} - \frac{\exp(c+dx)(44ab+16a^2+23b^2)}{12d(2\exp(2c+2dx) + \exp(4c+4dx) + 1)} + \frac{\exp(c+dx)(20ab+21b^2)}{3d(3\exp(2c+2dx) + 3\exp(4c+4dx) + \exp(6c+6dx) + 1)}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^2 \operatorname{sech}^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)

[Out] Integral((a + b*tanh(c + d*x)**2)**2*sech(c + d*x)**3, x)

3.96 $\int \operatorname{sech}^4(c + dx) (a + b \tanh^2(c + dx))^2 dx$

Optimal. Leaf size=76

$$\frac{a^2 \tanh(c + dx)}{d} - \frac{b(2a - b) \tanh^5(c + dx)}{5d} - \frac{a(a - 2b) \tanh^3(c + dx)}{3d} - \frac{b^2 \tanh^7(c + dx)}{7d}$$

[Out] $a^2 \tanh(d*x+c)/d - 1/3*a*(a-2*b)*\tanh(d*x+c)^3/d - 1/5*(2*a-b)*b*\tanh(d*x+c)^5/d - 1/7*b^2*\tanh(d*x+c)^7/d$

Rubi [A] time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3675, 373}

$$\frac{a^2 \tanh(c + dx)}{d} - \frac{b(2a - b) \tanh^5(c + dx)}{5d} - \frac{a(a - 2b) \tanh^3(c + dx)}{3d} - \frac{b^2 \tanh^7(c + dx)}{7d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2, x]

[Out] $(a^2 \tanh[c + d*x])/d - (a*(a - 2*b)*\tanh[c + d*x]^3)/(3*d) - ((2*a - b)*b*\tanh[c + d*x]^5)/(5*d) - (b^2*\tanh[c + d*x]^7)/(7*d)$

Rule 373

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m-1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2-1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^4(c + dx) (a + b \tanh^2(c + dx))^2 dx &= \frac{\operatorname{Subst}\left(\int (1 - x^2) (a + bx^2)^2 dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int (a^2 - a(a - 2b)x^2 - (2a - b)bx^4 - b^2x^6) dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{a^2 \tanh(c + dx)}{d} - \frac{a(a - 2b) \tanh^3(c + dx)}{3d} - \frac{(2a - b)b \tanh^5(c + dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.58, size = 83, normalized size = 1.09

$$\frac{\tanh(c + dx) \left((35a^2 + 14ab + 3b^2) \operatorname{sech}^2(c + dx) + 70a^2 - 6b(7a + 4b) \operatorname{sech}^4(c + dx) + 28ab + 15b^2 \operatorname{sech}^6(c + dx) \right)}{105d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2, x]

[Out] $((70a^2 + 28ab + 6b^2 + (35a^2 + 14ab + 3b^2)\text{Sech}[c + dx]^2 - 6b(7a + 4b)\text{Sech}[c + dx]^4 + 15b^2\text{Sech}[c + dx]^6)\text{Tanh}[c + dx]) / (105d)$

fricas [B] time = 0.40, size = 677, normalized size = 8.91

$$\frac{8(2(35a^2 + 56ab + 27b^2)\cosh(dx+c)^5 + 10(35a^2 + 56ab + 27b^2)\cosh(dx+c)\sinh(dx+c)^4 + (35a^2 + 98ab + 51b^2)\sinh(dx+c)^5 + 14(25a^2 + 16ab - 3b^2)\cosh(dx+c)^3 + (10(35a^2 + 98ab + 51b^2)\cosh(dx+c)^2 + 105a^2 + 126ab - 63b^2)\sinh(dx+c)^3 + 2(10(35a^2 + 56ab + 27b^2)\cosh(dx+c)^3 + 21(25a^2 + 16ab - 3b^2)\cosh(dx+c))\sinh(dx+c)^2 + 28(25a^2 + 4ab + 3b^2)\cosh(dx+c) + (5(35a^2 + 98ab + 51b^2)\cosh(dx+c)^4 + 63(5a^2 + 6ab - 3b^2)\cosh(dx+c)^2 + 70a^2 + 28ab + 126b^2)\sinh(dx+c))}{d(\cosh(dx+c)^9 + 9d\cosh(dx+c)\sinh(dx+c)^8 + d\sinh(dx+c)^9 + 7d\cosh(dx+c)^7 + (36d\cosh(dx+c)^2 + 7d)\sinh(dx+c)^7 + 7(12d\cosh(dx+c)^3 + 7d\cosh(dx+c))\sinh(dx+c)^6 + 22d\cosh(dx+c)^5 + (126d\cosh(dx+c)^4 + 147d\cosh(dx+c)^2 + 20d)\sinh(dx+c)^5 + (126d\cosh(dx+c)^5 + 245d\cosh(dx+c)^3 + 110d\cosh(dx+c))\sinh(dx+c)^4 + 42d\cosh(dx+c)^3 + (84d\cosh(dx+c)^6 + 245d\cosh(dx+c)^4 + 200d\cosh(dx+c)^2 + 28d)\sinh(dx+c)^3 + (36d\cosh(dx+c)^7 + 147d\cosh(dx+c)^5 + 220d\cosh(dx+c)^3 + 126d\cosh(dx+c))\sinh(dx+c)^2 + 56d\cosh(dx+c) + (9d\cosh(dx+c)^8 + 49d\cosh(dx+c)^6 + 100d\cosh(dx+c)^4 + 84d\cosh(dx+c)^2 + 14d)\sinh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")`

[Out] $-8/105*(2*(35a^2 + 56ab + 27b^2)\cosh(dx+c)^5 + 10(35a^2 + 56ab + 27b^2)\cosh(dx+c)\sinh(dx+c)^4 + (35a^2 + 98ab + 51b^2)\sinh(dx+c)^5 + 14(25a^2 + 16ab - 3b^2)\cosh(dx+c)^3 + (10(35a^2 + 98ab + 51b^2)\cosh(dx+c)^2 + 105a^2 + 126ab - 63b^2)\sinh(dx+c)^3 + 2(10(35a^2 + 56ab + 27b^2)\cosh(dx+c)^3 + 21(25a^2 + 16ab - 3b^2)\cosh(dx+c))\sinh(dx+c)^2 + 28(25a^2 + 4ab + 3b^2)\cosh(dx+c) + (5(35a^2 + 98ab + 51b^2)\cosh(dx+c)^4 + 63(5a^2 + 6ab - 3b^2)\cosh(dx+c)^2 + 70a^2 + 28ab + 126b^2)\sinh(dx+c)) / (d(\cosh(dx+c)^9 + 9d\cosh(dx+c)\sinh(dx+c)^8 + d\sinh(dx+c)^9 + 7d\cosh(dx+c)^7 + (36d\cosh(dx+c)^2 + 7d)\sinh(dx+c)^7 + 7(12d\cosh(dx+c)^3 + 7d\cosh(dx+c))\sinh(dx+c)^6 + 22d\cosh(dx+c)^5 + (126d\cosh(dx+c)^4 + 147d\cosh(dx+c)^2 + 20d)\sinh(dx+c)^5 + (126d\cosh(dx+c)^5 + 245d\cosh(dx+c)^3 + 110d\cosh(dx+c))\sinh(dx+c)^4 + 42d\cosh(dx+c)^3 + (84d\cosh(dx+c)^6 + 245d\cosh(dx+c)^4 + 200d\cosh(dx+c)^2 + 28d)\sinh(dx+c)^3 + (36d\cosh(dx+c)^7 + 147d\cosh(dx+c)^5 + 220d\cosh(dx+c)^3 + 126d\cosh(dx+c))\sinh(dx+c)^2 + 56d\cosh(dx+c) + (9d\cosh(dx+c)^8 + 49d\cosh(dx+c)^6 + 100d\cosh(dx+c)^4 + 84d\cosh(dx+c)^2 + 14d)\sinh(dx+c))$

giac [B] time = 0.24, size = 238, normalized size = 3.13

$$\frac{4(105a^2e^{(10dx+10c)} + 210abe^{(10dx+10c)} + 105b^2e^{(10dx+10c)} + 455a^2e^{(8dx+8c)} + 350abe^{(8dx+8c)} - 105b^2e^{(8dx+8c)})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")`

[Out] $-4/105*(105a^2e^{(10dx+10c)} + 210abbe^{(10dx+10c)} + 105b^2e^{(10dx+10c)} + 455a^2e^{(8dx+8c)} + 350abbe^{(8dx+8c)} - 105b^2e^{(8dx+8c)} + 770a^2e^{(6dx+6c)} + 140abbe^{(6dx+6c)} + 210b^2e^{(6dx+6c)} + 630a^2e^{(4dx+4c)} + 84abbe^{(4dx+4c)} - 42b^2e^{(4dx+4c)} + 245a^2e^{(2dx+2c)} + 98abbe^{(2dx+2c)} + 21b^2e^{(2dx+2c)} + 35a^2 + 14ab + 3b^2) / (d(e^{(2dx+2c)} + 1)^7)$

maple [B] time = 0.47, size = 158, normalized size = 2.08

$$\frac{a^2\left(\frac{2}{3} + \frac{\text{sech}(dx+c)^2}{3}\right)\tanh(dx+c) + 2ab\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5} + \frac{\left(\frac{8}{15} + \frac{\text{sech}(dx+c)^4}{5} + \frac{4\text{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{4}\right) + b^2\left(-\frac{\sinh^3(dx+c)}{4\cosh(dx+c)^7} - \frac{1}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x)`

```
[Out] 1/d*(a^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+2*a*b*(-1/4*sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))+b^2*(-1/4*sinh(d*x+c)^3/cosh(d*x+c)^7-1/8*sinh(d*x+c)/cosh(d*x+c)^7+1/8*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c)))
```

maxima [B] time = 0.34, size = 928, normalized size = 12.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 4/35*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 14*e^(-4*d*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 70*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 35*e^(-8*d*x - 8*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 35*e^(-10*d*x - 10*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 8/15*a*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

mupad [B] time = 1.22, size = 732, normalized size = 9.63

$$\frac{\frac{4(3a^2-2ab+3b^2)}{35d} + \frac{32e^{2c+2dx}(a^2-b^2)}{35d} + \frac{4e^{4c+4dx}(a+b)^2}{7d}}{4e^{2c+2dx} + 6e^{4c+4dx} + 4e^{6c+6dx} + e^{8c+8dx} + 1} - \frac{\frac{32(a^2-b^2)}{105d} + \frac{8e^{2c+2dx}(a+b)^2}{21d}}{3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1} - \frac{\frac{32(a^2-b^2)}{105d} + \frac{64e^{4c+4dx}}{35d}}{5e^{2c+2dx} + 10e^{4c+4dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)^2/cosh(c + d*x)^4,x)
```

```
[Out] - ((4*(3*a^2 - 2*a*b + 3*b^2))/(35*d) + (32*exp(2*c + 2*d*x)*(a^2 - b^2))/(35*d) + (4*exp(4*c + 4*d*x)*(a + b)^2)/(7*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((32*(a^2 - b^2))/(105*d) + (8*exp(2*c + 2*d*x)*(a + b)^2)/(21*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((32*(a^2 - b^2))/(105*d) + (64*exp(4*c + 4*d*x)*(a^2 - b^2))/(35*d) + (16*exp(6*c + 6*d*x)*(a + b)^2)/(21*d) + (16*exp(2*c + 2*d*x)*(3*a^2 - 2*a*b + 3*b^2))/(35*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((32*exp(4*c + 4*d*x)*(a^2 - b^2))/(7*d) + (32*exp(8*c + 8*d*x)*(a^2 - b^2))/(7*d) + (8*exp(2*c + 2*d*x)*(a + b)^2)/(7*d) + (8*exp(10*c + 10*d*x)*(a + b)^2)/(7*d) + (16*exp(6*c + 6*d*x)*(3*a^2 - 2*a*b + 3*b^2))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)
```

```
*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) +
exp(14*c + 14*d*x) + 1) - ((4*(a + b)^2)/(21*d) + (32*exp(2*c + 2*d*x)*(a^2
- b^2))/(21*d) + (64*exp(6*c + 6*d*x)*(a^2 - b^2))/(21*d) + (20*exp(8*c +
8*d*x)*(a + b)^2)/(21*d) + (8*exp(4*c + 4*d*x)*(3*a^2 - 2*a*b + 3*b^2))/(7*
d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*ex
p(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1) - (4*(a + b
)^2)/(21*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^2 \operatorname{sech}^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*sech(c + d*x)**4, x)
```

$$3.97 \quad \int \cosh^4(c + dx) \left(a + b \tanh^2(c + dx) \right)^3 dx$$

Optimal. Leaf size=91

$$\frac{3}{8}x(a+b)(a^2 - 2ab + 5b^2) + \frac{(a+b)^3 \sinh(c+dx) \cosh^3(c+dx)}{4d} + \frac{3(a-3b)(a+b)^2 \sinh(c+dx) \cosh(c+dx)}{8d}$$

[Out] 3/8*(a+b)*(a^2-2*a*b+5*b^2)*x+3/8*(a-3*b)*(a+b)^2*cosh(d*x+c)*sinh(d*x+c)/d+1/4*(a+b)^3*cosh(d*x+c)^3*sinh(d*x+c)/d-b^3*tanh(d*x+c)/d

Rubi [A] time = 0.13, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3675, 390, 1157, 385, 206}

$$\frac{3}{8}x(a+b)(a^2 - 2ab + 5b^2) + \frac{(a+b)^3 \sinh(c+dx) \cosh^3(c+dx)}{4d} + \frac{3(a-3b)(a+b)^2 \sinh(c+dx) \cosh(c+dx)}{8d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (3*(a + b)*(a^2 - 2*a*b + 5*b^2)*x)/8 + (3*(a - 3*b)*(a + b)^2*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + ((a + b)^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d) - (b^3*Tanh[c + d*x])/d

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 1157

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dis

```
t[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned} \int \cosh^4(c + dx) (a + b \tanh^2(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(-b^3 + \frac{a^3+b^3+3b(a^2-b^2)x^2+3b^2(a+b)x^4}{(1-x^2)^3}\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{b^3 \tanh(c + dx)}{d} + \frac{\text{Subst}\left(\int \frac{a^3+b^3+3b(a^2-b^2)x^2+3b^2(a+b)x^4}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{(a + b)^3 \cosh^3(c + dx) \sinh(c + dx)}{4d} - \frac{b^3 \tanh(c + dx)}{d} - \frac{\text{Subst}\left(\int \frac{a^3+b^3+3b(a^2-b^2)x^2+3b^2(a+b)x^4}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{3(a - 3b)(a + b)^2 \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{(a + b)^3 \cosh^3(c + dx)}{4d} \\ &= \frac{3}{8}(a + b)(a^2 - 2ab + 5b^2)x + \frac{3(a - 3b)(a + b)^2 \cosh(c + dx) \sinh(c + dx)}{8d} \end{aligned}$$

Mathematica [A] time = 0.70, size = 81, normalized size = 0.89

$$\frac{12(a^3 - a^2b + 3ab^2 + 5b^3)(c + dx) + 8(a - 2b)(a + b)^2 \sinh(2(c + dx)) + (a + b)^3 \sinh(4(c + dx)) - 32b^3 \tanh(c + dx)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (12*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*(c + d*x) + 8*(a - 2*b)*(a + b)^2*Sinh[2*(c + d*x)] + (a + b)^3*Sinh[4*(c + d*x)] - 32*b^3*Tanh[c + d*x])/(32*d)
```

fricas [B] time = 0.42, size = 227, normalized size = 2.49

$$\frac{(a^3 + 3a^2b + 3ab^2 + b^3) \sinh(dx + c)^5 + (9a^3 + 3a^2b - 21ab^2 - 15b^3 + 10(a^3 + 3a^2b + 3ab^2 + b^3) \cosh(dx + c)) \sinh(dx + c)^3}{d \cosh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^5 + (9*a^3 + 3*a^2*b - 21*a*b^2 - 15*b^3 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 8*(8*b^3 + 3*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*d*x)*cosh(d*x + c) + (5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 8*a^3 - 24*a*b^2 - 80*b^3 + 9*(3*a^3 + a^2*b - 7*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c))
```

giac [B] time = 0.76, size = 285, normalized size = 3.13

$$\frac{24(a^3 - a^2b + 3ab^2 + 5b^3)dx + \frac{128b^3}{e^{(2dx+2c)+1}} - (18a^3e^{(4dx+4c)} - 18a^2be^{(4dx+4c)} + 54ab^2e^{(4dx+4c)} + 90b^3e^{(4dx+4c)})}{e^{(2dx+2c)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out] $\frac{1}{64}*(24*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*d*x + 128*b^3/(e^{(2*d*x + 2*c)} + 1) - (18*a^3*e^{(4*d*x + 4*c)} - 18*a^2*b*e^{(4*d*x + 4*c)} + 54*a*b^2*e^{(4*d*x + 4*c)} + 90*b^3*e^{(4*d*x + 4*c)} + 8*a^3*e^{(2*d*x + 2*c)} - 24*a*b^2*e^{(2*d*x + 2*c)} - 16*b^3*e^{(2*d*x + 2*c)} + a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^{(-4*d*x - 4*c)} + (a^3*e^{(4*d*x + 20*c)} + 3*a^2*b*e^{(4*d*x + 20*c)} + 3*a*b^2*e^{(4*d*x + 20*c)} + b^3*e^{(4*d*x + 20*c)} + 8*a^3*e^{(2*d*x + 18*c)} - 24*a*b^2*e^{(2*d*x + 18*c)} - 16*b^3*e^{(2*d*x + 18*c)})*e^{(-16*c)})/d$

maple [B] time = 0.38, size = 184, normalized size = 2.02

$$a^3 \left(\left(\frac{\cosh^3(dx+c)}{4} + \frac{3 \cosh(dx+c)}{8} \right) \sinh(dx+c) + \frac{3dx}{8} + \frac{3c}{8} \right) + 3a^2b \left(\frac{\sinh(dx+c)\cosh^3(dx+c)}{4} - \frac{\cosh(dx+c)\sinh(dx+c)}{8} - \frac{dx}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x)

[Out] $\frac{1}{d}*(a^3*((1/4*\cosh(d*x+c)^3+3/8*\cosh(d*x+c))*\sinh(d*x+c)+3/8*d*x+3/8*c)+3*a^2*b*(1/4*\sinh(d*x+c)*\cosh(d*x+c)^3-1/8*\cosh(d*x+c)*\sinh(d*x+c)-1/8*d*x-1/8*c)+3*a*b^2*((1/4*\sinh(d*x+c)^3-3/8*\sinh(d*x+c))*\cosh(d*x+c)+3/8*d*x+3/8*c)+b^3*(1/4*\sinh(d*x+c)^5/\cosh(d*x+c)-5/8*\sinh(d*x+c)^3/\cosh(d*x+c)+15/8*d*x+15/8*c-15/8*\tanh(d*x+c)))$

maxima [B] time = 0.33, size = 267, normalized size = 2.93

$$\frac{1}{64} a^3 \left(24x + \frac{e^{(4dx+4c)}}{d} + \frac{8e^{(2dx+2c)}}{d} - \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) + \frac{3}{64} ab^2 \left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $\frac{1}{64}a^3*(24*x + e^{(4*d*x + 4*c)}/d + 8*e^{(2*d*x + 2*c)}/d - 8*e^{(-2*d*x - 2*c)}/d - e^{(-4*d*x - 4*c)}/d) + \frac{3}{64}a*b^2*(24*x + e^{(4*d*x + 4*c)}/d - 8*e^{(2*d*x + 2*c)}/d + 8*e^{(-2*d*x - 2*c)}/d - e^{(-4*d*x - 4*c)}/d) + \frac{1}{64}b^3*(120*(d*x + c)/d + (16*e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)})/d - (15*e^{(-2*d*x - 2*c)} + 144*e^{(-4*d*x - 4*c)} - 1)/(d*(e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)}))) - \frac{3}{64}a^2*b*(8*(d*x + c)/d - e^{(4*d*x + 4*c)}/d + e^{(-4*d*x - 4*c)}/d)$

mupad [B] time = 1.41, size = 133, normalized size = 1.46

$$x \left(\frac{3a^3}{8} - \frac{3a^2b}{8} + \frac{9ab^2}{8} + \frac{15b^3}{8} \right) + \frac{2b^3}{d(e^{2c+2dx} + 1)} - \frac{e^{-4c-4dx}(a+b)^3}{64d} + \frac{e^{4c+4dx}(a+b)^3}{64d} - \frac{e^{-2c-2dx}(a+b)^2}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3,x)

[Out] $x*((9*a*b^2)/8 - (3*a^2*b)/8 + (3*a^3)/8 + (15*b^3)/8) + (2*b^3)/(d*(\exp(2*c + 2*d*x) + 1)) - (\exp(-4*c - 4*d*x)*(a + b)^3)/(64*d) + (\exp(4*c + 4*d*x)*(a + b)^3)/(64*d) - (\exp(-2*c - 2*d*x)*(a + b)^2*(a - 2*b))/(8*d) + (\exp(2*c + 2*d*x)*(a + b)^2*(a - 2*b))/(8*d)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.98 $\int \cosh^3(c + dx) (a + b \tanh^2(c + dx))^3 dx$

Optimal. Leaf size=87

$$\frac{b^2(6a + 5b) \tan^{-1}(\sinh(c + dx))}{2d} + \frac{(a + b)^3 \sinh^3(c + dx)}{3d} + \frac{(a - 2b)(a + b)^2 \sinh(c + dx)}{d} - \frac{b^3 \tanh(c + dx) \operatorname{sech}(c + dx)}{2d}$$

[Out] 1/2*b^2*(6*a+5*b)*arctan(sinh(d*x+c))/d+(a-2*b)*(a+b)^2*sinh(d*x+c)/d+1/3*(a+b)^3*sinh(d*x+c)^3/d-1/2*b^3*sech(d*x+c)*tanh(d*x+c)/d

Rubi [A] time = 0.11, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3676, 390, 385, 203}

$$\frac{b^2(6a + 5b) \tan^{-1}(\sinh(c + dx))}{2d} + \frac{(a + b)^3 \sinh^3(c + dx)}{3d} + \frac{(a - 2b)(a + b)^2 \sinh(c + dx)}{d} - \frac{b^3 \tanh(c + dx) \operatorname{sech}(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (b^2*(6*a + 5*b)*ArcTan[Sinh[c + d*x]]/(2*d) + ((a - 2*b)*(a + b)^2*Sinh[c + d*x])/d + ((a + b)^3*Sinh[c + d*x]^3)/(3*d) - (b^3*Sech[c + d*x]*Tanh[c + d*x])/(2*d)

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 3676

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \cosh^3(c+dx) (a+b \tanh^2(c+dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{(a+(a+b)x^2)^3}{(1+x^2)^2} dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left((a-2b)(a+b)^2 + (a+b)^3 x^2 + \frac{b^2(3a+2b)+3b^2(a+b)x^2}{(1+x^2)^2}\right) dx, x\right)}{d} \\
&= \frac{(a-2b)(a+b)^2 \sinh(c+dx)}{d} + \frac{(a+b)^3 \sinh^3(c+dx)}{3d} + \frac{\text{Subst}\left(\int \frac{b^2 \text{sech}(c+dx)}{(1+x^2)^2} dx, x\right)}{d} \\
&= \frac{(a-2b)(a+b)^2 \sinh(c+dx)}{d} + \frac{(a+b)^3 \sinh^3(c+dx)}{3d} - \frac{b^3 \text{sech}(c+dx)}{d} \\
&= \frac{b^2(6a+5b) \tan^{-1}(\sinh(c+dx))}{2d} + \frac{(a-2b)(a+b)^2 \sinh(c+dx)}{d} + \frac{(a+b)^3 \sinh^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [C] time = 6.89, size = 494, normalized size = 5.68

$$\text{csch}^5(c+dx) \left(-256 \sinh^8(c+dx) (a \sinh^2(c+dx) + a + b \sinh^2(c+dx))^3 {}_5F_4\left(\frac{3}{2}, 2, 2, 2, 2; 1, 1, 1, \frac{11}{2}; -\sinh^2(c+dx)\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (Csch[c + d*x]^5*(-256*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8*(a + a*Sinh[c + d*x]^2 + b*Sinh[c + d*x]^2)^3 - (315*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(b^3*Sinh[c + d*x]^6*(2161 + 1875*Sinh[c + d*x]^2 + 243*Sinh[c + d*x]^4 + Sinh[c + d*x]^6) + a^3*Cosh[c + d*x]^6*(2401 + 1875*Sinh[c + d*x]^2 + 243*Sinh[c + d*x]^4 + Sinh[c + d*x]^6) + 3*a^2*b*(Sinh[c + d*x] + Sinh[c + d*x]^3)^2*(2401 + 1875*Sinh[c + d*x]^2 + 243*Sinh[c + d*x]^4 + Sinh[c + d*x]^6) + 3*a*b^2*Sinh[c + d*x]^4*(2401 + 4180*Sinh[c + d*x]^2 + 2118*Sinh[c + d*x]^4 + 244*Sinh[c + d*x]^6 + Sinh[c + d*x]^8))/Sqrt[-Sinh[c + d*x]^2] + 21*(b^3*Sinh[c + d*x]^6*(32415 + 17320*Sinh[c + d*x]^2 + 753*Sinh[c + d*x]^4) + 3*a*b^2*Sinh[c + d*x]^4*(36015 + 50695*Sinh[c + d*x]^2 + 18073*Sinh[c + d*x]^4 + 753*Sinh[c + d*x]^6) + 3*a^2*b*Sinh[c + d*x]^2*(36015 + 88150*Sinh[c + d*x]^2 + 69728*Sinh[c + d*x]^4 + 18826*Sinh[c + d*x]^6 + 753*Sinh[c + d*x]^8) + a^3*(36015 + 124165*Sinh[c + d*x]^2 + 157878*Sinh[c + d*x]^4 + 89514*Sinh[c + d*x]^6 + 19579*Sinh[c + d*x]^8 + 753*Sinh[c + d*x]^10))))/(30240*d)

fricas [B] time = 0.45, size = 1840, normalized size = 21.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/24*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^10 + (11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x + c)^8 + (11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3 + 45*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x + c)

```

)*sinh(d*x + c)^7 + 2*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^6
+ 2*(105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 5*a^3 - 3*a^2*b
- 21*a*b^2 - 25*b^3 + 14*(11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x +
c)^2)*sinh(d*x + c)^6 + 4*(63*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c
)^5 + 14*(11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x + c)^3 + 3*(5*a^3
- 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(5*a^3 -
3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^4 + 2*(105*(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*cosh(d*x + c)^6 + 35*(11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh
(d*x + c)^4 - 5*a^3 + 3*a^2*b + 21*a*b^2 + 25*b^3 + 15*(5*a^3 - 3*a^2*b - 2
1*a*b^2 - 25*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*cosh(d*x + c)^7 + 7*(11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*
cosh(d*x + c)^5 + 5*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^3 -
(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3
- 3*a^2*b - 3*a*b^2 - b^3 - (11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*
x + c)^2 + (45*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 28*(11*a^3
- 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x + c)^6 + 30*(5*a^3 - 3*a^2*b - 21*
a*b^2 - 25*b^3)*cosh(d*x + c)^4 - 11*a^3 + 3*a^2*b + 39*a*b^2 + 25*b^3 - 12
*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2
4*((6*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 7*(6*a*b^2 + 5*b^3)*cosh(d*x + c)*si
nh(d*x + c)^6 + (6*a*b^2 + 5*b^3)*sinh(d*x + c)^7 + 2*(6*a*b^2 + 5*b^3)*cos
h(d*x + c)^5 + (12*a*b^2 + 10*b^3 + 21*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*s
inh(d*x + c)^5 + 5*(7*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 2*(6*a*b^2 + 5*b^
3)*cosh(d*x + c))*sinh(d*x + c)^4 + (6*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (35
*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 6*a*b^2 + 5*b^3 + 20*(6*a*b^2 + 5*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (21*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^5 +
20*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 3*(6*a*b^2 + 5*b^3)*cosh(d*x + c))*
sinh(d*x + c)^2 + (7*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 10*(6*a*b^2 + 5*b^
3)*cosh(d*x + c)^4 + 3*(6*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*ar
ctan(cosh(d*x + c) + sinh(d*x + c)) + 2*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
cosh(d*x + c)^9 + 4*(11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x + c)^7
+ 6*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^5 - 4*(5*a^3 - 3*a^
2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^3 - (11*a^3 - 3*a^2*b - 39*a*b^2 - 2
5*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)
*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 2*d*cosh(d*x + c)^5 + (21*d*cosh(d*x
+ c)^2 + 2*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3 + 2*d*cosh(d*x + c)
)*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (35*d*cosh(d*x + c)^4 + 20*d*cosh(d
*x + c)^2 + d)*sinh(d*x + c)^3 + (21*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)
^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + (7*d*cosh(d*x + c)^6 + 10*d*cosh(
d*x + c)^4 + 3*d*cosh(d*x + c)^2)*sinh(d*x + c))

```

giac [B] time = 0.66, size = 279, normalized size = 3.21

$$24(6ab^2e^c + 5b^3e^c) \arctan(e^{(dx+c)})e^{-c} - (9a^3e^{(2dx+2c)} - 9a^2be^{(2dx+2c)} - 45ab^2e^{(2dx+2c)} - 27b^3e^{(2dx+2c)} + \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

```

[Out] 1/24*(24*(6*a*b^2*e^c + 5*b^3*e^c)*arctan(e^(d*x + c))*e^(-c) - (9*a^3*e^(
2*d*x + 2*c) - 9*a^2*b*e^(2*d*x + 2*c) - 45*a*b^2*e^(2*d*x + 2*c) - 27*b^3*e
^(2*d*x + 2*c) + a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-3*d*x - 3*c) + (a^3*e^(
3*d*x + 30*c) + 3*a^2*b*e^(3*d*x + 30*c) + 3*a*b^2*e^(3*d*x + 30*c) + b^3*e
^(3*d*x + 30*c) + 9*a^3*e^(d*x + 28*c) - 9*a^2*b*e^(d*x + 28*c) - 45*a*b^2*
e^(d*x + 28*c) - 27*b^3*e^(d*x + 28*c))*e^(-27*c) - 24*(b^3*e^(3*d*x + 3*c)
- b^3*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^2)/d

```

maple [B] time = 0.38, size = 206, normalized size = 2.37

$$\frac{2a^3 \sinh(dx+c)}{3d} + \frac{a^3 \sinh(dx+c) (\cosh^2(dx+c))}{3d} + \frac{a^2 b (\sinh^3(dx+c))}{d} + \frac{a b^2 (\sinh^3(dx+c))}{d} - \frac{3a b^2 \sinh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x)`

[Out] $\frac{2}{3}a^3\sinh(dx+c)/d + \frac{1}{3}d*a^3\sinh(dx+c)*\cosh(dx+c)^2 + a^2*b*\sinh(dx+c)^3/d + \frac{1}{d}*a*b^2*\sinh(dx+c)^3 - \frac{3}{d}*a*b^2*\sinh(dx+c) + \frac{6}{d}*a*b^2*\arctan(\exp(dx+c)) + \frac{1}{3}d*b^3*\sinh(dx+c)^5/\cosh(dx+c)^2 - \frac{5}{3}d*b^3*\sinh(dx+c)^3/\cosh(dx+c)^2 - \frac{5}{d*b^3*\sinh(dx+c)}/\cosh(dx+c)^2 + \frac{5}{2}d*b^3*\operatorname{sech}(dx+c)*\tanh(dx+c) + \frac{5}{d*b^3*\arctan(\exp(dx+c))}$

maxima [B] time = 0.43, size = 284, normalized size = 3.26

$$\frac{a^2 b (e^{dx+c} - e^{-dx-c})^3}{8d} - \frac{1}{8} a b^2 \left(\frac{(15 e^{(-2dx-2c)} - 1) e^{(3dx+3c)}}{d} - \frac{15 e^{(-dx-c)} - e^{(-3dx-3c)}}{d} + \frac{48 \arctan(e^{(-dx-c)})}{d} \right) + \frac{1}{24} b^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")`

[Out] $\frac{1}{8}a^2b*(e^{dx+c} - e^{-dx-c})^3/d - \frac{1}{8}ab^2*((15e^{(-2dx-2c)} - 1)e^{(3dx+3c)}/d - (15e^{(-dx-c)} - e^{(-3dx-3c)})/d + 48*\arctan(e^{(-dx-c)})/d) + \frac{1}{24}b^3*((27e^{(-dx-c)} - e^{(-3dx-3c)})/d - 12*\arctan(e^{(-dx-c)})/d - (25e^{(-2dx-2c)} + 77e^{(-4dx-4c)} + 3e^{(-6dx-6c)} - 1)/(d*(e^{(-3dx-3c)} + 2e^{(-5dx-5c)} + e^{(-7dx-7c)}))) + \frac{1}{24}a^3*(e^{(3dx+3c)}/d + 9e^{(dx+c)}/d - 9e^{(-dx-c)}/d - e^{(-3dx-3c)}/d)$

mupad [B] time = 0.36, size = 232, normalized size = 2.67

$$\frac{\operatorname{atan}\left(\frac{e^{dx} e^c (5b^3 \sqrt{d^2} + 6ab^2 \sqrt{d^2})}{d \sqrt{36a^2b^4 + 60ab^5 + 25b^6}}\right) \sqrt{36a^2b^4 + 60ab^5 + 25b^6}}{\sqrt{d^2}} - \frac{e^{-3c-3dx}(a+b)^3}{24d} + \frac{e^{3c+3dx}(a+b)^3}{24d} + \frac{3e^{c+dx}(a+b)^2}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(c+d*x)^3*(a+b*tanh(c+d*x)^2)^3,x)`

[Out] $\frac{\operatorname{atan}((\exp(dx)*\exp(c)*(5*b^3*(d^2)^{(1/2)} + 6*a*b^2*(d^2)^{(1/2)}))/(d*(60*a*b^5 + 25*b^6 + 36*a^2*b^4)^{(1/2)}))*(60*a*b^5 + 25*b^6 + 36*a^2*b^4)^{(1/2)}}{(d^2)^{(1/2)} - (\exp(-3*c - 3*d*x)*(a+b)^3)/(24*d) + (\exp(3*c + 3*d*x)*(a+b)^3)/(24*d) + (3*\exp(c+d*x)*(a+b)^2*(a-3*b))/(8*d) - (b^3*\exp(c+d*x))/(d*(\exp(2*c + 2*d*x) + 1)) - (3*\exp(-c-d*x)*(a+b)^2*(a-3*b))/(8*d) + (2*b^3*\exp(c+d*x))/(d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1))}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^3 \cosh^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)`

[Out] `Integral((a + b*tanh(c + d*x)**2)**3*cosh(c + d*x)**3, x)`

3.99 $\int \cosh^2(c + dx) (a + b \tanh^2(c + dx))^3 dx$

Optimal. Leaf size=78

$$\frac{b^2(3a + 2b) \tanh(c + dx)}{d} + \frac{(a + b)^3 \sinh(c + dx) \cosh(c + dx)}{2d} + \frac{1}{2} x^{(a-5b)(a+b)^2} + \frac{b^3 \tanh^3(c + dx)}{3d}$$

[Out] $1/2*(a-5*b)*(a+b)^2*x+1/2*(a+b)^3*\cosh(d*x+c)*\sinh(d*x+c)/d+b^2*(3*a+2*b)*\tanh(d*x+c)/d+1/3*b^3*\tanh(d*x+c)^3/d$

Rubi [A] time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3675, 390, 385, 206}

$$\frac{b^2(3a + 2b) \tanh(c + dx)}{d} + \frac{(a + b)^3 \sinh(c + dx) \cosh(c + dx)}{2d} + \frac{1}{2} x^{(a-5b)(a+b)^2} + \frac{b^3 \tanh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] $((a - 5*b)*(a + b)^2*x)/2 + ((a + b)^3*\cosh[c + d*x]*\sinh[c + d*x])/(2*d) + (b^2*(3*a + 2*b)*\tanh[c + d*x])/d + (b^3*\tanh[c + d*x]^3)/(3*d)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned}
\int \cosh^2(c+dx) (a+b \tanh^2(c+dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{(1-x^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(b^2(3a+2b) + b^3x^2 + \frac{(a-2b)(a+b)^2+3b(a+b)^2x^2}{(1-x^2)^2}\right) dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{b^2(3a+2b) \tanh(c+dx)}{d} + \frac{b^3 \tanh^3(c+dx)}{3d} + \frac{\text{Subst}\left(\int \frac{(a-2b)(a+b)^2}{(1-x^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{(a+b)^3 \cosh(c+dx) \sinh(c+dx)}{2d} + \frac{b^2(3a+2b) \tanh(c+dx)}{d} + \frac{b^3 \tanh^3(c+dx)}{3d} \\
&= \frac{1}{2}(a-5b)(a+b)^2x + \frac{(a+b)^3 \cosh(c+dx) \sinh(c+dx)}{2d} + \frac{b^2(3a+2b) \tanh(c+dx)}{d} + \frac{b^3 \tanh^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.97, size = 69, normalized size = 0.88

$$\frac{4b^2 \tanh(c+dx) (9a - b \operatorname{sech}^2(c+dx) + 7b) + 6(a-5b)(a+b)^2(c+dx) + 3(a+b)^3 \sinh(2(c+dx))}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (6*(a - 5*b)*(a + b)^2*(c + d*x) + 3*(a + b)^3*Sinh[2*(c + d*x)] + 4*b^2*(9*a + 7*b - b*Sech[c + d*x]^2)*Tanh[c + d*x])/(12*d)

fricas [B] time = 0.41, size = 369, normalized size = 4.73

$$\frac{3(a^3 + 3a^2b + 3ab^2 + b^3) \sinh(dx+c)^5 - 4(18ab^2 + 14b^3 - 3(a^3 - 3a^2b - 9ab^2 - 5b^3)dx) \cosh(dx+c)^3 - 12(a+b)^3 \cosh(dx+c) \sinh(dx+c)^2}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/24*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^5 - 4*(18*a*b^2 + 14*b^3 - 3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c)^3 - 12*(18*a*b^2 + 14*b^3 - 3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^2 + (9*a^3 + 27*a^2*b + 99*a*b^2 + 65*b^3 + 30*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 - 12*(18*a*b^2 + 14*b^3 - 3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c) + 3*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*a^3 + 6*a^2*b + 30*a*b^2 + 10*b^3 + (9*a^3 + 27*a^2*b + 99*a*b^2 + 65*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))

giac [B] time = 0.60, size = 267, normalized size = 3.42

$$\frac{12(a^3 - 3a^2b - 9ab^2 - 5b^3)dx - 3(2a^3e^{2dx+2c} - 6a^2be^{2dx+2c} - 18ab^2e^{2dx+2c} - 10b^3e^{2dx+2c} + a^3 + 3a^2b + 3ab^2 + b^3) \cosh(dx+c)^3 - 4(18ab^2 + 14b^3 - 3(a^3 - 3a^2b - 9ab^2 - 5b^3)dx) \cosh(dx+c)^3 - 12(a+b)^3 \cosh(dx+c) \sinh(dx+c)^2}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out] $\frac{1}{24} \cdot (12 \cdot (a^3 - 3 \cdot a^2 \cdot b - 9 \cdot a \cdot b^2 - 5 \cdot b^3) \cdot d \cdot x - 3 \cdot (2 \cdot a^3 \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} - 6 \cdot a^2 \cdot b \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} - 18 \cdot a \cdot b^2 \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} - 10 \cdot b^3 \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)}) + a^3 + 3 \cdot a^2 \cdot b + 3 \cdot a \cdot b^2 + b^3) \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} + 3 \cdot (a^3 \cdot e^{(2 \cdot d \cdot x + 12 \cdot c)} + 3 \cdot a^2 \cdot b \cdot e^{(2 \cdot d \cdot x + 12 \cdot c)} + 3 \cdot a \cdot b^2 \cdot e^{(2 \cdot d \cdot x + 12 \cdot c)} + b^3 \cdot e^{(2 \cdot d \cdot x + 12 \cdot c)}) \cdot e^{(-10 \cdot c)} - 16 \cdot (9 \cdot a \cdot b^2 \cdot e^{(4 \cdot d \cdot x + 4 \cdot c)} + 9 \cdot b^3 \cdot e^{(4 \cdot d \cdot x + 4 \cdot c)} + 18 \cdot a \cdot b^2 \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + 12 \cdot b^3 \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + 9 \cdot a \cdot b^2 + 7 \cdot b^3) / (e^{(2 \cdot d \cdot x + 2 \cdot c)} + 1)^3 / d$

maple [B] time = 0.24, size = 148, normalized size = 1.90

$$\frac{a^3 \left(\frac{\cosh(dx+c) \sinh(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) + 3a^2b \left(\frac{\cosh(dx+c) \sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2} \right) + 3ab^2 \left(\frac{\sinh^3(dx+c)}{2 \cosh(dx+c)} - \frac{3dx}{2} - \frac{3c}{2} + \frac{3 \tanh(dx+c)}{2} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)^2*(a+b*tanh(d*x+c))^3,x)`

[Out] $\frac{1}{d} \cdot (a^3 \cdot (1/2 \cdot \cosh(d \cdot x + c) \cdot \sinh(d \cdot x + c) + 1/2 \cdot d \cdot x + 1/2 \cdot c) + 3 \cdot a^2 \cdot b \cdot (1/2 \cdot \cosh(d \cdot x + c) \cdot \sinh(d \cdot x + c) - 1/2 \cdot d \cdot x - 1/2 \cdot c) + 3 \cdot a \cdot b^2 \cdot (1/2 \cdot \sinh(d \cdot x + c)^3 / \cosh(d \cdot x + c) - 3/2 \cdot d \cdot x - 3/2 \cdot c + 3/2 \cdot \tanh(d \cdot x + c)) + b^3 \cdot (1/2 \cdot \sinh(d \cdot x + c)^5 / \cosh(d \cdot x + c)^3 - 5/2 \cdot d \cdot x - 5/2 \cdot c + 5/2 \cdot \tanh(d \cdot x + c) + 5/6 \cdot \tanh(d \cdot x + c)^3))$

maxima [B] time = 0.41, size = 256, normalized size = 3.28

$$\frac{1}{8} a^3 \left(4x + \frac{e^{(2dx+2c)}}{d} - \frac{e^{(-2dx-2c)}}{d} \right) - \frac{3}{8} a^2 b \left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d} \right) - \frac{1}{24} b^3 \left(\frac{60(dx+c)}{d} + \frac{3e^{(-2dx-2c)}}{d} - \frac{1}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c))^3,x, algorithm="maxima")`

[Out] $\frac{1}{8} \cdot a^3 \cdot (4 \cdot x + e^{(2 \cdot d \cdot x + 2 \cdot c)} / d - e^{(-2 \cdot d \cdot x - 2 \cdot c)} / d) - \frac{3}{8} \cdot a^2 \cdot b \cdot (4 \cdot x - e^{(2 \cdot d \cdot x + 2 \cdot c)} / d + e^{(-2 \cdot d \cdot x - 2 \cdot c)} / d) - \frac{1}{24} \cdot b^3 \cdot (60 \cdot (d \cdot x + c) / d + 3 \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} / d - (121 \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} + 201 \cdot e^{(-4 \cdot d \cdot x - 4 \cdot c)} + 147 \cdot e^{(-6 \cdot d \cdot x - 6 \cdot c)} + 3) / (d \cdot (e^{(-2 \cdot d \cdot x - 2 \cdot c)} + 3 \cdot e^{(-4 \cdot d \cdot x - 4 \cdot c)} + 3 \cdot e^{(-6 \cdot d \cdot x - 6 \cdot c)} + e^{(-8 \cdot d \cdot x - 8 \cdot c)}))) - \frac{3}{8} \cdot a \cdot b^2 \cdot (12 \cdot (d \cdot x + c) / d + e^{(-2 \cdot d \cdot x - 2 \cdot c)} / d - (17 \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} + 1) / (d \cdot (e^{(-2 \cdot d \cdot x - 2 \cdot c)} + e^{(-4 \cdot d \cdot x - 4 \cdot c)})))$

mupad [B] time = 0.29, size = 243, normalized size = 3.12

$$\frac{e^{2c+2dx} (a+b)^3}{8d} - \frac{2(b^3+3ab^2)}{3d} + \frac{2e^{2c+2dx}(b^3+ab^2)}{d} - \frac{2(b^3+ab^2)}{d(e^{2c+2dx}+1)} - \frac{e^{-2c-2dx}(a+b)^3}{8d} - \frac{2(b^3+ab^2)}{3e^{2c+2dx}+3e^{4c+4dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(c+d*x)^2*(a+b*tanh(c+d*x))^3,x)`

[Out] $(\exp(2 \cdot c + 2 \cdot d \cdot x) \cdot (a + b)^3) / (8 \cdot d) - ((2 \cdot (3 \cdot a \cdot b^2 + b^3)) / (3 \cdot d) + (2 \cdot \exp(2 \cdot c + 2 \cdot d \cdot x) \cdot (a \cdot b^2 + b^3)) / d) / (2 \cdot \exp(2 \cdot c + 2 \cdot d \cdot x) + \exp(4 \cdot c + 4 \cdot d \cdot x) + 1) - (2 \cdot (a \cdot b^2 + b^3)) / (d \cdot (\exp(2 \cdot c + 2 \cdot d \cdot x) + 1)) - (\exp(-2 \cdot c - 2 \cdot d \cdot x) \cdot (a + b)^3) / (8 \cdot d) - ((2 \cdot (a \cdot b^2 + b^3)) / d + (4 \cdot \exp(2 \cdot c + 2 \cdot d \cdot x) \cdot (3 \cdot a \cdot b^2 + b^3)) / (3 \cdot d) + (2 \cdot \exp(4 \cdot c + 4 \cdot d \cdot x) \cdot (a \cdot b^2 + b^3)) / d) / (3 \cdot \exp(2 \cdot c + 2 \cdot d \cdot x) + 3 \cdot \exp(4 \cdot c + 4 \cdot d \cdot x) + \exp(6 \cdot c + 6 \cdot d \cdot x) + 1) + (x \cdot (a + b)^2 \cdot (a - 5 \cdot b)) / 2$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^3 \cosh^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)`

[Out] `Integral((a + b*tanh(c + d*x)**2)**3*cosh(c + d*x)**2, x)`

3.100 $\int \cosh(c + dx) (a + b \tanh^2(c + dx))^3 dx$

Optimal. Leaf size=99

$$\frac{3b^2(4a + 3b) \tanh(c + dx) \operatorname{sech}(c + dx)}{8d} + \frac{(a + b)^3 \sinh(c + dx)}{d} - \frac{3b(4(a + b)^2 + (2a + b)^2) \tan^{-1}(\sinh(c + dx))}{8d}$$

[Out] $-3/8*b*(4*(a+b)^2+(2*a+b)^2)*\arctan(\sinh(d*x+c))/d+(a+b)^3*\sinh(d*x+c)/d+3/8*b^2*(4*a+3*b)*\operatorname{sech}(d*x+c)*\tanh(d*x+c)/d-1/4*b^3*\operatorname{sech}(d*x+c)^3*\tanh(d*x+c)/d$

Rubi [A] time = 0.12, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3676, 390, 1157, 385, 203}

$$\frac{3b^2(4a + 3b) \tanh(c + dx) \operatorname{sech}(c + dx)}{8d} + \frac{(a + b)^3 \sinh(c + dx)}{d} - \frac{3b(4(a + b)^2 + (2a + b)^2) \tan^{-1}(\sinh(c + dx))}{8d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] $(-3*b*(4*(a + b)^2 + (2*a + b)^2)*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/(8*d) + ((a + b)^3*\operatorname{Sinh}[c + d*x])/d + (3*b^2*(4*a + 3*b)*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x])/(8*d) - (b^3*\operatorname{Sech}[c + d*x]^3*\operatorname{Tanh}[c + d*x])/(4*d)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 1157

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.))^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned} \int \cosh(c + dx) (a + b \tanh^2(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{(a+(b)x^2)^3}{(1+x^2)^3} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left((a+b)^3 - \frac{b(3a^2+3ab+b^2)+3b(a+b)(2a+b)x^2+3b(a+b)^2x^4}{(1+x^2)^3}\right) dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{(a+b)^3 \sinh(c + dx)}{d} - \frac{\text{Subst}\left(\int \frac{b(3a^2+3ab+b^2)+3b(a+b)(2a+b)x^2+3b(a+b)^2x^4}{(1+x^2)^3} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{(a+b)^3 \sinh(c + dx)}{d} - \frac{b^3 \text{sech}^3(c + dx) \tanh(c + dx)}{4d} + \frac{\text{Subst}\left(\int \frac{b(3a^2+3ab+b^2)+3b(a+b)(2a+b)x^2+3b(a+b)^2x^4}{(1+x^2)^3} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{(a+b)^3 \sinh(c + dx)}{d} + \frac{3b^2(4a + 3b) \text{sech}(c + dx) \tanh(c + dx)}{8d} \\ &= -\frac{3b(4(a+b)^2 + (2a+b)^2) \tan^{-1}(\sinh(c + dx))}{8d} + \frac{(a+b)^3 \sinh(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.42, size = 89, normalized size = 0.90

$$\frac{-3b(8a^2 + 12ab + 5b^2) \tan^{-1}(\sinh(c + dx)) + 3b^2(4a + 3b) \tanh(c + dx) \text{sech}(c + dx) + 8(a + b)^3 \sinh(c + dx)}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3, x]

[Out] (-3*b*(8*a^2 + 12*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]] + 8*(a + b)^3*Sinh[c + d*x] + 3*b^2*(4*a + 3*b)*Sech[c + d*x]*Tanh[c + d*x] - 2*b^3*Sech[c + d*x]^3*Tanh[c + d*x])/(8*d)

fricas [B] time = 0.45, size = 2411, normalized size = 24.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/4*(2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 + 20*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^10 + 3*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 3*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3 + 30*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 24*(10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + (4*a^3 + 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + (420*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*a^3 + 12*a^2*b

```

+ 24*a*b^2 + 5*b^3 + 84*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^6 + 6*(84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5
+ 28*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (4*a^3 + 12*a^2
*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - (4*a^3 + 12*a^2*b +
24*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + (420*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c
osh(d*x + c)^6 + 210*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^4 -
4*a^3 - 12*a^2*b - 24*a*b^2 - 5*b^3 + 15*(4*a^3 + 12*a^2*b + 24*a*b^2 + 5*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(60*(a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*cosh(d*x + c)^7 + 42*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^
5 + 5*(4*a^3 + 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^3 - (4*a^3 + 12*a
^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*a^3 - 6*a^2*b -
6*a*b^2 - 2*b^3 - 3*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^2 +
3*(30*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 28*(2*a^3 + 6*a^2*
b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 5*(4*a^3 + 12*a^2*b + 24*a*b^2 + 5*
b^3)*cosh(d*x + c)^4 - 2*a^3 - 6*a^2*b - 10*a*b^2 - 5*b^3 - 2*(4*a^3 + 12*a
^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*((8*a^2*b + 1
2*a*b^2 + 5*b^3)*cosh(d*x + c)^9 + 9*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x
+ c)*sinh(d*x + c)^8 + (8*a^2*b + 12*a*b^2 + 5*b^3)*sinh(d*x + c)^9 + 4*(8*
a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 4*(8*a^2*b + 12*a*b^2 + 5*b^3 +
9*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 28*(3*(8
*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (8*a^2*b + 12*a*b^2 + 5*b^3)*c
osh(d*x + c))*sinh(d*x + c)^6 + 6*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c
)^5 + 6*(21*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 8*a^2*b + 12*a*b
^2 + 5*b^3 + 14*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^5 + 2*(63*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 70*(8*a^2*b + 12*
a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 15*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x +
c))*sinh(d*x + c)^4 + 4*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 4*(
21*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 35*(8*a^2*b + 12*a*b^2 +
5*b^3)*cosh(d*x + c)^4 + 8*a^2*b + 12*a*b^2 + 5*b^3 + 15*(8*a^2*b + 12*a*b^
2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 12*(3*(8*a^2*b + 12*a*b^2 + 5
*b^3)*cosh(d*x + c)^7 + 7*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 5*
(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (8*a^2*b + 12*a*b^2 + 5*b^3)
*cosh(d*x + c))*sinh(d*x + c)^2 + (8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c
) + (9*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 28*(8*a^2*b + 12*a*b^
2 + 5*b^3)*cosh(d*x + c)^6 + 30*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^
4 + 8*a^2*b + 12*a*b^2 + 5*b^3 + 12*(8*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x +
c)^2)*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + 2*(10*(a^3 +
3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 12*(2*a^3 + 6*a^2*b + 10*a*b^2 +
5*b^3)*cosh(d*x + c)^7 + 3*(4*a^3 + 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x
+ c)^5 - 2*(4*a^3 + 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^3 - 3*(2*a^3
+ 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x +
c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + d*sinh(d*x + c)^9 + 4*d*cosh(d*x
+ c)^7 + 4*(9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^7 + 28*(3*d*cosh(d*x +
c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^6 + 6*d*cosh(d*x + c)^5 + 6*(21*d*cos
h(d*x + c)^4 + 14*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^5 + 2*(63*d*cosh(d*x
+ c)^5 + 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*d*
cosh(d*x + c)^3 + 4*(21*d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 + 15*d*cos
h(d*x + c)^2 + d)*sinh(d*x + c)^3 + 12*(3*d*cosh(d*x + c)^7 + 7*d*cosh(d*x
+ c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*
x + c) + (9*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4
+ 12*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))

```

giac [B] time = 0.38, size = 257, normalized size = 2.60

$$3(8a^2be^c + 12ab^2e^c + 5b^3e^c) \arctan(e^{(dx+c)})e^{(-c)} + 2(a^3 + 3a^2b + 3ab^2 + b^3)e^{(-dx-c)} - 2(a^3e^{(dx+12c)} + 3a^2be^c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out]
$$\frac{-1/4*(3*(8*a^2*b*e^c + 12*a*b^2*e^c + 5*b^3*e^c)*\arctan(e^{(d*x + c)})*e^{-c} + 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^{(-d*x - c)} - 2*(a^3*e^{(d*x + 12*c)} + 3*a^2*b*e^{(d*x + 12*c)} + 3*a*b^2*e^{(d*x + 12*c)} + b^3*e^{(d*x + 12*c)})*e^{(-11*c)} - (12*a*b^2*e^{(7*d*x + 7*c)} + 9*b^3*e^{(7*d*x + 7*c)} + 12*a*b^2*e^{(5*d*x + 5*c)} + b^3*e^{(5*d*x + 5*c)} - 12*a*b^2*e^{(3*d*x + 3*c)} - b^3*e^{(3*d*x + 3*c)} - 12*a*b^2*e^{(d*x + c)} - 9*b^3*e^{(d*x + c)})/(e^{(2*d*x + 2*c)} + 1)^4}{d}$$

maple [B] time = 0.40, size = 257, normalized size = 2.60

$$\frac{a^3 \sinh(dx + c)}{d} + \frac{3a^2b \sinh(dx + c)}{d} - \frac{6a^2b \arctan(e^{dx+c})}{d} + \frac{3ab^2 (\sinh^3(dx + c))}{d \cosh(dx + c)^2} + \frac{9ab^2 \sinh(dx + c)}{d \cosh(dx + c)^2} - \frac{9ab^2 \sinh^3(dx + c)}{d \cosh(dx + c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x)`

[Out]
$$a^3*\sinh(d*x+c)/d+3/d*a^2*b*\sinh(d*x+c)-6/d*a^2*b*\arctan(\exp(d*x+c))+3/d*a*b^2*\sinh(d*x+c)^3/\cosh(d*x+c)^2+9/d*a*b^2*\sinh(d*x+c)/\cosh(d*x+c)^2-9/2/d*a*b^2*\operatorname{sech}(d*x+c)*\tanh(d*x+c)-9/d*a*b^2*\arctan(\exp(d*x+c))+1/d*b^3*\sinh(d*x+c)^5/\cosh(d*x+c)^4+5/d*b^3*\sinh(d*x+c)^3/\cosh(d*x+c)^4+5/d*b^3*\sinh(d*x+c)/\cosh(d*x+c)^4-5/4/d*b^3*\tanh(d*x+c)*\operatorname{sech}(d*x+c)^3-15/8/d*b^3*\operatorname{sech}(d*x+c)*\tanh(d*x+c)-15/4/d*b^3*\arctan(\exp(d*x+c))$$

maxima [B] time = 0.70, size = 295, normalized size = 2.98

$$\frac{1}{4}b^3\left(\frac{15 \arctan(e^{-dx-c})}{d} - \frac{2e^{-dx-c}}{d} + \frac{17e^{-2dx-2c} + 13e^{-4dx-4c} + 7e^{-6dx-6c} - 7e^{-8dx-8c} + 2}{d(e^{-dx-c} + 4e^{-3dx-3c} + 6e^{-5dx-5c} + 4e^{-7dx-7c} + e^{-9dx-9c})}\right) + \frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")`

[Out]
$$\frac{1}{4}b^3*\left(\frac{15*\arctan(e^{(-d*x - c)})}{d} - \frac{2*e^{(-d*x - c)}}{d} + \frac{(17*e^{(-2*d*x - 2*c)} + 13*e^{(-4*d*x - 4*c)} + 7*e^{(-6*d*x - 6*c)} - 7*e^{(-8*d*x - 8*c)} + 2)/(d*(e^{(-d*x - c)} + 4*e^{(-3*d*x - 3*c)} + 6*e^{(-5*d*x - 5*c)} + 4*e^{(-7*d*x - 7*c)} + e^{(-9*d*x - 9*c)}))}{d} - \frac{e^{(-d*x - c)}}{d} + \frac{(4*e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)} + 1)/(d*(e^{(-d*x - c)} + 2*e^{(-3*d*x - 3*c)} + e^{(-5*d*x - 5*c)}))}{d} + \frac{3/2*a*b^2*(6*\arctan(e^{(-d*x - c)})/d - e^{(-d*x - c)}/d + (4*e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)} + 1)/(d*(e^{(-d*x - c)} + 2*e^{(-3*d*x - 3*c)} + e^{(-5*d*x - 5*c)})))}{d} + \frac{3/2*a^2*b*(4*\arctan(e^{(-d*x - c)})/d + e^{(-d*x - c)}/d - e^{(-d*x - c)}/d) + a^3*\sinh(d*x + c)/d}{d}\right)$$

mupad [B] time = 0.31, size = 355, normalized size = 3.59

$$\frac{e^{c+dx} (a+b)^3}{2d} - \frac{e^{-c-dx} (a+b)^3}{2d} - \frac{3 \operatorname{atan}\left(\frac{e^{dx} e^c (5b^3 \sqrt{d^2} + 12ab^2 \sqrt{d^2} + 8a^2 b \sqrt{d^2})}{d \sqrt{64a^4 b^2 + 192a^3 b^3 + 224a^2 b^4 + 120ab^5 + 25b^6}}\right)}{4\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^3,x)`

[Out]
$$\frac{(\exp(c + d*x)*(a + b)^3)/(2*d) - (\exp(-c - d*x)*(a + b)^3)/(2*d) - (3*\operatorname{atan}((\exp(d*x)*\exp(c)*(5*b^3*(d^2)^{(1/2)} + 12*a*b^2*(d^2)^{(1/2)} + 8*a^2*b*(d^2)^{(1/2)}))/d*(120*a*b^5 + 25*b^6 + 224*a^2*b^4 + 192*a^3*b^3 + 64*a^4*b^2)^{(1/2)}))*(120*a*b^5 + 25*b^6 + 224*a^2*b^4 + 192*a^3*b^3 + 64*a^4*b^2)^{(1/2)})/(4*(d^2)^{(1/2)}) + (3*\exp(c + d*x)*(4*a*b^2 + 3*b^3))/(4*d*(\exp(2*c + 2*d*x) + 1)) + (6*b^3*\exp(c + d*x))/(d*(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1)) - (\exp(c + d*x)*(12*a*b^2 + 13*b^3))/(2*d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1)) - (4*b^3*\exp(c + d*x))/(d*(4*\exp(2*c + 2*d*x) + 6*\exp(4*c + 4*d*x) + 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1))}{d}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^3 \cosh(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*cosh(c + d*x), x)
```

3.101 $\int \operatorname{sech}(c + dx) \left(a + b \tanh^2(c + dx) \right)^3 dx$

Optimal. Leaf size=149

$$\frac{(2a + b)(8a^2 + 8ab + 5b^2) \tan^{-1}(\sinh(c + dx))}{16d} - \frac{b(44a^2 + 44ab + 15b^2) \tanh(c + dx) \operatorname{sech}(c + dx)}{48d} - \frac{b \tanh(c + dx) \operatorname{sech}^3(c + dx)}{6d}$$

[Out] 1/16*(2*a+b)*(8*a^2+8*a*b+5*b^2)*arctan(sinh(d*x+c))/d-1/48*b*(44*a^2+44*a*b+15*b^2)*sech(d*x+c)*tanh(d*x+c)/d-5/24*b*(2*a+b)*sech(d*x+c)^3*(a+(a+b)*sinh(d*x+c)^2)*tanh(d*x+c)/d-1/6*b*sech(d*x+c)^5*(a+(a+b)*sinh(d*x+c)^2)^2*tanh(d*x+c)/d

Rubi [A] time = 0.15, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3676, 413, 526, 385, 203}

$$\frac{(2a + b)(8a^2 + 8ab + 5b^2) \tan^{-1}(\sinh(c + dx))}{16d} - \frac{b(44a^2 + 44ab + 15b^2) \tanh(c + dx) \operatorname{sech}(c + dx)}{48d} - \frac{b \tanh(c + dx) \operatorname{sech}^3(c + dx)}{6d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] ((2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]])/(16*d) - (b*(44*a^2 + 44*a*b + 15*b^2)*Sech[c + d*x]*Tanh[c + d*x])/(48*d) - (5*b*(2*a + b)*Sech[c + d*x]^3*(a + (a + b)*Sinh[c + d*x]^2)*Tanh[c + d*x])/(24*d) - (b*Sech[c + d*x]^5*(a + (a + b)*Sinh[c + d*x]^2)^2*Tanh[c + d*x])/(6*d)

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 413

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p + 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 526

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && GtQ[q, 0]

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \operatorname{sech}(c + dx) (a + b \tanh^2(c + dx))^3 dx = \frac{\operatorname{Subst}\left(\int \frac{(a+(a+b)x^2)^3}{(1+x^2)^4} dx, x, \sinh(c + dx)\right)}{d}$$

$$= -\frac{b \operatorname{sech}^5(c + dx) (a + (a + b) \sinh^2(c + dx))^2 \tanh(c + dx)}{6d} + \frac{\operatorname{Subst}\left(\int \frac{(a+(a+b)x^2)^3}{(1+x^2)^4} dx, x, \sinh(c + dx)\right)}{d}$$

$$= -\frac{5b(2a + b) \operatorname{sech}^3(c + dx) (a + (a + b) \sinh^2(c + dx)) \tanh(c + dx)}{24d} + \frac{\operatorname{Subst}\left(\int \frac{(a+(a+b)x^2)^3}{(1+x^2)^4} dx, x, \sinh(c + dx)\right)}{d}$$

$$= -\frac{b(44a^2 + 44ab + 15b^2) \operatorname{sech}(c + dx) \tanh(c + dx)}{48d} - \frac{5b(2a + b) \operatorname{sech}^3(c + dx) (a + (a + b) \sinh^2(c + dx)) \tanh(c + dx)}{24d} + \frac{\operatorname{Subst}\left(\int \frac{(a+(a+b)x^2)^3}{(1+x^2)^4} dx, x, \sinh(c + dx)\right)}{d}$$

$$= \frac{(2a + b)(8a^2 + 8ab + 5b^2) \tan^{-1}(\sinh(c + dx))}{16d} - \frac{b(44a^2 + 44ab + 15b^2) \operatorname{sech}(c + dx) \tanh(c + dx)}{48d} - \frac{5b(2a + b) \operatorname{sech}^3(c + dx) (a + (a + b) \sinh^2(c + dx)) \tanh(c + dx)}{24d}$$

Mathematica [C] time = 16.68, size = 1341, normalized size = 9.00

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Sinh[c + d*x]*((9514449*(a + b))/a + (135323370*(a + b)^2)/a^2 + (5800
9455*(a + b)^3)/a^3 + 4093425*Csch[c + d*x]^2 + (168951510*(a + b)*Csch[c +
d*x]^2)/a + (215549775*(a + b)^2*Csch[c + d*x]^2)/a^2 + 70189350*Csch[c +
d*x]^4 + (274542345*(a + b)*Csch[c + d*x]^4)/a + 117228825*Csch[c + d*x]^6
+ (7808535*(a + b)^2*Sinh[c + d*x]^2)/a^2 + (36772890*(a + b)^3*Sinh[c + d*
x]^2)/a^3 - 75520*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Si
nh[c + d*x]^2]*Sinh[c + d*x]^2 - 13824*HypergeometricPFQ[{3/2, 2, 2, 2, 2,
2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^2 - 1024*Hypergeome
tricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*S
inh[c + d*x]^2 + (2160711*(a + b)^3*Sinh[c + d*x]^4)/a^3 - (189696*(a + b)*
HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sin
h[c + d*x]^4)/a - (38400*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1
, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4)/a - (3072*(a + b)*Hype
rgeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x
]^2]*Sinh[c + d*x]^4)/a - (158976*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2, 2
, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6)/a^2 - (35328*(a +
b)^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c +
d*x]^2]*Sinh[c + d*x]^6)/a^2 - (3072*(a + b)^2*HypergeometricPFQ[{3/2, 2,
2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6)/a
^2 - (44800*(a + b)^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 11/2},
-Sinh[c + d*x]^2]*Sinh[c + d*x]^8)/a^3 - (10752*(a + b)^3*HypergeometricPF
Q[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x
^8)/a^3 - (1024*(a + b)^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1,
```


$$\begin{aligned}
& 1, 1, 1, 11/2\}, -\operatorname{Sinh}[c + d*x]^2 * \operatorname{Sinh}[c + d*x]^8 / a^3 + (142065 * \operatorname{ArcTanh}[\operatorname{Sqrt}[-\operatorname{Sinh}[c + d*x]^2]] * \operatorname{Sinh}[c + d*x]^8) / (-\operatorname{Sinh}[c + d*x]^2)^{9/2} + (1172288 \\
& 25 * \operatorname{ArcTanh}[\operatorname{Sqrt}[-\operatorname{Sinh}[c + d*x]^2]]) / (-\operatorname{Sinh}[c + d*x]^2)^{7/2} + (17069535 * \operatorname{ArcTanh}[\operatorname{Sqrt}[-\operatorname{Sinh}[c + d*x]^2]] * \operatorname{Sinh}[c + d*x]^4) / (-\operatorname{Sinh}[c + d*x]^2)^{7/2} + (\\
& 33756345 * (a + b)^2 * \operatorname{ArcTanh}[\operatorname{Sqrt}[-\operatorname{Sinh}[c + d*x]^2]] * \operatorname{Sinh}[c + d*x]^8) / (a^2 * (-\operatorname{Sinh}[c + d*x]^2)^{7/2}) + (56109375 * (a + b)^3 * \operatorname{ArcTanh}[\operatorname{Sqrt}[-\operatorname{Sinh}[c + d*x]^2] \\
&]) * \operatorname{Sinh}[c + d*x]^8) / (a^3 * (-\operatorname{Sinh}[c + d*x]^2)^{7/2}) - (109265625 * \operatorname{ArcTanh}[\operatorname{Sqrt}[-\operatorname{Sinh}[c + d*x]^2]]) / (-\operatorname{Sinh}[c + d*x]^2)^{5/2} - (274542345 * (a + b) * \operatorname{ArcTanh}[\operatorname{Sqrt}[-\operatorname{Sinh}[c + d*x]^2] \\
&]) / (a * (-\operatorname{Sinh}[c + d*x]^2)^{5/2}) + (260465625 * (a + b) * \operatorname{ArcTanh}[\operatorname{Sqrt}[-\operatorname{Sinh}[c + d*x]^2]]) / (a * (-\operatorname{Sinh}[c + d*x]^2)^{3/2}) + (215549775 * (a + b)^2 * \operatorname{ArcTanh}[\operatorname{Sqrt}[-\operatorname{Sinh}[c + d*x]^2] \\
&]) / (a^2 * (-\operatorname{Sinh}[c + d*x]^2)^{3/2}) + (174825 * (a + b)^2 * \operatorname{ArcTanh}[\operatorname{Sqrt}[-\operatorname{Sinh}[c + d*x]^2]] * \operatorname{Sinh}[c + d*x]^6) / (a^2 * (-\operatorname{Sinh}[c + d*x]^2)^{3/2}) + (9261945 * (a + b)^3 * \operatorname{ArcTanh}[\operatorname{Sqrt}[-\operatorname{Sinh}[c + d*x]^2] \\
&]) * \operatorname{Sinh}[c + d*x]^6) / (a^3 * (-\operatorname{Sinh}[c + d*x]^2)^{3/2}) + (48825 * (a + b)^3 * \operatorname{ArcTanh}[\operatorname{Sqrt}[-\operatorname{Sinh}[c + d*x]^2]] * \operatorname{Sinh}[c + d*x]^8) / (a^3 * (-\operatorname{Sinh}[c + d*x]^2)^{3/2}) \\
& - (41427855 * (a + b) * \operatorname{ArcTanh}[\operatorname{Sqrt}[-\operatorname{Sinh}[c + d*x]^2]]) / (a * \operatorname{Sqrt}[-\operatorname{Sinh}[c + d*x]^2]) - (207173295 * (a + b)^2 * \operatorname{ArcTanh}[\operatorname{Sqrt}[-\operatorname{Sinh}[c + d*x]^2]]) / (a^2 * \operatorname{Sqrt}[-\operatorname{Sinh}[c + d*x]^2]) - (58009455 * (a + b)^3 * \operatorname{ArcTanh}[\operatorname{Sqrt}[-\operatorname{Sinh}[c + d*x]^2]]) / (a^3 * \\
& \operatorname{Sqrt}[-\operatorname{Sinh}[c + d*x]^2]) + (210735 * (a + b) * \operatorname{ArcTanh}[\operatorname{Sqrt}[-\operatorname{Sinh}[c + d*x]^2]] * \operatorname{Sqrt}[-\operatorname{Sinh}[c + d*x]^2]) / a) / (725760 * d)
\end{aligned}$$

fricas [B] time = 0.45, size = 3465, normalized size = 23.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c))^3,x, algorithm="fricas")

[Out]
$$\begin{aligned}
& -1/24*(3*(24*a^2*b + 30*a*b^2 + 11*b^3)*\cosh(d*x + c)^{11} + 33*(24*a^2*b + 30*a*b^2 + 11*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^{10} + 3*(24*a^2*b + 30*a*b^2 + \\
& 11*b^3)*\sinh(d*x + c)^{11} + (216*a^2*b + 126*a*b^2 - 5*b^3)*\cosh(d*x + c)^9 \\
& + (216*a^2*b + 126*a*b^2 - 5*b^3 + 165*(24*a^2*b + 30*a*b^2 + 11*b^3)*\cosh \\
& (d*x + c)^2)*\sinh(d*x + c)^9 + 9*(55*(24*a^2*b + 30*a*b^2 + 11*b^3)*\cosh(d* \\
& x + c)^3 + (216*a^2*b + 126*a*b^2 - 5*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^8 + \\
& 18*(8*a^2*b + 2*a*b^2 + 5*b^3)*\cosh(d*x + c)^7 + 18*(55*(24*a^2*b + 30*a*b \\
& ^2 + 11*b^3)*\cosh(d*x + c)^4 + 8*a^2*b + 2*a*b^2 + 5*b^3 + 2*(216*a^2*b + 1 \\
& 26*a*b^2 - 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^7 + 42*(33*(24*a^2*b + 30* \\
& a*b^2 + 11*b^3)*\cosh(d*x + c)^5 + 2*(216*a^2*b + 126*a*b^2 - 5*b^3)*\cosh(d* \\
& x + c)^3 + 3*(8*a^2*b + 2*a*b^2 + 5*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^6 - 1 \\
& 8*(8*a^2*b + 2*a*b^2 + 5*b^3)*\cosh(d*x + c)^5 + 18*(77*(24*a^2*b + 30*a*b^2 \\
& + 11*b^3)*\cosh(d*x + c)^6 + 7*(216*a^2*b + 126*a*b^2 - 5*b^3)*\cosh(d*x + c \\
&)^4 - 8*a^2*b - 2*a*b^2 - 5*b^3 + 21*(8*a^2*b + 2*a*b^2 + 5*b^3)*\cosh(d*x + \\
& c)^2)*\sinh(d*x + c)^5 + 18*(55*(24*a^2*b + 30*a*b^2 + 11*b^3)*\cosh(d*x + c \\
&)^7 + 7*(216*a^2*b + 126*a*b^2 - 5*b^3)*\cosh(d*x + c)^5 + 35*(8*a^2*b + 2*a \\
& *b^2 + 5*b^3)*\cosh(d*x + c)^3 - 5*(8*a^2*b + 2*a*b^2 + 5*b^3)*\cosh(d*x + c \\
&)*\sinh(d*x + c)^4 - (216*a^2*b + 126*a*b^2 - 5*b^3)*\cosh(d*x + c)^3 + (495* \\
& (24*a^2*b + 30*a*b^2 + 11*b^3)*\cosh(d*x + c)^8 + 84*(216*a^2*b + 126*a*b^2 \\
& - 5*b^3)*\cosh(d*x + c)^6 + 630*(8*a^2*b + 2*a*b^2 + 5*b^3)*\cosh(d*x + c)^4 \\
& - 216*a^2*b - 126*a*b^2 + 5*b^3 - 180*(8*a^2*b + 2*a*b^2 + 5*b^3)*\cosh(d*x \\
& + c)^2)*\sinh(d*x + c)^3 + 3*(55*(24*a^2*b + 30*a*b^2 + 11*b^3)*\cosh(d*x + c \\
&)^9 + 12*(216*a^2*b + 126*a*b^2 - 5*b^3)*\cosh(d*x + c)^7 + 126*(8*a^2*b + 2 \\
& *a*b^2 + 5*b^3)*\cosh(d*x + c)^5 - 60*(8*a^2*b + 2*a*b^2 + 5*b^3)*\cosh(d*x + \\
& c)^3 - (216*a^2*b + 126*a*b^2 - 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 - 3* \\
& ((16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^{12} + 12*(16*a^3 + 24* \\
& a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^{11} + (16*a^3 + 24*a^2 \\
& *b + 18*a*b^2 + 5*b^3)*\sinh(d*x + c)^{12} + 6*(16*a^3 + 24*a^2*b + 18*a*b^2 + \\
& 5*b^3)*\cosh(d*x + c)^{10} + 6*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 + 11*(16 \\
& *a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{10} + 20* \\
& (11*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^3 + 3*(16*a^3 + 24 \\
& *a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^9 + 15*(16*a^3 + 24
\end{aligned}$$

$$\begin{aligned}
& *a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^8 + 15*(33*(16*a^3 + 24*a^2*b + 18* \\
& *a*b^2 + 5*b^3)*\cosh(d*x + c)^4 + 16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 + 18 \\
& *(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + \\
& 24*(33*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^5 + 30*(16*a^3 \\
& + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^3 + 5*(16*a^3 + 24*a^2*b + 18* \\
& a*b^2 + 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 20*(16*a^3 + 24*a^2*b + 18* \\
& a*b^2 + 5*b^3)*\cosh(d*x + c)^6 + 4*(231*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b \\
& ^3)*\cosh(d*x + c)^6 + 315*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + \\
& c)^4 + 80*a^3 + 120*a^2*b + 90*a*b^2 + 25*b^3 + 105*(16*a^3 + 24*a^2*b + 1 \\
& 8*a*b^2 + 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 24*(33*(16*a^3 + 24*a^2 \\
& *b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^7 + 63*(16*a^3 + 24*a^2*b + 18*a*b^2 + \\
& 5*b^3)*\cosh(d*x + c)^5 + 35*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d* \\
& x + c)^3 + 5*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c))*\sinh(d*x \\
& + c)^5 + 15*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^4 + 15*(3 \\
& 3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^8 + 84*(16*a^3 + 24* \\
& a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^6 + 70*(16*a^3 + 24*a^2*b + 18*a*b^ \\
& 2 + 5*b^3)*\cosh(d*x + c)^4 + 16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 + 20*(16* \\
& a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 20*(1 \\
& 1*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^9 + 36*(16*a^3 + 24* \\
& a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^7 + 42*(16*a^3 + 24*a^2*b + 18*a*b^ \\
& 2 + 5*b^3)*\cosh(d*x + c)^5 + 20*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh \\
& (d*x + c)^3 + 3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c))*\sinh(\\
& d*x + c)^3 + 16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3 + 6*(16*a^3 + 24*a^2*b + \\
& 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^2 + 6*(11*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5 \\
& *b^3)*\cosh(d*x + c)^10 + 45*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x \\
& + c)^8 + 70*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^6 + 50*(1 \\
& 6*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^4 + 16*a^3 + 24*a^2*b + \\
& 18*a*b^2 + 5*b^3 + 15*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^ \\
& 2)*\sinh(d*x + c)^2 + 12*((16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + \\
& c)^11 + 5*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^9 + 10*(16*a \\
& ^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^7 + 10*(16*a^3 + 24*a^2*b + \\
& 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^5 + 5*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^ \\
& 3)*\cosh(d*x + c)^3 + (16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*\cosh(d*x + c))* \\
& \sinh(d*x + c))*\arctan(\cosh(d*x + c) + \sinh(d*x + c)) - 3*(24*a^2*b + 30*a*b \\
& ^2 + 11*b^3)*\cosh(d*x + c) + 3*(11*(24*a^2*b + 30*a*b^2 + 11*b^3)*\cosh(d*x \\
& + c)^10 + 3*(216*a^2*b + 126*a*b^2 - 5*b^3)*\cosh(d*x + c)^8 + 42*(8*a^2*b + \\
& 2*a*b^2 + 5*b^3)*\cosh(d*x + c)^6 - 30*(8*a^2*b + 2*a*b^2 + 5*b^3)*\cosh(d*x \\
& + c)^4 - 24*a^2*b - 30*a*b^2 - 11*b^3 - (216*a^2*b + 126*a*b^2 - 5*b^3)*\co \\
& sh(d*x + c)^2)*\sinh(d*x + c))/(d*\cosh(d*x + c)^12 + 12*d*\cosh(d*x + c)*\sinh \\
& (d*x + c)^11 + d*\sinh(d*x + c)^12 + 6*d*\cosh(d*x + c)^10 + 6*(11*d*\cosh(d*x \\
& + c)^2 + d)*\sinh(d*x + c)^10 + 20*(11*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c \\
&))*\sinh(d*x + c)^9 + 15*d*\cosh(d*x + c)^8 + 15*(33*d*\cosh(d*x + c)^4 + 18*d \\
& *\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^8 + 24*(33*d*\cosh(d*x + c)^5 + 30*d*\cos \\
& h(d*x + c)^3 + 5*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 20*d*\cosh(d*x + c)^6 + \\
& 4*(231*d*\cosh(d*x + c)^6 + 315*d*\cosh(d*x + c)^4 + 105*d*\cosh(d*x + c)^2 + \\
& 5*d)*\sinh(d*x + c)^6 + 24*(33*d*\cosh(d*x + c)^7 + 63*d*\cosh(d*x + c)^5 + 35 \\
& *d*\cosh(d*x + c)^3 + 5*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 15*d*\cosh(d*x + c \\
&)^4 + 15*(33*d*\cosh(d*x + c)^8 + 84*d*\cosh(d*x + c)^6 + 70*d*\cosh(d*x + c)^ \\
& 4 + 20*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^4 + 20*(11*d*\cosh(d*x + c)^9 + \\
& 36*d*\cosh(d*x + c)^7 + 42*d*\cosh(d*x + c)^5 + 20*d*\cosh(d*x + c)^3 + 3*d*\co \\
& sh(d*x + c))*\sinh(d*x + c)^3 + 6*d*\cosh(d*x + c)^2 + 6*(11*d*\cosh(d*x + c)^ \\
& 10 + 45*d*\cosh(d*x + c)^8 + 70*d*\cosh(d*x + c)^6 + 50*d*\cosh(d*x + c)^4 + 1 \\
& 5*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^2 + 12*(d*\cosh(d*x + c)^11 + 5*d*\cos \\
& h(d*x + c)^9 + 10*d*\cosh(d*x + c)^7 + 10*d*\cosh(d*x + c)^5 + 5*d*\cosh(d*x + \\
& c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c) + d)
\end{aligned}$$

giac [B] time = 0.31, size = 321, normalized size = 2.15

$$3(16a^3e^c + 24a^2be^c + 18ab^2e^c + 5b^3e^c) \arctan(e^{(dx+c)})e^{(-c)} - \frac{72a^2be^{(11dx+11c)} + 90ab^2e^{(11dx+11c)} + 33b^3e^{(11dx+11c)} + 216a^3e^{(11dx+11c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

$$\begin{aligned} [Out] & \frac{1}{24} * (3 * (16 * a^3 * e^c + 24 * a^2 * b * e^c + 18 * a * b^2 * e^c + 5 * b^3 * e^c) * \arctan(e^{(d * x + c)}) * e^{(-c)} - (72 * a^2 * b * e^{(11 * d * x + 11 * c)} + 90 * a * b^2 * e^{(11 * d * x + 11 * c)} + \\ & 33 * b^3 * e^{(11 * d * x + 11 * c)} + 216 * a^2 * b * e^{(9 * d * x + 9 * c)} + 126 * a * b^2 * e^{(9 * d * x + 9 * c)} - 5 * b^3 * e^{(9 * d * x + 9 * c)} + 144 * a^2 * b * e^{(7 * d * x + 7 * c)} + 36 * a * b^2 * e^{(7 * d * x + 7 * c)} + \\ & 90 * b^3 * e^{(7 * d * x + 7 * c)} - 144 * a^2 * b * e^{(5 * d * x + 5 * c)} - 36 * a * b^2 * e^{(5 * d * x + 5 * c)} - 90 * b^3 * e^{(5 * d * x + 5 * c)} - 216 * a^2 * b * e^{(3 * d * x + 3 * c)} - 126 * \\ & a * b^2 * e^{(3 * d * x + 3 * c)} + 5 * b^3 * e^{(3 * d * x + 3 * c)} - 72 * a^2 * b * e^{(d * x + c)} - 90 * a * b^2 * e^{(d * x + c)} - 33 * b^3 * e^{(d * x + c)}) / (e^{(2 * d * x + 2 * c)} + 1)^6 / d \end{aligned}$$

maple [B] time = 0.42, size = 334, normalized size = 2.24

$$\frac{2a^3 \arctan(e^{dx+c})}{d} - \frac{3a^2b \sinh(dx+c)}{d \cosh(dx+c)^2} + \frac{3a^2b \operatorname{sech}(dx+c) \tanh(dx+c)}{2d} + \frac{3a^2b \arctan(e^{dx+c})}{d} - \frac{3ab^2 (\sinh^3(dx+c))}{d \cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x)

$$\begin{aligned} [Out] & \frac{2}{d} * a^3 * \arctan(\exp(d * x + c)) - \frac{3}{d} * a^2 * b * \sinh(d * x + c) / \cosh(d * x + c)^2 + \frac{3}{2} * \frac{2}{d} * a^2 * b * \operatorname{sech}(d * x + c) * \tanh(d * x + c) + \frac{3}{d} * a^2 * b * \arctan(\exp(d * x + c)) - \frac{3}{d} * a * b^2 * \sinh(d * x + c)^3 / \cosh(d * x + c)^4 - \frac{3}{d} * a * b^2 * \sinh(d * x + c) / \cosh(d * x + c)^4 + \frac{3}{4} * \frac{3}{d} * a * b^2 * \tanh(d * x + c) * \operatorname{sech}(d * x + c)^3 + \frac{9}{8} * \frac{3}{d} * a * b^2 * \operatorname{sech}(d * x + c) * \tanh(d * x + c) + \frac{9}{4} * \frac{3}{d} * a * b^2 * \arctan(\exp(d * x + c)) - \frac{1}{d} * b^3 * \sinh(d * x + c)^5 / \cosh(d * x + c)^6 - \frac{5}{3} * \frac{3}{d} * b^3 * \sinh(d * x + c)^3 / \cosh(d * x + c)^6 - \frac{1}{d} * b^3 * \sinh(d * x + c) / \cosh(d * x + c)^6 + \frac{1}{6} * \frac{3}{d} * b^3 * \tanh(d * x + c) * \operatorname{sech}(d * x + c)^5 + \frac{5}{24} * \frac{3}{d} * b^3 * \tanh(d * x + c) * \operatorname{sech}(d * x + c)^3 + \frac{5}{16} * \frac{3}{d} * b^3 * \operatorname{sech}(d * x + c) * \tanh(d * x + c) + \frac{5}{8} * \frac{3}{d} * b^3 * \arctan(\exp(d * x + c)) \end{aligned}$$

maxima [B] time = 0.43, size = 362, normalized size = 2.43

$$-\frac{1}{24} b^3 \left(\frac{15 \arctan(e^{(-dx-c)})}{d} + \frac{33e^{(-dx-c)} - 5e^{(-3dx-3c)} + 90e^{(-5dx-5c)} - 90e^{(-7dx-7c)} + 5e^{(-9dx-9c)} - 33e^{(-11dx-11c)}}{d(6e^{(-2dx-2c)} + 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} + 15e^{(-8dx-8c)} + 6e^{(-10dx-10c)} + e^{(-12dx-12c)} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

$$\begin{aligned} [Out] & -\frac{1}{24} * b^3 * (15 * \arctan(e^{(-d * x - c)}) / d + (33 * e^{(-d * x - c)} - 5 * e^{(-3 * d * x - 3 * c)} + 90 * e^{(-5 * d * x - 5 * c)} - 90 * e^{(-7 * d * x - 7 * c)} + 5 * e^{(-9 * d * x - 9 * c)} - 33 * e^{(-11 * d * x - 11 * c)}) / (d * (6 * e^{(-2 * d * x - 2 * c)} + 15 * e^{(-4 * d * x - 4 * c)} + 20 * e^{(-6 * d * x - 6 * c)} + 15 * e^{(-8 * d * x - 8 * c)} + 6 * e^{(-10 * d * x - 10 * c)} + e^{(-12 * d * x - 12 * c)} + 1))) - \frac{3}{4} * a * b^2 * (3 * \arctan(e^{(-d * x - c)}) / d + (5 * e^{(-d * x - c)} - 3 * e^{(-3 * d * x - 3 * c)} + 3 * e^{(-5 * d * x - 5 * c)} - 5 * e^{(-7 * d * x - 7 * c)}) / (d * (4 * e^{(-2 * d * x - 2 * c)} + 6 * e^{(-4 * d * x - 4 * c)} + 4 * e^{(-6 * d * x - 6 * c)} + e^{(-8 * d * x - 8 * c)} + 1))) - 3 * a^2 * b * (\arctan(e^{(-d * x - c)}) / d + (e^{(-d * x - c)} - e^{(-3 * d * x - 3 * c)}) / (d * (2 * e^{(-2 * d * x - 2 * c)} + e^{(-4 * d * x - 4 * c)} + 1)))) + a^3 * \arctan(\sinh(d * x + c)) / d \end{aligned}$$

mupad [B] time = 1.39, size = 535, normalized size = 3.59

$$\frac{\operatorname{atan}\left(\frac{e^{dx} e^c (16a^3 \sqrt{d^2} + 5b^3 \sqrt{d^2} + 18a^2 b \sqrt{d^2} + 24a^2 b \sqrt{d^2})}{d \sqrt{256a^6 + 768a^5 b + 1152a^4 b^2 + 1024a^3 b^3 + 564a^2 b^4 + 180a b^5 + 25b^6}}\right)}{8 \sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*tanh(c + d*x)^2)^3/cosh(c + d*x),x)`

[Out]
$$\frac{\operatorname{atan}\left(\frac{\exp(dx)\exp(c)\left(16a^3(d^2)^{1/2} + 5b^3(d^2)^{1/2} + 18ab^2(d^2)^{1/2} + 24a^2b(d^2)^{1/2}\right)}{d\left(180a^5b + 768a^5b + 256a^6 + 25b^6 + 564a^2b^4 + 1024a^3b^3 + 1152a^4b^2\right)^{1/2}}\right)\left(180a^5b + 768a^5b + 256a^6 + 25b^6 + 564a^2b^4 + 1024a^3b^3 + 1152a^4b^2\right)^{1/2}}{8(d^2)^{1/2}} - \frac{\exp(c + dx)(54ab^2 + 55b^3)}{3d(3\exp(2c + 2dx) + 3\exp(4c + 4dx) + \exp(6c + 6dx) + 1)} - \frac{80b^3\exp(c + dx)}{3d(5\exp(2c + 2dx) + 10\exp(4c + 4dx) + 10\exp(6c + 6dx) + 5\exp(8c + 8dx) + \exp(10c + 10dx) + 1)} + \frac{6\exp(c + dx)(2ab^2 + 5b^3)}{d(4\exp(2c + 2dx) + 6\exp(4c + 4dx) + 4\exp(6c + 6dx) + \exp(8c + 8dx) + 1)} + \frac{32b^3\exp(c + dx)}{3d(6\exp(2c + 2dx) + 15\exp(4c + 4dx) + 20\exp(6c + 6dx) + 15\exp(8c + 8dx) + 6\exp(10c + 10dx) + \exp(12c + 12dx) + 1)} - \frac{\exp(c + dx)(30ab^2 + 24a^2b + 11b^3)}{8d(\exp(2c + 2dx) + 1)} + \frac{\exp(c + dx)(162ab^2 + 72a^2b + 85b^3)}{12d(2\exp(2c + 2dx) + \exp(4c + 4dx) + 1)}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^3 \operatorname{sech}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)`

[Out] `Integral((a + b*tanh(c + d*x)**2)**3*sech(c + d*x), x)`

3.102 $\int \operatorname{sech}^2(c + dx) \left(a + b \tanh^2(c + dx) \right)^3 dx$

Optimal. Leaf size=67

$$\frac{a^3 \tanh(c + dx)}{d} + \frac{a^2 b \tanh^3(c + dx)}{d} + \frac{3ab^2 \tanh^5(c + dx)}{5d} + \frac{b^3 \tanh^7(c + dx)}{7d}$$

[Out] $a^3 \tanh(d*x+c)/d + a^2*b*\tanh(d*x+c)^3/d + 3/5*a*b^2*\tanh(d*x+c)^5/d + 1/7*b^3*\tanh(d*x+c)^7/d$

Rubi [A] time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3675, 194}

$$\frac{a^2 b \tanh^3(c + dx)}{d} + \frac{a^3 \tanh(c + dx)}{d} + \frac{3ab^2 \tanh^5(c + dx)}{5d} + \frac{b^3 \tanh^7(c + dx)}{7d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3, x]

[Out] $(a^3*\tanh[c + d*x])/d + (a^2*b*\tanh[c + d*x]^3)/d + (3*a*b^2*\tanh[c + d*x]^5)/(5*d) + (b^3*\tanh[c + d*x]^7)/(7*d)$

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m-1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2-1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^2(c + dx) \left(a + b \tanh^2(c + dx) \right)^3 dx &= \frac{\operatorname{Subst}\left(\int (a + bx^2)^3 dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int (a^3 + 3a^2bx^2 + 3ab^2x^4 + b^3x^6) dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{a^3 \tanh(c + dx)}{d} + \frac{a^2 b \tanh^3(c + dx)}{d} + \frac{3ab^2 \tanh^5(c + dx)}{5d} + \frac{b^3 \tanh^7(c + dx)}{7d} \end{aligned}$$

Mathematica [A] time = 0.18, size = 67, normalized size = 1.00

$$\frac{a^3 \tanh(c + dx)}{d} + \frac{a^2 b \tanh^3(c + dx)}{d} + \frac{3ab^2 \tanh^5(c + dx)}{5d} + \frac{b^3 \tanh^7(c + dx)}{7d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3, x]

[Out] $(a^3 \operatorname{Tanh}[c + d*x])/d + (a^2*b \operatorname{Tanh}[c + d*x]^3)/d + (3*a*b^2 \operatorname{Tanh}[c + d*x]^5)/(5*d) + (b^3 \operatorname{Tanh}[c + d*x]^7)/(7*d)$

fricas [B] time = 0.40, size = 786, normalized size = 11.73

$$\frac{4 \left((35 a^3 + 70 a^2 b + 63 a b^2 + 20 b^3) \cosh(dx + c)^6 + 6 (35 a^2 b + 42 a b^2 + 15 b^3) \cosh(dx + c) \sinh(dx + c)^5 + 35 (d \cosh(dx + c) \sinh(dx + c))^5 \right)}{35 (d \cosh(dx + c) \sinh(dx + c))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")`

[Out] $-4/35 * ((35*a^3 + 70*a^2*b + 63*a*b^2 + 20*b^3) * \cosh(d*x + c)^6 + 6 * (35*a^2*b + 42*a*b^2 + 15*b^3) * \cosh(d*x + c) * \sinh(d*x + c)^5 + (35*a^3 + 70*a^2*b + 63*a*b^2 + 20*b^3) * \sinh(d*x + c)^6 + 14 * (15*a^3 + 20*a^2*b + 9*a*b^2) * \cosh(d*x + c)^4 + (210*a^3 + 280*a^2*b + 126*a*b^2 + 15 * (35*a^3 + 70*a^2*b + 63*a*b^2 + 20*b^3) * \cosh(d*x + c)^2) * \sinh(d*x + c)^4 + 4 * (5 * (35*a^2*b + 42*a*b^2 + 15*b^3) * \cosh(d*x + c)^3 + 28 * (5*a^2*b + 3*a*b^2) * \cosh(d*x + c)) * \sinh(d*x + c)^3 + 350*a^3 + 280*a^2*b + 210*a*b^2 + 7 * (75*a^3 + 70*a^2*b + 39*a*b^2 + 20*b^3) * \cosh(d*x + c)^2 + (15 * (35*a^3 + 70*a^2*b + 63*a*b^2 + 20*b^3) * \cosh(d*x + c)^4 + 525*a^3 + 490*a^2*b + 273*a*b^2 + 140*b^3 + 84 * (15*a^3 + 20*a^2*b + 9*a*b^2) * \cosh(d*x + c)^2) * \sinh(d*x + c)^2 + 2 * (3 * (35*a^2*b + 42*a*b^2 + 15*b^3) * \cosh(d*x + c)^5 + 56 * (5*a^2*b + 3*a*b^2) * \cosh(d*x + c)^3 + 7 * (25*a^2*b + 6*a*b^2 + 5*b^3) * \cosh(d*x + c)) * \sinh(d*x + c)) / (d * \cosh(d*x + c)^8 + 8 * d * \cosh(d*x + c) * \sinh(d*x + c)^7 + d * \sinh(d*x + c)^8 + 8 * d * \cosh(d*x + c)^6 + 4 * (7 * d * \cosh(d*x + c)^2 + 2 * d) * \sinh(d*x + c)^6 + 4 * (14 * d * \cosh(d*x + c)^3 + 9 * d * \cosh(d*x + c)) * \sinh(d*x + c)^5 + 28 * d * \cosh(d*x + c)^4 + 2 * (35 * d * \cosh(d*x + c)^4 + 60 * d * \cosh(d*x + c)^2 + 14 * d) * \sinh(d*x + c)^4 + 8 * (7 * d * \cosh(d*x + c)^5 + 15 * d * \cosh(d*x + c)^3 + 7 * d * \cosh(d*x + c)) * \sinh(d*x + c)^3 + 56 * d * \cosh(d*x + c)^2 + 4 * (7 * d * \cosh(d*x + c)^6 + 30 * d * \cosh(d*x + c)^4 + 42 * d * \cosh(d*x + c)^2 + 14 * d) * \sinh(d*x + c)^2 + 4 * (2 * d * \cosh(d*x + c)^7 + 9 * d * \cosh(d*x + c)^5 + 14 * d * \cosh(d*x + c)^3 + 7 * d * \cosh(d*x + c)) * \sinh(d*x + c) + 35 * d)$

giac [B] time = 0.39, size = 347, normalized size = 5.18

$$\frac{2 \left(35 a^3 e^{(12 dx + 12 c)} + 105 a^2 b e^{(12 dx + 12 c)} + 105 a b^2 e^{(12 dx + 12 c)} + 35 b^3 e^{(12 dx + 12 c)} + 210 a^3 e^{(10 dx + 10 c)} + 420 a^2 b e^{(10 dx + 10 c)} \right)}{d \left(e^{(2 dx + 2 c)} + 1 \right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")`

[Out] $-2/35 * (35*a^3*e^{(12*d*x + 12*c)} + 105*a^2*b*e^{(12*d*x + 12*c)} + 105*a*b^2*e^{(12*d*x + 12*c)} + 35*b^3*e^{(12*d*x + 12*c)} + 210*a^3*e^{(10*d*x + 10*c)} + 420*a^2*b*e^{(10*d*x + 10*c)} + 210*a*b^2*e^{(10*d*x + 10*c)} + 525*a^3*e^{(8*d*x + 8*c)} + 665*a^2*b*e^{(8*d*x + 8*c)} + 315*a*b^2*e^{(8*d*x + 8*c)} + 175*b^3*e^{(8*d*x + 8*c)} + 700*a^3*e^{(6*d*x + 6*c)} + 560*a^2*b*e^{(6*d*x + 6*c)} + 420*a*b^2*e^{(6*d*x + 6*c)} + 525*a^3*e^{(4*d*x + 4*c)} + 315*a^2*b*e^{(4*d*x + 4*c)} + 231*a*b^2*e^{(4*d*x + 4*c)} + 105*b^3*e^{(4*d*x + 4*c)} + 210*a^3*e^{(2*d*x + 2*c)} + 140*a^2*b*e^{(2*d*x + 2*c)} + 42*a*b^2*e^{(2*d*x + 2*c)} + 35*a^3 + 35*a^2*b + 21*a*b^2 + 5*b^3) / (d * (e^{(2*d*x + 2*c)} + 1)^7)$

maple [B] time = 0.55, size = 227, normalized size = 3.39

$$a^3 \tanh(dx + c) + 3a^2b \left(-\frac{\sinh(dx+c)}{2 \cosh(dx+c)^3} + \frac{\left(\frac{2}{3} + \frac{\operatorname{sech}(dx+c)^2}{3}\right) \tanh(dx+c)}{2} \right) + 3ab^2 \left(-\frac{\sinh^3(dx+c)}{2 \cosh(dx+c)^5} - \frac{3 \sinh(dx+c)}{8 \cosh(dx+c)^5} + \frac{3 \left(\frac{8}{15} + \frac{\operatorname{sech}(dx+c)}{5}\right)}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^2*(a+b*tanh(d*x+c))^3,x)

[Out] $\frac{1}{d} \left(a^3 \tanh(d*x+c) + 3a^2b \left(-\frac{1}{2} \frac{\sinh(d*x+c)}{\cosh(d*x+c)^3} + \frac{1}{2} \left(\frac{2}{3} + \frac{1}{3} \operatorname{sech}(d*x+c)^2 \right) \tanh(d*x+c) \right) + 3ab^2 \left(-\frac{1}{2} \frac{\sinh(d*x+c)^3}{\cosh(d*x+c)^5} - \frac{3}{8} \frac{\sinh(d*x+c)}{\cosh(d*x+c)^5} + \frac{3}{8} \left(\frac{8}{15} + \frac{1}{5} \operatorname{sech}(d*x+c)^4 + \frac{4}{15} \operatorname{sech}(d*x+c)^2 \right) \tanh(d*x+c) \right) + b^3 \left(-\frac{1}{2} \frac{\sinh(d*x+c)^5}{\cosh(d*x+c)^7} - \frac{5}{8} \frac{\sinh(d*x+c)^3}{\cosh(d*x+c)^7} - \frac{5}{16} \frac{\sinh(d*x+c)}{\cosh(d*x+c)^7} + \frac{5}{16} \left(\frac{16}{35} + \frac{1}{7} \operatorname{sech}(d*x+c)^6 + \frac{6}{35} \operatorname{sech}(d*x+c)^4 + \frac{8}{35} \operatorname{sech}(d*x+c)^2 \right) \tanh(d*x+c) \right) \right)$

maxima [A] time = 0.33, size = 71, normalized size = 1.06

$$\frac{b^3 \tanh(dx+c)^7}{7d} + \frac{3ab^2 \tanh(dx+c)^5}{5d} + \frac{a^2b \tanh(dx+c)^3}{d} + \frac{2a^3}{d(e^{(-2dx-2c)}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c))^3,x, algorithm="maxima")

[Out] $\frac{1}{7} b^3 \tanh(d*x+c)^7/d + \frac{3}{5} a b^2 \tanh(d*x+c)^5/d + a^2 b \tanh(d*x+c)^3/d + 2a^3/(d(e^{-2d*x-2c}+1))$

mupad [B] time = 1.33, size = 1050, normalized size = 15.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tanh(c + d*x))^2)^3/cosh(c + d*x)^2,x)

[Out] $-\left(\frac{(2(3ab^2 - 3a^2b + 5a^3 - 5b^3))}{(35d)} + \frac{(2\exp(6c + 6d*x)(a+b)^3)}{(7d)} - \frac{(6\exp(2c + 2d*x)(ab^2 + a^2b - 5a^3 - 5b^3))}{(35d)} + \frac{(6\exp(4c + 4d*x)(a+b)^2(a-b))}{(7d)} \right) / \left(4\exp(2c + 2d*x) + 6\exp(4c + 4d*x) + 4\exp(6c + 6d*x) + \exp(8c + 8d*x) + 1 \right) - \left(\frac{(2(a+b)^3)}{(7d)} + \frac{(2\exp(12c + 12d*x)(a+b)^3)}{(7d)} - \frac{(6\exp(4c + 4d*x)(ab^2 + a^2b - 5a^3 - 5b^3))}{(7d)} - \frac{(6\exp(8c + 8d*x)(ab^2 + a^2b - 5a^3 - 5b^3))}{(7d)} + \frac{(8\exp(6c + 6d*x)(3ab^2 - 3a^2b + 5a^3 - 5b^3))}{(7d)} + \frac{(12\exp(2c + 2d*x)(a+b)^2(a-b))}{(7d)} + \frac{(12\exp(10c + 10d*x)(a+b)^2(a-b))}{(7d)} \right) / \left(7\exp(2c + 2d*x) + 21\exp(4c + 4d*x) + 35\exp(6c + 6d*x) + 35\exp(8c + 8d*x) + 21\exp(10c + 10d*x) + 7\exp(12c + 12d*x) + \exp(14c + 14d*x) + 1 \right) - \left(\frac{(2\exp(4c + 4d*x)(a+b)^3)}{(7d)} - \frac{(2(ab^2 + a^2b - 5a^3 - 5b^3))}{(35d)} + \frac{(4\exp(2c + 2d*x)(a+b)^2(a-b))}{(7d)} \right) / \left(3\exp(2c + 2d*x) + 3\exp(4c + 4d*x) + \exp(6c + 6d*x) + 1 \right) - \left(\frac{(2(a+b)^2(a-b))}{(7d)} + \frac{(2\exp(2c + 2d*x)(a+b)^3)}{(7d)} \right) / \left(2\exp(2c + 2d*x) + \exp(4c + 4d*x) + 1 \right) - \left(\frac{(2\exp(8c + 8d*x)(a+b)^3)}{(7d)} - \frac{(2(ab^2 + a^2b - 5a^3 - 5b^3))}{(35d)} - \frac{(12\exp(4c + 4d*x)(ab^2 + a^2b - 5a^3 - 5b^3))}{(35d)} + \frac{(8\exp(2c + 2d*x)(3ab^2 - 3a^2b + 5a^3 - 5b^3))}{(35d)} + \frac{(8\exp(6c + 6d*x)(a+b)^2(a-b))}{(7d)} \right) / \left(5\exp(2c + 2d*x) + 10\exp(4c + 4d*x) + 10\exp(6c + 6d*x) + 5\exp(8c + 8d*x) + \exp(10c + 10d*x) + 1 \right) - \left(\frac{(2(a+b)^2(a-b))}{(7d)} + \frac{(2\exp(10c + 10d*x)(a+b)^3)}{(7d)} - \frac{(2\exp(2c + 2d*x)(ab^2 + a^2b - 5a^3 - 5b^3))}{(7d)} - \frac{(4\exp(6c + 6d*x)(ab^2 + a^2b - 5a^3 - 5b^3))}{(7d)} + \frac{(4\exp(4c + 4d*x)(3ab^2 - 3a^2b + 5a^3 - 5b^3))}{(7d)} + \frac{(10\exp(8c + 8d*x)(a+b)^2(a-b))}{(7d)} \right) / \left(6\exp(2c + 2d*x) + 15\exp(4c + 4d*x) + 20\exp(6c + 6d*x) + 15\exp(8c + 8d*x) + 6\exp(10c + 10d*x) + \exp(12c + 12d*x) + 1 \right) - \frac{(2(a+b)^3)}{(7d(\exp(2c + 2d*x) + 1))}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^3 \operatorname{sech}^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*sech(c + d*x)**2, x)
```


3.103 $\int \operatorname{sech}^3(c + dx) \left(a + b \tanh^2(c + dx) \right)^3 dx$

Optimal. Leaf size=198

$$\frac{b(72a^2 + 52ab + 15b^2) \tanh(c + dx) \operatorname{sech}^3(c + dx)}{192d} + \frac{(64a^3 + 48a^2b + 24ab^2 + 5b^3) \tan^{-1}(\sinh(c + dx))}{128d} + \frac{(64a^3 + 48a^2b + 24ab^2 + 5b^3) \tan^{-1}(\sinh(c + dx))}{128d}$$

[Out] 1/128*(64*a^3+48*a^2*b+24*a*b^2+5*b^3)*arctan(sinh(d*x+c))/d+1/128*(64*a^3+48*a^2*b+24*a*b^2+5*b^3)*sech(d*x+c)*tanh(d*x+c)/d-1/192*b*(72*a^2+52*a*b+15*b^2)*sech(d*x+c)^3*tanh(d*x+c)/d-1/48*b*(12*a+5*b)*sech(d*x+c)^5*(a+(a+b)*sinh(d*x+c)^2)*tanh(d*x+c)/d-1/8*b*sech(d*x+c)^7*(a+(a+b)*sinh(d*x+c)^2)^2*tanh(d*x+c)/d

Rubi [A] time = 0.24, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3676, 413, 526, 385, 199, 203}

$$\frac{(48a^2b + 64a^3 + 24ab^2 + 5b^3) \tan^{-1}(\sinh(c + dx))}{128d} - \frac{b(72a^2 + 52ab + 15b^2) \tanh(c + dx) \operatorname{sech}^3(c + dx)}{192d} + \frac{(48a^2b + 64a^3 + 24ab^2 + 5b^3) \tan^{-1}(\sinh(c + dx))}{128d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3, x]

[Out] ((64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*ArcTan[Sinh[c + d*x]])/(128*d) + ((64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*Sech[c + d*x]*Tanh[c + d*x])/(128*d) - (b*(72*a^2 + 52*a*b + 15*b^2)*Sech[c + d*x]^3*Tanh[c + d*x])/(192*d) - (b*(12*a + 5*b)*Sech[c + d*x]^5*(a + (a + b)*Sinh[c + d*x]^2)*Tanh[c + d*x])/(48*d) - (b*Sech[c + d*x]^7*(a + (a + b)*Sinh[c + d*x]^2)^2*Tanh[c + d*x])/(8*d)

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 413

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p + 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p + 1) + 1)), x], x]

+ q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 526

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && GtQ[q, 0]

Rule 3676

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^3(c + dx) (a + b \tanh^2(c + dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+(a+b)x^2)^3}{(1+x^2)^5} dx, x, \sinh(c + dx)\right)}{d} \\ &= -\frac{b \operatorname{sech}^7(c + dx) (a + (a + b) \sinh^2(c + dx))^2 \tanh(c + dx)}{8d} + \frac{\operatorname{Subst}\left(\int \frac{(a+(a+b)x^2)^2}{(1+x^2)^5} dx, x, \sinh(c + dx)\right)}{d} \\ &= -\frac{b(12a + 5b) \operatorname{sech}^5(c + dx) (a + (a + b) \sinh^2(c + dx)) \tanh(c + dx)}{48d} + \frac{\operatorname{Subst}\left(\int \frac{(a+(a+b)x^2)}{(1+x^2)^5} dx, x, \sinh(c + dx)\right)}{d} \\ &= -\frac{b(72a^2 + 52ab + 15b^2) \operatorname{sech}^3(c + dx) \tanh(c + dx)}{192d} - \frac{b(12a + 5b)}{d} \\ &= \frac{(64a^3 + 48a^2b + 24ab^2 + 5b^3) \operatorname{sech}(c + dx) \tanh(c + dx)}{128d} - \frac{b(72a^2 + 52ab + 15b^2)}{d} \\ &= \frac{(64a^3 + 48a^2b + 24ab^2 + 5b^3) \tan^{-1}(\sinh(c + dx))}{128d} + \frac{(64a^3 + 48a^2b + 24ab^2 + 5b^3)}{d} \end{aligned}$$

Mathematica [A] time = 11.84, size = 158, normalized size = 0.80

$$\frac{-2b(144a^2 + 168ab + 59b^2) \tanh(c + dx) \operatorname{sech}^3(c + dx) + 6(64a^3 + 48a^2b + 24ab^2 + 5b^3) \tan^{-1}\left(\tanh\left(\frac{1}{2}(c + dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (6*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*ArcTan[Tanh[(c + d*x)/2]] + 3*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*Sech[c + d*x]*Tanh[c + d*x] - 2*b*(144*a^2 + 168*a*b + 59*b^2)*Sech[c + d*x]^3*Tanh[c + d*x] + 8*b^2*(24*a + 17*b)*Sech[c + d*x]^5*Tanh[c + d*x] - 48*b^3*Sech[c + d*x]^7*Tanh[c + d*x])/(384*d)

fricas [B] time = 1.18, size = 6114, normalized size = 30.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & 1/192*(3*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^{15} + 45*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^{14} + 3*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\sinh(d*x + c)^{15} + (960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*\cosh(d*x + c)^{13} + (960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3 + 315*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{13} + 13*(105*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^3 + (960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^{12} + (1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*\cosh(d*x + c)^{11} + (4095*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^4 + 1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3 + 78*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{11} + 11*(819*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^5 + 26*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*\cosh(d*x + c)^3 + (1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^{10} + (960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*\cosh(d*x + c)^9 + (15015*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^6 + 715*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*\cosh(d*x + c)^4 + 960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3 + 55*(1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^9 + 3*(6435*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^7 + 429*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*\cosh(d*x + c)^5 + 55*(1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*\cosh(d*x + c)^3 + 3*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^8 - (960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*\cosh(d*x + c)^7 + (19305*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^8 + 1716*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*\cosh(d*x + c)^6 + 330*(1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*\cosh(d*x + c)^4 - 960*a^3 + 1584*a^2*b - 744*a*b^2 + 1765*b^3 + 36*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^7 + (15015*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^9 + 1716*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*\cosh(d*x + c)^7 + 462*(1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*\cosh(d*x + c)^5 + 84*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*\cosh(d*x + c)^3 - 7*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^6 - (1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*\cosh(d*x + c)^5 + (9009*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^{10} + 1287*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*\cosh(d*x + c)^8 + 462*(1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*\cosh(d*x + c)^6 + 126*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*\cosh(d*x + c)^4 - 1728*a^3 + 2160*a^2*b + 312*a*b^2 - 895*b^3 - 21*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^5 + (4095*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^{11} + 715*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*\cosh(d*x + c)^9 + 330*(1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*\cosh(d*x + c)^7 + 126*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*\cosh(d*x + c)^5 - 35*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*\cosh(d*x + c)^3 - 5*(1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^4 - (960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*\cosh(d*x + c)^3 + (1365*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^{12} + 286*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*\cosh(d*x + c)^{10} + 165*(1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*\cosh(d*x + c)^8 + 84*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*\cosh(d*x + c)^6 - 35*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3)*\cosh(d*x + c)^4 - 960*a^3 + 432*a^2*b + 984*a*b^2 + 397*b^3 - 10*(1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^3 + (315*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^{13} + 78*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*\cosh(d*x + c)^{11} + 55*(1728*a^3 - 2160*a^2*b - 312$$

$$\begin{aligned}
& *a*b^2 + 895*b^3)*\cosh(d*x + c)^9 + 36*(960*a^3 - 1584*a^2*b + 744*a*b^2 - \\
& 1765*b^3)*\cosh(d*x + c)^7 - 21*(960*a^3 - 1584*a^2*b + 744*a*b^2 - 1765*b^3) \\
&)*\cosh(d*x + c)^5 - 10*(1728*a^3 - 2160*a^2*b - 312*a*b^2 + 895*b^3)*\cosh(d \\
& *x + c)^3 - 3*(960*a^3 - 432*a^2*b - 984*a*b^2 - 397*b^3)*\cosh(d*x + c))*\text{si} \\
& \text{nh}(d*x + c)^2 + 3*((64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^16 \\
& + 16*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^15 \\
& + (64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\sinh(d*x + c)^16 + 8*(64*a^3 + 48* \\
& a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^14 + 8*(64*a^3 + 48*a^2*b + 24*a*b^ \\
& 2 + 5*b^3 + 15*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^2)*\sinh \\
& (d*x + c)^14 + 112*(5*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^ \\
& 3 + (64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^13 \\
& + 28*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^12 + 28*(65*(64*a \\
& ^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^4 + 64*a^3 + 48*a^2*b + 24* \\
& a*b^2 + 5*b^3 + 26*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^2)* \\
& \sinh(d*x + c)^12 + 112*(39*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x \\
& + c)^5 + 26*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^3 + 3*(64* \\
& a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^11 + 56*(64 \\
& *a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^10 + 56*(143*(64*a^3 + 48 \\
& *a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^6 + 143*(64*a^3 + 48*a^2*b + 24*a* \\
& b^2 + 5*b^3)*\cosh(d*x + c)^4 + 64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3 + 33*(6 \\
& 4*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^10 + 16 \\
& *(715*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^7 + 1001*(64*a^3 \\
& + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^5 + 385*(64*a^3 + 48*a^2*b + \\
& 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^3 + 35*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^ \\
& 3)*\cosh(d*x + c))*\sinh(d*x + c)^9 + 70*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^ \\
& 3)*\cosh(d*x + c)^8 + 2*(6435*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d* \\
& x + c)^8 + 12012*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^6 + 6 \\
& 930*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^4 + 2240*a^3 + 168 \\
& 0*a^2*b + 840*a*b^2 + 175*b^3 + 1260*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3) \\
& *\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 16*(715*(64*a^3 + 48*a^2*b + 24*a*b^2 + \\
& 5*b^3)*\cosh(d*x + c)^9 + 1716*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(\\
& d*x + c)^7 + 1386*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^5 + \\
& 420*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^3 + 35*(64*a^3 + 4 \\
& 8*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 56*(64*a^3 + 4 \\
& 8*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^6 + 56*(143*(64*a^3 + 48*a^2*b + \\
& 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^10 + 429*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5* \\
& b^3)*\cosh(d*x + c)^8 + 462*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x \\
& + c)^6 + 210*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^4 + 64*a^ \\
& 3 + 48*a^2*b + 24*a*b^2 + 5*b^3 + 35*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3) \\
& *\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 112*(39*(64*a^3 + 48*a^2*b + 24*a*b^2 + \\
& 5*b^3)*\cosh(d*x + c)^11 + 143*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(\\
& d*x + c)^9 + 198*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^7 + 1 \\
& 26*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^5 + 35*(64*a^3 + 48 \\
& *a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^3 + 3*(64*a^3 + 48*a^2*b + 24*a*b^ \\
& 2 + 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 28*(64*a^3 + 48*a^2*b + 24*a*b^ \\
& 2 + 5*b^3)*\cosh(d*x + c)^4 + 28*(65*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)* \\
& \cosh(d*x + c)^12 + 286*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c) \\
& ^10 + 495*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^8 + 420*(64* \\
& a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^6 + 175*(64*a^3 + 48*a^2*b \\
& + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^4 + 64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3 \\
& + 30*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c) \\
& ^4 + 112*(5*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^13 + 26*(6 \\
& 4*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^11 + 55*(64*a^3 + 48*a^2 \\
& *b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^9 + 60*(64*a^3 + 48*a^2*b + 24*a*b^2 + \\
& 5*b^3)*\cosh(d*x + c)^7 + 35*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d* \\
& x + c)^5 + 10*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^3 + (64* \\
& a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 64*a^3 \\
& + 48*a^2*b + 24*a*b^2 + 5*b^3 + 8*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\text{co} \\
& \text{sh}(d*x + c)^2 + 8*(15*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*\cosh(d*x + c)^
\end{aligned}$$

$$\begin{aligned}
& 14 + 91(64a^3 + 48a^2b + 24ab^2 + 5b^3)\cosh(dx + c)^{12} + 231(64a^3 + 48a^2b + 24ab^2 + 5b^3)\cosh(dx + c)^{10} + 315(64a^3 + 48a^2b + 24ab^2 + 5b^3)\cosh(dx + c)^8 + 245(64a^3 + 48a^2b + 24ab^2 + 5b^3)\cosh(dx + c)^6 + 105(64a^3 + 48a^2b + 24ab^2 + 5b^3)\cosh(dx + c)^4 + 64a^3 + 48a^2b + 24ab^2 + 5b^3 + 21(64a^3 + 48a^2b + 24ab^2 + 5b^3)\cosh(dx + c)^2) \sinh(dx + c)^2 + 16((64a^3 + 48a^2b + 24ab^2 + 5b^3)\cosh(dx + c)^{15} + 7(64a^3 + 48a^2b + 24ab^2 + 5b^3)\cosh(dx + c)^{13} + 21(64a^3 + 48a^2b + 24ab^2 + 5b^3)\cosh(dx + c)^{11} + 35(64a^3 + 48a^2b + 24ab^2 + 5b^3)\cosh(dx + c)^9 + 35(64a^3 + 48a^2b + 24ab^2 + 5b^3)\cosh(dx + c)^7 + 21(64a^3 + 48a^2b + 24ab^2 + 5b^3)\cosh(dx + c)^5 + 7(64a^3 + 48a^2b + 24ab^2 + 5b^3)\cosh(dx + c)^3 + (64a^3 + 48a^2b + 24ab^2 + 5b^3)\cosh(dx + c)) \sinh(dx + c) \arctan(\cosh(dx + c) + \sinh(dx + c)) - 3(64a^3 + 48a^2b + 24ab^2 + 5b^3)\cosh(dx + c) + (45(64a^3 + 48a^2b + 24ab^2 + 5b^3)\cosh(dx + c)^{14} + 13(960a^3 - 432a^2b - 984ab^2 - 397b^3)\cosh(dx + c)^{12} + 11(1728a^3 - 2160a^2b - 312ab^2 + 895b^3)\cosh(dx + c)^{10} + 9(960a^3 - 1584a^2b + 744ab^2 - 1765b^3)\cosh(dx + c)^8 - 7(960a^3 - 1584a^2b + 744ab^2 - 1765b^3)\cosh(dx + c)^6 - 5(1728a^3 - 2160a^2b - 312ab^2 + 895b^3)\cosh(dx + c)^4 - 192a^3 - 144a^2b - 72ab^2 - 15b^3 - 3(960a^3 - 432a^2b - 984ab^2 - 397b^3)\cosh(dx + c)^2) \sinh(dx + c)) / (d \cosh(dx + c)^{16} + 16d \cosh(dx + c) \sinh(dx + c)^{15} + d \sinh(dx + c)^{16} + 8d \cosh(dx + c)^{14} + 8(15d \cosh(dx + c)^2 + d) \sinh(dx + c)^{14} + 112(5d \cosh(dx + c)^3 + d \cosh(dx + c)) \sinh(dx + c)^{13} + 28d \cosh(dx + c)^{12} + 28(65d \cosh(dx + c)^4 + 26d \cosh(dx + c)^2 + d) \sinh(dx + c)^{12} + 112(39d \cosh(dx + c)^5 + 26d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^{11} + 56d \cosh(dx + c)^{10} + 56(143d \cosh(dx + c)^6 + 143d \cosh(dx + c)^4 + 33d \cosh(dx + c)^2 + d) \sinh(dx + c)^{10} + 16(715d \cosh(dx + c)^7 + 1001d \cosh(dx + c)^5 + 385d \cosh(dx + c)^3 + 35d \cosh(dx + c)) \sinh(dx + c)^9 + 70d \cosh(dx + c)^8 + 2(6435d \cosh(dx + c)^8 + 12012d \cosh(dx + c)^6 + 6930d \cosh(dx + c)^4 + 1260d \cosh(dx + c)^2 + 35d) \sinh(dx + c)^8 + 16(715d \cosh(dx + c)^9 + 1716d \cosh(dx + c)^7 + 1386d \cosh(dx + c)^5 + 420d \cosh(dx + c)^3 + 35d \cosh(dx + c)) \sinh(dx + c)^7 + 56d \cosh(dx + c)^6 + 56(143d \cosh(dx + c)^{10} + 429d \cosh(dx + c)^8 + 462d \cosh(dx + c)^6 + 210d \cosh(dx + c)^4 + 35d \cosh(dx + c)^2 + d) \sinh(dx + c)^6 + 112(39d \cosh(dx + c)^{11} + 143d \cosh(dx + c)^9 + 198d \cosh(dx + c)^7 + 126d \cosh(dx + c)^5 + 35d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^5 + 28d \cosh(dx + c)^4 + 28(65d \cosh(dx + c)^{12} + 286d \cosh(dx + c)^{10} + 495d \cosh(dx + c)^8 + 420d \cosh(dx + c)^6 + 175d \cosh(dx + c)^4 + 30d \cosh(dx + c)^2 + d) \sinh(dx + c)^4 + 112(5d \cosh(dx + c)^{13} + 26d \cosh(dx + c)^{11} + 55d \cosh(dx + c)^9 + 60d \cosh(dx + c)^7 + 35d \cosh(dx + c)^5 + 10d \cosh(dx + c)^3 + d \cosh(dx + c)) \sinh(dx + c)^3 + 8d \cosh(dx + c)^2 + 8(15d \cosh(dx + c)^{14} + 91d \cosh(dx + c)^{12} + 231d \cosh(dx + c)^{10} + 315d \cosh(dx + c)^8 + 245d \cosh(dx + c)^6 + 105d \cosh(dx + c)^4 + 21d \cosh(dx + c)^2 + d) \sinh(dx + c)^2 + 16(d \cosh(dx + c)^{15} + 7d \cosh(dx + c)^{13} + 21d \cosh(dx + c)^{11} + 35d \cosh(dx + c)^9 + 35d \cosh(dx + c)^7 + 21d \cosh(dx + c)^5 + 7d \cosh(dx + c)^3 + d \cosh(dx + c)) \sinh(dx + c) + d)
\end{aligned}$$

giac [B] time = 0.41, size = 517, normalized size = 2.61

$$3(64a^3e^c + 48a^2be^c + 24ab^2e^c + 5b^3e^c) \arctan(e^{(dx+c)})e^{(-c)} + \frac{192a^3e^{(15dx+15c)} + 144a^2be^{(15dx+15c)} + 72ab^2e^{(15dx+15c)} + 15b^3e^{(15dx+15c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)^3*(a+b*tanh(dx+c)^2)^3,x, algorithm="giac")

[Out] 1/192*(3*(64*a^3*e^c + 48*a^2*b*e^c + 24*a*b^2*e^c + 5*b^3*e^c)*arctan(e^(dx + c))*e^(-c) + (192*a^3*e^(15*d*x + 15*c) + 144*a^2*b*e^(15*d*x + 15*c)

$$+ 72*a*b^2*e^{(15*d*x + 15*c)} + 15*b^3*e^{(15*d*x + 15*c)} + 960*a^3*e^{(13*d*x + 13*c)} - 432*a^2*b*e^{(13*d*x + 13*c)} - 984*a*b^2*e^{(13*d*x + 13*c)} - 397*b^3*e^{(13*d*x + 13*c)} + 1728*a^3*e^{(11*d*x + 11*c)} - 2160*a^2*b*e^{(11*d*x + 11*c)} - 312*a*b^2*e^{(11*d*x + 11*c)} + 895*b^3*e^{(11*d*x + 11*c)} + 960*a^3*e^{(9*d*x + 9*c)} - 1584*a^2*b*e^{(9*d*x + 9*c)} + 744*a*b^2*e^{(9*d*x + 9*c)} - 1765*b^3*e^{(9*d*x + 9*c)} - 960*a^3*e^{(7*d*x + 7*c)} + 1584*a^2*b*e^{(7*d*x + 7*c)} - 744*a*b^2*e^{(7*d*x + 7*c)} + 1765*b^3*e^{(7*d*x + 7*c)} - 1728*a^3*e^{(5*d*x + 5*c)} + 2160*a^2*b*e^{(5*d*x + 5*c)} + 312*a*b^2*e^{(5*d*x + 5*c)} - 895*b^3*e^{(5*d*x + 5*c)} - 960*a^3*e^{(3*d*x + 3*c)} + 432*a^2*b*e^{(3*d*x + 3*c)} + 984*a*b^2*e^{(3*d*x + 3*c)} + 397*b^3*e^{(3*d*x + 3*c)} - 192*a^3*e^{(d*x + c)} - 144*a^2*b*e^{(d*x + c)} - 72*a*b^2*e^{(d*x + c)} - 15*b^3*e^{(d*x + c)})/(e^{(2*d*x + 2*c)} + 1)^8/d$$

maple [B] time = 0.59, size = 421, normalized size = 2.13

$$\frac{a^3 \operatorname{sech}(dx+c) \tanh(dx+c)}{2d} + \frac{a^3 \arctan(e^{dx+c})}{d} - \frac{a^2 b \sinh(dx+c)}{d \cosh(dx+c)^4} + \frac{a^2 b \tanh(dx+c) \operatorname{sech}(dx+c)^3}{4d} + \frac{3a^2 b \operatorname{sech}(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x)

[Out] 1/2/d*a^3*sech(d*x+c)*tanh(d*x+c)+1/d*a^3*arctan(exp(d*x+c))-1/d*a^2*b*sinh(d*x+c)/cosh(d*x+c)^4+1/4/d*a^2*b*tanh(d*x+c)*sech(d*x+c)^3+3/8/d*a^2*b*sech(d*x+c)*tanh(d*x+c)+3/4/d*a^2*b*arctan(exp(d*x+c))-1/d*a*b^2*sinh(d*x+c)^3/cosh(d*x+c)^6-3/5/d*a*b^2*sinh(d*x+c)/cosh(d*x+c)^6+1/10/d*a*b^2*tanh(d*x+c)*sech(d*x+c)^5+1/8/d*a*b^2*tanh(d*x+c)*sech(d*x+c)^3+3/16/d*a*b^2*sech(d*x+c)*tanh(d*x+c)+3/8/d*a*b^2*arctan(exp(d*x+c))-1/3/d*b^3*sinh(d*x+c)^5/cosh(d*x+c)^8-1/3/d*b^3*sinh(d*x+c)^3/cosh(d*x+c)^8-1/7/d*b^3*sinh(d*x+c)/cosh(d*x+c)^8+1/56/d*b^3*tanh(d*x+c)*sech(d*x+c)^7+1/48/d*b^3*tanh(d*x+c)*sech(d*x+c)^5+5/192/d*b^3*tanh(d*x+c)*sech(d*x+c)^3+5/128/d*b^3*sech(d*x+c)*tanh(d*x+c)+5/64/d*b^3*arctan(exp(d*x+c))

maxima [B] time = 0.69, size = 553, normalized size = 2.79

$$-\frac{1}{192} b^3 \left(\frac{15 \arctan(e^{(-dx-c)})}{d} - \frac{15 e^{(-dx-c)} - 397 e^{(-3dx-3c)} + 895 e^{(-5dx-5c)} - 1765 e^{(-7dx-7c)} + 1765 e^{(-9dx-9c)} - 895 e^{(-11dx-11c)} + 397 e^{(-13dx-13c)} - 15 e^{(-15dx-15c)}}{d(8 e^{(-2dx-2c)} + 28 e^{(-4dx-4c)} + 56 e^{(-6dx-6c)} + 70 e^{(-8dx-8c)} + 56 e^{(-10dx-10c)} + 28 e^{(-12dx-12c)} + 8 e^{(-14dx-14c)} + e^{(-16dx-16c)} + 1)} \right) - \frac{1}{8} a b^2 (3 \arctan(e^{(-dx-c)})/d - (3 e^{(-dx-c)} - 47 e^{(-3dx-3c)} + 78 e^{(-5dx-5c)} - 78 e^{(-7dx-7c)} + 47 e^{(-9dx-9c)} - 3 e^{(-11dx-11c)})/(d(6 e^{(-2dx-2c)} + 15 e^{(-4dx-4c)} + 20 e^{(-6dx-6c)} + 15 e^{(-8dx-8c)} + 6 e^{(-10dx-10c)} + e^{(-12dx-12c)} + 1))) - \frac{3}{4} a^2 b (\arctan(e^{(-dx-c)})/d - (e^{(-dx-c)} - 7 e^{(-3dx-3c)} + 7 e^{(-5dx-5c)} - e^{(-7dx-7c)})/(d(4 e^{(-2dx-2c)} + 6 e^{(-4dx-4c)} + 4 e^{(-6dx-6c)} + e^{(-8dx-8c)} + 1))) - a^3 (\arctan(e^{(-dx-c)})/d - (e^{(-dx-c)} - e^{(-3dx-3c)})/(d(2 e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] -1/192*b^3*(15*arctan(e^{(-d*x - c)})/d - (15*e^{(-d*x - c)} - 397*e^{(-3*d*x - 3*c)} + 895*e^{(-5*d*x - 5*c)} - 1765*e^{(-7*d*x - 7*c)} + 1765*e^{(-9*d*x - 9*c)} - 895*e^{(-11*d*x - 11*c)} + 397*e^{(-13*d*x - 13*c)} - 15*e^{(-15*d*x - 15*c)})/(d*(8*e^{(-2*d*x - 2*c)} + 28*e^{(-4*d*x - 4*c)} + 56*e^{(-6*d*x - 6*c)} + 70*e^{(-8*d*x - 8*c)} + 56*e^{(-10*d*x - 10*c)} + 28*e^{(-12*d*x - 12*c)} + 8*e^{(-14*d*x - 14*c)} + e^{(-16*d*x - 16*c)} + 1))) - 1/8*a*b^2*(3*arctan(e^{(-d*x - c)})/d - (3*e^{(-d*x - c)} - 47*e^{(-3*d*x - 3*c)} + 78*e^{(-5*d*x - 5*c)} - 78*e^{(-7*d*x - 7*c)} + 47*e^{(-9*d*x - 9*c)} - 3*e^{(-11*d*x - 11*c)})/(d*(6*e^{(-2*d*x - 2*c)} + 15*e^{(-4*d*x - 4*c)} + 20*e^{(-6*d*x - 6*c)} + 15*e^{(-8*d*x - 8*c)} + 6*e^{(-10*d*x - 10*c)} + e^{(-12*d*x - 12*c)} + 1))) - 3/4*a^2*b*(arctan(e^{(-d*x - c)})/d - (e^{(-d*x - c)} - 7*e^{(-3*d*x - 3*c)} + 7*e^{(-5*d*x - 5*c)} - e^{(-7*d*x - 7*c)})/(d*(4*e^{(-2*d*x - 2*c)} + 6*e^{(-4*d*x - 4*c)} + 4*e^{(-6*d*x - 6*c)} + e^{(-8*d*x - 8*c)} + 1))) - a^3*(arctan(e^{(-d*x - c)})/d - (e^{(-d*x - c)} - e^{(-3*d*x - 3*c)})/(d*(2*e^{(-2*d*x - 2*c)} + e^{(-4*d*x - 4*c)} + 1)))

mupad [B] time = 1.36, size = 951, normalized size = 4.80

$$\operatorname{atan}\left(\frac{e^{dx} e^c (64 a^3 \sqrt{d^2} + 5 b^3 \sqrt{d^2} + 24 a b^2 \sqrt{d^2} + 48 a^2 b \sqrt{d^2})}{d \sqrt{4096 a^6 + 6144 a^5 b + 5376 a^4 b^2 + 2944 a^3 b^3 + 1056 a^2 b^4 + 240 a b^5 + 25 b^6}}\right) \sqrt{4096 a^6 + 6144 a^5 b + 5376 a^4 b^2 + 2944 a^3 b^3 + 1056 a^2 b^4 + 240 a b^5 + 25 b^6} / 64 \sqrt{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*tanh(c + d*x))^2)^3/cosh(c + d*x)^3,x)`

[Out] $(\operatorname{atan}(\frac{\exp(dx)\exp(c)(64a^3(d^2)^{1/2} + 5b^3(d^2)^{1/2} + 24ab^2(d^2)^{1/2} + 48a^2b(d^2)^{1/2})}{d(240ab^5 + 6144a^5b + 4096a^6 + 25b^6 + 1056a^2b^4 + 2944a^3b^3 + 5376a^4b^2)^{1/2}}) - \frac{\exp(c+dx)(a+b)^3}{2d} + \frac{\exp(13c+13dx)(a+b)^3}{2d} - \frac{3\exp(5c+5dx)(ab^2+a^2b-5a^3-5b^3)}{2d} - \frac{3\exp(9c+9dx)(ab^2+a^2b-5a^3-5b^3)}{2d} + \frac{2\exp(7c+7dx)(3ab^2-3a^2b+5a^3-5b^3)}{d} + \frac{3\exp(3c+3dx)(a+b)^2(a-b)}{d} + \frac{3\exp(11c+11dx)(a+b)^2(a-b)}{d}) / (8\exp(2c+2dx) + 28\exp(4c+4dx) + 56\exp(6c+6dx) + 70\exp(8c+8dx) + 56\exp(10c+10dx) + 28\exp(12c+12dx) + 8\exp(14c+14dx) + \exp(16c+16dx) + 1) + \frac{2\exp(c+dx)(48ab^2+85b^3)}{3d(5\exp(2c+2dx) + 10\exp(4c+4dx) + 10\exp(6c+6dx) + 5\exp(8c+8dx) + \exp(10c+10dx) + 1)} + \frac{16b^3\exp(c+dx)}{d(7\exp(2c+2dx) + 21\exp(4c+4dx) + 35\exp(6c+6dx) + 35\exp(8c+8dx) + 21\exp(10c+10dx) + 7\exp(12c+12dx) + \exp(14c+14dx) + 1)} + \frac{\exp(c+dx)(24ab^2+48a^2b+64a^3+5b^3)}{64d(\exp(2c+2dx) + 1)} - \frac{4\exp(c+dx)(6ab^2+35b^3)}{3d(6\exp(2c+2dx) + 15\exp(4c+4dx) + 20\exp(6c+6dx) + 15\exp(8c+8dx) + 6\exp(10c+10dx) + \exp(12c+12dx) + 1)} - \frac{\exp(c+dx)(600ab^2+576a^2b+144a^3+203b^3)}{96d(2\exp(2c+2dx) + \exp(4c+4dx) + 1)} + \frac{\exp(c+dx)(600ab^2+288a^2b+305b^3)}{24d(3\exp(2c+2dx) + 3\exp(4c+4dx) + \exp(6c+6dx) + 1)} - \frac{\exp(c+dx)(168ab^2+24a^2b+145b^3)}{4d(4\exp(2c+2dx) + 6\exp(4c+4dx) + 4\exp(6c+6dx) + \exp(8c+8dx) + 1)})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^3 \operatorname{sech}^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)`

[Out] `Integral((a + b*tanh(c + d*x)**2)**3*sech(c + d*x)**3, x)`

3.104 $\int \operatorname{sech}^4(c + dx) \left(a + b \tanh^2(c + dx) \right)^3 dx$

Optimal. Leaf size=102

$$\frac{a^3 \tanh(c + dx)}{d} - \frac{a^2(a - 3b) \tanh^3(c + dx)}{3d} - \frac{b^2(3a - b) \tanh^7(c + dx)}{7d} - \frac{3ab(a - b) \tanh^5(c + dx)}{5d} - \frac{b^3 \tanh^9(c + dx)}{9d}$$

[Out] $a^3 \tanh(d*x+c)/d - 1/3*a^2*(a-3*b)*\tanh(d*x+c)^3/d - 3/5*a*(a-b)*b*\tanh(d*x+c)^5/d - 1/7*(3*a-b)*b^2*\tanh(d*x+c)^7/d - 1/9*b^3*\tanh(d*x+c)^9/d$

Rubi [A] time = 0.09, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3675, 373}

$$-\frac{a^2(a - 3b) \tanh^3(c + dx)}{3d} + \frac{a^3 \tanh(c + dx)}{d} - \frac{b^2(3a - b) \tanh^7(c + dx)}{7d} - \frac{3ab(a - b) \tanh^5(c + dx)}{5d} - \frac{b^3 \tanh^9(c + dx)}{9d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] $(a^3 \operatorname{Tanh}[c + d*x])/d - (a^2*(a - 3*b)*\operatorname{Tanh}[c + d*x]^3)/(3*d) - (3*a*(a - b)*b*\operatorname{Tanh}[c + d*x]^5)/(5*d) - ((3*a - b)*b^2*\operatorname{Tanh}[c + d*x]^7)/(7*d) - (b^3*\operatorname{Tanh}[c + d*x]^9)/(9*d)$

Rule 373

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^4(c + dx) \left(a + b \tanh^2(c + dx) \right)^3 dx &= \frac{\operatorname{Subst} \left(\int (1 - x^2) (a + bx^2)^3 dx, x, \tanh(c + dx) \right)}{d} \\ &= \frac{\operatorname{Subst} \left(\int (a^3 - a^2(a - 3b)x^2 - 3a(a - b)bx^4 - (3a - b)b^2x^6 - b^3x^8) dx, x, \tanh(c + dx) \right)}{d} \\ &= \frac{a^3 \tanh(c + dx)}{d} - \frac{a^2(a - 3b) \tanh^3(c + dx)}{3d} - \frac{3a(a - b)b \tanh^5(c + dx)}{5d} - \frac{b^3 \tanh^9(c + dx)}{9d} \end{aligned}$$

Mathematica [B] time = 0.91, size = 218, normalized size = 2.14

$$\frac{\tanh(c + dx) \operatorname{sech}^8(c + dx) (1050a^3 \cosh(6(c + dx)) + 105a^3 \cosh(8(c + dx)) + 5775a^3 + 630a^2b \cosh(6(c + dx)))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] ((5775*a^3 - 1071*a^2*b + 621*a*b^2 - 725*b^3 + 10*(903*a^3 - 63*a^2*b - 27*a*b^2 + 107*b^3)*Cosh[2*(c + d*x)] + 8*(525*a^3 + 126*a^2*b - 81*a*b^2 - 50*b^3)*Cosh[4*(c + d*x)] + 1050*a^3*Cosh[6*(c + d*x)] + 630*a^2*b*Cosh[6*(c + d*x)] + 270*a*b^2*Cosh[6*(c + d*x)] + 50*b^3*Cosh[6*(c + d*x)] + 105*a^3*Cosh[8*(c + d*x)] + 63*a^2*b*Cosh[8*(c + d*x)] + 27*a*b^2*Cosh[8*(c + d*x)] + 5*b^3*Cosh[8*(c + d*x)])*Sech[c + d*x]^8*Tanh[c + d*x])/(20160*d)

fricas [B] time = 0.40, size = 1185, normalized size = 11.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -8/315*(2*(105*a^3 + 252*a^2*b + 243*a*b^2 + 80*b^3)*cosh(d*x + c)^7 + 14*(105*a^3 + 252*a^2*b + 243*a*b^2 + 80*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + (105*a^3 + 441*a^2*b + 459*a*b^2 + 155*b^3)*sinh(d*x + c)^7 + 6*(245*a^3 + 336*a^2*b + 99*a*b^2 - 40*b^3)*cosh(d*x + c)^5 + 3*(175*a^3 + 483*a^2*b + 117*a*b^2 - 95*b^3 + 7*(105*a^3 + 441*a^2*b + 459*a*b^2 + 155*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*(105*a^3 + 252*a^2*b + 243*a*b^2 + 80*b^3)*cosh(d*x + c)^3 + 3*(245*a^3 + 336*a^2*b + 99*a*b^2 - 40*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 18*(245*a^3 + 168*a^2*b + 27*a*b^2 + 40*b^3)*cosh(d*x + c)^3 + (35*(105*a^3 + 441*a^2*b + 459*a*b^2 + 155*b^3)*cosh(d*x + c)^4 + 945*a^3 + 1701*a^2*b + 459*a*b^2 + 855*b^3 + 30*(175*a^3 + 483*a^2*b + 117*a*b^2 - 95*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 6*(7*(105*a^3 + 252*a^2*b + 243*a*b^2 + 80*b^3)*cosh(d*x + c)^5 + 10*(245*a^3 + 336*a^2*b + 99*a*b^2 - 40*b^3)*cosh(d*x + c)^3 + 9*(245*a^3 + 168*a^2*b + 27*a*b^2 + 40*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 210*(35*a^3 + 12*a^2*b + 9*a*b^2)*cosh(d*x + c) + (7*(105*a^3 + 441*a^2*b + 459*a*b^2 + 155*b^3)*cosh(d*x + c)^6 + 15*(175*a^3 + 483*a^2*b + 117*a*b^2 - 95*b^3)*cosh(d*x + c)^4 + 525*a^3 + 693*a^2*b + 567*a*b^2 - 945*b^3 + 27*(105*a^3 + 189*a^2*b + 51*a*b^2 + 95*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^11 + 11*d*cosh(d*x + c)*sinh(d*x + c)^10 + d*sinh(d*x + c)^11 + 9*d*cosh(d*x + c)^9 + (55*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^9 + 3*(55*d*cosh(d*x + c)^3 + 27*d*cosh(d*x + c))*sinh(d*x + c)^8 + 37*d*cosh(d*x + c)^7 + (330*d*cosh(d*x + c)^4 + 324*d*cosh(d*x + c)^2 + 35*d)*sinh(d*x + c)^7 + 7*(66*d*cosh(d*x + c)^5 + 108*d*cosh(d*x + c)^3 + 37*d*cosh(d*x + c))*sinh(d*x + c)^6 + 93*d*cosh(d*x + c)^5 + 3*(154*d*cosh(d*x + c)^6 + 378*d*cosh(d*x + c)^4 + 245*d*cosh(d*x + c)^2 + 25*d)*sinh(d*x + c)^5 + (330*d*cosh(d*x + c)^7 + 1134*d*cosh(d*x + c)^5 + 1295*d*cosh(d*x + c)^3 + 465*d*cosh(d*x + c))*sinh(d*x + c)^4 + 162*d*cosh(d*x + c)^3 + (165*d*cosh(d*x + c)^8 + 756*d*cosh(d*x + c)^6 + 1225*d*cosh(d*x + c)^4 + 750*d*cosh(d*x + c)^2 + 90*d)*sinh(d*x + c)^3 + (55*d*cosh(d*x + c)^9 + 324*d*cosh(d*x + c)^7 + 777*d*cosh(d*x + c)^5 + 930*d*cosh(d*x + c)^3 + 486*d*cosh(d*x + c))*sinh(d*x + c)^2 + 210*d*cosh(d*x + c) + (11*d*cosh(d*x + c)^10 + 81*d*cosh(d*x + c)^8 + 245*d*cosh(d*x + c)^6 + 375*d*cosh(d*x + c)^4 + 270*d*cosh(d*x + c)^2 + 42*d)*sinh(d*x + c)

giac [B] time = 0.41, size = 447, normalized size = 4.38

$$\frac{4(315 a^3 e^{(14 dx+14 c)} + 945 a^2 b e^{(14 dx+14 c)} + 945 a b^2 e^{(14 dx+14 c)} + 315 b^3 e^{(14 dx+14 c)} + 1995 a^3 e^{(12 dx+12 c)} + 3465 a^2 b e^{(12 dx+12 c)} + 3465 a b^2 e^{(12 dx+12 c)} + 315 b^3 e^{(12 dx+12 c)})}{(d \cosh(d x + c))^{11} + 11 d \cosh(d x + c) \sinh(d x + c)^{10} + d \sinh(d x + c)^{11} + 9 d \cosh(d x + c)^9 + (55 d \cosh(d x + c)^2 + 9 d) \sinh(d x + c)^9 + 3(55 d \cosh(d x + c)^3 + 27 d \cosh(d x + c)) \sinh(d x + c)^8 + 37 d \cosh(d x + c)^7 + (330 d \cosh(d x + c)^4 + 324 d \cosh(d x + c)^2 + 35 d) \sinh(d x + c)^7 + 7(66 d \cosh(d x + c)^5 + 108 d \cosh(d x + c)^3 + 37 d \cosh(d x + c)) \sinh(d x + c)^6 + 93 d \cosh(d x + c)^5 + 3(154 d \cosh(d x + c)^6 + 378 d \cosh(d x + c)^4 + 245 d \cosh(d x + c)^2 + 25 d) \sinh(d x + c)^5 + (330 d \cosh(d x + c)^7 + 1134 d \cosh(d x + c)^5 + 1295 d \cosh(d x + c)^3 + 465 d \cosh(d x + c)) \sinh(d x + c)^4 + 162 d \cosh(d x + c)^3 + (165 d \cosh(d x + c)^8 + 756 d \cosh(d x + c)^6 + 1225 d \cosh(d x + c)^4 + 750 d \cosh(d x + c)^2 + 90 d) \sinh(d x + c)^3 + (55 d \cosh(d x + c)^9 + 324 d \cosh(d x + c)^7 + 777 d \cosh(d x + c)^5 + 930 d \cosh(d x + c)^3 + 486 d \cosh(d x + c)) \sinh(d x + c)^2 + 210 d \cosh(d x + c) + (11 d \cosh(d x + c)^{10} + 81 d \cosh(d x + c)^8 + 245 d \cosh(d x + c)^6 + 375 d \cosh(d x + c)^4 + 270 d \cosh(d x + c)^2 + 42 d) \sinh(d x + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out] -4/315*(315*a^3*e^(14*d*x + 14*c) + 945*a^2*b*e^(14*d*x + 14*c) + 945*a*b^2*e^(14*d*x + 14*c) + 315*b^3*e^(14*d*x + 14*c) + 1995*a^3*e^(12*d*x + 12*c)

$$\begin{aligned}
& + 3465*a^2*b*e^{(12*d*x + 12*c)} + 945*a*b^2*e^{(12*d*x + 12*c)} - 525*b^3*e^{(12*d*x + 12*c)} \\
& + 5355*a^3*e^{(10*d*x + 10*c)} + 4725*a^2*b*e^{(10*d*x + 10*c)} + 945*a*b^2*e^{(10*d*x + 10*c)} \\
& + 1575*b^3*e^{(10*d*x + 10*c)} + 7875*a^3*e^{(8*d*x + 8*c)} + 3213*a^2*b*e^{(8*d*x + 8*c)} \\
& + 2457*a*b^2*e^{(8*d*x + 8*c)} - 945*b^3*e^{(8*d*x + 8*c)} + 6825*a^3*e^{(6*d*x + 6*c)} \\
& + 1827*a^2*b*e^{(6*d*x + 6*c)} + 1323*a*b^2*e^{(6*d*x + 6*c)} + 945*b^3*e^{(6*d*x + 6*c)} \\
& + 3465*a^3*e^{(4*d*x + 4*c)} + 1323*a^2*b*e^{(4*d*x + 4*c)} + 27*a*b^2*e^{(4*d*x + 4*c)} - 135*b^3*e^{(4*d*x + 4*c)} \\
& + 945*a^3*e^{(2*d*x + 2*c)} + 567*a^2*b*e^{(2*d*x + 2*c)} + 243*a*b^2*e^{(2*d*x + 2*c)} \\
& + 45*b^3*e^{(2*d*x + 2*c)} + 105*a^3 + 63*a^2*b + 27*a*b^2 + 5*b^3)/(d*(e^{(2*d*x + 2*c)} + 1)^9)
\end{aligned}$$

maple [B] time = 0.56, size = 269, normalized size = 2.64

$$a^3 \left(\frac{2}{3} + \frac{\operatorname{sech}(dx+c)^2}{3} \right) \tanh(dx+c) + 3a^2b \left(-\frac{\sinh(dx+c)}{4 \cosh(dx+c)^5} + \frac{\left(\frac{8}{15} + \frac{\operatorname{sech}(dx+c)^4}{5} + \frac{4\operatorname{sech}(dx+c)^2}{15} \right) \tanh(dx+c)}{4} \right) + 3ab^2 \left(-\frac{\sinh^3(dx+c)}{4 \cosh(dx+c)^7} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x)

[Out] 1/d*(a^3*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+3*a^2*b*(-1/4*sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))+3*a*b^2*(-1/4*sinh(d*x+c)^3/cosh(d*x+c)^7-1/8*sinh(d*x+c)/cosh(d*x+c)^7+1/8*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c))+b^3*(-1/4*sinh(d*x+c)^5/cosh(d*x+c)^9-5/24*sinh(d*x+c)^3/cosh(d*x+c)^9-5/64*sinh(d*x+c)/cosh(d*x+c)^9+5/64*(128/315+1/9*sech(d*x+c)^8+8/63*sech(d*x+c)^6+16/105*sech(d*x+c)^4+64/315*sech(d*x+c)^2)*tanh(d*x+c)))

maxima [B] time = 0.35, size = 1847, normalized size = 18.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 4/63*b^3*(9*e^{(-2*d*x - 2*c)}/(d*(9*e^{(-2*d*x - 2*c)} + 36*e^{(-4*d*x - 4*c)} + 84*e^{(-6*d*x - 6*c)} + 126*e^{(-8*d*x - 8*c)} + 126*e^{(-10*d*x - 10*c)} + 84*e^{(-12*d*x - 12*c)} + 36*e^{(-14*d*x - 14*c)} + 9*e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1)) - 27*e^{(-4*d*x - 4*c)}/(d*(9*e^{(-2*d*x - 2*c)} + 36*e^{(-4*d*x - 4*c)} + 84*e^{(-6*d*x - 6*c)} + 126*e^{(-8*d*x - 8*c)} + 126*e^{(-10*d*x - 10*c)} + 84*e^{(-12*d*x - 12*c)} + 36*e^{(-14*d*x - 14*c)} + 9*e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1)) + 189*e^{(-6*d*x - 6*c)}/(d*(9*e^{(-2*d*x - 2*c)} + 36*e^{(-4*d*x - 4*c)} + 84*e^{(-6*d*x - 6*c)} + 126*e^{(-8*d*x - 8*c)} + 126*e^{(-10*d*x - 10*c)} + 84*e^{(-12*d*x - 12*c)} + 36*e^{(-14*d*x - 14*c)} + 9*e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1)) - 189*e^{(-8*d*x - 8*c)}/(d*(9*e^{(-2*d*x - 2*c)} + 36*e^{(-4*d*x - 4*c)} + 84*e^{(-6*d*x - 6*c)} + 126*e^{(-8*d*x - 8*c)} + 126*e^{(-10*d*x - 10*c)} + 84*e^{(-12*d*x - 12*c)} + 36*e^{(-14*d*x - 14*c)} + 9*e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1)) + 315*e^{(-10*d*x - 10*c)}/(d*(9*e^{(-2*d*x - 2*c)} + 36*e^{(-4*d*x - 4*c)} + 84*e^{(-6*d*x - 6*c)} + 126*e^{(-8*d*x - 8*c)} + 126*e^{(-10*d*x - 10*c)} + 84*e^{(-12*d*x - 12*c)} + 36*e^{(-14*d*x - 14*c)} + 9*e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1)) - 105*e^{(-12*d*x - 12*c)}/(d*(9*e^{(-2*d*x - 2*c)} + 36*e^{(-4*d*x - 4*c)} + 84*e^{(-6*d*x - 6*c)} + 126*e^{(-8*d*x - 8*c)} + 126*e^{(-10*d*x - 10*c)} + 84*e^{(-12*d*x - 12*c)} + 36*e^{(-14*d*x - 14*c)} + 9*e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1)) + 63*e^{(-14*d*x - 14*c)}/(d*(9*e^{(-2*d*x - 2*c)} + 36*e^{(-4*d*x - 4*c)} + 84*e^{(-6*d*x - 6*c)} + 126*e^{(-8*d*x - 8*c)} + 126*e^{(-10*d*x - 10*c)} + 84*e^{(-12*d*x - 12*c)} + 36*e^{(-14*d*x - 14*c)} + 9*e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1)) + 1/(d*(9*e^{(-2*d*x - 2*c)} + 36*e^{(-4*d*x - 4*c)} + 84*e^{(-6*d*x - 6*c)} + 126*e^{(-8*d*x - 8*c)} + 126*e^{(-10*d*x - 10*c)} + 84*e^{(-12*d*x - 12*c)} + 36*e^{(-14*d*x - 14*c)} + 9*e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1))

$$\begin{aligned}
& - 6*c) + 126*e^{(-8*d*x - 8*c)} + 126*e^{(-10*d*x - 10*c)} + 84*e^{(-12*d*x - 12*c)} + 36*e^{(-14*d*x - 14*c)} + 9*e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + \\
& 1))) + 12/35*a*b^2*(7*e^{(-2*d*x - 2*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) - 14*e^{(-4*d*x - 4*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) + 70*e^{(-6*d*x - 6*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) - 35*e^{(-8*d*x - 8*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) + 35*e^{(-10*d*x - 10*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) + 1/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1))) + 4/5*a^2*b*(5*e^{(-2*d*x - 2*c)}/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)) - 5*e^{(-4*d*x - 4*c)}/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)) + 15*e^{(-6*d*x - 6*c)}/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1)) + 1/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1))) + 4/3*a^3*(3*e^{(-2*d*x - 2*c)}/(d*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} + 1)) + 1/(d*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} + 1)))
\end{aligned}$$

mupad [B] time = 1.33, size = 1424, normalized size = 13.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*tanh(c + d*x))^2)^3/cosh(c + d*x)^4,x)`

[Out]
$$\begin{aligned}
& - ((4*(a + b)^2*(a - b))/(21*d) + (2*\exp(2*c + 2*d*x)*(a + b)^3)/(9*d))/(3*\exp(2*c + 2*d*x) + 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) + 1) - ((5*\exp(8*c + 8*d*x)*(a + b)^3)/(9*d) - (a*b^2 + a^2*b - 5*a^3 - 5*b^3)/(21*d) - (10*\exp(4*c + 4*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(21*d) + (16*\exp(2*c + 2*d*x)*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(63*d) + (40*\exp(6*c + 6*d*x)*(a + b)^2*(a - b))/(21*d))/(6*\exp(2*c + 2*d*x) + 15*\exp(4*c + 4*d*x) + 20*\exp(6*c + 6*d*x) + 15*\exp(8*c + 8*d*x) + 6*\exp(10*c + 10*d*x) + \exp(12*c + 12*d*x) + 1) - ((4*(a + b)^2*(a - b))/(21*d) + (2*\exp(10*c + 10*d*x)*(a + b)^3)/(3*d) - (2*\exp(2*c + 2*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(7*d) - (20*\exp(6*c + 6*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(21*d) + (16*\exp(4*c + 4*d*x)*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(21*d) + (20*\exp(8*c + 8*d*x)*(a + b)^2*(a - b))/(7*d))/(7*\exp(2*c + 2*d*x) + 21*\exp(4*c + 4*d*x) + 35*\exp(6*c + 6*d*x) + 35*\exp(8*c + 8*d*x) + 21*\exp(10*c + 10*d*x) + 7*\exp(12*c + 12*d*x) + \exp(14*c + 14*d*x) + 1) - ((16*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(315*d) + (4*\exp(6*c + 6*d*x)*(a + b)^3)/(9*d) - (4*\exp(2*c + 2*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(21*d) + (8*\exp(4*c + 4*d*x)*(a + b)^2*(a - b))/(7*d))/(5*\exp(2*c + 2*d*x) + 10*\exp(4*c + 4*d*x) + 10*\exp(6*c + 6*d*x) + 5*\exp(8*c + 8*d*x) + \exp(10*c + 10*d*x) + 1) - ((8*\exp(2*c + 2*d*x)*(a + b)^3)/(9*d) + (8*\exp(14*c + 14*d*x)*(a + b)^3)/(9*d) - (8*\exp(6*c + 6*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(3*d) - (8*\exp(10*c + 10*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(3*d) + (32*\exp(8*c + 8*d*x)*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(9*d) + (16*\exp(4*c + 4*d*x)*(a + b)^2*(a - b))/(3*d) + (16*\exp(12*c + 12*d*x)*(a + b)^2*(a - b))/(3*d))/(9*\exp(2*c + 2*d*x) + 36*\exp(4*c + 4*d*x) + 84*\exp(6*c + 6*d*x) + 126*\exp(8*c + 8*d*x) + 126*\exp(10*c + 10*d*x) + 84*\exp(12*c + 12*d*x) + 36*\exp(14*c + 14*d*x) + 9*\exp(16*c + 16*d*x) + \exp
\end{aligned}$$

```
(18*c + 18*d*x) + 1) - ((a + b)^3/(9*d) + (7*exp(12*c + 12*d*x)*(a + b)^3)/
(9*d) - (exp(4*c + 4*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/d - (5*exp(8*c +
8*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(3*d) + (16*exp(6*c + 6*d*x)*(3*a*
b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(9*d) + (4*exp(2*c + 2*d*x)*(a + b)^2*(a -
b))/(3*d) + (4*exp(10*c + 10*d*x)*(a + b)^2*(a - b))/d)/(8*exp(2*c + 2*d*x)
+ 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp
(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c +
16*d*x) + 1) - ((exp(4*c + 4*d*x)*(a + b)^3)/(3*d) - (a*b^2 + a^2*b - 5*a^
3 - 5*b^3)/(21*d) + (4*exp(2*c + 2*d*x)*(a + b)^2*(a - b))/(7*d))/(4*exp(2*
c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1
) - (a + b)^3/(9*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^3 \operatorname{sech}^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)

[Out] Integral((a + b*tanh(c + d*x)**2)**3*sech(c + d*x)**4, x)

$$3.105 \quad \int \frac{\cosh^4(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=120

$$\frac{x(3a^2 + 10ab + 15b^2)}{8(a+b)^3} + \frac{b^{5/2} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} d(a+b)^3} + \frac{\sinh(c+dx) \cosh^3(c+dx)}{4d(a+b)} + \frac{(3a+7b) \sinh(c+dx) \cosh(c+dx)}{8d(a+b)^2}$$

[Out] 1/8*(3*a^2+10*a*b+15*b^2)*x/(a+b)^3+1/8*(3*a+7*b)*cosh(d*x+c)*sinh(d*x+c)/(a+b)^2/d+1/4*cosh(d*x+c)^3*sinh(d*x+c)/(a+b)/d+b^(5/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/(a+b)^3/d/a^(1/2)

Rubi [A] time = 0.17, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3675, 414, 527, 522, 206, 205}

$$\frac{x(3a^2 + 10ab + 15b^2)}{8(a+b)^3} + \frac{b^{5/2} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} d(a+b)^3} + \frac{\sinh(c+dx) \cosh^3(c+dx)}{4d(a+b)} + \frac{(3a+7b) \sinh(c+dx) \cosh(c+dx)}{8d(a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^4/(a + b*Tanh[c + d*x]^2), x]

[Out] ((3*a^2 + 10*a*b + 15*b^2)*x)/(8*(a + b)^3) + (b^(5/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^3*d) + ((3*a + 7*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*(a + b)^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*(a + b)*d)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p+1)*(c +

$d*x^n)^{(q+1)}/(a*n*(b*c - a*d)*(p+1)), x] + \text{Dist}[1/(a*n*(b*c - a*d)*(p+1)), \text{Int}[(a + b*x^n)^{(p+1)}*(c + d*x^n)^q*\text{Simp}[c*(b*e - a*f) + e*n*(b*c - a*d)*(p+1) + d*(b*e - a*f)*(n*(p+q+2) + 1)*x^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n, q\}, x] \&\& \text{LtQ}[p, -1]$

Rule 3675

$\text{Int}[\text{sec}[(e_.) + (f_.)*(x_)]^{(m_)}*((a_.) + (b_.)*((c_.)*\text{tan}[(e_.) + (f_.)*(x_)]^{(n_)}))^{(p_)}, x_Symbol] :> \text{With}[\{ff = \text{FreeFactors}[\text{Tan}[e + f*x], x]\}, \text{Dist}[ff/(c^{(m-1)}*f), \text{Subst}[\text{Int}[(c^2 + ff^2*x^2)^{(m/2-1)}*(a + b*(ff*x)^n)^p, x], x, (c*\text{Tan}[e + f*x])/ff], x] /; \text{FreeQ}[\{a, b, c, e, f, n, p\}, x] \&\& \text{IntegerQ}[m/2] \&\& (\text{IntegersQ}[n, p] \|\ \text{IGtQ}[m, 0] \|\ \text{IGtQ}[p, 0] \|\ \text{EqQ}[n^2, 4] \|\ \text{EqQ}[n^2, 16])$

Rubi steps

$$\begin{aligned} \int \frac{\cosh^4(c+dx)}{a+b \tanh^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3(a+bx^2)} dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\cosh^3(c+dx) \sinh(c+dx)}{4(a+b)d} + \frac{\text{Subst}\left(\int \frac{3a+4b+3bx^2}{(1-x^2)^2(a+bx^2)} dx, x, \tanh(c+dx)\right)}{4(a+b)d} \\ &= \frac{(3a+7b) \cosh(c+dx) \sinh(c+dx)}{8(a+b)^2d} + \frac{\cosh^3(c+dx) \sinh(c+dx)}{4(a+b)d} + \frac{\text{Subst}\left(\int \frac{3a^2+7ab}{a+bx^2} dx, x, \tanh(c+dx)\right)}{4(a+b)d} \\ &= \frac{(3a+7b) \cosh(c+dx) \sinh(c+dx)}{8(a+b)^2d} + \frac{\cosh^3(c+dx) \sinh(c+dx)}{4(a+b)d} + \frac{b^3 \text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \tanh(c+dx)\right)}{4(a+b)d} \\ &= \frac{(3a^2+10ab+15b^2)x}{8(a+b)^3} + \frac{b^{5/2} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^3d} + \frac{(3a+7b) \cosh(c+dx) \sinh(c+dx)}{8(a+b)^2d} \end{aligned}$$

Mathematica [A] time = 0.30, size = 115, normalized size = 0.96

$$\frac{(3a^2+10ab+15b^2)(c+dx)}{8d(a+b)^3} + \frac{b^{5/2} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a+b)^3} + \frac{(a+2b) \sinh(2(c+dx))}{4d(a+b)^2} + \frac{\sinh(4(c+dx))}{32d(a+b)}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^4/(a + b*Tanh[c + d*x]^2), x]

[Out] ((3*a^2 + 10*a*b + 15*b^2)*(c + d*x))/(8*(a + b)^3*d) + (b^(5/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^3*d) + ((a + 2*b)*Sinh[2*(c + d*x)])/(4*(a + b)^2*d) + Sinh[4*(c + d*x)]/(32*(a + b)*d)

fricas [B] time = 0.48, size = 2180, normalized size = 18.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] [1/64*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 8*(3*a^2 + 10

```

*a*b + 15*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^
6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 + 6*a*b + 4*b^2)*sinh(
d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 6*(a^2 + 3*a*b + 2*
b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x +
c)^4 + 4*(3*a^2 + 10*a*b + 15*b^2)*d*x + 60*(a^2 + 3*a*b + 2*b^2)*cosh(d*x
+ c)^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 4*(3*a
^2 + 10*a*b + 15*b^2)*d*x*cosh(d*x + c) + 20*(a^2 + 3*a*b + 2*b^2)*cosh(d*x
+ c)^3)*sinh(d*x + c)^3 - 8*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^2 + 4*(7*(
a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 12*(3*a^2 + 10*a*b + 15*b^2)*d*x*cosh(
d*x + c)^2 + 30*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^4 - 2*a^2 - 6*a*b - 4*b
^2)*sinh(d*x + c)^2 + 32*(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)^3*sinh(
d*x + c) + 6*b^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^2*cosh(d*x + c)*sinh
(d*x + c)^3 + b^2*sinh(d*x + c)^4)*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh
(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 +
2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 +
2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b
^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*si
nh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*
sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a +
b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh
(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a
- b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*
sinh(d*x + c) + a + b)) - a^2 - 2*a*b - b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(d
*x + c)^7 + 4*(3*a^2 + 10*a*b + 15*b^2)*d*x*cosh(d*x + c)^3 + 6*(a^2 + 3*a*
b + 2*b^2)*cosh(d*x + c)^5 - 2*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*
x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^4 + 4*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3 + 3*a^2*b + 3*a*b^2
+ b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d
*sinh(d*x + c)^4), 1/64*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a
*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)
^8 + 8*(3*a^2 + 10*a*b + 15*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2 + 3*a*b + 2*b
^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 + 6*
a*b + 4*b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 6
*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^4 + 4*(3*a^2 + 10*a*b + 15*b^2)*d*x + 60*(a^2 + 3*a*b
+ 2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)^5 + 4*(3*a^2 + 10*a*b + 15*b^2)*d*x*cosh(d*x + c) + 20*(a^2 + 3*a*b
+ 2*b^2)*cosh(d*x + c)^3)*sinh(d*x + c)^3 - 8*(a^2 + 3*a*b + 2*b^2)*cosh(d
*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 12*(3*a^2 + 10*a*b +
15*b^2)*d*x*cosh(d*x + c)^2 + 30*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^4 - 2
*a^2 - 6*a*b - 4*b^2)*sinh(d*x + c)^2 + 64*(b^2*cosh(d*x + c)^4 + 4*b^2*cos
h(d*x + c)^3*sinh(d*x + c) + 6*b^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^2*
cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4)*sqrt(b/a)*arctan(1/2*(
(a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*s
inh(d*x + c)^2 + a - b)*sqrt(b/a)/b) - a^2 - 2*a*b - b^2 + 8*((a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^7 + 4*(3*a^2 + 10*a*b + 15*b^2)*d*x*cosh(d*x + c)^3 +
6*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^5 - 2*(a^2 + 3*a*b + 2*b^2)*cosh(d*x
+ c))*sinh(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^4 + 4
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^3 +
3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*d*sinh(d*x + c)^4)]

```

giac [B] time = 1.07, size = 326, normalized size = 2.72

$$\frac{64 b^3 \arctan\left(\frac{ae^{(2dx+2c)+be^{(2dx+2c)+a-b}}}{2\sqrt{ab}}\right)}{(a^3+3a^2b+3ab^2+b^3)\sqrt{ab}} + \frac{8(3a^2+10ab+15b^2)dx}{a^3+3a^2b+3ab^2+b^3} - \frac{(18a^2e^{(4dx+4c)}+60abe^{(4dx+4c)}+90b^2e^{(4dx+4c)}+8a^2e^{(2dx+2c)}+24abe^{(2dx+2c)})}{a^3e^{(4c)}+3a^2be^{(4c)}+3ab^2e^{(4c)}+b^3e^{(4c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] $\frac{1}{64} \cdot \frac{(64b^3 \arctan(\frac{1}{2}(ae^{2dx} + 2c) + be^{2dx} + 2c) + a - b) \sqrt{a^3 + 3a^2b + 3ab^2 + b^3} + 8(3a^2 + 10ab + 15b^2)dx + (a^3 + 3a^2b + 3ab^2 + b^3) - (18a^2e^{4dx} + 4c) + 60ab e^{4dx} + 4c + 90b^2e^{4dx} + 4c + 8a^2e^{2dx} + 2c + 24ab b e^{2dx} + 2c + 16b^2e^{2dx} + 2c + a^2 + 2ab + b^2)e^{-4dx}}{(a^3e^{4c} + 3a^2b e^{4c} + 3ab^2e^{4c} + b^3e^{4c}) + (ae^{4dx} + 20c) + b e^{4dx} + 20c + 8a e^{2dx} + 18c + 16b e^{2dx} + 18c)} / (a^2e^{16c} + 2ab e^{16c} + b^2e^{16c}) / d$

maple [B] time = 0.46, size = 857, normalized size = 7.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x)

[Out] $\frac{1}{2} \cdot \frac{d}{(2b+2a)} \cdot \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) - 1)^4} + \frac{2}{d} \cdot \frac{1}{(4a+4b)} \cdot \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) - 1)^3} + \frac{7}{8} \cdot \frac{d}{(a+b)^2} \cdot \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) - 1)^2} + \frac{11}{8} \cdot \frac{d}{(a+b)^2} \cdot \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) - 1)} + \frac{5}{8} \cdot \frac{d}{(a+b)^2} \cdot \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) - 1)} + \frac{9}{8} \cdot \frac{d}{(a+b)^2} \cdot \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) - 1)} + \frac{3}{8} \cdot \frac{d}{a^2} \cdot \frac{1}{(a+b)^3} \cdot \ln(\tanh(\frac{1}{2}dx + \frac{1}{2}c) - 1) - \frac{5}{4} \cdot \frac{d}{a} \cdot \frac{1}{(a+b)^3} \cdot \ln(\tanh(\frac{1}{2}dx + \frac{1}{2}c) - 1) + \frac{15}{8} \cdot \frac{d}{(a+b)^3} \cdot \ln(\tanh(\frac{1}{2}dx + \frac{1}{2}c) - 1) + \frac{1}{2} \cdot \frac{d}{(2b+2a)} \cdot \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) + 1)^4} + \frac{2}{d} \cdot \frac{1}{(4a+4b)} \cdot \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) + 1)^3} + \frac{5}{8} \cdot \frac{d}{(a+b)^2} \cdot \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) + 1)} + \frac{9}{8} \cdot \frac{d}{(a+b)^2} \cdot \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) + 1)} + \frac{7}{8} \cdot \frac{d}{(a+b)^2} \cdot \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) + 1)} + \frac{11}{8} \cdot \frac{d}{(a+b)^2} \cdot \frac{1}{(\tanh(\frac{1}{2}dx + \frac{1}{2}c) + 1)} + \frac{3}{8} \cdot \frac{d}{(a+b)^3} \cdot \ln(\tanh(\frac{1}{2}dx + \frac{1}{2}c) + 1) + \frac{5}{4} \cdot \frac{d}{(a+b)^3} \cdot \ln(\tanh(\frac{1}{2}dx + \frac{1}{2}c) + 1) + \frac{15}{8} \cdot \frac{d}{(a+b)^3} \cdot \ln(\tanh(\frac{1}{2}dx + \frac{1}{2}c) + 1) + \frac{1}{d} \cdot \frac{b^3}{(a+b)^3} \cdot \frac{1}{(b(a+b))^{\frac{1}{2}}} \cdot \frac{1}{((2(b(a+b))^{\frac{1}{2}} - a - 2b)a)^{\frac{1}{2}}} \cdot \frac{1}{\arctanh(a \tanh(\frac{1}{2}dx + \frac{1}{2}c))} + \frac{1}{d} \cdot \frac{b^3}{(a+b)^3} \cdot \frac{1}{((2(b(a+b))^{\frac{1}{2}} - a - 2b)a)^{\frac{1}{2}}} \cdot \frac{1}{\arctanh(a \tanh(\frac{1}{2}dx + \frac{1}{2}c))} + \frac{1}{d} \cdot \frac{b^3}{(a+b)^3} \cdot \frac{1}{(b(a+b))^{\frac{1}{2}}} \cdot \frac{1}{((2(b(a+b))^{\frac{1}{2}} - a - 2b)a)^{\frac{1}{2}}} \cdot \frac{1}{\arctanh(a \tanh(\frac{1}{2}dx + \frac{1}{2}c))} + \frac{1}{d} \cdot \frac{b^3}{(a+b)^3} \cdot \frac{1}{(b(a+b))^{\frac{1}{2}}} \cdot \frac{1}{((2(b(a+b))^{\frac{1}{2}} - a - 2b)a)^{\frac{1}{2}}} \cdot \frac{1}{\arctanh(a \tanh(\frac{1}{2}dx + \frac{1}{2}c))} + \frac{1}{d} \cdot \frac{b^3}{(a+b)^3} \cdot \frac{1}{(b(a+b))^{\frac{1}{2}}} \cdot \frac{1}{((2(b(a+b))^{\frac{1}{2}} + a + 2b)a)^{\frac{1}{2}}} \cdot \frac{1}{\arctan(a \tanh(\frac{1}{2}dx + \frac{1}{2}c))} + \frac{1}{d} \cdot \frac{b^3}{(a+b)^3} \cdot \frac{1}{((2(b(a+b))^{\frac{1}{2}} + a + 2b)a)^{\frac{1}{2}}} \cdot \frac{1}{\arctan(a \tanh(\frac{1}{2}dx + \frac{1}{2}c))} + \frac{1}{d} \cdot \frac{b^3}{(a+b)^3} \cdot \frac{1}{(b(a+b))^{\frac{1}{2}}} \cdot \frac{1}{((2(b(a+b))^{\frac{1}{2}} + a + 2b)a)^{\frac{1}{2}}} \cdot \frac{1}{\arctan(a \tanh(\frac{1}{2}dx + \frac{1}{2}c))} + \frac{1}{d} \cdot \frac{b^3}{(a+b)^3} \cdot \frac{1}{(b(a+b))^{\frac{1}{2}}} \cdot \frac{1}{((2(b(a+b))^{\frac{1}{2}} + a + 2b)a)^{\frac{1}{2}}} \cdot \frac{1}{\arctan(a \tanh(\frac{1}{2}dx + \frac{1}{2}c))}$

maxima [B] time = 0.51, size = 514, normalized size = 4.28

$$\frac{(ab - b^2)(dx + c)}{2(a^3 + 3a^2b + 3ab^2 + b^3)d} + \frac{(8be^{-2dx-2c} + a + b)e^{4dx+4c}}{64(a^2 + 2ab + b^2)d} + \frac{b \log((a + b)e^{4dx+4c} + 2(a - b)e^{2dx+2c} + a + b)}{4(a^2 + 2ab + b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] $-\frac{1}{2} \cdot \frac{(ab - b^2)(dx + c)}{(a^3 + 3a^2b + 3ab^2 + b^3)d} + \frac{1}{64} \cdot \frac{(8b e^{-2dx} - 2c) + a + b}{(a^2 + 2ab + b^2)d} \cdot \frac{e^{4dx} + 4c}{(a^2 + 2ab + b^2)d} + \frac{1}{4} \cdot \frac{b \cdot \log((a + b)e^{4dx} + 4c) + 2(a - b)e^{2dx} + 2c + a + b}{(a^2 + 2ab + b^2)d} - \frac{1}{4} \cdot \frac{b \cdot \log(2(a - b)e^{-2dx} - 2c) + (a + b)e^{-4dx} - 4c + a + b}{(a^2 + 2ab + b^2)d} - \frac{1}{4} \cdot \frac{(ab - b^2) \arctan(\frac{1}{2}((a + b)e^{2dx} + 2c) + a - b) \sqrt{a^3 + 3a^2b + 3ab^2 + b^3}}{(a^2 + 2ab + b^2) \sqrt{a^3 + 3a^2b + 3ab^2 + b^3}} + \frac{1}{4} \cdot \frac{(ab - b^2) \arctan(\frac{1}{2}((a + b)e^{-2dx} - 2c) + a - b) \sqrt{a^3 + 3a^2b + 3ab^2 + b^3}}{(a^2 + 2ab + b^2) \sqrt{a^3 + 3a^2b + 3ab^2 + b^3}}$

$\text{qrt}(a*b)*d) - 3/8*b*\arctan(1/2*((a + b)*e^{(-2*d*x - 2*c) + a - b})/\text{sqrt}(a*b)) / (\text{sqrt}(a*b)*(a + b)*d) - 1/64*(8*b*e^{(-2*d*x - 2*c) + (a + b)*e^{(-4*d*x - 4*c)}})/((a^2 + 2*a*b + b^2)*d) + 3/8*(d*x + c)/((a + b)*d) + 1/8*e^{(2*d*x + 2*c)}/((a + b)*d) - 1/8*e^{(-2*d*x - 2*c)}/((a + b)*d)$

mupad [B] time = 1.93, size = 967, normalized size = 8.06

$$\frac{x(3a^2 + 10ab + 15b^2)}{8(a+b)^3} - \frac{e^{-4c-4dx}}{64d(a+b)} + \frac{e^{4c+4dx}}{64d(a+b)} + \frac{\text{atan}\left(\left(e^{2c}e^{2dx}\left(\frac{4b^3}{d(a+b)^5\sqrt{b^5}(a^3+3a^2b+3ab^2+b^3)} + \frac{1}{b^3(a+b)^2}\right)\right)}{\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)^4/(a + b*tanh(c + d*x)^2), x)

[Out] $(x*(10*a*b + 3*a^2 + 15*b^2))/(8*(a + b)^3) - \exp(-4*c - 4*d*x)/(64*d*(a + b)) + \exp(4*c + 4*d*x)/(64*d*(a + b)) + (\text{atan}((\exp(2*c)*\exp(2*d*x))*((4*b^3)/(d*(a + b)^5*(b^5)^{(1/2)}*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + ((a - b)*(a^4*d*(b^5)^{(1/2)} - b^4*d*(b^5)^{(1/2)} - 2*a*b^3*d*(b^5)^{(1/2)} + 2*a^3*b*d*(b^5)^{(1/2)})))/(b^3*(a + b)^2*(a*d^2*(a + b)^6)^{(1/2)}*(3*a*b^2 + 3*a^2*b + a^3 + b^3)*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^{(1/2)})) + ((a - b)*(a^4*d*(b^5)^{(1/2)} + b^4*d*(b^5)^{(1/2)} + 4*a*b^3*d*(b^5)^{(1/2)} + 4*a^3*b*d*(b^5)^{(1/2)} + 6*a^2*b^2*d*(b^5)^{(1/2)})))/(b^3*(a + b)^2*(a*d^2*(a + b)^6)^{(1/2)}*(3*a*b^2 + 3*a^2*b + a^3 + b^3)*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^{(1/2)})) * ((a^4*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^{(1/2)})/2 + (b^4*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^{(1/2)})/2 + 2*a*b^3*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^{(1/2)} + 2*a^3*b*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^{(1/2)} + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^{(1/2)} + 3*a^2*b^2*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^{(1/2)})) * (b^5)^{(1/2)})/(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^{(1/2)} - (\exp(-2*c - 2*d*x)*(a + 2*b))/(8*d*(a + b)^2) + (\exp(2*c + 2*d*x)*(a + 2*b))/(8*d*(a + b)^2)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^4(c + dx)}{a + b \tanh^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**4/(a+b*tanh(d*x+c)**2), x)

[Out] Integral(cosh(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)

$$3.106 \quad \int \frac{\cosh^3(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=80

$$\frac{b^2 \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} d(a+b)^{5/2}} + \frac{\sinh^3(c+dx)}{3d(a+b)} + \frac{(a+2b) \sinh(c+dx)}{d(a+b)^2}$$

[Out] (a+2*b)*sinh(d*x+c)/(a+b)^2/d+1/3*sinh(d*x+c)^3/(a+b)/d+b^2*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/(a+b)^(5/2)/d/a^(1/2)

Rubi [A] time = 0.11, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3676, 390, 205}

$$\frac{b^2 \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} d(a+b)^{5/2}} + \frac{\sinh^3(c+dx)}{3d(a+b)} + \frac{(a+2b) \sinh(c+dx)}{d(a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^3/(a + b*Tanh[c + d*x]^2), x]

[Out] (b^2*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*(a + b)^(5/2)*d) + ((a + 2*b)*Sinh[c + d*x])/((a + b)^2*d) + Sinh[c + d*x]^3/(3*(a + b)*d)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 3676

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(c+dx)}{a+b \tanh^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{a+(a+b)x^2} dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a+2b}{(a+b)^2} + \frac{x^2}{a+b} + \frac{b^2}{(a+b)^2(a+(a+b)x^2)}\right) dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{(a+2b) \sinh(c+dx)}{(a+b)^2 d} + \frac{\sinh^3(c+dx)}{3(a+b)d} + \frac{b^2 \text{Subst}\left(\int \frac{1}{a+(a+b)x^2} dx, x, \sinh(c+dx)\right)}{(a+b)^2 d} \\
&= \frac{b^2 \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{5/2} d} + \frac{(a+2b) \sinh(c+dx)}{(a+b)^2 d} + \frac{\sinh^3(c+dx)}{3(a+b)d}
\end{aligned}$$

Mathematica [A] time = 0.43, size = 79, normalized size = 0.99

$$\frac{-\frac{12b^2 \tan^{-1}\left(\frac{\sqrt{a} \text{csch}(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{5/2}} + \frac{3(3a+7b) \sinh(c+dx)}{(a+b)^2} + \frac{\sinh(3(c+dx))}{a+b}}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^3/(a + b*Tanh[c + d*x]^2), x]

[Out] ((-12*b^2*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5/2)) + (3*(3*a + 7*b)*Sinh[c + d*x])/(a + b)^2 + Sinh[3*(c + d*x)]/(a + b))/(12*d)

fricas [B] time = 0.49, size = 1850, normalized size = 23.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] [1/24*((a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^6 + 6*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^3 + 2*a^2*b + a*b^2)*sinh(d*x + c)^6 + 3*(3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c)^4 + 3*(3*a^3 + 10*a^2*b + 7*a*b^2 + 5*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^3 + 3*(3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - a^3 - 2*a^2*b - a*b^2 - 3*(3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c)^2 + 3*(5*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^4 - 3*a^3 - 10*a^2*b - 7*a*b^2 + 6*(3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 12*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)^2*sinh(d*x + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x + c)^3)*sqrt(-a^2 - a*b)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 6*((a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^5 + 2*(3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c)^3 - (3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c))*sinh(d*x + c)]/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 + 3*(a^4 + 3*a^3*

$b + 3a^2b^2 + ab^3)d\cosh(dx + c)^2\sinh(dx + c) + 3(a^4 + 3a^3b + 3a^2b^2 + ab^3)d\cosh(dx + c)\sinh(dx + c)^2 + (a^4 + 3a^3b + 3a^2b^2 + ab^3)d\sinh(dx + c)^3, 1/24*((a^3 + 2a^2b + ab^2)\cosh(dx + c)^6 + 6(a^3 + 2a^2b + ab^2)\cosh(dx + c)\sinh(dx + c)^5 + (a^3 + 2a^2b + ab^2)\sinh(dx + c)^6 + 3(3a^3 + 10a^2b + 7ab^2)\cosh(dx + c)^4 + 3(3a^3 + 10a^2b + 7ab^2 + 5(a^3 + 2a^2b + ab^2)\cosh(dx + c)^2)\sinh(dx + c)^4 + 4(5(a^3 + 2a^2b + ab^2)\cosh(dx + c)^3 + 3(3a^3 + 10a^2b + 7ab^2)\cosh(dx + c))\sinh(dx + c)^3 - a^3 - 2a^2b - ab^2 - 3(3a^3 + 10a^2b + 7ab^2)\cosh(dx + c)^2 + 3(5(a^3 + 2a^2b + ab^2)\cosh(dx + c)^4 - 3a^3 - 10a^2b - 7ab^2 + 6(3a^3 + 10a^2b + 7ab^2)\cosh(dx + c)^2)\sinh(dx + c)^2 + 24(b^2\cosh(dx + c)^3 + 3b^2\cosh(dx + c)^2\sinh(dx + c) + 3b^2\cosh(dx + c)\sinh(dx + c)^2 + b^2\sinh(dx + c)^3)\sqrt{a^2 + ab}\arctan(1/2*((a + b)\cosh(dx + c)^3 + 3(a + b)\cosh(dx + c)\sinh(dx + c)^2 + (a + b)\sinh(dx + c)^3 + (3a - b)\cosh(dx + c) + (3(a + b)\cosh(dx + c)^2 + 3a - b)\sinh(dx + c)))/\sqrt{a^2 + ab}) + 24(b^2\cosh(dx + c)^3 + 3b^2\cosh(dx + c)^2\sinh(dx + c) + 3b^2\cosh(dx + c)\sinh(dx + c)^2 + b^2\sinh(dx + c)^3)\sqrt{a^2 + ab}\arctan(1/2*\sqrt{a^2 + ab}*(\cosh(dx + c) + \sinh(dx + c))/a) + 6*((a^3 + 2a^2b + ab^2)\cosh(dx + c)^5 + 2(3a^3 + 10a^2b + 7ab^2)\cosh(dx + c)^3 - (3a^3 + 10a^2b + 7ab^2)\cosh(dx + c))\sinh(dx + c))/((a^4 + 3a^3b + 3a^2b^2 + ab^3)d\cosh(dx + c)^3 + 3(a^4 + 3a^3b + 3a^2b^2 + ab^3)d\cosh(dx + c)^2\sinh(dx + c) + 3(a^4 + 3a^3b + 3a^2b^2 + ab^3)d\sinh(dx + c)^2 + (a^4 + 3a^3b + 3a^2b^2 + ab^3)d\sinh(dx + c)^3)]$

giac [B] time = 0.49, size = 829, normalized size = 10.36

$$24(ab^2 + \sqrt{-ab}b^2)\sqrt{a^2 - b^2 + 2\sqrt{-ab}(a+b)}|ae^{(2c)} + be^{(2c)}|\arctan\left(\frac{e^{(dx)}}{\sqrt{\frac{a^3e^{(2c)} + a^2be^{(2c)} - ab^2e^{(2c)} - b^3e^{(2c)} + \sqrt{(a^3e^{(2c)} + a^2be^{(2c)} - ab^2e^{(2c)} - b^3e^{(2c)})^2 - (a^3e^{(4c)} + 3a^2be^{(4c)} - ab^2e^{(4c)} + b^3e^{(4c)})}}{a^3e^{(4c)} + 3a^2be^{(4c)} + 3ab^2e^{(4c)} + b^3e^{(4c)}}}}}\right)$$

$$a^6 + 3a^5b + 2a^4b^2 - 2a^3b^3 - 3a^2b^4 - ab^5 + 2(a^5 + 4a^4b + 6a^3b^2 + 4a^2b^3 + ab^4)\sqrt{-ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(dx+c)^3/(a+b*tanh(dx+c)^2),x, algorithm="giac")

[Out] $1/24*(24*(ab^2 + \sqrt{-ab}b^2)\sqrt{a^2 - b^2 + 2\sqrt{-ab}(a+b)}*abs(ae^{(2*c)} + be^{(2*c)})*\arctan(e^{(d*x)}/\sqrt{(a^3e^{(2*c)} + a^2b*be^{(2*c)} - a*b^2e^{(2*c)} - b^3e^{(2*c)} + \sqrt{(a^3e^{(2*c)} + a^2b*be^{(2*c)} - a*b^2e^{(2*c)} - b^3e^{(2*c)})^2 - (a^3e^{(4*c)} + 3a^2b*be^{(4*c)} + 3a*b^2e^{(4*c)} + b^3e^{(4*c)})})})/(a^3e^{(4*c)} + 3a^2b*be^{(4*c)} + 3a*b^2e^{(4*c)} + b^3e^{(4*c)})))*e^{(-2*c)}/(a^6 + 3a^5b + 2a^4b^2 - 2a^3b^3 - 3a^2b^4 - ab^5 + 2*(a^5 + 4a^4b + 6a^3b^2 + 4a^2b^3 + a*b^4)*\sqrt{-ab}) + 24*(a^2b^2 - 3a*b^3 - (3a*b^2 - b^3)*\sqrt{-ab})*abs(ae^{(2*c)} + be^{(2*c)})*\arctan(e^{(d*x)}/\sqrt{(a^3e^{(2*c)} + a^2b*be^{(2*c)} - a*b^2e^{(2*c)} - b^3e^{(2*c)} - \sqrt{(a^3e^{(2*c)} + a^2b*be^{(2*c)} - a*b^2e^{(2*c)} - b^3e^{(2*c)})^2 - (a^3e^{(4*c)} + 3a^2b*be^{(4*c)} + 3a*b^2e^{(4*c)} + b^3e^{(4*c)})})})/(a^3e^{(4*c)} + 3a^2b*be^{(4*c)} + 3a*b^2e^{(4*c)} + b^3e^{(4*c)})))*e^{(-2*c)}/((a^5 + 2a^4b - 2a^2b^3 - a*b^4 - 2*(a^4 + 3a^3b + 3a^2b^2 + a*b^3)*\sqrt{-ab})*\sqrt{a^2 - b^2 - 2*\sqrt{-ab}(a+b)}) - (9*a*e^{(2*d*x + 2*c)} + 21*b*be^{(2*d*x + 2*c)} + a + b)*e^{(-3*d*x)}/(a^2e^{(3*c)} + 2a*b*be^{(3*c)} + b^2e^{(3*c)}) + (a^2e^{(3*d*x + 24*c)} + 2a*b*be^{(3*d*x + 24*c)} + b^2e^{(3*d*x + 24*c)} + 9*a^2e^{(d*x + 22*c)} + 30*a*b*be^{(d*x + 22*c)} + 21*b^2e^{(d*x + 22*c)})/(a^3e^{(21*c)} + 3a^2b*be^{(21*c)} + 3a*b^2e^{(21*c)} + b^3e^{(21*c)}))/d$

maple [B] time = 0.44, size = 468, normalized size = 5.85

$$\frac{3d \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^3 (2b + 2a)}{d(2b + 2a) \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^2} \frac{1}{d(a + b)^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)} \frac{a}{d(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2), x)

[Out]
$$-2/3/d/(\tanh(1/2*d*x+1/2*c)-1)^3/(2*b+2*a)-1/d/(2*b+2*a)/(\tanh(1/2*d*x+1/2*c)-1)^2-1/d/(a+b)^2/(\tanh(1/2*d*x+1/2*c)-1)*a-2/d/(a+b)^2/(\tanh(1/2*d*x+1/2*c)-1)*b-2/3/d/(\tanh(1/2*d*x+1/2*c)+1)^3/(2*b+2*a)+1/d/(2*b+2*a)/(\tanh(1/2*d*x+1/2*c)+1)^2-1/d/(a+b)^2/(\tanh(1/2*d*x+1/2*c)+1)*a-2/d/(a+b)^2/(\tanh(1/2*d*x+1/2*c)+1)*b-1/d*b^2/(a+b)^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*\arctan(h(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*b^3/(a+b)^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*b^2/(a+b)^2/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/d*b^3/(a+b)^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left((ae^{6c} + be^{6c})e^{6dx} + 3(3ae^{4c} + 7be^{4c})e^{4dx} - 3(3ae^{2c} + 7be^{2c})e^{2dx} - a - b \right) e^{-3dx}}{24(a^2de^{3c} + 2abde^{3c} + b^2de^{3c})} + \frac{1}{8} \int \frac{1}{a^3 + 3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2), x, algorithm="maxima")

[Out]
$$1/24*((a*e^{6*c} + b*e^{6*c})*e^{6*d*x} + 3*(3*a*e^{4*c} + 7*b*e^{4*c})*e^{4*d*x} - 3*(3*a*e^{2*c} + 7*b*e^{2*c})*e^{2*d*x} - a - b)*e^{-3*d*x}/(a^2*d*e^{3*c} + 2*a*b*d*e^{3*c} + b^2*d*e^{3*c}) + 1/8*\integrate(16*(b^2*e^{3*d*x + 3*c} + b^2*e^{d*x + c}))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3 + (a^3*e^{4*c} + 3*a^2*b*e^{4*c} + 3*a*b^2*e^{4*c} + b^3*e^{4*c})*e^{4*d*x} + 2*(a^3*e^{2*c} + a^2*b*e^{2*c} - a*b^2*e^{2*c} - b^3*e^{2*c})*e^{2*d*x}), x)$$

mupad [B] time = 2.88, size = 2194, normalized size = 27.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)^3/(a + b*tanh(c + d*x)^2), x)

[Out]
$$\exp(3*c + 3*d*x)/(24*d*(a + b)) - \exp(-3*c - 3*d*x)/(24*d*(a + b)) + ((b^4)^{1/2}*(2*\operatorname{atan}(\exp(d*x)*\exp(c)*((4*(10*a^2*d*(b^4)^{5/2} + 12*a^6*d*(b^4)^{3/2} + 2*a*b^9*d*(b^4)^{1/2} + 10*a^3*b^3*d*(b^4)^{3/2} + 2*a^2*b^8*d*(b^4)^{1/2} + 20*a^3*b^7*d*(b^4)^{1/2} + 40*a^4*b^6*d*(b^4)^{1/2} + 30*a^5*b^5*d*(b^4)^{1/2} + 2*a^7*b^3*d*(b^4)^{1/2}))/((a*b^5*(a + b)^5*(a*d^2*(a + b)^5)^{1/2}*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)*(5*a*b^4 + 5*a^4*b + a^5 + b^5 + 10*a^2*b^3 + 10*a^3*b^2)*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{1/2}) - (2*(b^9*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{1/2} + 4*a*b^8*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{1/2} + 6*a^2*b^7*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{1/2} + 4*a^3*b^6*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{1/2} + 4*a^4*b^5*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{1/2} + 4*a^5*b^4*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{1/2} + 4*a^6*b^3*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{1/2} + 4*a^7*b^2*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{1/2} + 4*a^8*b*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{1/2} + 4*a^9*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{1/2}))$$

$$\begin{aligned}
& *b^2*d^2)^{(1/2)} + a^4*b^5*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 \\
& + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{(1/2)))/(a^2*b^3*d*(a + b)^7*(b^4)^{(1/2)} \\
& *(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)*(5*a*b^4 + 5*a^4*b + a^5 + b^5 \\
& + 10*a^2*b^3 + 10*a^3*b^2)*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 \\
& + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{(1/2)) + (2*\exp(3*c)*\exp(3*d*x)*(\\
& b^9*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 1 \\
& 0*a^4*b^2*d^2)^{(1/2)} + 4*a*b^8*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 \\
& + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{(1/2)} + 6*a^2*b^7*(a^6*d^2 + a*b^5*d^2 \\
& + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{(1/2)} \\
&) + 4*a^3*b^6*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 \\
& + 10*a^4*b^2*d^2)^{(1/2)} + a^4*b^5*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 \\
& + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{(1/2)))/(a^2*b^3*d*(a + \\
& b)^7*(b^4)^{(1/2)}*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)*(5*a*b^4 + 5* \\
& a^4*b + a^5 + b^5 + 10*a^2*b^3 + 10*a^3*b^2)*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b \\
& *d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{(1/2)))*((a^11*(a^6 \\
& *d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2 \\
& *d^2)^{(1/2)))/4 + (a*b^10*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 \\
& + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{(1/2)))/4 + (5*a^10*b*(a^6*d^2 + a*b^5*d^2 \\
& + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{(1/2)) \\
& /2 + (5*a^2*b^9*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3 \\
& *b^3*d^2 + 10*a^4*b^2*d^2)^{(1/2)))/2 + (45*a^3*b^8*(a^6*d^2 + a*b^5*d^2 + 5* \\
& a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{(1/2)))/4 + 30* \\
& a^4*b^7*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 \\
& + 10*a^4*b^2*d^2)^{(1/2)} + (105*a^5*b^6*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 \\
& + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{(1/2)))/2 + 63*a^6*b^5*(a \\
& ^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2 \\
& *d^2)^{(1/2)} + (105*a^7*b^4*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4 \\
& *d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{(1/2)))/2 + 30*a^8*b^3*(a^6*d^2 + a \\
& *b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{(\\
& 1/2)} + (45*a^9*b^2*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10* \\
& a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{(1/2)))/4) + 2*atan((b^2*\exp(d*x)*\exp(c)*(a*d \\
& ^2*(a + b)^5)^{(1/2)))/(2*a*d*(a + b)^2*(b^4)^{(1/2)))))/(2*(a^6*d^2 + a*b^5*d \\
& ^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^{(1/2)) \\
& - (\exp(-c - d*x)*(3*a + 7*b))/(8*d*(a + b)^2) + (\exp(c + d*x)*(3*a + 7*b)) \\
& /(8*d*(a + b)^2)
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(c + dx)}{a + b \tanh^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**3/(a+b*tanh(d*x+c)**2), x)

[Out] Integral(cosh(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)

$$3.107 \quad \int \frac{\cosh^2(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=77

$$\frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} d(a+b)^2} + \frac{\sinh(c+dx) \cosh(c+dx)}{2d(a+b)} + \frac{x(a+3b)}{2(a+b)^2}$$

[Out] 1/2*(a+3*b)*x/(a+b)^2+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d+b^(3/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/(a+b)^2/d/a^(1/2)

Rubi [A] time = 0.10, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3675, 414, 522, 206, 205}

$$\frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} d(a+b)^2} + \frac{\sinh(c+dx) \cosh(c+dx)}{2d(a+b)} + \frac{x(a+3b)}{2(a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]

[Out] ((a + 3*b)*x)/(2*(a + b)^2) + (b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*(a + b)*d)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m-1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && In

tegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \frac{\cosh^2(c+dx)}{a+b \tanh^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+bx^2)} dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\cosh(c+dx) \sinh(c+dx)}{2(a+b)d} + \frac{\text{Subst}\left(\int \frac{a+2b+bx^2}{(1-x^2)(a+bx^2)} dx, x, \tanh(c+dx)\right)}{2(a+b)d} \\ &= \frac{\cosh(c+dx) \sinh(c+dx)}{2(a+b)d} + \frac{b^2 \text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \tanh(c+dx)\right)}{(a+b)^2d} + \frac{(a+3b) \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c+dx)\right)}{2(a+b)d} \\ &= \frac{(a+3b)x}{2(a+b)^2} + \frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^2d} + \frac{\cosh(c+dx) \sinh(c+dx)}{2(a+b)d} \end{aligned}$$

Mathematica [A] time = 0.16, size = 77, normalized size = 1.00

$$\frac{4b^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right) + 2\sqrt{a}(a+3b)(c+dx) + \sqrt{a}(a+b) \sinh(2(c+dx))}{4\sqrt{a}d(a+b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2), x]

[Out] (2*Sqrt[a]*(a + 3*b)*(c + d*x) + 4*b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*(a + b)*Sinh[2*(c + d*x)])/(4*Sqrt[a]*(a + b)^2*d)

fricas [B] time = 0.45, size = 948, normalized size = 12.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] [1/8*(4*(a + 3*b)*d*x*cosh(d*x + c)^2 + (a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(2*(a + 3*b)*d*x + 3*(a + b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2)*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a)]/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b) + 4*(2*(a + 3*b)*d*x*cosh(d*x + c) + (a + b)*cosh(d*x + c)^3)*sinh(d*x + c) - a - b)/((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^2), 1/8*(4*(a + 3*b)*d*x*cosh(d*x + c)^2 + (a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*

$$\sinh(dx + c)^4 + 2*(2*(a + 3*b)*dx + 3*(a + b)*\cosh(dx + c)^2)*\sinh(dx + c)^2 + 8*(b*\cosh(dx + c)^2 + 2*b*\cosh(dx + c)*\sinh(dx + c) + b*\sinh(dx + c)^2)*\sqrt{b/a}*\arctan(1/2*((a + b)*\cosh(dx + c)^2 + 2*(a + b)*\cosh(dx + c)*\sinh(dx + c) + (a + b)*\sinh(dx + c)^2 + a - b)*\sqrt{b/a}/b) + 4*(2*(a + 3*b)*dx*\cosh(dx + c) + (a + b)*\cosh(dx + c)^3)*\sinh(dx + c) - a - b)/((a^2 + 2*a*b + b^2)*d*\cosh(dx + c)^2 + 2*(a^2 + 2*a*b + b^2)*d*\cosh(dx + c)*\sinh(dx + c) + (a^2 + 2*a*b + b^2)*d*\sinh(dx + c)^2)]$$

giac [B] time = 0.56, size = 172, normalized size = 2.23

$$\frac{\frac{4(a+3b)dx}{a^2+2ab+b^2} + \frac{8b^2 \arctan\left(\frac{ae^{2dx+2c}+be^{2dx+2c}+a-b}{2\sqrt{ab}}\right)}{(a^2+2ab+b^2)\sqrt{ab}} - \frac{(2ae^{2dx+2c}+6be^{2dx+2c}+a+b)e^{-2dx}}{a^2e^{2c}+2abe^{2c}+b^2e^{2c}} + \frac{e^{2dx+8c}}{ae^{6c}+be^{6c}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(dx+c)^2/(a+b*tanh(dx+c)^2),x, algorithm="giac")

[Out] 1/8*(4*(a + 3*b)*dx/(a^2 + 2*a*b + b^2) + 8*b^2*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b)))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) - (2*a*e^(2*d*x + 2*c) + 6*b*e^(2*d*x + 2*c) + a + b)*e^(-2*d*x)/(a^2*e^(2*c) + 2*a*b*e^(2*c) + b^2*e^(2*c)) + e^(2*d*x + 8*c)/(a*e^(6*c) + b*e^(6*c)))/d

maple [B] time = 0.43, size = 608, normalized size = 7.90

$$\frac{1}{d(2b+2a)\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)^2} + \frac{2}{d(4a+4b)\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)} - \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)a}{2d(a+b)^2} - \frac{3\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)b}{2d(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(dx+c)^2/(a+b*tanh(dx+c)^2),x)

[Out] 1/d/(2*b+2*a)/(tanh(1/2*d*x+1/2*c)-1)^2+2/d/(4*a+4*b)/(tanh(1/2*d*x+1/2*c)-1)-1/2/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)-1)*a-3/2/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)-1)*b-1/d/(2*b+2*a)/(tanh(1/2*d*x+1/2*c)+1)^2+2/d/(4*a+4*b)/(tanh(1/2*d*x+1/2*c)+1)+1/2/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)+1)*a+3/2/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)+1)*b-1/d*a*b^2/(a+b)^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*b^2/(a+b)^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/d*b^3/(a+b)^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/d*a*b^2/(a+b)^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/d*b^2/(a+b)^2/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/d*b^3/(a+b)^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))

maxima [B] time = 0.62, size = 316, normalized size = 4.10

$$\frac{b \log\left((a+b)e^{4dx+4c} + 2(a-b)e^{2dx+2c} + a+b\right)}{4(a^2+2ab+b^2)d} - \frac{b \log\left(2(a-b)e^{-2dx-2c} + (a+b)e^{-4dx-4c} + a+b\right)}{4(a^2+2ab+b^2)d} - \frac{(ab)^2}{4(a^2+2ab+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(dx+c)^2/(a+b*tanh(dx+c)^2),x, algorithm="maxima")

```
[Out] 1/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/4*(a*b - b^2)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d) + 1/4*(a*b - b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d) - 1/2*b*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a + b)*d) + 1/2*(d*x + c)/((a + b)*d) + 1/8*e^(2*d*x + 2*c)/((a + b)*d) - 1/8*e^(-2*d*x - 2*c)/((a + b)*d)
```

mupad [B] time = 1.93, size = 880, normalized size = 11.43

$$\frac{e^{2c+2dx}}{8d(a+b)} - \frac{e^{-2c-2dx}}{8d(a+b)} + \frac{x(a+3b)}{2(a+b)^2} + \frac{\operatorname{atan}\left(\left(e^{2c} e^{2dx} \left(\frac{2\left(2b^3 \sqrt{a^5 d^2 + 4a^4 b d^2 + 6a^3 b^2 d^2 + 4a^2 b^3 d^2 + a b^4 d^2} + 2ab^2 \sqrt{a^5 d^2 + 4a^4 b d^2 + 6a^3 b^2 d^2 + 4a^2 b^3 d^2 + a b^4 d^2}\right)}{d(a+b)^5 \sqrt{b^3} (a^3 + 3a^2 b + 3ab^2 + b^3)} \sqrt{a^5 d^2 + 4a^4 b d^2 + 6a^3 b^2 d^2 + 4a^2 b^3 d^2 + a b^4 d^2}\right)}\right)}{2(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2), x)
```

```
[Out] exp(2*c + 2*d*x)/(8*d*(a + b)) - exp(- 2*c - 2*d*x)/(8*d*(a + b)) + (x*(a + 3*b))/(2*(a + b)^2) + (atan((exp(2*c)*exp(2*d*x))*((2*(2*b^3*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) + 2*a*b^2*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)))/(d*(a + b)^5*(b^3)^(1/2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)) - ((a - b)*(2*a*d*(b^3)^(3/2) + b*d*(b^3)^(3/2) - a^4*d*(b^3)^(1/2) - 2*a^3*b*d*(b^3)^(1/2)))/(b^2*(a + b)^3*(a*d^2*(a + b)^4)^(1/2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))) + ((a - b)*(4*a*d*(b^3)^(3/2) + b*d*(b^3)^(3/2) + a^4*d*(b^3)^(1/2) + 4*a^3*b*d*(b^3)^(1/2) + 6*a^2*b^2*d*(b^3)^(1/2)))/(b^2*(a + b)^3*(a*d^2*(a + b)^4)^(1/2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)))*((a^4*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))/2 + (b^4*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))/2 + 3*a^2*b^2*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) + 2*a*b^3*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)))*(b^3)^(1/2))/(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(c + dx)}{a + b \tanh^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*tanh(d*x+c)**2), x)
```

```
[Out] Integral(cosh(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

$$3.108 \quad \int \frac{\cosh(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=53

$$\frac{\sinh(c+dx)}{d(a+b)} + \frac{b \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} d(a+b)^{3/2}}$$

[Out] $\sinh(d*x+c)/(a+b)/d+b*\arctan(\sinh(d*x+c)*(a+b)^{(1/2)/a^{(1/2)})/(a+b)^{(3/2)/d/a^{(1/2)}}$

Rubi [A] time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3676, 388, 205}

$$\frac{\sinh(c+dx)}{d(a+b)} + \frac{b \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2), x]

[Out] $(b*\text{ArcTan}[(\text{Sqrt}[a + b]*\text{Sinh}[c + d*x])/(\text{Sqrt}[a])]) / (\text{Sqrt}[a]*(a + b)^{(3/2)*d}) + \text{Sinh}[c + d*x] / ((a + b)*d)$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p+1))/(b*(n*(p+1)+1)), x] - Dist[(a*d - b*c*(n*(p+1)+1))/(b*(n*(p+1)+1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p+1)+1, 0]

Rule 3676

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2)], x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\cosh(c+dx)}{a+b \tanh^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{a+(a+b)x^2} dx, x, \sinh(c+dx)\right)}{d} \\ &= \frac{\sinh(c+dx)}{(a+b)d} + \frac{b \text{Subst}\left(\int \frac{1}{a+(a+b)x^2} dx, x, \sinh(c+dx)\right)}{(a+b)d} \\ &= \frac{b \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} (a+b)^{3/2} d} + \frac{\sinh(c+dx)}{(a+b)d} \end{aligned}$$

Mathematica [A] time = 0.11, size = 53, normalized size = 1.00

$$\frac{\sinh(c + dx)}{d(a + b)} + \frac{b \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} d(a + b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2), x]

[Out] (b*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(3/2)*d) + Sinh[c + d*x]/((a + b)*d)

fricas [B] time = 0.45, size = 766, normalized size = 14.45

$$\left[\frac{(a^2 + ab) \cosh(dx + c)^2 + 2(a^2 + ab) \cosh(dx + c) \sinh(dx + c) + (a^2 + ab) \sinh(dx + c)^2 - \sqrt{-a^2 - ab} (b \cosh(dx + c) + \sinh(dx + c))}{(a + b)^2 \cosh(dx + c)^2 + 4(a + b) \cosh(dx + c) \sinh(dx + c) + (a + b)^2 \sinh(dx + c)^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] [1/2*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 - sqrt(-a^2 - a*b)*(b*cosh(d*x + c) + b*sinh(d*x + c))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b) - a^2 - a*b)/((a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c) + (a^3 + 2*a^2*b + a*b^2)*d*sinh(d*x + c)), 1/2*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + 2*sqrt(a^2 + a*b)*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + 2*sqrt(a^2 + a*b)*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/a - a^2 - a*b)/((a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c) + (a^3 + 2*a^2*b + a*b^2)*d*sinh(d*x + c)))]

giac [B] time = 0.27, size = 1117, normalized size = 21.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2), x, algorithm="giac")

[Out] -1/2*(2*(2*(2*a^2*b^2 - (a^2*b - a*b^2)*sqrt(-a*b))*sqrt(a^2 - b^2 + 2*sqrt(-a*b)*(a + b))*(a*e^(2*c) + b*e^(2*c))^2*abs(a*e^(2*c) + b*e^(2*c)) - (a^4*b + a^3*b^2 - a^2*b^3 - a*b^4 + 2*(a^3*b + 2*a^2*b^2 + a*b^3)*sqrt(-a*b))*sqrt(a^2 - b^2 + 2*sqrt(-a*b)*(a + b))*abs(a*e^(2*c) + b*e^(2*c))*abs(-a*e^(2*c) - b*e^(2*c))*e^(2*c) - (2*a^4*b^2 + 2*a^3*b^3 - 2*a^2*b^4 - 2*a*b^5 - (a^4*b - 2*a^2*b^3 + b^5)*sqrt(-a*b))*sqrt(a^2 - b^2 + 2*sqrt(-a*b)*(a + b))*abs(a*e^(2*c) + b*e^(2*c))*e^(4*c))*arctan(e^(d*x)/sqrt((a^2*e^(2*c) - b

$$\begin{aligned} & \frac{2}{d(2b+2a)\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} - \frac{b \operatorname{arctanh}\left(\frac{a \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}}\right)}{d(a+b)\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}} + \frac{b^2 \operatorname{arctanh}\left(\frac{a \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}}\right)}{d(a+b)\sqrt{b(a+b)}\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}} \\ & \frac{e^{2dx+2c} - 1}{2(ad e^c + bde^c)} + \frac{1}{2} \int \frac{4\left(b e^{3dx+3c} + b e^{(dx+c)}\right)}{a^2 + 2ab + b^2 + \left(a^2 e^{4c} + 2abe^{4c} + b^2 e^{4c}\right)e^{4dx} + 2\left(a^2 e^{2c} - b^2 e^{2c}\right)e^{2dx}} dx \end{aligned}$$

maple [B] time = 0.38, size = 315, normalized size = 5.94

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/(a+b*tanh(d*x+c)^2), x)

[Out]
$$\begin{aligned} & -2/d/(2*b+2*a)/(tanh(1/2*d*x+1/2*c)+1)-1/d*b/(a+b)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*b^2/(a+b)/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*b/(a+b)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/d*b^2/(a+b)/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-2/d/(2*b+2*a)/(tanh(1/2*d*x+1/2*c)-1) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2), x, algorithm="maxima")

[Out]
$$\frac{1}{2}*(e^{2*d*x + 2*c} - 1)*e^{-d*x}/(a*d*e^c + b*d*e^c) + \frac{1}{2}*integrate(4*(b*e^{3*d*x + 3*c} + b*e^{(d*x + c)})/(a^2 + 2*a*b + b^2 + (a^2*e^{4*c} + 2*a*b*e^{4*c} + b^2*e^{4*c})*e^{4*d*x} + 2*(a^2*e^{2*c} - b^2*e^{2*c})*e^{2*d*x}), x)$$

mupad [B] time = 1.68, size = 154, normalized size = 2.91

$$\frac{e^{c+dx}}{2d(a+b)} - \frac{e^{-c-dx}}{2d(a+b)} - \frac{b \ln\left(\sqrt{-a} \sqrt{a+b} + 2ae^{c+dx} - \sqrt{-a} e^{2c+2dx} \sqrt{a+b}\right)}{2\sqrt{-a} d(a+b)^{3/2}} + \frac{b \ln\left(2ae^{c+dx} - \sqrt{-a} \sqrt{a+b}\right)}{2\sqrt{-a} d(a+b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2),x)
```

```
[Out] exp(c + d*x)/(2*d*(a + b)) - exp(- c - d*x)/(2*d*(a + b)) - (b*log((-a)^(1/2)*(a + b)^(1/2) + 2*a*exp(c + d*x) - (-a)^(1/2)*exp(2*c + 2*d*x)*(a + b)^(1/2)))/(2*(-a)^(1/2)*d*(a + b)^(3/2)) + (b*log(2*a*exp(c + d*x) - (-a)^(1/2)*(a + b)^(1/2) + (-a)^(1/2)*exp(2*c + 2*d*x)*(a + b)^(1/2)))/(2*(-a)^(1/2)*d*(a + b)^(3/2))
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{\cosh(c + dx)}{a + b \tanh^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(cosh(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

$$3.109 \quad \int \frac{\operatorname{sech}(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=36

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} d \sqrt{a+b}}$$

[Out] arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/d/a^(1/2)/(a+b)^(1/2)

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3676, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} d \sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2), x]

[Out] ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]*d)

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3676

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_.))^p, x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}(c+dx)}{a+b \tanh^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{a+(a+b)x^2} dx, x, \sinh(c+dx)\right)}{d} \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{a+b} d} \end{aligned}$$

Mathematica [A] time = 0.05, size = 36, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} d \sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2), x]

[Out] ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]*d)

fricas [B] time = 0.42, size = 511, normalized size = 14.19

$$\left[\frac{\sqrt{-a^2 - ab} \log \left(\frac{(a+b) \cosh(dx+c)^4 + 4(a+b) \cosh(dx+c) \sinh(dx+c)^3 + (a+b) \sinh(dx+c)^4 - 2(3a+b) \cosh(dx+c)^2 + 2(3(a+b) \cosh(dx+c)^2 - 3a - b) \sinh(dx+c)}{(a+b) \cosh(dx+c)^4 + 4(a+b) \cosh(dx+c) \sinh(dx+c)^3 + (a+b) \sinh(dx+c)^4} \right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")

[Out] [-1/2*sqrt(-a^2 - a*b)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b))/((a^2 + a*b)*d), (sqrt(a^2 + a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + sqrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/a))/((a^2 + a*b)*d)]

giac [B] time = 0.18, size = 480, normalized size = 13.33

$$\frac{(a^3 - 10a^2b + 5ab^2 + (5a^2 - 10ab + b^2)\sqrt{-ab})\sqrt{a^2 - b^2 + 2\sqrt{-ab}(a+b)} |ae^{(2c)} + be^{(2c)}| \arctan \left(\frac{e^{(dx)}}{\sqrt{\frac{ae^{(2c)} - be^{(2c)} + \sqrt{(ae^{(2c)} - be^{(2c)})^2 - (ae^{(4c)} + be^{(4c)})(a+b)}}{ae^{(4c)} + be^{(4c)}}}} \right) e^{(-2c)}}{a^6 - 13a^5b - 14a^4b^2 + 14a^3b^3 + 13a^2b^4 - ab^5 + 2(3a^5 - 4a^4b - 14a^3b^2 - 4a^2b^3 + 3ab^4)\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] ((a^3 - 10*a^2*b + 5*a*b^2 + (5*a^2 - 10*a*b + b^2)*sqrt(-a*b))*sqrt(a^2 - b^2 + 2*sqrt(-a*b)*(a + b))*abs(a*e^(2*c) + b*e^(2*c))*arctan(e^(d*x)/sqrt((a*e^(2*c) - b*e^(2*c) + sqrt((a*e^(2*c) - b*e^(2*c))^2 - (a*e^(4*c) + b*e^(4*c))*(a + b)))/(a*e^(4*c) + b*e^(4*c))))*e^(-2*c)/(a^6 - 13*a^5*b - 14*a^4*b^2 + 14*a^3*b^3 + 13*a^2*b^4 - a*b^5 + 2*(3*a^5 - 4*a^4*b - 14*a^3*b^2 - 4*a^2*b^3 + 3*a*b^4)*sqrt(-a*b)) + (a^3 - 10*a^2*b + 5*a*b^2 - (5*a^2 - 10*a*b + b^2)*sqrt(-a*b))*sqrt(a^2 - b^2 - 2*sqrt(-a*b)*(a + b))*abs(a*e^(2*c) + b*e^(2*c))*arctan(e^(d*x)/sqrt((a*e^(2*c) - b*e^(2*c) - sqrt((a*e^(2*c) - b*e^(2*c))^2 - (a*e^(4*c) + b*e^(4*c))*(a + b)))/(a*e^(4*c) + b*e^(4*c))))*e^(-2*c)/(a^6 - 13*a^5*b - 14*a^4*b^2 + 14*a^3*b^3 + 13*a^2*b^4 - a*b^5 - 2*(3*a^5 - 4*a^4*b - 14*a^3*b^2 - 4*a^2*b^3 + 3*a*b^4)*sqrt(-a*b)))/d

maple [B] time = 0.32, size = 235, normalized size = 6.53

$$\frac{\operatorname{arctanh} \left(\frac{a \tanh \left(\frac{dx}{2} + \frac{c}{2} \right)}{\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}} \right)}{d\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}} + \frac{\operatorname{arctanh} \left(\frac{a \tanh \left(\frac{dx}{2} + \frac{c}{2} \right)}{\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}} \right) b}{d\sqrt{b(a+b)}\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}} + \frac{\operatorname{arctan} \left(\frac{a \tanh \left(\frac{dx}{2} + \frac{c}{2} \right)}{\sqrt{(2\sqrt{b(a+b)} + a + 2b)a}} \right)}{d\sqrt{(2\sqrt{b(a+b)} + a + 2b)a}} + \frac{\operatorname{arctan} \left(\frac{a \tanh \left(\frac{dx}{2} + \frac{c}{2} \right)}{\sqrt{(2\sqrt{b(a+b)} + a + 2b)a}} \right)}{d\sqrt{b(a+b)}\sqrt{(2\sqrt{b(a+b)} + a + 2b)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x)

[Out]
$$-1/d/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}))+1/d/(b*(a+b))^{1/2}/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}))+b+1/d/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}))+1/d/(b*(a+b))^{1/2}/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}))*b$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(dx+c)}{b \tanh(dx+c)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] integrate(sech(d*x + c)/(b*tanh(d*x + c)^2 + a), x)

mupad [B] time = 0.34, size = 147, normalized size = 4.08

$$\frac{\operatorname{atan}\left(\frac{4a^2d^2e^{dx}e^c - e^{dx}e^c\sqrt{a^2d^2+bad^2}\sqrt{ad^2(a+b)} + e^{3c}e^{3dx}\sqrt{a^2d^2+bad^2}\sqrt{ad^2(a+b)}}{2ad\sqrt{ad^2(a+b)}}\right) + \operatorname{atan}\left(\frac{e^{dx}e^c\sqrt{ad^2(a+b)}}{2ad}\right)}{\sqrt{a^2d^2+bad^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)),x)

[Out]
$$\left(\operatorname{atan}\left(\frac{4a^2d^2\exp(dx)\exp(c) - \exp(dx)\exp(c)(a^2d^2 + a*b*d^2)^{1/2}}{(a*d^2*(a + b))^{1/2} + \exp(3*c)*\exp(3*d*x)*(a^2d^2 + a*b*d^2)^{1/2}}\right) + \operatorname{atan}\left(\frac{\exp(dx)\exp(c)*\sqrt{a*d^2*(a + b)}}{2*a*d}\right)\right) / (a^2*d^2 + a*b*d^2)^{1/2}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)**2),x)

[Out] Integral(sech(c + d*x)/(a + b*tanh(c + d*x)**2), x)

$$3.110 \quad \int \frac{\operatorname{sech}^2(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=32

$$\frac{\tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} d}$$

[Out] arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/d/a^(1/2)/b^(1/2)

Rubi [A] time = 0.06, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3675, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2), x]

[Out] ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m-1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2-1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}^2(c+dx)}{a+b \tanh^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} d} \end{aligned}$$

Mathematica [A] time = 0.06, size = 32, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} \sqrt{b} d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2), x]

[Out] ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)

fricas [B] time = 0.46, size = 455, normalized size = 14.22

$$\sqrt{-ab} \log \left(\frac{(a^2+2ab+b^2) \cosh(dx+c)^4 + 4(a^2+2ab+b^2) \cosh(dx+c) \sinh(dx+c)^3 + (a^2+2ab+b^2) \sinh(dx+c)^4 + 2(a^2-b^2) \cosh(dx+c)^2 + 2(3(a+b) \cosh(dx+c)^4 + 4(a+b) \cosh(dx+c) \sinh(dx+c)^3 + (a-b) \cosh(dx+c)^2 + 2(a+b) \cosh(dx+c) \sinh(dx+c) + (a+b) \sinh(dx+c)^2 + a - b) \sqrt{-ab}}{(a+b) \cosh(dx+c)^4 + 4(a+b) \cosh(dx+c) \sinh(dx+c)^3 + (a-b) \cosh(dx+c)^2 + 2(a+b) \cosh(dx+c) \sinh(dx+c) + (a+b) \sinh(dx+c)^2 + a - b) \sqrt{-ab}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")

[Out]
$$\frac{-1/2 \sqrt{-ab} \log \left(\frac{(a^2 + 2ab + b^2) \cosh(dx+c)^4 + 4(a^2 + 2ab + b^2) \cosh(dx+c) \sinh(dx+c)^3 + (a^2 + 2ab + b^2) \sinh(dx+c)^4 + 2(a^2 - b^2) \cosh(dx+c)^2 + 2(3(a+b) \cosh(dx+c)^4 + 4(a+b) \cosh(dx+c) \sinh(dx+c)^3 + (a-b) \cosh(dx+c)^2 + 2(a+b) \cosh(dx+c) \sinh(dx+c) + (a+b) \sinh(dx+c)^2 + a - b) \sqrt{-ab}}{(a+b) \cosh(dx+c)^4 + 4(a+b) \cosh(dx+c) \sinh(dx+c)^3 + (a-b) \cosh(dx+c)^2 + 2(a+b) \cosh(dx+c) \sinh(dx+c) + (a+b) \sinh(dx+c)^2 + a - b) \sqrt{-ab}} \right)}{(a+b) \cosh(dx+c)^4 + 4(a+b) \cosh(dx+c) \sinh(dx+c)^3 + (a-b) \cosh(dx+c)^2 + 2(a+b) \cosh(dx+c) \sinh(dx+c) + (a+b) \sinh(dx+c)^2 + a - b) \sqrt{-ab}}$$

giac [A] time = 0.33, size = 44, normalized size = 1.38

$$\frac{\arctan \left(\frac{ae^{(2dx+2c)} + be^{(2dx+2c)} + a - b}{2\sqrt{ab}} \right)}{\sqrt{ab}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out]
$$\arctan \left(\frac{1/2(ae^{(2dx+2c)} + be^{(2dx+2c)} + a - b)/\sqrt{ab}}{\sqrt{ab}d} \right)$$

maple [B] time = 0.36, size = 363, normalized size = 11.34

$$\frac{a \operatorname{arctanh} \left(\frac{a \tanh \left(\frac{dx+c}{2} \right)}{\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}} \right)}{d\sqrt{b(a+b)} \sqrt{(2\sqrt{b(a+b)} - a - 2b)a}} + \frac{\operatorname{arctanh} \left(\frac{a \tanh \left(\frac{dx+c}{2} \right)}{\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}} \right)}{d\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}} - \frac{\operatorname{arctanh} \left(\frac{a \tanh \left(\frac{dx+c}{2} \right)}{\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}} \right) b}{d\sqrt{b(a+b)} \sqrt{(2\sqrt{b(a+b)} - a - 2b)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x)

[Out]
$$\frac{-1/d*a/(b*(a+b))^{1/2}/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2})+1/d/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2})-1/d/(b*(a+b))^{1/2}/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2})*b-1/d*a/(b*(a+b))^{1/2}/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2})-1/d/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2})-1/d/(b*(a+b))^{1/2}/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2})*b}{(b*(a+b))^{1/2}/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2})+1/d/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2})-1/d/(b*(a+b))^{1/2}/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2})*b-1/d*a/(b*(a+b))^{1/2}/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2})-1/d/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2})-1/d/(b*(a+b))^{1/2}/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2})*b}$$

maxima [A] time = 0.52, size = 36, normalized size = 1.12

$$\frac{\arctan \left(\frac{(a+b)e^{(-2dx-2c)} + a - b}{2\sqrt{ab}} \right)}{\sqrt{ab}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] -arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*d)

mupad [B] time = 1.46, size = 81, normalized size = 2.53

$$\frac{\operatorname{atan}\left(\frac{a\sqrt{abd^2}-b\sqrt{abd^2}+ae^{2c}e^{2dx}\sqrt{abd^2}+be^{2c}e^{2dx}\sqrt{abd^2}}{2abd}\right)}{\sqrt{abd^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)),x)

[Out] atan((a*(a*b*d^2)^(1/2) - b*(a*b*d^2)^(1/2) + a*exp(2*c)*exp(2*d*x)*(a*b*d^2)^(1/2) + b*exp(2*c)*exp(2*d*x)*(a*b*d^2)^(1/2))/(2*a*b*d))/(a*b*d^2)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(c + dx)}{a + b \tanh^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)

[Out] Integral(sech(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)

$$3.111 \quad \int \frac{\operatorname{sech}^3(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=55

$$\frac{\sqrt{a+b} \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} bd} - \frac{\tan^{-1}(\sinh(c+dx))}{bd}$$

[Out] $-\arctan(\sinh(d*x+c))/b/d+\arctan(\sinh(d*x+c)*(a+b)^{(1/2)/a^{(1/2)}}*(a+b)^{(1/2)}/b/d/a^{(1/2)})$

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3676, 391, 203, 205}

$$\frac{\sqrt{a+b} \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} bd} - \frac{\tan^{-1}(\sinh(c+dx))}{bd}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2), x]

[Out] $-(\text{ArcTan}[\text{Sinh}[c + d*x]]/(b*d)) + (\text{Sqrt}[a + b]*\text{ArcTan}[(\text{Sqrt}[a + b]*\text{Sinh}[c + d*x])/(\text{Sqrt}[a])]) / (\text{Sqrt}[a]*b*d)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 391

Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]

Rule 3676

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b \tanh^2(c+dx)} dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+(a+b)x^2)} dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(c+dx)\right)}{bd} + \frac{(a+b) \operatorname{Subst}\left(\int \frac{1}{a+(a+b)x^2} dx, x, \sinh(c+dx)\right)}{bd}$$

$$= -\frac{\tan^{-1}(\sinh(c+dx))}{bd} + \frac{\sqrt{a+b} \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} bd}$$

Mathematica [A] time = 0.21, size = 55, normalized size = 1.00

$$\frac{\frac{\sqrt{a+b} \tan^{-1}\left(\frac{\sqrt{a} \operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}} + 2 \tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{bd}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2), x]

[Out] -(((Sqrt[a + b]*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/Sqrt[a] + 2*ArcTan[Tanh[(c + d*x)/2]])/(b*d))

fricas [B] time = 0.45, size = 540, normalized size = 9.82

$$\left[\sqrt{-\frac{a+b}{a}} \log\left(\frac{(a+b) \cosh(dx+c)^4 + 4(a+b) \cosh(dx+c) \sinh(dx+c)^3 + (a+b) \sinh(dx+c)^4 - 2(3a+b) \cosh(dx+c)^2 + 2(3(a+b) \cosh(dx+c)^2 - 3a-b) \sinh(dx+c)}{(a+b) \cosh(dx+c)^4 + 4(a+b) \cosh(dx+c) \sinh(dx+c)^3 + (a+b) \sinh(dx+c)^4}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] [1/2*(sqrt(-(a + b)/a)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c))*sqrt(-(a + b)/a) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 4*arctan(cosh(d*x + c) + sinh(d*x + c)))/(b*d), (sqrt((a + b)/a)*arctan(1/2*sqrt((a + b)/a)*(cosh(d*x + c) + sinh(d*x + c))) + sqrt((a + b)/a)*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))*sqrt((a + b)/a)/(a + b)) - 2*arctan(cosh(d*x + c) + sinh(d*x + c)))/(b*d)]

giac [B] time = 0.40, size = 563, normalized size = 10.24

$$\left(2 \sqrt{a^2-b^2+2 \sqrt{-ab}(a+b)} ab^2 |ae^{(2c)}+be^{(2c)}| + \sqrt{a^2-b^2+2 \sqrt{-ab}(a+b)} \sqrt{-ab}(a+b) |ae^{(2c)}+be^{(2c)}| |b| - (ab^2-b^3) \sqrt{a^2-b^2+2 \sqrt{-ab}(a+b)} |ae^{(2c)}+be^{(2c)}| \right) \operatorname{arctan}$$

$$(a^3b+3a^2b^2+3ab^3+b^4) \sqrt{-ab} |b|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out]
$$\begin{aligned} & ((2*\sqrt{a^2 - b^2 + 2*\sqrt{-a*b}*(a + b)})*a*b^2*abs(a*e^{(2*c)} + b*e^{(2*c)}) \\ & + \sqrt{a^2 - b^2 + 2*\sqrt{-a*b}*(a + b)}*\sqrt{-a*b}*(a + b)*abs(a*e^{(2*c)} \\ & + b*e^{(2*c)})*abs(b) - (a*b^2 - b^3)*\sqrt{a^2 - b^2 + 2*\sqrt{-a*b}*(a + b)}* \\ & abs(a*e^{(2*c)} + b*e^{(2*c)}))*\arctan(e^{(d*x)}/\sqrt{(a*b*e^{(2*c)} - b^2*e^{(2*c)} \\ & + \sqrt{(a*b*e^{(2*c)} - b^2*e^{(2*c)})^2 - (a*b*e^{(4*c)} + b^2*e^{(4*c)})*(a*b + b^2)})) \\ & + \sqrt{(a*b*e^{(2*c)} - b^2*e^{(2*c)})^2 - (a*b*e^{(4*c)} + b^2*e^{(4*c)})*(a*b + b^2)})) \\ & *e^{(-2*c)}/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*\sqrt{-a*b}*abs(b)) - (2*\sqrt{a^2 - b^2 - 2*\sqrt{-a*b}*(a + b)})*a*b^2 \\ & *abs(a*e^{(2*c)} + b*e^{(2*c)}) - \sqrt{a^2 - b^2 - 2*\sqrt{-a*b}*(a + b)}*\sqrt{-a*b}*(a + b) \\ & *abs(a*e^{(2*c)} + b*e^{(2*c)})*abs(b) - (a*b^2 - b^3)*\sqrt{a^2 - b^2 - 2*\sqrt{-a*b}*(a + b)} \\ & *abs(a*e^{(2*c)} + b*e^{(2*c)}))*\arctan(e^{(d*x)}/\sqrt{(a*b*e^{(2*c)} - b^2*e^{(2*c)} - \sqrt{(a*b*e^{(2*c)} - b^2*e^{(2*c)})^2 - (a*b*e^{(4*c)} + b^2*e^{(4*c)})*(a*b + b^2)})) \\ & + \sqrt{(a*b*e^{(2*c)} - b^2*e^{(2*c)})^2 - (a*b*e^{(4*c)} + b^2*e^{(4*c)})*(a*b + b^2)})) \\ & *e^{(-2*c)}/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*\sqrt{-a*b}*abs(b)) - 2*\arctan(e^{(d*x + c)})/b)/d \end{aligned}$$

maple [B] time = 0.34, size = 494, normalized size = 8.98

$$\frac{a \operatorname{arctanh}\left(\frac{a \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}}\right)}{db\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}} + \frac{a \operatorname{arctanh}\left(\frac{a \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}}\right)}{d\sqrt{b(a+b)}\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}} + \frac{a \operatorname{arctan}\left(\frac{a \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(2\sqrt{b(a+b)} + a + 2b)a}}\right)}{db\sqrt{(2\sqrt{b(a+b)} + a + 2b)a}} + \frac{a \operatorname{arctan}\left(\frac{a \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(2\sqrt{b(a+b)} + a + 2b)a}}\right)}{d\sqrt{b(a+b)}\sqrt{(2\sqrt{b(a+b)} + a + 2b)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2),x)

[Out]
$$\begin{aligned} & -1/d*a/b/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/ \\ & ((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)})+1/d*a/(b*(a+b))^{(1/2)}/((2*(b*(a+b))^{(1/2)}-a-2*b) \\ & *a)^{(1/2)}+1/d*a/b/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\operatorname{arctan}(a*\tanh(1/2*d \\ & *x+1/2*c)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)})+1/d*a/(b*(a+b))^{(1/2)}/((2*(b \\ & *(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}+a+2*b) \\ & *a)^{(1/2)})-1/d/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}*\operatorname{arctanh}(a*\tanh \\ & (1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)})+1/d/(b*(a+b))^{(1/2)}/((\\ & 2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b) \\ &))^{(1/2)}-a-2*b)*a)^{(1/2)})*b+1/d/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\operatorname{arctan} \\ & (a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)})+1/d/(b*(a+b))^{(1/2)}/((2*(b \\ & *(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b \\ & *(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)})*b-2/d/b*\operatorname{arctan}(\tanh(1/2*d*x+1/2*c)) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2 \operatorname{arctan}\left(e^{(dx+c)}\right)}{bd} + 8 \int \frac{(ae^{(3c)} + be^{(3c)})e^{(3dx)} + (ae^c + be^c)e^{(dx)}}{4(ab + b^2 + (abe^{(4c)} + b^2e^{(4c)})e^{(4dx)} + 2(abe^{(2c)} - b^2e^{(2c)})e^{(2dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out]
$$\begin{aligned} & -2*\operatorname{arctan}(e^{(d*x + c)})/(b*d) + 8*\operatorname{integrate}(1/4*((a*e^{(3*c)} + b*e^{(3*c)})*e^{(3*d*x)} \\ & + (a*e^c + b*e^c)*e^{(d*x)})/(a*b + b^2 + (a*b*e^{(4*c)} + b^2*e^{(4*c)})* \\ & e^{(4*d*x)} + 2*(a*b*e^{(2*c)} - b^2*e^{(2*c)})*e^{(2*d*x)}), x) \end{aligned}$$

mupad [B] time = 1.70, size = 449, normalized size = 8.16

$$\sqrt{a+b} \left(2 \operatorname{atan} \left(\frac{e^{dx} e^c \sqrt{a+b} \sqrt{ab^2 d^2}}{2abd} \right) - 2 \operatorname{atan} \left(\frac{e^{dx} e^c \left(\frac{64(2ab^2 d \sqrt{a+b} - 6a^2 b d \sqrt{a+b})}{a^3 b^3 d^2 (a+b)^2 (a^2 + 2ab + b^2)} + \frac{32(3a^2 \sqrt{ab^2 d^2} - b^2 \sqrt{ab^2 d^2} + 2ab \sqrt{ab^2 d^2})}{a^3 b^2 d (a+b)^{3/2} (a^2 + 2ab + b^2) \sqrt{ab^2 d^2}} \right)}{32e^{3c}} \right) \right) \frac{1}{2\sqrt{ab^2 d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)), x)`

[Out] $((a + b)^{1/2} * (2 * \operatorname{atan}((\exp(dx) * \exp(c) * (a + b)^{1/2} * (a * b^2 * d^2)^{1/2})) / (2 * a * b * d)) - 2 * \operatorname{atan}(((\exp(dx) * \exp(c) * (64 * (2 * a * b^2 * d * (a + b)^{1/2} - 6 * a^2 * b * d * (a + b)^{1/2})) / (a^3 * b^3 * d^2 * (a + b)^2 * (2 * a * b + a^2 + b^2)) + (32 * (3 * a^2 * (a * b^2 * d^2)^{1/2} - b^2 * (a * b^2 * d^2)^{1/2} + 2 * a * b * (a * b^2 * d^2)^{1/2})) / (a^3 * b^2 * d * (a + b)^{3/2} * (2 * a * b + a^2 + b^2) * (a * b^2 * d^2)^{1/2})) - (32 * \exp(3 * c) * \exp(3 * d * x) * (3 * a^2 * (a * b^2 * d^2)^{1/2} - b^2 * (a * b^2 * d^2)^{1/2} + 2 * a * b * (a * b^2 * d^2)^{1/2})) / (a^3 * b^2 * d * (a + b)^{3/2} * (2 * a * b + a^2 + b^2) * (a * b^2 * d^2)^{1/2})) * (a^4 * b * (a + b) * (a * b^2 * d^2)^{1/2} + a^2 * b^3 * (a + b) * (a * b^2 * d^2)^{1/2} + 2 * a^3 * b^2 * (a + b) * (a * b^2 * d^2)^{1/2})) / (192 * a - 64 * b))) / (2 * (a * b^2 * d^2)^{1/2})) - (2 * \operatorname{atan}((\exp(dx) * \exp(c) * (9 * a^2 * (b^2 * d^2)^{1/2} + b^2 * (b^2 * d^2)^{1/2} - 6 * a * b * (b^2 * d^2)^{1/2})) / (b^3 * d - 6 * a * b^2 * d + 9 * a^2 * b * d))) / (b^2 * d^2)^{1/2}))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(c + dx)}{a + b \tanh^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)**3/(a+b*tanh(d*x+c)**2), x)`

[Out] `Integral(sech(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)`

$$3.112 \quad \int \frac{\operatorname{sech}^4(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=50

$$\frac{(a+b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} b^{3/2} d} - \frac{\tanh(c+dx)}{bd}$$

[Out] (a+b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/b^(3/2)/d/a^(1/2)-tanh(d*x+c)/b/d

Rubi [A] time = 0.07, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3675, 388, 205}

$$\frac{(a+b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} b^{3/2} d} - \frac{\tanh(c+dx)}{bd}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2), x]

[Out] ((a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d) - Tanh[c + d*x]/(b*d)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p+1))/(b*(n*(p+1)+1)), x] - Dist[(a*d - b*c*(n*(p+1)+1))/(b*(n*(p+1)+1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p+1)+1, 0]

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m-1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2-1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegerQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}^4(c+dx)}{a+b \tanh^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{a+bx^2} dx, x, \tanh(c+dx)\right)}{d} \\ &= -\frac{\tanh(c+dx)}{bd} + \frac{(a+b) \operatorname{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \tanh(c+dx)\right)}{bd} \\ &= \frac{(a+b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} b^{3/2} d} - \frac{\tanh(c+dx)}{bd} \end{aligned}$$

Mathematica [A] time = 0.15, size = 50, normalized size = 1.00

$$\frac{(a+b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} b^{3/2} d} - \frac{\tanh(c+dx)}{bd}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2), x]

[Out] ((a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d) - Tanh[c + d*x]/(b*d)

fricas [B] time = 0.44, size = 649, normalized size = 12.98

$$\left[\frac{\left((a+b) \cosh(dx+c)^2 + 2(a+b) \cosh(dx+c) \sinh(dx+c) + (a+b) \sinh(dx+c)^2 + a+b \right) \sqrt{-ab} \log\left(\frac{a^2+2ab+b^2}{(a+b) \cosh(dx+c)^2 + 2(a+b) \cosh(dx+c) \sinh(dx+c) + (a+b) \sinh(dx+c)^2 + a+b}\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] [-1/2*(((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a + b)*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b)))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b) - 4*a*b)/(a*b^2*d*cosh(d*x + c)^2 + 2*a*b^2*d*cosh(d*x + c)*sinh(d*x + c) + a*b^2*d*sinh(d*x + c)^2 + a*b^2*d), ((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a + b)*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b)) + 2*a*b)/(a*b^2*d*cosh(d*x + c)^2 + 2*a*b^2*d*cosh(d*x + c)*sinh(d*x + c) + a*b^2*d*sinh(d*x + c)^2 + a*b^2*d)]

giac [A] time = 0.34, size = 84, normalized size = 1.68

$$\frac{\frac{(ae^{2c}+be^{2c}) \arctan\left(\frac{ae^{2dx+2c}+be^{2dx+2c}+a-b}{2\sqrt{ab}}\right)e^{-2c}}{\sqrt{ab}b} + \frac{2}{b(e^{2dx+2c}+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2), x, algorithm="giac")

[Out] ((a*e^(2*c) + b*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))*e^(-2*c)/(sqrt(a*b)*b) + 2/(b*(e^(2*d*x + 2*c) + 1)))/d

maple [B] time = 0.30, size = 648, normalized size = 12.96

$$\frac{a^2 \operatorname{arctanh}\left(\frac{a \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}}\right)}{db\sqrt{b(a+b)}\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}} + \frac{a \operatorname{arctanh}\left(\frac{a \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}}\right)}{db\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}} - \frac{2a \operatorname{arctanh}\left(\frac{a \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}}\right)}{d\sqrt{b(a+b)}\sqrt{(2\sqrt{b(a+b)} - a - 2b)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2),x)`

[Out]
$$-1/d*a^2/b/(b*(a+b))^{1/2}/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}*\operatorname{arctanh}(a*\operatorname{tanh}(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2})+1/d*a/b/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}*\operatorname{arctanh}(a*\operatorname{tanh}(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2})-2/d*a/(b*(a+b))^{1/2}/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}*\operatorname{arctanh}(a*\operatorname{tanh}(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2})-1/d*a^2/b/(b*(a+b))^{1/2}/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}*\operatorname{arctan}(a*\operatorname{tanh}(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2})-1/d*a/b/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}*\operatorname{arctan}(a*\operatorname{tanh}(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2})+1/d/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}*\operatorname{arctanh}(a*\operatorname{tanh}(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2})-1/d/(b*(a+b))^{1/2}/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}*\operatorname{arctanh}(a*\operatorname{tanh}(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2})*b-1/d/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}*\operatorname{arctan}(a*\operatorname{tanh}(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2})-1/d/(b*(a+b))^{1/2}/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}*\operatorname{arctan}(a*\operatorname{tanh}(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2})*b-2/d/b*\operatorname{tanh}(1/2*d*x+1/2*c)/(\operatorname{tanh}(1/2*d*x+1/2*c)^2+1)$$

maxima [A] time = 0.45, size = 63, normalized size = 1.26

$$\frac{(a+b)\operatorname{arctan}\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}bd}-\frac{2}{(be^{(-2dx-2c)}+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")`

[Out]
$$-(a+b)\operatorname{arctan}(1/2*((a+b)*e^{(-2*d*x-2*c)}+a-b)/\operatorname{sqrt}(a*b))/(\operatorname{sqrt}(a*b)*b*d)-2/((b*e^{(-2*d*x-2*c)}+b)*d)$$

mupad [B] time = 1.61, size = 176, normalized size = 3.52

$$\frac{2}{bd(e^{2c+2dx}+1)}+\frac{\ln\left(-\frac{4e^{2c+2dx}}{b}-\frac{2(ad+bd+ade^{2c+2dx}-bde^{2c+2dx})}{\sqrt{-a}b^{3/2}d}\right)(a+b)}{2\sqrt{-a}b^{3/2}d}-\frac{\ln\left(\frac{2(ad+bd+ade^{2c+2dx}-bde^{2c+2dx})}{\sqrt{-a}b^{3/2}d}\right)}{2\sqrt{-a}b^{3/2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(cosh(c+d*x)^4*(a+b*tanh(c+d*x)^2)),x)`

[Out]
$$2/(b*d*(\exp(2*c+2*d*x)+1))+(\log(-(4*\exp(2*c+2*d*x))/b-(2*(a*d+b*d+a*d*\exp(2*c+2*d*x)-b*d*\exp(2*c+2*d*x)))/((-a)^{1/2}*b^{3/2}*d)))/(2*(-a)^{1/2}*b^{3/2}*d)-(\log((2*(a*d+b*d+a*d*\exp(2*c+2*d*x))-b*d*\exp(2*c+2*d*x)))/((-a)^{1/2}*b^{3/2}*d)-(4*\exp(2*c+2*d*x))/b*(a+b))/(2*(-a)^{1/2}*b^{3/2}*d)$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)`

[Out] `Integral(sech(c+d*x)**4/(a+b*tanh(c+d*x)**2),x)`

$$3.113 \quad \int \frac{\operatorname{sech}^5(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=86

$$\frac{(a+b)^{3/2} \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} b^2 d} - \frac{(2a+3b) \tan^{-1}(\sinh(c+dx))}{2b^2 d} - \frac{\tanh(c+dx) \operatorname{sech}(c+dx)}{2bd}$$

[Out] $-1/2*(2*a+3*b)*\arctan(\sinh(d*x+c))/b^2/d+(a+b)^{(3/2)*\arctan(\sinh(d*x+c)*(a+b)^{(1/2)/a^{(1/2)})/b^2/d/a^{(1/2)}-1/2*\operatorname{sech}(d*x+c)*\tanh(d*x+c)/b/d}$

Rubi [A] time = 0.12, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3676, 414, 522, 203, 205}

$$\frac{(a+b)^{3/2} \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} b^2 d} - \frac{(2a+3b) \tan^{-1}(\sinh(c+dx))}{2b^2 d} - \frac{\tanh(c+dx) \operatorname{sech}(c+dx)}{2bd}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2), x]

[Out] $-\left(\frac{(2a+3b)\operatorname{ArcTan}[\operatorname{Sinh}[c+d*x]]}{2b^2d} + \frac{(a+b)^{3/2}\operatorname{ArcTan}\left[\frac{\sqrt{a+b}\operatorname{Sinh}[c+d*x]}{\sqrt{a}}\right]}{\sqrt{a}b^2d} - \frac{\operatorname{Sech}[c+d*x]\operatorname{Tanh}[c+d*x]}{2bd}\right)$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 3676

Int[sec[(e_) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*

$x^2)^{(m+n*p+1)/2}, x], x, \text{Sin}[e+f*x]/ff], x]] /; \text{FreeQ}[\{a, b, e, f\}, x] \&\& \text{IntegerQ}[(m-1)/2] \&\& \text{IntegerQ}[n/2] \&\& \text{IntegerQ}[p]$

Rubi steps

$$\begin{aligned} \int \frac{\text{sech}^5(c+dx)}{a+b \tanh^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2(a+(a+b)x^2)} dx, x, \sinh(c+dx)\right)}{d} \\ &= -\frac{\text{sech}(c+dx) \tanh(c+dx)}{2bd} + \frac{\text{Subst}\left(\int \frac{a+2b+(-a-b)x^2}{(1+x^2)(a+(a+b)x^2)} dx, x, \sinh(c+dx)\right)}{2bd} \\ &= -\frac{\text{sech}(c+dx) \tanh(c+dx)}{2bd} + \frac{(a+b)^2 \text{Subst}\left(\int \frac{1}{a+(a+b)x^2} dx, x, \sinh(c+dx)\right)}{b^2d} \\ &= -\frac{(2a+3b) \tan^{-1}(\sinh(c+dx))}{2b^2d} + \frac{(a+b)^{3/2} \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} b^2d} - \frac{\text{sech}(c+dx)}{b^2d} \end{aligned}$$

Mathematica [A] time = 0.60, size = 79, normalized size = 0.92

$$\frac{2(2a+3b) \tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + \frac{2(a+b)^{3/2} \tan^{-1}\left(\frac{\sqrt{a} \text{csch}(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}} + b \tanh(c+dx) \text{sech}(c+dx)}{2b^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2), x]

[Out] $-\frac{1}{2} * ((2 * (a + b)^{(3/2)} * \text{ArcTan}[(\text{Sqrt}[a] * \text{Csch}[c + d*x]) / \text{Sqrt}[a + b]]) / \text{Sqrt}[a] + 2 * (2 * a + 3 * b) * \text{ArcTan}[\text{Tanh}[(c + d*x) / 2]] + b * \text{Sech}[c + d*x] * \text{Tanh}[c + d*x]) / (b^2 * d)$

fricas [B] time = 0.47, size = 1584, normalized size = 18.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] $[-1/2 * (2 * b * \cosh(d*x + c)^3 + 6 * b * \cosh(d*x + c) * \sinh(d*x + c)^2 + 2 * b * \sinh(d*x + c)^3 - ((a + b) * \cosh(d*x + c)^4 + 4 * (a + b) * \cosh(d*x + c) * \sinh(d*x + c)^3 + (a + b) * \sinh(d*x + c)^4 + 2 * (a + b) * \cosh(d*x + c)^2 + 2 * (3 * (a + b) * \cosh(d*x + c)^2 + a + b) * \sinh(d*x + c)^2 + 4 * ((a + b) * \cosh(d*x + c)^3 + (a + b) * \cosh(d*x + c)) * \sinh(d*x + c) + a + b) * \sqrt{-(a + b) / a} * \log(((a + b) * \cosh(d*x + c)^4 + 4 * (a + b) * \cosh(d*x + c) * \sinh(d*x + c)^3 + (a + b) * \sinh(d*x + c)^4 - 2 * (3 * a + b) * \cosh(d*x + c)^2 + 2 * (3 * (a + b) * \cosh(d*x + c)^2 - 3 * a - b) * \sinh(d*x + c)^2 + 4 * ((a + b) * \cosh(d*x + c)^3 - (3 * a + b) * \cosh(d*x + c)) * \sinh(d*x + c) + 4 * (a * \cosh(d*x + c)^3 + 3 * a * \cosh(d*x + c) * \sinh(d*x + c)^2 + a * \sinh(d*x + c)^3 - a * \cosh(d*x + c) + (3 * a * \cosh(d*x + c)^2 - a) * \sinh(d*x + c))) * \sqrt{-(a + b) / a} + a + b) / ((a + b) * \cosh(d*x + c)^4 + 4 * (a + b) * \cosh(d*x + c) * \sinh(d*x + c)^3 + (a + b) * \sinh(d*x + c)^4 + 2 * (a - b) * \cosh(d*x + c)^2 + 2 * (3 * (a + b) * \cosh(d*x + c)^2 + a - b) * \sinh(d*x + c)^2 + 4 * ((a + b) * \cosh(d*x + c)^3 + (a - b) * \cosh(d*x + c)) * \sinh(d*x + c) + a + b) + 2 * ((2 * a + 3 * b) * \cosh(d*x + c)^4 + 4 * (2 * a + 3 * b) * \cosh(d*x + c) * \sinh(d*x + c)^3 + (2 * a + 3 * b) * \sinh(d*x + c)^4 + 2 * (2 * a + 3 * b) * \cosh(d*x + c)^2 + 2 * (3 * (2 * a + 3 * b) * \cosh(d*x + c)^2 + 2 * a + 3 * b) * \sinh(d*x + c)^2 + 4 * ((2 * a + 3 * b) * \cosh(d*x + c)^3 + ($

$2*a + 3*b)*\cosh(d*x + c))*\sinh(d*x + c) + 2*a + 3*b)*\arctan(\cosh(d*x + c) + \sinh(d*x + c)) - 2*b*\cosh(d*x + c) + 2*(3*b*\cosh(d*x + c)^2 - b)*\sinh(d*x + c))/(b^2*d*\cosh(d*x + c)^4 + 4*b^2*d*\cosh(d*x + c)*\sinh(d*x + c)^3 + b^2*d*\sinh(d*x + c)^4 + 2*b^2*d*\cosh(d*x + c)^2 + b^2*d + 2*(3*b^2*d*\cosh(d*x + c)^2 + b^2*d)*\sinh(d*x + c)^2 + 4*(b^2*d*\cosh(d*x + c)^3 + b^2*d*\cosh(d*x + c))*\sinh(d*x + c)), -(b*\cosh(d*x + c)^3 + 3*b*\cosh(d*x + c)*\sinh(d*x + c)^2 + b*\sinh(d*x + c)^3 - ((a + b)*\cosh(d*x + c)^4 + 4*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a + b)*\sinh(d*x + c)^4 + 2*(a + b)*\cosh(d*x + c)^2 + 2*(3*(a + b)*\cosh(d*x + c)^2 + a + b)*\sinh(d*x + c)^2 + 4*((a + b)*\cosh(d*x + c)^3 + (a + b)*\cosh(d*x + c))*\sinh(d*x + c) + a + b)*\sqrt((a + b)/a)*\arctan(1/2*\sqrt((a + b)/a)*(cosh(d*x + c) + sinh(d*x + c))) - ((a + b)*\cosh(d*x + c)^4 + 4*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a + b)*\sinh(d*x + c)^4 + 2*(a + b)*\cosh(d*x + c)^2 + 2*(3*(a + b)*\cosh(d*x + c)^2 + a + b)*\sinh(d*x + c)^2 + 4*((a + b)*\cosh(d*x + c)^3 + (a + b)*\cosh(d*x + c))*\sinh(d*x + c) + a + b)*\sqrt((a + b)/a)*\arctan(1/2*((a + b)*\cosh(d*x + c)^3 + 3*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a + b)*\sinh(d*x + c)^3 + (3*a - b)*\cosh(d*x + c) + (3*(a + b)*\cosh(d*x + c)^2 + 3*a - b)*\sinh(d*x + c))*\sqrt((a + b)/a)/(a + b)) + ((2*a + 3*b)*\cosh(d*x + c)^4 + 4*(2*a + 3*b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (2*a + 3*b)*\sinh(d*x + c)^4 + 2*(2*a + 3*b)*\cosh(d*x + c)^2 + 2*(3*(2*a + 3*b)*\cosh(d*x + c)^2 + 2*a + 3*b)*\sinh(d*x + c)^2 + 4*((2*a + 3*b)*\cosh(d*x + c)^3 + (2*a + 3*b)*\cosh(d*x + c))*\sinh(d*x + c) + 2*a + 3*b)*\arctan(cosh(d*x + c) + sinh(d*x + c)) - b*\cosh(d*x + c) + (3*b*\cosh(d*x + c)^2 - b)*\sinh(d*x + c))/(b^2*d*\cosh(d*x + c)^4 + 4*b^2*d*\cosh(d*x + c)*\sinh(d*x + c)^3 + b^2*d*\sinh(d*x + c)^4 + 2*b^2*d*\cosh(d*x + c)^2 + b^2*d + 2*(3*b^2*d*\cosh(d*x + c)^2 + b^2*d)*\sinh(d*x + c)^2 + 4*(b^2*d*\cosh(d*x + c)^3 + b^2*d*\cosh(d*x + c))*\sinh(d*x + c))]$

giac [B] time = 0.40, size = 428, normalized size = 4.98

$$\frac{\sqrt{a^2-b^2+2\sqrt{-ab}(a+b)}(a+\sqrt{-ab})|ae^{(2c)}+be^{(2c)}|\arctan\left(\frac{e^{(dx)}}{\sqrt{\frac{ab^2e^{(2c)}-b^3e^{(2c)}+\sqrt{(ab^2e^{(2c)}-b^3e^{(2c)})^2-(ab^2e^{(4c)}+b^3e^{(4c)})(ab^2+b^3)}}}{ab^2e^{(4c)}+b^3e^{(4c)}}}\right)}e^{(-2c)} + \sqrt{a^2-b^2-2\sqrt{-ab}(a+b)}}{a^2b^2-ab^3+2\sqrt{-ab}ab^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
[Out] (sqrt(a^2 - b^2 + 2*sqrt(-a*b)*(a + b))*(a + sqrt(-a*b))*abs(a*e^(2*c) + b*e^(2*c))*arctan(e^(d*x)/sqrt((a*b^2*e^(2*c) - b^3*e^(2*c) + sqrt((a*b^2*e^(2*c) - b^3*e^(2*c))^2 - (a*b^2*e^(4*c) + b^3*e^(4*c))*(a*b^2 + b^3)))/(a*b^2*e^(4*c) + b^3*e^(4*c))))*e^(-2*c)/(a^2*b^2 - a*b^3 + 2*sqrt(-a*b)*a*b^2) + sqrt(a^2 - b^2 - 2*sqrt(-a*b)*(a + b))*(a - sqrt(-a*b))*abs(a*e^(2*c) + b*e^(2*c))*arctan(e^(d*x)/sqrt((a*b^2*e^(2*c) - b^3*e^(2*c) - sqrt((a*b^2*e^(2*c) - b^3*e^(2*c))^2 - (a*b^2*e^(4*c) + b^3*e^(4*c))*(a*b^2 + b^3)))/(a*b^2*e^(4*c) + b^3*e^(4*c))))*e^(-2*c)/(a^2*b^2 - a*b^3 - 2*sqrt(-a*b)*a*b^2) - (2*a*e^c + 3*b*e^c)*arctan(e^(d*x + c))*e^(-c)/b^2 - (e^(3*d*x + 3*c) - e^(d*x + c))/(b*(e^(2*d*x + 2*c) + 1)^2)/d

maple [B] time = 0.33, size = 836, normalized size = 9.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2),x)
[Out] -1/d*a^2/b^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-2/d*a/b/((2*(b*(a+b))^(1/2)-a-2*b)

$$\begin{aligned}
& *a)^{(1/2)} * \operatorname{arctanh}(a * \tanh(1/2 * d * x + 1/2 * c) / ((2 * (b * (a + b))^{(1/2)} - a - 2 * b) * a)^{(1/2)} \\
&) - 1/d / ((2 * (b * (a + b))^{(1/2)} - a - 2 * b) * a)^{(1/2)} * \operatorname{arctanh}(a * \tanh(1/2 * d * x + 1/2 * c) / ((2 \\
& * (b * (a + b))^{(1/2)} - a - 2 * b) * a)^{(1/2)}) + 1/d * a^2/b / (b * (a + b))^{(1/2)} / ((2 * (b * (a + b))^{(1/2)} - a - 2 * \\
& b) * a)^{(1/2)} * \operatorname{arctanh}(a * \tanh(1/2 * d * x + 1/2 * c) / ((2 * (b * (a + b))^{(1/2)} - a - 2 * b) * a)^{(1/2)}) + 2/d * a / (b * (a + b))^{(1/2)} / ((2 * (b * (a + b))^{(1/2)} - a - 2 * b) * a)^{(1/2)} * \operatorname{arctan} \\
& \operatorname{anh}(a * \tanh(1/2 * d * x + 1/2 * c) / ((2 * (b * (a + b))^{(1/2)} - a - 2 * b) * a)^{(1/2)}) + 1/d / (b * (a + b) \\
&)^{(1/2)} / ((2 * (b * (a + b))^{(1/2)} - a - 2 * b) * a)^{(1/2)} * \operatorname{arctanh}(a * \tanh(1/2 * d * x + 1/2 * c) / ((2 * (b * (a + b))^{(1/2)} - a - 2 * b) * a)^{(1/2)}) * b + 1/d * a^2/b^2 / ((2 * (b * (a + b))^{(1/2)} + a + 2 * b) \\
& * a)^{(1/2)} * \operatorname{arctan}(a * \tanh(1/2 * d * x + 1/2 * c) / ((2 * (b * (a + b))^{(1/2)} + a + 2 * b) * a)^{(1/2)}) \\
&) + 2/d * a/b / ((2 * (b * (a + b))^{(1/2)} + a + 2 * b) * a)^{(1/2)} * \operatorname{arctan}(a * \tanh(1/2 * d * x + 1/2 * c) / \\
& ((2 * (b * (a + b))^{(1/2)} + a + 2 * b) * a)^{(1/2)}) + 1/d / ((2 * (b * (a + b))^{(1/2)} + a + 2 * b) * a)^{(1/2)} \\
&) * \operatorname{arctan}(a * \tanh(1/2 * d * x + 1/2 * c) / ((2 * (b * (a + b))^{(1/2)} + a + 2 * b) * a)^{(1/2)}) + 1/d * a^2 \\
& / b / (b * (a + b))^{(1/2)} / ((2 * (b * (a + b))^{(1/2)} + a + 2 * b) * a)^{(1/2)} * \operatorname{arctan}(a * \tanh(1/2 * d * x \\
& + 1/2 * c) / ((2 * (b * (a + b))^{(1/2)} + a + 2 * b) * a)^{(1/2)}) + 2/d * a / (b * (a + b))^{(1/2)} / ((2 * (b * (a + b))^{(1/2)} + a + 2 * b) * a)^{(1/2)} * \operatorname{arctan}(a * \tanh(1/2 * d * x + 1/2 * c) / ((2 * (b * (a + b))^{(1/2)} + a + 2 * b) * a)^{(1/2)}) + 1/d / (b * (a + b))^{(1/2)} / ((2 * (b * (a + b))^{(1/2)} + a + 2 * b) * a)^{(1/2)} * \operatorname{arctan}(a * \tanh(1/2 * d * x + 1/2 * c) / ((2 * (b * (a + b))^{(1/2)} + a + 2 * b) * a)^{(1/2)}) * b + 1/d/b / (\tanh(1/2 * d * x + 1/2 * c)^2 + 1)^2 * \tanh(1/2 * d * x + 1/2 * c)^3 - 1/d/b / (\tanh(1/2 * d * x + 1/2 * c)^2 + 1)^2 * \tanh(1/2 * d * x + 1/2 * c) - 3/d/b * \operatorname{arctan}(\tanh(1/2 * d * x + 1/2 * c)) - 2/d/b^2 * \operatorname{arctan}(\tanh(1/2 * d * x + 1/2 * c)) * a
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{e^{(3dx+3c)} - e^{(dx+c)}}{bde^{(4dx+4c)} + 2bde^{(2dx+2c)} + bd} - \frac{(2ae^c + 3be^c) \operatorname{arctan}(e^{(dx+c)}) e^{(-c)}}{b^2d} + 32 \int \frac{(a^2e^{(3c)} + 2abe^{(3c)} + b^2e^{(3c)})e^{(3dx)}}{16(ab^2 + b^3 + (ab^2e^{(4c)} + b^3e^{(4c)}))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] $-(e^{(3dx+3c)} - e^{(dx+c)}) / (bde^{(4dx+4c)} + 2bde^{(2dx+2c)} + bd) - (2ae^c + 3be^c) \operatorname{arctan}(e^{(dx+c)}) e^{(-c)} / (b^2d) + 32 \int \operatorname{egrate}(1/16 * ((a^2e^{(3c)} + 2a * b * e^{(3c)} + b^2e^{(3c)}) * e^{(3dx)} + (a^2e^c + 2a * b * e^c + b^2e^c) * e^{(dx)}) / (a * b^2 + b^3 + (a * b^2 * e^{(4c)} + b^3 * e^{(4c)})) * e^{(4dx)} + 2 * (a * b^2 * e^{(2c)} - b^3 * e^{(2c)}) * e^{(2dx)}), x$

mupad [B] time = 2.10, size = 1012, normalized size = 11.77

$$\left(2 \operatorname{atan} \left(\frac{e^{dx} e^c \left(\frac{64 \left(12 a^2 b^4 d \sqrt{a^3+3a^2b+3ab^2+b^3} - 2ab^5d \sqrt{a^3+3a^2b+3ab^2+b^3} + 18a^3b^3d \sqrt{a^3+3a^2b+3ab^2+b^3} + 6a^4b^2d \sqrt{a^3+3a^2b+3ab^2+b^3} \right)}{a^3b^9d^2(a+b)^2(a^2+2ab+b^2)} \right)}{32(3a^5} \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)),x)

[Out] $((2 * \operatorname{atan}(((\exp(dx) * \exp(c)) * ((64 * (12 * a^2 * b^4 * d * (3 * a * b^2 + 3 * a^2 * b + a^3 + b^3)^{(1/2)} - 2 * a * b^5 * d * (3 * a * b^2 + 3 * a^2 * b + a^3 + b^3)^{(1/2)} + 18 * a^3 * b^3 * d * (3 * a * b^2 + 3 * a^2 * b + a^3 + b^3)^{(1/2)} + 6 * a^4 * b^2 * d * (3 * a * b^2 + 3 * a^2 * b + a^3 + b^3)^{(1/2)})) / (a^3 * b^9 * d^2 * (a + b)^2 * (2 * a * b + a^2 + b^2)) - (32 * (3 * a^5 * (a * b^4 * d^2)^{(1/2)} - b^5 * (a * b^4 * d^2)^{(1/2)} + 4 * a * b^4 * (a * b^4 * d^2)^{(1/2)} + 15 * a^4 * b * (a * b^4 * d^2)^{(1/2)} + 20 * a^2 * b^3 * (a * b^4 * d^2)^{(1/2)} + 27 * a^3 * b^2 * (a * b^4 * d^2)^{(1/2)})) / (a^3 * b^7 * d * ((a + b)^3)^{(1/2)} * (2 * a * b + a^2 + b^2) * (a * b^4 * d^2)^{(1/2)})) + (32 * \exp(3c) * \exp(3 * d * x) * (3 * a^5 * (a * b^4 * d^2)^{(1/2)} - b^5 * (a * b^4 * d^2)^{(1/2)} + 4 * a * b^4 * (a * b^4 * d^2)^{(1/2)} + 15 * a^4 * b * (a * b^4 * d^2)^{(1/2)} + 20 * a^2 * b^3 * (a * b^4 * d^2)^{(1/2)} + 27 * a^3 * b^2 * (a * b^4 * d^2)^{(1/2)})) / (a^3 * b^7 * d * ((a + b)^3)^{(1/2)} * (2 * a * b + a^2 + b^2) * (a * b^4 * d^2)^{(1/2)})) * (a^2 * b^7 * (a * b^4 * d^2)^{(1/2)} + 2 * a^3 * b^6 * (a * b^4 * d^2)^{(1/2)} + a^4 * b^5 * (a * b^4 * d^2)^{(1/2)})) / (384 * a * b^2 + 576 * a$

$$\begin{aligned} & ^2*b + 192*a^3 - 64*b^3)) + 2*\operatorname{atan}((\exp(d*x)*\exp(c)*(a + b)^2*(a*b^4*d^2)^{(1/2)})/(2*a*b^2*d*((a + b)^3)^{(1/2)})))*(3*a*b^2 + 3*a^2*b + a^3 + b^3)^{(1/2)} \\ &)/(2*(a*b^4*d^2)^{(1/2)}) - (\operatorname{atan}((\exp(d*x)*\exp(c)*(18*a^7*(b^4*d^2)^{(1/2)} + \\ & 3*b^7*(b^4*d^2)^{(1/2)} + 30*a^2*b^5*(b^4*d^2)^{(1/2)} + 342*a^3*b^4*(b^4*d^2)^{(1/2)} + \\ & 555*a^4*b^3*(b^4*d^2)^{(1/2)} + 396*a^5*b^2*(b^4*d^2)^{(1/2)} - 34*a*b^6*(b^4*d^2)^{(1/2)} + \\ & 135*a^6*b*(b^4*d^2)^{(1/2)})))/(b^8*d*(12*a*b + 4*a^2 + 9*b^2)^{(1/2)} - 12*a*b^7*d*(12*a*b + \\ & 4*a^2 + 9*b^2)^{(1/2)} + 18*a^2*b^6*d*(12*a*b + 4*a^2 + 9*b^2)^{(1/2)} + 102*a^3*b^5*d*(12*a*b + \\ & 4*a^2 + 9*b^2)^{(1/2)} + 117*a^4*b^4*d*(12*a*b + 4*a^2 + 9*b^2)^{(1/2)} + 54*a^5*b^3*d*(12*a*b + \\ & 4*a^2 + 9*b^2)^{(1/2)} + 9*a^6*b^2*d*(12*a*b + 4*a^2 + 9*b^2)^{(1/2)}))*(12*a*b + 4*a^2 + \\ & 9*b^2)^{(1/2)})/(b^4*d^2)^{(1/2)} - \exp(c + d*x)/(b*d*(\exp(2*c + 2*d*x) + 1)) + (2*\exp(c + d*x))/(b*d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1)) \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^5(c + dx)}{a + b \tanh^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**5/(a+b*tanh(d*x+c)**2), x)

[Out] Integral(sech(c + d*x)**5/(a + b*tanh(c + d*x)**2), x)

$$3.114 \quad \int \frac{\operatorname{sech}^6(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=75

$$\frac{(a+b)^2 \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} b^{5/2} d} - \frac{(a+2b) \tanh(c+dx)}{b^2 d} + \frac{\tanh^3(c+dx)}{3bd}$$

[Out] $(a+b)^2 \arctan(b^{(1/2)} \tanh(dx+c)/a^{(1/2)})/b^{(5/2)}/d/a^{(1/2)} - (a+2*b) \tanh(dx+c)/b^2/d + 1/3 \tanh(dx+c)^3/b/d$

Rubi [A] time = 0.09, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3675, 390, 205}

$$-\frac{(a+2b) \tanh(c+dx)}{b^2 d} + \frac{(a+b)^2 \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} b^{5/2} d} + \frac{\tanh^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2), x]

[Out] $((a+b)^2 \operatorname{ArcTan}[(\operatorname{Sqrt}[b] \operatorname{Tanh}[c+d*x])/\operatorname{Sqrt}[a]])/(\operatorname{Sqrt}[a] * b^{(5/2)} * d) - ((a+2*b) \operatorname{Tanh}[c+d*x])/(b^2 * d) + \operatorname{Tanh}[c+d*x]^3/(3*b*d)$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m-1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegerQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^6(c+dx)}{a+b \tanh^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{a+bx^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \left(-\frac{a+2b}{b^2} + \frac{x^2}{b} + \frac{a^2+2ab+b^2}{b^2(a+bx^2)}\right) dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{(a+2b) \tanh(c+dx)}{b^2 d} + \frac{\tanh^3(c+dx)}{3bd} + \frac{(a+b)^2 \operatorname{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \tanh(c+dx)\right)}{b^2 d} \\
&= \frac{(a+b)^2 \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} b^{5/2} d} - \frac{(a+2b) \tanh(c+dx)}{b^2 d} + \frac{\tanh^3(c+dx)}{3bd}
\end{aligned}$$

Mathematica [A] time = 0.40, size = 71, normalized size = 0.95

$$\frac{(a+b)^2 \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} b^{5/2} d} - \frac{\tanh(c+dx) (3a + b \operatorname{sech}^2(c+dx) + 5b)}{3b^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2), x]

[Out] ((a + b)^2*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*b^(5/2)*d) - (3*a + 5*b + b*Sech[c + d*x]^2)*Tanh[c + d*x])/(3*b^2*d)

fricas [B] time = 0.48, size = 2032, normalized size = 27.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] [1/6*(12*(a^2*b + a*b^2)*cosh(d*x + c)^4 + 48*(a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 12*(a^2*b + a*b^2)*sinh(d*x + c)^4 + 12*a^2*b + 20*a*b^2 + 24*(a^2*b + 2*a*b^2)*cosh(d*x + c)^2 + 24*(a^2*b + 2*a*b^2 + 3*(a^2*b + a*b^2))*cosh(d*x + c)^2*sinh(d*x + c)^2 - 3*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 6*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b) + 48*((a^2*b + a*b^2)*cosh(d*x + c)^3 + (a^2*b + 2*a*b^2)*cosh(d*x + c))*

$$\frac{\sinh(dx + c)}{(a^3 d \cosh(dx + c)^6 + 6 a^3 d \cosh(dx + c) \sinh(dx + c)^5 + a^3 d \sinh(dx + c)^6 + 3 a^3 d \cosh(dx + c)^4 + 3 a^3 d \cosh(dx + c)^2 + a^3 d + 3(5 a^3 d \cosh(dx + c)^2 + a^3 d) \sinh(dx + c)^4 + 4(5 a^3 d \cosh(dx + c)^3 + 3 a^3 d \cosh(dx + c)) \sinh(dx + c)^3 + 3(5 a^3 d \cosh(dx + c)^4 + 6 a^3 d \cosh(dx + c)^2 + a^3 d) \sinh(dx + c)^2 + 6(a^3 d \cosh(dx + c)^5 + 2 a^3 d \cosh(dx + c)^3 + a^3 d \cosh(dx + c)) \sinh(dx + c)}, \frac{1}{3}(6(a^2 b + a b^2) \cosh(dx + c)^4 + 24(a^2 b + a b^2) \cosh(dx + c) \sinh(dx + c)^3 + 6(a^2 b + a b^2) \sinh(dx + c)^4 + 6 a^2 b + 10 a b^2 + 12(a^2 b + 2 a a b^2) \cosh(dx + c)^2 + 12(a^2 b + 2 a a b^2 + 3(a^2 b + a b^2) \cosh(dx + c)^2) \sinh(dx + c)^2 + 3((a^2 + 2 a a b + b^2) \cosh(dx + c)^6 + 6(a^2 + 2 a a b + b^2) \cosh(dx + c) \sinh(dx + c)^5 + (a^2 + 2 a a b + b^2) \sinh(dx + c)^6 + 3(a^2 + 2 a a b + b^2) \cosh(dx + c)^4 + 3(5(a^2 + 2 a a b + b^2) \cosh(dx + c)^2 + a^2 + 2 a a b + b^2) \sinh(dx + c)^4 + 4(5(a^2 + 2 a a b + b^2) \cosh(dx + c)^3 + 3(a^2 + 2 a a b + b^2) \cosh(dx + c)) \sinh(dx + c)^3 + 3(a^2 + 2 a a b + b^2) \cosh(dx + c)^2 + 3(5(a^2 + 2 a a b + b^2) \cosh(dx + c)^4 + 6(a^2 + 2 a a b + b^2) \cosh(dx + c)^2 + a^2 + 2 a a b + b^2) \sinh(dx + c)^2 + a^2 + 2 a a b + b^2 + 6((a^2 + 2 a a b + b^2) \cosh(dx + c)^5 + 2(a^2 + 2 a a b + b^2) \cosh(dx + c) \sinh(dx + c)^3 + (a^2 + 2 a a b + b^2) \cosh(dx + c) \sinh(dx + c)) \sqrt{a b} \arctan(1/2((a + b) \cosh(dx + c)^2 + 2(a + b) \cosh(dx + c) \sinh(dx + c) + (a + b) \sinh(dx + c)^2 + a - b) \sqrt{a b} / (a b)) + 24((a^2 b + a b^2) \cosh(dx + c)^3 + (a^2 b + 2 a a b^2) \cosh(dx + c) \sinh(dx + c)) / (a^3 d \cosh(dx + c)^6 + 6 a^3 d \cosh(dx + c) \sinh(dx + c)^5 + a^3 d \sinh(dx + c)^6 + 3 a^3 d \cosh(dx + c)^4 + 3 a^3 d \cosh(dx + c)^2 + a^3 d + 3(5 a^3 d \cosh(dx + c)^2 + a^3 d) \sinh(dx + c)^4 + 4(5 a^3 d \cosh(dx + c)^3 + 3 a^3 d \cosh(dx + c)) \sinh(dx + c)^3 + 3(5 a^3 d \cosh(dx + c)^4 + 6 a^3 d \cosh(dx + c)^2 + a^3 d) \sinh(dx + c)^2 + 6(a^3 d \cosh(dx + c)^5 + 2 a^3 d \cosh(dx + c)^3 + a^3 d \cosh(dx + c)) \sinh(dx + c)]$$

giac [B] time = 0.36, size = 153, normalized size = 2.04

$$\frac{3(a^2 e^{2c} + 2 a b e^{2c} + b^2 e^{2c}) \arctan\left(\frac{a e^{2dx+2c} + b e^{2dx+2c} + a - b}{2 \sqrt{a b}}\right) e^{-2c} + \frac{2(3 a e^{4dx+4c} + 3 b e^{4dx+4c} + 6 a e^{2dx+2c} + 12 b e^{2dx+2c} + 3 a + 5 b)}{b^2 (e^{2dx+2c} + 1)^3}}{\sqrt{a b} b^2} \quad \frac{1}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)^6/(a+b*tanh(dx+c)^2),x, algorithm="giac")

[Out] 1/3*(3*(a^2*e^(2*c) + 2*a*b*e^(2*c) + b^2*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))*e^(-2*c)/(sqrt(a*b)*b^2) + 2*(3*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) + 12*b*e^(2*d*x + 2*c) + 3*a + 5*b)/(b^2*(e^(2*d*x + 2*c) + 1)^3)/d

maple [B] time = 0.35, size = 1077, normalized size = 14.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(dx+c)^6/(a+b*tanh(dx+c)^2),x)

[Out] -1/d/b^2*a^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-3/d*a^2/b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-3/d*a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d*a^2/b^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+2/d*a/b/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))

) * a^(1/2)) + 1/d / ((2 * (b * (a + b))^(1/2) - a - 2 * b) * a)^(1/2) * arctanh(a * tanh(1/2 * d * x + 1/2 * c)) / ((2 * (b * (a + b))^(1/2) - a - 2 * b) * a)^(1/2)) - 1/d / (b * (a + b))^(1/2) / ((2 * (b * (a + b))^(1/2) - a - 2 * b) * a)^(1/2) * arctanh(a * tanh(1/2 * d * x + 1/2 * c)) / ((2 * (b * (a + b))^(1/2) - a - 2 * b) * a)^(1/2)) * b - 1/d / b^2 * a^3 / (b * (a + b))^(1/2) / ((2 * (b * (a + b))^(1/2) + a + 2 * b) * a)^(1/2) * arctan(a * tanh(1/2 * d * x + 1/2 * c)) / ((2 * (b * (a + b))^(1/2) + a + 2 * b) * a)^(1/2)) - 3 / d * a^2 / b / (b * (a + b))^(1/2) / ((2 * (b * (a + b))^(1/2) + a + 2 * b) * a)^(1/2) * arctan(a * tanh(1/2 * d * x + 1/2 * c)) / ((2 * (b * (a + b))^(1/2) + a + 2 * b) * a)^(1/2)) - 3 / d * a / (b * (a + b))^(1/2) / ((2 * (b * (a + b))^(1/2) + a + 2 * b) * a)^(1/2) * arctan(a * tanh(1/2 * d * x + 1/2 * c)) / ((2 * (b * (a + b))^(1/2) + a + 2 * b) * a)^(1/2)) - 1/d * a^2 / b^2 / ((2 * (b * (a + b))^(1/2) + a + 2 * b) * a)^(1/2) * arctan(a * tanh(1/2 * d * x + 1/2 * c)) / ((2 * (b * (a + b))^(1/2) + a + 2 * b) * a)^(1/2)) - 2/d * a / b / ((2 * (b * (a + b))^(1/2) + a + 2 * b) * a)^(1/2) * arctan(a * tanh(1/2 * d * x + 1/2 * c)) / ((2 * (b * (a + b))^(1/2) + a + 2 * b) * a)^(1/2)) - 1/d / ((2 * (b * (a + b))^(1/2) + a + 2 * b) * a)^(1/2) * arctan(a * tanh(1/2 * d * x + 1/2 * c)) / ((2 * (b * (a + b))^(1/2) + a + 2 * b) * a)^(1/2)) - 1/d / (b * (a + b))^(1/2) / ((2 * (b * (a + b))^(1/2) + a + 2 * b) * a)^(1/2) * arctan(a * tanh(1/2 * d * x + 1/2 * c)) / ((2 * (b * (a + b))^(1/2) + a + 2 * b) * a)^(1/2)) * b - 2/d / b^2 / (tanh(1/2 * d * x + 1/2 * c)^2 + 1)^3 * tanh(1/2 * d * x + 1/2 * c)^5 * a - 4/d / b / (tanh(1/2 * d * x + 1/2 * c)^2 + 1)^3 * tanh(1/2 * d * x + 1/2 * c)^5 - 4/d / b^2 / (tanh(1/2 * d * x + 1/2 * c)^2 + 1)^3 * tanh(1/2 * d * x + 1/2 * c)^3 * a - 16/3/d / b / (tanh(1/2 * d * x + 1/2 * c)^2 + 1)^3 * tanh(1/2 * d * x + 1/2 * c)^3 - 2/d / b^2 / (tanh(1/2 * d * x + 1/2 * c)^2 + 1)^3 * tanh(1/2 * d * x + 1/2 * c) * a - 4/d / b / (tanh(1/2 * d * x + 1/2 * c)^2 + 1)^3 * tanh(1/2 * d * x + 1/2 * c)

maxima [B] time = 0.46, size = 140, normalized size = 1.87

$$\frac{2 \left(6(a + 2b)e^{(-2dx - 2c)} + 3(a + b)e^{(-4dx - 4c)} + 3a + 5b \right) \left(a^2 + 2ab + b^2 \right) \arctan \left(\frac{(a+b)e^{(-2dx - 2c)} + a - b}{2\sqrt{ab}} \right)}{3 \left(3b^2e^{(-2dx - 2c)} + 3b^2e^{(-4dx - 4c)} + b^2e^{(-6dx - 6c)} + b^2 \right) d} \sqrt{ab} b^2 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2), x, algorithm="maxima")

[Out] -2/3*(6*(a + 2*b)*e^(-2*d*x - 2*c) + 3*(a + b)*e^(-4*d*x - 4*c) + 3*a + 5*b) / ((3*b^2*e^(-2*d*x - 2*c) + 3*b^2*e^(-4*d*x - 4*c) + b^2*e^(-6*d*x - 6*c) + b^2)*d) - (a^2 + 2*a*b + b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b)) / (sqrt(a*b)*b^2*d)

mupad [B] time = 1.65, size = 252, normalized size = 3.36

$$\frac{4}{bd \left(2e^{2c+2dx} + e^{4c+4dx} + 1 \right)} - \frac{8}{3bd \left(3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1 \right)} + \frac{2(a+b)}{b^2 d \left(e^{2c+2dx} + 1 \right)} + \frac{\ln \left(-\frac{4e^{2c+2dx}(a-b)}{b^2} \right)}{b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)), x)

[Out] 4/(b*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - 8/(3*b*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (2*(a + b))/(b^2*d*(exp(2*c + 2*d*x) + 1)) + (log(- (4*exp(2*c + 2*d*x)*(a + b))/b^2 - (2*(a + b)*(a + b + a*exp(2*c + 2*d*x) - b*exp(2*c + 2*d*x)))/((-a)^(1/2)*b^(5/2))))*(a + b)^2/(2*(-a)^(1/2)*b^(5/2)*d) - (log((2*(a + b)*(a + b + a*exp(2*c + 2*d*x) - b*exp(2*c + 2*d*x)))/((-a)^(1/2)*b^(5/2)) - (4*exp(2*c + 2*d*x)*(a + b))/b^2)*(a + b)^2/(2*(-a)^(1/2)*b^(5/2)*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^6(c + dx)}{a + b \tanh^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**6/(a + b*tanh(c + d*x)**2), x)
```

$$3.115 \quad \int \frac{\cosh^3(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=128

$$\frac{b^2(6a+b) \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{7/2}} + \frac{b^3 \sinh(c+dx)}{2ad(a+b)^3((a+b) \sinh^2(c+dx) + a)} + \frac{\sinh^3(c+dx)}{3d(a+b)^2} + \frac{(a+3b) \sinh(c+dx)}{d(a+b)^3}$$

[Out] 1/2*b^2*(6*a+b)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(3/2)/(a+b)^(7/2)/d+(a+3*b)*sinh(d*x+c)/(a+b)^3/d+1/3*sinh(d*x+c)^3/(a+b)^2/d+1/2*b^3*sinh(d*x+c)/a/(a+b)^3/d/(a+(a+b)*sinh(d*x+c)^2)

Rubi [A] time = 0.18, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3676, 390, 385, 205}

$$\frac{b^2(6a+b) \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{7/2}} + \frac{b^3 \sinh(c+dx)}{2ad(a+b)^3((a+b) \sinh^2(c+dx) + a)} + \frac{\sinh^3(c+dx)}{3d(a+b)^2} + \frac{(a+3b) \sinh(c+dx)}{d(a+b)^3}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (b^2*(6*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*(a + b)^(7/2)*d) + ((a + 3*b)*Sinh[c + d*x])/((a + b)^3*d) + Sinh[c + d*x]^3/(3*(a + b)^2*d) + (b^3*Sinh[c + d*x])/(2*a*(a + b)^3*d*(a + (a + b)*Sinh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 3676

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(c+dx)}{(a+b \tanh^2(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{(a+(a+b)x^2)^2} dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a+3b}{(a+b)^3} + \frac{x^2}{(a+b)^2} + \frac{b^2(3a+b)+3b^2(a+b)x^2}{(a+b)^3(a+(a+b)x^2)^2}\right) dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{(a+3b) \sinh(c+dx)}{(a+b)^3 d} + \frac{\sinh^3(c+dx)}{3(a+b)^2 d} + \frac{\text{Subst}\left(\int \frac{b^2(3a+b)+3b^2(a+b)x^2}{(a+(a+b)x^2)^2} dx, x, \sinh(c+dx)\right)}{(a+b)^3 d} \\
&= \frac{(a+3b) \sinh(c+dx)}{(a+b)^3 d} + \frac{\sinh^3(c+dx)}{3(a+b)^2 d} + \frac{b^3 \sinh(c+dx)}{2a(a+b)^3 d (a+(a+b) \sinh^2(c+dx))} \\
&= \frac{b^2(6a+b) \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{7/2} d} + \frac{(a+3b) \sinh(c+dx)}{(a+b)^3 d} + \frac{\sinh^3(c+dx)}{3(a+b)^2 d} + \dots
\end{aligned}$$

Mathematica [A] time = 1.19, size = 111, normalized size = 0.87

$$\frac{-\frac{6b^2(6a+b) \tan^{-1}\left(\frac{\sqrt{a} \operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{a^{3/2}(a+b)^{7/2}} + \frac{3 \sinh(c+dx) \left(\frac{4b^3}{a((a+b) \cosh(2(c+dx))+a-b)} + 3a+11b\right)}{(a+b)^3} + \frac{\sinh(3(c+dx))}{(a+b)^2}}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]

[Out] ((-6*b^2*(6*a + b)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/(a^(3/2)*(a + b)^(7/2)) + (3*(3*a + 11*b + (4*b^3)/(a*(a - b + (a + b)*Cosh[2*(c + d*x)]))))*Sinh[c + d*x]/(a + b)^3 + Sinh[3*(c + d*x)]/(a + b)^2/(12*d)

fricas [B] time = 0.56, size = 6934, normalized size = 54.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/24*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^10 + 10*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sinh(d*x + c)^10 + (11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*cosh(d*x + c)^8 + (11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3 + 45*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^3 + (11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^6 + 2*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4 + 105*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^4 + 14*(11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^5 + 14*(11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*cosh(d*x + c)^3 + 3*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 - a^5 - 3*a^4*b - 3*a^3*b^2 - a^2*b^3 - 2*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4)*cosh(d*x + c)^4 + 2*(105*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^6 - 5*a^5

$$\begin{aligned}
& - 9a^4b + 45a^3b^2 + 37a^2b^3 - 12ab^4 + 35(11a^5 + 53a^4b + 73 \\
& a^3b^2 + 31a^2b^3) \cosh(dx + c)^4 + 15(5a^5 + 9a^4b - 45a^3b^2 - \\
& 37a^2b^3 + 12ab^4) \cosh(dx + c)^2 \sinh(dx + c)^4 + 8(15(a^5 + 3a \\
& ^4b + 3a^3b^2 + a^2b^3) \cosh(dx + c)^7 + 7(11a^5 + 53a^4b + 73a^3 \\
& b^2 + 31a^2b^3) \cosh(dx + c)^5 + 5(5a^5 + 9a^4b - 45a^3b^2 - 37a \\
& ^2b^3 + 12ab^4) \cosh(dx + c)^3 - (5a^5 + 9a^4b - 45a^3b^2 - 37a^2 \\
& b^3 + 12ab^4) \cosh(dx + c) \sinh(dx + c)^3 - (11a^5 + 53a^4b + 73a \\
& ^3b^2 + 31a^2b^3) \cosh(dx + c)^2 + (45(a^5 + 3a^4b + 3a^3b^2 + a^2 \\
& b^3) \cosh(dx + c)^8 + 28(11a^5 + 53a^4b + 73a^3b^2 + 31a^2b^3) \co \\
& sh(dx + c)^6 - 11a^5 - 53a^4b - 73a^3b^2 - 31a^2b^3 + 30(5a^5 + 9 \\
& a^4b - 45a^3b^2 - 37a^2b^3 + 12ab^4) \cosh(dx + c)^4 - 12(5a^5 + \\
& 9a^4b - 45a^3b^2 - 37a^2b^3 + 12ab^4) \cosh(dx + c)^2 \sinh(dx + c \\
&)^2 - 6((6a^2b^2 + 7ab^3 + b^4) \cosh(dx + c)^7 + 7(6a^2b^2 + 7ab \\
& ^3 + b^4) \cosh(dx + c) \sinh(dx + c)^6 + (6a^2b^2 + 7ab^3 + b^4) \sinh(\\
& dx + c)^7 + 2(6a^2b^2 - 5ab^3 - b^4) \cosh(dx + c)^5 + (12a^2b^2 - \\
& 10ab^3 - 2b^4 + 21(6a^2b^2 + 7ab^3 + b^4) \cosh(dx + c)^2) \sinh(dx \\
& + c)^5 + 5(7(6a^2b^2 + 7ab^3 + b^4) \cosh(dx + c)^3 + 2(6a^2b^2 - \\
& 5ab^3 - b^4) \cosh(dx + c)) \sinh(dx + c)^4 + (6a^2b^2 + 7ab^3 + b^4 \\
&) \cosh(dx + c)^3 + (35(6a^2b^2 + 7ab^3 + b^4) \cosh(dx + c)^4 + 6a^2 \\
& b^2 + 7ab^3 + b^4 + 20(6a^2b^2 - 5ab^3 - b^4) \cosh(dx + c)^2) \sinh \\
& (dx + c)^3 + (21(6a^2b^2 + 7ab^3 + b^4) \cosh(dx + c)^5 + 20(6a^2b \\
& ^2 - 5ab^3 - b^4) \cosh(dx + c)^3 + 3(6a^2b^2 + 7ab^3 + b^4) \cosh(dx \\
& + c)) \sinh(dx + c)^2 + (7(6a^2b^2 + 7ab^3 + b^4) \cosh(dx + c)^6 + \\
& 10(6a^2b^2 - 5ab^3 - b^4) \cosh(dx + c)^4 + 3(6a^2b^2 + 7ab^3 + b \\
& ^4) \cosh(dx + c)^2) \sinh(dx + c) \sqrt{-a^2 - ab} \log(((a + b) \cosh(dx \\
& + c)^4 + 4(a + b) \cosh(dx + c) \sinh(dx + c)^3 + (a + b) \sinh(dx + c)^4 \\
& - 2(3a + b) \cosh(dx + c)^2 + 2(3(a + b) \cosh(dx + c)^2 - 3a - b) \sin \\
& h(dx + c)^2 + 4((a + b) \cosh(dx + c)^3 - (3a + b) \cosh(dx + c)) \sinh(dx \\
& + c) - 4(\cosh(dx + c)^3 + 3 \cosh(dx + c) \sinh(dx + c)^2 + \sinh(dx + \\
& c)^3 + (3 \cosh(dx + c)^2 - 1) \sinh(dx + c) - \cosh(dx + c)) \sqrt{-a^2 - \\
& ab} + a + b) / ((a + b) \cosh(dx + c)^4 + 4(a + b) \cosh(dx + c) \sinh(dx + \\
& c)^3 + (a + b) \sinh(dx + c)^4 + 2(a - b) \cosh(dx + c)^2 + 2(3(a + b) \\
& \cosh(dx + c)^2 + a - b) \sinh(dx + c)^2 + 4((a + b) \cosh(dx + c)^3 + (a \\
& - b) \cosh(dx + c)) \sinh(dx + c) + a + b) + 2(5(a^5 + 3a^4b + 3a^3b \\
& ^2 + a^2b^3) \cosh(dx + c)^9 + 4(11a^5 + 53a^4b + 73a^3b^2 + 31a^2b \\
& b^3) \cosh(dx + c)^7 + 6(5a^5 + 9a^4b - 45a^3b^2 - 37a^2b^3 + 12a \\
& b^4) \cosh(dx + c)^5 - 4(5a^5 + 9a^4b - 45a^3b^2 - 37a^2b^3 + 12a \\
& b^4) \cosh(dx + c)^3 - (11a^5 + 53a^4b + 73a^3b^2 + 31a^2b^3) \cosh(dx \\
& + c)) \sinh(dx + c) / ((a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 + 5a^3b \\
& ^4 + a^2b^5) d \cosh(dx + c)^7 + 7(a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 \\
& + 5a^3b^4 + a^2b^5) d \cosh(dx + c) \sinh(dx + c)^6 + (a^7 + 5a^6b + \\
& 10a^5b^2 + 10a^4b^3 + 5a^3b^4 + a^2b^5) d \sinh(dx + c)^7 + 2(a^7 \\
& + 3a^6b + 2a^5b^2 - 2a^4b^3 - 3a^3b^4 - a^2b^5) d \cosh(dx + c)^5 \\
& + (21(a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 + 5a^3b^4 + a^2b^5) d \cos \\
& h(dx + c)^2 + 2(a^7 + 3a^6b + 2a^5b^2 - 2a^4b^3 - 3a^3b^4 - a^2b \\
& ^5) d) \sinh(dx + c)^5 + (a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 + 5a^3b \\
& ^4 + a^2b^5) d \cosh(dx + c)^3 + 5(7(a^7 + 5a^6b + 10a^5b^2 + 10a^4 \\
& b^3 + 5a^3b^4 + a^2b^5) d \cosh(dx + c)^3 + 2(a^7 + 3a^6b + 2a^5b^2 \\
& - 2a^4b^3 - 3a^3b^4 - a^2b^5) d \cosh(dx + c)) \sinh(dx + c)^4 + (35 \\
& (a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 + 5a^3b^4 + a^2b^5) d \cosh(dx \\
& + c)^4 + 20(a^7 + 3a^6b + 2a^5b^2 - 2a^4b^3 - 3a^3b^4 - a^2b^5) \\
& d \cosh(dx + c)^2 + (a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 + 5a^3b^4 + \\
& a^2b^5) d) \sinh(dx + c)^3 + (21(a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 \\
& + 5a^3b^4 + a^2b^5) d \cosh(dx + c)^5 + 20(a^7 + 3a^6b + 2a^5b^2 - \\
& 2a^4b^3 - 3a^3b^4 - a^2b^5) d \cosh(dx + c)^3 + 3(a^7 + 5a^6b + 10 \\
& a^5b^2 + 10a^4b^3 + 5a^3b^4 + a^2b^5) d \cosh(dx + c)) \sinh(dx + c)^ \\
& 2 + (7(a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 + 5a^3b^4 + a^2b^5) d \co \\
& sh(dx + c)^6 + 10(a^7 + 3a^6b + 2a^5b^2 - 2a^4b^3 - 3a^3b^4 - a^2 \\
& b^5) d \cosh(dx + c)^4 + 3(a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 + 5a^
\end{aligned}$$

$$\begin{aligned}
& 3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^2)*\sinh(d*x + c)), 1/24*((a^5 + 3*a^4*b + \\
& 3*a^3*b^2 + a^2*b^3)*\cosh(d*x + c)^{10} + 10*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3) \\
& *b^3)*\cosh(d*x + c)*\sinh(d*x + c)^9 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3) \\
& *\sinh(d*x + c)^{10} + (11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*\cosh(d*x \\
& + c)^8 + (11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3 + 45*(a^5 + 3*a^4*b + \\
& 3*a^3*b^2 + a^2*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 8*(15*(a^5 + 3*a^4 \\
& *b + 3*a^3*b^2 + a^2*b^3)*\cosh(d*x + c)^3 + (11*a^5 + 53*a^4*b + 73*a^3*b^2 \\
& + 31*a^2*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(5*a^5 + 9*a^4*b - 45*a^3 \\
& *b^2 - 37*a^2*b^3 + 12*a*b^4)*\cosh(d*x + c)^6 + 2*(5*a^5 + 9*a^4*b - 45*a^3 \\
& *b^2 - 37*a^2*b^3 + 12*a*b^4 + 105*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*\cosh \\
& (d*x + c)^4 + 14*(11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*\cosh(d*x + \\
& c)^2)*\sinh(d*x + c)^6 + 4*(63*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*\cosh(d \\
& *x + c)^5 + 14*(11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*\cosh(d*x + c)^ \\
& 3 + 3*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4)*\cosh(d*x + c)) \\
& *\sinh(d*x + c)^5 - a^5 - 3*a^4*b - 3*a^3*b^2 - a^2*b^3 - 2*(5*a^5 + 9*a^4*b \\
& - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4)*\cosh(d*x + c)^4 + 2*(105*(a^5 + 3*a^ \\
& 4*b + 3*a^3*b^2 + a^2*b^3)*\cosh(d*x + c)^6 - 5*a^5 - 9*a^4*b + 45*a^3*b^2 + \\
& 37*a^2*b^3 - 12*a*b^4 + 35*(11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*\c \\
& osh(d*x + c)^4 + 15*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4)* \\
& \cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(15*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b \\
& ^3)*\cosh(d*x + c)^7 + 7*(11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*\cosh(\\
& d*x + c)^5 + 5*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4)*\cosh(\\
& d*x + c)^3 - (5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37*a^2*b^3 + 12*a*b^4)*\cosh(d* \\
& x + c))*\sinh(d*x + c)^3 - (11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*\cos \\
& h(d*x + c)^2 + (45*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*\cosh(d*x + c)^8 + \\
& 28*(11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^3)*\cosh(d*x + c)^6 - 11*a^5 - \\
& 53*a^4*b - 73*a^3*b^2 - 31*a^2*b^3 + 30*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 37 \\
& *a^2*b^3 + 12*a*b^4)*\cosh(d*x + c)^4 - 12*(5*a^5 + 9*a^4*b - 45*a^3*b^2 - 3 \\
& 7*a^2*b^3 + 12*a*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 12*((6*a^2*b^2 + 7 \\
& *a*b^3 + b^4)*\cosh(d*x + c)^7 + 7*(6*a^2*b^2 + 7*a*b^3 + b^4)*\cosh(d*x + c) \\
& *\sinh(d*x + c)^6 + (6*a^2*b^2 + 7*a*b^3 + b^4)*\sinh(d*x + c)^7 + 2*(6*a^2*b \\
& ^2 - 5*a*b^3 - b^4)*\cosh(d*x + c)^5 + (12*a^2*b^2 - 10*a*b^3 - 2*b^4 + 21*(\\
& 6*a^2*b^2 + 7*a*b^3 + b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^5 + 5*(7*(6*a^2*b \\
& ^2 + 7*a*b^3 + b^4)*\cosh(d*x + c)^3 + 2*(6*a^2*b^2 - 5*a*b^3 - b^4)*\cosh(d* \\
& x + c))*\sinh(d*x + c)^4 + (6*a^2*b^2 + 7*a*b^3 + b^4)*\cosh(d*x + c)^3 + (35 \\
& *(6*a^2*b^2 + 7*a*b^3 + b^4)*\cosh(d*x + c)^4 + 6*a^2*b^2 + 7*a*b^3 + b^4 + \\
& 20*(6*a^2*b^2 - 5*a*b^3 - b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^3 + (21*(6*a^ \\
& 2*b^2 + 7*a*b^3 + b^4)*\cosh(d*x + c)^5 + 20*(6*a^2*b^2 - 5*a*b^3 - b^4)*\cos \\
& h(d*x + c)^3 + 3*(6*a^2*b^2 + 7*a*b^3 + b^4)*\cosh(d*x + c))*\sinh(d*x + c)^2 \\
& + (7*(6*a^2*b^2 + 7*a*b^3 + b^4)*\cosh(d*x + c)^6 + 10*(6*a^2*b^2 - 5*a*b^3 \\
& - b^4)*\cosh(d*x + c)^4 + 3*(6*a^2*b^2 + 7*a*b^3 + b^4)*\cosh(d*x + c)^2)*\si \\
& nh(d*x + c))*\sqrt{a^2 + a*b}*\arctan(1/2*((a + b)*\cosh(d*x + c)^3 + 3*(a + b) \\
&)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a + b)*\sinh(d*x + c)^3 + (3*a - b)*\cosh(\\
& d*x + c) + (3*(a + b)*\cosh(d*x + c)^2 + 3*a - b)*\sinh(d*x + c))/\sqrt{a^2 + \\
& a*b}) + 12*((6*a^2*b^2 + 7*a*b^3 + b^4)*\cosh(d*x + c)^7 + 7*(6*a^2*b^2 + 7* \\
& a*b^3 + b^4)*\cosh(d*x + c)*\sinh(d*x + c)^6 + (6*a^2*b^2 + 7*a*b^3 + b^4)*\si \\
& nh(d*x + c)^7 + 2*(6*a^2*b^2 - 5*a*b^3 - b^4)*\cosh(d*x + c)^5 + (12*a^2*b^2 \\
& - 10*a*b^3 - 2*b^4 + 21*(6*a^2*b^2 + 7*a*b^3 + b^4)*\cosh(d*x + c)^2)*\sinh(\\
& d*x + c)^5 + 5*(7*(6*a^2*b^2 + 7*a*b^3 + b^4)*\cosh(d*x + c)^3 + 2*(6*a^2*b^ \\
& 2 - 5*a*b^3 - b^4)*\cosh(d*x + c))*\sinh(d*x + c)^4 + (6*a^2*b^2 + 7*a*b^3 + \\
& b^4)*\cosh(d*x + c)^3 + (35*(6*a^2*b^2 + 7*a*b^3 + b^4)*\cosh(d*x + c)^4 + 6* \\
& a^2*b^2 + 7*a*b^3 + b^4 + 20*(6*a^2*b^2 - 5*a*b^3 - b^4)*\cosh(d*x + c)^2)*\s \\
& inh(d*x + c)^3 + (21*(6*a^2*b^2 + 7*a*b^3 + b^4)*\cosh(d*x + c)^5 + 20*(6*a^ \\
& 2*b^2 - 5*a*b^3 - b^4)*\cosh(d*x + c)^3 + 3*(6*a^2*b^2 + 7*a*b^3 + b^4)*\cosh \\
& (d*x + c))*\sinh(d*x + c)^2 + (7*(6*a^2*b^2 + 7*a*b^3 + b^4)*\cosh(d*x + c)^6 \\
& + 10*(6*a^2*b^2 - 5*a*b^3 - b^4)*\cosh(d*x + c)^4 + 3*(6*a^2*b^2 + 7*a*b^3 \\
& + b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c))*\sqrt{a^2 + a*b}*\arctan(1/2*\sqrt{a^2 \\
& + a*b}*(\cosh(d*x + c) + \sinh(d*x + c))/a) + 2*(5*(a^5 + 3*a^4*b + 3*a^3*b^2 \\
& + a^2*b^3)*\cosh(d*x + c)^9 + 4*(11*a^5 + 53*a^4*b + 73*a^3*b^2 + 31*a^2*b^
\end{aligned}$$

$$\begin{aligned}
& 3) * \cosh(dx + c)^7 + 6 * (5 * a^5 + 9 * a^4 * b - 45 * a^3 * b^2 - 37 * a^2 * b^3 + 12 * a * b^4) * \cosh(dx + c)^5 - 4 * (5 * a^5 + 9 * a^4 * b - 45 * a^3 * b^2 - 37 * a^2 * b^3 + 12 * a * b^4) * \cosh(dx + c)^3 - (11 * a^5 + 53 * a^4 * b + 73 * a^3 * b^2 + 31 * a^2 * b^3) * \cosh(dx + c) * \sinh(dx + c) / ((a^7 + 5 * a^6 * b + 10 * a^5 * b^2 + 10 * a^4 * b^3 + 5 * a^3 * b^4 + a^2 * b^5) * d * \cosh(dx + c)^7 + 7 * (a^7 + 5 * a^6 * b + 10 * a^5 * b^2 + 10 * a^4 * b^3 + 5 * a^3 * b^4 + a^2 * b^5) * d * \cosh(dx + c) * \sinh(dx + c)^6 + (a^7 + 5 * a^6 * b + 10 * a^5 * b^2 + 10 * a^4 * b^3 + 5 * a^3 * b^4 + a^2 * b^5) * d * \sinh(dx + c)^7 + 2 * (a^7 + 3 * a^6 * b + 2 * a^5 * b^2 - 2 * a^4 * b^3 - 3 * a^3 * b^4 - a^2 * b^5) * d * \cosh(dx + c)^5 + (21 * (a^7 + 5 * a^6 * b + 10 * a^5 * b^2 + 10 * a^4 * b^3 + 5 * a^3 * b^4 + a^2 * b^5) * d * \cosh(dx + c)^2 + 2 * (a^7 + 3 * a^6 * b + 2 * a^5 * b^2 - 2 * a^4 * b^3 - 3 * a^3 * b^4 - a^2 * b^5) * d) * \sinh(dx + c)^5 + (a^7 + 5 * a^6 * b + 10 * a^5 * b^2 + 10 * a^4 * b^3 + 5 * a^3 * b^4 + a^2 * b^5) * d * \cosh(dx + c)^3 + 5 * (7 * (a^7 + 5 * a^6 * b + 10 * a^5 * b^2 + 10 * a^4 * b^3 + 5 * a^3 * b^4 + a^2 * b^5) * d * \cosh(dx + c)^3 + 2 * (a^7 + 3 * a^6 * b + 2 * a^5 * b^2 - 2 * a^4 * b^3 - 3 * a^3 * b^4 - a^2 * b^5) * d * \cosh(dx + c)) * \sinh(dx + c)^4 + (35 * (a^7 + 5 * a^6 * b + 10 * a^5 * b^2 + 10 * a^4 * b^3 + 5 * a^3 * b^4 + a^2 * b^5) * d * \cosh(dx + c)^4 + 20 * (a^7 + 3 * a^6 * b + 2 * a^5 * b^2 - 2 * a^4 * b^3 - 3 * a^3 * b^4 - a^2 * b^5) * d * \cosh(dx + c)^2 + (a^7 + 5 * a^6 * b + 10 * a^5 * b^2 + 10 * a^4 * b^3 + 5 * a^3 * b^4 + a^2 * b^5) * d) * \sinh(dx + c)^3 + (21 * (a^7 + 5 * a^6 * b + 10 * a^5 * b^2 + 10 * a^4 * b^3 + 5 * a^3 * b^4 + a^2 * b^5) * d * \cosh(dx + c)^5 + 20 * (a^7 + 3 * a^6 * b + 2 * a^5 * b^2 - 2 * a^4 * b^3 - 3 * a^3 * b^4 - a^2 * b^5) * d * \cosh(dx + c)^3 + 3 * (a^7 + 5 * a^6 * b + 10 * a^5 * b^2 + 10 * a^4 * b^3 + 5 * a^3 * b^4 + a^2 * b^5) * d * \cosh(dx + c)) * \sinh(dx + c)^2 + (7 * (a^7 + 5 * a^6 * b + 10 * a^5 * b^2 + 10 * a^4 * b^3 + 5 * a^3 * b^4 + a^2 * b^5) * d * \cosh(dx + c)^6 + 10 * (a^7 + 3 * a^6 * b + 2 * a^5 * b^2 - 2 * a^4 * b^3 - 3 * a^3 * b^4 - a^2 * b^5) * d * \cosh(dx + c)^4 + 3 * (a^7 + 5 * a^6 * b + 10 * a^5 * b^2 + 10 * a^4 * b^3 + 5 * a^3 * b^4 + a^2 * b^5) * d * \cosh(dx + c)^2) * \sinh(dx + c))]]
\end{aligned}$$

giac [B] time = 0.94, size = 2074, normalized size = 16.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(dx+c)^3/(a+b*tanh(dx+c)^2)^2,x, algorithm="giac")

[Out]
$$\begin{aligned}
& -1/24 * ((9 * a * e^{(2 * dx + 2 * c)} + 33 * b * e^{(2 * dx + 2 * c)} + a + b) * e^{(-3 * dx)} / (a^3 * e^{(3 * c)} + 3 * a^2 * b * e^{(3 * c)} + 3 * a * b^2 * e^{(3 * c)} + b^3 * e^{(3 * c)}) + 12 * (2 * (12 * a^2 * b^3 + 2 * a * b^4 + (6 * a^2 * b^2 - 5 * a * b^3 - b^4) * \sqrt{-a * b}) * (a^4 * e^{(2 * c)} + 3 * a^3 * b * e^{(2 * c)} + 3 * a^2 * b^2 * e^{(2 * c)} + a * b^3 * e^{(2 * c)})^2 * \text{abs}(a * e^{(2 * c)} + b * e^{(2 * c)}) - (6 * a^7 * b^2 + 19 * a^6 * b^3 + 15 * a^5 * b^4 - 10 * a^4 * b^5 - 20 * a^3 * b^6 - 9 * a^2 * b^7 - a * b^8 - 2 * (6 * a^6 * b^2 + 25 * a^5 * b^3 + 40 * a^4 * b^4 + 30 * a^3 * b^5 + 10 * a^2 * b^6 + a * b^7) * \sqrt{-a * b}) * \text{abs}(-a^4 * e^{(2 * c)} - 3 * a^3 * b * e^{(2 * c)} - 3 * a^2 * b^2 * e^{(2 * c)} - a * b^3 * e^{(2 * c)}) * \text{abs}(a * e^{(2 * c)} + b * e^{(2 * c)}) * e^{(2 * c)} - (12 * a^{10} * b^3 + 62 * a^9 * b^4 + 118 * a^8 * b^5 + 78 * a^7 * b^6 - 50 * a^6 * b^7 - 118 * a^5 * b^8 - 78 * a^4 * b^9 - 22 * a^3 * b^{10} - 2 * a^2 * b^{11} + (6 * a^{10} * b^2 + 25 * a^9 * b^3 + 28 * a^8 * b^4 - 20 * a^7 * b^5 - 64 * a^6 * b^6 - 34 * a^5 * b^7 + 20 * a^4 * b^8 + 28 * a^3 * b^9 + 10 * a^2 * b^{10} + a * b^{11}) * \sqrt{-a * b}) * \text{abs}(a * e^{(2 * c)} + b * e^{(2 * c)}) * e^{(4 * c)}) * \arctan(e^{(dx)} / \sqrt{(a^5 * e^{(2 * c)} + 2 * a^4 * b * e^{(2 * c)} - 2 * a^2 * b^3 * e^{(2 * c)} - a * b^4 * e^{(2 * c)} + \sqrt{(a^5 * e^{(2 * c)} + 2 * a^4 * b * e^{(2 * c)} - 2 * a^2 * b^3 * e^{(2 * c)} - a * b^4 * e^{(2 * c)})^2 - (a^5 * e^{(4 * c)} + 4 * a^4 * b * e^{(4 * c)} + 6 * a^3 * b^2 * e^{(4 * c)} + 4 * a^2 * b^3 * e^{(4 * c)} + a * b^4 * e^{(4 * c)}) * (a^5 + 4 * a^4 * b + 6 * a^3 * b^2 + 4 * a^2 * b^3 + a * b^4))} / (a^5 * e^{(4 * c)} + 4 * a^4 * b * e^{(4 * c)} + 6 * a^3 * b^2 * e^{(4 * c)} + 4 * a^2 * b^3 * e^{(4 * c)} + a * b^4 * e^{(4 * c)})) * e^{(-4 * c)} / ((a^{11} + 9 * a^{10} * b + 36 * a^9 * b^2 + 84 * a^8 * b^3 + 126 * a^7 * b^4 + 126 * a^6 * b^5 + 84 * a^5 * b^6 + 36 * a^4 * b^7 + 9 * a^3 * b^8 + a^2 * b^9) * \sqrt{a^2 - b^2 - 2 * \sqrt{-a * b}} * (a + b)) * \text{abs}(-a^4 * e^{(2 * c)} - 3 * a^3 * b * e^{(2 * c)} - 3 * a^2 * b^2 * e^{(2 * c)} - a * b^3 * e^{(2 * c)})) + 12 * (2 * (12 * a^2 * b^3 + 2 * a * b^4 - (6 * a^2 * b^2 - 5 * a * b^3 - b^4) * \sqrt{-a * b}) * (a^4 * e^{(2 * c)} + 3 * a^3 * b * e^{(2 * c)} + 3 * a^2 * b^2 * e^{(2 * c)} + a * b^3 * e^{(2 * c)})^2 * \text{abs}(a * e^{(2 * c)} + b * e^{(2 * c)}) - (6 * a^7 * b^2 + 19 * a^6 * b^3 + 15 * a^5 * b^4 - 10 * a^4 * b^5 - 20 * a^3 * b^6 - 9 * a^2 * b^7 - a * b^8 + 2 * (6 * a^6 * b^2 + 25 * a^5 * b^3 + 40 * a^4 * b^4 + 30 * a^3 * b^5 + 10 * a^2 * b^6 + a * b^7) * \sqrt{-a * b}) * \text{abs}(-a^4 * e^{(2 * c)} - 3 * a^3 * b * e^{(2 * c)} - 3 * a^2 * b^2 * e^{(2 * c)} - a * b^3 * e^{(2 * c)}) * \text{abs}(a * e^{(2 * c)} + b * e^{(2 * c)}) * e^{(2 * c)} - (12 * a^{10} * b^3 + 62 * a^9 * b^4 + 118 * a^8 * b^5 + 78 * a^7 * b^6 -
\end{aligned}$$

$$50a^6b^7 - 118a^5b^8 - 78a^4b^9 - 22a^3b^{10} - 2a^2b^{11} - (6a^{10}b^2 + 25a^9b^3 + 28a^8b^4 - 20a^7b^5 - 64a^6b^6 - 34a^5b^7 + 20a^4b^8 + 28a^3b^9 + 10a^2b^{10} + ab^{11})\sqrt{-ab})\operatorname{abs}(ae^{(2c)} + be^{(2c)})e^{(4c)}\operatorname{arctan}(e^{(dx)}/\sqrt{(a^5e^{(2c)} + 2a^4be^{(2c)} - 2a^2b^3e^{(2c)} - ab^4e^{(2c)})^2 - (a^5e^{(4c)} + 4a^4be^{(4c)} + 6a^3b^2e^{(4c)} + 4a^2b^3e^{(4c)} + ab^4e^{(4c)})}(a^5 + 4a^4b + 6a^3b^2 + 4a^2b^3 + ab^4)))/((a^5e^{(4c)} + 4a^4be^{(4c)} + 6a^3b^2e^{(4c)} + 4a^2b^3e^{(4c)} + ab^4e^{(4c)}))e^{(-4c)}/((a^{11} + 9a^{10}b + 36a^9b^2 + 84a^8b^3 + 126a^7b^4 + 126a^6b^5 + 84a^5b^6 + 36a^4b^7 + 9a^3b^8 + a^2b^9)\sqrt{a^2 - b^2 + 2\sqrt{-ab}}(a + b))\operatorname{abs}(-a^4e^{(2c)} - 3a^3be^{(2c)} - 3a^2b^2e^{(2c)} - ab^3e^{(2c)}) - (a^4e^{(3dx + 36c)} + 4a^3be^{(3dx + 36c)} + 6a^2b^2e^{(3dx + 36c)} + 4a^2b^3e^{(3dx + 36c)} + b^4e^{(3dx + 36c)} + 9a^4e^{(dx + 34c)} + 60a^3be^{(dx + 34c)} + 126a^2b^2e^{(dx + 34c)} + 108a^2b^3e^{(dx + 34c)} + 33b^4e^{(dx + 34c)})/(a^6e^{(33c)} + 6a^5be^{(33c)} + 15a^4b^2e^{(33c)} + 20a^3b^3e^{(33c)} + 15a^2b^4e^{(33c)} + 6a^2b^5e^{(33c)} + b^6e^{(33c)}) - 24(b^3e^{(3dx + 3c)} - b^3e^{(dx + c)})/((a^4 + 3a^3b + 3a^2b^2 + ab^3)(ae^{(4dx + 4c)} + be^{(4dx + 4c)} + 2ae^{(2dx + 2c)} - 2be^{(2dx + 2c)} + a + b)))/d$$

maple [B] time = 0.52, size = 875, normalized size = 6.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (\cosh(dx+c)^3/(a+b*\tanh(dx+c)^2)^2, x)$

[Out]
$$\begin{aligned} & -1/3/d/(a+b)^2/(\tanh(1/2*dx+1/2*c)-1)^3-1/2/d/(a+b)^2/(\tanh(1/2*dx+1/2*c)-1)^2-1/d/(a+b)^3/(\tanh(1/2*dx+1/2*c)-1)*a-3/d/(a+b)^3/(\tanh(1/2*dx+1/2*c)-1)*b-1/3/d/(a+b)^2/(\tanh(1/2*dx+1/2*c)+1)^3+1/2/d/(a+b)^2/(\tanh(1/2*dx+1/2*c)+1)^2-1/d/(a+b)^3/(\tanh(1/2*dx+1/2*c)+1)*a-3/d/(a+b)^3/(\tanh(1/2*dx+1/2*c)+1)*b-1/d*b^3/(a+b)^3/(\tanh(1/2*dx+1/2*c)^4+a+2*\tanh(1/2*dx+1/2*c)^2*a+4*\tanh(1/2*dx+1/2*c)^2*b+a)/a*\tanh(1/2*dx+1/2*c)^3+1/d*b^3/(a+b)^3/(\tanh(1/2*dx+1/2*c)^4+a+2*\tanh(1/2*dx+1/2*c)^2*a+4*\tanh(1/2*dx+1/2*c)^2*b+a)/a*\tanh(1/2*dx+1/2*c)-3/d*b^2/(a+b)^3/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*dx+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/d*b^3/(a+b)^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*dx+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/d*b^2/(a+b)^3/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*dx+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+3/d*b^3/(a+b)^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*dx+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/2/d*b^4/(a+b)^3/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*dx+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2/d*b^3/(a+b)^3/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*dx+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/2/d*b^4/(a+b)^3/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*dx+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^3 + 2a^2b + ab^2 - (a^3e^{(10c)} + 2a^2be^{(10c)} + ab^2e^{(10c)})e^{(10dx)} - (11a^3e^{(8c)} + 42a^2be^{(8c)} + 31ab^2e^{(8c)})e^{(8dx)} - 24((a^5de^{(7c)} + 4a^4bde^{(7c)} + 6a^3b^2de^{(7c)} + 4a^2b^3de^{(7c)} + ab^4de^{(7c)})e^{(7dx)} + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (\cosh(dx+c)^3/(a+b*\tanh(dx+c)^2)^2, x, \text{algorithm}="maxima")$

```
[Out] -1/24*(a^3 + 2*a^2*b + a*b^2 - (a^3*e^(10*c) + 2*a^2*b*e^(10*c) + a*b^2*e^(10*c))*e^(10*d*x) - (11*a^3*e^(8*c) + 42*a^2*b*e^(8*c) + 31*a*b^2*e^(8*c))*e^(8*d*x) - 2*(5*a^3*e^(6*c) + 4*a^2*b*e^(6*c) - 49*a*b^2*e^(6*c) + 12*b^3*e^(6*c))*e^(6*d*x) + 2*(5*a^3*e^(4*c) + 4*a^2*b*e^(4*c) - 49*a*b^2*e^(4*c) + 12*b^3*e^(4*c))*e^(4*d*x) + (11*a^3*e^(2*c) + 42*a^2*b*e^(2*c) + 31*a*b^2*e^(2*c))*e^(2*d*x))/((a^5*d*e^(7*c) + 4*a^4*b*d*e^(7*c) + 6*a^3*b^2*d*e^(7*c) + 4*a^2*b^3*d*e^(7*c) + a*b^4*d*e^(7*c))*e^(7*d*x) + 2*(a^5*d*e^(5*c) + 2*a^4*b*d*e^(5*c) - 2*a^2*b^3*d*e^(5*c) - a*b^4*d*e^(5*c))*e^(5*d*x) + (a^5*d*e^(3*c) + 4*a^4*b*d*e^(3*c) + 6*a^3*b^2*d*e^(3*c) + 4*a^2*b^3*d*e^(3*c) + a*b^4*d*e^(3*c))*e^(3*d*x)) + 1/8*integrate(8*((6*a*b^2*e^(3*c) + b^3*e^(3*c))*e^(3*d*x) + (6*a*b^2*e^c + b^3*e^c)*e^(d*x))/(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + (a^5*e^(4*c) + 4*a^4*b*e^(4*c) + 6*a^3*b^2*e^(4*c) + 4*a^2*b^3*e^(4*c) + a*b^4*e^(4*c))*e^(4*d*x) + 2*(a^5*e^(2*c) + 2*a^4*b*e^(2*c) - 2*a^2*b^3*e^(2*c) - a*b^4*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^3}{(b \tanh(c + dx)^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2,x)
```

```
[Out] int(cosh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

$$3.116 \quad \int \frac{\cosh^2(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=140

$$\frac{b^{3/2}(5a+b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^3} - \frac{b(a-b) \tanh(c+dx)}{2ad(a+b)^2(a+b \tanh^2(c+dx))} + \frac{\sinh(c+dx) \cosh(c+dx)}{2d(a+b)(a+b \tanh^2(c+dx))} + \frac{x(a+b)}{2(a+b)}$$

[Out] 1/2*(a+5*b)*x/(a+b)^3+1/2*b^(3/2)*(5*a+b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/(a+b)^3/d+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c)^2)-1/2*(a-b)*b*tanh(d*x+c)/a/(a+b)^2/d/(a+b*tanh(d*x+c)^2)

Rubi [A] time = 0.19, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3675, 414, 527, 522, 206, 205}

$$\frac{b^{3/2}(5a+b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^3} - \frac{b(a-b) \tanh(c+dx)}{2ad(a+b)^2(a+b \tanh^2(c+dx))} + \frac{\sinh(c+dx) \cosh(c+dx)}{2d(a+b)(a+b \tanh^2(c+dx))} + \frac{x(a+b)}{2(a+b)}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2, x]

[Out] ((a + 5*b)*x)/(2*(a + b)^3) + (b^(3/2)*(5*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^3*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*(a + b)*d*(a + b*Tanh[c + d*x]^2)) - ((a - b)*b*Tanh[c + d*x])/(2*a*(a + b)^2*d*(a + b*Tanh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3675

```
Int[sec[(e_) + (f_)*(x_)]^(m_)*((a_) + (b_)*((c_)*tan[(e_) + (f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\cosh^2(c + dx)}{(a + b \tanh^2(c + dx))^2} dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+bx^2)^2} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{\cosh(c + dx) \sinh(c + dx)}{2(a + b)d(a + b \tanh^2(c + dx))} + \frac{\text{Subst}\left(\int \frac{a+2b+3bx^2}{(1-x^2)(a+bx^2)^2} dx, x, \tanh(c + dx)\right)}{2(a + b)d}$$

$$= \frac{\cosh(c + dx) \sinh(c + dx)}{2(a + b)d(a + b \tanh^2(c + dx))} - \frac{(a - b)b \tanh(c + dx)}{2a(a + b)^2d(a + b \tanh^2(c + dx))} - \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^2)^2} dx, x, \tanh(c + dx)\right)}{2(a + b)d}$$

$$= \frac{\cosh(c + dx) \sinh(c + dx)}{2(a + b)d(a + b \tanh^2(c + dx))} - \frac{(a - b)b \tanh(c + dx)}{2a(a + b)^2d(a + b \tanh^2(c + dx))} + \frac{(b^2(5a + b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right) + (a + 5b)x)}{2(a + b)^3} + \frac{b^{3/2}(5a + b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a + b)^3d} + \frac{\cosh(c + dx) \sinh(c + dx)}{2(a + b)d(a + b \tanh^2(c + dx))}$$

Mathematica [A] time = 0.84, size = 110, normalized size = 0.79

$$\frac{2b^{3/2}(5a+b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{2b^2(a+b) \sinh(2(c+dx))}{a((a+b) \cosh(2(c+dx))+a-b)} + \frac{2(a + 5b)(c + dx) + (a + b) \sinh(2(c + dx))}{4d(a + b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2, x]
[Out] (2*(a + 5*b)*(c + d*x) + (2*b^(3/2)*(5*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2) + (a + b)*Sinh[2*(c + d*x)] + (2*b^2*(a + b)*Sinh[2*(c + d*x)])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(4*(a + b)^3*d)
```

fricas [B] time = 0.53, size = 4324, normalized size = 30.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.


```

+ c)^2)*sinh(d*x + c)^6 + 4*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^3 +
3*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x +
c)^5 - 8*(a*b^2 - b^3 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^4 + 2
*(35*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^4 - 4*a*b^2 + 4*b^3 + 4*(a^3 + 4
*a^2*b - 5*a*b^2)*d*x + 15*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*
cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x +
c)^5 + 5*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^3 -
4*(a*b^2 - b^3 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c
)^3 - a^3 - 2*a^2*b - a*b^2 - 2*(a^3 + 3*a*b^2 + 4*b^3 - 2*(a^3 + 6*a^2*b +
5*a*b^2)*d*x)*cosh(d*x + c)^2 + 2*(14*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c
)^6 + 15*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^4 -
a^3 - 3*a*b^2 - 4*b^3 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x - 24*(a*b^2 - b^3 -
(a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((5*a^
2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^6 + 6*(5*a^2*b + 6*a*b^2 + b^3)*cosh(d*x
+ c)*sinh(d*x + c)^5 + (5*a^2*b + 6*a*b^2 + b^3)*sinh(d*x + c)^6 + 2*(5*a^
2*b - 4*a*b^2 - b^3)*cosh(d*x + c)^4 + (10*a^2*b - 8*a*b^2 - 2*b^3 + 15*(5*
a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(5*a^2*b + 6
*a*b^2 + b^3)*cosh(d*x + c)^3 + 2*(5*a^2*b - 4*a*b^2 - b^3)*cosh(d*x + c))*
sinh(d*x + c)^3 + (5*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^2 + (15*(5*a^2*b
+ 6*a*b^2 + b^3)*cosh(d*x + c)^4 + 5*a^2*b + 6*a*b^2 + b^3 + 12*(5*a^2*b -
4*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(5*a^2*b + 6*a*b^2 +
b^3)*cosh(d*x + c)^5 + 4*(5*a^2*b - 4*a*b^2 - b^3)*cosh(d*x + c)^3 + (5*a^
2*b + 6*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*((a
+ b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sin
h(d*x + c)^2 + a - b)*sqrt(b/a)/b) + 4*(2*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x
+ c)^7 + 3*(a^3 - a*b^2 + 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^5
- 8*(a*b^2 - b^3 - (a^3 + 4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^3 - (a^3 +
3*a*b^2 + 4*b^3 - 2*(a^3 + 6*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x
+ c))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^6 +
6*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x
+ c)^5 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*sinh(d*x + c)^6
+ 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^4 + (15*(a^5 + 4*a^
4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + 2*(a^5 + 2*a^4*b -
2*a^2*b^3 - a*b^4)*d)*sinh(d*x + c)^4 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2
*b^3 + a*b^4)*d*cosh(d*x + c)^2 + 4*(5*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b
^3 + a*b^4)*d*cosh(d*x + c)^3 + 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*d*cos
h(d*x + c))*sinh(d*x + c)^3 + (15*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 +
a*b^4)*d*cosh(d*x + c)^4 + 12*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*d*cosh(d*
x + c)^2 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d)*sinh(d*x + c)
^2 + 2*(3*(a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^5
+ 4*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^3 + (a^5 + 4*a^4*b
+ 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c))*sinh(d*x + c))]

```

giac [B] time = 1.10, size = 449, normalized size = 3.21

$$\frac{12(a+5b)dx}{a^3+3a^2b+3ab^2+b^3} + \frac{12(5ab^2e^{2c}+b^3e^{2c})\arctan\left(\frac{ae^{2dx+2c}+be^{2dx+2c}+a-b}{2\sqrt{ab}}\right)e^{-2c}}{(a^4+3a^3b+3a^2b^2+ab^3)\sqrt{ab}} + \frac{3e^{2dx+12c}}{a^2e^{10c}+2abe^{10c}+b^2e^{10c}} - \frac{2a^3e^{6dx+6c}+12a^2be^{6dx+6c}}{a^2e^{10c}+2abe^{10c}+b^2e^{10c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

```

[Out] 1/24*(12*(a + 5*b)*d*x/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 12*(5*a*b^2*e^(2*c)
+ b^3*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)
/sqrt(a*b))*e^(-2*c)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)) + 3*e^
(2*d*x + 12*c)/(a^2*e^(10*c) + 2*a*b*e^(10*c) + b^2*e^(10*c)) - (2*a^3*e^(6
*d*x + 6*c) + 12*a^2*b*e^(6*d*x + 6*c) + 10*a*b^2*e^(6*d*x + 6*c) + 7*a^3*e
^(4*d*x + 4*c) + 22*a^2*b*e^(4*d*x + 4*c) + 7*a*b^2*e^(4*d*x + 4*c) - 24*b^
3*e^(4*d*x + 4*c) + 8*a^3*e^(2*d*x + 2*c) + 12*a^2*b*e^(2*d*x + 2*c) + 28*a

```


$$\frac{b^2 e^{(2dx+2c)} + 24b^3 e^{(2dx+2c)} + 3a^3 + 6a^2 b + 3a b^2}{((a^4 e^{(2c)} + 3a^3 b e^{(2c)} + 3a^2 b^2 e^{(2c)} + a b^3 e^{(2c)}) (a e^{(2dx)} + b e^{(2dx)} + a e^{(6dx+4c)} + b e^{(6dx+4c)} + 2a e^{(4dx+2c)} - 2b e^{(4dx+2c)}))} / d$$

maple [B] time = 0.47, size = 1146, normalized size = 8.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(dx+c)^2/(a+b*tanh(dx+c))^2,x)

[Out] $\frac{1}{2} \frac{d}{(a+b)^2} \frac{1}{(\tanh(\frac{1}{2}dx+\frac{1}{2}c)-1)^2} + \frac{1}{2} \frac{d}{(a+b)^2} \frac{1}{(\tanh(\frac{1}{2}dx+\frac{1}{2}c)+1)^2} - \frac{1}{2} \frac{d}{(a+b)^3} \ln(\tanh(\frac{1}{2}dx+\frac{1}{2}c)-1) * a^{-5/2} \frac{d}{(a+b)^3} \ln(\tanh(\frac{1}{2}dx+\frac{1}{2}c)+1) * b^{-1/2} \frac{d}{(a+b)^2} \frac{1}{(\tanh(\frac{1}{2}dx+\frac{1}{2}c)+1)^2} + \frac{1}{2} \frac{d}{(a+b)^2} \frac{1}{(\tanh(\frac{1}{2}dx+\frac{1}{2}c)+1)} + \frac{1}{2} \frac{d}{(a+b)^3} \ln(\tanh(\frac{1}{2}dx+\frac{1}{2}c)+1) * a^{5/2} \frac{d}{(a+b)^3} \ln(\tanh(\frac{1}{2}dx+\frac{1}{2}c)+1) * b^{1/2} \frac{d}{(a+b)^3} \frac{1}{(\tanh(\frac{1}{2}dx+\frac{1}{2}c)+1)^4} * a^{2} \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{2a+4} \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{2b+a} * \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{3+1/d} * b^{3/(a+b)^3} \frac{1}{(\tanh(\frac{1}{2}dx+\frac{1}{2}c)+1)^4} * a^{2} \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{2a+4} \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{2b+a} / a * \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{3+1/d} * b^{2/(a+b)^3} \frac{1}{(\tanh(\frac{1}{2}dx+\frac{1}{2}c)+1)^4} * a^{2} \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{2a+4} \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{2b+a} * \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{1/d} * b^{3/(a+b)^3} \frac{1}{(\tanh(\frac{1}{2}dx+\frac{1}{2}c)+1)^4} * a^{2} \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{2a+4} \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{2b+a} / a * \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{-5/2} \frac{d}{(a+b)^3} \frac{a}{(b*(a+b))^{1/2}} \frac{1}{((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}} * \operatorname{arctanh}(a * \tanh(\frac{1}{2}dx+\frac{1}{2}c)) / ((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2} + \frac{5}{2} \frac{d}{(a+b)^3} \frac{1}{((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}} * \operatorname{arctanh}(a * \tanh(\frac{1}{2}dx+\frac{1}{2}c)) / ((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2} - \frac{3}{d} \frac{d}{(a+b)^3} \frac{1}{(b*(a+b))^{1/2}} \frac{1}{((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}} * a * \operatorname{rctanh}(a * \tanh(\frac{1}{2}dx+\frac{1}{2}c)) / ((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2} - \frac{5}{2} \frac{d}{(a+b)^3} \frac{a}{(b*(a+b))^{1/2}} \frac{1}{((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}} * \operatorname{arctan}(a * \tanh(\frac{1}{2}dx+\frac{1}{2}c)) / ((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2} - \frac{3}{d} \frac{d}{(a+b)^3} \frac{1}{(b*(a+b))^{1/2}} \frac{1}{((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}} * a * \operatorname{arctan}(a * \tanh(\frac{1}{2}dx+\frac{1}{2}c)) / ((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2} + \frac{1}{2} \frac{d}{(a+b)^3} \frac{a}{((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}} * \operatorname{arctanh}(a * \tanh(\frac{1}{2}dx+\frac{1}{2}c)) / ((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2} - \frac{1}{2} \frac{d}{(a+b)^3} \frac{a}{(b*(a+b))^{1/2}} \frac{1}{((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}} * \operatorname{arctanh}(a * \tanh(\frac{1}{2}dx+\frac{1}{2}c)) / ((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2} - \frac{1}{2} \frac{d}{(a+b)^3} \frac{a}{((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}} * \operatorname{arctan}(a * \tanh(\frac{1}{2}dx+\frac{1}{2}c)) / ((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2} - \frac{1}{2} \frac{d}{(a+b)^3} \frac{a}{(b*(a+b))^{1/2}} \frac{1}{((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}} * \operatorname{arctan}(a * \tanh(\frac{1}{2}dx+\frac{1}{2}c)) / ((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}$

maxima [B] time = 0.70, size = 840, normalized size = 6.00

$$\frac{b \log((a+b)e^{(4dx+4c)} + 2(a-b)e^{(2dx+2c)} + a+b)}{2(a^3 + 3a^2b + 3ab^2 + b^3)d} - \frac{b \log(2(a-b)e^{(-2dx-2c)} + (a+b)e^{(-4dx-4c)} + a+b)}{2(a^3 + 3a^2b + 3ab^2 + b^3)d} - \frac{(3a^3 + 3a^2b + 3ab^2 + b^3)d}{2(a^3 + 3a^2b + 3ab^2 + b^3)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(dx+c)^2/(a+b*tanh(dx+c))^2,x, algorithm="maxima")

[Out] $\frac{1}{2} b * \log((a+b) e^{(4dx+4c)} + 2(a-b) e^{(2dx+2c)} + a+b) / ((a^3 + 3a^2b + 3ab^2 + b^3) * d) - \frac{1}{2} b * \log(2(a-b) e^{(-2dx-2c)} + (a+b) e^{(-4dx-4c)} + a+b) / ((a^3 + 3a^2b + 3ab^2 + b^3) * d) - \frac{1}{8} * (3a^2b - 6ab^2 - b^3) * \operatorname{arctan}(1/2 * ((a+b) e^{(2dx+2c)} + a-b) / \sqrt{a*b}) / ((a^4 + 3a^3b + 3a^2b^2 + ab^3) * \sqrt{a*b} * d) + \frac{1}{8} * (3a^2b - 6ab^2 - b^3) * \operatorname{arctan}(1/2 * ((a+b) e^{(-2dx-2c)} + a-b) / \sqrt{a*b}) / ((a^4 + 3a^3b + 3a^2b^2 + ab^3) * \sqrt{a*b} * d) - \frac{1}{4} * (3ab + b^2) * \operatorname{arctan}(1/2 * ((a+b) e^{(-2dx-2c)} + a-b) / \sqrt{a*b}) / ((a^3 + 2a^2b + ab^2) * \sqrt{a*b} * d)$

$t(a*b)*d) + 1/4*(a^2*b - b^3 + (a^2*b - 6*a*b^2 + b^3)*e^{(2*d*x + 2*c)})/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*e^{(4*d*x + 4*c)} + 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*e^{(2*d*x + 2*c)})*d) - 1/4*(a^2*b - b^3 + (a^2*b - 6*a*b^2 + b^3)*e^{(-2*d*x - 2*c)})/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + 2*(a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*e^{(-2*d*x - 2*c)} + (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*e^{(-4*d*x - 4*c)})*d) + 1/2*(a*b + b^2 + (a*b - b^2)*e^{(-2*d*x - 2*c)})/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*e^{(-2*d*x - 2*c)} + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*e^{(-4*d*x - 4*c)})*d) + 1/2*(d*x + c)/((a^2 + 2*a*b + b^2)*d) + 1/8*e^{(2*d*x + 2*c)}/((a^2 + 2*a*b + b^2)*d) - 1/8*e^{(-2*d*x - 2*c)}/((a^2 + 2*a*b + b^2)*d)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^2}{(b \tanh(c + dx)^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2,x)

[Out] int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)

[Out] Timed out

$$3.117 \quad \int \frac{\cosh(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=101

$$\frac{b(4a+b) \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{5/2}} + \frac{b^2 \sinh(c+dx)}{2ad(a+b)^2((a+b) \sinh^2(c+dx) + a)} + \frac{\sinh(c+dx)}{d(a+b)^2}$$

[Out] 1/2*b*(4*a+b)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(3/2)/(a+b)^(5/2)/d +sinh(d*x+c)/(a+b)^2/d+1/2*b^2*sinh(d*x+c)/a/(a+b)^2/d/(a+(a+b)*sinh(d*x+c)^2)

Rubi [A] time = 0.14, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3676, 390, 385, 205}

$$\frac{b(4a+b) \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{5/2}} + \frac{b^2 \sinh(c+dx)}{2ad(a+b)^2((a+b) \sinh^2(c+dx) + a)} + \frac{\sinh(c+dx)}{d(a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (b*(4*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*(a + b)^(5/2)*d) + Sinh[c + d*x]/((a + b)^2*d) + (b^2*Sinh[c + d*x])/(2*a*(a + b)^2*d*(a + (a + b)*Sinh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 3676

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
& 2*b - 3*a*b^2 - b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c))*\sqrt{-a^2 - a*b}*\log((\\
& (a + b)*\cosh(d*x + c)^4 + 4*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a + b) \\
& *\sinh(d*x + c)^4 - 2*(3*a + b)*\cosh(d*x + c)^2 + 2*(3*(a + b)*\cosh(d*x + c) \\
& ^2 - 3*a - b)*\sinh(d*x + c)^2 + 4*((a + b)*\cosh(d*x + c)^3 - (3*a + b)*\cosh \\
& (d*x + c))*\sinh(d*x + c) - 4*(\cosh(d*x + c)^3 + 3*\cosh(d*x + c)*\sinh(d*x + \\
& c)^2 + \sinh(d*x + c)^3 + (3*\cosh(d*x + c)^2 - 1)*\sinh(d*x + c) - \cosh(d*x + \\
& c))*\sqrt{-a^2 - a*b} + a + b)/((a + b)*\cosh(d*x + c)^4 + 4*(a + b)*\cosh(d* \\
& x + c)*\sinh(d*x + c)^3 + (a + b)*\sinh(d*x + c)^4 + 2*(a - b)*\cosh(d*x + c)^ \\
& 2 + 2*(3*(a + b)*\cosh(d*x + c)^2 + a - b)*\sinh(d*x + c)^2 + 4*((a + b)*\cosh \\
& (d*x + c)^3 + (a - b)*\cosh(d*x + c))*\sinh(d*x + c) + a + b)) + 4*(3*(a^4 + \\
& 2*a^3*b + a^2*b^2)*\cosh(d*x + c)^5 + 2*(a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)* \\
& \cosh(d*x + c)^3 - (a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*\cosh(d*x + c))*\sinh(d \\
& *x + c))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*\cosh(d*x + c) \\
& ^5 + 5*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*\cosh(d*x + c)*\si \\
& nh(d*x + c)^4 + (a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*\sinh(d* \\
& x + c)^5 + 2*(a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*d*\cosh(d*x + c)^3 + 2*(5 \\
& *(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*\cosh(d*x + c)^2 + (a^6 \\
& + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*d)*\sinh(d*x + c)^3 + (a^6 + 4*a^5*b + 6*a \\
& ^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*\cosh(d*x + c) + 2*(5*(a^6 + 4*a^5*b + 6*a^4 \\
& *b^2 + 4*a^3*b^3 + a^2*b^4)*d*\cosh(d*x + c)^3 + 3*(a^6 + 2*a^5*b - 2*a^3*b^ \\
& 3 - a^2*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^2 + (5*(a^6 + 4*a^5*b + 6*a^4*b \\
& ^2 + 4*a^3*b^3 + a^2*b^4)*d*\cosh(d*x + c)^4 + 6*(a^6 + 2*a^5*b - 2*a^3*b^3 \\
& - a^2*b^4)*d*\cosh(d*x + c)^2 + (a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2 \\
& *b^4)*d)*\sinh(d*x + c)), 1/2*((a^4 + 2*a^3*b + a^2*b^2)*\cosh(d*x + c)^6 + 6 \\
& *(a^4 + 2*a^3*b + a^2*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^5 + (a^4 + 2*a^3*b + \\
& a^2*b^2)*\sinh(d*x + c)^6 + (a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*\cosh(d*x + \\
& c)^4 + (a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3 + 15*(a^4 + 2*a^3*b + a^2*b^2))*\co \\
& sh(d*x + c)^2)*\sinh(d*x + c)^4 - a^4 - 2*a^3*b - a^2*b^2 + 4*(5*(a^4 + 2*a^ \\
& 3*b + a^2*b^2)*\cosh(d*x + c)^3 + (a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*\cosh(d \\
& *x + c))*\sinh(d*x + c)^3 - (a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*\cosh(d*x + c \\
&)^2 + (15*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(d*x + c)^4 - a^4 + 2*a^3*b + a^2*b \\
& ^2 - 2*a*b^3 + 6*(a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*\cosh(d*x + c)^2)*\sinh(\\
& d*x + c)^2 + ((4*a^2*b + 5*a*b^2 + b^3)*\cosh(d*x + c)^5 + 5*(4*a^2*b + 5*a* \\
& b^2 + b^3)*\cosh(d*x + c)*\sinh(d*x + c)^4 + (4*a^2*b + 5*a*b^2 + b^3)*\sinh(d \\
& *x + c)^5 + 2*(4*a^2*b - 3*a*b^2 - b^3)*\cosh(d*x + c)^3 + 2*(4*a^2*b - 3*a* \\
& b^2 - b^3 + 5*(4*a^2*b + 5*a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^3 + \\
& 2*(5*(4*a^2*b + 5*a*b^2 + b^3)*\cosh(d*x + c)^3 + 3*(4*a^2*b - 3*a*b^2 - b^3 \\
&)*\cosh(d*x + c))*\sinh(d*x + c)^2 + (4*a^2*b + 5*a*b^2 + b^3)*\cosh(d*x + c) \\
& + (5*(4*a^2*b + 5*a*b^2 + b^3)*\cosh(d*x + c)^4 + 4*a^2*b + 5*a*b^2 + b^3 + \\
& 6*(4*a^2*b - 3*a*b^2 - b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c))*\sqrt{a^2 + a*b} \\
& *\arctan(1/2*((a + b)*\cosh(d*x + c)^3 + 3*(a + b)*\cosh(d*x + c)*\sinh(d*x + c \\
&)^2 + (a + b)*\sinh(d*x + c)^3 + (3*a - b)*\cosh(d*x + c) + (3*(a + b)*\cosh(d \\
& *x + c)^2 + 3*a - b)*\sinh(d*x + c))/\sqrt{a^2 + a*b})) + ((4*a^2*b + 5*a*b^2 \\
& + b^3)*\cosh(d*x + c)^5 + 5*(4*a^2*b + 5*a*b^2 + b^3)*\cosh(d*x + c)*\sinh(d*x \\
& + c)^4 + (4*a^2*b + 5*a*b^2 + b^3)*\sinh(d*x + c)^5 + 2*(4*a^2*b - 3*a*b^2 \\
& - b^3)*\cosh(d*x + c)^3 + 2*(4*a^2*b - 3*a*b^2 - b^3 + 5*(4*a^2*b + 5*a*b^2 \\
& + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^3 + 2*(5*(4*a^2*b + 5*a*b^2 + b^3)*\co \\
& sh(d*x + c)^3 + 3*(4*a^2*b - 3*a*b^2 - b^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 \\
& + (4*a^2*b + 5*a*b^2 + b^3)*\cosh(d*x + c) + (5*(4*a^2*b + 5*a*b^2 + b^3)*\co \\
& sh(d*x + c)^4 + 4*a^2*b + 5*a*b^2 + b^3 + 6*(4*a^2*b - 3*a*b^2 - b^3)*\cosh(\\
& d*x + c)^2)*\sinh(d*x + c))*\sqrt{a^2 + a*b}*\arctan(1/2*\sqrt{a^2 + a*b}*(\cosh \\
& (d*x + c) + \sinh(d*x + c))/a) + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(d*x + c \\
&)^5 + 2*(a^4 - 2*a^3*b - a^2*b^2 + 2*a*b^3)*\cosh(d*x + c)^3 - (a^4 - 2*a^3* \\
& b - a^2*b^2 + 2*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c))/((a^6 + 4*a^5*b + 6*a^ \\
& 4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*\cosh(d*x + c)^5 + 5*(a^6 + 4*a^5*b + 6*a^4*b \\
& ^2 + 4*a^3*b^3 + a^2*b^4)*d*\cosh(d*x + c)*\sinh(d*x + c)^4 + (a^6 + 4*a^5*b \\
& + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*\sinh(d*x + c)^5 + 2*(a^6 + 2*a^5*b - 2 \\
& *a^3*b^3 - a^2*b^4)*d*\cosh(d*x + c)^3 + 2*(5*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4 \\
& *a^3*b^3 + a^2*b^4)*d*\cosh(d*x + c)^2 + (a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^
\end{aligned}$$

```
4)*d)*sinh(d*x + c)^3 + (a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d
*cosh(d*x + c) + 2*(5*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*c
osh(d*x + c)^3 + 3*(a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c))*s
inh(d*x + c)^2 + (5*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d*cos
h(d*x + c)^4 + 6*(a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^2 +
(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*d)*sinh(d*x + c))]
```

giac [B] time = 0.45, size = 1838, normalized size = 18.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*(8*a^2*b^2 + 2*a*b^3 - (4*a^2*b - 3*a*b^2 - b^3)*sqrt(-a*b))*(a^3*e
^(2*c) + 2*a^2*b*e^(2*c) + a*b^2*e^(2*c))^2*sqrt(a^2 - b^2 + 2*sqrt(-a*b)*(
a + b))*abs(a*e^(2*c) + b*e^(2*c)) + (4*a^6*b + 9*a^5*b^2 + 2*a^4*b^3 - 8*a
^3*b^4 - 6*a^2*b^5 - a*b^6 + 2*(4*a^5*b + 13*a^4*b^2 + 15*a^3*b^3 + 7*a^2*b
^4 + a*b^5)*sqrt(-a*b))*sqrt(a^2 - b^2 + 2*sqrt(-a*b)*(a + b))*abs(a^3*e^(2
*c) + 2*a^2*b*e^(2*c) + a*b^2*e^(2*c))*abs(a*e^(2*c) + b*e^(2*c))*e^(2*c) -
(8*a^8*b^2 + 26*a^7*b^3 + 22*a^6*b^4 - 12*a^5*b^5 - 28*a^4*b^6 - 14*a^3*b^
7 - 2*a^2*b^8 - (4*a^8*b + 9*a^7*b^2 - 2*a^6*b^3 - 17*a^5*b^4 - 8*a^4*b^5 +
7*a^3*b^6 + 6*a^2*b^7 + a*b^8)*sqrt(-a*b))*sqrt(a^2 - b^2 + 2*sqrt(-a*b)*(
a + b))*abs(a*e^(2*c) + b*e^(2*c))*e^(4*c))*arctan(e^(d*x)/sqrt((a^4*e^(2*c
) + a^3*b*e^(2*c) - a^2*b^2*e^(2*c) - a*b^3*e^(2*c) + sqrt((a^4*e^(2*c) + a
^3*b*e^(2*c) - a^2*b^2*e^(2*c) - a*b^3*e^(2*c))^2 - (a^4*e^(4*c) + 3*a^3*b*
e^(4*c) + 3*a^2*b^2*e^(4*c) + a*b^3*e^(4*c)))*(a^4 + 3*a^3*b + 3*a^2*b^2 + a
*b^3)))/(a^4*e^(4*c) + 3*a^3*b*e^(4*c) + 3*a^2*b^2*e^(4*c) + a*b^3*e^(4*c)
))*e^(-4*c)/((a^11 + 7*a^10*b + 20*a^9*b^2 + 28*a^8*b^3 + 14*a^7*b^4 - 14*a
^6*b^5 - 28*a^5*b^6 - 20*a^4*b^7 - 7*a^3*b^8 - a^2*b^9 + 2*(a^10 + 8*a^9*b
+ 28*a^8*b^2 + 56*a^7*b^3 + 70*a^6*b^4 + 56*a^5*b^5 + 28*a^4*b^6 + 8*a^3*b^
7 + a^2*b^8)*sqrt(-a*b))*abs(a^3*e^(2*c) + 2*a^2*b*e^(2*c) + a*b^2*e^(2*c)
) + (2*(16*a^3*b^2 - 12*a^2*b^3 - 4*a*b^4 + (4*a^3*b - 23*a^2*b^2 - 2*a*b^3
+ b^4)*sqrt(-a*b))*(a^3*e^(2*c) + 2*a^2*b*e^(2*c) + a*b^2*e^(2*c))^2*abs(a
*e^(2*c) + b*e^(2*c)) + (4*a^7*b - 11*a^6*b^2 - 59*a^5*b^3 - 70*a^4*b^4 - 2
6*a^3*b^5 + a^2*b^6 + a*b^7 - 4*(4*a^6*b + 9*a^5*b^2 + 2*a^4*b^3 - 8*a^3*b^
4 - 6*a^2*b^5 - a*b^6)*sqrt(-a*b))*abs(a^3*e^(2*c) + 2*a^2*b*e^(2*c) + a*b^
2*e^(2*c))*abs(a*e^(2*c) + b*e^(2*c))*e^(2*c) - (16*a^9*b^2 + 36*a^8*b^3 -
8*a^7*b^4 - 68*a^6*b^5 - 32*a^5*b^6 + 28*a^4*b^7 + 24*a^3*b^8 + 4*a^2*b^9 +
(4*a^9*b - 11*a^8*b^2 - 63*a^7*b^3 - 59*a^6*b^4 + 33*a^5*b^5 + 71*a^4*b^6
+ 27*a^3*b^7 - a^2*b^8 - a*b^9)*sqrt(-a*b))*abs(a*e^(2*c) + b*e^(2*c))*e^(4
*c))*arctan(e^(d*x)/sqrt((a^4*e^(2*c) + a^3*b*e^(2*c) - a^2*b^2*e^(2*c) - a
*b^3*e^(2*c) - sqrt((a^4*e^(2*c) + a^3*b*e^(2*c) - a^2*b^2*e^(2*c) - a*b^3*
e^(2*c))^2 - (a^4*e^(4*c) + 3*a^3*b*e^(4*c) + 3*a^2*b^2*e^(4*c) + a*b^3*e^(
4*c)))*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)))/(a^4*e^(4*c) + 3*a^3*b*e^(4*c)
+ 3*a^2*b^2*e^(4*c) + a*b^3*e^(4*c)))*e^(-4*c)/((a^10 + 6*a^9*b + 14*a^8*b
^2 + 14*a^7*b^3 - 14*a^5*b^5 - 14*a^4*b^6 - 6*a^3*b^7 - a^2*b^8 - 2*(a^9 +
7*a^8*b + 21*a^7*b^2 + 35*a^6*b^3 + 35*a^5*b^4 + 21*a^4*b^5 + 7*a^3*b^6 + a
^2*b^7)*sqrt(-a*b))*sqrt(a^2 - b^2 - 2*sqrt(-a*b)*(a + b))*abs(a^3*e^(2*c)
+ 2*a^2*b*e^(2*c) + a*b^2*e^(2*c))) + e^(d*x + 10*c)/(a^2*e^(9*c) + 2*a*b*e
^(9*c) + b^2*e^(9*c)) - (a^2*e^(4*d*x + 4*c) + a*b*e^(4*d*x + 4*c) - 2*b^2*
e^(4*d*x + 4*c) + 2*a^2*e^(2*d*x + 2*c) - 2*a*b*e^(2*d*x + 2*c) + 2*b^2*e^(
2*d*x + 2*c) + a^2 + a*b)/((a^3*e^c + 2*a^2*b*e^c + a*b^2*e^c)*(a*e^(5*d*x
+ 4*c) + b*e^(5*d*x + 4*c) + 2*a*e^(3*d*x + 2*c) - 2*b*e^(3*d*x + 2*c) + a
e^(d*x) + b*e^(d*x))))/d
```

maple [B] time = 0.42, size = 729, normalized size = 7.22

$$\frac{1}{d(a+b)^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)} - \frac{1}{d(a+b)^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)} - \frac{b^2 \left(\tanh^3\left(\frac{dx}{2}\right) \right)}{d(a+b)^2 \left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right) \right) a + 2 \left(\tanh^2\left(\frac{dx}{2}\right) \right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x)

[Out]
$$\begin{aligned} & -1/d/(a+b)^2/(\tanh(1/2*d*x+1/2*c)-1)-1/d/(a+b)^2/(\tanh(1/2*d*x+1/2*c)+1)-1/ \\ & d*b^2/(a+b)^2/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2 \\ & *d*x+1/2*c)^2*b+a)/a*\tanh(1/2*d*x+1/2*c)^3+1/d*b^2/(a+b)^2/(\tanh(1/2*d*x+1/ \\ & 2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)/a*\tanh(1/2* \\ & d*x+1/2*c)-2/d*b/(a+b)^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh \\ & (1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+2/d*b^2/(a+b)^2/(b*(a+ \\ & b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c) \\ & /((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+2/d*b/(a+b)^2/((2*(b*(a+b))^(1/2)+a+2 \\ & *b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/ \\ & 2))+2/d*b^2/(a+b)^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*\operatorname{arc} \\ & \operatorname{tan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/2/d*b^2/(a \\ & +b)^2/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/ \\ & ((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/2/d*b^3/(a+b)^2/a/(b*(a+b))^(1/2)/((2 \\ & *(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b) \\ &)^(1/2)-a-2*b)*a)^(1/2))+1/2/d*b^2/(a+b)^2/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(\\ & 1/2)*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/2 \\ & /d*b^3/(a+b)^2/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*\operatorname{arctan} \\ & (a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 + ab - (a^2 e^{6c} + ab e^{6c}) e^{6dx} - (a^2 e^{4c} - 3 ab e^{4c} + 2 b^2 e^{4c}) e^{4dx} + (a^2 e^{2c} - 3 ab e^{2c} + 2 b^2 e^{2c}) e^{2dx}}{2 \left((a^4 d e^{5c} + 3 a^3 b d e^{5c} + 3 a^2 b^2 d e^{5c} + ab^3 d e^{5c}) e^{5dx} + 2 (a^4 d e^{3c} + a^3 b d e^{3c} - a^2 b^2 d e^{3c} - ab^3 d e^{3c}) e^{3dx} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/2*(a^2 + a*b - (a^2*e^{6*c} + a*b*e^{6*c})*e^{6*d*x} - (a^2*e^{4*c} - 3*a*b*e^{4*c} + 2*b \\ & *b^2*e^{4*c})*e^{4*d*x} + (a^2*e^{2*c} - 3*a*b*e^{2*c} + 2*b^2*e^{2*c})*e^{2*d*x})/((a^4*d*e^{5*c} + 3*a^3*b*d*e^{5*c} + 3*a^2*b^2*d*e^{5*c} \\ & + a*b^3*d*e^{5*c})*e^{5*d*x} + 2*(a^4*d*e^{3*c} + a^3*b*d*e^{3*c} - a^2*b^2*d*e^{3*c} - a*b^3*d*e^{3*c})*e^{3*d*x} + (a^4*d*e^c + 3*a^3*b*d*e^c \\ & + 3*a^2*b^2*d*e^c + a*b^3*d*e^c)*e^{d*x}) + 1/2*\operatorname{integrate}(2*((4*a*b*e^{3*c} + b^2*e^{3*c})*e^{3*d*x} + (4*a*b*e^c + b^2*e^c)*e^{d*x}))/ \\ & (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + (a^4*e^{4*c} + 3*a^3*b*e^{4*c} + 3*a^2*b^2*e^{4*c} + a*b^3*e^{4*c})*e^{4*d*x} + 2*(a^4*e^{2*c} + a^3*b*e^{2*c} - a^2*b^2*e^{2*c} \\ & - a*b^3*e^{2*c})*e^{2*d*x}), x) \end{aligned}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)}{(b \tanh(c + dx)^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2)^2,x)

[Out] int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2)^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)

[Out] Timed out

$$3.118 \quad \int \frac{\operatorname{sech}(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=83

$$\frac{(2a+b) \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{3/2}} + \frac{b \sinh(c+dx)}{2ad(a+b)((a+b) \sinh^2(c+dx) + a)}$$

[Out] 1/2*(2*a+b)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(3/2)/(a+b)^(3/2)/d+1/2*b*sinh(d*x+c)/a/(a+b)/d/(a+(a+b)*sinh(d*x+c)^2)

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3676, 385, 205}

$$\frac{(2a+b) \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{3/2}} + \frac{b \sinh(c+dx)}{2ad(a+b)((a+b) \sinh^2(c+dx) + a)}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2), x]

[Out] ((2*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(3/2)*d) + (b*Sinh[c + d*x])/(2*a*(a + b)*d*(a + (a + b)*Sinh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 3676

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^(m + n*p + 1)/2], x], x, Sin[e + f*x]/ff, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$d*x + c)) * \sinh(d*x + c)), 1/2*(2*(a^2*b + a*b^2)*\cosh(d*x + c)^3 + 6*(a^2*b + a*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^2 + 2*(a^2*b + a*b^2)*\sinh(d*x + c)^3 + ((2*a^2 + 3*a*b + b^2)*\cosh(d*x + c)^4 + 4*(2*a^2 + 3*a*b + b^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (2*a^2 + 3*a*b + b^2)*\sinh(d*x + c)^4 + 2*(2*a^2 - a*b - b^2)*\cosh(d*x + c)^2 + 2*(3*(2*a^2 + 3*a*b + b^2)*\cosh(d*x + c)^2 + 2*a^2 - a*b - b^2)*\sinh(d*x + c)^2 + 2*a^2 + 3*a*b + b^2 + 4*((2*a^2 + 3*a*b + b^2)*\cosh(d*x + c)^3 + (2*a^2 - a*b - b^2)*\cosh(d*x + c))*\sinh(d*x + c)) * \sqrt{a^2 + a*b} * \arctan(1/2*((a + b)*\cosh(d*x + c)^3 + 3*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a + b)*\sinh(d*x + c)^3 + (3*a - b)*\cosh(d*x + c) + (3*(a + b)*\cosh(d*x + c)^2 + 3*a - b)*\sinh(d*x + c))/\sqrt{a^2 + a*b})) + ((2*a^2 + 3*a*b + b^2)*\cosh(d*x + c)^4 + 4*(2*a^2 + 3*a*b + b^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (2*a^2 + 3*a*b + b^2)*\sinh(d*x + c)^4 + 2*(2*a^2 - a*b - b^2)*\cosh(d*x + c)^2 + 2*(3*(2*a^2 + 3*a*b + b^2)*\cosh(d*x + c)^2 + 2*a^2 - a*b - b^2)*\sinh(d*x + c)^2 + 2*a^2 + 3*a*b + b^2 + 4*((2*a^2 + 3*a*b + b^2)*\cosh(d*x + c)^3 + (2*a^2 - a*b - b^2)*\cosh(d*x + c))*\sinh(d*x + c)) * \sqrt{a^2 + a*b} * \arctan(1/2*\sqrt{a^2 + a*b}*(\cosh(d*x + c) + \sinh(d*x + c))/a) - 2*(a^2*b + a*b^2)*\cosh(d*x + c) - 2*(a^2*b + a*b^2 - 3*(a^2*b + a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*\cosh(d*x + c)^4 + 4*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*\sinh(d*x + c)^4 + 2*(a^5 + a^4*b - a^3*b^2 - a^2*b^3)*d*\cosh(d*x + c)^2 + 2*(3*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*\cosh(d*x + c)^2 + (a^5 + a^4*b - a^3*b^2 - a^2*b^3)*d)*\sinh(d*x + c)^2 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d + 4*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*\cosh(d*x + c)^3 + (a^5 + a^4*b - a^3*b^2 - a^2*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c))]$$

giac [B] time = 0.32, size = 1309, normalized size = 15.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out]
$$-1/2*((2*(4*a^2*b + 2*a*b^2 - (2*a^2 - a*b - b^2)*\sqrt{-a*b})*(a^2*e^{(2*c)} + a*b*e^{(2*c)})^2*\sqrt{a^2 - b^2 + 2*\sqrt{-a*b}*(a + b)}*abs(a*e^{(2*c)} + b*e^{(2*c)}) - (2*a^5 + 3*a^4*b - a^3*b^2 - 3*a^2*b^3 - a*b^4 + 2*(2*a^4 + 5*a^3*b + 4*a^2*b^2 + a*b^3)*\sqrt{-a*b})*\sqrt{a^2 - b^2 + 2*\sqrt{-a*b}*(a + b)}*abs(-a^2*e^{(2*c)} - a*b*e^{(2*c)})*abs(a*e^{(2*c)} + b*e^{(2*c)})*e^{(2*c)} - (4*a^6*b + 6*a^5*b^2 - 2*a^4*b^3 - 6*a^3*b^4 - 2*a^2*b^5 - (2*a^6 + a^5*b - 4*a^4*b^2 - 2*a^3*b^3 + 2*a^2*b^4 + a*b^5)*\sqrt{-a*b})*\sqrt{a^2 - b^2 + 2*\sqrt{-a*b}*(a + b)}*abs(a*e^{(2*c)} + b*e^{(2*c)})*e^{(4*c)}*\arctan(e^{(d*x)}/\sqrt{(a^3*e^{(2*c)} - a*b^2*e^{(2*c)} - \sqrt{(a^3*e^{(2*c)} - a*b^2*e^{(2*c)})^2 - (a^3*e^{(4*c)} + 2*a^2*b*e^{(4*c)} + a*b^2*e^{(4*c)})*(a^3 + 2*a^2*b + a*b^2))})/(a^3*e^{(4*c)} + 2*a^2*b*e^{(4*c)} + a*b^2*e^{(4*c)}))) * e^{(-4*c)}/((a^9 + 5*a^8*b + 9*a^7*b^2 + 5*a^6*b^3 - 5*a^5*b^4 - 9*a^4*b^5 - 5*a^3*b^6 - a^2*b^7 + 2*(a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*\sqrt{-a*b})*abs(-a^2*e^{(2*c)} - a*b*e^{(2*c)})) + (2*(8*a^3*b - 4*a^2*b^2 - 4*a*b^3 + (2*a^3 - 11*a^2*b - 4*a*b^2 + b^3)*\sqrt{-a*b})*(a^2*e^{(2*c)} + a*b*e^{(2*c)})^2*abs(a*e^{(2*c)} + b*e^{(2*c)}) - (2*a^6 - 7*a^5*b - 24*a^4*b^2 - 18*a^3*b^3 - 2*a^2*b^4 + a*b^5 - 4*(2*a^5 + 3*a^4*b - a^3*b^2 - 3*a^2*b^3 - a*b^4)*\sqrt{-a*b})*abs(-a^2*e^{(2*c)} - a*b*e^{(2*c)})*abs(a*e^{(2*c)} + b*e^{(2*c)})*e^{(2*c)} - (8*a^7*b + 4*a^6*b^2 - 16*a^5*b^3 - 8*a^4*b^4 + 8*a^3*b^5 + 4*a^2*b^6 + (2*a^7 - 9*a^6*b - 17*a^5*b^2 + 6*a^4*b^3 + 16*a^3*b^4 + 3*a^2*b^5 - a*b^6)*\sqrt{-a*b})*abs(a*e^{(2*c)} + b*e^{(2*c)})*e^{(4*c)}*\arctan(e^{(d*x)}/\sqrt{(a^3*e^{(2*c)} - a*b^2*e^{(2*c)} + \sqrt{(a^3*e^{(2*c)} - a*b^2*e^{(2*c)})^2 - (a^3*e^{(4*c)} + 2*a^2*b*e^{(4*c)} + a*b^2*e^{(4*c)})*(a^3 + 2*a^2*b + a*b^2))})/(a^3*e^{(4*c)} + 2*a^2*b*e^{(4*c)} + a*b^2*e^{(4*c)}))) * e^{(-4*c)}/((a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6 - 2*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*\sqrt{-a*b})*\sqrt{a^2 - b^2 - 2*\sqrt{-a*b}*(a + b)}*abs(-a^2*e^{(2*c)} - a*b*e^{(2*c)}))) - 2*(b*e^{(3*d*x + 3*c)} - b*e^{(d*x + c)})/(($$

$a^2 + a*b)*(a*e^{(4*d*x + 4*c)} + b*e^{(4*d*x + 4*c)} + 2*a*e^{(2*d*x + 2*c)} - 2*b*e^{(2*d*x + 2*c)} + a + b))/d$

maple [B] time = 0.39, size = 666, normalized size = 8.02

$$\frac{b \left(\tanh^3 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{d \left(\left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a + 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a + 4 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b + a \right) a (a + b)} + \frac{1}{d \left(\left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a + 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a + 4 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b + a \right) a (a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x)

[Out] $-1/d/(\tanh(1/2*d*x+1/2*c)^{4*a+2*\tanh(1/2*d*x+1/2*c)^{2*a+4*\tanh(1/2*d*x+1/2*c)^{2*b+a}}*b/a/(a+b)*\tanh(1/2*d*x+1/2*c)^{3+1/d/(\tanh(1/2*d*x+1/2*c)^{4*a+2*\tanh(1/2*d*x+1/2*c)^{2*a+4*\tanh(1/2*d*x+1/2*c)^{2*b+a}}*b/a/(a+b)*\tanh(1/2*d*x+1/2*c)-1/d/(a+b)/((2*(b*(a+b))^{(1/2)-a-2*b})*a)^{(1/2)}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)-a-2*b})*a)^{(1/2)}))+1/d/(a+b)/(b*(a+b))^{(1/2)}/((2*(b*(a+b))^{(1/2)-a-2*b})*a)^{(1/2)}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)-a-2*b})*a)^{(1/2)}))*b+1/d/(a+b)/((2*(b*(a+b))^{(1/2)+a+2*b})*a)^{(1/2)}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)+a+2*b})*a)^{(1/2)}))+1/d/(a+b)/(b*(a+b))^{(1/2)}/((2*(b*(a+b))^{(1/2)+a+2*b})*a)^{(1/2)}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)+a+2*b})*a)^{(1/2)}))*b-1/2/d/(a+b)*b/a/((2*(b*(a+b))^{(1/2)-a-2*b})*a)^{(1/2)}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)-a-2*b})*a)^{(1/2)}))+1/2/d/(a+b)/a/(b*(a+b))^{(1/2)}/((2*(b*(a+b))^{(1/2)-a-2*b})*a)^{(1/2)}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)-a-2*b})*a)^{(1/2)}))*b^2+1/2/d/(a+b)*b/a/((2*(b*(a+b))^{(1/2)+a+2*b})*a)^{(1/2)}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)+a+2*b})*a)^{(1/2)}))+1/2/d/(a+b)/a/(b*(a+b))^{(1/2)}/((2*(b*(a+b))^{(1/2)+a+2*b})*a)^{(1/2)}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)+a+2*b})*a)^{(1/2)}))*b^2$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{b e^{(3 d x+3 c)} - b e^{(d x+c)}}{a^3 d + 2 a^2 b d + a b^2 d + (a^3 d e^{(4 c)} + 2 a^2 b d e^{(4 c)} + a b^2 d e^{(4 c)}) e^{(4 d x)} + 2 (a^3 d e^{(2 c)} - a b^2 d e^{(2 c)}) e^{(2 d x)}} + 2 \int \frac{1}{2 (a^3 + 2 a^2 b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $(b*e^{(3*d*x + 3*c)} - b*e^{(d*x + c)})/(a^3*d + 2*a^2*b*d + a*b^2*d + (a^3*d*e^{(4*c)} + 2*a^2*b*d*e^{(4*c)} + a*b^2*d*e^{(4*c)})*e^{(4*d*x)} + 2*(a^3*d*e^{(2*c)} - a*b^2*d*e^{(2*c)})*e^{(2*d*x)}) + 2*\operatorname{integrate}(1/2*((2*a*e^{(3*c)} + b*e^{(3*c)})*e^{(3*d*x)} + (2*a*e^c + b*e^c)*e^{(d*x)})/(a^3 + 2*a^2*b + a*b^2 + (a^3*e^{(4*c)} + 2*a^2*b*e^{(4*c)} + a*b^2*e^{(4*c)})*e^{(4*d*x)} + 2*(a^3*e^{(2*c)} - a*b^2*e^{(2*c)})*e^{(2*d*x)}), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + d x) (b \tanh(c + d x)^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^2),x)

[Out] int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)

[Out] Integral(sech(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)

$$3.119 \quad \int \frac{\operatorname{sech}^2(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=66

$$\frac{\tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d} + \frac{\tanh(c+dx)}{2ad(a+b \tanh^2(c+dx))}$$

[Out] 1/2*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/d/b^(1/2)+1/2*tanh(d*x+c)/a/d/(a+b*tanh(d*x+c)^2)

Rubi [A] time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3675, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d} + \frac{\tanh(c+dx)}{2ad(a+b \tanh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2), x]

[Out] ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]*d) + Tanh[c + d*x]/(2*a*d*(a + b*Tanh[c + d*x]^2))

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegerQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b \tanh^2(c+dx))^2} dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(a+bx^2)^2} dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)}{2ad(a+b \tanh^2(c+dx))} + \frac{\operatorname{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \tanh(c+dx)\right)}{2ad}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d} + \frac{\tanh(c+dx)}{2ad(a+b \tanh^2(c+dx))}$$

Mathematica [A] time = 0.27, size = 63, normalized size = 0.95

$$\frac{\frac{\sqrt{a} \tanh(c+dx)}{a+b \tanh^2(c+dx)} + \frac{\tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]

[Out] (ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/Sqrt[b] + (Sqrt[a]*Tanh[c + d*x])/(a + b*Tanh[c + d*x]^2))/(2*a^(3/2)*d)

fricas [B] time = 0.43, size = 1515, normalized size = 22.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")

[Out] [-1/4*(4*a^2*b + 4*a*b^2 + 4*(a^2*b - a*b^2)*cosh(d*x + c)^2 + 8*(a^2*b - a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(a^2*b - a*b^2)*sinh(d*x + c)^2 + ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b))/((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^4*b - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^4*b - a^2*b^3)*d)*sinh(d*x + c)^2 + (a^4*b + 2*a^3*b^2 + a^2*b^3)*d + 4*((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (a^4*b - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*a^2*b + 2*a*b^2 + 2*(a^2*b - a*b^2)*cosh(d*x + c)^2 + 4*(a^2*b - a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 2*(a^2*b - a*b^2)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b)))/(2*a^(3/2)*d)

$x + c)^3 + (a^2 + 2ab + b^2) \sinh(dx + c)^4 + 2(a^2 - b^2) \cosh(dx + c)^2 + 2(3(a^2 + 2ab + b^2) \cosh(dx + c)^2 + a^2 - b^2) \sinh(dx + c)^2 + a^2 + 2ab + b^2 + 4((a^2 + 2ab + b^2) \cosh(dx + c)^3 + (a^2 - b^2) \cosh(dx + c)) \sinh(dx + c) \sqrt{ab} \arctan(1/2((a + b) \cosh(dx + c)^2 + 2(a + b) \cosh(dx + c) \sinh(dx + c) + (a + b) \sinh(dx + c)^2 + a - b) \sqrt{ab} / (ab)) / ((a^4b + 2a^3b^2 + a^2b^3) d \cosh(dx + c)^4 + 4(a^4b + 2a^3b^2 + a^2b^3) d \cosh(dx + c) \sinh(dx + c)^3 + (a^4b + 2a^3b^2 + a^2b^3) d \sinh(dx + c)^4 + 2(a^4b - a^2b^3) d \cosh(dx + c)^2 + 2(3(a^4b + 2a^3b^2 + a^2b^3) d \cosh(dx + c)^2 + (a^4b - a^2b^3) d) \sinh(dx + c)^2 + (a^4b + 2a^3b^2 + a^2b^3) d + 4((a^4b + 2a^3b^2 + a^2b^3) d \cosh(dx + c)^3 + (a^4b - a^2b^3) d \cosh(dx + c)) \sinh(dx + c))]$

giac [B] time = 0.56, size = 138, normalized size = 2.09

$$\frac{\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{2(ae^{(2dx+2c)}-be^{(2dx+2c)}+a+b)}{(a^2+ab)(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b)}$$

$2d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)^2/(a+b*tanh(dx+c)^2)^2,x, algorithm="giac")

[Out] $1/2 * (\arctan(1/2 * (a * e^{(2 * dx + 2 * c)} + b * e^{(2 * dx + 2 * c)} + a - b) / \sqrt{ab})) / (\sqrt{ab} * a) - 2 * (a * e^{(2 * dx + 2 * c)} - b * e^{(2 * dx + 2 * c)} + a + b) / ((a^2 + a * b) * (a * e^{(4 * dx + 4 * c)} + b * e^{(4 * dx + 4 * c)} + 2 * a * e^{(2 * dx + 2 * c)} - 2 * b * e^{(2 * dx + 2 * c)} + a + b)) / d$

maple [B] time = 0.40, size = 498, normalized size = 7.55

$$\frac{\tanh^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{d\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + 4\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b + a\right)a} + \frac{\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)}{d\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + 4\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b + a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(dx+c)^2/(a+b*tanh(dx+c)^2)^2,x)

[Out] $1/d / (\tanh(1/2 * dx + 1/2 * c)^4 * a + 2 * \tanh(1/2 * dx + 1/2 * c)^2 * a + 4 * \tanh(1/2 * dx + 1/2 * c)^2 * b + a) / a * \tanh(1/2 * dx + 1/2 * c)^3 + 1/d / (\tanh(1/2 * dx + 1/2 * c)^4 * a + 2 * \tanh(1/2 * dx + 1/2 * c)^2 * a + 4 * \tanh(1/2 * dx + 1/2 * c)^2 * b + a) / a * \tanh(1/2 * dx + 1/2 * c) - 1/2 / d / (b * (a + b))^{(1/2)} / ((2 * (b * (a + b))^{(1/2)} - a - 2 * b) * a)^{(1/2)} * \arctanh(a * \tanh(1/2 * dx + 1/2 * c)) / ((2 * (b * (a + b))^{(1/2)} - a - 2 * b) * a)^{(1/2)} + 1/2 / d / a / ((2 * (b * (a + b))^{(1/2)} - a - 2 * b) * a)^{(1/2)} * \arctanh(a * \tanh(1/2 * dx + 1/2 * c)) / ((2 * (b * (a + b))^{(1/2)} - a - 2 * b) * a)^{(1/2)} - 1/2 / d / (b * (a + b))^{(1/2)} / a / ((2 * (b * (a + b))^{(1/2)} - a - 2 * b) * a)^{(1/2)} * \arctanh(a * \tanh(1/2 * dx + 1/2 * c)) / ((2 * (b * (a + b))^{(1/2)} - a - 2 * b) * a)^{(1/2)} * b - 1/2 / d / (b * (a + b))^{(1/2)} / ((2 * (b * (a + b))^{(1/2)} + a + 2 * b) * a)^{(1/2)} * \arctan(a * \tanh(1/2 * dx + 1/2 * c)) / ((2 * (b * (a + b))^{(1/2)} + a + 2 * b) * a)^{(1/2)} - 1/2 / d / a / ((2 * (b * (a + b))^{(1/2)} + a + 2 * b) * a)^{(1/2)} * \arctan(a * \tanh(1/2 * dx + 1/2 * c)) / ((2 * (b * (a + b))^{(1/2)} + a + 2 * b) * a)^{(1/2)} - 1/2 / d / (b * (a + b))^{(1/2)} / a / ((2 * (b * (a + b))^{(1/2)} + a + 2 * b) * a)^{(1/2)} * \arctan(a * \tanh(1/2 * dx + 1/2 * c)) / ((2 * (b * (a + b))^{(1/2)} + a + 2 * b) * a)^{(1/2)} * b$

maxima [B] time = 0.52, size = 125, normalized size = 1.89

$$\frac{(a - b)e^{(-2dx - 2c)} + a + b}{(a^3 + 2a^2b + ab^2 + 2(a^3 - ab^2)e^{(-2dx - 2c)} + (a^3 + 2a^2b + ab^2)e^{(-4dx - 4c)})d} - \frac{\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] ((a - b)*e^(-2*d*x - 2*c) + a + b)/((a^3 + 2*a^2*b + a*b^2 + 2*(a^3 - a*b^2)*e^(-2*d*x - 2*c) + (a^3 + 2*a^2*b + a*b^2)*e^(-4*d*x - 4*c))*d) - 1/2*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a*d)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cosh(c + dx)^2 (b \tanh(c + dx)^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2),x)

[Out] int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)

[Out] Integral(sech(c + d*x)**2/(a + b*tanh(c + d*x)**2)**2, x)

$$3.120 \quad \int \frac{\operatorname{sech}^3(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=72

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d\sqrt{a+b}} + \frac{\sinh(c+dx)}{2ad((a+b)\sinh^2(c+dx)+a)}$$

[Out] 1/2*sinh(d*x+c)/a/d/(a+(a+b)*sinh(d*x+c)^2)+1/2*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(3/2)/d/(a+b)^(1/2)

Rubi [A] time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3676, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d\sqrt{a+b}} + \frac{\sinh(c+dx)}{2ad((a+b)\sinh^2(c+dx)+a)}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2), x]

[Out] ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a + b]*d) + Sinh[c + d*x]/(2*a*d*(a + (a + b)*Sinh[c + d*x]^2))

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3676

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{(a+b \tanh^2(c+dx))^2} dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(a+(a+b)x^2)^2} dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\sinh(c+dx)}{2ad(a+(a+b)\sinh^2(c+dx))} + \frac{\operatorname{Subst}\left(\int \frac{1}{a+(a+b)x^2} dx, x, \sinh(c+dx)\right)}{2ad}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}d} + \frac{\sinh(c+dx)}{2ad(a+(a+b)\sinh^2(c+dx))}$$

Mathematica [A] time = 0.14, size = 69, normalized size = 0.96

$$\frac{\frac{\sqrt{a}\sinh(c+dx)}{(a+b)\sinh^2(c+dx)+a} + \frac{\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/Sqrt[a + b] + (Sqrt[a]*Sinh[c + d*x])/(a + (a + b)*Sinh[c + d*x]^2))/(2*a^(3/2)*d)

fricas [B] time = 0.44, size = 1555, normalized size = 21.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/4*(4*(a^2 + a*b)*cosh(d*x + c)^3 + 12*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^2 + 4*(a^2 + a*b)*sinh(d*x + c)^3 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*sqrt(-a^2 - a*b)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b) - 4*(a^2 + a*b)*cosh(d*x + c) + 4*(3*(a^2 + a*b)*cosh(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*b^2)*d*sinh(d*x + c)^4 + 2*(a^4 - a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 - a^2*b^2)*d)*sinh(d*x + c)^2 + (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^3 + (a^4 - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*(a^2 + a*b)*cosh(d*x + c)^3 + 6*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2 + a*b)*sinh(d*x + c)^3 + ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^

```

3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*sqrt(a^2 + a*b)*arctan(1/
2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a +
b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2
+ 3*a - b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + ((a + b)*cosh(d*x + c)^4 + 4*(
a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*
cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4
*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*s
qrt(a^2 + a*b)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/a
) - 2*(a^2 + a*b)*cosh(d*x + c) + 2*(3*(a^2 + a*b)*cosh(d*x + c)^2 - a^2 -
a*b)*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^4 + 4*(a^4 +
2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4 + 2*a^3*b + a^2*
b^2)*d*sinh(d*x + c)^4 + 2*(a^4 - a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^4 +
2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 - a^2*b^2)*d)*sinh(d*x + c)^2 +
(a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)
^3 + (a^4 - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))]

```

giac [B] time = 0.60, size = 978, normalized size = 13.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

```

[Out] -1/2*((2*(4*a^2*b - 4*a*b^2 + (a^2 - 6*a*b + b^2)*sqrt(-a*b))*sqrt(a^2 - b^
2 - 2*sqrt(-a*b)*(a + b))*a^2*abs(a*e^(2*c) + b*e^(2*c)) - (a^4 - 5*a^3*b -
5*a^2*b^2 + a*b^3 - 4*(a^3 - a*b^2)*sqrt(-a*b))*sqrt(a^2 - b^2 - 2*sqrt(-a
*b)*(a + b))*abs(a*e^(2*c) + b*e^(2*c))*abs(a) - (4*a^4*b - 8*a^3*b^2 + 4*a
^2*b^3 + (a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*sqrt(-a*b))*sqrt(a^2 - b^2 - 2
*sqrt(-a*b)*(a + b))*abs(a*e^(2*c) + b*e^(2*c)))*arctan(e^(d*x)/sqrt((a^2*e
^(2*c) - a*b*e^(2*c) + sqrt((a^2*e^(2*c) - a*b*e^(2*c))^2 - (a^2*e^(4*c) +
a*b*e^(4*c))*(a^2 + a*b)))/(a^2*e^(4*c) + a*b*e^(4*c))))*e^(-2*c)/((a^8 - 2
*a^7*b - 17*a^6*b^2 - 28*a^5*b^3 - 17*a^4*b^4 - 2*a^3*b^5 + a^2*b^6 - 4*(a^
7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*sqrt(-a*b))*abs(
a)) + (2*(4*a^2*b - 4*a*b^2 - (a^2 - 6*a*b + b^2)*sqrt(-a*b))*sqrt(a^2 - b^
2 + 2*sqrt(-a*b)*(a + b))*a^2*abs(a*e^(2*c) + b*e^(2*c)) - (a^4 - 5*a^3*b -
5*a^2*b^2 + a*b^3 + 4*(a^3 - a*b^2)*sqrt(-a*b))*sqrt(a^2 - b^2 + 2*sqrt(-a
*b)*(a + b))*abs(a*e^(2*c) + b*e^(2*c))*abs(a) - (4*a^4*b - 8*a^3*b^2 + 4*a
^2*b^3 - (a^4 - 7*a^3*b + 7*a^2*b^2 - a*b^3)*sqrt(-a*b))*sqrt(a^2 - b^2 + 2
*sqrt(-a*b)*(a + b))*abs(a*e^(2*c) + b*e^(2*c)))*arctan(e^(d*x)/sqrt((a^2*e
^(2*c) - a*b*e^(2*c) - sqrt((a^2*e^(2*c) - a*b*e^(2*c))^2 - (a^2*e^(4*c) +
a*b*e^(4*c))*(a^2 + a*b)))/(a^2*e^(4*c) + a*b*e^(4*c))))*e^(-2*c)/((a^8 - 2
*a^7*b - 17*a^6*b^2 - 28*a^5*b^3 - 17*a^4*b^4 - 2*a^3*b^5 + a^2*b^6 + 4*(a^
7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*sqrt(-a*b))*abs(
a)) - 2*(e^(3*d*x + 3*c) - e^(d*x + c))/((a*e^(4*d*x + 4*c) + b*e^(4*d*x +
4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*a))/d

```

maple [B] time = 0.39, size = 375, normalized size = 5.21

$$\frac{\tanh^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{d\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + 4\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b + a\right)a} + \frac{\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)}{d\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + 4\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b + a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x)

```

[Out] -1/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*
c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)^3+1/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d
*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)-1/2/d/a/((

```

$$2*(b*(a+b))^{(1/2)-a-2*b}*a)^{(1/2)}*\operatorname{arctanh}(a*\operatorname{tanh}(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)-a-2*b}*a)^{(1/2)}+1/2/d/(b*(a+b))^{(1/2)}/a/((2*(b*(a+b))^{(1/2)-a-2*b}*a)^{(1/2)}*\operatorname{arctanh}(a*\operatorname{tanh}(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)-a-2*b}*a)^{(1/2)})))*b+1/2/d/a/((2*(b*(a+b))^{(1/2)+a+2*b}*a)^{(1/2)}*\operatorname{arctan}(a*\operatorname{tanh}(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)+a+2*b}*a)^{(1/2)})))+1/2/d/(b*(a+b))^{(1/2)}/a/((2*(b*(a+b))^{(1/2)+a+2*b}*a)^{(1/2)}*\operatorname{arctan}(a*\operatorname{tanh}(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)+a+2*b}*a)^{(1/2)})))*b$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(ae^{3c} + be^{3c})e^{3dx} - (ae^c + be^c)e^{dx}}{a^3d + 2a^2bd + ab^2d + (a^3de^{4c} + 2a^2bde^{4c} + ab^2de^{4c})e^{4dx} + 2(a^3de^{2c} - ab^2de^{2c})e^{2dx}} + 8 \int \frac{1}{8(a^2 + a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] ((a*e^(3*c) + b*e^(3*c))*e^(3*d*x) - (a*e^c + b*e^c)*e^(d*x))/(a^3*d + 2*a^2*b*d + a*b^2*d + (a^3*d*e^(4*c) + 2*a^2*b*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(a^3*d*e^(2*c) - a*b^2*d*e^(2*c))*e^(2*d*x)) + 8*integrate(1/8*(e^(3*d*x + 3*c) + e^(d*x + c))/(a^2 + a*b + (a^2*e^(4*c) + a*b*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) - a*b*e^(2*c))*e^(2*d*x)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + dx)^3 (b \tanh(c + dx)^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2), x)

[Out] int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)

[Out] Integral(sech(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)

$$3.121 \quad \int \frac{\operatorname{sech}^4(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=77

$$\frac{(a+b) \tanh(c+dx)}{2abd(a+b \tanh^2(c+dx))} - \frac{(a-b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d}$$

[Out] $-1/2*(a-b)*\arctan(b^{(1/2)}*\tanh(d*x+c)/a^{(1/2)})/a^{(3/2)}/b^{(3/2)}/d+1/2*(a+b)*\tanh(d*x+c)/a/b/d/(a+b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3675, 385, 205}

$$\frac{(a+b) \tanh(c+dx)}{2abd(a+b \tanh^2(c+dx))} - \frac{(a-b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2), x]

[Out] $-(a-b)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Tanh}[c+d*x])/\text{Sqrt}[a]]/(2*a^{(3/2)}*b^{(3/2)}*d) + ((a+b)*\text{Tanh}[c+d*x])/(2*a*b*d*(a+b*\text{Tanh}[c+d*x]^2))$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p+1))/(a*b*n*(p+1)), x] - Dist[(a*d - b*c*(n*(p+1) + 1))/(a*b*n*(p+1)), Int[(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m-1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b \tanh^2(c+dx))^2} dx = \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{(a+bx^2)^2} dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{(a+b) \tanh(c+dx)}{2abd(a+b \tanh^2(c+dx))} - \frac{(a-b) \operatorname{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \tanh(c+dx)\right)}{2abd}$$

$$= -\frac{(a-b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d} + \frac{(a+b) \tanh(c+dx)}{2abd(a+b \tanh^2(c+dx))}$$

Mathematica [A] time = 0.33, size = 83, normalized size = 1.08

$$\frac{(b-a) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right) + \frac{\sqrt{a} \sqrt{b} (a+b) \sinh(2(c+dx))}{(a+b) \cosh(2(c+dx))+a-b}}{2a^{3/2}b^{3/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]

[Out] ((-a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] + (Sqrt[a]*Sqrt[b]*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(2*a^(3/2)*b^(3/2)*d)

fricas [B] time = 0.44, size = 1443, normalized size = 18.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [-1/4*(4*a^2*b + 4*a*b^2 + 4*(a^2*b - a*b^2)*cosh(d*x + c)^2 + 8*(a^2*b - a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(a^2*b - a*b^2)*sinh(d*x + c)^2 - ((a^2 - b^2)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - b^2)*sinh(d*x + c)^4 + 2*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - b^2)*cosh(d*x + c)^2 + a^2 - 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b))/((a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^2 + (a^3*b^2 + a^2*b^3)*d + 4*((a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*a^2*b + 2*a*b^2 + 2*(a^2*b - a*b^2)*cosh(d*x + c)^2 + 4*(a^2*b - a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 2*(a^2*b - a*b^2)*sinh(d*x + c)^2 + ((a^2 - b^2)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 -

$$b^2) \sinh(dx + c)^4 + 2(a^2 - 2ab + b^2) \cosh(dx + c)^2 + 2(3(a^2 - b^2) \cosh(dx + c)^2 + a^2 - 2ab + b^2) \sinh(dx + c)^2 + a^2 - b^2 + 4((a^2 - b^2) \cosh(dx + c)^3 + (a^2 - 2ab + b^2) \cosh(dx + c)) \sinh(dx + c) \sqrt{ab} \arctan(1/2((a + b) \cosh(dx + c)^2 + 2(a + b) \cosh(dx + c) \sinh(dx + c) + (a + b) \sinh(dx + c)^2 + a - b) \sqrt{ab}) / (a^3 b^2 + a^2 b^3) d \cosh(dx + c)^4 + 4(a^3 b^2 + a^2 b^3) d \cosh(dx + c) \sinh(dx + c)^3 + (a^3 b^2 + a^2 b^3) d \sinh(dx + c)^4 + 2(a^3 b^2 - a^2 b^3) d \cosh(dx + c)^2 + 2(3(a^3 b^2 + a^2 b^3) d \cosh(dx + c)^2 + (a^3 b^2 - a^2 b^3) d) \sinh(dx + c)^2 + (a^3 b^2 + a^2 b^3) d + 4((a^3 b^2 + a^2 b^3) d \cosh(dx + c)^3 + (a^3 b^2 - a^2 b^3) d \cosh(dx + c)) \sinh(dx + c))]$$

giac [B] time = 0.55, size = 156, normalized size = 2.03

$$\frac{(ae^{2c} - be^{2c}) \arctan\left(\frac{ae^{2dx+2c} + be^{2dx+2c} + a - b}{2\sqrt{ab}}\right) e^{-2c}}{\sqrt{ab} ab} + \frac{2(ae^{2dx+2c} - be^{2dx+2c} + a + b)}{(ae^{4dx+4c} + be^{4dx+4c} + 2ae^{2dx+2c} - 2be^{2dx+2c} + a + b) ab}$$

$$2d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)^4/(a+b*tanh(dx+c)^2)^2,x, algorithm="giac")

[Out] $-1/2 * ((a * e^{2c}) - b * e^{2c}) * \arctan(1/2 * (a * e^{2dx+2c} + b * e^{2dx+2c} + a - b) / \sqrt{ab}) * e^{-2c} / (\sqrt{ab} * a * b) + 2 * (a * e^{2dx+2c} - b * e^{2dx+2c} + a + b) / ((a * e^{4dx+4c} + b * e^{4dx+4c} + 2 * a * e^{2dx+2c} - 2 * b * e^{2dx+2c} + a + b) * a * b) / d$

maple [B] time = 0.37, size = 746, normalized size = 9.69

$$\frac{\tanh^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{d \left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right) \right) a + 2 \left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) a + 4 \left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) b + a \right) b} + \frac{\tanh^3}{d \left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right) \right) a + 2 \left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) a + 4 \left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right) \right) b + a \right) b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(dx+c)^4/(a+b*tanh(dx+c)^2)^2,x)

[Out] $1/d / (\tanh(1/2 * dx + 1/2 * c)^4 * a + 2 * \tanh(1/2 * dx + 1/2 * c)^2 * a + 4 * \tanh(1/2 * dx + 1/2 * c)^2 * b + a) / b * \tanh(1/2 * dx + 1/2 * c)^3 + 1/d / (\tanh(1/2 * dx + 1/2 * c)^4 * a + 2 * \tanh(1/2 * dx + 1/2 * c)^2 * a + 4 * \tanh(1/2 * dx + 1/2 * c)^2 * b + a) / a * \tanh(1/2 * dx + 1/2 * c)^3 + 1/d / (\tanh(1/2 * dx + 1/2 * c)^4 * a + 2 * \tanh(1/2 * dx + 1/2 * c)^2 * a + 4 * \tanh(1/2 * dx + 1/2 * c)^2 * b + a) / b * \tanh(1/2 * dx + 1/2 * c) + 1/d / (\tanh(1/2 * dx + 1/2 * c)^4 * a + 2 * \tanh(1/2 * dx + 1/2 * c)^2 * a + 4 * \tanh(1/2 * dx + 1/2 * c)^2 * b + a) / a * \tanh(1/2 * dx + 1/2 * c) + 1/2 / d / b * a / (b * (a + b))^{1/2} / ((2 * (b * (a + b))^{1/2} - a - 2 * b) * a)^{1/2} * \operatorname{arctanh}(a * \tanh(1/2 * dx + 1/2 * c)) / ((2 * (b * (a + b))^{1/2} - a - 2 * b) * a)^{1/2} - 1/2 / d / b / ((2 * (b * (a + b))^{1/2} - a - 2 * b) * a)^{1/2} * \operatorname{arctanh}(a * \tanh(1/2 * dx + 1/2 * c)) / ((2 * (b * (a + b))^{1/2} - a - 2 * b) * a)^{1/2} + 1/2 / d / b * a / (b * (a + b))^{1/2} / ((2 * (b * (a + b))^{1/2} + a + 2 * b) * a)^{1/2} * \operatorname{arctan}(a * \tanh(1/2 * dx + 1/2 * c)) / ((2 * (b * (a + b))^{1/2} + a + 2 * b) * a)^{1/2} + 1/2 / d / b / ((2 * (b * (a + b))^{1/2} + a + 2 * b) * a)^{1/2} * \operatorname{arctan}(a * \tanh(1/2 * dx + 1/2 * c)) / ((2 * (b * (a + b))^{1/2} + a + 2 * b) * a)^{1/2} + 1/2 / d / a / ((2 * (b * (a + b))^{1/2} - a - 2 * b) * a)^{1/2} * \operatorname{arctanh}(a * \tanh(1/2 * dx + 1/2 * c)) / ((2 * (b * (a + b))^{1/2} - a - 2 * b) * a)^{1/2} - 1/2 / d / (b * (a + b))^{1/2} / a / ((2 * (b * (a + b))^{1/2} - a - 2 * b) * a)^{1/2} * \operatorname{arctanh}(a * \tanh(1/2 * dx + 1/2 * c)) / ((2 * (b * (a + b))^{1/2} - a - 2 * b) * a)^{1/2} * b - 1/2 / d / a / ((2 * (b * (a + b))^{1/2} + a + 2 * b) * a)^{1/2} * \operatorname{arctan}(a * \tanh(1/2 * dx + 1/2 * c)) / ((2 * (b * (a + b))^{1/2} + a + 2 * b) * a)^{1/2} - 1/2 / d / (b * (a + b))^{1/2} / a / ((2 * (b * (a + b))^{1/2} + a + 2 * b) * a)^{1/2} * \operatorname{arctan}(a * \tanh(1/2 * dx + 1/2 * c)) / ((2 * (b * (a + b))^{1/2} + a + 2 * b) * a)^{1/2} * b$

maxima [A] time = 0.54, size = 127, normalized size = 1.65

$$\frac{(a-b)e^{(-2dx-2c)} + a + b}{(a^2b + ab^2 + 2(a^2b - ab^2)e^{(-2dx-2c)} + (a^2b + ab^2)e^{(-4dx-4c)})d} + \frac{(a-b) \arctan\left(\frac{(a+b)e^{(-2dx-2c)} + a - b}{2\sqrt{ab}}\right)}{2\sqrt{ab}abd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] ((a - b)*e^(-2*d*x - 2*c) + a + b)/((a^2*b + a*b^2 + 2*(a^2*b - a*b^2)*e^(-2*d*x - 2*c) + (a^2*b + a*b^2)*e^(-4*d*x - 4*c))*d) + 1/2*(a - b)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a*b*d)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + dx)^4 (b \tanh(c + dx)^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2), x)

[Out] int(1/(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^4(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)

[Out] Integral(sech(c + d*x)**4/(a + b*tanh(c + d*x)**2)**2, x)

$$3.122 \quad \int \frac{\operatorname{sech}^5(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=102

$$\frac{(2a-b)\sqrt{a+b} \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d} + \frac{(a+b) \sinh(c+dx)}{2abd((a+b) \sinh^2(c+dx) + a)} + \frac{\tan^{-1}(\sinh(c+dx))}{b^2d}$$

[Out] arctan(sinh(d*x+c))/b^2/d+1/2*(a+b)*sinh(d*x+c)/a/b/d/(a+(a+b)*sinh(d*x+c)^2)-1/2*(2*a-b)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))*(a+b)^(1/2)/a^(3/2)/b^2/d

Rubi [A] time = 0.13, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3676, 414, 522, 203, 205}

$$\frac{(2a-b)\sqrt{a+b} \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d} + \frac{(a+b) \sinh(c+dx)}{2abd((a+b) \sinh^2(c+dx) + a)} + \frac{\tan^{-1}(\sinh(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^2,x]

[Out] ArcTan[Sinh[c + d*x]]/(b^2*d) - ((2*a - b)*Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^2*d) + ((a + b)*Sinh[c + d*x])/(2*a*b*d*(a + (a + b)*Sinh[c + d*x]^2))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 3676

$$\begin{aligned}
& (dx + c)^2 + 4*((a + b)*\cosh(dx + c)^3 - (3*a + b)*\cosh(dx + c))*\sinh(dx \\
& x + c) + 4*(a*\cosh(dx + c)^3 + 3*a*\cosh(dx + c)*\sinh(dx + c)^2 + a*\sinh(\\
& dx + c)^3 - a*\cosh(dx + c) + (3*a*\cosh(dx + c)^2 - a)*\sinh(dx + c))*\text{sq} \\
& \text{rt}(-(a + b)/a) + a + b)/((a + b)*\cosh(dx + c)^4 + 4*(a + b)*\cosh(dx + c)*\text{s} \\
& \text{inh}(dx + c)^3 + (a + b)*\sinh(dx + c)^4 + 2*(a - b)*\cosh(dx + c)^2 + 2*(3 \\
& *(a + b)*\cosh(dx + c)^2 + a - b)*\sinh(dx + c)^2 + 4*((a + b)*\cosh(dx + c \\
&)^3 + (a - b)*\cosh(dx + c))*\sinh(dx + c) + a + b)) + 8*((a^2 + a*b)*\cosh(\\
& dx + c)^4 + 4*(a^2 + a*b)*\cosh(dx + c)*\sinh(dx + c)^3 + (a^2 + a*b)*\sinh \\
& (dx + c)^4 + 2*(a^2 - a*b)*\cosh(dx + c)^2 + 2*(3*(a^2 + a*b)*\cosh(dx + c \\
&)^2 + a^2 - a*b)*\sinh(dx + c)^2 + a^2 + a*b + 4*((a^2 + a*b)*\cosh(dx + c) \\
& ^3 + (a^2 - a*b)*\cosh(dx + c))*\sinh(dx + c))*\arctan(\cosh(dx + c) + \sinh(\\
& dx + c)) - 4*(a*b + b^2)*\cosh(dx + c) + 4*(3*(a*b + b^2)*\cosh(dx + c)^2 \\
& - a*b - b^2)*\sinh(dx + c))/((a^2*b^2 + a*b^3)*d*\cosh(dx + c)^4 + 4*(a^2*b \\
& ^2 + a*b^3)*d*\cosh(dx + c)*\sinh(dx + c)^3 + (a^2*b^2 + a*b^3)*d*\sinh(dx \\
& + c)^4 + 2*(a^2*b^2 - a*b^3)*d*\cosh(dx + c)^2 + 2*(3*(a^2*b^2 + a*b^3)*d*c \\
& \text{osh}(dx + c)^2 + (a^2*b^2 - a*b^3)*d)*\sinh(dx + c)^2 + (a^2*b^2 + a*b^3)*d \\
& + 4*((a^2*b^2 + a*b^3)*d*\cosh(dx + c)^3 + (a^2*b^2 - a*b^3)*d*\cosh(dx + \\
& c))*\sinh(dx + c)), 1/2*(2*(a*b + b^2)*\cosh(dx + c)^3 + 6*(a*b + b^2)*\cosh \\
& (dx + c)*\sinh(dx + c)^2 + 2*(a*b + b^2)*\sinh(dx + c)^3 - ((2*a^2 + a*b - \\
& b^2)*\cosh(dx + c)^4 + 4*(2*a^2 + a*b - b^2)*\cosh(dx + c)*\sinh(dx + c)^3 \\
& + (2*a^2 + a*b - b^2)*\sinh(dx + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*\cosh(dx + \\
& c)^2 + 2*(3*(2*a^2 + a*b - b^2)*\cosh(dx + c)^2 + 2*a^2 - 3*a*b + b^2)*\text{sin} \\
& \text{h}(dx + c)^2 + 2*a^2 + a*b - b^2 + 4*((2*a^2 + a*b - b^2)*\cosh(dx + c)^3 + \\
& (2*a^2 - 3*a*b + b^2)*\cosh(dx + c))*\sinh(dx + c))*\text{sqrt}((a + b)/a)*\arctan \\
& (1/2*\text{sqrt}((a + b)/a)*(\cosh(dx + c) + \sinh(dx + c))) - ((2*a^2 + a*b - b^2 \\
&)*\cosh(dx + c)^4 + 4*(2*a^2 + a*b - b^2)*\cosh(dx + c)*\sinh(dx + c)^3 + (\\
& 2*a^2 + a*b - b^2)*\sinh(dx + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*\cosh(dx + c)^ \\
& 2 + 2*(3*(2*a^2 + a*b - b^2)*\cosh(dx + c)^2 + 2*a^2 - 3*a*b + b^2)*\text{sin} \\
& \text{h}(dx + c)^2 + 2*a^2 + a*b - b^2 + 4*((2*a^2 + a*b - b^2)*\cosh(dx + c)^3 + (2* \\
& a^2 - 3*a*b + b^2)*\cosh(dx + c))*\sinh(dx + c))*\text{sqrt}((a + b)/a)*\arctan(1/2 \\
& *((a + b)*\cosh(dx + c)^3 + 3*(a + b)*\cosh(dx + c)*\sinh(dx + c)^2 + (a + \\
& b)*\sinh(dx + c)^3 + (3*a - b)*\cosh(dx + c) + (3*(a + b)*\cosh(dx + c)^2 + \\
& 3*a - b)*\sinh(dx + c))*\text{sqrt}((a + b)/a)/(a + b)) + 4*((a^2 + a*b)*\cosh(dx \\
& + c)^4 + 4*(a^2 + a*b)*\cosh(dx + c)*\sinh(dx + c)^3 + (a^2 + a*b)*\sinh(dx \\
& x + c)^4 + 2*(a^2 - a*b)*\cosh(dx + c)^2 + 2*(3*(a^2 + a*b)*\cosh(dx + c)^2 \\
& + a^2 - a*b)*\sinh(dx + c)^2 + a^2 + a*b + 4*((a^2 + a*b)*\cosh(dx + c)^3 \\
& + (a^2 - a*b)*\cosh(dx + c))*\sinh(dx + c))*\arctan(\cosh(dx + c) + \sinh(dx \\
& + c)) - 2*(a*b + b^2)*\cosh(dx + c) + 2*(3*(a*b + b^2)*\cosh(dx + c)^2 - a \\
& *b - b^2)*\sinh(dx + c))/((a^2*b^2 + a*b^3)*d*\cosh(dx + c)^4 + 4*(a^2*b^2 \\
& + a*b^3)*d*\cosh(dx + c)*\sinh(dx + c)^3 + (a^2*b^2 + a*b^3)*d*\sinh(dx + c \\
&)^4 + 2*(a^2*b^2 - a*b^3)*d*\cosh(dx + c)^2 + 2*(3*(a^2*b^2 + a*b^3)*d*\cosh \\
& (dx + c)^2 + (a^2*b^2 - a*b^3)*d)*\sinh(dx + c)^2 + (a^2*b^2 + a*b^3)*d + \\
& 4*((a^2*b^2 + a*b^3)*d*\cosh(dx + c)^3 + (a^2*b^2 - a*b^3)*d*\cosh(dx + c)) \\
& *\sinh(dx + c))]
\end{aligned}$$

giac [B] time = 0.67, size = 1035, normalized size = 10.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)^5/(a+b*tanh(dx+c)^2)^2,x, algorithm="giac")

[Out] $1/2*((2*(4*a^2*b - 2*a*b^2 - (2*a^2 - 3*a*b + b^2)*\text{sqrt}(-a*b))*\text{sqrt}(a^2 - b^2 + 2*\text{sqrt}(-a*b)*(a + b))*a^2*\text{abs}(a*e^{(2*c)} + b*e^{(2*c)}) - (2*a^4 - a^3*b - 2*a^2*b^2 + a*b^3 + 2*(2*a^3 + a^2*b - a*b^2)*\text{sqrt}(-a*b))*\text{sqrt}(a^2 - b^2 + 2*\text{sqrt}(-a*b)*(a + b))*\text{abs}(a*e^{(2*c)} + b*e^{(2*c)})*\text{abs}(a) - (4*a^4*b - 6*a^3*b^2 + 2*a^2*b^3 - (2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*\text{sqrt}(-a*b))*\text{sqrt}(a^2 - b^2 + 2*\text{sqrt}(-a*b)*(a + b))*\text{abs}(a*e^{(2*c)} + b*e^{(2*c)}))*\arctan(e^{(dx)}/\text{sqrt}((a^2*b^2*e^{(2*c)} - a*b^3*e^{(2*c)} - \text{sqrt}((a^2*b^2*e^{(2*c)} - a*b^3*e^{(2*c)})^2 - (a^2*b^2*e^{(4*c)} + a*b^3*e^{(4*c)}))*(a^2*b^2 + a*b^3)))/(a^2*b^2*e^{(2*c)})$

$$(4*c) + a*b^3*e^{(4*c)})) * e^{(-2*c)} / ((a^6*b^2 + 2*a^5*b^3 - 2*a^3*b^5 - a^2*b^6 + 2*(a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*\sqrt{-a*b})*\text{abs}(a)) + (2*(8*a^3*b - 12*a^2*b^2 + 4*a*b^3 + (2*a^3 - 13*a^2*b + 8*a*b^2 - b^3)*\sqrt{-a*b})*a^2*\text{abs}(a*e^{(2*c)} + b*e^{(2*c)}) - (2*a^5 - 11*a^4*b - 5*a^3*b^2 + 7*a^2*b^3 - a*b^4 - 4*(2*a^4 - a^3*b - 2*a^2*b^2 + a*b^3)*\sqrt{-a*b})*\text{abs}(a*e^{(2*c)} + b*e^{(2*c)})*\text{abs}(a) - (8*a^5*b - 20*a^4*b^2 + 16*a^3*b^3 - 4*a^2*b^4 + (2*a^5 - 15*a^4*b + 21*a^3*b^2 - 9*a^2*b^3 + a*b^4)*\sqrt{-a*b})*\text{abs}(a*e^{(2*c)} + b*e^{(2*c)})) * \arctan(e^{(d*x)} / \sqrt{(a^2*b^2*e^{(2*c)} - a*b^3*e^{(2*c)} + \sqrt{(a^2*b^2*e^{(2*c)} - a*b^3*e^{(2*c)})^2 - (a^2*b^2*e^{(4*c)} + a*b^3*e^{(4*c)})}) * (a^2*b^2 + a*b^3)}) / (a^2*b^2*e^{(4*c)} + a*b^3*e^{(4*c)})) * e^{(-2*c)} / ((a^5*b^2 + a^4*b^3 - a^3*b^4 - a^2*b^5 - 2*(a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*\sqrt{-a*b})*\sqrt{a^2 - b^2 - 2*\sqrt{-a*b}*(a + b)}*\text{abs}(a)) + 4*\arctan(e^{(d*x + c)} / b^2 + 2*(a*e^{(3*d*x + 3*c)} + b*e^{(3*d*x + 3*c)} - a*e^{(d*x + c)} - b*e^{(d*x + c)}) / ((a*e^{(4*d*x + 4*c)} + b*e^{(4*d*x + 4*c)} + 2*a*e^{(2*d*x + 2*c)} - 2*b*e^{(2*d*x + 2*c)} + a + b)*a*b)) / d$$

maple [B] time = 0.36, size = 1007, normalized size = 9.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x)

[Out]
$$-1/d / (\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a) / b*\tanh(1/2*d*x+1/2*c)^3 - 1/d / (\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a) / a*\tanh(1/2*d*x+1/2*c)^3 + 1/d / (\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a) / b*\tanh(1/2*d*x+1/2*c) + 1/d / (\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a) / a*\tanh(1/2*d*x+1/2*c) + 1/d / b^2*a / ((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c)) / ((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2) - 1/d / b*a / (b*(a+b))^(1/2) / ((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2) * \arctanh(a*\tanh(1/2*d*x+1/2*c)) / ((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2) - 1/d / b^2*a / ((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*\arctan(a*\tanh(1/2*d*x+1/2*c)) / ((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2) - 1/d / b*a / (b*(a+b))^(1/2) / ((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2) * \arctan(a*\tanh(1/2*d*x+1/2*c)) / ((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2) + 1/2/d / b / ((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2) * \arctanh(a*\tanh(1/2*d*x+1/2*c)) / ((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2) - 1/2/d / (b*(a+b))^(1/2) / ((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2) * \arctanh(a*\tanh(1/2*d*x+1/2*c)) / ((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2) - 1/2/d / b / ((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2) * \arctan(a*\tanh(1/2*d*x+1/2*c)) / ((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2) - 1/2/d / a / ((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2) * \arctanh(a*\tanh(1/2*d*x+1/2*c)) / ((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2) + 1/2/d / (b*(a+b))^(1/2) / a / ((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2) * \arctan(a*\tanh(1/2*d*x+1/2*c)) / ((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2) + 1/2/d / (b*(a+b))^(1/2) / a / ((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2) * \arctan(a*\tanh(1/2*d*x+1/2*c)) / ((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2) * b + 2/d / b^2 * \arctan(\tanh(1/2*d*x+1/2*c))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(ae^{(3c)} + be^{(3c)})e^{(3dx)} - (ae^c + be^c)e^{(dx)}}{a^2bd + ab^2d + (a^2bde^{(4c)} + ab^2de^{(4c)})e^{(4dx)} + 2(a^2bde^{(2c)} - ab^2de^{(2c)})e^{(2dx)}} + \frac{2 \arctan(e^{(dx+c)})}{b^2d} - 32 \int \frac{1}{32(a^2b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out]
$$((a*e^{(3*c)} + b*e^{(3*c)}) * e^{(3*d*x)} - (a*e^c + b*e^c) * e^{(d*x)}) / (a^2*b*d + a*b^2*d + (a^2*b*d*e^{(4*c)} + a*b^2*d*e^{(4*c)}) * e^{(4*d*x)} + 2*(a^2*b*d*e^{(2*c)} - a*b^2*d*e^{(2*c)}) * e^{(2*d*x)})$$

- a*b^2*d*e^(2*c))*e^(2*d*x)) + 2*arctan(e^(d*x + c))/(b^2*d) - 32*integrate(1/32*((2*a^2*e^(3*c) + a*b*e^(3*c) - b^2*e^(3*c))*e^(3*d*x) + (2*a^2*e^c + a*b*e^c - b^2*e^c)*e^(d*x))/(a^2*b^2 + a*b^3 + (a^2*b^2*e^(4*c) + a*b^3*e^(4*c))*e^(4*d*x) + 2*(a^2*b^2*e^(2*c) - a*b^3*e^(2*c))*e^(2*d*x)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + dx)^5 (b \tanh(c + dx)^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)^2), x)

[Out] int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^5(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**5/(a+b*tanh(d*x+c)**2)**2, x)

[Out] Integral(sech(c + d*x)**5/(a + b*tanh(c + d*x)**2)**2, x)

$$3.123 \quad \int \frac{\operatorname{sech}^6(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=97

$$-\frac{(3a-b)(a+b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d} + \frac{(a+b)^2 \tanh(c+dx)}{2ab^2d(a+b \tanh^2(c+dx))} + \frac{\tanh(c+dx)}{b^2d}$$

[Out] $-1/2*(3*a-b)*(a+b)*\arctan(b^{(1/2)}*\tanh(d*x+c)/a^{(1/2)})/a^{(3/2)}/b^{(5/2)}/d+\tanh(d*x+c)/b^2/d+1/2*(a+b)^2*\tanh(d*x+c)/a/b^2/d/(a+b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.13, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3675, 390, 385, 205}

$$-\frac{(3a-b)(a+b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d} + \frac{(a+b)^2 \tanh(c+dx)}{2ab^2d(a+b \tanh^2(c+dx))} + \frac{\tanh(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^2,x]

[Out] $-((3*a - b)*(a + b)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Tanh}[c + d*x])/\text{Sqrt}[a]])/(2*a^{(3/2)}*b^{(5/2)}*d) + \text{Tanh}[c + d*x]/(b^2*d) + ((a + b)^2*\text{Tanh}[c + d*x])/(2*a*b^2*d*(a + b*\text{Tanh}[c + d*x]^2))$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^6(c+dx)}{(a+b \tanh^2(c+dx))^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{(a+bx^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{b^2} - \frac{a^2-b^2+2b(a+b)x^2}{b^2(a+bx^2)^2}\right) dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\tanh(c+dx)}{b^2 d} - \frac{\operatorname{Subst}\left(\int \frac{a^2-b^2+2b(a+b)x^2}{(a+bx^2)^2} dx, x, \tanh(c+dx)\right)}{b^2 d} \\
&= \frac{\tanh(c+dx)}{b^2 d} + \frac{(a+b)^2 \tanh(c+dx)}{2ab^2 d (a+b \tanh^2(c+dx))} - \frac{((3a-b)(a+b)) \operatorname{Subst}\left(\int \frac{1}{a+bx^2} dx\right)}{2ab^2 d} \\
&= -\frac{(3a-b)(a+b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2} b^{5/2} d} + \frac{\tanh(c+dx)}{b^2 d} + \frac{(a+b)^2 \tanh(c+dx)}{2ab^2 d (a+b \tanh^2(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.65, size = 102, normalized size = 1.05

$$\frac{-\frac{(3a-b)(a+b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\sqrt{b}(a+b)^2 \sinh(2(c+dx))}{a((a+b) \cosh(2(c+dx))+a-b)} + 2\sqrt{b} \tanh(c+dx)}{2b^{5/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (-(((3*a - b)*(a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2)) + (Sqrt[b]*(a + b)^2*Sinh[2*(c + d*x)]/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])) + 2*Sqrt[b]*Tanh[c + d*x])/(2*b^(5/2)*d)

fricas [B] time = 0.50, size = 2869, normalized size = 29.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [-1/4*(4*(3*a^3*b + 2*a^2*b^2 - a*b^3)*cosh(d*x + c)^4 + 16*(3*a^3*b + 2*a^2*b^2 - a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(3*a^3*b + 2*a^2*b^2 - a*b^3)*sinh(d*x + c)^4 + 12*a^3*b + 16*a^2*b^2 + 4*a*b^3 + 8*(3*a^3*b - a^2*b^2)*cosh(d*x + c)^2 + 8*(3*a^3*b - a^2*b^2 + 3*(3*a^3*b + 2*a^2*b^2 - a*b^3))*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^6 + 6*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^3 + 5*a^2*b + a*b^2 - b^3)*sinh(d*x + c)^6 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^4 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3 + 15*(3*a^3 + 5*a^2*b + a*b^2 - b^3))*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(3*a^3 + 5*a^2*b + a*b^2 - b^3))*cosh(d*x + c)^3 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a^3 + 5*a^2*b + a*b^2 - b^3 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^2 + (15*(3*a^3 + 5*a^2*b + a*b^2 - b^3))*cosh(d*x + c)^4 + 9*a^3 + 3*a^2*b - 5*a*b^2 + b^3 + 6*(9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(3*a^3 + 5*a^2*b + a*b^2 - b^3))*cosh(d*x + c)^5 + 2*(9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^3 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 4*(a^2 + 2*a*b + b^2))


```

osh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2
- b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 -
b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c
)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a -
b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x
+ c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b
)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (
a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 16*((3*a^3*b + 2*a^2*b^2 -
a*b^3)*cosh(d*x + c)^3 + (3*a^3*b - a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))/
((a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^6 + 6*(a^3*b^3 + a^2*b^4)*d*cosh(d*x +
c)*sinh(d*x + c)^5 + (a^3*b^3 + a^2*b^4)*d*sinh(d*x + c)^6 + (3*a^3*b^3 -
a^2*b^4)*d*cosh(d*x + c)^4 + (15*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^2 + (3
*a^3*b^3 - a^2*b^4)*d)*sinh(d*x + c)^4 + (3*a^3*b^3 - a^2*b^4)*d*cosh(d*x +
c)^2 + 4*(5*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^3 + (3*a^3*b^3 - a^2*b^4)*
d*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^
4 + 6*(3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^2 + (3*a^3*b^3 - a^2*b^4)*d)*si
nh(d*x + c)^2 + (a^3*b^3 + a^2*b^4)*d + 2*(3*(a^3*b^3 + a^2*b^4)*d*cosh(d*x
+ c)^5 + 2*(3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^3 + (3*a^3*b^3 - a^2*b^4)
*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(3*a^3*b + 2*a^2*b^2 - a*b^3)*cos
h(d*x + c)^4 + 8*(3*a^3*b + 2*a^2*b^2 - a*b^3)*cosh(d*x + c)*sinh(d*x + c)^
3 + 2*(3*a^3*b + 2*a^2*b^2 - a*b^3)*sinh(d*x + c)^4 + 6*a^3*b + 8*a^2*b^2 +
2*a*b^3 + 4*(3*a^3*b - a^2*b^2)*cosh(d*x + c)^2 + 4*(3*a^3*b - a^2*b^2 + 3
*(3*a^3*b + 2*a^2*b^2 - a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^3 +
5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^6 + 6*(3*a^3 + 5*a^2*b + a*b^2 - b^3)
*cosh(d*x + c)*sinh(d*x + c)^5 + (3*a^3 + 5*a^2*b + a*b^2 - b^3)*sinh(d*x +
c)^6 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^4 + (9*a^3 + 3*a^2*
b - 5*a*b^2 + b^3 + 15*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^4 + 4*(5*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^3 + (9*a^
3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a^3 + 5*a^2
*b + a*b^2 - b^3 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^2 + (15*
(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^4 + 9*a^3 + 3*a^2*b - 5*a*b^2
+ b^3 + 6*(9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)
^2 + 2*(3*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^5 + 2*(9*a^3 + 3*a^
2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^3 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*co
sh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 +
2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*s
qrt(a*b)/(a*b)) + 8*((3*a^3*b + 2*a^2*b^2 - a*b^3)*cosh(d*x + c)^3 + (3*a^3
*b - a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^3*b^3 + a^2*b^4)*d*cosh(d*x
+ c)^6 + 6*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^3*b^3
+ a^2*b^4)*d*sinh(d*x + c)^6 + (3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^4 + (1
5*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^2 + (3*a^3*b^3 - a^2*b^4)*d)*sinh(d*x
+ c)^4 + (3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^2 + 4*(5*(a^3*b^3 + a^2*b^4)
)*d*cosh(d*x + c)^3 + (3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^
3 + (15*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^4 + 6*(3*a^3*b^3 - a^2*b^4)*d*c
osh(d*x + c)^2 + (3*a^3*b^3 - a^2*b^4)*d)*sinh(d*x + c)^2 + (a^3*b^3 + a^2*
b^4)*d + 2*(3*(a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^5 + 2*(3*a^3*b^3 - a^2*b^
4)*d*cosh(d*x + c)^3 + (3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)
)]

```

giac [B] time = 0.60, size = 248, normalized size = 2.56

$$\frac{(3a^2e^{2c} + 2abe^{2c} - b^2e^{2c}) \arctan\left(\frac{ae^{2dx+2c} + be^{2dx+2c} + a-b}{2\sqrt{ab}}\right) e^{(-2c)}}{\sqrt{ab}ab^2} + \frac{2(3a^2e^{4dx+4c} + 2abe^{4dx+4c} - b^2e^{4dx+4c} + 6a^2e^{2dx+2c} - 2abe^{2dx+2c} - b^2e^{2dx+2c})}{(ae^{6dx+6c} + be^{6dx+6c} + 3ae^{4dx+4c} - be^{4dx+4c} + 3ae^{2dx+2c} - be^{2dx+2c})}$$

2d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

```
[Out] -1/2*((3*a^2*e^(2*c) + 2*a*b*e^(2*c) - b^2*e^(2*c))*arctan(1/2*(a*e^(2*d*x
+ 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))*e^(-2*c)/(sqrt(a*b)*a*b^2) +
  2*(3*a^2*e^(4*d*x + 4*c) + 2*a*b*e^(4*d*x + 4*c) - b^2*e^(4*d*x + 4*c) + 6
*a^2*e^(2*d*x + 2*c) - 2*a*b*e^(2*d*x + 2*c) + 3*a^2 + 4*a*b + b^2)/((a*e^(
6*d*x + 6*c) + b*e^(6*d*x + 6*c) + 3*a*e^(4*d*x + 4*c) - b*e^(4*d*x + 4*c)
+ 3*a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + a + b)*a*b^2))/d
```

maple [B] time = 0.36, size = 1283, normalized size = 13.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] 1/d/b^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1
/2*c)^2*b+a)*a*tanh(1/2*d*x+1/2*c)^3+2/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/
2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/b*tanh(1/2*d*x+1/2*c)^3+1/d/(
tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b
+a)/a*tanh(1/2*d*x+1/2*c)^3+1/d/b^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x
+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*a*tanh(1/2*d*x+1/2*c)+2/d/(tanh(1/
2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/b*t
anh(1/2*d*x+1/2*c)+1/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4
*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)+3/2/d/b^2*a^2/(b*(a+b))^(
1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c))/((2*
(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-3/2/d/b^2*a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(
1/2)*arctanh(a*tanh(1/2*d*x+1/2*c))/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+5/
2/d/b*a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(
1/2*d*x+1/2*c))/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/2/d/b^2*a^2/(b*(a+b))
^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c))/((2
*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+3/2/d/b^2*a/((2*(b*(a+b))^(1/2)+a+2*b)*a)
^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c))/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+5/
2/d/b*a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1
/2*d*x+1/2*c))/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/d/b/((2*(b*(a+b))^(1/2
)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c))/((2*(b*(a+b))^(1/2)-a-2*b)*
a)^(1/2))+1/2/d/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh
(a*tanh(1/2*d*x+1/2*c))/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/d/b/((2*(b*(a
+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c))/((2*(b*(a+b))^(1/2
)+a+2*b)*a)^(1/2))+1/2/d/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2
)*arctan(a*tanh(1/2*d*x+1/2*c))/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/2/d/a/
((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c))/((2*(b*(a
+b))^(1/2)-a-2*b)*a)^(1/2))-1/2/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)-a-2
*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c))/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1
/2))*b-1/2/d/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/
2*c))/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/2/d/(b*(a+b))^(1/2)/a/((2*(b*(a
+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c))/((2*(b*(a+b))^(1/2
)+a+2*b)*a)^(1/2))*b+2/d/b^2*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+1)
```

maxima [B] time = 0.60, size = 209, normalized size = 2.15

$$\frac{3a^2 + 4ab + b^2 + 2(3a^2 - ab)e^{(-2dx-2c)} + (3a^2 + 2ab - b^2)e^{(-4dx-4c)}}{(a^2b^2 + ab^3 + (3a^2b^2 - ab^3)e^{(-2dx-2c)} + (3a^2b^2 - ab^3)e^{(-4dx-4c)} + (a^2b^2 + ab^3)e^{(-6dx-6c)})d} + \frac{(3a^2 + 2ab - b^2)a}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] (3*a^2 + 4*a*b + b^2 + 2*(3*a^2 - a*b)*e^(-2*d*x - 2*c) + (3*a^2 + 2*a*b -
b^2)*e^(-4*d*x - 4*c))/((a^2*b^2 + a*b^3 + (3*a^2*b^2 - a*b^3)*e^(-2*d*x -
2*c) + (3*a^2*b^2 - a*b^3)*e^(-4*d*x - 4*c) + (a^2*b^2 + a*b^3)*e^(-6*d*x -
```

$6*c)) * d) + 1/2*(3*a^2 + 2*a*b - b^2) * \arctan(1/2*((a + b)*e^{(-2*d*x - 2*c)} + a - b)/\sqrt{a*b}) / (\sqrt{a*b}) * a*b^2*d)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + dx)^6 (b \tanh(c + dx)^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)^2), x)

[Out] int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^6(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**6/(a+b*tanh(d*x+c)**2)**2, x)

[Out] Integral(sech(c + d*x)**6/(a + b*tanh(c + d*x)**2)**2, x)

$$3.124 \quad \int \frac{\operatorname{sech}^7(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=155

$$\frac{(4a-b)(a+b)^{3/2} \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d} + \frac{(4a+5b) \tan^{-1}(\sinh(c+dx))}{2b^3d} + \frac{(2a+b)(a+b) \sinh(c+dx)}{2ab^2d((a+b) \sinh^2(c+dx)+a)} - \frac{2b}{2b^3d}$$

[Out] 1/2*(4*a+5*b)*arctan(sinh(d*x+c))/b^3/d-1/2*(4*a-b)*(a+b)^(3/2)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(3/2)/b^3/d+1/2*(a+b)*(2*a+b)*sinh(d*x+c)/a/b^2/d/(a+(a+b)*sinh(d*x+c)^2)-1/2*sech(d*x+c)*tanh(d*x+c)/b/d/(a+(a+b)*sinh(d*x+c)^2)

Rubi [A] time = 0.25, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3676, 414, 527, 522, 203, 205}

$$\frac{(4a-b)(a+b)^{3/2} \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d} + \frac{(2a+b)(a+b) \sinh(c+dx)}{2ab^2d((a+b) \sinh^2(c+dx)+a)} + \frac{(4a+5b) \tan^{-1}(\sinh(c+dx))}{2b^3d} - \frac{2b}{2b^3d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^7/(a + b*Tanh[c + d*x]^2), x]

[Out] ((4*a + 5*b)*ArcTan[Sinh[c + d*x]])/(2*b^3*d) - ((4*a - b)*(a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^3*d) + ((a + b)*(2*a + b)*Sinh[c + d*x])/(2*a*b^2*d*(a + (a + b)*Sinh[c + d*x]^2)) - (Sech[c + d*x]*Tanh[c + d*x])/(2*b*d*(a + (a + b)*Sinh[c + d*x]^2))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^(m + n*p + 1)/2, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^7(c + dx)}{(a + b \tanh^2(c + dx))^2} dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^2(a+(a+b)x^2)^2} dx, x, \sinh(c + dx)\right)}{d}$$

$$= -\frac{\operatorname{sech}(c + dx) \tanh(c + dx)}{2bd(a + (a + b) \sinh^2(c + dx))} + \frac{\operatorname{Subst}\left(\int \frac{a+2b-3(a+b)x^2}{(1+x^2)(a+(a+b)x^2)^2} dx, x, \sinh(c + dx)\right)}{2bd}$$

$$= \frac{(a + b)(2a + b) \sinh(c + dx)}{2ab^2d(a + (a + b) \sinh^2(c + dx))} - \frac{\operatorname{sech}(c + dx) \tanh(c + dx)}{2bd(a + (a + b) \sinh^2(c + dx))} - \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(c + dx)\right)}{2bd}$$

$$= \frac{(a + b)(2a + b) \sinh(c + dx)}{2ab^2d(a + (a + b) \sinh^2(c + dx))} - \frac{\operatorname{sech}(c + dx) \tanh(c + dx)}{2bd(a + (a + b) \sinh^2(c + dx))} - \frac{\operatorname{ArcTan}\left(\frac{\sinh(c + dx)}{\sqrt{a}}\right)}{2bd}$$

$$= \frac{(4a + 5b) \tan^{-1}(\sinh(c + dx))}{2b^3d} - \frac{(4a - b)(a + b)^{3/2} \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d} + \frac{\operatorname{sech}(c + dx) \tanh(c + dx)}{2bd}$$

Mathematica [A] time = 1.41, size = 265, normalized size = 1.71

$$(a - b) \left(2a^{3/2}(4a + 5b)\sqrt{a + b} \tan^{-1}\left(\tanh\left(\frac{1}{2}(c + dx)\right)\right) \right) + a^{3/2}b\sqrt{a + b} \tanh(c + dx) \operatorname{sech}(c + dx) + (4a - b)(a + b) \operatorname{sech}(c + dx) \tanh(c + dx)$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^7/(a + b*Tanh[c + d*x]^2)^2, x]

[Out] (2*sqrt[a]*b*(a + b)^(5/2)*Sinh[c + d*x] + (a - b)*((4*a - b)*(a + b)^2*ArcTan[(sqrt[a]*Csch[c + d*x])/sqrt[a + b]] + 2*a^(3/2)*sqrt[a + b]*(4*a + 5*b)*ArcTan[Tanh[(c + d*x)/2]] + a^(3/2)*b*sqrt[a + b]*Sech[c + d*x]*Tanh[c + d*x]) + (a + b)*Cosh[2*(c + d*x)]*((4*a - b)*(a + b)^2*ArcTan[(sqrt[a]*Csch[c + d*x])/sqrt[a + b]] + 2*a^(3/2)*sqrt[a + b]*(4*a + 5*b)*ArcTan[Tanh[(c + d*x)/2]] + a^(3/2)*b*sqrt[a + b]*Sech[c + d*x]*Tanh[c + d*x]))/(2*a^(3/2)*b^3*sqrt[a + b]*d*(a - b + (a + b)*Cosh[2*(c + d*x)]))

fricas [B] time = 0.57, size = 6396, normalized size = 41.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/4*(4*(2*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^7 + 28*(2*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)*\sinh(d*x + c)^6 + 4*(2*a^2*b + 3*a*b^2 + b^3)*\sinh(d*x + c)^7 + 4*(2*a^2*b - a*b^2 + b^3)*\cosh(d*x + c)^5 + 4*(2*a^2*b - a*b^2 + b^3 + 21*(2*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^5 + 20*(7*(2*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^3 + (2*a^2*b - a*b^2 + b^3)*\cosh(d*x + c))*\sinh(d*x + c)^4 - 4*(2*a^2*b - a*b^2 + b^3)*\cosh(d*x + c)^3 + 4*(35*(2*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^4 - 2*a^2*b + a*b^2 - b^3 + 10*(2*a^2*b - a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^3 + 4*(21*(2*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^5 + 10*(2*a^2*b - a*b^2 + b^3)*\cosh(d*x + c)^3 - 3*(2*a^2*b - a*b^2 + b^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 - ((4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*\cosh(d*x + c)^8 + 8*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*\sinh(d*x + c)^8 + 4*(4*a^3 + 3*a^2*b - a*b^2)*\cosh(d*x + c)^6 + 4*(4*a^3 + 3*a^2*b - a*b^2 + 7*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*\cosh(d*x + c)^3 + 3*(4*a^3 + 3*a^2*b - a*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(12*a^3 + 5*a^2*b - 6*a*b^2 + b^3)*\cosh(d*x + c)^4 + 2*(35*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*\cosh(d*x + c)^4 + 12*a^3 + 5*a^2*b - 6*a*b^2 + b^3 + 30*(4*a^3 + 3*a^2*b - a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(7*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*\cosh(d*x + c)^5 + 10*(4*a^3 + 3*a^2*b - a*b^2)*\cosh(d*x + c)^3 + (12*a^3 + 5*a^2*b - 6*a*b^2 + b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*a^3 + 7*a^2*b + 2*a*b^2 - b^3 + 4*(4*a^3 + 3*a^2*b - a*b^2)*\cosh(d*x + c)^2 + 4*(7*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*\cosh(d*x + c)^6 + 15*(4*a^3 + 3*a^2*b - a*b^2)*\cosh(d*x + c)^4 + 4*a^3 + 3*a^2*b - a*b^2 + 3*(12*a^3 + 5*a^2*b - 6*a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*\cosh(d*x + c)^7 + 3*(4*a^3 + 3*a^2*b - a*b^2)*\cosh(d*x + c)^5 + (12*a^3 + 5*a^2*b - 6*a*b^2 + b^3)*\cosh(d*x + c)^3 + (4*a^3 + 3*a^2*b - a*b^2)*\cosh(d*x + c))*\sinh(d*x + c)*\sqrt{-(a + b)/a}*\log(((a + b)*\cosh(d*x + c)^4 + 4*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a + b)*\sinh(d*x + c)^4 - 2*(3*a + b)*\cosh(d*x + c)^2 + 2*(3*(a + b)*\cosh(d*x + c)^2 - 3*a - b)*\sinh(d*x + c)^2 + 4*((a + b)*\cosh(d*x + c)^3 - (3*a + b)*\cosh(d*x + c))*\sinh(d*x + c) + 4*(a*\cosh(d*x + c)^3 + 3*a*\cosh(d*x + c)*\sinh(d*x + c)^2 + a*\sinh(d*x + c)^3 - a*\cosh(d*x + c) + (3*a*\cosh(d*x + c)^2 - a)*\sinh(d*x + c))*\sqrt{-(a + b)/(a + b)} / ((a + b)*\cosh(d*x + c)^4 + 4*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a + b)*\sinh(d*x + c)^4 + 2*(a - b)*\cosh(d*x + c)^2 + 2*(3*(a + b)*\cosh(d*x + c)^2 + a - b)*\sinh(d*x + c)^2 + 4*((a + b)*\cosh(d*x + c)^3 + (a - b)*\cosh(d*x + c))*\sinh(d*x + c) + a + b)) + 4*((4*a^3 + 9*a^2*b + 5*a*b^2)*\cosh(d*x + c)^8 + 8*(4*a^3 + 9*a^2*b + 5*a*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (4*a^3 + 9*a^2*b + 5*a*b^2)*\sinh(d*x + c)^8 + 4*(4*a^3 + 5*a^2*b)*\cosh(d*x + c)^6 + 4*(4*a^3 + 5*a^2*b + 7*(4*a^3 + 9*a^2*b + 5*a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(4*a^3 + 9*a^2*b + 5*a*b^2)*\cosh(d*x + c)^3 + 3*(4*a^3 + 5*a^2*b)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(12*a^3 + 11*a^2*b - 5*a*b^2)*\cosh(d*x + c)^4 + 2*(35*(4*a^3 + 9*a^2*b + 5*a*b^2)*\cosh(d*x + c)^4 + 12*a^3 + 11*a^2*b - 5*a*b^2 + 30*(4*a^3 + 5*a^2*b)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(7*(4*a^3 + 9*a^2*b + 5*a*b^2)*\cosh(d*x + c)^5 + 10*(4*a^3 + 5*a^2*b)*\cosh(d*x + c)^3 + (12*a^3 + 11*a^2*b - 5*a*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*a^3 + 9*a^2*b + 5*a*b^2 + 4*(4*a^3 + 5*a^2*b)*\cosh(d*x + c)^2 + 4*(7*(4*a^3 + 9*a^2*b + 5*a*b^2)*\cosh(d*x + c)^6 + 15*(4*a^3 + 5*a^2*b)*\cosh(d*x + c)^4 + 4*a^3 + 5*a^2*b + 3*(12*a^3 + 11*a^2*b - 5*a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((4*a^3 + 9*a^2*b + 5*a*b^2)*\cosh(d*x + c)^7 + 3*(4*a^3 + 5*a^2*b)*\cosh(d*x + c)^5 + (12*a^3 + 11*a^2*b - 5*a*b^2)*\cosh(d*x + c)^3 + (4*a^3 + 5*a^2*b)*\cosh(d*x + c))*\sinh(d*x + c))*\arctan(\cosh(d*x + c) + \sinh(d*x + c)) - 4*(2*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c) + 4*(7*(2*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^6 + 5*(2*a^2*b - a*b^2 + b^3)*\cosh(d*x + c)^4 - 2*a^2*b - 3*a*b^2 - b^3 - 3*(2*a^2*b - a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)) / (4*a^2*b^3*d*\cosh(d*x + c)^6 + (a^2*b^3 + a*b^$$

$$\begin{aligned}
& 4)*d*\cosh(d*x + c)^8 + 8*(a^2*b^3 + a*b^4)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 \\
& + (a^2*b^3 + a*b^4)*d*\sinh(d*x + c)^8 + 4*a^2*b^3*d*\cosh(d*x + c)^2 + 4*(a^2 \\
& *b^3*d + 7*(a^2*b^3 + a*b^4)*d*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 2*(3*a^2 \\
& *b^3 - a*b^4)*d*\cosh(d*x + c)^4 + 8*(3*a^2*b^3*d*\cosh(d*x + c) + 7*(a^2*b^3 \\
& + a*b^4)*d*\cosh(d*x + c)^3)*\sinh(d*x + c)^5 + 2*(30*a^2*b^3*d*\cosh(d*x + c \\
&)^2 + 35*(a^2*b^3 + a*b^4)*d*\cosh(d*x + c)^4 + (3*a^2*b^3 - a*b^4)*d)*\sinh(\\
& d*x + c)^4 + 8*(10*a^2*b^3*d*\cosh(d*x + c)^3 + 7*(a^2*b^3 + a*b^4)*d*\cosh(d \\
& *x + c)^5 + (3*a^2*b^3 - a*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(15*a^2 \\
& *b^3*d*\cosh(d*x + c)^4 + 7*(a^2*b^3 + a*b^4)*d*\cosh(d*x + c)^6 + a^2*b^3*d \\
& + 3*(3*a^2*b^3 - a*b^4)*d*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + (a^2*b^3 + a \\
& b^4)*d + 8*(3*a^2*b^3*d*\cosh(d*x + c)^5 + (a^2*b^3 + a*b^4)*d*\cosh(d*x + c) \\
& ^7 + a^2*b^3*d*\cosh(d*x + c) + (3*a^2*b^3 - a*b^4)*d*\cosh(d*x + c)^3)*\sinh(\\
& d*x + c)), 1/2*(2*(2*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^7 + 14*(2*a^2*b + \\
& 3*a*b^2 + b^3)*\cosh(d*x + c)*\sinh(d*x + c)^6 + 2*(2*a^2*b + 3*a*b^2 + b^3) \\
& *\sinh(d*x + c)^7 + 2*(2*a^2*b - a*b^2 + b^3)*\cosh(d*x + c)^5 + 2*(2*a^2*b - \\
& a*b^2 + b^3 + 21*(2*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^5 \\
& + 10*(7*(2*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^3 + (2*a^2*b - a*b^2 + b^3) \\
& *\cosh(d*x + c))*\sinh(d*x + c)^4 - 2*(2*a^2*b - a*b^2 + b^3)*\cosh(d*x + c) \\
& ^3 + 2*(35*(2*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^4 - 2*a^2*b + a*b^2 - b^3 \\
& + 10*(2*a^2*b - a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^3 + 2*(21*(2* \\
& a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^5 + 10*(2*a^2*b - a*b^2 + b^3)*\cosh(d* \\
& x + c)^3 - 3*(2*a^2*b - a*b^2 + b^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 - ((4*a \\
& ^3 + 7*a^2*b + 2*a*b^2 - b^3)*\cosh(d*x + c)^8 + 8*(4*a^3 + 7*a^2*b + 2*a*b^2 \\
& - b^3)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)* \\
& \sinh(d*x + c)^8 + 4*(4*a^3 + 3*a^2*b - a*b^2)*\cosh(d*x + c)^6 + 4*(4*a^3 + \\
& 3*a^2*b - a*b^2 + 7*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*\cosh(d*x + c)^2)*\sinh \\
& (d*x + c)^6 + 8*(7*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*\cosh(d*x + c)^3 + 3*(4 \\
& *a^3 + 3*a^2*b - a*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(12*a^3 + 5*a^2* \\
& b - 6*a*b^2 + b^3)*\cosh(d*x + c)^4 + 2*(35*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3) \\
&)*\cosh(d*x + c)^4 + 12*a^3 + 5*a^2*b - 6*a*b^2 + b^3 + 30*(4*a^3 + 3*a^2*b \\
& - a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(7*(4*a^3 + 7*a^2*b + 2*a*b^2 \\
& - b^3)*\cosh(d*x + c)^5 + 10*(4*a^3 + 3*a^2*b - a*b^2)*\cosh(d*x + c)^3 + (1 \\
& 2*a^3 + 5*a^2*b - 6*a*b^2 + b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*a^3 + 7 \\
& *a^2*b + 2*a*b^2 - b^3 + 4*(4*a^3 + 3*a^2*b - a*b^2)*\cosh(d*x + c)^2 + 4*(7 \\
& *(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*\cosh(d*x + c)^6 + 15*(4*a^3 + 3*a^2*b - \\
& a*b^2)*\cosh(d*x + c)^4 + 4*a^3 + 3*a^2*b - a*b^2 + 3*(12*a^3 + 5*a^2*b - 6* \\
& a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((4*a^3 + 7*a^2*b + 2*a*b \\
& ^2 - b^3)*\cosh(d*x + c)^7 + 3*(4*a^3 + 3*a^2*b - a*b^2)*\cosh(d*x + c)^5 + (\\
& 12*a^3 + 5*a^2*b - 6*a*b^2 + b^3)*\cosh(d*x + c)^3 + (4*a^3 + 3*a^2*b - a*b^2) \\
& *\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a + b)/a}*\arctan(1/2*\sqrt{(a + b)/a} \\
&)*(\cosh(d*x + c) + \sinh(d*x + c))) - ((4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*\cosh \\
& (d*x + c)^8 + 8*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*\cosh(d*x + c)*\sinh(d*x + \\
& c)^7 + (4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*\sinh(d*x + c)^8 + 4*(4*a^3 + 3*a^2 \\
& *b - a*b^2)*\cosh(d*x + c)^6 + 4*(4*a^3 + 3*a^2*b - a*b^2 + 7*(4*a^3 + 7*a^2 \\
& *b + 2*a*b^2 - b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(4*a^3 + 7*a^2* \\
& b + 2*a*b^2 - b^3)*\cosh(d*x + c)^3 + 3*(4*a^3 + 3*a^2*b - a*b^2)*\cosh(d*x + \\
& c))*\sinh(d*x + c)^5 + 2*(12*a^3 + 5*a^2*b - 6*a*b^2 + b^3)*\cosh(d*x + c)^4 \\
& + 2*(35*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*\cosh(d*x + c)^4 + 12*a^3 + 5*a^2 \\
& *b - 6*a*b^2 + b^3 + 30*(4*a^3 + 3*a^2*b - a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x \\
& + c)^4 + 8*(7*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*\cosh(d*x + c)^5 + 10*(4*a^ \\
& 3 + 3*a^2*b - a*b^2)*\cosh(d*x + c)^3 + (12*a^3 + 5*a^2*b - 6*a*b^2 + b^3)*c \\
& osh(d*x + c))*\sinh(d*x + c)^3 + 4*a^3 + 7*a^2*b + 2*a*b^2 - b^3 + 4*(4*a^3 \\
& + 3*a^2*b - a*b^2)*\cosh(d*x + c)^2 + 4*(7*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3) \\
& *\cosh(d*x + c)^6 + 15*(4*a^3 + 3*a^2*b - a*b^2)*\cosh(d*x + c)^4 + 4*a^3 + 3 \\
& *a^2*b - a*b^2 + 3*(12*a^3 + 5*a^2*b - 6*a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh \\
& (d*x + c)^2 + 8*((4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*\cosh(d*x + c)^7 + 3*(4*a \\
& ^3 + 3*a^2*b - a*b^2)*\cosh(d*x + c)^5 + (12*a^3 + 5*a^2*b - 6*a*b^2 + b^3)* \\
& \cosh(d*x + c)^3 + (4*a^3 + 3*a^2*b - a*b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{(a + b)/a}*\arctan(1/2*((a + b)*\cosh(d*x + c)^3 + 3*(a + b)*\cosh(d*x + c)
\end{aligned}$$

```

)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*
(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))*sqrt((a + b)/a)/(a + b))
+ 2*((4*a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^8 + 8*(4*a^3 + 9*a^2*b + 5*a
*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a^3 + 9*a^2*b + 5*a*b^2)*sinh(d*x
+ c)^8 + 4*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^6 + 4*(4*a^3 + 5*a^2*b + 7*(4*a^
3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(4*a^3 + 9*a
^2*b + 5*a*b^2)*cosh(d*x + c)^3 + 3*(4*a^3 + 5*a^2*b)*cosh(d*x + c))*sinh(d
*x + c)^5 + 2*(12*a^3 + 11*a^2*b - 5*a*b^2)*cosh(d*x + c)^4 + 2*(35*(4*a^3
+ 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^4 + 12*a^3 + 11*a^2*b - 5*a*b^2 + 30*(4*
a^3 + 5*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(4*a^3 + 9*a^2*b + 5
*a*b^2)*cosh(d*x + c)^5 + 10*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^3 + (12*a^3 +
11*a^2*b - 5*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a^3 + 9*a^2*b + 5*a*
b^2 + 4*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^2 + 4*(7*(4*a^3 + 9*a^2*b + 5*a*b^2
))*cosh(d*x + c)^6 + 15*(4*a^3 + 5*a^2*b)*cosh(d*x + c)^4 + 4*a^3 + 5*a^2*b
+ 3*(12*a^3 + 11*a^2*b - 5*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((4*
a^3 + 9*a^2*b + 5*a*b^2)*cosh(d*x + c)^7 + 3*(4*a^3 + 5*a^2*b)*cosh(d*x + c
)^5 + (12*a^3 + 11*a^2*b - 5*a*b^2)*cosh(d*x + c)^3 + (4*a^3 + 5*a^2*b)*cos
h(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 2*(2*a^2
*b + 3*a*b^2 + b^3)*cosh(d*x + c) + 2*(7*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x
+ c)^6 + 5*(2*a^2*b - a*b^2 + b^3)*cosh(d*x + c)^4 - 2*a^2*b - 3*a*b^2 - b
^3 - 3*(2*a^2*b - a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(4*a^2*b^3*d
*cosh(d*x + c)^6 + (a^2*b^3 + a*b^4)*d*cosh(d*x + c)^8 + 8*(a^2*b^3 + a*b^4
)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b^3 + a*b^4)*d*sinh(d*x + c)^8 + 4
*a^2*b^3*d*cosh(d*x + c)^2 + 4*(a^2*b^3*d + 7*(a^2*b^3 + a*b^4)*d*cosh(d*x
+ c)^2)*sinh(d*x + c)^6 + 2*(3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^4 + 8*(3*a^
2*b^3*d*cosh(d*x + c) + 7*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3)*sinh(d*x + c
)^5 + 2*(30*a^2*b^3*d*cosh(d*x + c)^2 + 35*(a^2*b^3 + a*b^4)*d*cosh(d*x + c
)^4 + (3*a^2*b^3 - a*b^4)*d)*sinh(d*x + c)^4 + 8*(10*a^2*b^3*d*cosh(d*x + c
)^3 + 7*(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^5 + (3*a^2*b^3 - a*b^4)*d*cosh(d*
x + c))*sinh(d*x + c)^3 + 4*(15*a^2*b^3*d*cosh(d*x + c)^4 + 7*(a^2*b^3 + a*
b^4)*d*cosh(d*x + c)^6 + a^2*b^3*d + 3*(3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^
2)*sinh(d*x + c)^2 + (a^2*b^3 + a*b^4)*d + 8*(3*a^2*b^3*d*cosh(d*x + c)^5 +
(a^2*b^3 + a*b^4)*d*cosh(d*x + c)^7 + a^2*b^3*d*cosh(d*x + c) + (3*a^2*b^3
- a*b^4)*d*cosh(d*x + c)^3)*sinh(d*x + c))]

```

giac [B] time = 0.69, size = 870, normalized size = 5.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```

[Out] 1/2*(2*(4*a*e^c + 5*b*e^c)*arctan(e^(d*x + c))*e^(-c)/b^3 - (2*sqrt(a^2 - b
^2 - 2*sqrt(-a*b)*(a + b))*(4*a - b)*a^2*b^2*abs(a*e^(2*c) + b*e^(2*c)) + (
4*a^2 + 3*a*b - b^2)*sqrt(a^2 - b^2 - 2*sqrt(-a*b)*(a + b))*sqrt(-a*b)*abs(
a*e^(2*c) + b*e^(2*c))*abs(a)*abs(b) - (4*a^3*b^2 - 5*a^2*b^3 + a*b^4)*sqrt
(a^2 - b^2 - 2*sqrt(-a*b)*(a + b))*abs(a*e^(2*c) + b*e^(2*c)))*arctan(e^(d*
x)/sqrt((a^2*b^3*e^(2*c) - a*b^4*e^(2*c) + sqrt((a^2*b^3*e^(2*c) - a*b^4*e^
(2*c))^2 - (a^2*b^3*e^(4*c) + a*b^4*e^(4*c))*(a^2*b^3 + a*b^4)))/(a^2*b^3*e
^(4*c) + a*b^4*e^(4*c))))*e^(-2*c)/((a^3*b^3 + 2*a^2*b^4 + a*b^5)*sqrt(-a*b
)*abs(a)*abs(b)) + (2*sqrt(a^2 - b^2 + 2*sqrt(-a*b)*(a + b))*(4*a - b)*a^2*
b^2*abs(a*e^(2*c) + b*e^(2*c)) - (4*a^2 + 3*a*b - b^2)*sqrt(a^2 - b^2 + 2*s
qrt(-a*b)*(a + b))*sqrt(-a*b)*abs(a*e^(2*c) + b*e^(2*c))*abs(a)*abs(b) - (4
*a^3*b^2 - 5*a^2*b^3 + a*b^4)*sqrt(a^2 - b^2 + 2*sqrt(-a*b)*(a + b))*abs(a*
e^(2*c) + b*e^(2*c)))*arctan(e^(d*x)/sqrt((a^2*b^3*e^(2*c) - a*b^4*e^(2*c)
- sqrt((a^2*b^3*e^(2*c) - a*b^4*e^(2*c))^2 - (a^2*b^3*e^(4*c) + a*b^4*e^(4*
c))*(a^2*b^3 + a*b^4)))/(a^2*b^3*e^(4*c) + a*b^4*e^(4*c))))*e^(-2*c)/((a^3*
b^3 + 2*a^2*b^4 + a*b^5)*sqrt(-a*b)*abs(a)*abs(b)) + 2*(a^2*e^(3*d*x + 3*c)
+ 2*a*b*e^(3*d*x + 3*c) + b^2*e^(3*d*x + 3*c) - a^2*e^(d*x + c) - 2*a*b*e^
(d*x + c) - b^2*e^(d*x + c))/((a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*

```


$$e^{(2dx + 2c)} - 2be^{(2dx + 2c)} + (a + b)ab^2 + 2(e^{(3dx + 3c)} - e^{(dx + c)}) / (b^2(e^{(2dx + 2c)} + 1)^2) / d$$

maple [B] time = 0.40, size = 1477, normalized size = 9.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(dx+c)^7/(a+b*tanh(dx+c))^2,x)

[Out]
$$\begin{aligned} & -7/2/d/b*a/(b*(a+b))^{1/2}/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}) \\ & -7/2/d/b*a/(b*(a+b))^{1/2}/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}) \\ & -1/d/b^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)^3+1/d/b^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)+4/d \\ & /b^3*arctan(tanh(1/2*d*x+1/2*c))*a+1/2/d/(b*(a+b))^{1/2}/a/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}) \\ & *b-1/d/b^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*a*tanh(1/2*d*x+1/2*c)^3-2/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/b*tanh(1/2*d*x+1/2*c)^3+7/2/d/b^2*a/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}) \\ & -2/d/b^2*a^2/(b*(a+b))^{1/2}/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}) \\ & -2/d/b^2*a^2/(b*(a+b))^{1/2}/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}) \\ & +2/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/b*tanh(1/2*d*x+1/2*c)+1/d/b/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}) \\ & -1/d/b/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}) \\ & -1/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)-1/d/(b*(a+b))^{1/2}/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2} \\ & *arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2})-1/2/d/a/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}) \\ & -1/d/(b*(a+b))^{1/2}/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}) \\ & +1/2/d/a/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}) \\ & +5/d/b^2*arctan(tanh(1/2*d*x+1/2*c))+1/2/d/(b*(a+b))^{1/2}/a/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}) \\ & *b-7/2/d/b^2*a/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}) \\ & -2/d/b^3*a^2/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}+a+2*b)*a)^{1/2}) \\ & +2/d/b^3*a^2/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{1/2}-a-2*b)*a)^{1/2}) \\ & +1/d/b^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*a*tanh(1/2*d*x+1/2*c) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(2a^2e^{7c} + 3abe^{7c} + b^2e^{7c})e^{7dx} + (2a^2e^{5c} - abe^{5c} + b^2e^{5c})e^{5dx} - (2a^2e^{3c} - abe^{3c} + b^2e^{3c})e^{3dx}}{4a^2b^2de^{6dx+6c} + 4a^2b^2de^{2dx+2c} + a^2b^2d + ab^3d + (a^2b^2de^{8c} + ab^3de^{8c})e^{8dx} + 2(3a^2b^2de^{4c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)^7/(a+b*tanh(dx+c))^2,x, algorithm="maxima")

[Out]
$$\begin{aligned} & ((2*a^2*e^{(7*c)} + 3*a*b*e^{(7*c)} + b^2*e^{(7*c)})*e^{(7*d*x)} + (2*a^2*e^{(5*c)} - a*b*e^{(5*c)} + b^2*e^{(5*c)})*e^{(5*d*x)} - (2*a^2*e^{(3*c)} - a*b*e^{(3*c)} + b^2*e^{(3*c)})*e^{(3*d*x)} \\ & - (2*a^2*e^c + 3*a*b*e^c + b^2*e^c)*e^{(d*x)}) / (4*a^2*b^2*d \end{aligned}$$

$d \cdot e^{(6 \cdot d \cdot x + 6 \cdot c)} + 4 \cdot a^2 \cdot b^2 \cdot d \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} + a^2 \cdot b^2 \cdot d + a \cdot b^3 \cdot d + (a^2 \cdot b^2 \cdot d \cdot e^{(8 \cdot c)} + a \cdot b^3 \cdot d \cdot e^{(8 \cdot c)}) \cdot e^{(8 \cdot d \cdot x)} + 2 \cdot (3 \cdot a^2 \cdot b^2 \cdot d \cdot e^{(4 \cdot c)} - a \cdot b^3 \cdot d \cdot e^{(4 \cdot c)}) \cdot e^{(4 \cdot d \cdot x)} + (4 \cdot a \cdot e^c + 5 \cdot b \cdot e^c) \cdot \arctan(e^{(d \cdot x + c)}) \cdot e^{-c} / (b^3 \cdot d) - 128 \cdot \text{integrate}(1/128 \cdot ((4 \cdot a^3 \cdot e^{(3 \cdot c)} + 7 \cdot a^2 \cdot b \cdot e^{(3 \cdot c)} + 2 \cdot a \cdot b^2 \cdot e^{(3 \cdot c)} - b^3 \cdot e^{(3 \cdot c)}) \cdot e^{(3 \cdot d \cdot x)} + (4 \cdot a^3 \cdot e^c + 7 \cdot a^2 \cdot b \cdot e^c + 2 \cdot a \cdot b^2 \cdot e^c - b^3 \cdot e^c) \cdot e^{(d \cdot x)}) / (a^2 \cdot b^3 + a \cdot b^4 + (a^2 \cdot b^3 \cdot e^{(4 \cdot c)} + a \cdot b^4 \cdot e^{(4 \cdot c)}) \cdot e^{(4 \cdot d \cdot x)} + 2 \cdot (a^2 \cdot b^3 \cdot e^{(2 \cdot c)} - a \cdot b^4 \cdot e^{(2 \cdot c)}) \cdot e^{(2 \cdot d \cdot x)}), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + dx)^7 (b \tanh(c + dx)^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^7*(a + b*tanh(c + d*x)^2)^2), x)

[Out] int(1/(cosh(c + d*x)^7*(a + b*tanh(c + d*x)^2)^2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**7/(a+b*tanh(d*x+c)**2)**2, x)

[Out] Timed out

$$3.125 \quad \int \frac{\cosh^2(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=198

$$-\frac{b(a-3b)(4a+b) \tanh(c+dx)}{8a^2d(a+b)^3(a+b \tanh^2(c+dx))} + \frac{b^{3/2}(35a^2+14ab+3b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^4} - \frac{b(2a-b) \tanh(c+dx)}{4ad(a+b)^2(a+b \tanh^2(c+dx))}$$

[Out] 1/2*(a+7*b)*x/(a+b)^4+1/8*b^(3/2)*(35*a^2+14*a*b+3*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/(a+b)^4/d+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c)^2)^2-1/4*(2*a-b)*b*tanh(d*x+c)/a/(a+b)^2/d/(a+b*tanh(d*x+c)^2)^2-1/8*(a-3*b)*b*(4*a+b)*tanh(d*x+c)/a^2/(a+b)^3/d/(a+b*tanh(d*x+c)^2)

Rubi [A] time = 0.31, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3675, 414, 527, 522, 206, 205}

$$\frac{b^{3/2}(35a^2+14ab+3b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^4} - \frac{b(a-3b)(4a+b) \tanh(c+dx)}{8a^2d(a+b)^3(a+b \tanh^2(c+dx))} - \frac{b(2a-b) \tanh(c+dx)}{4ad(a+b)^2(a+b \tanh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] ((a + 7*b)*x)/(2*(a + b)^4) + (b^(3/2)*(35*a^2 + 14*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a + b)^4*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) - ((2*a - b)*b*Tanh[c + d*x])/(4*a*(a + b)^2*d*(a + b*Tanh[c + d*x]^2)^2) - ((a - 3*b)*b*(4*a + b)*Tanh[c + d*x])/(8*a^2*(a + b)^3*d*(a + b*Tanh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3675

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh^2(c + dx)}{(a + b \tanh^2(c + dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+bx^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\cosh(c + dx) \sinh(c + dx)}{2(a + b)d (a + b \tanh^2(c + dx))^2} + \frac{\text{Subst}\left(\int \frac{a+2b+5bx^2}{(1-x^2)(a+bx^2)^3} dx, x, \tanh(c + dx)\right)}{2(a + b)d} \\ &= \frac{\cosh(c + dx) \sinh(c + dx)}{2(a + b)d (a + b \tanh^2(c + dx))^2} - \frac{(2a - b)b \tanh(c + dx)}{4a(a + b)^2d (a + b \tanh^2(c + dx))^2} - \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^2)^3} dx, x, \tanh(c + dx)\right)}{8a^2(a + b)d} \\ &= \frac{\cosh(c + dx) \sinh(c + dx)}{2(a + b)d (a + b \tanh^2(c + dx))^2} - \frac{(2a - b)b \tanh(c + dx)}{4a(a + b)^2d (a + b \tanh^2(c + dx))^2} - \frac{(a - 3b)}{8a^2(a + b)d} \\ &= \frac{\cosh(c + dx) \sinh(c + dx)}{2(a + b)d (a + b \tanh^2(c + dx))^2} - \frac{(2a - b)b \tanh(c + dx)}{4a(a + b)^2d (a + b \tanh^2(c + dx))^2} - \frac{(a - 3b)}{8a^2(a + b)d} \\ &= \frac{(a + 7b)x}{2(a + b)^4} + \frac{b^{3/2} (35a^2 + 14ab + 3b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a + b)^4d} + \frac{\cosh(c + dx) \sinh(c + dx)}{2(a + b)d (a + b \tanh^2(c + dx))^2} \end{aligned}$$

Mathematica [A] time = 1.42, size = 164, normalized size = 0.83

$$\frac{b^2(a+b)(13a+3b) \sinh(2(c+dx))}{a^2((a+b) \cosh(2(c+dx))+a-b)} + \frac{b^{3/2}(35a^2+14ab+3b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}} + \frac{4b^3(a+b) \sinh(2(c+dx))}{a((a+b) \cosh(2(c+dx))+a-b)^2} + \frac{4(a+7b)(c+dx) + 2(a-b)}{8d(a+b)^4}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (4*(a + 7*b)*(c + d*x) + (b^(3/2)*(35*a^2 + 14*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(5/2) + 2*(a + b)*Sinh[2*(c + d*x)] + (4*b^3*(a

$$+ b) * \text{Sinh}[2*(c + d*x)] / (a*(a - b + (a + b) * \text{Cosh}[2*(c + d*x)])^2) + (b^2*(a + b) * (13*a + 3*b) * \text{Sinh}[2*(c + d*x)] / (a^2*(a - b + (a + b) * \text{Cosh}[2*(c + d*x)]))) / (8*(a + b)^4*d)$$

fricas [B] time = 0.71, size = 13887, normalized size = 70.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(2*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^12 + 24*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)^11 + 2*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sinh(d*x + c)^12 + 8*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^10 + 4*(2*a^5 + 2*a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + 2*(a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x + 33*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 40*(11*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^3 + 2*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 + 2*(5*a^5 - a^4*b - 27*a^3*b^2 + 7*a^2*b^3 + 34*a*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c)^8 + 2*(5*a^5 - a^4*b - 27*a^3*b^2 + 7*a^2*b^3 + 34*a*b^4 + 6*b^5 + 495*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^4 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x + 180*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(99*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^5 + 60*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^3 + (5*a^5 - a^4*b - 27*a^3*b^2 + 7*a^2*b^3 + 34*a*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(39*a^3*b^2 - 17*a^2*b^3 + 33*a*b^4 + 9*b^5 - 4*(3*a^5 + 19*a^4*b - 11*a^3*b^2 + 21*a^2*b^3)*d*x)*cosh(d*x + c)^6 + 4*(462*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^6 - 39*a^3*b^2 + 17*a^2*b^3 - 33*a*b^4 - 9*b^5 + 420*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^4 + 4*(3*a^5 + 19*a^4*b - 11*a^3*b^2 + 21*a^2*b^3)*d*x + 14*(5*a^5 - a^4*b - 27*a^3*b^2 + 7*a^2*b^3 + 34*a*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(198*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^7 + 252*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^5 + 14*(5*a^5 - a^4*b - 27*a^3*b^2 + 7*a^2*b^3 + 34*a*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c)^3 - 3*(39*a^3*b^2 - 17*a^2*b^3 + 33*a*b^4 + 9*b^5 - 4*(3*a^5 + 19*a^4*b - 11*a^3*b^2 + 21*a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*a^5 - 6*a^4*b - 6*a^3*b^2 - 2*a^2*b^3 - 2*(5*a^5 - a^4*b + 77*a^3*b^2 + 31*a^2*b^3 - 70*a*b^4 - 18*b^5 - 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c)^4 + 2*(495*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^8 + 840*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^6 - 5*a^5 + a^4*b - 77*a^3*b^2 - 31*a^2*b^3 + 70*a*b^4 + 18*b^5 + 70*(5*a^5 - a^4*b - 27*a^3*b^2 + 7*a^2*b^3 + 34*a*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c)^4 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x - 30*(39*a^3*b^2 - 17*a^2*b^3 + 33*a*b^4 + 9*b^5 - 4*(3*a^5 + 19*a^4*b - 11*a^3*b^2 + 21*a^2*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(55*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(d*x + c)^9 + 120*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*cosh(d*x + c)^7 + 14*(5*a^5 - a^4*b - 27*a^3*b^2 + 7*a^2*b^3 + 34*a*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c)^5 - 10*(39*a^3*b^2 - 17*a^2*b^3 + 33*a*b^4 + 9*b^5 - 4*(3*a^5 + 19*a^4*b - 11*a^3*b^2 + 21*a^2*b^3)*d*x)*cosh(d*x + c)^3 - (5*a^5 - a^4*b + 77*a^3*b^2 + 31*a^2*b^3 - 70*a*b^4 - 18*b^5 - 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(2*a^5 + 2*a^4*b + 11*a^3*b^2 + 27*a^2*b^3 + 19*a*b^4 + 3*b^5 - 2*(a^5 + 9*a^4*b +

$$\begin{aligned}
& *b^3 + 34*a*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x) * \cos \\
& h(d*x + c)^7 - 3*(39*a^3*b^2 - 17*a^2*b^3 + 33*a*b^4 + 9*b^5 - 4*(3*a^5 + 1 \\
& 9*a^4*b - 11*a^3*b^2 + 21*a^2*b^3)*d*x) * \cosh(d*x + c)^5 - (5*a^5 - a^4*b + \\
& 77*a^3*b^2 + 31*a^2*b^3 - 70*a*b^4 - 18*b^5 - 16*(a^5 + 7*a^4*b - a^3*b^2 - \\
& 7*a^2*b^3)*d*x) * \cosh(d*x + c)^3 - (2*a^5 + 2*a^4*b + 11*a^3*b^2 + 27*a^2*b \\
& ^3 + 19*a*b^4 + 3*b^5 - 2*(a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x) * \cos \\
& h(d*x + c)) * \sinh(d*x + c)) / ((a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a \\
& ^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d * \cosh(d*x + c)^10 + 10*(a^8 + 6*a^7*b + 15*a \\
& ^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d * \cosh(d*x + c) * \sin \\
& h(d*x + c)^9 + (a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^ \\
& 3*b^5 + a^2*b^6)*d * \sinh(d*x + c)^10 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b \\
& b^4 - 4*a^3*b^5 - a^2*b^6)*d * \cosh(d*x + c)^8 + (45*(a^8 + 6*a^7*b + 15*a^6* \\
& b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d * \cosh(d*x + c)^2 + 4* \\
& (a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d) * \sinh(d*x + \\
& c)^8 + 2*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3 \\
& *b^5 + 3*a^2*b^6)*d * \cosh(d*x + c)^6 + 8*(15*(a^8 + 6*a^7*b + 15*a^6*b^2 + 2 \\
& 0*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d * \cosh(d*x + c)^3 + 4*(a^8 + \\
& 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d * \cosh(d*x + c)) * \sin \\
& h(d*x + c)^7 + 2*(105*(a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 \\
& + 6*a^3*b^5 + a^2*b^6)*d * \cosh(d*x + c)^4 + 56*(a^8 + 4*a^7*b + 5*a^6*b^2 - \\
& 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d * \cosh(d*x + c)^2 + (3*a^8 + 10*a^7*b + 1 \\
& 3*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*d) * \sinh(d*x + \\
& c)^6 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d * c \\
& osh(d*x + c)^4 + 4*(63*(a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^ \\
& 4 + 6*a^3*b^5 + a^2*b^6)*d * \cosh(d*x + c)^5 + 56*(a^8 + 4*a^7*b + 5*a^6*b^2 \\
& - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d * \cosh(d*x + c)^3 + 3*(3*a^8 + 10*a^7*b \\
& + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*d * \cosh(d*x \\
& + c)) * \sinh(d*x + c)^5 + 2*(105*(a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + \\
& 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d * \cosh(d*x + c)^6 + 140*(a^8 + 4*a^7*b + \\
& 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d * \cosh(d*x + c)^4 + 15*(3*a^8 \\
& + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6) \\
& *d * \cosh(d*x + c)^2 + 2*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - \\
& a^2*b^6)*d) * \sinh(d*x + c)^4 + (a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 1 \\
& 5*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d * \cosh(d*x + c)^2 + 8*(15*(a^8 + 6*a^7*b + \\
& 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d * \cosh(d*x + c \\
&)^7 + 28*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d * co \\
& sh(d*x + c)^5 + 5*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 \\
& + 10*a^3*b^5 + 3*a^2*b^6)*d * \cosh(d*x + c)^3 + 2*(a^8 + 4*a^7*b + 5*a^6*b^2 \\
& - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d * \cosh(d*x + c)) * \sinh(d*x + c)^3 + (45*(\\
& a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6) \\
& *d * \cosh(d*x + c)^8 + 112*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 \\
& - a^2*b^6)*d * \cosh(d*x + c)^6 + 30*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5* \\
& b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*d * \cosh(d*x + c)^4 + 24*(a^8 + 4* \\
& a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d * \cosh(d*x + c)^2 + (a \\
& ^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)* \\
& d) * \sinh(d*x + c)^2 + 2*(5*(a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4 \\
& *b^4 + 6*a^3*b^5 + a^2*b^6)*d * \cosh(d*x + c)^9 + 16*(a^8 + 4*a^7*b + 5*a^6*b \\
& ^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d * \cosh(d*x + c)^7 + 6*(3*a^8 + 10*a^7 \\
& *b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*d * \cosh(\\
& d*x + c)^5 + 8*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6 \\
&) *d * \cosh(d*x + c)^3 + (a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 \\
& + 6*a^3*b^5 + a^2*b^6)*d * \cosh(d*x + c)) * \sinh(d*x + c)), 1/8*((a^5 + 3*a^4*b \\
& + 3*a^3*b^2 + a^2*b^3) * \cosh(d*x + c)^12 + 12*(a^5 + 3*a^4*b + 3*a^3*b^2 + \\
& a^2*b^3) * \cosh(d*x + c) * \sinh(d*x + c)^11 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2 \\
& *b^3) * \sinh(d*x + c)^12 + 4*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + (a^5 + 9*a^4*b \\
& + 15*a^3*b^2 + 7*a^2*b^3)*d*x) * \cosh(d*x + c)^10 + 2*(2*a^5 + 2*a^4*b - 2* \\
& a^3*b^2 - 2*a^2*b^3 + 2*(a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x + 33*(\\
& a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3) * \cosh(d*x + c)^2) * \sinh(d*x + c)^10 + 20 \\
& *(11*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3) * \cosh(d*x + c)^3 + 2*(a^5 + a^4*b
\end{aligned}$$

$$\begin{aligned}
& - a^3 b^2 - a^2 b^3 + (a^5 + 9a^4 b + 15a^3 b^2 + 7a^2 b^3) d x) \cosh(d x + c) \\
& * \sinh(d x + c)^9 + (5a^5 - a^4 b - 27a^3 b^2 + 7a^2 b^3 + 34a b^4 + 6b^5 + 16(a^5 + 7a^4 b - a^3 b^2 - 7a^2 b^3) d x) \cosh(d x + c)^8 \\
& + (5a^5 - a^4 b - 27a^3 b^2 + 7a^2 b^3 + 34a b^4 + 6b^5 + 495(a^5 + 3a^4 b + 3a^3 b^2 + a^2 b^3) \cosh(d x + c)^4 + 16(a^5 + 7a^4 b - a^3 b^2 - 7a^2 b^3) d x + 180(a^5 + a^4 b - a^3 b^2 - a^2 b^3 + (a^5 + 9a^4 b + 15a^3 b^2 + 7a^2 b^3) d x) \cosh(d x + c)^2) \sinh(d x + c)^8 + 8(99(a^5 + 3a^4 b + 3a^3 b^2 + a^2 b^3) \cosh(d x + c)^5 + 60(a^5 + a^4 b - a^3 b^2 - a^2 b^3 + (a^5 + 9a^4 b + 15a^3 b^2 + 7a^2 b^3) d x) \cosh(d x + c)^3 + (5a^5 - a^4 b - 27a^3 b^2 + 7a^2 b^3 + 34a b^4 + 6b^5 + 16(a^5 + 7a^4 b - a^3 b^2 - 7a^2 b^3) d x) \cosh(d x + c)) \sinh(d x + c)^7 - 2(39a^3 b^2 - 17a^2 b^3 + 33a b^4 + 9b^5 - 4(3a^5 + 19a^4 b - 11a^3 b^2 + 21a^2 b^3) d x) \cosh(d x + c)^6 + 2(462(a^5 + 3a^4 b + 3a^3 b^2 + a^2 b^3) \cosh(d x + c)^6 - 39a^3 b^2 + 17a^2 b^3 - 33a b^4 - 9b^5 + 420(a^5 + a^4 b - a^3 b^2 - a^2 b^3 + (a^5 + 9a^4 b + 15a^3 b^2 + 7a^2 b^3) d x) \cosh(d x + c)^4 + 4(3a^5 + 19a^4 b - 11a^3 b^2 + 21a^2 b^3) d x + 14(5a^5 - a^4 b - 27a^3 b^2 + 7a^2 b^3 + 34a b^4 + 6b^5 + 16(a^5 + 7a^4 b - a^3 b^2 - 7a^2 b^3) d x) \cosh(d x + c)^2) \sinh(d x + c)^6 + 4(198(a^5 + 3a^4 b + 3a^3 b^2 + a^2 b^3) \cosh(d x + c)^7 + 252(a^5 + a^4 b - a^3 b^2 - a^2 b^3 + (a^5 + 9a^4 b + 15a^3 b^2 + 7a^2 b^3) d x) \cosh(d x + c)^5 + 14(5a^5 - a^4 b - 27a^3 b^2 + 7a^2 b^3 + 34a b^4 + 6b^5 + 16(a^5 + 7a^4 b - a^3 b^2 - 7a^2 b^3) d x) \cosh(d x + c)^3 - 3(39a^3 b^2 - 17a^2 b^3 + 33a b^4 + 9b^5 - 4(3a^5 + 19a^4 b - 11a^3 b^2 + 21a^2 b^3) d x) \cosh(d x + c)) \sinh(d x + c)^5 - a^5 - 3a^4 b - 3a^3 b^2 - a^2 b^3 - (5a^5 - a^4 b + 77a^3 b^2 + 31a^2 b^3 - 70a b^4 - 18b^5 - 16(a^5 + 7a^4 b - a^3 b^2 - 7a^2 b^3) d x) \cosh(d x + c)^4 + (495(a^5 + 3a^4 b + 3a^3 b^2 + a^2 b^3) \cosh(d x + c)^8 + 840(a^5 + a^4 b - a^3 b^2 - a^2 b^3 + (a^5 + 9a^4 b + 15a^3 b^2 + 7a^2 b^3) d x) \cosh(d x + c)^6 - 5a^5 + a^4 b - 77a^3 b^2 - 31a^2 b^3 + 70a b^4 + 18b^5 + 70(5a^5 - a^4 b - 27a^3 b^2 + 7a^2 b^3 + 34a b^4 + 6b^5 + 16(a^5 + 7a^4 b - a^3 b^2 - 7a^2 b^3) d x) \cosh(d x + c)^4 + 16(a^5 + 7a^4 b - a^3 b^2 - 7a^2 b^3) d x - 30(39a^3 b^2 - 17a^2 b^3 + 33a b^4 + 9b^5 - 4(3a^5 + 19a^4 b - 11a^3 b^2 + 21a^2 b^3) d x) \cosh(d x + c)^2) \sinh(d x + c)^4 + 4(55(a^5 + 3a^4 b + 3a^3 b^2 + a^2 b^3) \cosh(d x + c)^9 + 120(a^5 + a^4 b - a^3 b^2 - a^2 b^3 + (a^5 + 9a^4 b + 15a^3 b^2 + 7a^2 b^3) d x) \cosh(d x + c)^7 + 14(5a^5 - a^4 b - 27a^3 b^2 + 7a^2 b^3 + 34a b^4 + 6b^5 + 16(a^5 + 7a^4 b - a^3 b^2 - 7a^2 b^3) d x) \cosh(d x + c)^5 - 10(39a^3 b^2 - 17a^2 b^3 + 33a b^4 + 9b^5 - 4(3a^5 + 19a^4 b - 11a^3 b^2 + 21a^2 b^3) d x) \cosh(d x + c)^3 - (5a^5 - a^4 b + 77a^3 b^2 + 31a^2 b^3 - 70a b^4 - 18b^5 - 16(a^5 + 7a^4 b - a^3 b^2 - 7a^2 b^3) d x) \cosh(d x + c)) \sinh(d x + c)^3 - 2(2a^5 + 2a^4 b + 11a^3 b^2 + 27a^2 b^3 + 19a b^4 + 3b^5 - 2(a^5 + 9a^4 b + 15a^3 b^2 + 7a^2 b^3) d x) \cosh(d x + c)^2 + 2(33(a^5 + 3a^4 b + 3a^3 b^2 + a^2 b^3) \cosh(d x + c)^10 + 90(a^5 + a^4 b - a^3 b^2 - a^2 b^3 + (a^5 + 9a^4 b + 15a^3 b^2 + 7a^2 b^3) d x) \cosh(d x + c)^8 + 14(5a^5 - a^4 b - 27a^3 b^2 + 7a^2 b^3 + 34a b^4 + 6b^5 + 16(a^5 + 7a^4 b - a^3 b^2 - 7a^2 b^3) d x) \cosh(d x + c)^6 - 2a^5 - 2a^4 b - 11a^3 b^2 - 27a^2 b^3 - 19a b^4 - 3b^5 - 15(39a^3 b^2 - 17a^2 b^3 + 33a b^4 + 9b^5 - 4(3a^5 + 19a^4 b - 11a^3 b^2 + 21a^2 b^3) d x) \cosh(d x + c)^4 + 2(a^5 + 9a^4 b + 15a^3 b^2 + 7a^2 b^3) d x - 3(5a^5 - a^4 b + 77a^3 b^2 + 31a^2 b^3 - 70a b^4 - 18b^5 - 16(a^5 + 7a^4 b - a^3 b^2 - 7a^2 b^3) d x) \cosh(d x + c)^2) \sinh(d x + c)^2 + ((35a^4 b + 84a^3 b^2 + 66a^2 b^3 + 20a b^4 + 3b^5) \cosh(d x + c)^10 + 10(35a^4 b + 84a^3 b^2 + 66a^2 b^3 + 20a b^4 + 3b^5) \cosh(d x + c) \sinh(d x + c)^9 + (35a^4 b + 84a^3 b^2 + 66a^2 b^3 + 20a b^4 + 3b^5) \sinh(d x + c)^10 + 4(35a^4 b + 14a^3 b^2 - 32a^2 b^3 - 14a b^4 - 3b^5) \cosh(d x + c)^8 + (140a^4 b + 56a^3 b^2 - 128a^2 b^3 - 56a b^4 - 12b^5 + 45(35a^4 b + 84a^3 b^2 + 66a^2 b^3 + 20a b^4 + 3b^5) \cosh(d x + c)^2) \sinh(d x + c)^8 + 8(15(35a^4 b + 84a^3 b^2 + 66a^2 b^3 + 20a b^4 + 3b^5) \cosh(d x + c)^3 + 4(35a^4 b + 14a^3 b^2 - 32a^2 b^3 - 14
\end{aligned}$$

$$\begin{aligned}
& *a*b^4 - 3*b^5)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(105*a^4*b - 28*a^3*b^2 \\
& + 86*a^2*b^3 + 36*a*b^4 + 9*b^5)*\cosh(d*x + c)^6 + 2*(105*a^4*b - 28*a^3*b^2 \\
& + 86*a^2*b^3 + 36*a*b^4 + 9*b^5 + 105*(35*a^4*b + 84*a^3*b^2 + 66*a^2*b^3 \\
& + 20*a*b^4 + 3*b^5)*\cosh(d*x + c)^4 + 56*(35*a^4*b + 14*a^3*b^2 - 32*a^2*b^3 \\
& - 14*a*b^4 - 3*b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 4*(63*(35*a^4*b + \\
& 84*a^3*b^2 + 66*a^2*b^3 + 20*a*b^4 + 3*b^5)*\cosh(d*x + c)^5 + 56*(35*a^4*b \\
& + 14*a^3*b^2 - 32*a^2*b^3 - 14*a*b^4 - 3*b^5)*\cosh(d*x + c)^3 + 3*(105*a^4 \\
& *b - 28*a^3*b^2 + 86*a^2*b^3 + 36*a*b^4 + 9*b^5)*\cosh(d*x + c))*\sinh(d*x + \\
& c)^5 + 4*(35*a^4*b + 14*a^3*b^2 - 32*a^2*b^3 - 14*a*b^4 - 3*b^5)*\cosh(d*x + \\
& c)^4 + 2*(105*(35*a^4*b + 84*a^3*b^2 + 66*a^2*b^3 + 20*a*b^4 + 3*b^5)*\cosh \\
& (d*x + c)^6 + 70*a^4*b + 28*a^3*b^2 - 64*a^2*b^3 - 28*a*b^4 - 6*b^5 + 140*(\\
& 35*a^4*b + 14*a^3*b^2 - 32*a^2*b^3 - 14*a*b^4 - 3*b^5)*\cosh(d*x + c)^4 + 15 \\
& *(105*a^4*b - 28*a^3*b^2 + 86*a^2*b^3 + 36*a*b^4 + 9*b^5)*\cosh(d*x + c)^2)* \\
& \sinh(d*x + c)^4 + 8*(15*(35*a^4*b + 84*a^3*b^2 + 66*a^2*b^3 + 20*a*b^4 + 3* \\
& b^5)*\cosh(d*x + c)^7 + 28*(35*a^4*b + 14*a^3*b^2 - 32*a^2*b^3 - 14*a*b^4 - \\
& 3*b^5)*\cosh(d*x + c)^5 + 5*(105*a^4*b - 28*a^3*b^2 + 86*a^2*b^3 + 36*a*b^4 \\
& + 9*b^5)*\cosh(d*x + c)^3 + 2*(35*a^4*b + 14*a^3*b^2 - 32*a^2*b^3 - 14*a*b^4 \\
& - 3*b^5)*\cosh(d*x + c))*\sinh(d*x + c)^3 + (35*a^4*b + 84*a^3*b^2 + 66*a^2* \\
& b^3 + 20*a*b^4 + 3*b^5)*\cosh(d*x + c)^2 + (45*(35*a^4*b + 84*a^3*b^2 + 66*a \\
& ^2*b^3 + 20*a*b^4 + 3*b^5)*\cosh(d*x + c)^8 + 112*(35*a^4*b + 14*a^3*b^2 - 3 \\
& 2*a^2*b^3 - 14*a*b^4 - 3*b^5)*\cosh(d*x + c)^6 + 35*a^4*b + 84*a^3*b^2 + 66* \\
& a^2*b^3 + 20*a*b^4 + 3*b^5 + 30*(105*a^4*b - 28*a^3*b^2 + 86*a^2*b^3 + 36*a \\
& *b^4 + 9*b^5)*\cosh(d*x + c)^4 + 24*(35*a^4*b + 14*a^3*b^2 - 32*a^2*b^3 - 14 \\
& *a*b^4 - 3*b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 2*(5*(35*a^4*b + 84*a^3* \\
& b^2 + 66*a^2*b^3 + 20*a*b^4 + 3*b^5)*\cosh(d*x + c)^9 + 16*(35*a^4*b + 14*a^ \\
& 3*b^2 - 32*a^2*b^3 - 14*a*b^4 - 3*b^5)*\cosh(d*x + c)^7 + 6*(105*a^4*b - 28* \\
& a^3*b^2 + 86*a^2*b^3 + 36*a*b^4 + 9*b^5)*\cosh(d*x + c)^5 + 8*(35*a^4*b + 14 \\
& *a^3*b^2 - 32*a^2*b^3 - 14*a*b^4 - 3*b^5)*\cosh(d*x + c)^3 + (35*a^4*b + 84* \\
& a^3*b^2 + 66*a^2*b^3 + 20*a*b^4 + 3*b^5)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{ \\
& (b/a)*\arctan(1/2*((a + b)*\cosh(d*x + c)^2 + 2*(a + b)*\cosh(d*x + c)*\sinh(d* \\
& x + c) + (a + b)*\sinh(d*x + c)^2 + a - b)*\sqrt{b/a}/b) + 4*(3*(a^5 + 3*a^4* \\
& b + 3*a^3*b^2 + a^2*b^3)*\cosh(d*x + c)^11 + 10*(a^5 + a^4*b - a^3*b^2 - a^2 \\
& *b^3 + (a^5 + 9*a^4*b + 15*a^3*b^2 + 7*a^2*b^3)*d*x)*\cosh(d*x + c)^9 + 2*(5 \\
& *a^5 - a^4*b - 27*a^3*b^2 + 7*a^2*b^3 + 34*a*b^4 + 6*b^5 + 16*(a^5 + 7*a^4*b \\
& b - a^3*b^2 - 7*a^2*b^3)*d*x)*\cosh(d*x + c)^7 - 3*(39*a^3*b^2 - 17*a^2*b^3 \\
& + 33*a*b^4 + 9*b^5 - 4*(3*a^5 + 19*a^4*b - 11*a^3*b^2 + 21*a^2*b^3)*d*x)*\co \\
& sh(d*x + c)^5 - (5*a^5 - a^4*b + 77*a^3*b^2 + 31*a^2*b^3 - 70*a*b^4 - 18*b^ \\
& 5 - 16*(a^5 + 7*a^4*b - a^3*b^2 - 7*a^2*b^3)*d*x)*\cosh(d*x + c)^3 - (2*a^5 \\
& + 2*a^4*b + 11*a^3*b^2 + 27*a^2*b^3 + 19*a*b^4 + 3*b^5 - 2*(a^5 + 9*a^4*b + \\
& 15*a^3*b^2 + 7*a^2*b^3)*d*x)*\cosh(d*x + c))*\sinh(d*x + c))/((a^8 + 6*a^7*b \\
& + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*\cosh(d*x + \\
& c)^10 + 10*(a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b \\
& ^5 + a^2*b^6)*d*\cosh(d*x + c))*\sinh(d*x + c)^9 + (a^8 + 6*a^7*b + 15*a^6*b^2 \\
& + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*\sinh(d*x + c)^10 + 4*(a \\
& ^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*\cosh(d*x + c) \\
& ^8 + (45*(a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 \\
& + a^2*b^6)*d*\cosh(d*x + c)^2 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4 \\
& *a^3*b^5 - a^2*b^6)*d)*\sinh(d*x + c)^8 + 2*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + \\
& 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*d*\cosh(d*x + c)^6 + 8*(1 \\
& 5*(a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b \\
& ^6)*d*\cosh(d*x + c)^3 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^ \\
& 5 - a^2*b^6)*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(105*(a^8 + 6*a^7*b + 15* \\
& a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*\cosh(d*x + c)^4 \\
& + 56*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*\cosh(d \\
& *x + c)^2 + (3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a \\
& ^3*b^5 + 3*a^2*b^6)*d)*\sinh(d*x + c)^6 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a \\
& ^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*\cosh(d*x + c)^4 + 4*(63*(a^8 + 6*a^7*b + 15 \\
& *a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*\cosh(d*x + c)^5 \\
& + 56*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*\cosh(
\end{aligned}$$

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d*x + c)^3 + 3*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 1
0*a^3*b^5 + 3*a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^8 + 6*a
^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*cosh(d
*x + c)^6 + 140*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^
6)*d*cosh(d*x + c)^4 + 15*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*
a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*d*cosh(d*x + c)^2 + 2*(a^8 + 4*a^7*b + 5*
a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d)*sinh(d*x + c)^4 + (a^8 + 6*a^
7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*cosh(d*
x + c)^2 + 8*(15*(a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*
a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^7 + 28*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^
4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^5 + 5*(3*a^8 + 10*a^7*b + 13*a
^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*d*cosh(d*x + c)^
3 + 2*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*cosh(
d*x + c))*sinh(d*x + c)^3 + (45*(a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 +
15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^8 + 112*(a^8 + 4*a^7*b +
5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^6 + 30*(3*a^8
+ 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)
*d*cosh(d*x + c)^4 + 24*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5
- a^2*b^6)*d*cosh(d*x + c)^2 + (a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 1
5*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d)*sinh(d*x + c)^2 + 2*(5*(a^8 + 6*a^7*b +
15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c
)^9 + 16*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*co
sh(d*x + c)^7 + 6*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4
+ 10*a^3*b^5 + 3*a^2*b^6)*d*cosh(d*x + c)^5 + 8*(a^8 + 4*a^7*b + 5*a^6*b^2
- 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^3 + (a^8 + 6*a^7*b + 15*
a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c))*s
inh(d*x + c)]]

```

giac [B] time = 1.91, size = 595, normalized size = 3.01

$$\frac{4(a+7b)dx}{a^4+4a^3b+6a^2b^2+4ab^3+b^4} + \frac{(35a^2b^2e^{(2c)}+14ab^3e^{(2c)}+3b^4e^{(2c)})\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)e^{(-2c)}}{(a^6+4a^5b+6a^4b^2+4a^3b^3+a^2b^4)\sqrt{ab}} - \frac{(2ae^{(2dx+2c)}+14be^{(2dx+2c)}+a+b)}{a^4e^{(2c)}+4a^3be^{(2c)}+6a^2b^2e^{(2c)}+4ab^3e^{(2c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

```

[Out] 1/8*(4*(a + 7*b)*d*x/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + (35*a^2*
b^2*e^(2*c) + 14*a*b^3*e^(2*c) + 3*b^4*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*
c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))*e^(-2*c)/((a^6 + 4*a^5*b + 6*a^4
*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b)) - (2*a*e^(2*d*x + 2*c) + 14*b*e^(2*d
*x + 2*c) + a + b)*e^(-2*d*x)/(a^4*e^(2*c) + 4*a^3*b*e^(2*c) + 6*a^2*b^2*e^
(2*c) + 4*a*b^3*e^(2*c) + b^4*e^(2*c)) + e^(2*d*x + 16*c)/(a^3*e^(14*c) + 3
*a^2*b*e^(14*c) + 3*a*b^2*e^(14*c) + b^3*e^(14*c)) - 2*(13*a^3*b^2*e^(6*d*x
+ 6*c) - a^2*b^3*e^(6*d*x + 6*c) - 17*a*b^4*e^(6*d*x + 6*c) - 3*b^5*e^(6*d
*x + 6*c) + 39*a^3*b^2*e^(4*d*x + 4*c) - 17*a^2*b^3*e^(4*d*x + 4*c) + 33*a*
b^4*e^(4*d*x + 4*c) + 9*b^5*e^(4*d*x + 4*c) + 39*a^3*b^2*e^(2*d*x + 2*c) +
13*a^2*b^3*e^(2*d*x + 2*c) - 35*a*b^4*e^(2*d*x + 2*c) - 9*b^5*e^(2*d*x + 2*
c) + 13*a^3*b^2 + 29*a^2*b^3 + 19*a*b^4 + 3*b^5)/((a^6 + 4*a^5*b + 6*a^4*b^
2 + 4*a^3*b^3 + a^2*b^4)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*
d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2))/d

```

maple [B] time = 0.51, size = 2132, normalized size = 10.77

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x)

```
[Out] -35/8/d*b^2/(a+b)^4*a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*a
rctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-35/8/d*b^
2/(a+b)^4*a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*ta
nh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-17/8/d*b^4/(a+b)^4/a
/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x
+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-3/8/d*b^5/(a+b)^4/a^2/(b*(a+b)
)^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((
2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-3/8/d*b^5/(a+b)^4/a^2/(b*(a+b))^(1/2)/((
2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b)
))^(1/2)-a-2*b)*a)^(1/2))-17/8/d*b^4/(a+b)^4/a/(b*(a+b))^(1/2)/((2*(b*(a+b)
)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+
2*b)*a)^(1/2))+1/2/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/(a+b)^3/(tanh(
1/2*d*x+1/2*c)-1)-1/2/d/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/(a+b)^3/(ta
nh(1/2*d*x+1/2*c)+1)-7/2/d/(a+b)^4*ln(tanh(1/2*d*x+1/2*c)-1)*b-49/8/d*b^3/(
a+b)^4/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1
/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-49/8/d*b^3/(a+b)^4/(b*(a
+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)
/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+5/4/d*b^4/(a+b)^4/(tanh(1/2*d*x+1/2*c
)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d
*x+1/2*c)^7+5/4/d*b^4/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c
)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)-1/2/d/(a+b)^4*ln
(tanh(1/2*d*x+1/2*c)-1)*a+1/2/d/(a+b)^4*ln(tanh(1/2*d*x+1/2*c)+1)*a+7/2/d/(
a+b)^4*ln(tanh(1/2*d*x+1/2*c)+1)*b-7/4/d*b^3/(a+b)^4/a/((2*(b*(a+b))^(1/2)+
a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(
1/2))+13/4/d*b^2/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*
a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*d*x+1/2*c)^7+39/4/d*b^2/(a+b)^4
/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2
*b+a)^2*a*tanh(1/2*d*x+1/2*c)^5+39/4/d*b^2/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a
+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*d*x+1/
2*c)^3+3/8/d*b^4/(a+b)^4/a^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*
tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-3/8/d*b^4/(a+b)^4/a
^2/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b
*(a+b))^(1/2)+a+2*b)*a)^(1/2))+71/4/d*b^4/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+
2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2
*c)^3+3/d*b^5/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*
tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^3+71/4/d*b^4/(a+b)^4/(
tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b
+a)^2/a*tanh(1/2*d*x+1/2*c)^5+13/4/d*b^2/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2
*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*d*x+1/2*
c)+7/4/d*b^3/(a+b)^4/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1
/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/d*b^5/(a+b)^4/(tanh(1/
2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a
^2*tanh(1/2*d*x+1/2*c)^5+35/8/d*b^2/(a+b)^4/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(
1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-35/
8/d*b^2/(a+b)^4/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1
/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+9/2/d*b^3/(a+b)^4/(tanh(1/2*d*x+
1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/
2*d*x+1/2*c)^7+49/2/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1
/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5+49/2/d*b^3/(
a+b)^4/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/
2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3+9/2/d*b^3/(a+b)^4/(tanh(1/2*d*x+1/2*c)^
4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1
/2*c)
```

maxima [B] time = 1.05, size = 1806, normalized size = 9.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c))^3,x, algorithm="maxima")

```
[Out] 3/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 3/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 3/32*(5*a^3*b - 15*a^2*b^2 - 5*a*b^3 - b^4)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) + 3/32*(5*a^3*b - 15*a^2*b^2 - 5*a*b^3 - b^4)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) - 1/16*(15*a^2*b + 10*a*b^2 + 3*b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)*d) + 1/16*(9*a^4*b + 4*a^3*b^2 - 22*a^2*b^3 - 20*a*b^4 - 3*b^5 + 3*(3*a^4*b - 22*a^3*b^2 - 20*a^2*b^3 + 6*a*b^4 + b^5)*e^(6*d*x + 6*c) + (27*a^4*b - 156*a^3*b^2 + 110*a^2*b^3 - 36*a*b^4 - 9*b^5)*e^(4*d*x + 4*c) + (27*a^4*b - 86*a^3*b^2 - 84*a^2*b^3 + 38*a*b^4 + 9*b^5)*e^(2*d*x + 2*c))/((a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6 + (a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*e^(8*d*x + 8*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*e^(6*d*x + 6*c) + 2*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*e^(4*d*x + 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*e^(2*d*x + 2*c))*d) - 1/16*(9*a^4*b + 4*a^3*b^2 - 22*a^2*b^3 - 20*a*b^4 - 3*b^5 + (27*a^4*b - 86*a^3*b^2 - 84*a^2*b^3 + 38*a*b^4 + 9*b^5)*e^(-2*d*x - 2*c) + (27*a^4*b - 156*a^3*b^2 + 110*a^2*b^3 - 36*a*b^4 - 9*b^5)*e^(-4*d*x - 4*c) + 3*(3*a^4*b - 22*a^3*b^2 - 20*a^2*b^3 + 6*a*b^4 + b^5)*e^(-6*d*x - 6*c))/((a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6 + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*e^(-2*d*x - 2*c) + 2*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5 + 3*a^2*b^6)*e^(-4*d*x - 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*e^(-6*d*x - 6*c) + (a^8 + 6*a^7*b + 15*a^6*b^2 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*e^(-8*d*x - 8*c))*d) + 1/8*(9*a^3*b + 21*a^2*b^2 + 15*a*b^3 + 3*b^4 + (27*a^3*b + 13*a^2*b^2 - 23*a*b^3 - 9*b^4)*e^(-2*d*x - 2*c) + 3*(9*a^3*b - 3*a^2*b^2 + 7*a*b^3 + 3*b^4)*e^(-4*d*x - 4*c) + (9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4)*e^(-6*d*x - 6*c))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*e^(-4*d*x - 4*c) + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(-6*d*x - 6*c) + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*e^(-8*d*x - 8*c))*d) + 1/2*(d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 1/8*e^(2*d*x + 2*c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/8*e^(-2*d*x - 2*c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)^2}{(b \tanh(c + dx)^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3,x)
```

```
[Out] int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

$$3.126 \quad \int \frac{\cosh(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=154

$$\frac{3b^2(4a+b)\sinh(c+dx)}{8a^2d(a+b)^3((a+b)\sinh^2(c+dx)+a)} + \frac{3b(8a^2+4ab+b^2)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^{7/2}} + \frac{b^3\sinh(c+dx)}{4ad(a+b)^3((a+b)\sinh^2(c+dx)+a)}$$

[Out] 3/8*b*(8*a^2+4*a*b+b^2)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(5/2)/(a+b)^(7/2)/d+sinh(d*x+c)/(a+b)^3/d+1/4*b^3*sinh(d*x+c)/a/(a+b)^3/d/(a+(a+b)*sinh(d*x+c)^2)^2+3/8*b^2*(4*a+b)*sinh(d*x+c)/a^2/(a+b)^3/d/(a+(a+b)*sinh(d*x+c)^2)

Rubi [A] time = 0.22, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3676, 390, 1157, 385, 205}

$$\frac{3b^2(4a+b)\sinh(c+dx)}{8a^2d(a+b)^3((a+b)\sinh^2(c+dx)+a)} + \frac{3b(8a^2+4ab+b^2)\tan^{-1}\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^{7/2}} + \frac{b^3\sinh(c+dx)}{4ad(a+b)^3((a+b)\sinh^2(c+dx)+a)}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (3*b*(8*a^2 + 4*a*b + b^2)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a + b)^(7/2)*d) + Sinh[c + d*x]/((a + b)^3*d) + (b^3*Sinh[c + d*x])/((4*a*(a + b)^3*d*(a + (a + b)*Sinh[c + d*x]^2)^2) + (3*b^2*(4*a + b)*Sinh[c + d*x])/((8*a^2*(a + b)^3*d*(a + (a + b)*Sinh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 1157

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -

$b*d*e + a*e^2, 0] \&\& \text{IGtQ}[p, 0] \&\& \text{LtQ}[q, -1]$

Rule 3676

$\text{Int}[\sec[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_.)]^{(n_.)})^{(p_.)}, x_Symbol] \text{:> With}[\{ff = \text{FreeFactors}[\text{Sin}[e + f*x], x]\}, \text{Dist}[ff/f, \text{Subst}[\text{Int}[\text{ExpandToSum}[b*(ff*x)^n + a*(1 - ff^2*x^2)^{(n/2)}, x]^p/(1 - ff^2*x^2)^{((m + n*p + 1)/2)}, x], x, \text{Sin}[e + f*x]/ff], x]] /; \text{FreeQ}[\{a, b, e, f\}, x] \&\& \text{IntegerQ}[(m - 1)/2] \&\& \text{IntegerQ}[n/2] \&\& \text{IntegerQ}[p]$

Rubi steps

$$\begin{aligned} \int \frac{\cosh(c + dx)}{(a + b \tanh^2(c + dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{(a+(a+b)x^2)^3} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)^3} + \frac{b(3a^2+3ab+b^2)+3b(a+b)(2a+b)x^2+3b(a+b)^2x^4}{(a+b)^3(a+(a+b)x^2)^3}\right) dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{\sinh(c + dx)}{(a + b)^3 d} + \frac{\text{Subst}\left(\int \frac{b(3a^2+3ab+b^2)+3b(a+b)(2a+b)x^2+3b(a+b)^2x^4}{(a+(a+b)x^2)^3} dx, x, \sinh(c + dx)\right)}{(a + b)^3 d} \\ &= \frac{\sinh(c + dx)}{(a + b)^3 d} + \frac{b^3 \sinh(c + dx)}{4a(a + b)^3 d (a + (a + b) \sinh^2(c + dx))^2} - \frac{\text{Subst}\left(\int \frac{-3b(2a+b)^2-12a}{(a+(a+b)x^2)} dx, x, \sinh(c + dx)\right)}{4a(a + b)^3 d} \\ &= \frac{\sinh(c + dx)}{(a + b)^3 d} + \frac{b^3 \sinh(c + dx)}{4a(a + b)^3 d (a + (a + b) \sinh^2(c + dx))^2} + \frac{3b^2(4a + b) \sinh(c + dx)}{8a^2(a + b)^3 d (a + (a + b) \sinh^2(c + dx))} \\ &= \frac{3b(8a^2 + 4ab + b^2) \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{7/2}d} + \frac{\sinh(c + dx)}{(a + b)^3 d} + \frac{b^3 \sinh(c + dx)}{4a(a + b)^3 d (a + (a + b) \sinh^2(c + dx))} \end{aligned}$$

Mathematica [A] time = 2.14, size = 136, normalized size = 0.88

$$\frac{\sinh(c+dx) \left(\frac{3b^3}{a^2(a+b)\sinh^2(c+dx)+a} + \frac{2b^2(6(a+b)\sinh^2(c+dx)+6a+b)}{a(a+b)\sinh^2(c+dx)+a} + 8 \right)}{(a+b)^3} + \frac{3b(8a^2+4ab+b^2) \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a+b)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3, x]

[Out] ((3*b*(8*a^2 + 4*a*b + b^2)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(a^(5/2)*(a + b)^(7/2)) + (Sinh[c + d*x]*(8 + (3*b^3)/(a^2*(a + (a + b)*Sinh[c + d*x]^2))) + (2*b^2*(6*a + b + 6*(a + b)*Sinh[c + d*x]^2))/(a*(a + (a + b)*Sinh[c + d*x]^2)^2))/(a + b)^3)/(8*d)

fricas [B] time = 0.65, size = 11392, normalized size = 73.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

$$\begin{aligned}
& b^5) * \cosh(dx + c)^8 + 28*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5) \\
&) * \cosh(dx + c)^6 + 8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5 + 10* \\
& (24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 + 10*a*b^4 + 3*b^5) * \cosh(dx + c)^4 + 12 \\
& *(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5) * \cosh(dx + c)^2) * \sinh(dx \\
& x + c)) * \sqrt{-a^2 - a*b} * \log(((a + b) * \cosh(dx + c)^4 + 4*(a + b) * \cosh(dx \\
& + c) * \sinh(dx + c)^3 + (a + b) * \sinh(dx + c)^4 - 2*(3*a + b) * \cosh(dx + c)^ \\
& 2 + 2*(3*(a + b) * \cosh(dx + c)^2 - 3*a - b) * \sinh(dx + c)^2 + 4*((a + b) * \co \\
& sh(dx + c)^3 - (3*a + b) * \cosh(dx + c)) * \sinh(dx + c) - 4*(\cosh(dx + c)^3 \\
& + 3*\cosh(dx + c) * \sinh(dx + c)^2 + \sinh(dx + c)^3 + (3*\cosh(dx + c)^2 - \\
& 1) * \sinh(dx + c) - \cosh(dx + c)) * \sqrt{-a^2 - a*b} + a + b) / ((a + b) * \cosh(\\
& dx + c)^4 + 4*(a + b) * \cosh(dx + c) * \sinh(dx + c)^3 + (a + b) * \sinh(dx + c \\
&)^4 + 2*(a - b) * \cosh(dx + c)^2 + 2*(3*(a + b) * \cosh(dx + c)^2 + a - b) * \sin \\
& h(dx + c)^2 + 4*((a + b) * \cosh(dx + c)^3 + (a - b) * \cosh(dx + c)) * \sinh(dx \\
& + c) + a + b)) + 8*(10*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3) * \cosh(dx + c) \\
& ^9 + 4*(6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5) * \co \\
& sh(dx + c)^7 + 3*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - \\
& 9*a*b^5) * \cosh(dx + c)^5 - 2*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34 \\
& *a^2*b^4 - 9*a*b^5) * \cosh(dx + c)^3 - (6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3 \\
& *b^3 + 18*a^2*b^4 + 3*a*b^5) * \cosh(dx + c)) * \sinh(dx + c)) / ((a^9 + 6*a^8*b \\
& + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6) * d * \cosh(dx + \\
& c)^9 + 9*(a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 \\
& + a^3*b^6) * d * \cosh(dx + c) * \sinh(dx + c)^8 + (a^9 + 6*a^8*b + 15*a^7*b^2 + \\
& 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6) * d * \sinh(dx + c)^9 + 4*(a^9 + \\
& 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3*b^6) * d * \cosh(dx + c)^7 + \\
& 4*(9*(a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a \\
& ^3*b^6) * d * \cosh(dx + c)^2 + (a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b \\
& ^5 - a^3*b^6) * d) * \sinh(dx + c)^7 + 2*(3*a^9 + 10*a^8*b + 13*a^7*b^2 + 12*a \\
& ^6*b^3 + 13*a^5*b^4 + 10*a^4*b^5 + 3*a^3*b^6) * d * \cosh(dx + c)^5 + 28*(3*(a^ \\
& 9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6) * d \\
& * \cosh(dx + c)^3 + (a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3 \\
& *b^6) * d * \cosh(dx + c)) * \sinh(dx + c)^6 + 2*(63*(a^9 + 6*a^8*b + 15*a^7*b^2 \\
& + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6) * d * \cosh(dx + c)^4 + 42*(a^ \\
& 9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3*b^6) * d * \cosh(dx + c)^ \\
& 2 + (3*a^9 + 10*a^8*b + 13*a^7*b^2 + 12*a^6*b^3 + 13*a^5*b^4 + 10*a^4*b^5 + \\
& 3*a^3*b^6) * d) * \sinh(dx + c)^5 + 4*(a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - \\
& 4*a^4*b^5 - a^3*b^6) * d * \cosh(dx + c)^3 + 2*(63*(a^9 + 6*a^8*b + 15*a^7*b^2 \\
& + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6) * d * \cosh(dx + c)^5 + 70*(a \\
& ^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3*b^6) * d * \cosh(dx + c) \\
& ^3 + 5*(3*a^9 + 10*a^8*b + 13*a^7*b^2 + 12*a^6*b^3 + 13*a^5*b^4 + 10*a^4*b^ \\
& 5 + 3*a^3*b^6) * d * \cosh(dx + c)) * \sinh(dx + c)^4 + 4*(21*(a^9 + 6*a^8*b + 15 \\
& *a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6) * d * \cosh(dx + c)^6 \\
& + 35*(a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3*b^6) * d * \cosh(\\
& dx + c)^4 + 5*(3*a^9 + 10*a^8*b + 13*a^7*b^2 + 12*a^6*b^3 + 13*a^5*b^4 + 1 \\
& 0*a^4*b^5 + 3*a^3*b^6) * d * \cosh(dx + c)^2 + (a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a \\
& ^5*b^4 - 4*a^4*b^5 - a^3*b^6) * d) * \sinh(dx + c)^3 + (a^9 + 6*a^8*b + 15*a^7* \\
& b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6) * d * \cosh(dx + c) + 4*(9 \\
& *(a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^ \\
& 6) * d * \cosh(dx + c)^7 + 21*(a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^ \\
& 5 - a^3*b^6) * d * \cosh(dx + c)^5 + 5*(3*a^9 + 10*a^8*b + 13*a^7*b^2 + 12*a^6* \\
& b^3 + 13*a^5*b^4 + 10*a^4*b^5 + 3*a^3*b^6) * d * \cosh(dx + c)^3 + 3*(a^9 + 4*a \\
& ^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3*b^6) * d * \cosh(dx + c)) * \sinh(d \\
& *x + c)^2 + (9*(a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^ \\
& 4*b^5 + a^3*b^6) * d * \cosh(dx + c)^8 + 28*(a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5* \\
& b^4 - 4*a^4*b^5 - a^3*b^6) * d * \cosh(dx + c)^6 + 10*(3*a^9 + 10*a^8*b + 13*a^ \\
& 7*b^2 + 12*a^6*b^3 + 13*a^5*b^4 + 10*a^4*b^5 + 3*a^3*b^6) * d * \cosh(dx + c)^4 \\
& + 12*(a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3*b^6) * d * \cosh(\\
& dx + c)^2 + (a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4* \\
& b^5 + a^3*b^6) * d) * \sinh(dx + c)), 1/8*(4*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b \\
& ^3) * \cosh(dx + c)^10 + 40*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3) * \cosh(dx +
\end{aligned}$$

$$\begin{aligned}
& c) * \sinh(dx + c)^9 + 4*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3) * \sinh(dx + c)^{10} \\
& + 2*(6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5) * \cosh(dx + c)^8 \\
& + 2*(6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5 + 90*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3) * \cosh(dx + c)^2) * \sinh(dx + c)^8 \\
& + 16*(30*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3) * \cosh(dx + c)^3 + (6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5) * \cosh(dx + c)) * \sinh(dx + c)^7 \\
& + 2*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5) * \cosh(dx + c)^6 + 2*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5 + 420*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3) * \cosh(dx + c)^4 \\
& + 28*(6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5) * \cosh(dx + c)^2) * \sinh(dx + c)^6 - 4*a^6 - 12*a^5*b - 12*a^4*b^2 - 4*a^3*b^3 \\
& + 4*(252*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3) * \cosh(dx + c)^5 + 28*(6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5) * \cosh(dx + c)^3 + 3*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5) * \cosh(dx + c)) * \sinh(dx + c)^5 \\
& - 2*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5) * \cosh(dx + c)^4 + 2*(420*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3) * \cosh(dx + c)^6 - 4*a^6 + 4*a^5*b - 24*a^4*b^2 - 7*a^3*b^3 + 34*a^2*b^4 + 9*a*b^5 + 70*(6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5) * \cosh(dx + c)^4 \\
& + 15*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5) * \cosh(dx + c)^2) * \sinh(dx + c)^4 + 8*(60*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3) * \cosh(dx + c)^7 + 14*(6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5) * \cosh(dx + c)^5 + 5*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5) * \cosh(dx + c)^3 - (4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5) * \cosh(dx + c)) * \sinh(dx + c)^3 \\
& - 2*(6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5) * \cosh(dx + c)^2 + 2*(90*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3) * \cosh(dx + c)^8 + 28*(6*a^6 + 2*a^5*b - 2*a^4*b^2 + 17*a^3*b^3 + 18*a^2*b^4 + 3*a*b^5) * \cosh(dx + c)^6 - 6*a^6 - 2*a^5*b + 2*a^4*b^2 - 17*a^3*b^3 - 18*a^2*b^4 - 3*a*b^5 + 15*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5) * \cosh(dx + c)^4 - 6*(4*a^6 - 4*a^5*b + 24*a^4*b^2 + 7*a^3*b^3 - 34*a^2*b^4 - 9*a*b^5) * \cosh(dx + c)^2) * \sinh(dx + c)^2 + 3*((8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5) * \cosh(dx + c)^9 + 9*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5) * \cosh(dx + c) * \sinh(dx + c)^8 + (8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5) * \sinh(dx + c)^9 + 4*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5) * \cosh(dx + c)^7 + 4*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5 + 9*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5) * \cosh(dx + c)^2) * \sinh(dx + c)^7 + 28*(3*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5) * \cosh(dx + c)^3 + (8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5) * \cosh(dx + c)) * \sinh(dx + c)^6 + 2*(24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 + 10*a*b^4 + 3*b^5) * \cosh(dx + c)^5 + 2*(24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 + 10*a*b^4 + 3*b^5 + 63*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5) * \cosh(dx + c)^4 + 42*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5) * \cosh(dx + c)^2) * \sinh(dx + c)^5 + 2*(63*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5) * \cosh(dx + c)^5 + 70*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5) * \cosh(dx + c)^3 + 5*(24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 + 10*a*b^4 + 3*b^5) * \cosh(dx + c)) * \sinh(dx + c)^4 + 4*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5) * \cosh(dx + c)^3 + 4*(21*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5) * \cosh(dx + c)^6 + 8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5 + 35*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5) * \cosh(dx + c)^4 + 5*(24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 + 10*a*b^4 + 3*b^5) * \cosh(dx + c)^2) * \sinh(dx + c)^3 + 4*(9*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5) * \cosh(dx + c)^7 + 21*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5) * \cosh(dx + c)^5 + 5*(24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 + 10*a*b^4 + 3*b^5) * \cosh(dx + c)^3 + 3*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5) * \cosh(dx + c)) * \sinh(dx + c)^2 + (8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5) * \cosh(dx + c) + (9*(8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5) * \cosh(dx + c)^8 + 28*(8*a^4*b + 4*a^3*b^2 - 7*a^2*b^3 - 4*a*b^4 - b^5) * \cosh(dx + c)^6 + 8*a^4*b + 20*a^3*b^2 + 17*a^2*b^3 + 6*a*b^4 + b^5 + 10*(24*a^4*b - 4*a^3*b^2 + 19*a^2*b^3 + 10*a*b^4
\end{aligned}$$

$$\begin{aligned}
& 4 + 3b^5) \cosh(dx + c)^4 + 12(8a^4b + 4a^3b^2 - 7a^2b^3 - 4ab^4 - b^5) \cosh(dx + c)^2 \sinh(dx + c) \sqrt{a^2 + ab} \arctan(1/2((a + b) \cosh(dx + c)^3 + 3(a + b) \cosh(dx + c) \sinh(dx + c)^2 + (a + b) \sinh(dx + c)^3 + (3a - b) \cosh(dx + c) + (3(a + b) \cosh(dx + c)^2 + 3a - b) \sinh(dx + c)) / \sqrt{a^2 + ab})) + 3((8a^4b + 20a^3b^2 + 17a^2b^3 + 6ab^4 + b^5) \cosh(dx + c)^9 + 9(8a^4b + 20a^3b^2 + 17a^2b^3 + 6ab^4 + b^5) \cosh(dx + c) \sinh(dx + c)^8 + (8a^4b + 20a^3b^2 + 17a^2b^3 + 6ab^4 + b^5) \sinh(dx + c)^9 + 4(8a^4b + 4a^3b^2 - 7a^2b^3 - 4ab^4 - b^5) \cosh(dx + c)^7 + 4(8a^4b + 4a^3b^2 - 7a^2b^3 - 4ab^4 - b^5) \cosh(dx + c)^2 \sinh(dx + c)^7 + 28(3(8a^4b + 20a^3b^2 + 17a^2b^3 + 6ab^4 + b^5) \cosh(dx + c)^3 + (8a^4b + 4a^3b^2 - 7a^2b^3 - 4ab^4 - b^5) \cosh(dx + c)) \sinh(dx + c)^6 + 2(24a^4b - 4a^3b^2 + 19a^2b^3 + 10ab^4 + 3b^5) \cosh(dx + c)^5 + 2(24a^4b - 4a^3b^2 + 19a^2b^3 + 10ab^4 + 3b^5 + 63(8a^4b + 20a^3b^2 + 17a^2b^3 + 6ab^4 + b^5) \cosh(dx + c)^4 + 42(8a^4b + 4a^3b^2 - 7a^2b^3 - 4ab^4 - b^5) \cosh(dx + c)^2 \sinh(dx + c)^5 + 2(63(8a^4b + 20a^3b^2 + 17a^2b^3 + 6ab^4 + b^5) \cosh(dx + c)^5 + 70(8a^4b + 4a^3b^2 - 7a^2b^3 - 4ab^4 - b^5) \cosh(dx + c)^3 + 5(24a^4b - 4a^3b^2 + 19a^2b^3 + 10ab^4 + 3b^5) \cosh(dx + c)) \sinh(dx + c)^4 + 4(8a^4b + 4a^3b^2 - 7a^2b^3 - 4ab^4 - b^5) \cosh(dx + c)^3 + 4(21(8a^4b + 20a^3b^2 + 17a^2b^3 + 6ab^4 + b^5) \cosh(dx + c)^6 + 8a^4b + 4a^3b^2 - 7a^2b^3 - 4ab^4 - b^5 + 35(8a^4b + 4a^3b^2 - 7a^2b^3 - 4ab^4 - b^5) \cosh(dx + c)^4 + 5(24a^4b - 4a^3b^2 + 19a^2b^3 + 10ab^4 + 3b^5) \cosh(dx + c)^2 \sinh(dx + c)^3 + 4(9(8a^4b + 20a^3b^2 + 17a^2b^3 + 6ab^4 + b^5) \cosh(dx + c)^7 + 21(8a^4b + 4a^3b^2 - 7a^2b^3 - 4ab^4 - b^5) \cosh(dx + c)^5 + 5(24a^4b - 4a^3b^2 + 19a^2b^3 + 10ab^4 + 3b^5) \cosh(dx + c)^3 + 3(8a^4b + 4a^3b^2 - 7a^2b^3 - 4ab^4 - b^5) \cosh(dx + c)) \sinh(dx + c)^2 + (8a^4b + 20a^3b^2 + 17a^2b^3 + 6ab^4 + b^5) \cosh(dx + c) + (9(8a^4b + 20a^3b^2 + 17a^2b^3 + 6ab^4 + b^5) \cosh(dx + c)^8 + 28(8a^4b + 4a^3b^2 - 7a^2b^3 - 4ab^4 - b^5) \cosh(dx + c)^6 + 8a^4b + 20a^3b^2 + 17a^2b^3 + 6ab^4 + b^5 + 10(24a^4b - 4a^3b^2 + 19a^2b^3 + 10ab^4 + 3b^5) \cosh(dx + c)^4 + 12(8a^4b + 4a^3b^2 - 7a^2b^3 - 4ab^4 - b^5) \cosh(dx + c)^2 \sinh(dx + c)) \sqrt{a^2 + ab} \arctan(1/2 \sqrt{a^2 + ab} (\cosh(dx + c) + \sinh(dx + c)) / a) + 4(10(a^6 + 3a^5b + 3a^4b^2 + a^3b^3) \cosh(dx + c)^9 + 4(6a^6 + 2a^5b - 2a^4b^2 + 17a^3b^3 + 18a^2b^4 + 3ab^5) \cosh(dx + c)^7 + 3(4a^6 - 4a^5b + 24a^4b^2 + 7a^3b^3 - 34a^2b^4 - 9ab^5) \cosh(dx + c)^5 - 2(4a^6 - 4a^5b + 24a^4b^2 + 7a^3b^3 - 34a^2b^4 - 9ab^5) \cosh(dx + c)^3 - (6a^6 + 2a^5b - 2a^4b^2 + 17a^3b^3 + 18a^2b^4 + 3ab^5) \cosh(dx + c)) \sinh(dx + c)) / ((a^9 + 6a^8b + 15a^7b^2 + 20a^6b^3 + 15a^5b^4 + 6a^4b^5 + a^3b^6) d \cosh(dx + c)^9 + 9(a^9 + 6a^8b + 15a^7b^2 + 20a^6b^3 + 15a^5b^4 + 6a^4b^5 + a^3b^6) d \cosh(dx + c) \sinh(dx + c)^8 + (a^9 + 6a^8b + 15a^7b^2 + 20a^6b^3 + 15a^5b^4 + 6a^4b^5 + a^3b^6) d \sinh(dx + c)^9 + 4(a^9 + 4a^8b + 5a^7b^2 - 5a^5b^4 - 4a^4b^5 - a^3b^6) d \cosh(dx + c)^7 + 4(9(a^9 + 6a^8b + 15a^7b^2 + 20a^6b^3 + 15a^5b^4 + 6a^4b^5 + a^3b^6) d \cosh(dx + c)^2 + (a^9 + 4a^8b + 5a^7b^2 - 5a^5b^4 - 4a^4b^5 - a^3b^6) d) \sinh(dx + c)^7 + 2(3a^9 + 10a^8b + 13a^7b^2 + 12a^6b^3 + 13a^5b^4 + 10a^4b^5 + 3a^3b^6) d \cosh(dx + c)^5 + 28(3(a^9 + 6a^8b + 15a^7b^2 + 20a^6b^3 + 15a^5b^4 + 6a^4b^5 + a^3b^6) d \cosh(dx + c)^3 + (a^9 + 4a^8b + 5a^7b^2 - 5a^5b^4 - 4a^4b^5 - a^3b^6) d \cosh(dx + c)) \sinh(dx + c)^6 + 2(63(a^9 + 6a^8b + 15a^7b^2 + 20a^6b^3 + 15a^5b^4 + 6a^4b^5 + a^3b^6) d \cosh(dx + c)^4 + 42(a^9 + 4a^8b + 5a^7b^2 - 5a^5b^4 - 4a^4b^5 - a^3b^6) d \cosh(dx + c)^2 + (3a^9 + 10a^8b + 13a^7b^2 + 12a^6b^3 + 13a^5b^4 + 10a^4b^5 + 3a^3b^6) d) \sinh(dx + c)^5 + 4(a^9 + 4a^8b + 5a^7b^2 - 5a^5b^4 - 4a^4b^5 - a^3b^6) d \cosh(dx + c)^3 + 2(63(a^9 + 6a^8b + 15a^7b^2 + 20a^6b^3 + 15a^5b^4 + 6a^4b^5 + a^3b^6) d \cosh(dx + c)^5 + 70(a^9 + 4a^
\end{aligned}$$

$$\begin{aligned} &^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3*b^6)*d*\cosh(d*x + c)^3 + 5*(\\ &3*a^9 + 10*a^8*b + 13*a^7*b^2 + 12*a^6*b^3 + 13*a^5*b^4 + 10*a^4*b^5 + 3*a^ \\ &3*b^6)*d*\cosh(d*x + c))*\sinh(d*x + c)^4 + 4*(21*(a^9 + 6*a^8*b + 15*a^7*b^2 \\ &+ 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*d*\cosh(d*x + c)^6 + 35*(a \\ &^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3*b^6)*d*\cosh(d*x + c) \\ &^4 + 5*(3*a^9 + 10*a^8*b + 13*a^7*b^2 + 12*a^6*b^3 + 13*a^5*b^4 + 10*a^4*b^ \\ &5 + 3*a^3*b^6)*d*\cosh(d*x + c)^2 + (a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - \\ &4*a^4*b^5 - a^3*b^6)*d*\sinh(d*x + c)^3 + (a^9 + 6*a^8*b + 15*a^7*b^2 + 20 \\ &*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*d*\cosh(d*x + c) + 4*(9*(a^9 + \\ &6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^3*b^6)*d*\cos \\ &h(d*x + c)^7 + 21*(a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3* \\ &b^6)*d*\cosh(d*x + c)^5 + 5*(3*a^9 + 10*a^8*b + 13*a^7*b^2 + 12*a^6*b^3 + 13 \\ &*a^5*b^4 + 10*a^4*b^5 + 3*a^3*b^6)*d*\cosh(d*x + c)^3 + 3*(a^9 + 4*a^8*b + 5 \\ &*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3*b^6)*d*\cosh(d*x + c))*\sinh(d*x + c)^ \\ &2 + (9*(a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + \\ &a^3*b^6)*d*\cosh(d*x + c)^8 + 28*(a^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4* \\ &a^4*b^5 - a^3*b^6)*d*\cosh(d*x + c)^6 + 10*(3*a^9 + 10*a^8*b + 13*a^7*b^2 + \\ &12*a^6*b^3 + 13*a^5*b^4 + 10*a^4*b^5 + 3*a^3*b^6)*d*\cosh(d*x + c)^4 + 12*(a \\ &^9 + 4*a^8*b + 5*a^7*b^2 - 5*a^5*b^4 - 4*a^4*b^5 - a^3*b^6)*d*\cosh(d*x + c) \\ &^2 + (a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5 + a^ \\ &3*b^6)*d*\sinh(d*x + c))] \end{aligned}$$

giac [B] time = 0.68, size = 2145, normalized size = 13.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} &-1/8*(3*(2*(a^5*e^{(2*c)} + 3*a^4*b*e^{(2*c)} + 3*a^3*b^2*e^{(2*c)} + a^2*b^3*e^{(2*c)} \\ &^2*(16*a^3*b^2 + 8*a^2*b^3 + 2*a*b^4 + (8*a^3*b - 4*a^2*b^2 - 3*a*b^3 \\ &- b^4)*\sqrt{-a*b})*\abs(a*e^{(2*c)} + b*e^{(2*c)}) - (8*a^9*b + 28*a^8*b^2 + 29* \\ &a^7*b^3 - 5*a^6*b^4 - 30*a^5*b^5 - 22*a^4*b^6 - 7*a^3*b^7 - a^2*b^8 - 2*(8* \\ &a^8*b + 36*a^7*b^2 + 65*a^6*b^3 + 60*a^5*b^4 + 30*a^4*b^5 + 8*a^3*b^6 + a^2 \\ &*b^7)*\sqrt{-a*b})*\abs(-a^5*e^{(2*c)} - 3*a^4*b*e^{(2*c)} - 3*a^3*b^2*e^{(2*c)} - \\ &a^2*b^3*e^{(2*c)})*\abs(a*e^{(2*c)} + b*e^{(2*c)})*e^{(2*c)} - (16*a^{13}*b^2 + 88*a^{1 \\ &2}*b^3 + 186*a^{11}*b^4 + 162*a^{10}*b^5 - 22*a^9*b^6 - 174*a^8*b^7 - 162*a^7*b^ \\ &8 - 74*a^6*b^9 - 18*a^5*b^{10} - 2*a^4*b^{11} + (8*a^{13}*b + 36*a^{12}*b^2 + 49*a^ \\ &11*b^3 - 12*a^{10}*b^4 - 92*a^9*b^5 - 76*a^8*b^6 + 6*a^7*b^7 + 44*a^6*b^8 + 2 \\ &8*a^5*b^9 + 8*a^4*b^{10} + a^3*b^{11})*\sqrt{-a*b})*\abs(a*e^{(2*c)} + b*e^{(2*c)})*e \\ &^{(4*c)})*\arctan(e^{(d*x)}/\sqrt{(a^6*e^{(2*c)} + 2*a^5*b*e^{(2*c)} - 2*a^3*b^3*e^{(2 \\ &*c)} - a^2*b^4*e^{(2*c)} + \sqrt{(a^6*e^{(2*c)} + 2*a^5*b*e^{(2*c)} - 2*a^3*b^3*e^{(2 \\ &^2*c)} - a^2*b^4*e^{(2*c)})^2 - (a^6*e^{(4*c)} + 4*a^5*b*e^{(4*c)} + 6*a^4*b^2*e^{(4 \\ &*c)} + 4*a^3*b^3*e^{(4*c)} + a^2*b^4*e^{(4*c)})*(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a \\ &^3*b^3 + a^2*b^4)))/((a^6*e^{(4*c)} + 4*a^5*b*e^{(4*c)} + 6*a^4*b^2*e^{(4*c)} + 4* \\ &a^3*b^3*e^{(4*c)} + a^2*b^4*e^{(4*c)}))) * e^{(-4*c)} / ((a^{13} + 9*a^{12}*b + 36*a^{11}*b \\ &^2 + 84*a^{10}*b^3 + 126*a^9*b^4 + 126*a^8*b^5 + 84*a^7*b^6 + 36*a^6*b^7 + 9* \\ &a^5*b^8 + a^4*b^9)*\sqrt{a^2 - b^2 - 2*\sqrt{-a*b}*(a + b)}*\abs(-a^5*e^{(2*c)} \\ &- 3*a^4*b*e^{(2*c)} - 3*a^3*b^2*e^{(2*c)} - a^2*b^3*e^{(2*c)})) + 3*(2*(a^5*e^{(2 \\ &c)} + 3*a^4*b*e^{(2*c)} + 3*a^3*b^2*e^{(2*c)} + a^2*b^3*e^{(2*c)})^2*(16*a^3*b^2 + \\ &8*a^2*b^3 + 2*a*b^4 - (8*a^3*b - 4*a^2*b^2 - 3*a*b^3 - b^4)*\sqrt{-a*b})*\abs \\ &(a*e^{(2*c)} + b*e^{(2*c)}) - (8*a^9*b + 28*a^8*b^2 + 29*a^7*b^3 - 5*a^6*b^4 - \\ &30*a^5*b^5 - 22*a^4*b^6 - 7*a^3*b^7 - a^2*b^8 + 2*(8*a^8*b + 36*a^7*b^2 + \\ &65*a^6*b^3 + 60*a^5*b^4 + 30*a^4*b^5 + 8*a^3*b^6 + a^2*b^7)*\sqrt{-a*b})*\abs \\ &(-a^5*e^{(2*c)} - 3*a^4*b*e^{(2*c)} - 3*a^3*b^2*e^{(2*c)} - a^2*b^3*e^{(2*c)})*\abs(\\ &a*e^{(2*c)} + b*e^{(2*c)})*e^{(2*c)} - (16*a^{13}*b^2 + 88*a^{12}*b^3 + 186*a^{11}*b^4 \\ &+ 162*a^{10}*b^5 - 22*a^9*b^6 - 174*a^8*b^7 - 162*a^7*b^8 - 74*a^6*b^9 - 18*a \\ &^5*b^{10} - 2*a^4*b^{11} - (8*a^{13}*b + 36*a^{12}*b^2 + 49*a^{11}*b^3 - 12*a^{10}*b^4 \\ &- 92*a^9*b^5 - 76*a^8*b^6 + 6*a^7*b^7 + 44*a^6*b^8 + 28*a^5*b^9 + 8*a^4*b^{1 \\ &0} + a^3*b^{11})*\sqrt{-a*b})*\abs(a*e^{(2*c)} + b*e^{(2*c)})*e^{(4*c)})*\arctan(e^{(d*x} \end{aligned}$$

$$\frac{\sqrt{(a^6 e^{2c} + 2a^5 b e^{2c} - 2a^3 b^3 e^{2c} - a^2 b^4 e^{2c}) - \sqrt{(a^6 e^{4c} + 4a^5 b e^{4c} + 6a^4 b^2 e^{4c} + 4a^3 b^3 e^{4c} + a^2 b^4 e^{4c})}}{(a^6 + 4a^5 b + 6a^4 b^2 + 4a^3 b^3 + a^2 b^4)} \sqrt{(a^6 e^{4c} + 4a^5 b e^{4c} + 6a^4 b^2 e^{4c} + 4a^3 b^3 e^{4c} + a^2 b^4 e^{4c})} e^{-4c} / ((a^{13} + 9a^{12}b + 36a^{11}b^2 + 84a^{10}b^3 + 126a^9b^4 + 126a^8b^5 + 84a^7b^6 + 36a^6b^7 + 9a^5b^8 + a^4b^9) \sqrt{(a^2 - b^2 + 2\sqrt{-ab}(a+b))} \operatorname{abs}(-a^5 e^{2c} - 3a^4 b e^{2c} - 3a^3 b^2 e^{2c} - a^2 b^3 e^{2c})) - 4e^{(dx+14c)} / (a^3 e^{13c} + 3a^2 b e^{13c} + 3a b^2 e^{13c} + b^3 e^{13c})) + 4e^{(-dx)} / (a^3 e^c + 3a^2 b e^c + 3a b^2 e^c + b^3 e^c) - 2(12a^2 b^2 e^{(7dx+7c)} + 15a b^3 e^{(7dx+7c)} + 3b^4 e^{(7dx+7c)} + 12a^2 b^2 e^{(5dx+5c)} - 25a b^3 e^{(5dx+5c)} - 9b^4 e^{(5dx+5c)} - 12a^2 b^2 e^{(3dx+3c)} + 25a b^3 e^{(3dx+3c)} + 9b^4 e^{(3dx+3c)} - 12a^2 b^2 e^{(dx+c)} - 15a b^3 e^{(dx+c)} - 3b^4 e^{(dx+c)}) / ((a^5 + 3a^4 b + 3a^3 b^2 + a^2 b^3) (a e^{(4dx+4c)} + b e^{(4dx+4c)} + 2a e^{(2dx+2c)} - 2b e^{(2dx+2c)} + a + b)^2) / d$$

maple [B] time = 0.49, size = 1570, normalized size = 10.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(dx+c)/(a+b*tanh(dx+c)^2)^3,x)`

[Out]
$$\begin{aligned} & -1/d/(a+b)^3/(\tanh(1/2*dx+1/2*c)-1)-1/d/(a+b)^3/(\tanh(1/2*dx+1/2*c)+1)-3/ \\ & d*b^2/(a+b)^3/(\tanh(1/2*dx+1/2*c)^4+a+2*\tanh(1/2*dx+1/2*c)^2*a+4*\tanh(1/2 \\ & *dx+1/2*c)^2*b+a)^2*\tanh(1/2*dx+1/2*c)^7-5/4/d*b^3/(a+b)^3/(\tanh(1/2*dx+ \\ & 1/2*c)^4*a+2*\tanh(1/2*dx+1/2*c)^2*a+4*\tanh(1/2*dx+1/2*c)^2*b+a)^2/a*\tanh(\\ & 1/2*dx+1/2*c)^7-3/d*b^2/(a+b)^3/(\tanh(1/2*dx+1/2*c)^4*a+2*\tanh(1/2*dx+1/ \\ & 2*c)^2*a+4*\tanh(1/2*dx+1/2*c)^2*b+a)^2*\tanh(1/2*dx+1/2*c)^5-45/4/d*b^3/(a \\ & +b)^3/(\tanh(1/2*dx+1/2*c)^4*a+2*\tanh(1/2*dx+1/2*c)^2*a+4*\tanh(1/2*dx+1/2 \\ & *c)^2*b+a)^2/a*\tanh(1/2*dx+1/2*c)^5-3/d*b^4/(a+b)^3/(\tanh(1/2*dx+1/2*c)^4 \\ & *a+2*\tanh(1/2*dx+1/2*c)^2*a+4*\tanh(1/2*dx+1/2*c)^2*b+a)^2/a^2*\tanh(1/2*d \\ & x+1/2*c)^5+3/d*b^2/(a+b)^3/(\tanh(1/2*dx+1/2*c)^4*a+2*\tanh(1/2*dx+1/2*c)^2 \\ & *a+4*\tanh(1/2*dx+1/2*c)^2*b+a)^2*\tanh(1/2*dx+1/2*c)^3+45/4/d*b^3/(a+b)^3/ \\ & (\tanh(1/2*dx+1/2*c)^4*a+2*\tanh(1/2*dx+1/2*c)^2*a+4*\tanh(1/2*dx+1/2*c)^2* \\ & b+a)^2/a*\tanh(1/2*dx+1/2*c)^3+3/d*b^4/(a+b)^3/(\tanh(1/2*dx+1/2*c)^4*a+2*t \\ & anh(1/2*dx+1/2*c)^2*a+4*\tanh(1/2*dx+1/2*c)^2*b+a)^2/a^2*\tanh(1/2*d \\ & x+1/2*c)^3+3/d*b^2/(a+b)^3/(\tanh(1/2*dx+1/2*c)^4*a+2*\tanh(1/2*dx+1/2*c)^2*a+4*t \\ & anh(1/2*dx+1/2*c)^2*b+a)^2*\tanh(1/2*dx+1/2*c)+5/4/d*b^3/(a+b)^3/(\tanh(1/2 \\ & *dx+1/2*c)^4*a+2*\tanh(1/2*dx+1/2*c)^2*a+4*\tanh(1/2*dx+1/2*c)^2*b+a)^2/a* \\ & \tanh(1/2*dx+1/2*c)-3/d*b/(a+b)^3/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*\operatorname{arctan} \\ & h(a*\tanh(1/2*dx+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-3/2/d*b^2/(a+ \\ & b)^3/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*dx+1/2*c)/((\\ & 2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-3/8/d*b^3/(a+b)^3/a^2/((2*(b*(a+b))^(1/2 \\ &)-a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*dx+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)* \\ & a)^(1/2))+3/d*b^2/(a+b)^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/ \\ & 2)*\operatorname{arctanh}(a*\tanh(1/2*dx+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/2/d \\ & *b^3/(a+b)^3/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*\operatorname{arctanh}(\\ & a*\tanh(1/2*dx+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/8/d*b^4/(a+b)^ \\ & 3/a^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/ \\ & 2*dx+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/d*b/(a+b)^3/((2*(b*(a+b) \\ &))^(1/2)+a+2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*dx+1/2*c)/((2*(b*(a+b))^(1/2)+a \\ & +2*b)*a)^(1/2))+3/2/d*b^2/(a+b)^3/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*\operatorname{arc} \\ & \tan(a*\tanh(1/2*dx+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+3/8/d*b^3/(a \\ & +b)^3/a^2/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*dx+1/2*c)/ \\ & ((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+3/d*b^2/(a+b)^3/(b*(a+b))^(1/2)/((2*(b \\ & *(a+b))^(1/2)+a+2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*dx+1/2*c)/((2*(b*(a+b))^(1 \\ & /2)+a+2*b)*a)^(1/2))+3/2/d*b^3/(a+b)^3/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2) \end{aligned}$$

$) + a + 2b) * a)^{(1/2)} * \arctan(a * \tanh(1/2 * d * x + 1/2 * c) / ((2 * (b * (a + b))^{(1/2)} + a + 2b) * a)^{(1/2)}) + 3/8 / d * b^4 / (a + b)^3 / a^2 / (b * (a + b))^{(1/2)} / ((2 * (b * (a + b))^{(1/2)} + a + 2b) * a)^{(1/2)} * \arctan(a * \tanh(1/2 * d * x + 1/2 * c) / ((2 * (b * (a + b))^{(1/2)} + a + 2b) * a)^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $-1/4 * (2 * a^4 + 4 * a^3 * b + 2 * a^2 * b^2 - 2 * (a^4 * e^{(10 * c)} + 2 * a^3 * b * e^{(10 * c)} + a^2 * b^2 * e^{(10 * c)}) * e^{(10 * d * x)} - (6 * a^4 * e^{(8 * c)} - 4 * a^3 * b * e^{(8 * c)} + 2 * a^2 * b^2 * e^{(8 * c)} + 15 * a * b^3 * e^{(8 * c)} + 3 * b^4 * e^{(8 * c)}) * e^{(8 * d * x)} - (4 * a^4 * e^{(6 * c)} - 8 * a^3 * b * e^{(6 * c)} + 32 * a^2 * b^2 * e^{(6 * c)} - 25 * a * b^3 * e^{(6 * c)} - 9 * b^4 * e^{(6 * c)}) * e^{(6 * d * x)} + (4 * a^4 * e^{(4 * c)} - 8 * a^3 * b * e^{(4 * c)} + 32 * a^2 * b^2 * e^{(4 * c)} - 25 * a * b^3 * e^{(4 * c)} - 9 * b^4 * e^{(4 * c)}) * e^{(4 * d * x)} + (6 * a^4 * e^{(2 * c)} - 4 * a^3 * b * e^{(2 * c)} + 2 * a^2 * b^2 * e^{(2 * c)} + 15 * a * b^3 * e^{(2 * c)} + 3 * b^4 * e^{(2 * c)}) * e^{(2 * d * x)}) / ((a^7 * d * e^{(9 * c)} + 5 * a^6 * b * d * e^{(9 * c)} + 10 * a^5 * b^2 * d * e^{(9 * c)} + 10 * a^4 * b^3 * d * e^{(9 * c)} + 5 * a^3 * b^4 * d * e^{(9 * c)} + a^2 * b^5 * d * e^{(9 * c)}) * e^{(9 * d * x)} + 4 * (a^7 * d * e^{(7 * c)} + 3 * a^6 * b * d * e^{(7 * c)} + 2 * a^5 * b^2 * d * e^{(7 * c)} - 2 * a^4 * b^3 * d * e^{(7 * c)} - 3 * a^3 * b^4 * d * e^{(7 * c)} - a^2 * b^5 * d * e^{(7 * c)}) * e^{(7 * d * x)} + 2 * (3 * a^7 * d * e^{(5 * c)} + 7 * a^6 * b * d * e^{(5 * c)} + 6 * a^5 * b^2 * d * e^{(5 * c)} + 6 * a^4 * b^3 * d * e^{(5 * c)} + 7 * a^3 * b^4 * d * e^{(5 * c)} + 3 * a^2 * b^5 * d * e^{(5 * c)}) * e^{(5 * d * x)} + 4 * (a^7 * d * e^{(3 * c)} + 3 * a^6 * b * d * e^{(3 * c)} + 2 * a^5 * b^2 * d * e^{(3 * c)} - 2 * a^4 * b^3 * d * e^{(3 * c)} - 3 * a^3 * b^4 * d * e^{(3 * c)} - a^2 * b^5 * d * e^{(3 * c)}) * e^{(3 * d * x)} + (a^7 * d * e^c + 5 * a^6 * b * d * e^c + 10 * a^5 * b^2 * d * e^c + 10 * a^4 * b^3 * d * e^c + 5 * a^3 * b^4 * d * e^c + a^2 * b^5 * d * e^c) * e^{(d * x)}) + 1/2 * integrate(3/2 * ((8 * a^2 * b * e^{(3 * c)} + 4 * a * b^2 * e^{(3 * c)} + b^3 * e^{(3 * c)}) * e^{(3 * d * x)} + (8 * a^2 * b * e^c + 4 * a * b^2 * e^c + b^3 * e^c) * e^{(d * x)}) / (a^6 + 4 * a^5 * b + 6 * a^4 * b^2 + 4 * a^3 * b^3 + a^2 * b^4 + (a^6 * e^{(4 * c)} + 4 * a^5 * b * e^{(4 * c)} + 6 * a^4 * b^2 * e^{(4 * c)} + 4 * a^3 * b^3 * e^{(4 * c)} + a^2 * b^4 * e^{(4 * c)}) * e^{(4 * d * x)} + 2 * (a^6 * e^{(2 * c)} + 2 * a^5 * b * e^{(2 * c)} - 2 * a^3 * b^3 * e^{(2 * c)} - a^2 * b^4 * e^{(2 * c)}) * e^{(2 * d * x)}), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(c + dx)}{(b \tanh(c + dx)^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2)^3,x)

[Out] int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2)^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)

[Out] Timed out

$$3.127 \quad \int \frac{\operatorname{sech}(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=144

$$\frac{3b(2a+b) \sinh(c+dx)}{8a^2d(a+b)^2((a+b) \sinh^2(c+dx)+a)} + \frac{(8a^2+8ab+3b^2) \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^{5/2}} + \frac{b \sinh(c+dx) \cosh^2(c+dx)}{4ad(a+b)((a+b) \sinh^2(c+dx)+a)}$$

[Out] 1/8*(8*a^2+8*a*b+3*b^2)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(5/2)/(a+b)^(5/2)/d+1/4*b*cosh(d*x+c)^2*sinh(d*x+c)/a/(a+b)/d/(a+(a+b)*sinh(d*x+c)^2)+3/8*b*(2*a+b)*sinh(d*x+c)/a^2/(a+b)^2/d/(a+(a+b)*sinh(d*x+c)^2)

Rubi [A] time = 0.14, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3676, 413, 385, 205}

$$\frac{(8a^2+8ab+3b^2) \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^{5/2}} + \frac{3b(2a+b) \sinh(c+dx)}{8a^2d(a+b)^2((a+b) \sinh^2(c+dx)+a)} + \frac{b \sinh(c+dx) \cosh^2(c+dx)}{4ad(a+b)((a+b) \sinh^2(c+dx)+a)}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2)^3, x]

[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a + b)^(5/2)*d) + (b*Cosh[c + d*x]^2*Sinh[c + d*x])/(4*a*(a + b)*d*(a + (a + b)*Sinh[c + d*x]^2)^2) + (3*b*(2*a + b)*Sinh[c + d*x])/(8*a^2*(a + b)^2*d*(a + (a + b)*Sinh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 413

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p + 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p + q) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 3676

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}(c+dx)}{(a+b \tanh^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1+x^2)^2}{(a+(a+b)x^2)^3} dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{b \cosh^2(c+dx) \sinh(c+dx)}{4a(a+b)d \left(a+(a+b) \sinh^2(c+dx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{4a+3b+(4a+b)x^2}{(a+(a+b)x^2)^2} dx, x, \sinh(c+dx)\right)}{4a(a+b)d} \\
&= \frac{b \cosh^2(c+dx) \sinh(c+dx)}{4a(a+b)d \left(a+(a+b) \sinh^2(c+dx)\right)^2} + \frac{3b(2a+b) \sinh(c+dx)}{8a^2(a+b)^2d \left(a+(a+b) \sinh^2(c+dx)\right)} \\
&= \frac{(8a^2+8ab+3b^2) \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{5/2}d} + \frac{b \cosh^2(c+dx) \sinh(c+dx)}{4a(a+b)d \left(a+(a+b) \sinh^2(c+dx)\right)}
\end{aligned}$$

Mathematica [A] time = 1.18, size = 134, normalized size = 0.93

$$\frac{2\sqrt{a} b \sinh(c+dx) \left((8a^2+11ab+3b^2) \cosh(2(c+dx)) + 8a^2 - ab - 3b^2 \right)}{(a+b)^2 \left((a+b) \cosh(2(c+dx)) + a - b \right)^2} - \frac{(8a^2+8ab+3b^2) \tan^{-1}\left(\frac{\sqrt{a} \operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2)^3, x]

[Out] $-\left(\frac{(8a^2+8ab+3b^2) \operatorname{ArcTan}\left(\frac{\sqrt{a} \operatorname{Csch}[c+dx]}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + \frac{(2\sqrt{a} b \sinh(c+dx) \left((8a^2+11ab+3b^2) \cosh(2(c+dx)) + 8a^2 - ab - 3b^2 \right)) \operatorname{Cosh}[2(c+dx)] \operatorname{Sinh}[c+dx]}{(a+b)^2 \left((a+b) \cosh(2(c+dx)) + a - b \right)^2}\right) / (8a^{5/2}d)$

fricas [B] time = 0.56, size = 7909, normalized size = 54.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^3, x, algorithm="fricas")

[Out] $[1/16*(4*(8a^4b + 19a^3b^2 + 14a^2b^3 + 3ab^4) \cosh(dx+c)^7 + 28*(8a^4b + 19a^3b^2 + 14a^2b^3 + 3ab^4) \cosh(dx+c) \sinh(dx+c)^6 + 4*(8a^4b + 19a^3b^2 + 14a^2b^3 + 3ab^4) \sinh(dx+c)^7 + 4*(8a^4b - 5a^3b^2 - 22a^2b^3 - 9ab^4) \cosh(dx+c)^5 + 4*(8a^4b - 5a^3b^2 - 22a^2b^3 - 9ab^4) \cosh(dx+c)^2 \sinh(dx+c)^5 + 20*(7*(8a^4b + 19a^3b^2 + 14a^2b^3 + 3ab^4) \cosh(dx+c)^3 + (8a^4b - 5a^3b^2 - 22a^2b^3 - 9ab^4) \cosh(dx+c)) \sinh(dx+c)^4 - 4*(8a^4b - 5a^3b^2 - 22a^2b^3 - 9ab^4) \cosh(dx+c)^3 - 4*(8a^4b - 5a^3b^2 - 22a^2b^3 - 9ab^4) \cosh(dx+c)^2 \sinh(dx+c)^3 - 35*(8a^4b + 19a^3b^2 + 14a^2b^3 + 3ab^4) \cosh(dx+c)^4 - 10*(8a^4b - 5a^3b^2 - 22a^2b^3 - 9ab^4) \cosh(dx+c)^2 \sinh(dx+c)^3 + 4*(21*(8a^4b + 19a^3b^2 + 14a^2b^3 + 3ab^4) \cosh(dx+c)^5 + 10*(8a^4b - 5a^3b^2 - 22a^2b^3 - 9ab^4) \cosh(dx+c)^3 - 3*(8a^4b - 5a^3b^2 - 22a^2b^3 - 9ab^4) \cosh(dx+c)) \sinh(dx+c)^2 - ((8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4) \cosh(dx+c)^8 + 8*(8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4) \cosh(dx+c) \sinh(dx+c)^7 + (8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4) \sinh(dx+c)^8 + 4*(8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4) \cosh(dx+c) \sinh(dx+c)^7 + 4*(8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4) \sinh(dx+c)^8) / (8a^{5/2}d)$

$$\begin{aligned}
& + 10a^5b^3 + 5a^4b^4 + a^3b^5) d \cosh(dx + c)^7 + 3(a^8 + 3a^7b + 2a^6b^2 - 2a^5b^3 - 3a^4b^4 - a^3b^5) d \cosh(dx + c)^5 + (3a^8 + 7a^7b + 6a^6b^2 + 6a^5b^3 + 7a^4b^4 + 3a^3b^5) d \cosh(dx + c)^3 \\
& + (a^8 + 3a^7b + 2a^6b^2 - 2a^5b^3 - 3a^4b^4 - a^3b^5) d \cosh(dx + c) \sinh(dx + c), \frac{1}{8}(2(8a^4b + 19a^3b^2 + 14a^2b^3 + 3ab^4) \cosh(dx + c)^7 + 14(8a^4b + 19a^3b^2 + 14a^2b^3 + 3ab^4) \cosh(dx + c) \sinh(dx + c)^6 + 2(8a^4b + 19a^3b^2 + 14a^2b^3 + 3ab^4) \sinh(dx + c)^7 + 2(8a^4b - 5a^3b^2 - 22a^2b^3 - 9ab^4) \cosh(dx + c)^5 + 2(8a^4b - 5a^3b^2 - 22a^2b^3 - 9ab^4 + 21(8a^4b + 19a^3b^2 + 14a^2b^3 + 3ab^4) \cosh(dx + c)^2) \sinh(dx + c)^5 + 10(7(8a^4b + 19a^3b^2 + 14a^2b^3 + 3ab^4) \cosh(dx + c)^3 + (8a^4b - 5a^3b^2 - 22a^2b^3 - 9ab^4) \cosh(dx + c)) \sinh(dx + c)^4 - 2(8a^4b - 5a^3b^2 - 22a^2b^3 - 9ab^4) \cosh(dx + c)^3 - 2(8a^4b - 5a^3b^2 - 22a^2b^3 - 9ab^4 - 35(8a^4b + 19a^3b^2 + 14a^2b^3 + 3ab^4) \cosh(dx + c)^4 - 10(8a^4b - 5a^3b^2 - 22a^2b^3 - 9ab^4) \cosh(dx + c)^2) \sinh(dx + c)^3 + 2(21(8a^4b + 19a^3b^2 + 14a^2b^3 + 3ab^4) \cosh(dx + c)^5 + 10(8a^4b - 5a^3b^2 - 22a^2b^3 - 9ab^4) \cosh(dx + c)^3 - 3(8a^4b - 5a^3b^2 - 22a^2b^3 - 9ab^4) \cosh(dx + c)) \sinh(dx + c)^2 + ((8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4) \cosh(dx + c)^8 + 8(8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4) \cosh(dx + c) \sinh(dx + c)^7 + (8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4) \sinh(dx + c)^8 + 4(8a^4 + 8a^3b - 5a^2b^2 - 8ab^3 - 3b^4) \cosh(dx + c)^6 + 4(8a^4 + 8a^3b - 5a^2b^2 - 8ab^3 - 3b^4 + 7(8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4) \cosh(dx + c)^2) \sinh(dx + c)^6 + 8(7(8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4) \cosh(dx + c)^3 + 3(8a^4 + 8a^3b - 5a^2b^2 - 8ab^3 - 3b^4) \cosh(dx + c)) \sinh(dx + c)^5 + 2(24a^4 + 8a^3b + 17a^2b^2 + 18ab^3 + 9b^4) \cosh(dx + c)^4 + 2(35(8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4) \cosh(dx + c)^4 + 24a^4 + 8a^3b + 17a^2b^2 + 18ab^3 + 9b^4 + 30(8a^4 + 8a^3b - 5a^2b^2 - 8ab^3 - 3b^4) \cosh(dx + c)^2) \sinh(dx + c)^4 + 8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4 + 8(7(8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4) \cosh(dx + c)^5 + 10(8a^4 + 8a^3b - 5a^2b^2 - 8ab^3 - 3b^4) \cosh(dx + c)^3 + (24a^4 + 8a^3b + 17a^2b^2 + 18ab^3 + 9b^4) \cosh(dx + c)) \sinh(dx + c)^3 + 4(8a^4 + 8a^3b - 5a^2b^2 - 8ab^3 - 3b^4) \cosh(dx + c)^2 + 4(7(8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4) \cosh(dx + c)^6 + 15(8a^4 + 8a^3b - 5a^2b^2 - 8ab^3 - 3b^4) \cosh(dx + c)^4 + 8a^4 + 8a^3b - 5a^2b^2 - 8ab^3 - 3b^4 + 3(24a^4 + 8a^3b + 17a^2b^2 + 18ab^3 + 9b^4) \cosh(dx + c)^2) \sinh(dx + c)^2 + 8((8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4) \cosh(dx + c)^7 + 3(8a^4 + 8a^3b - 5a^2b^2 - 8ab^3 - 3b^4) \cosh(dx + c)^5 + (24a^4 + 8a^3b + 17a^2b^2 + 18ab^3 + 9b^4) \cosh(dx + c)^3 + (8a^4 + 8a^3b - 5a^2b^2 - 8ab^3 - 3b^4) \cosh(dx + c)) \sinh(dx + c)) \sqrt{a^2 + ab} \arctan\left(\frac{1}{2}((a + b) \cosh(dx + c)^3 + 3(a + b) \cosh(dx + c) \sinh(dx + c)^2 + (a + b) \sinh(dx + c)^3 + (3a - b) \cosh(dx + c) + (3(a + b) \cosh(dx + c)^2 + 3a - b) \sinh(dx + c)) / \sqrt{a^2 + ab})\right) + ((8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4) \cosh(dx + c)^8 + 8(8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4) \cosh(dx + c) \sinh(dx + c)^7 + (8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4) \sinh(dx + c)^8 + 4(8a^4 + 8a^3b - 5a^2b^2 - 8ab^3 - 3b^4) \cosh(dx + c)^6 + 4(8a^4 + 8a^3b - 5a^2b^2 - 8ab^3 - 3b^4 + 7(8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4) \cosh(dx + c)^2) \sinh(dx + c)^6 + 8(7(8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4) \cosh(dx + c)^3 + 3(8a^4 + 8a^3b - 5a^2b^2 - 8ab^3 - 3b^4) \cosh(dx + c)) \sinh(dx + c)^5 + 2(24a^4 + 8a^3b + 17a^2b^2 + 18ab^3 + 9b^4) \cosh(dx + c)^4 + 2(35(8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4) \cosh(dx + c)^4 + 24a^4 + 8a^3b + 17a^2b^2 + 18ab^3 + 9b^4 + 30(8a^4 + 8a^3b - 5a^2b^2 - 8ab^3 - 3b^4) \cosh(dx + c)^2) \sinh(dx + c)^4 + 8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4 + 8(7(8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4) \cosh(dx + c)^5 + 10(8a^4 + 8a^3b - 5a^2b^2 - 8ab^3 - 3b^4) \cosh(dx + c)
\end{aligned}$$

$$\begin{aligned}
& c)^3 + (24a^4 + 8a^3b + 17a^2b^2 + 18ab^3 + 9b^4) \cosh(dx + c) \sinh(dx + c)^3 + 4(8a^4 + 8a^3b - 5a^2b^2 - 8ab^3 - 3b^4) \cosh(dx + c)^2 + 4(7(8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4) \cosh(dx + c)^6 + 15(8a^4 + 8a^3b - 5a^2b^2 - 8ab^3 - 3b^4) \cosh(dx + c)^4 + 8a^4 + 8a^3b - 5a^2b^2 - 8ab^3 - 3b^4 + 3(24a^4 + 8a^3b + 17a^2b^2 + 18ab^3 + 9b^4) \cosh(dx + c)^2) \sinh(dx + c)^2 + 8((8a^4 + 24a^3b + 27a^2b^2 + 14ab^3 + 3b^4) \cosh(dx + c)^7 + 3(8a^4 + 8a^3b - 5a^2b^2 - 8ab^3 - 3b^4) \cosh(dx + c)^5 + (24a^4 + 8a^3b + 17a^2b^2 + 18ab^3 + 9b^4) \cosh(dx + c)^3 + (8a^4 + 8a^3b - 5a^2b^2 - 8ab^3 - 3b^4) \cosh(dx + c)) \sinh(dx + c) \sqrt{a^2 + ab} \arctan(1/2\sqrt{a^2 + ab}(\cosh(dx + c) + \sinh(dx + c))/a) - 2(8a^4b + 19a^3b^2 + 14a^2b^3 + 3ab^4) \cosh(dx + c) + 2(7(8a^4b + 19a^3b^2 + 14a^2b^3 + 3ab^4) \cosh(dx + c)^6 - 8a^4b - 19a^3b^2 - 14a^2b^3 - 3ab^4 + 5(8a^4b - 5a^3b^2 - 22a^2b^3 - 9ab^4) \cosh(dx + c)^4 - 3(8a^4b - 5a^3b^2 - 22a^2b^3 - 9ab^4) \cosh(dx + c)^2) \sinh(dx + c)) / ((a^8 + 5a^7b + 10a^6b^2 + 10a^5b^3 + 5a^4b^4 + a^3b^5) d \cosh(dx + c)^8 + 8(a^8 + 5a^7b + 10a^6b^2 + 10a^5b^3 + 5a^4b^4 + a^3b^5) d \cosh(dx + c) \sinh(dx + c)^7 + (a^8 + 5a^7b + 10a^6b^2 + 10a^5b^3 + 5a^4b^4 + a^3b^5) d \sinh(dx + c)^8 + 4(a^8 + 3a^7b + 2a^6b^2 - 2a^5b^3 - 3a^4b^4 - a^3b^5) d \cosh(dx + c)^6 + 4(7(a^8 + 5a^7b + 10a^6b^2 + 10a^5b^3 + 5a^4b^4 + a^3b^5) d \cosh(dx + c)^2 + (a^8 + 3a^7b + 2a^6b^2 - 2a^5b^3 - 3a^4b^4 - a^3b^5) d) \sinh(dx + c)^6 + 2(3a^8 + 7a^7b + 6a^6b^2 + 6a^5b^3 + 7a^4b^4 + 3a^3b^5) d \cosh(dx + c)^4 + 8(7(a^8 + 5a^7b + 10a^6b^2 + 10a^5b^3 + 5a^4b^4 + a^3b^5) d \cosh(dx + c)^3 + 3(a^8 + 3a^7b + 2a^6b^2 - 2a^5b^3 - 3a^4b^4 - a^3b^5) d \cosh(dx + c)) \sinh(dx + c)^5 + 2(35(a^8 + 5a^7b + 10a^6b^2 + 10a^5b^3 + 5a^4b^4 + a^3b^5) d \cosh(dx + c)^4 + 30(a^8 + 3a^7b + 2a^6b^2 - 2a^5b^3 - 3a^4b^4 - a^3b^5) d \cosh(dx + c)^2 + (3a^8 + 7a^7b + 6a^6b^2 + 6a^5b^3 + 7a^4b^4 + 3a^3b^5) d) \sinh(dx + c)^4 + 4(a^8 + 3a^7b + 2a^6b^2 - 2a^5b^3 - 3a^4b^4 - a^3b^5) d \cosh(dx + c)^2 + 8(7(a^8 + 5a^7b + 10a^6b^2 + 10a^5b^3 + 5a^4b^4 + a^3b^5) d \cosh(dx + c)^5 + 10(a^8 + 3a^7b + 2a^6b^2 - 2a^5b^3 - 3a^4b^4 - a^3b^5) d \cosh(dx + c)^3 + (3a^8 + 7a^7b + 6a^6b^2 + 6a^5b^3 + 7a^4b^4 + 3a^3b^5) d \cosh(dx + c)) \sinh(dx + c)^3 + 4(7(a^8 + 5a^7b + 10a^6b^2 + 10a^5b^3 + 5a^4b^4 + a^3b^5) d \cosh(dx + c)^6 + 15(a^8 + 3a^7b + 2a^6b^2 - 2a^5b^3 - 3a^4b^4 - a^3b^5) d \cosh(dx + c)^4 + 3(3a^8 + 7a^7b + 6a^6b^2 + 6a^5b^3 + 7a^4b^4 + 3a^3b^5) d \cosh(dx + c)^2 + (a^8 + 3a^7b + 2a^6b^2 - 2a^5b^3 - 3a^4b^4 - a^3b^5) d) \sinh(dx + c)^2 + (a^8 + 5a^7b + 10a^6b^2 + 10a^5b^3 + 5a^4b^4 + a^3b^5) d + 8((a^8 + 5a^7b + 10a^6b^2 + 10a^5b^3 + 5a^4b^4 + a^3b^5) d \cosh(dx + c)^7 + 3(a^8 + 3a^7b + 2a^6b^2 - 2a^5b^3 - 3a^4b^4 - a^3b^5) d \cosh(dx + c)^5 + (3a^8 + 7a^7b + 6a^6b^2 + 6a^5b^3 + 7a^4b^4 + 3a^3b^5) d \cosh(dx + c)^3 + (a^8 + 3a^7b + 2a^6b^2 - 2a^5b^3 - 3a^4b^4 - a^3b^5) d \cosh(dx + c)) \sinh(dx + c)]
\end{aligned}$$

giac [B] time = 0.45, size = 1137, normalized size = 7.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)/(a+b*tanh(dx+c))^2)^3,x, algorithm="giac")

[Out] $1/8 * ((8a^5 - 72a^4b - 37a^3b^2 + 10a^2b^3 + 15ab^4 + (40a^4 - 40a^3b - 57a^2b^2 - 22ab^3 + 3b^4) \sqrt{-ab}) \sqrt{a^2 - b^2 + 2\sqrt{-ab}} (a + b) \operatorname{abs}(a e^{2c} + b e^{2c}) \operatorname{arctan}(e^{dx} / \sqrt{(a^5 e^{2c} + a^4 b e^{2c} - a^3 b^2 e^{2c} - a^2 b^3 e^{2c} + \sqrt{(a^5 e^{2c} + a^4 b e^{2c} - a^3 b^2 e^{2c} - a^2 b^3 e^{2c}))^2 - (a^5 e^{4c} + 3a^4 b e^{4c} + 3a^3 b^2 e^{4c} + a^2 b^3 e^{4c})}) / (a^5 e^{4c} + 3a^4 b e^{4c} + 3a^3 b^2 e^{4c} + a^2 b^3 e^{4c}))$

$$e^{(4*c)})) * e^{(-2*c)} / (a^{10} - 11*a^9*b - 39*a^8*b^2 - 27*a^7*b^3 + 27*a^6*b^4 + 39*a^5*b^5 + 11*a^4*b^6 - a^3*b^7 + 2*(3*a^9 + 2*a^8*b - 19*a^7*b^2 - 36*a^6*b^3 - 19*a^5*b^4 + 2*a^4*b^5 + 3*a^3*b^6) * \sqrt{-a*b}) + (8*a^5 - 72*a^4*b - 37*a^3*b^2 + 10*a^2*b^3 + 15*a*b^4 - (40*a^4 - 40*a^3*b - 57*a^2*b^2 - 22*a*b^3 + 3*b^4) * \sqrt{-a*b}) * \sqrt{a^2 - b^2 - 2*\sqrt{-a*b}*(a + b)} * \text{abs}(a * e^{(2*c)} + b * e^{(2*c)}) * \arctan(e^{(d*x)} / \sqrt{(a^5 * e^{(2*c)} + a^4 * b * e^{(2*c)} - a^3 * b^2 * e^{(2*c)} - a^2 * b^3 * e^{(2*c)} - \sqrt{(a^5 * e^{(2*c)} + a^4 * b * e^{(2*c)} - a^3 * b^2 * e^{(2*c)} - a^2 * b^3 * e^{(2*c)})^2 - (a^5 * e^{(4*c)} + 3*a^4 * b * e^{(4*c)} + 3*a^3 * b^2 * e^{(4*c)} + a^2 * b^3 * e^{(4*c)})} * (a^5 + 3*a^4 * b + 3*a^3 * b^2 + a^2 * b^3)))) / (a^5 * e^{(4*c)} + 3*a^4 * b * e^{(4*c)} + 3*a^3 * b^2 * e^{(4*c)} + a^2 * b^3 * e^{(4*c)})) * e^{(-2*c)} / (a^{10} - 11*a^9*b - 39*a^8*b^2 - 27*a^7*b^3 + 27*a^6*b^4 + 39*a^5*b^5 + 11*a^4*b^6 - a^3*b^7 - 2*(3*a^9 + 2*a^8*b - 19*a^7*b^2 - 36*a^6*b^3 - 19*a^5*b^4 + 2*a^4*b^5 + 3*a^3*b^6) * \sqrt{-a*b}) + 2*(8*a^2*b * e^{(7*d*x + 7*c)} + 11*a*b^2 * e^{(7*d*x + 7*c)} + 3*b^3 * e^{(7*d*x + 7*c)} + 8*a^2*b * e^{(5*d*x + 5*c)} - 13*a*b^2 * e^{(5*d*x + 5*c)} - 9*b^3 * e^{(5*d*x + 5*c)} - 8*a^2*b * e^{(3*d*x + 3*c)} + 13*a*b^2 * e^{(3*d*x + 3*c)} + 9*b^3 * e^{(3*d*x + 3*c)} - 8*a^2*b * e^{(d*x + c)} - 11*a*b^2 * e^{(d*x + c)} - 3*b^3 * e^{(d*x + c)}) / ((a^4 + 2*a^3*b + a^2*b^2) * (a * e^{(4*d*x + 4*c)} + b * e^{(4*d*x + 4*c)} + 2*a * e^{(2*d*x + 2*c)} - 2*b * e^{(2*d*x + 2*c)} + a + b)^2) / d$$

maple [B] time = 0.45, size = 1676, normalized size = 11.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x)

[Out]
$$-2/d / (\tanh(1/2*d*x+1/2*c)^4 * a + 2 * \tanh(1/2*d*x+1/2*c)^2 * a + 4 * \tanh(1/2*d*x+1/2*c)^2 * b + a)^2 * b / (a^2 + 2*a*b + b^2) * \tanh(1/2*d*x+1/2*c)^7 - 5/4/d / (\tanh(1/2*d*x+1/2*c)^4 * a + 2 * \tanh(1/2*d*x+1/2*c)^2 * a + 4 * \tanh(1/2*d*x+1/2*c)^2 * b + a)^2 * b^2 / a / (a^2 + 2*a*b + b^2) * \tanh(1/2*d*x+1/2*c)^7 - 2/d / (\tanh(1/2*d*x+1/2*c)^4 * a + 2 * \tanh(1/2*d*x+1/2*c)^2 * a + 4 * \tanh(1/2*d*x+1/2*c)^2 * b + a)^2 * b / (a^2 + 2*a*b + b^2) * \tanh(1/2*d*x+1/2*c)^5 - 29/4/d / (\tanh(1/2*d*x+1/2*c)^4 * a + 2 * \tanh(1/2*d*x+1/2*c)^2 * a + 4 * \tanh(1/2*d*x+1/2*c)^2 * b + a)^2 * a * b^2 / (a^2 + 2*a*b + b^2) * \tanh(1/2*d*x+1/2*c)^5 - 3/d / (\tanh(1/2*d*x+1/2*c)^4 * a + 2 * \tanh(1/2*d*x+1/2*c)^2 * a + 4 * \tanh(1/2*d*x+1/2*c)^2 * b + a)^2 * a^2 * b^3 / (a^2 + 2*a*b + b^2) * \tanh(1/2*d*x+1/2*c)^5 + 2/d / (\tanh(1/2*d*x+1/2*c)^4 * a + 2 * \tanh(1/2*d*x+1/2*c)^2 * a + 4 * \tanh(1/2*d*x+1/2*c)^2 * b + a)^2 * b / (a^2 + 2*a*b + b^2) * \tanh(1/2*d*x+1/2*c)^3 + 29/4/d / (\tanh(1/2*d*x+1/2*c)^4 * a + 2 * \tanh(1/2*d*x+1/2*c)^2 * a + 4 * \tanh(1/2*d*x+1/2*c)^2 * b + a)^2 * a * b^2 / (a^2 + 2*a*b + b^2) * \tanh(1/2*d*x+1/2*c)^3 + 3/d / (\tanh(1/2*d*x+1/2*c)^4 * a + 2 * \tanh(1/2*d*x+1/2*c)^2 * a + 4 * \tanh(1/2*d*x+1/2*c)^2 * b + a)^2 * a^2 * b^3 / (a^2 + 2*a*b + b^2) * \tanh(1/2*d*x+1/2*c)^3 + 2/d / (\tanh(1/2*d*x+1/2*c)^4 * a + 2 * \tanh(1/2*d*x+1/2*c)^2 * a + 4 * \tanh(1/2*d*x+1/2*c)^2 * b + a)^2 * b / (a^2 + 2*a*b + b^2) * \tanh(1/2*d*x+1/2*c)^5 + 5/4/d / (\tanh(1/2*d*x+1/2*c)^4 * a + 2 * \tanh(1/2*d*x+1/2*c)^2 * a + 4 * \tanh(1/2*d*x+1/2*c)^2 * b + a)^2 * b^2 / a / (a^2 + 2*a*b + b^2) * \tanh(1/2*d*x+1/2*c) - 1/d / (a^2 + 2*a*b + b^2) / ((2*(b*(a+b))^(1/2) - a - 2*b)*a)^(1/2) * \arctanh(a * \tanh(1/2*d*x+1/2*c) / ((2*(b*(a+b))^(1/2) - a - 2*b)*a)^(1/2)) - 1/d / a / (a^2 + 2*a*b + b^2) / ((2*(b*(a+b))^(1/2) - a - 2*b)*a)^(1/2) * \arctanh(a * \tanh(1/2*d*x+1/2*c) / ((2*(b*(a+b))^(1/2) - a - 2*b)*a)^(1/2)) * b - 3/8/d / a^2 / (a^2 + 2*a*b + b^2) / ((2*(b*(a+b))^(1/2) - a - 2*b)*a)^(1/2) * \arctanh(a * \tanh(1/2*d*x+1/2*c) / ((2*(b*(a+b))^(1/2) - a - 2*b)*a)^(1/2)) * b^2 + 1/d / (a^2 + 2*a*b + b^2) / (b*(a+b))^(1/2) / ((2*(b*(a+b))^(1/2) - a - 2*b)*a)^(1/2) * \arctanh(a * \tanh(1/2*d*x+1/2*c) / ((2*(b*(a+b))^(1/2) - a - 2*b)*a)^(1/2)) * b + 1/d / a / (a^2 + 2*a*b + b^2) / (b*(a+b))^(1/2) / ((2*(b*(a+b))^(1/2) - a - 2*b)*a)^(1/2) * \arctanh(a * \tanh(1/2*d*x+1/2*c) / ((2*(b*(a+b))^(1/2) - a - 2*b)*a)^(1/2)) * b^2 + 3/8/d / a^2 / (a^2 + 2*a*b + b^2) / (b*(a+b))^(1/2) / ((2*(b*(a+b))^(1/2) - a - 2*b)*a)^(1/2) * \arctanh(a * \tanh(1/2*d*x+1/2*c) / ((2*(b*(a+b))^(1/2) - a - 2*b)*a)^(1/2)) * b^3 + 1/d / (a^2 + 2*a*b + b^2) / ((2*(b*(a+b))^(1/2) + a + 2*b)*a)^(1/2) * \arctan(a * \tanh(1/2*d*x+1/2*c) / ((2*(b*(a+b))^(1/2) + a + 2*b)*a)^(1/2)) + 1/d / a / (a^2 + 2*a*b + b^2) / ((2*(b*(a+b))^(1/2) + a + 2*b)*a)^(1/2) * \arctan(a * \tanh(1/2*d*x+1/2*c) / ((2*(b*(a+b))^(1/2) + a + 2*b)*a)^(1/2)) * b + 3/8/d / a^2 / (a^2 + 2*a*b + b^2) / ((2*(b*(a+b))^(1/2) + a + 2*b)*a)^(1/2) * \arctan(a * \tanh(1/2*d*x+1/2*c) / ((2*(b*(a+b))^(1/2) + a + 2*b)*a)^(1/2))$$

$$\begin{aligned} & *b)*a)^{(1/2)} *b^2 + 1/d / (a^2 + 2*a*b + b^2) / (b*(a+b))^{(1/2)} / ((2*(b*(a+b))^{(1/2)} + a \\ & + 2*b)*a)^{(1/2)} * \arctan(a*\tanh(1/2*d*x + 1/2*c) / ((2*(b*(a+b))^{(1/2)} + a + 2*b)*a)^{(1/2)} \\ & *b + 1/d / a / (a^2 + 2*a*b + b^2) / (b*(a+b))^{(1/2)} / ((2*(b*(a+b))^{(1/2)} + a + 2*b)*a)^{(1/2)} \\ & * \arctan(a*\tanh(1/2*d*x + 1/2*c) / ((2*(b*(a+b))^{(1/2)} + a + 2*b)*a)^{(1/2)}) *b^2 \\ & + 3/8/d / a^2 / (a^2 + 2*a*b + b^2) / (b*(a+b))^{(1/2)} / ((2*(b*(a+b))^{(1/2)} + a + 2*b)*a)^{(1/2)} \\ & * \arctan(a*\tanh(1/2*d*x + 1/2*c) / ((2*(b*(a+b))^{(1/2)} + a + 2*b)*a)^{(1/2)}) *b^3 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(8a^2be^{7c} + 11ab^2e^{7c} + 3b^3e^{7c})e^{(8c)}}{4(a^6d + 4a^5bd + 6a^4b^2d + 4a^3b^3d + a^2b^4d + (a^6de^{8c} + 4a^5bde^{8c} + 6a^4b^2de^{8c} + 4a^3b^3de^{8c} + a^2b^4de^{8c}))e^{(8c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $\frac{1}{4} * ((8*a^2*b*e^{(7*c)} + 11*a*b^2*e^{(7*c)} + 3*b^3*e^{(7*c)}) * e^{(7*d*x)} + (8*a^2*b*e^{(5*c)} - 13*a*b^2*e^{(5*c)} - 9*b^3*e^{(5*c)}) * e^{(5*d*x)} - (8*a^2*b*e^{(3*c)} - 13*a*b^2*e^{(3*c)} - 9*b^3*e^{(3*c)}) * e^{(3*d*x)} - (8*a^2*b*e^c + 11*a*b^2*e^c + 3*b^3*e^c) * e^{(d*x)}) / (a^6*d + 4*a^5*b*d + 6*a^4*b^2*d + 4*a^3*b^3*d + a^2*b^4*d + (a^6*d*e^{(8*c)} + 4*a^5*b*d*e^{(8*c)} + 6*a^4*b^2*d*e^{(8*c)} + 4*a^3*b^3*d*e^{(8*c)} + a^2*b^4*d*e^{(8*c)}) * e^{(8*d*x)} + 4*(a^6*d*e^{(6*c)} + 2*a^5*b*d*e^{(6*c)} - 2*a^3*b^3*d*e^{(6*c)} - a^2*b^4*d*e^{(6*c)}) * e^{(6*d*x)} + 2*(3*a^6*d*e^{(4*c)} + 4*a^5*b*d*e^{(4*c)} + 2*a^4*b^2*d*e^{(4*c)} + 4*a^3*b^3*d*e^{(4*c)} + 3*a^2*b^4*d*e^{(4*c)}) * e^{(4*d*x)} + 4*(a^6*d*e^{(2*c)} + 2*a^5*b*d*e^{(2*c)} - 2*a^3*b^3*d*e^{(2*c)} - a^2*b^4*d*e^{(2*c)}) * e^{(2*d*x)}) + 2*integrate(1/8*((8*a^2*b*e^{(3*c)} + 8*a*b*e^{(3*c)} + 3*b^2*e^{(3*c)}) * e^{(3*d*x)} + (8*a^2*b*e^c + 8*a*b*e^c + 3*b^2*e^c) * e^{(d*x)}) / (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + (a^5*e^{(4*c)} + 3*a^4*b*e^{(4*c)} + 3*a^3*b^2*e^{(4*c)} + a^2*b^3*e^{(4*c)}) * e^{(4*d*x)} + 2*(a^5*e^{(2*c)} + a^4*b*e^{(2*c)} - a^3*b^2*e^{(2*c)} - a^2*b^3*e^{(2*c)}) * e^{(2*d*x)}), x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + dx) (b \tanh(c + dx)^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^3), x)

[Out] int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^3), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)

[Out] Timed out

$$3.128 \quad \int \frac{\operatorname{sech}^2(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=96

$$\frac{3 \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d} + \frac{3 \tanh(c+dx)}{8a^2d(a+b \tanh^2(c+dx))} + \frac{\tanh(c+dx)}{4ad(a+b \tanh^2(c+dx))^2}$$

[Out] $3/8*\arctan(b^{(1/2)}*\tanh(d*x+c)/a^{(1/2)})/a^{(5/2)}/d/b^{(1/2)}+1/4*\tanh(d*x+c)/a/d/(a+b*\tanh(d*x+c)^2)^2+3/8*\tanh(d*x+c)/a^2/d/(a+b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.08, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3675, 199, 205}

$$\frac{3 \tanh(c+dx)}{8a^2d(a+b \tanh^2(c+dx))} + \frac{3 \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d} + \frac{\tanh(c+dx)}{4ad(a+b \tanh^2(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3, x]

[Out] $(3*\text{ArcTan}[(\text{Sqrt}[b]*\text{Tanh}[c + d*x])/(\text{Sqrt}[a])])/(8*a^{(5/2)}*\text{Sqrt}[b]*d) + \text{Tanh}[c + d*x]/(4*a*d*(a + b*\text{Tanh}[c + d*x]^2)^2) + (3*\text{Tanh}[c + d*x])/(8*a^2*d*(a + b*\text{Tanh}[c + d*x]^2))$

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegerQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^2(c+dx)}{(a+b \tanh^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(a+bx^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\tanh(c+dx)}{4ad(a+b \tanh^2(c+dx))^2} + \frac{3 \operatorname{Subst}\left(\int \frac{1}{(a+bx^2)^2} dx, x, \tanh(c+dx)\right)}{4ad} \\
&= \frac{\tanh(c+dx)}{4ad(a+b \tanh^2(c+dx))^2} + \frac{3 \tanh(c+dx)}{8a^2d(a+b \tanh^2(c+dx))} + \frac{3 \operatorname{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \tanh(c+dx)\right)}{8a^2d} \\
&= \frac{3 \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d} + \frac{\tanh(c+dx)}{4ad(a+b \tanh^2(c+dx))^2} + \frac{3 \tanh(c+dx)}{8a^2d(a+b \tanh^2(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.83, size = 77, normalized size = 0.80

$$\frac{\frac{3 \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}} + \frac{\tanh(c+dx)(5a+3b \tanh^2(c+dx))}{a^2(a+b \tanh^2(c+dx))^2}}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] ((3*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[b])) + (Tanh[c + d*x]*(5*a + 3*b*Tanh[c + d*x]^2))/(a^2*(a + b*Tanh[c + d*x]^2)^2)/(8*d)

fricas [B] time = 0.54, size = 5840, normalized size = 60.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [-1/16*(4*(5*a^4*b - a^3*b^2 - 9*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^6 + 24*(5*a^4*b - a^3*b^2 - 9*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(5*a^4*b - a^3*b^2 - 9*a^2*b^3 - 3*a*b^4)*sinh(d*x + c)^6 + 20*a^4*b + 52*a^3*b^2 + 44*a^2*b^3 + 12*a*b^4 + 4*(15*a^4*b - a^3*b^2 + 9*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^4 + 4*(15*a^4*b - a^3*b^2 + 9*a^2*b^3 + 9*a*b^4 + 15*(5*a^4*b - a^3*b^2 - 9*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(5*a^4*b - a^3*b^2 - 9*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^3 + (15*a^4*b - a^3*b^2 + 9*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^4*b + 13*a^3*b^2 - 11*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^2 + 4*(15*a^4*b + 13*a^3*b^2 - 11*a^2*b^3 - 9*a*b^4 + 15*(5*a^4*b - a^3*b^2 - 9*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^4 + 6*(15*a^4*b - a^3*b^2 + 9*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^8 + 8*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sinh(d*x + c)^8 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(a^4 + 2*a^3*b - 2*a*b^3 - b^4 + 7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*cosh(d*x + c)^3 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 2*

$$\begin{aligned}
& (35*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\cosh(d*x + c)^4 + 3*a^4 + 4 \\
& *a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4 + 30*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\c \\
& \cosh(d*x + c)^2*\sinh(d*x + c)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 \\
& + 8*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*\cosh(d*x + c)^5 + 10*(a \\
& ^4 + 2*a^3*b - 2*a*b^3 - b^4)*\cosh(d*x + c)^3 + (3*a^4 + 4*a^3*b + 2*a^2*b^ \\
& 2 + 4*a*b^3 + 3*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(a^4 + 2*a^3*b - 2* \\
& a*b^3 - b^4)*\cosh(d*x + c)^2 + 4*(7*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + \\
& b^4)*\cosh(d*x + c)^6 + 15*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\cosh(d*x + c)^4 + \\
& a^4 + 2*a^3*b - 2*a*b^3 - b^4 + 3*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + \\
& 3*b^4)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 8*((a^4 + 4*a^3*b + 6*a^2*b^2 + \\
& 4*a*b^3 + b^4)*\cosh(d*x + c)^7 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\cosh(d*x \\
& + c)^5 + (3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*\cosh(d*x + c)^3 + \\
& (a^4 + 2*a^3*b - 2*a*b^3 - b^4)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-a*b}*l \\
& \text{og}(((a^2 + 2*a*b + b^2)*\cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*\cosh(d*x + \\
& c)*\sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*\sinh(d*x + c)^4 + 2*(a^2 - b^2)*\c \\
& \cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)^2 + a^2 - b^2)*\sinh(\\
& d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*\cosh(d*x + c)^3 + (\\
& a^2 - b^2)*\cosh(d*x + c))*\sinh(d*x + c) - 4*((a + b)*\cosh(d*x + c)^2 + 2*(a \\
& + b)*\cosh(d*x + c)*\sinh(d*x + c) + (a + b)*\sinh(d*x + c)^2 + a - b)*\sqrt{- \\
& a*b})/((a + b)*\cosh(d*x + c)^4 + 4*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + \\
& (a + b)*\sinh(d*x + c)^4 + 2*(a - b)*\cosh(d*x + c)^2 + 2*(3*(a + b)*\cosh(d*x \\
& + c)^2 + a - b)*\sinh(d*x + c)^2 + 4*((a + b)*\cosh(d*x + c)^3 + (a - b)*\c \\
& \cosh(d*x + c))*\sinh(d*x + c) + a + b)) + 8*(3*(5*a^4*b - a^3*b^2 - 9*a^2*b^3 - \\
& 3*a*b^4)*\cosh(d*x + c)^5 + 2*(15*a^4*b - a^3*b^2 + 9*a^2*b^3 + 9*a*b^4)*\c \\
& \cosh(d*x + c)^3 + (15*a^4*b + 13*a^3*b^2 - 11*a^2*b^3 - 9*a*b^4)*\cosh(d*x + c \\
&))*\sinh(d*x + c))/((a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5)*d \\
& *\cosh(d*x + c)^8 + 8*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5)*d \\
& *\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 \\
& + a^3*b^5)*d*\sinh(d*x + c)^8 + 4*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5) \\
& *d*\cosh(d*x + c)^6 + 4*(7*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3* \\
& b^5)*d*\cosh(d*x + c)^2 + (a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d)*\sinh(\\
& d*x + c)^6 + 2*(3*a^7*b + 4*a^6*b^2 + 2*a^5*b^3 + 4*a^4*b^4 + 3*a^3*b^5)*d* \\
& \cosh(d*x + c)^4 + 8*(7*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5 \\
&)*d*\cosh(d*x + c)^3 + 3*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d*\cosh(d* \\
& x + c))*\sinh(d*x + c)^5 + 2*(35*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 \\
& + a^3*b^5)*d*\cosh(d*x + c)^4 + 30*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5) \\
& *d*\cosh(d*x + c)^2 + (3*a^7*b + 4*a^6*b^2 + 2*a^5*b^3 + 4*a^4*b^4 + 3*a^3*b \\
& ^5)*d)*\sinh(d*x + c)^4 + 4*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d*\cosh \\
& (d*x + c)^2 + 8*(7*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5)*d* \\
& \cosh(d*x + c)^5 + 10*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d*\cosh(d*x + \\
& c)^3 + (3*a^7*b + 4*a^6*b^2 + 2*a^5*b^3 + 4*a^4*b^4 + 3*a^3*b^5)*d*\cosh(d* \\
& x + c))*\sinh(d*x + c)^3 + 4*(7*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + \\
& a^3*b^5)*d*\cosh(d*x + c)^6 + 15*(a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)* \\
& d*\cosh(d*x + c)^4 + 3*(3*a^7*b + 4*a^6*b^2 + 2*a^5*b^3 + 4*a^4*b^4 + 3*a^3* \\
& b^5)*d*\cosh(d*x + c)^2 + (a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d)*\sinh(\\
& d*x + c)^2 + (a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5)*d + 8*((\\
& a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c)^7 + 3* \\
& (a^7*b + 2*a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c)^5 + (3*a^7*b + 4* \\
& a^6*b^2 + 2*a^5*b^3 + 4*a^4*b^4 + 3*a^3*b^5)*d*\cosh(d*x + c)^3 + (a^7*b + 2 \\
& *a^6*b^2 - 2*a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)), -1/8*(2*(5 \\
& *a^4*b - a^3*b^2 - 9*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c)^6 + 12*(5*a^4*b - a^3 \\
& *b^2 - 9*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^5 + 2*(5*a^4*b - a^ \\
& 3*b^2 - 9*a^2*b^3 - 3*a*b^4)*\sinh(d*x + c)^6 + 10*a^4*b + 26*a^3*b^2 + 22*a \\
& ^2*b^3 + 6*a*b^4 + 2*(15*a^4*b - a^3*b^2 + 9*a^2*b^3 + 9*a*b^4)*\cosh(d*x + \\
& c)^4 + 2*(15*a^4*b - a^3*b^2 + 9*a^2*b^3 + 9*a*b^4 + 15*(5*a^4*b - a^3*b^2 \\
& - 9*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(5*(5*a^4*b - a \\
& ^3*b^2 - 9*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c)^3 + (15*a^4*b - a^3*b^2 + 9*a^2 \\
& *b^3 + 9*a*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 2*(15*a^4*b + 13*a^3*b^2 - \\
& 11*a^2*b^3 - 9*a*b^4)*\cosh(d*x + c)^2 + 2*(15*a^4*b + 13*a^3*b^2 - 11*a^2*
\end{aligned}$$

$$\begin{aligned}
& b^3 - 9ab^4 + 15(5a^4b - a^3b^2 - 9a^2b^3 - 3ab^4) \cosh(dx + c)^4 + 6(15a^4b - a^3b^2 + 9a^2b^3 + 9ab^4) \cosh(dx + c)^2 \sinh(dx + c)^2 - 3((a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4) \cosh(dx + c)^8 + 8(a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4) \cosh(dx + c) \sinh(dx + c)^7 + (a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4) \sinh(dx + c)^8 + 4(a^4 + 2a^3b - 2ab^3 - b^4) \cosh(dx + c)^6 + 4(a^4 + 2a^3b - 2ab^3 - b^4 + 7(a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4) \cosh(dx + c)^2) \sinh(dx + c)^6 + 8(7(a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4) \cosh(dx + c)^3 + 3(a^4 + 2a^3b - 2ab^3 - b^4) \cosh(dx + c)) \sinh(dx + c)^5 + 2(3a^4 + 4a^3b + 2a^2b^2 + 4ab^3 + 3b^4) \cosh(dx + c)^4 + 2(35(a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4) \cosh(dx + c)^4 + 3a^4 + 4a^3b + 2a^2b^2 + 4ab^3 + 3b^4 + 30(a^4 + 2a^3b - 2ab^3 - b^4) \cosh(dx + c)^2) \sinh(dx + c)^4 + a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4 + 8(7(a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4) \cosh(dx + c)^5 + 10(a^4 + 2a^3b - 2ab^3 - b^4) \cosh(dx + c)^3 + (3a^4 + 4a^3b + 2a^2b^2 + 4ab^3 + 3b^4) \cosh(dx + c)) \sinh(dx + c)^3 + 4(a^4 + 2a^3b - 2ab^3 - b^4) \cosh(dx + c)^2 + 4(7(a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4) \cosh(dx + c)^6 + 15(a^4 + 2a^3b - 2ab^3 - b^4) \cosh(dx + c)^4 + a^4 + 2a^3b - 2ab^3 - b^4 + 3(3a^4 + 4a^3b + 2a^2b^2 + 4ab^3 + 3b^4) \cosh(dx + c)^2) \sinh(dx + c)^2 + 8((a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4) \cosh(dx + c)^7 + 3(a^4 + 2a^3b - 2ab^3 - b^4) \cosh(dx + c)^5 + (3a^4 + 4a^3b + 2a^2b^2 + 4ab^3 + 3b^4) \cosh(dx + c)^3 + (a^4 + 2a^3b - 2ab^3 - b^4) \cosh(dx + c)) \sinh(dx + c) \sqrt{ab} \arctan(1/2((a + b) \cosh(dx + c)^2 + 2(a + b) \cosh(dx + c) \sinh(dx + c) + (a + b) \sinh(dx + c)^2 + a - b) \sqrt{ab}) / (ab)) + 4(3(5a^4b - a^3b^2 - 9a^2b^3 - 3ab^4) \cosh(dx + c)^5 + 2(15a^4b - a^3b^2 + 9a^2b^3 + 9ab^4) \cosh(dx + c)^3 + (15a^4b + 13a^3b^2 - 11a^2b^3 - 9ab^4) \cosh(dx + c)) \sinh(dx + c) / ((a^7b + 4a^6b^2 + 6a^5b^3 + 4a^4b^4 + a^3b^5) d \cosh(dx + c)^8 + 8(a^7b + 4a^6b^2 + 6a^5b^3 + 4a^4b^4 + a^3b^5) d \cosh(dx + c) \sinh(dx + c)^7 + (a^7b + 4a^6b^2 + 6a^5b^3 + 4a^4b^4 + a^3b^5) d \sinh(dx + c)^8 + 4(a^7b + 2a^6b^2 - 2a^4b^4 - a^3b^5) d \cosh(dx + c)^6 + 4(7(a^7b + 4a^6b^2 + 6a^5b^3 + 4a^4b^4 + a^3b^5) d \cosh(dx + c)^2 + (a^7b + 2a^6b^2 - 2a^4b^4 - a^3b^5) d) \sinh(dx + c)^6 + 2(3a^7b + 4a^6b^2 + 2a^5b^3 + 4a^4b^4 + 3a^3b^5) d \cosh(dx + c)^4 + 8(7(a^7b + 4a^6b^2 + 6a^5b^3 + 4a^4b^4 + a^3b^5) d \cosh(dx + c)^3 + 3(a^7b + 2a^6b^2 - 2a^4b^4 - a^3b^5) d \cosh(dx + c)) \sinh(dx + c)^5 + 2(35(a^7b + 4a^6b^2 + 6a^5b^3 + 4a^4b^4 + a^3b^5) d \cosh(dx + c)^4 + 30(a^7b + 2a^6b^2 - 2a^4b^4 - a^3b^5) d \cosh(dx + c)^2 + (3a^7b + 4a^6b^2 + 2a^5b^3 + 4a^4b^4 + 3a^3b^5) d) \sinh(dx + c)^4 + 4(a^7b + 2a^6b^2 - 2a^4b^4 - a^3b^5) d \cosh(dx + c)^2 + 8(7(a^7b + 4a^6b^2 + 6a^5b^3 + 4a^4b^4 + a^3b^5) d \cosh(dx + c)^5 + 10(a^7b + 2a^6b^2 - 2a^4b^4 - a^3b^5) d \cosh(dx + c)^3 + (3a^7b + 4a^6b^2 + 2a^5b^3 + 4a^4b^4 + 3a^3b^5) d \cosh(dx + c)) \sinh(dx + c)^3 + 4(7(a^7b + 4a^6b^2 + 6a^5b^3 + 4a^4b^4 + a^3b^5) d \cosh(dx + c)^6 + 15(a^7b + 2a^6b^2 - 2a^4b^4 - a^3b^5) d \cosh(dx + c)^4 + 3(3a^7b + 4a^6b^2 + 2a^5b^3 + 4a^4b^4 + 3a^3b^5) d \cosh(dx + c)^2 + (a^7b + 2a^6b^2 - 2a^4b^4 - a^3b^5) d) \sinh(dx + c)^2 + (a^7b + 4a^6b^2 + 6a^5b^3 + 4a^4b^4 + a^3b^5) d + 8((a^7b + 4a^6b^2 + 6a^5b^3 + 4a^4b^4 + a^3b^5) d \cosh(dx + c)^7 + 3(a^7b + 2a^6b^2 - 2a^4b^4 - a^3b^5) d \cosh(dx + c)^5 + (3a^7b + 4a^6b^2 + 2a^5b^3 + 4a^4b^4 + 3a^3b^5) d \cosh(dx + c)^3 + (a^7b + 2a^6b^2 - 2a^4b^4 - a^3b^5) d \cosh(dx + c)) \sinh(dx + c)]
\end{aligned}$$

giac [B] time = 0.75, size = 320, normalized size = 3.33

$$\frac{3 \arctan\left(\frac{ae^{2dx+2c} + be^{2dx+2c} + a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}a^2} - \frac{2\left(5a^3e^{6dx+6c} - a^2be^{6dx+6c} - 9ab^2e^{6dx+6c} - 3b^3e^{6dx+6c} + 15a^3e^{4dx+4c} - a^2be^{4dx+4c} + 9ab^2e^{4dx+4c} + 9\right)}{(a^4+2a^3b+a^2b^2)(ae^{4dx+4c}+be^{4dx+4c})}$$

8d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c))^3,x, algorithm="giac")

[Out] $\frac{1}{8} \cdot \frac{(3 \arctan(\frac{1}{2} \cdot (a \cdot e^{2dx+2c}) + b \cdot e^{2dx+2c}) + a - b) \sqrt{a \cdot b}}{(\sqrt{a \cdot b} \cdot a^2 - 2 \cdot (5a^3 e^{6dx+6c} - a^2 b e^{6dx+6c} - 9a \cdot b^2 e^{6dx+6c} - 3b^3 e^{6dx+6c}) + 15a^3 e^{4dx+4c} - a^2 b e^{4dx+4c} + 9a \cdot b^2 e^{4dx+4c} + 9b^3 e^{4dx+4c}) + 15a^3 e^{2dx+2c} + 13a^2 b e^{2dx+2c} - 11a \cdot b^2 e^{2dx+2c} - 9b^3 e^{2dx+2c} + 5a^3 + 13a^2 b + 11a \cdot b^2 + 3b^3) / ((a^4 + 2a^3 b + a^2 b^2) \cdot (a \cdot e^{4dx+4c} + b \cdot e^{4dx+4c} + 2a \cdot e^{2dx+2c} - 2b \cdot e^{2dx+2c} + a + b)^2)} / d$

maple [B] time = 0.48, size = 764, normalized size = 7.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^2/(a+b*tanh(d*x+c))^3,x)

[Out] $\frac{5}{4} \cdot \frac{1}{d} \cdot \frac{(\tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^{4a+2} \cdot \tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^{2a+4} \cdot \tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^{2b+a})^2 / a \cdot \tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^{7+15/4} / d \cdot (\tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^{4a+2} \cdot \tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^{2b+a})^2 / a \cdot \tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^{5+3/d} / a^2 \cdot b / (\tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^{4a+2} \cdot \tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^{2b+a})^2 \cdot \tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^{5+15/4} / d \cdot (\tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^{4a+2} \cdot \tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^{2b+a})^2 \cdot \tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^{3+3/d} / a^2 \cdot b / (\tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^{4a+2} \cdot \tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^{2b+a})^2 \cdot \tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^3 + 5/4 / d \cdot (\tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^{4a+2} \cdot \tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^{2b+a})^2 / a \cdot \tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)^{-3/8} / d \cdot (b \cdot (a+b))^{1/2} / a \cdot ((2 \cdot (b \cdot (a+b))^{1/2} - a - 2b) \cdot a)^{1/2} \cdot \arctanh(a \cdot \tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)) / ((2 \cdot (b \cdot (a+b))^{1/2} - a - 2b) \cdot a)^{1/2} + 3/8 / d \cdot a^2 / ((2 \cdot (b \cdot (a+b))^{1/2} - a - 2b) \cdot a)^{1/2} \cdot \arctanh(a \cdot \tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)) / ((2 \cdot (b \cdot (a+b))^{1/2} - a - 2b) \cdot a)^{1/2} - 3/8 / d \cdot a^2 \cdot b / (b \cdot (a+b))^{1/2} / ((2 \cdot (b \cdot (a+b))^{1/2} - a - 2b) \cdot a)^{1/2} \cdot \arctanh(a \cdot \tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)) / ((2 \cdot (b \cdot (a+b))^{1/2} - a - 2b) \cdot a)^{1/2} - 3/8 / d \cdot (b \cdot (a+b))^{1/2} / a \cdot ((2 \cdot (b \cdot (a+b))^{1/2} + a + 2b) \cdot a)^{1/2} \cdot \arctan(a \cdot \tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)) / ((2 \cdot (b \cdot (a+b))^{1/2} + a + 2b) \cdot a)^{1/2} - 3/8 / d \cdot a^2 / ((2 \cdot (b \cdot (a+b))^{1/2} + a + 2b) \cdot a)^{1/2} \cdot \arctan(a \cdot \tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)) / ((2 \cdot (b \cdot (a+b))^{1/2} + a + 2b) \cdot a)^{1/2} \cdot \arctan(a \cdot \tanh(\frac{1}{2} \cdot d \cdot x + \frac{1}{2} \cdot c)) / ((2 \cdot (b \cdot (a+b))^{1/2} + a + 2b) \cdot a)^{1/2}}$

maxima [B] time = 0.63, size = 366, normalized size = 3.81

$$\frac{5a^3 + 13a^2b + 11ab^2 + 3b^3 + (15a^3 + 13a^2b - 11ab^2 - 9b^3)e^{(-2dx-2c)}}{4(a^6 + 4a^5b + 6a^4b^2 + 4a^3b^3 + a^2b^4 + 4(a^6 + 2a^5b - 2a^3b^3 - a^2b^4)e^{(-2dx-2c)} + 2(3a^6 + 4a^5b + 2a^4b^2 + 4a^3b^3 + a^2b^4)e^{(-4dx-4c)} + (5a^3 - a^2b - 9a \cdot b^2 - 3b^3)e^{(-6dx-6c)}) / ((a^6 + 4a^5b + 6a^4b^2 + 4a^3b^3 + a^2b^4 + 4(a^6 + 2a^5b - 2a^3b^3 - a^2b^4)e^{(-2dx-2c)} + 2(3a^6 + 4a^5b + 2a^4b^2 + 4a^3b^3 + 3a^2b^4)e^{(-4dx-4c)} + 4(a^6 + 2a^5b - 2a^3b^3 - a^2b^4)e^{(-6dx-6c)} + (a^6 + 4a^5b + 6a^4b^2 + 4a^3b^3 + a^2b^4)e^{(-8dx-8c)})) \cdot d - 3/8 \cdot \arctan(1/2 \cdot ((a+b) \cdot e^{(-2dx-2c)} + a - b) / \sqrt{a \cdot b}) / (\sqrt{a \cdot b} \cdot a^2 \cdot d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c))^3,x, algorithm="maxima")

[Out] $\frac{1}{4} \cdot \frac{(5a^3 + 13a^2b + 11a \cdot b^2 + 3b^3 + (15a^3 + 13a^2b - 11a \cdot b^2 - 9b^3) \cdot e^{(-2dx-2c)} + (15a^3 - a^2b + 9a \cdot b^2 + 9b^3) \cdot e^{(-4dx-4c)} + (5a^3 - a^2b - 9a \cdot b^2 - 3b^3) \cdot e^{(-6dx-6c)}) / ((a^6 + 4a^5b + 6a^4b^2 + 4a^3b^3 + a^2b^4 + 4(a^6 + 2a^5b - 2a^3b^3 - a^2b^4) \cdot e^{(-2dx-2c)} + 2(3a^6 + 4a^5b + 2a^4b^2 + 4a^3b^3 + 3a^2b^4) \cdot e^{(-4dx-4c)} + 4(a^6 + 2a^5b - 2a^3b^3 - a^2b^4) \cdot e^{(-6dx-6c)} + (a^6 + 4a^5b + 6a^4b^2 + 4a^3b^3 + a^2b^4) \cdot e^{(-8dx-8c)})) \cdot d - 3/8 \cdot \arctan(1/2 \cdot ((a+b) \cdot e^{(-2dx-2c)} + a - b) / \sqrt{a \cdot b}) / (\sqrt{a \cdot b} \cdot a^2 \cdot d)}$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c+dx)^2 (b \tanh(c+dx)^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3), x)
```

```
[Out] int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3, x)
```

```
[Out] Timed out
```

$$3.129 \quad \int \frac{\operatorname{sech}^3(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=129

$$\frac{(4a+3b) \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^{3/2}} + \frac{(4a+3b) \sinh(c+dx)}{8a^2d(a+b)((a+b) \sinh^2(c+dx)+a)} + \frac{b \sinh(c+dx)}{4ad(a+b)((a+b) \sinh^2(c+dx)+a)}$$

[Out] 1/8*(4*a+3*b)*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(5/2)/(a+b)^(3/2)/d + 1/4*b*sinh(d*x+c)/a/(a+b)/d/(a+(a+b)*sinh(d*x+c)^2)^2+1/8*(4*a+3*b)*sinh(d*x+c)/a^2/(a+b)/d/(a+(a+b)*sinh(d*x+c)^2)

Rubi [A] time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, number of rules / integrand size = 0.174, Rules used = {3676, 385, 199, 205}

$$\frac{(4a+3b) \sinh(c+dx)}{8a^2d(a+b)((a+b) \sinh^2(c+dx)+a)} + \frac{(4a+3b) \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^{3/2}} + \frac{b \sinh(c+dx)}{4ad(a+b)((a+b) \sinh^2(c+dx)+a)}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3, x]

[Out] ((4*a + 3*b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(8*a^(5/2)*(a + b)^(3/2)*d) + (b*Sinh[c + d*x])/(4*a*(a + b)*d*(a + (a + b)*Sinh[c + d*x]^2)^2) + ((4*a + 3*b)*Sinh[c + d*x])/(8*a^2*(a + b)*d*(a + (a + b)*Sinh[c + d*x]^2))

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 3676

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2)], x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

$$\begin{aligned}
& a^2b + 10ab^2 + 3b^3) \sinh(dx + c)^8 + 4(4a^3 + 3a^2b - 4ab^2 - \\
& 3b^3) \cosh(dx + c)^6 + 4(4a^3 + 3a^2b - 4ab^2 - 3b^3 + 7(4a^3 + \\
& 11a^2b + 10ab^2 + 3b^3) \cosh(dx + c)^2) \sinh(dx + c)^6 + 8(7(4a^3 + \\
& 11a^2b + 10ab^2 + 3b^3) \cosh(dx + c)^3 + 3(4a^3 + 3a^2b - 4ab^2 - \\
& b^2 - 3b^3) \cosh(dx + c)) \sinh(dx + c)^5 + 2(12a^3 + a^2b + 6ab^2 + \\
& 9b^3) \cosh(dx + c)^4 + 2(35(4a^3 + 11a^2b + 10ab^2 + 3b^3) \cosh(dx + c)^4 + \\
& 12a^3 + a^2b + 6ab^2 + 9b^3 + 30(4a^3 + 3a^2b - 4ab^2 - 3b^3) \cosh(dx + c)^2) \sinh(dx + c)^4 + \\
& 8(7(4a^3 + 11a^2b + 10ab^2 + 3b^3) \cosh(dx + c)^5 + 10(4a^3 + 3a^2b - 4ab^2 - 3b^3) \cosh(dx + c)^3 + \\
& (12a^3 + a^2b + 6ab^2 + 9b^3) \cosh(dx + c)) \sinh(dx + c)^3 + 4a^3 + 11a^2b + 10ab^2 + 3b^3 + \\
& 4(4a^3 + 3a^2b - 4ab^2 - 3b^3) \cosh(dx + c)^2 + 4(7(4a^3 + 11a^2b + 10ab^2 + 3b^3) \cosh(dx + c)^6 + \\
& 15(4a^3 + 3a^2b - 4ab^2 - 3b^3) \cosh(dx + c)^4 + 4a^3 + 3a^2b - 4ab^2 - 3b^3 + 3(12a^3 + a^2b + \\
& 6ab^2 + 9b^3) \cosh(dx + c)^2) \sinh(dx + c)^2 + 8((4a^3 + 11a^2b + 10ab^2 + 3b^3) \cosh(dx + c)^7 + \\
& 3(4a^3 + 3a^2b - 4ab^2 - 3b^3) \cosh(dx + c)^5 + (12a^3 + a^2b + 6ab^2 + 9b^3) \cosh(dx + c)^3 + \\
& (4a^3 + 3a^2b - 4ab^2 - 3b^3) \cosh(dx + c)) \sinh(dx + c) \sqrt{-a^2 - ab} \log((a + b) \cosh(dx + c)^4 + \\
& 4(a + b) \cosh(dx + c) \sinh(dx + c)^3 + (a + b) \sinh(dx + c)^4 - 2(3a + b) \cosh(dx + c)^2 + \\
& 2(3(a + b) \cosh(dx + c)^2 - 3a - b) \sinh(dx + c)^2 + 4((a + b) \cosh(dx + c)^3 - (3a + b) \cosh(dx + c)) \sinh(dx + c) - \\
& 4(\cosh(dx + c)^3 + 3 \cosh(dx + c) \sinh(dx + c)^2 + \sinh(dx + c)^3 + (3 \cosh(dx + c)^2 - 1) \sinh(dx + c) - \cosh(dx + c)) \sqrt{-a^2 - ab} + \\
& a + b) / ((a + b) \cosh(dx + c)^4 + 4(a + b) \cosh(dx + c) \sinh(dx + c)^3 + (a + b) \sinh(dx + c)^4 + 2(a - b) \cosh(dx + c)^2 + \\
& 2(3(a + b) \cosh(dx + c)^2 + a - b) \sinh(dx + c)^2 + 4((a + b) \cosh(dx + c)^3 + (a - b) \cosh(dx + c)) \sinh(dx + c) + a + b) - \\
& 4(4a^4 + 11a^3b + 10a^2b^2 + 3ab^3) \cosh(dx + c) + 4(7(4a^4 + 11a^3b + 10a^2b^2 + 3ab^3) \cosh(dx + c)^6 + 5(4a^4 + 3a^3b - 10a^2b^2 - 9ab^3) \cosh(dx + c)^4 - 4a^4 - 11a^3b - 10a^2b^2 - 3ab^3 - 3(4a^4 + 3a^3b - 10a^2b^2 - 9ab^3) \cosh(dx + c)^2) \sinh(dx + c) / ((a^7 + 4a^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4) d \cosh(dx + c)^8 + 8(a^7 + 4a^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4) d \cosh(dx + c) \sinh(dx + c)^7 + (a^7 + 4a^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4) d \sinh(dx + c)^8 + 4(a^7 + 2a^6b - 2a^4b^3 - a^3b^4) d \cosh(dx + c)^6 + 4(7(a^7 + 4a^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4) d \cosh(dx + c)^2 + (a^7 + 2a^6b - 2a^4b^3 - a^3b^4) d) \sinh(dx + c)^6 + 2(3a^7 + 4a^6b + 2a^5b^2 + 4a^4b^3 + 3a^3b^4) d \cosh(dx + c)^4 + 8(7(a^7 + 4a^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4) d \cosh(dx + c)^3 + 3(a^7 + 2a^6b - 2a^4b^3 - a^3b^4) d \cosh(dx + c)) \sinh(dx + c)^5 + 2(35(a^7 + 4a^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4) d \cosh(dx + c)^4 + 30(a^7 + 2a^6b - 2a^4b^3 - a^3b^4) d \cosh(dx + c)^2 + (3a^7 + 4a^6b + 2a^5b^2 + 4a^4b^3 + 3a^3b^4) d) \sinh(dx + c)^4 + 4(a^7 + 2a^6b - 2a^4b^3 - a^3b^4) d \cosh(dx + c)^2 + 8(7(a^7 + 4a^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4) d \cosh(dx + c)^5 + 10(a^7 + 2a^6b - 2a^4b^3 - a^3b^4) d \cosh(dx + c)^3 + (3a^7 + 4a^6b + 2a^5b^2 + 4a^4b^3 + 3a^3b^4) d \cosh(dx + c)) \sinh(dx + c)^3 + 4(7(a^7 + 4a^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4) d \cosh(dx + c)^6 + 15(a^7 + 2a^6b - 2a^4b^3 - a^3b^4) d \cosh(dx + c)^4 + 3(3a^7 + 4a^6b + 2a^5b^2 + 4a^4b^3 + 3a^3b^4) d \cosh(dx + c)^2 + (a^7 + 2a^6b - 2a^4b^3 - a^3b^4) d) \sinh(dx + c)^2 + (a^7 + 4a^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4) d + 8((a^7 + 4a^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4) d \cosh(dx + c)^7 + 3(a^7 + 2a^6b - 2a^4b^3 - a^3b^4) d \cosh(dx + c)^5 + (3a^7 + 4a^6b + 2a^5b^2 + 4a^4b^3 + 3a^3b^4) d \cosh(dx + c)^3 + (a^7 + 2a^6b - 2a^4b^3 - a^3b^4) d \cosh(dx + c)) \sinh(dx + c)), 1/8 * (2(4a^4 + 11a^3b + 10a^2b^2 + 3ab^3) \cosh(dx + c)^7 + 14(4a^4 + 11a^3b + 10a^2b^2 + 3ab^3) \cosh(dx + c) \sinh(dx + c)^6 + 2(4a^4 + 11a^3b + 10a^2b^2 + 3ab^3) \sinh(dx + c)^7 + 2(4a^4 + 3a^3b - 10a^2b^2 - 9ab^3) \cosh(dx + c)^5 + 2(4a^4 + 3a^3b - 10a^2b^2 - 9ab^3 + 21(4a^4 + 11a^3b + 10a^2b^2 + 3ab^3) \cosh(dx + c)^2) \sinh(dx + c)^4 + 4(4a^4 + 11a^3b + 10a^2b^2 + 3ab^3) \cosh(dx + c)^3 + 4(4a^4 + 11a^3b + 10a^2b^2 + 3ab^3) \sinh(dx + c)^4 + 2(4a^4 + 11a^3b + 10a^2b^2 + 3ab^3) \cosh(dx + c)^2 + 2(4a^4 + 11a^3b + 10a^2b^2 + 3ab^3) \sinh(dx + c)^3 + 2(4a^4 + 11a^3b + 10a^2b^2 + 3ab^3) \cosh(dx + c) + 2(4a^4 + 11a^3b + 10a^2b^2 + 3ab^3) \sinh(dx + c) + 4a^4 + 11a^3b + 10a^2b^2 + 3ab^3) \sinh(dx + c)
\end{aligned}$$

$$\begin{aligned}
& d*x + c)^5 + 10*(7*(4*a^4 + 11*a^3*b + 10*a^2*b^2 + 3*a*b^3)*\cosh(d*x + c)^3 \\
& + (4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^4 \\
& - 2*(4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^3)*\cosh(d*x + c)^3 + 2*(35*(4*a^4 \\
& + 11*a^3*b + 10*a^2*b^2 + 3*a*b^3)*\cosh(d*x + c)^4 - 4*a^4 - 3*a^3*b + 10 \\
& *a^2*b^2 + 9*a*b^3 + 10*(4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^3)*\cosh(d*x + \\
& c)^2)*\sinh(d*x + c)^3 + 2*(21*(4*a^4 + 11*a^3*b + 10*a^2*b^2 + 3*a*b^3)*\co \\
& sh(d*x + c)^5 + 10*(4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^3)*\cosh(d*x + c)^3 \\
& - 3*(4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 \\
& + ((4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*\cosh(d*x + c)^8 + 8*(4*a^3 + 11* \\
& a^2*b + 10*a*b^2 + 3*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (4*a^3 + 11*a^2*b \\
& + 10*a*b^2 + 3*b^3)*\sinh(d*x + c)^8 + 4*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3 \\
&)*\cosh(d*x + c)^6 + 4*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3 + 7*(4*a^3 + 11*a^2 \\
& *b + 10*a*b^2 + 3*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(4*a^3 + 11 \\
& *a^2*b + 10*a*b^2 + 3*b^3)*\cosh(d*x + c)^3 + 3*(4*a^3 + 3*a^2*b - 4*a*b^2 - \\
& 3*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(12*a^3 + a^2*b + 6*a*b^2 + 9*b^3 \\
&)*\cosh(d*x + c)^4 + 2*(35*(4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*\cosh(d*x + \\
& c)^4 + 12*a^3 + a^2*b + 6*a*b^2 + 9*b^3 + 30*(4*a^3 + 3*a^2*b - 4*a*b^2 - \\
& 3*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(7*(4*a^3 + 11*a^2*b + 10*a*b^2 \\
& + 3*b^3)*\cosh(d*x + c)^5 + 10*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3)*\cosh(d*x \\
& + c)^3 + (12*a^3 + a^2*b + 6*a*b^2 + 9*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 \\
& + 4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3 + 4*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b \\
& ^3)*\cosh(d*x + c)^2 + 4*(7*(4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*\cosh(d*x + \\
& c)^6 + 15*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3)*\cosh(d*x + c)^4 + 4*a^3 + 3* \\
& a^2*b - 4*a*b^2 - 3*b^3 + 3*(12*a^3 + a^2*b + 6*a*b^2 + 9*b^3)*\cosh(d*x + c \\
&)^2)*\sinh(d*x + c)^2 + 8*((4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*\cosh(d*x + \\
& c)^7 + 3*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3)*\cosh(d*x + c)^5 + (12*a^3 + a^ \\
& 2*b + 6*a*b^2 + 9*b^3)*\cosh(d*x + c)^3 + (4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3 \\
&)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{a^2 + a*b}*\arctan(1/2*((a + b)*\cosh(d* \\
& x + c)^3 + 3*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a + b)*\sinh(d*x + c)^ \\
& 3 + (3*a - b)*\cosh(d*x + c) + (3*(a + b)*\cosh(d*x + c)^2 + 3*a - b)*\sinh(d* \\
& x + c))/\sqrt{a^2 + a*b})) + ((4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*\cosh(d*x \\
& + c)^8 + 8*(4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*\cosh(d*x + c)*\sinh(d*x + c \\
&)^7 + (4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*\sinh(d*x + c)^8 + 4*(4*a^3 + 3* \\
& a^2*b - 4*a*b^2 - 3*b^3)*\cosh(d*x + c)^6 + 4*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3 \\
& *b^3 + 7*(4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + \\
& c)^6 + 8*(7*(4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*\cosh(d*x + c)^3 + 3*(4*a^ \\
& 3 + 3*a^2*b - 4*a*b^2 - 3*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(12*a^3 + \\
& a^2*b + 6*a*b^2 + 9*b^3)*\cosh(d*x + c)^4 + 2*(35*(4*a^3 + 11*a^2*b + 10*a* \\
& b^2 + 3*b^3)*\cosh(d*x + c)^4 + 12*a^3 + a^2*b + 6*a*b^2 + 9*b^3 + 30*(4*a^3 \\
& + 3*a^2*b - 4*a*b^2 - 3*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(7*(4*a^ \\
& 3 + 11*a^2*b + 10*a*b^2 + 3*b^3)*\cosh(d*x + c)^5 + 10*(4*a^3 + 3*a^2*b - 4* \\
& a*b^2 - 3*b^3)*\cosh(d*x + c)^3 + (12*a^3 + a^2*b + 6*a*b^2 + 9*b^3)*\cosh(d* \\
& x + c))*\sinh(d*x + c)^3 + 4*a^3 + 11*a^2*b + 10*a*b^2 + 3*b^3 + 4*(4*a^3 + \\
& 3*a^2*b - 4*a*b^2 - 3*b^3)*\cosh(d*x + c)^2 + 4*(7*(4*a^3 + 11*a^2*b + 10*a* \\
& b^2 + 3*b^3)*\cosh(d*x + c)^6 + 15*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3)*\cosh(\\
& d*x + c)^4 + 4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3 + 3*(12*a^3 + a^2*b + 6*a*b^ \\
& 2 + 9*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((4*a^3 + 11*a^2*b + 10*a*b \\
& ^2 + 3*b^3)*\cosh(d*x + c)^7 + 3*(4*a^3 + 3*a^2*b - 4*a*b^2 - 3*b^3)*\cosh(d* \\
& x + c)^5 + (12*a^3 + a^2*b + 6*a*b^2 + 9*b^3)*\cosh(d*x + c)^3 + (4*a^3 + 3* \\
& a^2*b - 4*a*b^2 - 3*b^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{a^2 + a*b}*\arct \\
& an(1/2*\sqrt{a^2 + a*b}*(\cosh(d*x + c) + \sinh(d*x + c))/a) - 2*(4*a^4 + 11*a \\
& ^3*b + 10*a^2*b^2 + 3*a*b^3)*\cosh(d*x + c) + 2*(7*(4*a^4 + 11*a^3*b + 10*a^ \\
& 2*b^2 + 3*a*b^3)*\cosh(d*x + c)^6 + 5*(4*a^4 + 3*a^3*b - 10*a^2*b^2 - 9*a*b^ \\
& 3)*\cosh(d*x + c)^4 - 4*a^4 - 11*a^3*b - 10*a^2*b^2 - 3*a*b^3 - 3*(4*a^4 + 3 \\
& *a^3*b - 10*a^2*b^2 - 9*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c))/((a^7 + 4*a^ \\
& 6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^8 + 8*(a^7 + 4*a^6*b \\
& + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^7 \\
& + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\sinh(d*x + c)^8 + 4*(a^7 + 2 \\
& *a^6*b - 2*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c)^6 + 4*(7*(a^7 + 4*a^6*b + 6*a
\end{aligned}$$

$$\begin{aligned} &^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^2 + (a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d)*\sinh(d*x + c)^6 + 2*(3*a^7 + 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)*d*\cosh(d*x + c)^4 + 8*(7*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^3 + 3*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^4 + 30*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c)^2 + (3*a^7 + 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)*d)*\sinh(d*x + c)^4 + 4*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c)^2 + 8*(7*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^5 + 10*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c)^3 + (3*a^7 + 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^6 + 15*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c)^4 + 3*(3*a^7 + 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)*d*\cosh(d*x + c)^2 + (a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d)*\sinh(d*x + c)^2 + (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d + 8*((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^7 + 3*(a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c)^5 + (3*a^7 + 4*a^6*b + 2*a^5*b^2 + 4*a^4*b^3 + 3*a^3*b^4)*d*\cosh(d*x + c)^3 + (a^7 + 2*a^6*b - 2*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c))] \end{aligned}$$

giac [B] time = 0.99, size = 1495, normalized size = 11.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} &-1/8*((2*(a^3*e^{(2*c)} + a^2*b*e^{(2*c)})^2*(8*a^2*b + 6*a*b^2 - (4*a^2 - a*b - 3*b^2)*\sqrt{-a*b})*\sqrt{a^2 - b^2 + 2*\sqrt{-a*b}*(a + b)}*abs(a*e^{(2*c)} + b*e^{(2*c)}) - (4*a^6 + 7*a^5*b - a^4*b^2 - 7*a^3*b^3 - 3*a^2*b^4 + 2*(4*a^5 + 11*a^4*b + 10*a^3*b^2 + 3*a^2*b^3)*\sqrt{-a*b})*\sqrt{a^2 - b^2 + 2*\sqrt{-a*b}*(a + b)}*abs(-a^3*e^{(2*c)} - a^2*b*e^{(2*c)})*abs(a*e^{(2*c)} + b*e^{(2*c)})*e^{(2*c)} - (8*a^8*b + 14*a^7*b^2 - 2*a^6*b^3 - 14*a^5*b^4 - 6*a^4*b^5 - (4*a^8 + 3*a^7*b - 8*a^6*b^2 - 6*a^5*b^3 + 4*a^4*b^4 + 3*a^3*b^5)*\sqrt{-a*b})*\sqrt{a^2 - b^2 + 2*\sqrt{-a*b}*(a + b)}*abs(a*e^{(2*c)} + b*e^{(2*c)})*e^{(4*c)})*\arctan(e^{(d*x)}/\sqrt{(a^4*e^{(2*c)} - a^2*b^2*e^{(2*c)} - \sqrt{(a^4*e^{(2*c)} - a^2*b^2*e^{(2*c)})^2 - (a^4*e^{(4*c)} + 2*a^3*b*e^{(4*c)} + a^2*b^2*e^{(4*c)})*(a^4 + 2*a^3*b + a^2*b^2)})))/(a^4*e^{(4*c)} + 2*a^3*b*e^{(4*c)} + a^2*b^2*e^{(4*c)})))*e^{(-4*c)}/((a^{11} + 5*a^{10}*b + 9*a^9*b^2 + 5*a^8*b^3 - 5*a^7*b^4 - 9*a^6*b^5 - 5*a^5*b^6 - a^4*b^7 + 2*(a^{10} + 6*a^9*b + 15*a^8*b^2 + 20*a^7*b^3 + 15*a^6*b^4 + 6*a^5*b^5 + a^4*b^6)*\sqrt{-a*b})*abs(-a^3*e^{(2*c)} - a^2*b*e^{(2*c)})) + (2*(16*a^3*b - 4*a^2*b^2 - 12*a*b^3 + (4*a^3 - 21*a^2*b - 14*a*b^2 + 3*b^3)*\sqrt{-a*b})*abs(a^3*e^{(2*c)} + a^2*b*e^{(2*c)})^2*abs(a*e^{(2*c)} + b*e^{(2*c)}) - (4*a^7 - 13*a^6*b - 52*a^5*b^2 - 46*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5 - 4*(4*a^6 + 7*a^5*b - a^4*b^2 - 7*a^3*b^3 - 3*a^2*b^4)*\sqrt{-a*b})*abs(-a^3*e^{(2*c)} - a^2*b*e^{(2*c)})*abs(a*e^{(2*c)} + b*e^{(2*c)})*e^{(2*c)} - (16*a^9*b + 12*a^8*b^2 - 32*a^7*b^3 - 24*a^6*b^4 + 16*a^5*b^5 + 12*a^4*b^6 + (4*a^9 - 17*a^8*b - 39*a^7*b^2 + 6*a^6*b^3 + 38*a^5*b^4 + 11*a^4*b^5 - 3*a^3*b^6)*\sqrt{-a*b})*abs(a*e^{(2*c)} + b*e^{(2*c)})*e^{(4*c)})*\arctan(e^{(d*x)}/\sqrt{(a^4*e^{(2*c)} - a^2*b^2*e^{(2*c)} + \sqrt{(a^4*e^{(2*c)} - a^2*b^2*e^{(2*c)})^2 - (a^4*e^{(4*c)} + 2*a^3*b*e^{(4*c)} + a^2*b^2*e^{(4*c)})*(a^4 + 2*a^3*b + a^2*b^2)})))/(a^4*e^{(4*c)} + 2*a^3*b*e^{(4*c)} + a^2*b^2*e^{(4*c)})))*e^{(-4*c)}/((a^{10} + 4*a^9*b + 5*a^8*b^2 - 5*a^6*b^4 - 4*a^5*b^5 - a^4*b^6 - 2*(a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4 + a^4*b^5)*\sqrt{-a*b})*\sqrt{a^2 - b^2 - 2*\sqrt{-a*b}*(a + b)}*abs(-a^3*e^{(2*c)} - a^2*b*e^{(2*c)})) - 2*(4*a^2*e^{(7*d*x + 7*c)} + 7*a*b*e^{(7*d*x + 7*c)} + 3*b^2*e^{(7*d*x + 7*c)} + 4*a^2*e^{(5*d*x + 5*c)} - a*b*e^{(5*d*x + 5*c)} - 9*b^2*e^{(5*d*x + 5*c)} - 4*a^2*e^{(3*d*x + 3*c)} + a*b*e^{(3*d*x + 3*c)} + 9*b^2*e^{(3*d*x + 3*c)} - 4*a^2*e^{(d*x + c)} - 7*a*b*e^{(d*x + c)} - 3*b^2 \end{aligned}$$

$*e^{(4*c)} + 2*a^3*b*e^{(4*c)} + a^2*b^2*e^{(4*c)})*e^{(4*d*x)} + 2*(a^4*e^{(2*c)} - a^2*b^2*e^{(2*c)})*e^{(2*d*x)}, x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + dx)^3 (b \tanh(c + dx)^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3), x)

[Out] int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3, x)

[Out] Timed out

$$3.130 \quad \int \frac{\operatorname{sech}^4(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=115

$$-\frac{(a-3b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d} - \frac{(a-3b) \tanh(c+dx)}{8a^2bd(a+b \tanh^2(c+dx))} + \frac{(a+b) \tanh(c+dx)}{4abd(a+b \tanh^2(c+dx))^2}$$

[Out] $-1/8*(a-3*b)*\arctan(b^{(1/2)}*\tanh(d*x+c)/a^{(1/2)})/a^{(5/2)}/b^{(3/2)}/d+1/4*(a+b)*\tanh(d*x+c)/a/b/d/(a+b*\tanh(d*x+c)^2)^2-1/8*(a-3*b)*\tanh(d*x+c)/a^2/b/d/(a+b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3675, 385, 199, 205}

$$-\frac{(a-3b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d} - \frac{(a-3b) \tanh(c+dx)}{8a^2bd(a+b \tanh^2(c+dx))} + \frac{(a+b) \tanh(c+dx)}{4abd(a+b \tanh^2(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] $-((a-3*b)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Tanh}[c+d*x])/\text{Sqrt}[a]])/(8*a^{(5/2)}*b^{(3/2)}*d) + ((a+b)*\text{Tanh}[c+d*x])/(4*a*b*d*(a+b*\text{Tanh}[c+d*x]^2)^2) - ((a-3*b)*\text{Tanh}[c+d*x])/(8*a^2*b*d*(a+b*\text{Tanh}[c+d*x]^2))$

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegerQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^4(c+dx)}{(a+b \tanh^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{(a+bx^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{(a+b) \tanh(c+dx)}{4abd (a+b \tanh^2(c+dx))^2} - \frac{(a-3b) \operatorname{Subst}\left(\int \frac{1}{(a+bx^2)^2} dx, x, \tanh(c+dx)\right)}{4abd} \\
&= \frac{(a+b) \tanh(c+dx)}{4abd (a+b \tanh^2(c+dx))^2} - \frac{(a-3b) \tanh(c+dx)}{8a^2bd (a+b \tanh^2(c+dx))} - \frac{(a-3b) \operatorname{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \tanh(c+dx)\right)}{8a^2bd (a+b \tanh^2(c+dx))} \\
&= -\frac{(a-3b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d} + \frac{(a+b) \tanh(c+dx)}{4abd (a+b \tanh^2(c+dx))^2} - \frac{(a-3b) \operatorname{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \tanh(c+dx)\right)}{8a^2bd (a+b \tanh^2(c+dx))}
\end{aligned}$$

Mathematica [A] time = 1.05, size = 115, normalized size = 1.00

$$\frac{\frac{\sqrt{a} \sinh(2(c+dx))((a^2+4ab+3b^2) \cosh(2(c+dx))+a^2+6ab-3b^2)}{b((a+b) \cosh(2(c+dx))+a-b)^2} + \frac{(3b-a) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (((-a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/b^(3/2) + (Sqrt[a]*(a^2 + 6*a*b - 3*b^2 + (a^2 + 4*a*b + 3*b^2)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(b*(a - b + (a + b)*Cosh[2*(c + d*x)]^2))/(8*a^(5/2)*d)

fricas [B] time = 0.53, size = 5659, normalized size = 49.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [-1/16*(4*(a^4*b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^6 + 24*(a^4*b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(a^4*b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*sinh(d*x + c)^6 + 4*a^4*b + 20*a^3*b^2 + 28*a^2*b^3 + 12*a*b^4 + 4*(3*a^4*b + 7*a^3*b^2 - 3*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^4 + 4*(3*a^4*b + 7*a^3*b^2 - 3*a^2*b^3 + 9*a*b^4 + 15*(a^4*b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(a^4*b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^3 + (3*a^4*b + 7*a^3*b^2 - 3*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^4*b + 13*a^3*b^2 + a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^2 + 4*(3*a^4*b + 13*a^3*b^2 + a^2*b^3 - 9*a*b^4 + 15*(a^4*b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^4 + 6*(3*a^4*b + 7*a^3*b^2 - 3*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^8 + 8*(a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*sinh(d*x + c)^8 + 4*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 4*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4 + 7*(a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + 3*(a^4 - 2*a^3*b -

$$\begin{aligned}
& 4a^2b^2 + 2ab^3 + 3b^4) \cosh(dx + c) \sinh(dx + c)^5 + 2(3a^4 - 8a^3b - 2a^2b^2 - 9b^4) \cosh(dx + c)^4 + 2(35(a^4 - 6a^2b^2 - 8ab^3 - 3b^4) \cosh(dx + c)^4 + 3a^4 - 8a^3b - 2a^2b^2 - 9b^4 + 30(a^4 - 2a^3b - 4a^2b^2 + 2ab^3 + 3b^4) \cosh(dx + c)^2) \sinh(dx + c)^4 \\
& + a^4 - 6a^2b^2 - 8ab^3 - 3b^4 + 8(7(a^4 - 6a^2b^2 - 8ab^3 - 3b^4) \cosh(dx + c)^5 + 10(a^4 - 2a^3b - 4a^2b^2 + 2ab^3 + 3b^4) \cosh(dx + c)^3 + (3a^4 - 8a^3b - 2a^2b^2 - 9b^4) \cosh(dx + c) \sinh(dx + c)^3 + 4(a^4 - 2a^3b - 4a^2b^2 + 2ab^3 + 3b^4) \cosh(dx + c)^2 \\
& + 4(7(a^4 - 6a^2b^2 - 8ab^3 - 3b^4) \cosh(dx + c)^6 + 15(a^4 - 2a^3b - 4a^2b^2 + 2ab^3 + 3b^4) \cosh(dx + c)^4 + a^4 - 2a^3b - 4a^2b^2 + 2ab^3 + 3b^4 + 3(3a^4 - 8a^3b - 2a^2b^2 - 9b^4) \cosh(dx + c)^2) \sinh(dx + c)^2 + 8((a^4 - 6a^2b^2 - 8ab^3 - 3b^4) \cosh(dx + c)^7 + 3(a^4 - 2a^3b - 4a^2b^2 + 2ab^3 + 3b^4) \cosh(dx + c)^5 + (3a^4 - 8a^3b - 2a^2b^2 - 9b^4) \cosh(dx + c)^3 + (a^4 - 2a^3b - 4a^2b^2 + 2ab^3 + 3b^4) \cosh(dx + c)) \sqrt{-ab} \log(((a^2 + 2ab + b^2) \cosh(dx + c)^4 + 4(a^2 + 2ab + b^2) \cosh(dx + c) \sinh(dx + c)^3 + (a^2 + 2ab + b^2) \sinh(dx + c)^4 + 2(a^2 - b^2) \cosh(dx + c)^2 + 2(3(a^2 + 2ab + b^2) \cosh(dx + c)^2 + a^2 - b^2) \sinh(dx + c)^2 + a^2 - 6ab + b^2 + 4((a^2 + 2ab + b^2) \cosh(dx + c)^3 + (a^2 - b^2) \cosh(dx + c)) \sinh(dx + c) - 4((a + b) \cosh(dx + c)^2 + 2(a + b) \cosh(dx + c) \sinh(dx + c) + (a + b) \sinh(dx + c)^2 + a - b) \sqrt{-ab})) / ((a + b) \cosh(dx + c)^4 + 4(a + b) \cosh(dx + c) \sinh(dx + c)^3 + (a + b) \sinh(dx + c)^4 + 2(a - b) \cosh(dx + c)^2 + 2(3(a + b) \cosh(dx + c)^2 + a - b) \sinh(dx + c)^2 + 4((a + b) \cosh(dx + c)^3 + (a - b) \cosh(dx + c)) \sinh(dx + c) + a + b)) + 8(3(a^4b - a^3b^2 - 5a^2b^3 - 3ab^4) \cosh(dx + c)^5 + 2(3a^4b + 7a^3b^2 - 3a^2b^3 + 9ab^4) \cosh(dx + c)^3 + (3a^4b + 13a^3b^2 + a^2b^3 - 9ab^4) \cosh(dx + c)) \sinh(dx + c)) / ((a^6b^2 + 3a^5b^3 + 3a^4b^4 + a^3b^5) d \cosh(dx + c)^8 + 8(a^6b^2 + 3a^5b^3 + 3a^4b^4 + a^3b^5) d \cosh(dx + c) \sinh(dx + c)^7 + (a^6b^2 + 3a^5b^3 + 3a^4b^4 + a^3b^5) d \sinh(dx + c)^8 + 4(a^6b^2 + a^5b^3 - a^4b^4 - a^3b^5) d \cosh(dx + c)^6 + 4(7(a^6b^2 + 3a^5b^3 + 3a^4b^4 + a^3b^5) d \cosh(dx + c)^2 + (a^6b^2 + a^5b^3 - a^4b^4 - a^3b^5) d) \sinh(dx + c)^6 + 2(3a^6b^2 + a^5b^3 + a^4b^4 + 3a^3b^5) d \cosh(dx + c)^4 + 8(7(a^6b^2 + 3a^5b^3 + 3a^4b^4 + a^3b^5) d \cosh(dx + c)^3 + 3(a^6b^2 + a^5b^3 - a^4b^4 - a^3b^5) d \cosh(dx + c)) \sinh(dx + c)^5 + 2(35(a^6b^2 + 3a^5b^3 + 3a^4b^4 + a^3b^5) d \cosh(dx + c)^4 + 30(a^6b^2 + a^5b^3 - a^4b^4 - a^3b^5) d \cosh(dx + c)^2 + (3a^6b^2 + a^5b^3 + a^4b^4 + 3a^3b^5) d) \sinh(dx + c)^4 + 4(a^6b^2 + a^5b^3 - a^4b^4 - a^3b^5) d \cosh(dx + c)^2 + 8(7(a^6b^2 + 3a^5b^3 + 3a^4b^4 + a^3b^5) d \cosh(dx + c)^5 + 10(a^6b^2 + a^5b^3 - a^4b^4 - a^3b^5) d \cosh(dx + c)^3 + (3a^6b^2 + a^5b^3 + a^4b^4 + 3a^3b^5) d \cosh(dx + c)) \sinh(dx + c)^3 + 4(7(a^6b^2 + 3a^5b^3 + 3a^4b^4 + a^3b^5) d \cosh(dx + c)^6 + 15(a^6b^2 + a^5b^3 - a^4b^4 - a^3b^5) d \cosh(dx + c)^4 + 3(3a^6b^2 + a^5b^3 + a^4b^4 + 3a^3b^5) d \cosh(dx + c)^2 + (a^6b^2 + a^5b^3 - a^4b^4 - a^3b^5) d) \sinh(dx + c)^2 + (a^6b^2 + 3a^5b^3 + 3a^4b^4 + a^3b^5) d + 8((a^6b^2 + 3a^5b^3 + 3a^4b^4 + a^3b^5) d \cosh(dx + c)^7 + 3(a^6b^2 + a^5b^3 - a^4b^4 - a^3b^5) d \cosh(dx + c)^5 + (3a^6b^2 + a^5b^3 + a^4b^4 + 3a^3b^5) d \cosh(dx + c)^3 + (a^6b^2 + a^5b^3 - a^4b^4 - a^3b^5) d \cosh(dx + c)) \sinh(dx + c)), -1/8(2(a^4b - a^3b^2 - 5a^2b^3 - 3ab^4) \cosh(dx + c)^6 + 12(a^4b - a^3b^2 - 5a^2b^3 - 3ab^4) \cosh(dx + c) \sinh(dx + c)^5 + 2(a^4b - a^3b^2 - 5a^2b^3 - 3ab^4) \sinh(dx + c)^6 + 2a^4b + 10a^3b^2 + 14a^2b^3 + 6ab^4 + 2(3a^4b + 7a^3b^2 - 3a^2b^3 + 9ab^4) \cosh(dx + c)^4 + 2(3a^4b + 7a^3b^2 - 3a^2b^3 + 9ab^4 + 15(a^4b - a^3b^2 - 5a^2b^3 - 3ab^4) \cosh(dx + c)^2) \sinh(dx + c)^4 + 8(5(a^4b - a^3b^2 - 5a^2b^3 - 3ab^4) \cosh(dx + c)^3 + (3a^4b + 7a^3b^2 - 3a^2b^3 + 9ab^4) \cosh(dx + c)) \sinh(dx + c)^3 + 2(3a^4b + 13a^3b^2 + a^2b^3 - 9ab^4) \cosh(dx + c)^2 + 2(3a^4b + 13a^3b^2 + a^2b^3 - 9ab^4) \cosh(dx + c) + 15(a^4b - a^3b^2 - 5a^2b^3 - 3ab^4) \cosh(dx + c)
\end{aligned}$$

$c)^4 + 6*(3*a^4*b + 7*a^3*b^2 - 3*a^2*b^3 + 9*a*b^4)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + ((a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*\cosh(d*x + c)^8 + 8*(a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*\sinh(d*x + c)^8 + 4*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^6 + 4*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4 + 7*(a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*\cosh(d*x + c)^3 + 3*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(3*a^4 - 8*a^3*b - 2*a^2*b^2 - 9*b^4)*\cosh(d*x + c)^4 + 2*(35*(a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*\cosh(d*x + c)^4 + 3*a^4 - 8*a^3*b - 2*a^2*b^2 - 9*b^4 + 30*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4 + 8*(7*(a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*\cosh(d*x + c)^5 + 10*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^3 + (3*a^4 - 8*a^3*b - 2*a^2*b^2 - 9*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^2 + 4*(7*(a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*\cosh(d*x + c)^6 + 15*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^4 + a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4 + 3*(3*a^4 - 8*a^3*b - 2*a^2*b^2 - 9*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((a^4 - 6*a^2*b^2 - 8*a*b^3 - 3*b^4)*\cosh(d*x + c)^7 + 3*(a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^5 + (3*a^4 - 8*a^3*b - 2*a^2*b^2 - 9*b^4)*\cosh(d*x + c)^3 + (a^4 - 2*a^3*b - 4*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{a*b}*\arctan(1/2*((a + b)*\cosh(d*x + c)^2 + 2*(a + b)*\cosh(d*x + c)*\sinh(d*x + c) + (a + b)*\sinh(d*x + c)^2 + a - b)*\sqrt{a*b}/(a*b)) + 4*(3*(a^4*b - a^3*b^2 - 5*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c)^5 + 2*(3*a^4*b + 7*a^3*b^2 - 3*a^2*b^3 + 9*a*b^4)*\cosh(d*x + c)^3 + (3*a^4*b + 13*a^3*b^2 + a^2*b^3 - 9*a*b^4)*\cosh(d*x + c))*\sinh(d*x + c))/((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c)^8 + 8*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*\sinh(d*x + c)^8 + 4*(a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c)^6 + 4*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c)^2 + (a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d)*\sinh(d*x + c)^6 + 2*(3*a^6*b^2 + a^5*b^3 + a^4*b^4 + 3*a^3*b^5)*d*\cosh(d*x + c)^4 + 8*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c)^3 + 3*(a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c)^4 + 30*(a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c)^2 + (3*a^6*b^2 + a^5*b^3 + a^4*b^4 + 3*a^3*b^5)*d)*\sinh(d*x + c)^4 + 4*(a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c)^2 + 8*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c)^5 + 10*(a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c)^3 + (3*a^6*b^2 + a^5*b^3 + a^4*b^4 + 3*a^3*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c)^6 + 15*(a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c)^4 + 3*(3*a^6*b^2 + a^5*b^3 + a^4*b^4 + 3*a^3*b^5)*d*\cosh(d*x + c)^2 + (a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d)*\sinh(d*x + c)^2 + (a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d + 8*((a^6*b^2 + 3*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c)^7 + 3*(a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c)^5 + (3*a^6*b^2 + a^5*b^3 + a^4*b^4 + 3*a^3*b^5)*d*\cosh(d*x + c)^3 + (a^6*b^2 + a^5*b^3 - a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c))]$

giac [B] time = 0.73, size = 332, normalized size = 2.89

$$\frac{(ae^{2c}-3be^{2c})\arctan\left(\frac{ae^{2dx+2c}+be^{2dx+2c}+a-b}{2\sqrt{ab}}\right)e^{(-2c)}}{\sqrt{ab}a^2b} + \frac{2(a^3e^{6dx+6c}-a^2be^{6dx+6c}-5ab^2e^{6dx+6c}-3b^3e^{6dx+6c}+3a^3e^{4dx+4c}+7a^2be^{4dx+4c})}{(a^3b+a^2b^2)(ae^{4dx+4c}+7a^2be^{4dx+4c})}$$

8d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

```
[Out] -1/8*((a*e^(2*c) - 3*b*e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))*e^(-2*c)/(sqrt(a*b)*a^2*b) + 2*(a^3*e^(6*d*x + 6*c) - a^2*b*e^(6*d*x + 6*c) - 5*a*b^2*e^(6*d*x + 6*c) - 3*b^3*e^(6*d*x + 6*c) + 3*a^3*e^(4*d*x + 4*c) + 7*a^2*b*e^(4*d*x + 4*c) - 3*a*b^2*e^(4*d*x + 4*c) + 9*b^3*e^(4*d*x + 4*c) + 3*a^3*e^(2*d*x + 2*c) + 13*a^2*b*e^(2*d*x + 2*c) + a*b^2*e^(2*d*x + 2*c) - 9*b^3*e^(2*d*x + 2*c) + a^3 + 5*a^2*b + 7*a*b^2 + 3*b^3)/((a^3*b + a^2*b^2)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2))/d
```

maple [B] time = 0.40, size = 1270, normalized size = 11.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x)
```

```
[Out] 1/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/b*tanh(1/2*d*x+1/2*c)^7+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^7+3/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/b*tanh(1/2*d*x+1/2*c)^5+11/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^5+3/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5+3/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/b*tanh(1/2*d*x+1/2*c)^3+11/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^3+3/d/a^2*b/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3+1/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/b*tanh(1/2*d*x+1/2*c)+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)+1/8/d/b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c))/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/8/d/a/b/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c))/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-1/4/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c))/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+1/8/d/b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c))/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+1/8/d/a/b/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c))/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/4/d/(b*(a+b))^(1/2)/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c))/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+3/8/d/a^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c))/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-3/8/d/a^2/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c))/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-3/8/d/a^2/b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c))/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))
```

maxima [B] time = 0.70, size = 360, normalized size = 3.13

$$\frac{a^3 + 5a^2b + 7ab^2 + 3b^3 + (3a^3 + 13a^2b + ab^2 - 9b^3)e^{(-2dx-2c)} + (3a^3 + 7a^2b - 3a^2b^2 + 3a^3b + 3a^4b^2 + 3a^3b^3 + a^2b^4 + 4(a^5b + a^4b^2 - a^3b^3 - a^2b^4)e^{(-2dx-2c)} + 2(3a^5b + a^4b^2 + a^3b^3 + 3a^2b^4)e^{(-4dx-4c)})}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(a^3 + 5*a^2*b + 7*a*b^2 + 3*b^3 + (3*a^3 + 13*a^2*b + a*b^2 - 9*b^3))*e^(-2*d*x - 2*c) + (3*a^3 + 7*a^2*b - 3*a*b^2 + 9*b^3)*e^(-4*d*x - 4*c) + (a
```

$$\begin{aligned} &^3 - a^2b - 5ab^2 - 3b^3)e^{(-6dx - 6c)} / ((a^5b + 3a^4b^2 + 3a^3 \\ &b^3 + a^2b^4 + 4(a^5b + a^4b^2 - a^3b^3 - a^2b^4)e^{(-2dx - 2c)} + \\ &2(3a^5b + a^4b^2 + a^3b^3 + 3a^2b^4)e^{(-4dx - 4c)} + 4(a^5b + \\ &a^4b^2 - a^3b^3 - a^2b^4)e^{(-6dx - 6c)} + (a^5b + 3a^4b^2 + 3a^3 \\ &b^3 + a^2b^4)e^{(-8dx - 8c)})d) + 1/8(a - 3b) \arctan(1/2((a + b)e^{(-2dx - 2c)} + a - b) / \sqrt{ab}) / (\sqrt{ab} a^2 b d) \end{aligned}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + dx)^4 (b \tanh(c + dx)^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3), x)

[Out] int(1/(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3, x)

[Out] Timed out

$$3.131 \quad \int \frac{\operatorname{sech}^5(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=104

$$\frac{3 \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d\sqrt{a+b}} + \frac{3 \sinh(c+dx)}{8a^2d((a+b) \sinh^2(c+dx) + a)} + \frac{\sinh(c+dx)}{4ad((a+b) \sinh^2(c+dx) + a)^2}$$

[Out] 1/4*sinh(d*x+c)/a/d/(a+(a+b)*sinh(d*x+c)^2)^2+3/8*sinh(d*x+c)/a^2/d/(a+(a+b)*sinh(d*x+c)^2)+3/8*arctan(sinh(d*x+c)*(a+b)^(1/2)/a^(1/2))/a^(5/2)/d/(a+b)^(1/2)

Rubi [A] time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3676, 199, 205}

$$\frac{3 \sinh(c+dx)}{8a^2d((a+b) \sinh^2(c+dx) + a)} + \frac{3 \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d\sqrt{a+b}} + \frac{\sinh(c+dx)}{4ad((a+b) \sinh^2(c+dx) + a)^2}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (3*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[a + b]*d) + Sinh[c + d*x]/(4*a*d*(a + (a + b)*Sinh[c + d*x]^2)^2) + (3*Sinh[c + d*x])/(8*a^2*d*(a + (a + b)*Sinh[c + d*x]^2))

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3676

Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^5(c+dx)}{(a+b \tanh^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(a+(a+b)x^2)^3} dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{\sinh(c+dx)}{4ad(a+(a+b)\sinh^2(c+dx))^2} + \frac{3 \operatorname{Subst}\left(\int \frac{1}{(a+(a+b)x^2)^2} dx, x, \sinh(c+dx)\right)}{4ad} \\
&= \frac{\sinh(c+dx)}{4ad(a+(a+b)\sinh^2(c+dx))^2} + \frac{3 \sinh(c+dx)}{8a^2d(a+(a+b)\sinh^2(c+dx))} + \frac{3 \operatorname{Subst}\left(\int \frac{1}{a+(a+b)x^2} dx, x, \sinh(c+dx)\right)}{8a^2d(a+(a+b)\sinh^2(c+dx))} \\
&= \frac{3 \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{a+b}d} + \frac{\sinh(c+dx)}{4ad(a+(a+b)\sinh^2(c+dx))^2} + \frac{3 \operatorname{Subst}\left(\int \frac{1}{a+(a+b)x^2} dx, x, \sinh(c+dx)\right)}{8a^2d(a+(a+b)\sinh^2(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.31, size = 88, normalized size = 0.85

$$\frac{3 \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}} + \frac{3(a+b) \sinh^3(c+dx) + 5a \sinh(c+dx)}{a((a+b) \sinh^2(c+dx) + a)^2}$$

$8ad$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] ((3*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(a^(3/2)*Sqrt[a + b])) + (5*a*Sinh[c + d*x] + 3*(a + b)*Sinh[c + d*x]^3)/(a*(a + (a + b)*Sinh[c + d*x]^2)^2)/(8*a*d)

fricas [B] time = 0.52, size = 5077, normalized size = 48.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(12*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^7 + 84*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c)^6 + 12*(a^3 + 2*a^2*b + a*b^2)*sinh(d*x + c)^7 + 4*(11*a^3 + 2*a^2*b - 9*a*b^2)*cosh(d*x + c)^5 + 4*(11*a^3 + 2*a^2*b - 9*a*b^2 + 63*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(21*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^3 + (11*a^3 + 2*a^2*b - 9*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(11*a^3 + 2*a^2*b - 9*a*b^2)*cosh(d*x + c)^3 + 4*(105*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^4 - 11*a^3 - 2*a^2*b + 9*a*b^2 + 10*(11*a^3 + 2*a^2*b - 9*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(63*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^5 + 10*(11*a^3 + 2*a^2*b - 9*a*b^2)*cosh(d*x + c)^3 - 3*(11*a^3 + 2*a^2*b - 9*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 3*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 4*(a^2 - b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 30*(a^2 - b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a*b + 3*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 10*(a^2 - b^2)*cosh(d*x + c)^3 + (3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 3*(a^2 - b^2)*sinh(d*x + c)^3 + 3*(a^2 - b^2)*sinh(d*x + c)^2)]

$$\begin{aligned}
& b^2) * \cosh(dx + c)) * \sinh(dx + c)^3 + 4 * (a^2 - b^2) * \cosh(dx + c)^2 + 4 * (7 * \\
& (a^2 + 2 * a * b + b^2) * \cosh(dx + c)^6 + 15 * (a^2 - b^2) * \cosh(dx + c)^4 + 3 * (3 \\
& * a^2 - 2 * a * b + 3 * b^2) * \cosh(dx + c)^2 + a^2 - b^2) * \sinh(dx + c)^2 + a^2 + \\
& 2 * a * b + b^2 + 8 * ((a^2 + 2 * a * b + b^2) * \cosh(dx + c)^7 + 3 * (a^2 - b^2) * \cosh(dx \\
& * x + c)^5 + (3 * a^2 - 2 * a * b + 3 * b^2) * \cosh(dx + c)^3 + (a^2 - b^2) * \cosh(dx \\
& + c)) * \sinh(dx + c)) * \sqrt{-a^2 - a * b} * \log(((a + b) * \cosh(dx + c)^4 + 4 * (a + \\
& b) * \cosh(dx + c) * \sinh(dx + c)^3 + (a + b) * \sinh(dx + c)^4 - 2 * (3 * a + b) * c \\
& osh(dx + c)^2 + 2 * (3 * (a + b) * \cosh(dx + c)^2 - 3 * a - b) * \sinh(dx + c)^2 + \\
& 4 * ((a + b) * \cosh(dx + c)^3 - (3 * a + b) * \cosh(dx + c)) * \sinh(dx + c) - 4 * (co \\
& sh(dx + c)^3 + 3 * \cosh(dx + c) * \sinh(dx + c)^2 + \sinh(dx + c)^3 + (3 * \cosh \\
& (dx + c)^2 - 1) * \sinh(dx + c) - \cosh(dx + c)) * \sqrt{-a^2 - a * b} + a + b) / (\\
& (a + b) * \cosh(dx + c)^4 + 4 * (a + b) * \cosh(dx + c) * \sinh(dx + c)^3 + (a + b) \\
& * \sinh(dx + c)^4 + 2 * (a - b) * \cosh(dx + c)^2 + 2 * (3 * (a + b) * \cosh(dx + c)^2 \\
& + a - b) * \sinh(dx + c)^2 + 4 * ((a + b) * \cosh(dx + c)^3 + (a - b) * \cosh(dx + \\
& c)) * \sinh(dx + c) + a + b)) - 12 * (a^3 + 2 * a^2 * b + a * b^2) * \cosh(dx + c) + 4 \\
& * (21 * (a^3 + 2 * a^2 * b + a * b^2) * \cosh(dx + c)^6 + 5 * (11 * a^3 + 2 * a^2 * b - 9 * a * b \\
& ^2) * \cosh(dx + c)^4 - 3 * a^3 - 6 * a^2 * b - 3 * a * b^2 - 3 * (11 * a^3 + 2 * a^2 * b - 9 * a * \\
& b^2) * \cosh(dx + c)^2) * \sinh(dx + c)) / ((a^6 + 3 * a^5 * b + 3 * a^4 * b^2 + a^3 * b^3) \\
& * d * \cosh(dx + c)^8 + 8 * (a^6 + 3 * a^5 * b + 3 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c \\
&) * \sinh(dx + c)^7 + (a^6 + 3 * a^5 * b + 3 * a^4 * b^2 + a^3 * b^3) * d * \sinh(dx + c)^8 \\
& + 4 * (a^6 + a^5 * b - a^4 * b^2 - a^3 * b^3) * d * \cosh(dx + c)^6 + 4 * (7 * (a^6 + 3 * a^ \\
& 5 * b + 3 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c)^2 + (a^6 + a^5 * b - a^4 * b^2 - a^3 \\
& * b^3) * d) * \sinh(dx + c)^6 + 2 * (3 * a^6 + a^5 * b + a^4 * b^2 + 3 * a^3 * b^3) * d * \cosh(dx \\
& * x + c)^4 + 8 * (7 * (a^6 + 3 * a^5 * b + 3 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c)^3 + \\
& 3 * (a^6 + a^5 * b - a^4 * b^2 - a^3 * b^3) * d * \cosh(dx + c)) * \sinh(dx + c)^5 + 2 * (3 \\
& 5 * (a^6 + 3 * a^5 * b + 3 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c)^4 + 30 * (a^6 + a^5 * b \\
& - a^4 * b^2 - a^3 * b^3) * d * \cosh(dx + c)^2 + (3 * a^6 + a^5 * b + a^4 * b^2 + 3 * a^3 * \\
& b^3) * d) * \sinh(dx + c)^4 + 4 * (a^6 + a^5 * b - a^4 * b^2 - a^3 * b^3) * d * \cosh(dx + \\
& c)^2 + 8 * (7 * (a^6 + 3 * a^5 * b + 3 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c)^5 + 10 * (a \\
& ^6 + a^5 * b - a^4 * b^2 - a^3 * b^3) * d * \cosh(dx + c)^3 + (3 * a^6 + a^5 * b + a^4 * b^ \\
& 2 + 3 * a^3 * b^3) * d * \cosh(dx + c)) * \sinh(dx + c)^3 + 4 * (7 * (a^6 + 3 * a^5 * b + 3 * a \\
& ^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c)^6 + 15 * (a^6 + a^5 * b - a^4 * b^2 - a^3 * b^3) * \\
& d * \cosh(dx + c)^4 + 3 * (3 * a^6 + a^5 * b + a^4 * b^2 + 3 * a^3 * b^3) * d * \cosh(dx + c) \\
& ^2 + (a^6 + a^5 * b - a^4 * b^2 - a^3 * b^3) * d) * \sinh(dx + c)^2 + (a^6 + 3 * a^5 * b \\
& + 3 * a^4 * b^2 + a^3 * b^3) * d + 8 * ((a^6 + 3 * a^5 * b + 3 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx \\
& + c)^7 + 3 * (a^6 + a^5 * b - a^4 * b^2 - a^3 * b^3) * d * \cosh(dx + c)^5 + (3 * a^6 \\
& + a^5 * b + a^4 * b^2 + 3 * a^3 * b^3) * d * \cosh(dx + c)^3 + (a^6 + a^5 * b - a^4 * b^2 \\
& - a^3 * b^3) * d * \cosh(dx + c)) * \sinh(dx + c)), 1/8 * (6 * (a^3 + 2 * a^2 * b + a * b^2) * \\
& \cosh(dx + c)^7 + 42 * (a^3 + 2 * a^2 * b + a * b^2) * \cosh(dx + c) * \sinh(dx + c)^6 \\
& + 6 * (a^3 + 2 * a^2 * b + a * b^2) * \sinh(dx + c)^7 + 2 * (11 * a^3 + 2 * a^2 * b - 9 * a * b^2 \\
&) * \cosh(dx + c)^5 + 2 * (11 * a^3 + 2 * a^2 * b - 9 * a * b^2 + 63 * (a^3 + 2 * a^2 * b + a * b \\
& ^2) * \cosh(dx + c)^2) * \sinh(dx + c)^5 + 10 * (21 * (a^3 + 2 * a^2 * b + a * b^2) * \cosh(dx \\
& + c)^3 + (11 * a^3 + 2 * a^2 * b - 9 * a * b^2) * \cosh(dx + c)) * \sinh(dx + c)^4 - \\
& 2 * (11 * a^3 + 2 * a^2 * b - 9 * a * b^2) * \cosh(dx + c)^3 + 2 * (105 * (a^3 + 2 * a^2 * b + a * \\
& b^2) * \cosh(dx + c)^4 - 11 * a^3 - 2 * a^2 * b + 9 * a * b^2 + 10 * (11 * a^3 + 2 * a^2 * b - \\
& 9 * a * b^2) * \cosh(dx + c)^2) * \sinh(dx + c)^3 + 2 * (63 * (a^3 + 2 * a^2 * b + a * b^2) * c \\
& osh(dx + c)^5 + 10 * (11 * a^3 + 2 * a^2 * b - 9 * a * b^2) * \cosh(dx + c)^3 - 3 * (11 * a^ \\
& 3 + 2 * a^2 * b - 9 * a * b^2) * \cosh(dx + c)) * \sinh(dx + c)^2 + 3 * ((a^2 + 2 * a * b + b \\
& ^2) * \cosh(dx + c)^8 + 8 * (a^2 + 2 * a * b + b^2) * \cosh(dx + c) * \sinh(dx + c)^7 + \\
& (a^2 + 2 * a * b + b^2) * \sinh(dx + c)^8 + 4 * (a^2 - b^2) * \cosh(dx + c)^6 + 4 * (7 \\
& * (a^2 + 2 * a * b + b^2) * \cosh(dx + c)^2 + a^2 - b^2) * \sinh(dx + c)^6 + 8 * (7 * (a \\
& ^2 + 2 * a * b + b^2) * \cosh(dx + c)^3 + 3 * (a^2 - b^2) * \cosh(dx + c)) * \sinh(dx + \\
& c)^5 + 2 * (3 * a^2 - 2 * a * b + 3 * b^2) * \cosh(dx + c)^4 + 2 * (35 * (a^2 + 2 * a * b + b \\
& ^2) * \cosh(dx + c)^4 + 30 * (a^2 - b^2) * \cosh(dx + c)^2 + 3 * a^2 - 2 * a * b + 3 * b^2 \\
&) * \sinh(dx + c)^4 + 8 * (7 * (a^2 + 2 * a * b + b^2) * \cosh(dx + c)^5 + 10 * (a^2 - b \\
& ^2) * \cosh(dx + c)^3 + (3 * a^2 - 2 * a * b + 3 * b^2) * \cosh(dx + c)) * \sinh(dx + c)^3 \\
& + 4 * (a^2 - b^2) * \cosh(dx + c)^2 + 4 * (7 * (a^2 + 2 * a * b + b^2) * \cosh(dx + c)^6 \\
& + 15 * (a^2 - b^2) * \cosh(dx + c)^4 + 3 * (3 * a^2 - 2 * a * b + 3 * b^2) * \cosh(dx + c) \\
& ^2 + a^2 - b^2) * \sinh(dx + c)^2 + a^2 + 2 * a * b + b^2 + 8 * ((a^2 + 2 * a * b + b^2)
\end{aligned}$$

```

)*cosh(d*x + c)^7 + 3*(a^2 - b^2)*cosh(d*x + c)^5 + (3*a^2 - 2*a*b + 3*b^2)
*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + a*b
)*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x +
c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(
d*x + c)^2 + 3*a - b)*sinh(d*x + c))/sqrt(a^2 + a*b)) + 3*((a^2 + 2*a*b + b
^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 +
(a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 4*(a^2 - b^2)*cosh(d*x + c)^6 + 4*(7
*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^6 + 8*(7*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 - b^2)*cosh(d*x + c))*sinh(d*x +
c)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^
2)*cosh(d*x + c)^4 + 30*(a^2 - b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a*b + 3*b^2
)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 10*(a^2 - b^
2)*cosh(d*x + c)^3 + (3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3
+ 4*(a^2 - b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6
+ 15*(a^2 - b^2)*cosh(d*x + c)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)
^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2
)*cosh(d*x + c)^7 + 3*(a^2 - b^2)*cosh(d*x + c)^5 + (3*a^2 - 2*a*b + 3*b^2)
*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 + a*b
)*arctan(1/2*sqrt(a^2 + a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) - 6*(a^3 +
2*a^2*b + a*b^2)*cosh(d*x + c) + 2*(21*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c
)^6 + 5*(11*a^3 + 2*a^2*b - 9*a*b^2)*cosh(d*x + c)^4 - 3*a^3 - 6*a^2*b - 3*
a*b^2 - 3*(11*a^3 + 2*a^2*b - 9*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^
6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^8 + 8*(a^6 + 3*a^5*b + 3
*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^6 + 3*a^5*b + 3*a^
4*b^2 + a^3*b^3)*d*sinh(d*x + c)^8 + 4*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*
cosh(d*x + c)^6 + 4*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c
)^2 + (a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d)*sinh(d*x + c)^6 + 2*(3*a^6 + a^5
*b + a^4*b^2 + 3*a^3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b
^2 + a^3*b^3)*d*cosh(d*x + c)^3 + 3*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*cos
h(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d
*cosh(d*x + c)^4 + 30*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^2 +
(3*a^6 + a^5*b + a^4*b^2 + 3*a^3*b^3)*d)*sinh(d*x + c)^4 + 4*(a^6 + a^5*b
- a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^2 + 8*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 +
a^3*b^3)*d*cosh(d*x + c)^5 + 10*(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*cosh(d
*x + c)^3 + (3*a^6 + a^5*b + a^4*b^2 + 3*a^3*b^3)*d*cosh(d*x + c))*sinh(d*x
+ c)^3 + 4*(7*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^6 + 15*
(a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^4 + 3*(3*a^6 + a^5*b + a^
4*b^2 + 3*a^3*b^3)*d*cosh(d*x + c)^2 + (a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d)
*sinh(d*x + c)^2 + (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d + 8*((a^6 + 3*a^
5*b + 3*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^7 + 3*(a^6 + a^5*b - a^4*b^2 - a
^3*b^3)*d*cosh(d*x + c)^5 + (3*a^6 + a^5*b + a^4*b^2 + 3*a^3*b^3)*d*cosh(d
*x + c)^3 + (a^6 + a^5*b - a^4*b^2 - a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)
)]

```

giac [B] time = 0.82, size = 690, normalized size = 6.63

$$\frac{3(a^3 - 10a^2b + 5ab^2 + (5a^2 - 10ab + b^2)\sqrt{-ab})\sqrt{a^2 - b^2 + 2\sqrt{-ab}(a+b)}|ae^{(2c)} + be^{(2c)}|\arctan\left(\frac{e^{(dx)}}{\sqrt{\frac{a^3e^{(2c)} - a^2be^{(2c)} + \sqrt{(a^3e^{(2c)} - a^2be^{(2c)})^2 - (a^3e^{(4c)} + a^2be^{(4c)})}}{a^3e^{(4c)} + a^2be^{(4c)}}}}}\right)}{a^8 - 13a^7b - 14a^6b^2 + 14a^5b^3 + 13a^4b^4 - a^3b^5 + 2(3a^7 - 4a^6b - 14a^5b^2 - 4a^4b^3 + 3a^3b^4)\sqrt{-ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/8*(3*(a^3 - 10*a^2*b + 5*a*b^2 + (5*a^2 - 10*a*b + b^2)*sqrt(-a*b))*sqrt(a^2 - b^2 + 2*sqrt(-a*b)*(a + b))*abs(a*e^(2*c) + b*e^(2*c))*arctan(e^(d*x)/sqrt((a^3*e^(2*c) - a^2*b*e^(2*c) + sqrt((a^3*e^(2*c) - a^2*b*e^(2*c)))^2 -

$$\frac{(a^3 e^{4c} + a^2 b e^{4c})(a^3 + a^2 b)}{(a^3 e^{4c} + a^2 b e^{4c})} e^{-2c} / (a^8 - 13a^7 b - 14a^6 b^2 + 14a^5 b^3 + 13a^4 b^4 - a^3 b^5 + 2(3a^7 - 4a^6 b - 14a^5 b^2 - 4a^4 b^3 + 3a^3 b^4) \sqrt{-ab}) + 3(a^3 - 10a^2 b + 5ab^2 - (5a^2 - 10ab + b^2) \sqrt{-ab}) \sqrt{a^2 - b^2 - 2\sqrt{-ab}(a+b)} \operatorname{abs}(a e^{2c} + b e^{2c}) \arctan(e^{d x} / \sqrt{(a^3 e^{2c} - a^2 b e^{2c} - \sqrt{(a^3 e^{2c} - a^2 b e^{2c})^2 - (a^3 e^{4c} + a^2 b e^{4c})(a^3 + a^2 b)})}) e^{-2c} / (a^8 - 13a^7 b - 14a^6 b^2 + 14a^5 b^3 + 13a^4 b^4 - a^3 b^5 - 2(3a^7 - 4a^6 b - 14a^5 b^2 - 4a^4 b^3 + 3a^3 b^4) \sqrt{-ab}) + 2(3a e^{7d x + 7c} + 3b e^{7d x + 7c} + 11a e^{5d x + 5c} - 9b e^{5d x + 5c} - 11a e^{3d x + 3c} + 9b e^{3d x + 3c} - 3a e^{d x + c} - 3b e^{d x + c}) / ((a e^{4d x + 4c} + b e^{4d x + 4c} + 2a e^{2d x + 2c} - 2b e^{2d x + 2c} + a + b)^2 a^2) / d$$

maple [B] time = 0.40, size = 634, normalized size = 6.10

$$\frac{5 \left(\tanh^7 \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{4d \left(\left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a + 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a + 4 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b + a \right)^2 a} + \frac{3 \left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a + 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a + 4 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b + a}{4d \left(\left(\tanh^4 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a + 2 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) a + 4 \left(\tanh^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b + a \right)^2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x)`

[Out]
$$-5/4/d/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*\tanh(1/2*d*x+1/2*c)^7+3/4/d/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*\tanh(1/2*d*x+1/2*c)^5-3/d/a^2*b/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)^5-3/4/d/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*\tanh(1/2*d*x+1/2*c)^3+3/d/a^2*b/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)^3+5/4/d/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*\tanh(1/2*d*x+1/2*c)-3/8/d/a^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/8/d/a^2*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/8/d/a^2/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+3/8/d/a^2*b/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3(ae^{7c} + be^{7c})e^{7dx} + (11ae^{5c} - 9be^{5c})e^{5dx} - (11ae^{3c} - 9be^{3c})e^{3dx} - 3(ae^c + be^c)e^{dx}}{4(a^4d + 2a^3bd + a^2b^2d + (a^4de^{8c} + 2a^3bde^{8c} + a^2b^2de^{8c})e^{8dx} + 4(a^4de^{6c} - a^2b^2de^{6c})e^{6dx} + 2(3a^4de^{4c} - 2a^3bde^{4c} + a^2b^2de^{4c})e^{4dx} + 4(a^4de^{2c} - a^2b^2de^{2c})e^{2dx}) + 32 \int \frac{3(128*(e^{3d x + 3c} + e^{d x + c}))}{(a^3 + a^2 b + (a^3 e^{4c} + a^2 b e^{4c}) e^{4d x} + 2(a^3 e^{2c} - a^2 b e^{2c}) e^{2d x})} dx, x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")`

[Out]
$$1/4*(3*(a e^{7c} + b e^{7c}) e^{7d x} + (11 a e^{5c} - 9 b e^{5c}) e^{5d x} - (11 a e^{3c} - 9 b e^{3c}) e^{3d x} - 3(a e^c + b e^c) e^{d x}) / (a^4 d + 2 a^3 b d + a^2 b^2 d + (a^4 d e^{8c} + 2 a^3 b d e^{8c} + a^2 b^2 d e^{8c}) e^{8d x} + 4(a^4 d e^{6c} - a^2 b^2 d e^{6c}) e^{6d x} + 2(3 a^4 d e^{4c} - 2 a^3 b d e^{4c} + a^2 b^2 d e^{4c}) e^{4d x} + 4(a^4 d e^{2c} - a^2 b^2 d e^{2c}) e^{2d x}) + 32 \int \frac{3(128*(e^{3d x + 3c} + e^{d x + c}))}{(a^3 + a^2 b + (a^3 e^{4c} + a^2 b e^{4c}) e^{4d x} + 2(a^3 e^{2c} - a^2 b e^{2c}) e^{2d x})} dx, x$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + dx)^5 (b \tanh(c + dx)^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)^3), x)

[Out] int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)^3), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**5/(a+b*tanh(d*x+c)**2)**3, x)

[Out] Timed out

$$3.132 \quad \int \frac{\operatorname{sech}^6(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=131

$$\frac{3\left(\frac{1}{a^2} - \frac{1}{b^2}\right) \tanh(c+dx)}{8d(a+b \tanh^2(c+dx))} + \frac{(3a^2 - 2ab + 3b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d} + \frac{(a+b) \tanh(c+dx) \operatorname{sech}^2(c+dx)}{4abd(a+b \tanh^2(c+dx))^2}$$

[Out] 1/8*(3*a^2-2*a*b+3*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/b^(5/2)/d+1/4*(a+b)*sech(d*x+c)^2*tanh(d*x+c)/a/b/d/(a+b*tanh(d*x+c)^2)^2+3/8*(1/a^2-1/b^2)*tanh(d*x+c)/d/(a+b*tanh(d*x+c)^2)

Rubi [A] time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3675, 413, 385, 205}

$$\frac{3\left(\frac{1}{a^2} - \frac{1}{b^2}\right) \tanh(c+dx)}{8d(a+b \tanh^2(c+dx))} + \frac{(3a^2 - 2ab + 3b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d} + \frac{(a+b) \tanh(c+dx) \operatorname{sech}^2(c+dx)}{4abd(a+b \tanh^2(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] ((3*a^2 - 2*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*b^(5/2)*d) + ((a + b)*Sech[c + d*x]^2*Tanh[c + d*x])/(4*a*b*d*(a + b*Tanh[c + d*x]^2)^2) + (3*(a^(-2) - b^(-2))*Tanh[c + d*x])/(8*d*(a + b*Tanh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p+1))/(a*b*n*(p+1)), x] - Dist[(a*d - b*c*(n*(p+1) + 1))/(a*b*n*(p+1)), Int[(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 413

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[((a*d - c*b)*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q-1))/(a*b*n*(p+1)), x] - Dist[1/(a*b*n*(p+1)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^(q-2)*Simp[c*(a*d - c*b*(n*(p+1) + 1)) + d*(a*d*(n*(q-1) + 1) - b*c*(n*(p+q) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m-1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4])

|| EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}^6(c+dx)}{(a+b \tanh^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{(a+bx^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{(a+b)\operatorname{sech}^2(c+dx) \tanh(c+dx)}{4abd(a+b \tanh^2(c+dx))^2} + \frac{\operatorname{Subst}\left(\int \frac{-a+3b+(3a-b)x^2}{(a+bx^2)^2} dx, x, \tanh(c+dx)\right)}{4abd} \\ &= \frac{(a+b)\operatorname{sech}^2(c+dx) \tanh(c+dx)}{4abd(a+b \tanh^2(c+dx))^2} - \frac{3(a^2-b^2) \tanh(c+dx)}{8a^2b^2d(a+b \tanh^2(c+dx))} + \frac{(3a^2-2ab+3b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d} \\ &= \frac{(3a^2-2ab+3b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d} + \frac{(a+b)\operatorname{sech}^2(c+dx) \tanh(c+dx)}{4abd(a+b \tanh^2(c+dx))^2} \end{aligned}$$

Mathematica [A] time = 1.00, size = 128, normalized size = 0.98

$$\frac{(3a^2 - 2ab + 3b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right) - \frac{\sqrt{a} \sqrt{b} (a+b) \sinh(2(c+dx))(3(a^2-b^2) \cosh(2(c+dx))+3a^2-10ab+3b^2)}{((a+b) \cosh(2(c+dx))+a-b)^2}}{8a^{5/2}b^{5/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] ((3*a^2 - 2*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] - (Sqrt[a]*Sqrt[b]*(a + b)*(3*a^2 - 10*a*b + 3*b^2 + 3*(a^2 - b^2)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)]/(a - b + (a + b)*Cosh[2*(c + d*x)]^2)/(8*a^(5/2)*b^(5/2)*d)

fricas [B] time = 0.55, size = 5233, normalized size = 39.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(4*(3*a^4*b + a^3*b^2 + a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^6 + 24*(3*a^4*b + a^3*b^2 + a^2*b^3 + 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(3*a^4*b + a^3*b^2 + a^2*b^3 + 3*a*b^4)*sinh(d*x + c)^6 + 12*a^4*b + 12*a^3*b^2 - 12*a^2*b^3 - 12*a*b^4 + 12*(3*a^4*b - 5*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^4 + 12*(3*a^4*b - 5*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4 + 5*(3*a^4*b + a^3*b^2 + a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(3*a^4*b + a^3*b^2 + a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^3 + 3*(3*a^4*b - 5*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(9*a^4*b - 13*a^3*b^2 - 13*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^2 + 4*(9*a^4*b - 13*a^3*b^2 - 13*a^2*b^3 + 9*a*b^4 + 15*(3*a^4*b + a^3*b^2 + a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^4 + 18*(3*a^4*b - 5*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*sinh(d*x + c

$$\begin{aligned}
&)^8 + 4*(3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*\cosh(d*x + c)^6 + 4*(3*a^4 - 2* \\
& a^3*b + 2*a*b^3 - 3*b^4 + 7*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4) \\
& *\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a \\
& *b^3 + 3*b^4)*\cosh(d*x + c)^3 + 3*(3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*\cosh(\\
& d*x + c))*\sinh(d*x + c)^5 + 2*(9*a^4 - 12*a^3*b + 22*a^2*b^2 - 12*a*b^3 + 9 \\
& *b^4)*\cosh(d*x + c)^4 + 2*(35*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^ \\
& 4)*\cosh(d*x + c)^4 + 9*a^4 - 12*a^3*b + 22*a^2*b^2 - 12*a*b^3 + 9*b^4 + 30* \\
& (3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 3*a^ \\
& 4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4 + 8*(7*(3*a^4 + 4*a^3*b + 2*a^2*b \\
& ^2 + 4*a*b^3 + 3*b^4)*\cosh(d*x + c)^5 + 10*(3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b \\
& ^4)*\cosh(d*x + c)^3 + (9*a^4 - 12*a^3*b + 22*a^2*b^2 - 12*a*b^3 + 9*b^4)*co \\
& sh(d*x + c))*\sinh(d*x + c)^3 + 4*(3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*\cosh(d \\
& *x + c)^2 + 4*(7*(3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4*a*b^3 + 3*b^4)*\cosh(d*x + \\
& c)^6 + 15*(3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*\cosh(d*x + c)^4 + 3*a^4 - 2* \\
& a^3*b + 2*a*b^3 - 3*b^4 + 3*(9*a^4 - 12*a^3*b + 22*a^2*b^2 - 12*a*b^3 + 9*b \\
& ^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((3*a^4 + 4*a^3*b + 2*a^2*b^2 + 4* \\
& a*b^3 + 3*b^4)*\cosh(d*x + c)^7 + 3*(3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*\cosh \\
& (d*x + c)^5 + (9*a^4 - 12*a^3*b + 22*a^2*b^2 - 12*a*b^3 + 9*b^4)*\cosh(d*x + \\
& c)^3 + (3*a^4 - 2*a^3*b + 2*a*b^3 - 3*b^4)*\cosh(d*x + c))*\sinh(d*x + c))*s \\
& qrt(-a*b)*\log(((a^2 + 2*a*b + b^2)*\cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)* \\
& cosh(d*x + c)*\sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*\sinh(d*x + c)^4 + 2*(a^ \\
& 2 - b^2)*\cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)^2 + a^2 - \\
& b^2)*\sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*\cosh(d*x \\
& + c)^3 + (a^2 - b^2)*\cosh(d*x + c))*\sinh(d*x + c) - 4*((a + b)*\cosh(d*x + \\
& c)^2 + 2*(a + b)*\cosh(d*x + c)*\sinh(d*x + c) + (a + b)*\sinh(d*x + c)^2 + a \\
& - b)*\sqrt{-a*b}))/((a + b)*\cosh(d*x + c)^4 + 4*(a + b)*\cosh(d*x + c)*\sinh(d \\
& x + c)^3 + (a + b)*\sinh(d*x + c)^4 + 2*(a - b)*\cosh(d*x + c)^2 + 2*(3*(a + \\
& b)*\cosh(d*x + c)^2 + a - b)*\sinh(d*x + c)^2 + 4*((a + b)*\cosh(d*x + c)^3 + \\
& (a - b)*\cosh(d*x + c))*\sinh(d*x + c) + a + b)) + 8*(3*(3*a^4*b + a^3*b^2 + \\
& a^2*b^3 + 3*a*b^4)*\cosh(d*x + c)^5 + 6*(3*a^4*b - 5*a^3*b^2 + 5*a^2*b^3 - 3 \\
& *a*b^4)*\cosh(d*x + c)^3 + (9*a^4*b - 13*a^3*b^2 - 13*a^2*b^3 + 9*a*b^4)*cos \\
& h(d*x + c))*\sinh(d*x + c))/((a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c) \\
& ^8 + 8*(a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a \\
& ^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d*\sinh(d*x + c)^8 + 4*(a^5*b^3 - a^3*b^5)*d*c \\
& osh(d*x + c)^6 + 4*(7*(a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c)^2 + (\\
& a^5*b^3 - a^3*b^5)*d)*\sinh(d*x + c)^6 + 2*(3*a^5*b^3 - 2*a^4*b^4 + 3*a^3*b^ \\
& 5)*d*\cosh(d*x + c)^4 + 8*(7*(a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c) \\
& ^3 + 3*(a^5*b^3 - a^3*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^5*b^ \\
& 3 + 2*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c)^4 + 30*(a^5*b^3 - a^3*b^5)*d*\cosh(\\
& d*x + c)^2 + (3*a^5*b^3 - 2*a^4*b^4 + 3*a^3*b^5)*d)*\sinh(d*x + c)^4 + 4*(a^ \\
& 5*b^3 - a^3*b^5)*d*\cosh(d*x + c)^2 + 8*(7*(a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d \\
& *\cosh(d*x + c)^5 + 10*(a^5*b^3 - a^3*b^5)*d*\cosh(d*x + c)^3 + (3*a^5*b^3 - \\
& 2*a^4*b^4 + 3*a^3*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^5*b^3 + 2 \\
& *a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c)^6 + 15*(a^5*b^3 - a^3*b^5)*d*\cosh(d*x + \\
& c)^4 + 3*(3*a^5*b^3 - 2*a^4*b^4 + 3*a^3*b^5)*d*\cosh(d*x + c)^2 + (a^5*b^3 \\
& - a^3*b^5)*d)*\sinh(d*x + c)^2 + (a^5*b^3 + 2*a^4*b^4 + a^3*b^5)*d + 8*((a^5 \\
& *b^3 + 2*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c)^7 + 3*(a^5*b^3 - a^3*b^5)*d*cos \\
& h(d*x + c)^5 + (3*a^5*b^3 - 2*a^4*b^4 + 3*a^3*b^5)*d*\cosh(d*x + c)^3 + (a^5 \\
& *b^3 - a^3*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)), 1/8*(2*(3*a^4*b + a^3*b^2 \\
& + a^2*b^3 + 3*a*b^4)*\cosh(d*x + c)^6 + 12*(3*a^4*b + a^3*b^2 + a^2*b^3 + 3* \\
& a*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^5 + 2*(3*a^4*b + a^3*b^2 + a^2*b^3 + 3*a \\
& *b^4)*\sinh(d*x + c)^6 + 6*a^4*b + 6*a^3*b^2 - 6*a^2*b^3 - 6*a*b^4 + 6*(3*a^ \\
& 4*b - 5*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c)^4 + 6*(3*a^4*b - 5*a^3 \\
& *b^2 + 5*a^2*b^3 - 3*a*b^4 + 5*(3*a^4*b + a^3*b^2 + a^2*b^3 + 3*a*b^4)*\cosh \\
& (d*x + c)^2)*\sinh(d*x + c)^4 + 8*(5*(3*a^4*b + a^3*b^2 + a^2*b^3 + 3*a*b^4) \\
& *\cosh(d*x + c)^3 + 3*(3*a^4*b - 5*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*\cosh(d*x + \\
& c))*\sinh(d*x + c)^3 + 2*(9*a^4*b - 13*a^3*b^2 - 13*a^2*b^3 + 9*a*b^4)*\cosh \\
& (d*x + c)^2 + 2*(9*a^4*b - 13*a^3*b^2 - 13*a^2*b^3 + 9*a*b^4 + 15*(3*a^4*b \\
& + a^3*b^2 + a^2*b^3 + 3*a*b^4)*\cosh(d*x + c)^4 + 18*(3*a^4*b - 5*a^3*b^2 +
\end{aligned}$$

$$\begin{aligned}
& 5a^2b^3 - 3ab^4) \cosh(dx + c)^2 \sinh(dx + c)^2 + ((3a^4 + 4a^3b + 2a^2b^2 + 4ab^3 + 3b^4) \cosh(dx + c)^8 + 8(3a^4 + 4a^3b + 2a^2b^2 + 4ab^3 + 3b^4) \cosh(dx + c) \sinh(dx + c)^7 + (3a^4 + 4a^3b + 2a^2b^2 + 4ab^3 + 3b^4) \sinh(dx + c)^8 + 4(3a^4 - 2a^3b + 2ab^3 - 3b^4) \cosh(dx + c)^6 + 4(3a^4 - 2a^3b + 2ab^3 - 3b^4 + 7(3a^4 + 4a^3b + 2a^2b^2 + 4ab^3 + 3b^4) \cosh(dx + c)^2) \sinh(dx + c)^6 + 8(7(3a^4 + 4a^3b + 2a^2b^2 + 4ab^3 + 3b^4) \cosh(dx + c)^3 + 3(3a^4 - 2a^3b + 2ab^3 - 3b^4) \cosh(dx + c)) \sinh(dx + c)^5 + 2(9a^4 - 12a^3b + 22a^2b^2 - 12ab^3 + 9b^4) \cosh(dx + c)^4 + 2(35(3a^4 + 4a^3b + 2a^2b^2 + 4ab^3 + 3b^4) \cosh(dx + c)^4 + 9a^4 - 12a^3b + 22a^2b^2 - 12ab^3 + 9b^4 + 30(3a^4 - 2a^3b + 2ab^3 - 3b^4) \cosh(dx + c)^2) \sinh(dx + c)^4 + 3a^4 + 4a^3b + 2a^2b^2 + 4ab^3 + 3b^4 + 8(7(3a^4 + 4a^3b + 2a^2b^2 + 4ab^3 + 3b^4) \cosh(dx + c)^5 + 10(3a^4 - 2a^3b + 2ab^3 - 3b^4) \cosh(dx + c)^3 + (9a^4 - 12a^3b + 22a^2b^2 - 12ab^3 + 9b^4) \cosh(dx + c)) \sinh(dx + c)^3 + 4(3a^4 - 2a^3b + 2ab^3 - 3b^4) \cosh(dx + c)^2 + 4(7(3a^4 + 4a^3b + 2a^2b^2 + 4ab^3 + 3b^4) \cosh(dx + c)^6 + 15(3a^4 - 2a^3b + 2ab^3 - 3b^4) \cosh(dx + c)^4 + 3a^4 - 2a^3b + 2ab^3 - 3b^4 + 3(9a^4 - 12a^3b + 22a^2b^2 - 12ab^3 + 9b^4) \cosh(dx + c)^2) \sinh(dx + c)^2 + 8((3a^4 + 4a^3b + 2a^2b^2 + 4ab^3 + 3b^4) \cosh(dx + c)^7 + 3(3a^4 - 2a^3b + 2ab^3 - 3b^4) \cosh(dx + c)^5 + (9a^4 - 12a^3b + 22a^2b^2 - 12ab^3 + 9b^4) \cosh(dx + c)^3 + (3a^4 - 2a^3b + 2ab^3 - 3b^4) \cosh(dx + c)) \sinh(dx + c)) \sqrt{ab} \arctan(1/2((a + b) \cosh(dx + c)^2 + 2(a + b) \cosh(dx + c) \sinh(dx + c) + (a + b) \sinh(dx + c)^2 + a - b) \sqrt{ab}) / (ab)) + 4(3(3a^4b + a^3b^2 + a^2b^3 + 3ab^4) \cosh(dx + c)^5 + 6(3a^4b - 5a^3b^2 + 5a^2b^3 - 3ab^4) \cosh(dx + c)^3 + (9a^4b - 13a^3b^2 - 13a^2b^3 + 9ab^4) \cosh(dx + c)) \sinh(dx + c)) / ((a^5b^3 + 2a^4b^4 + a^3b^5) d \cosh(dx + c)^8 + 8(a^5b^3 + 2a^4b^4 + a^3b^5) d \cosh(dx + c) \sinh(dx + c)^7 + (a^5b^3 + 2a^4b^4 + a^3b^5) d \sinh(dx + c)^8 + 4(a^5b^3 - a^3b^5) d \cosh(dx + c)^6 + 4(7(a^5b^3 + 2a^4b^4 + a^3b^5) d \cosh(dx + c)^2 + (a^5b^3 - a^3b^5) d) \sinh(dx + c)^6 + 2(3a^5b^3 - 2a^4b^4 + 3a^3b^5) d \cosh(dx + c)^4 + 8(7(a^5b^3 + 2a^4b^4 + a^3b^5) d \cosh(dx + c)^3 + 3(a^5b^3 - a^3b^5) d \cosh(dx + c)) \sinh(dx + c)^5 + 2(35(a^5b^3 + 2a^4b^4 + a^3b^5) d \cosh(dx + c)^4 + 30(a^5b^3 - a^3b^5) d \cosh(dx + c)^2 + (3a^5b^3 - 2a^4b^4 + 3a^3b^5) d) \sinh(dx + c)^4 + 4(a^5b^3 - a^3b^5) d \cosh(dx + c)^2 + 8(7(a^5b^3 + 2a^4b^4 + a^3b^5) d \cosh(dx + c)^5 + 10(a^5b^3 - a^3b^5) d \cosh(dx + c)^3 + (3a^5b^3 - 2a^4b^4 + 3a^3b^5) d \cosh(dx + c)) \sinh(dx + c)^3 + 4(7(a^5b^3 + 2a^4b^4 + a^3b^5) d \cosh(dx + c)^6 + 15(a^5b^3 - a^3b^5) d \cosh(dx + c)^4 + 3(3a^5b^3 - 2a^4b^4 + 3a^3b^5) d \cosh(dx + c)^2 + (a^5b^3 - a^3b^5) d) \sinh(dx + c)^2 + (a^5b^3 + 2a^4b^4 + a^3b^5) d + 8((a^5b^3 + 2a^4b^4 + a^3b^5) d \cosh(dx + c)^7 + 3(a^5b^3 - a^3b^5) d \cosh(dx + c)^5 + (3a^5b^3 - 2a^4b^4 + 3a^3b^5) d \cosh(dx + c)^3 + (a^5b^3 - a^3b^5) d \cosh(dx + c)) \sinh(dx + c))]
\end{aligned}$$

giac [B] time = 0.75, size = 338, normalized size = 2.58

$$\frac{(3a^2e^{2c} - 2abe^{2c} + 3b^2e^{2c}) \arctan\left(\frac{ae^{2dx+2c} + be^{2dx+2c} + a - b}{2\sqrt{ab}}\right) e^{-2c}}{\sqrt{ab} a^2 b^2} + \frac{2(3a^3e^{6dx+6c} + a^2be^{6dx+6c} + ab^2e^{6dx+6c} + 3b^3e^{6dx+6c} + 9a^3e^{4dx+4c})}{\sqrt{ab} a^2 b^2}$$

8d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)^6/(a+b*tanh(dx+c)^2)^3,x, algorithm="giac")

[Out] 1/8*((3a^2e^(2*c) - 2a*b*e^(2*c) + 3b^2e^(2*c))*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))*e^(-2*c)/(sqrt(a*b)*a^2*b^2) + 2*(3a^3e^(6*d*x + 6*c) + a^2*b*e^(6*d*x + 6*c) + a*b^2e^(6*d*x + 6*c) + 3b^3e^(6*d*x + 6*c) + 9a^3e^(4*d*x + 4*c) - 15a^2*b*e^(4*d*x + 4*c)

$$) + 15*a*b^2*e^{(4*d*x + 4*c)} - 9*b^3*e^{(4*d*x + 4*c)} + 9*a^3*e^{(2*d*x + 2*c)} - 13*a^2*b*e^{(2*d*x + 2*c)} - 13*a*b^2*e^{(2*d*x + 2*c)} + 9*b^3*e^{(2*d*x + 2*c)} + 3*a^3 + 3*a^2*b - 3*a*b^2 - 3*b^3)/((a*e^{(4*d*x + 4*c)} + b*e^{(4*d*x + 4*c)} + 2*a*e^{(2*d*x + 2*c)} - 2*b*e^{(2*d*x + 2*c)} + a + b)^2*a^2*b^2))/d$$

maple [B] time = 0.42, size = 1776, normalized size = 13.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x)

[Out]
$$\begin{aligned} & -1/8/d/(b*(a+b))^{(1/2)}/a/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh \\ & (1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)})-1/8/d/(b*(a+b))^{(1/2)}/ \\ & a/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(\\ & a+b))^{(1/2)}+a+2*b)*a)^{(1/2)})-3/8/d*a/b^2/(b*(a+b))^{(1/2)}/((2*(b*(a+b))^{(1/2) \\ &)-a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}-a-2*b)* \\ & a)^{(1/2)})-3/8/d*a/b^2/(b*(a+b))^{(1/2)}/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*a \\ & rctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)})+1/2/d/(\tanh \\ & (1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a \\ & ^2/b*\tanh(1/2*d*x+1/2*c)^7-7/2/d/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/ \\ & 2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/b*\tanh(1/2*d*x+1/2*c)^5-7/2/d/(\tanh \\ & (1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^ \\ & 2/b*\tanh(1/2*d*x+1/2*c)^3+1/2/d/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2 \\ & *c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/b*\tanh(1/2*d*x+1/2*c)-3/8/d/a^2*b/(b \\ & *(a+b))^{(1/2)}/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2 \\ & *c)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)})-3/8/d/a^2*b/(b*(a+b))^{(1/2)}/((2*(b \\ & *(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(\\ & 1/2)}-a-2*b)*a)^{(1/2)})+3/8/d/b^2/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}*\arctanh \\ & (a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)})-3/8/d/b^2/((2*(\\ & b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(\\ & 1/2)}+a+2*b)*a)^{(1/2)})+5/4/d/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^ \\ & 2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*\tanh(1/2*d*x+1/2*c)^7+7/4/d/(\tanh(1/2* \\ & d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*t \\ & anh(1/2*d*x+1/2*c)^5+7/4/d/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2 \\ & *a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*\tanh(1/2*d*x+1/2*c)^3+5/4/d/(\tanh(1/2*d \\ & *x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*t \\ & anh(1/2*d*x+1/2*c)+3/8/d/a^2/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}*\arctanh(a*t \\ & anh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)})-3/8/d/a^2/((2*(b*(a \\ & +b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2) \\ & +a+2*b)*a)^{(1/2)})-9/4/d/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+ \\ & 4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*a/b^2*\tanh(1/2*d*x+1/2*c)^3+3/d/a^2*b/(\tanh(\\ & 1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2 \\ & *tanh(1/2*d*x+1/2*c)^5+3/d/a^2*b/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/ \\ & 2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3-3/4/d/(\tanh(1 \\ & /2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2* \\ & a/b^2*\tanh(1/2*d*x+1/2*c)-3/4/d/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2 \\ & *c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*a/b^2*\tanh(1/2*d*x+1/2*c)^7-9/4/d/(\tanh \\ & (1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+ \\ & a)^2*a/b^2*\tanh(1/2*d*x+1/2*c)^5-1/8/d/b/(b*(a+b))^{(1/2)}/((2*(b*(a+b))^{(1/2) \\ &)-a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}-a-2*b)* \\ & a)^{(1/2)})-1/4/d/a/b/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2* \\ & d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)})-1/8/d/b/(b*(a+b))^{(1/2)}/((2 \\ & *(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b)) \\ & ^{(1/2)}+a+2*b)*a)^{(1/2)})+1/4/d/a/b/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\arctan \\ & n(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}) \end{aligned}$$

maxima [B] time = 0.68, size = 332, normalized size = 2.53

$$\frac{3a^3 + 3a^2b - 3ab^2 - 3b^3 + (9a^3 - 13a^2b - 13ab^2 + 9b^3)e^{(-2dx-2c)} + 3(3a^3 - 5a^2b + 5ab^2 - 3b^3)e^{(-4dx-4c)}}{4(a^4b^2 + 2a^3b^3 + a^2b^4 + 4(a^4b^2 - a^2b^4)e^{(-2dx-2c)} + 2(3a^4b^2 - 2a^3b^3 + 3a^2b^4)e^{(-4dx-4c)} + 4(a^4b^2 - a^2b^4)e^{(-6dx-6c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $-1/4*(3*a^3 + 3*a^2*b - 3*a*b^2 - 3*b^3 + (9*a^3 - 13*a^2*b - 13*a*b^2 + 9*b^3)*e^{(-2*d*x - 2*c)} + 3*(3*a^3 - 5*a^2*b + 5*a*b^2 - 3*b^3)*e^{(-4*d*x - 4*c)} + (3*a^3 + a^2*b + a*b^2 + 3*b^3)*e^{(-6*d*x - 6*c)})/((a^4*b^2 + 2*a^3*b^3 + a^2*b^4 + 4*(a^4*b^2 - a^2*b^4)*e^{(-2*d*x - 2*c)} + 2*(3*a^4*b^2 - 2*a^3*b^3 + 3*a^2*b^4)*e^{(-4*d*x - 4*c)} + 4*(a^4*b^2 - a^2*b^4)*e^{(-6*d*x - 6*c)}) + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*e^{(-8*d*x - 8*c)})*d - 1/8*(3*a^2 - 2*a*b + 3*b^2)*arctan(1/2*((a + b)*e^{(-2*d*x - 2*c)} + a - b)/sqrt(a*b))/sqrt(a*b)*a^2*b^2*d)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + dx)^6 (b \tanh(c + dx)^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)^3),x)

[Out] int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)^3), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**6/(a+b*tanh(d*x+c)**2)**3,x)

[Out] Timed out

$$3.133 \quad \int \frac{\operatorname{sech}^7(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=156

$$\frac{(4a-3b)(a+b) \sinh(c+dx)}{8a^2b^2d((a+b) \sinh^2(c+dx)+a)} + \frac{\sqrt{a+b} (8a^2-4ab+3b^2) \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d} + \frac{(a+b) \sinh(c+dx)}{4abd((a+b) \sinh^2(c+dx)+a)}$$

[Out] $-\arctan(\sinh(d*x+c))/b^3/d+1/4*(a+b)*\sinh(d*x+c)/a/b/d/(a+(a+b)*\sinh(d*x+c)^2)^2-1/8*(4*a-3*b)*(a+b)*\sinh(d*x+c)/a^2/b^2/d/(a+(a+b)*\sinh(d*x+c)^2)+1/8*(8*a^2-4*a*b+3*b^2)*\arctan(\sinh(d*x+c)*(a+b)^{(1/2)}/a^{(1/2)})*(a+b)^{(1/2)}/a^{(5/2)}/b^3/d$

Rubi [A] time = 0.23, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3676, 414, 527, 522, 203, 205}

$$\frac{(4a-3b)(a+b) \sinh(c+dx)}{8a^2b^2d((a+b) \sinh^2(c+dx)+a)} + \frac{\sqrt{a+b} (8a^2-4ab+3b^2) \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d} + \frac{(a+b) \sinh(c+dx)}{4abd((a+b) \sinh^2(c+dx)+a)}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^7/(a + b*Tanh[c + d*x]^2)^3, x]

[Out] $-(\text{ArcTan}[\text{Sinh}[c + d*x]]/(b^3*d)) + (\text{Sqrt}[a + b]*(8*a^2 - 4*a*b + 3*b^2)*\text{ArcTan}[(\text{Sqrt}[a + b]*\text{Sinh}[c + d*x])/(\text{Sqrt}[a])])/(8*a^{(5/2)}*b^3*d) + ((a + b)*\text{Sinh}[c + d*x])/(4*a*b*d*(a + (a + b)*\text{Sinh}[c + d*x]^2)^2) - ((4*a - 3*b)*(a + b)*\text{Sinh}[c + d*x])/(8*a^2*b^2*d*(a + (a + b)*\text{Sinh}[c + d*x]^2))$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3676

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2*x^2)^(m + n*p + 1)/2, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\operatorname{sech}^7(c + dx)}{(a + b \tanh^2(c + dx))^3} dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+(a+b)x^2)^3} dx, x, \sinh(c + dx)\right)}{d}$$

$$= \frac{(a + b) \sinh(c + dx)}{4abd (a + (a + b) \sinh^2(c + dx))^2} - \frac{\operatorname{Subst}\left(\int \frac{a-3b-3(a+b)x^2}{(1+x^2)(a+(a+b)x^2)^2} dx, x, \sinh(c + dx)\right)}{4abd}$$

$$= \frac{(a + b) \sinh(c + dx)}{4abd (a + (a + b) \sinh^2(c + dx))^2} - \frac{(4a - 3b)(a + b) \sinh(c + dx)}{8a^2b^2d (a + (a + b) \sinh^2(c + dx))} + \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(c + dx)\right)}{4abd}$$

$$= \frac{(a + b) \sinh(c + dx)}{4abd (a + (a + b) \sinh^2(c + dx))^2} - \frac{(4a - 3b)(a + b) \sinh(c + dx)}{8a^2b^2d (a + (a + b) \sinh^2(c + dx))} - \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(c + dx)\right)}{4abd}$$

$$= -\frac{\tan^{-1}(\sinh(c + dx))}{b^3d} + \frac{\sqrt{a+b} (8a^2 - 4ab + 3b^2) \tan^{-1}\left(\frac{\sqrt{a+b} \sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d} + \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(c + dx)\right)}{4abd}$$

Mathematica [C] time = 3.38, size = 317, normalized size = 2.03

$$\frac{8b(4a^2+ab-3b^2) \sinh(c+dx)}{a^2((a+b) \cosh(2(c+dx))+a-b)} + \frac{i\sqrt{a+b}(8a^2-4ab+3b^2) \log((a+b) \cosh(2(c+dx))+a-b)}{a^{5/2}} + \frac{2\sqrt{a+b}(8a^2-4ab+3b^2) \tan^{-1}\left(\frac{\sqrt{a} \operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{a^{5/2}} - \frac{i(8a^2-4ab+3b^2) \operatorname{ArcTan}\left(\frac{\sqrt{a} \operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{4abd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^7/(a + b*Tanh[c + d*x]^2)^3, x]
```

```
[Out] -1/32*((2*Sqrt[a + b]*(8*a^2 - 4*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/a^(5/2) + (2*(8*a^3 + 4*a^2*b - a*b^2 + 3*b^3)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]) + 64*ArcTan[Tanh[(c + d*x)/2]] + (I*Sqrt[a + b]*(8*a^2 - 4*a*b + 3*b^2)*Log[a - b + (a + b)*Cosh[2*(c + d*x]])/a^(5/2) - (I*(8*a^3 + 4*a^2*b - a*b^2 + 3*b^3)*Log[a - b + (a + b)*Cosh[2*(c + d*x]])/(a^(5/2)*Sqrt[a + b]) - (32*b^2*(a + b)*Sinh[c + d*x])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])^2) + (8*b*(4*a^2 + a*b - 3*b^2)*Sinh[c + d*x])/(a^2*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(b^3*d)
```

fricas [B] time = 0.60, size = 8070, normalized size = 51.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/16*(4*(4*a^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*\cosh(d*x + c)^7 + 28*(4*a^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^6 + 4*(4*a^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*\sinh(d*x + c)^7 + 4*(4*a^3*b - 19*a^2*b^2 - 14*a*b^3 + 9*b^4)*\cosh(d*x + c)^5 + 4*(4*a^3*b - 19*a^2*b^2 - 14*a*b^3 + 9*b^4 + 21*(4*a^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^5 + 20*(7*(4*a^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*\cosh(d*x + c)^3 + (4*a^3*b - 19*a^2*b^2 - 14*a*b^3 + 9*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^4 - 4*(4*a^3*b - 19*a^2*b^2 - 14*a*b^3 + 9*b^4)*\cosh(d*x + c)^3 + 4*(35*(4*a^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*\cosh(d*x + c)^4 - 4*a^3*b + 19*a^2*b^2 + 14*a*b^3 - 9*b^4 + 10*(4*a^3*b - 19*a^2*b^2 - 14*a*b^3 + 9*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^3 + 4*(21*(4*a^3*b + 5*a^2*b^2 - 2*a*b^3 - 3*b^4)*\cosh(d*x + c)^5 + 10*(4*a^3*b - 19*a^2*b^2 - 14*a*b^3 + 9*b^4)*\cosh(d*x + c)^3 - 3*(4*a^3*b - 19*a^2*b^2 - 14*a*b^3 + 9*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^2 - ((8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^8 + 8*(8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\sinh(d*x + c)^8 + 4*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c)^6 + 4*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4 + 7*(8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^3 + 3*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(24*a^4 - 28*a^3*b + 41*a^2*b^2 - 18*a*b^3 + 9*b^4)*\cosh(d*x + c)^4 + 2*(35*(8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^4 + 24*a^4 - 28*a^3*b + 41*a^2*b^2 - 18*a*b^3 + 9*b^4 + 30*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4 + 8*(7*(8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^5 + 10*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c)^3 + (24*a^4 - 28*a^3*b + 41*a^2*b^2 - 18*a*b^3 + 9*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c)^2 + 4*(7*(8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^6 + 15*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c)^4 + 8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4 + 3*(24*a^4 - 28*a^3*b + 41*a^2*b^2 - 18*a*b^3 + 9*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((8*a^4 + 12*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^7 + 3*(8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c)^5 + (24*a^4 - 28*a^3*b + 41*a^2*b^2 - 18*a*b^3 + 9*b^4)*\cosh(d*x + c)^3 + (8*a^4 - 4*a^3*b - 5*a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c))*\sinh(d*x + c))^2 + 4*(a*\cosh(d*x + c)^3 + 3*a*\cosh(d*x + c)*\sinh(d*x + c)^2 + a*\sinh(d*x + c)^3 - a*\cosh(d*x + c) + (3*a*\cosh(d*x + c)^2 - a)*\sinh(d*x + c))*\sqrt{-(a + b)/a} + a + b)/((a + b)*\cosh(d*x + c)^4 + 4*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a + b)*\sinh(d*x + c)^4 - 2*(3*a + b)*\cosh(d*x + c)^2 + 2*(3*(a + b)*\cosh(d*x + c)^2 - 3*a - b)*\sinh(d*x + c)^2 + 4*((a + b)*\cosh(d*x + c)^3 - (3*a + b)*\cosh(d*x + c))*\sinh(d*x + c) + 4*(a*\cosh(d*x + c)^3 + 3*a*\cosh(d*x + c)*\sinh(d*x + c)^2 + a*\sinh(d*x + c)^3 - a*\cosh(d*x + c) + (3*a*\cosh(d*x + c)^2 - a)*\sinh(d*x + c))*\sqrt{-(a + b)/a} + a + b)/((a + b)*\cosh(d*x + c)^4 + 4*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a + b)*\sinh(d*x + c)^4 + 2*(a - b)*\cosh(d*x + c)^2 + 2*(3*(a + b)*\cosh(d*x + c)^2 + a - b)*\sinh(d*x + c)^2 + 4*((a + b)*\cosh(d*x + c)^3 + (a - b)*\cosh(d*x + c))*\sinh(d*x + c) + a + b)) + 32*((a^4 + 2*a^3*b + a^2*b^2)*\cosh(d*x + c)^8 + 8*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^4 + 2*a^3*b + a^2*b^2)*\sinh(d*x + c)^8 + 4*(a^4 - a^2*b^2)*\cosh(d*x + c)^6 + 4*(a^4 - a^2*b^2 + 7*(a^4 + 2*a^3*b + a^2*b^2))*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(d*x + c)^3 + 3*(a^4 - a^2*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^4 + 2*(35*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(d$$

$$\begin{aligned}
& a^2b^2 + 4ab^3 - 3b^4) \cosh(dx + c)^2 + 4(7(8a^4 + 12a^3b + 3a^2b^2 + 2ab^3 + 3b^4) \cosh(dx + c)^6 + 15(8a^4 - 4a^3b - 5a^2b^2 + 4ab^3 - 3b^4) \cosh(dx + c)^4 + 8a^4 - 4a^3b - 5a^2b^2 + 4ab^3 - 3b^4 + 3(24a^4 - 28a^3b + 41a^2b^2 - 18ab^3 + 9b^4) \cosh(dx + c)^2) \sinh(dx + c)^2 + 8((8a^4 + 12a^3b + 3a^2b^2 + 2ab^3 + 3b^4) \cosh(dx + c)^7 + 3(8a^4 - 4a^3b - 5a^2b^2 + 4ab^3 - 3b^4) \cosh(dx + c)^5 + (24a^4 - 28a^3b + 41a^2b^2 - 18ab^3 + 9b^4) \cosh(dx + c)^3 + (8a^4 - 4a^3b - 5a^2b^2 + 4ab^3 - 3b^4) \cosh(dx + c)) \sinh(dx + c)) \sqrt{(a + b)/a} \arctan(1/2 \sqrt{(a + b)/a} (\cosh(dx + c) + \sinh(dx + c))) - ((8a^4 + 12a^3b + 3a^2b^2 + 2ab^3 + 3b^4) \cosh(dx + c)^8 + 8(8a^4 + 12a^3b + 3a^2b^2 + 2ab^3 + 3b^4) \cosh(dx + c) \sinh(dx + c)^7 + (8a^4 + 12a^3b + 3a^2b^2 + 2ab^3 + 3b^4) \sinh(dx + c)^8 + 4(8a^4 - 4a^3b - 5a^2b^2 + 4ab^3 - 3b^4) \cosh(dx + c)^6 + 4(8a^4 - 4a^3b - 5a^2b^2 + 4ab^3 - 3b^4 + 7(8a^4 + 12a^3b + 3a^2b^2 + 2ab^3 + 3b^4) \cosh(dx + c)^2) \sinh(dx + c)^6 + 8(7(8a^4 + 12a^3b + 3a^2b^2 + 2ab^3 + 3b^4) \cosh(dx + c)^3 + 3(8a^4 - 4a^3b - 5a^2b^2 + 4ab^3 - 3b^4) \cosh(dx + c)) \sinh(dx + c)^5 + 2(24a^4 - 28a^3b + 41a^2b^2 - 18ab^3 + 9b^4) \cosh(dx + c)^4 + 2(35(8a^4 + 12a^3b + 3a^2b^2 + 2ab^3 + 3b^4) \cosh(dx + c)^4 + 24a^4 - 28a^3b + 41a^2b^2 - 18ab^3 + 9b^4 + 30(8a^4 - 4a^3b - 5a^2b^2 + 4ab^3 - 3b^4) \cosh(dx + c)^2) \sinh(dx + c)^4 + 8a^4 + 12a^3b + 3a^2b^2 + 2ab^3 + 3b^4 + 8(7(8a^4 + 12a^3b + 3a^2b^2 + 2ab^3 + 3b^4) \cosh(dx + c)^5 + 10(8a^4 - 4a^3b - 5a^2b^2 + 4ab^3 - 3b^4) \cosh(dx + c)^3 + (24a^4 - 28a^3b + 41a^2b^2 - 18ab^3 + 9b^4) \cosh(dx + c)) \sinh(dx + c)^3 + 4(8a^4 - 4a^3b - 5a^2b^2 + 4ab^3 - 3b^4) \cosh(dx + c)^2 + 4(7(8a^4 + 12a^3b + 3a^2b^2 + 2ab^3 + 3b^4) \cosh(dx + c)^6 + 15(8a^4 - 4a^3b - 5a^2b^2 + 4ab^3 - 3b^4) \cosh(dx + c)^4 + 8a^4 - 4a^3b - 5a^2b^2 + 4ab^3 - 3b^4 + 3(24a^4 - 28a^3b + 41a^2b^2 - 18ab^3 + 9b^4) \cosh(dx + c)^2) \sinh(dx + c)^2 + 8((8a^4 + 12a^3b + 3a^2b^2 + 2ab^3 + 3b^4) \cosh(dx + c)^7 + 3(8a^4 - 4a^3b - 5a^2b^2 + 4ab^3 - 3b^4) \cosh(dx + c)^5 + (24a^4 - 28a^3b + 41a^2b^2 - 18ab^3 + 9b^4) \cosh(dx + c)^3 + (8a^4 - 4a^3b - 5a^2b^2 + 4ab^3 - 3b^4) \cosh(dx + c)) \sinh(dx + c)) \sqrt{(a + b)/a} \arctan(1/2((a + b) \cosh(dx + c)^3 + 3(a + b) \cosh(dx + c) \sinh(dx + c)^2 + (a + b) \sinh(dx + c)^3 + (3a - b) \cosh(dx + c) + (3(a + b) \cosh(dx + c)^2 + 3a - b) \sinh(dx + c)) \sqrt{(a + b)/a} / (a + b)) + 16((a^4 + 2a^3b + a^2b^2) \cosh(dx + c)^8 + 8(a^4 + 2a^3b + a^2b^2) \cosh(dx + c) \sinh(dx + c)^7 + (a^4 + 2a^3b + a^2b^2) \sinh(dx + c)^8 + 4(a^4 - a^2b^2) \cosh(dx + c)^6 + 4(a^4 - a^2b^2 + 7(a^4 + 2a^3b + a^2b^2) \cosh(dx + c)^2) \sinh(dx + c)^6 + 8(7(a^4 + 2a^3b + a^2b^2) \cosh(dx + c)^3 + 3(a^4 - a^2b^2) \cosh(dx + c)) \sinh(dx + c)^5 + 2(3a^4 - 2a^3b + 3a^2b^2) \cosh(dx + c)^4 + 2(35(a^4 + 2a^3b + a^2b^2) \cosh(dx + c)^4 + 3a^4 - 2a^3b + 3a^2b^2 + 30(a^4 - a^2b^2) \cosh(dx + c)^2) \sinh(dx + c)^4 + a^4 + 2a^3b + a^2b^2 + 8(7(a^4 + 2a^3b + a^2b^2) \cosh(dx + c)^5 + 10(a^4 - a^2b^2) \cosh(dx + c)^3 + (3a^4 - 2a^3b + 3a^2b^2) \cosh(dx + c)) \sinh(dx + c)^3 + 4(a^4 - a^2b^2) \cosh(dx + c)^2 + 4(7(a^4 + 2a^3b + a^2b^2) \cosh(dx + c)^6 + 15(a^4 - a^2b^2) \cosh(dx + c)^4 + a^4 - a^2b^2 + 3(3a^4 - 2a^3b + 3a^2b^2) \cosh(dx + c)^2) \sinh(dx + c)^2 + 8((a^4 + 2a^3b + a^2b^2) \cosh(dx + c)^7 + 3(a^4 - a^2b^2) \cosh(dx + c)^5 + (3a^4 - 2a^3b + 3a^2b^2) \cosh(dx + c)^3 + (a^4 - a^2b^2) \cosh(dx + c)) \sinh(dx + c)) \arctan(\cosh(dx + c) + \sinh(dx + c)) - 2(4a^3b + 5a^2b^2 - 2ab^3 - 3b^4) \cosh(dx + c) + 2(7(4a^3b + 5a^2b^2 - 2ab^3 - 3b^4) \cosh(dx + c)^6 + 5(4a^3b - 19a^2b^2 - 14ab^3 + 9b^4) \cosh(dx + c)^4 - 4a^3b - 5a^2b^2 + 2ab^3 + 3b^4 - 3(4a^3b - 19a^2b^2 - 14ab^3 + 9b^4) \cosh(dx + c)^2) \sinh(dx + c)) / ((a^4b^3 + 2a^3b^4 + a^2b^5) d \cosh(dx + c)^8 + 8(a^4b^3 + 2a^3b^4 + a^2b^5) d \cosh(dx + c) \sinh(dx + c)^7 + (a^4b^3 + 2a^3b^4 + a^2b^5) d \sinh(dx + c)^8 + 4(a^4b^3 - a^2b^5) d \cosh(dx + c)^6 + 4(7(a^4b^3 + 2a^3b^4 + a^2b^5) d \cosh(dx + c)^2 + (a^4b^3 - a^2b^5) d
\end{aligned}$$

$$d) \sinh(dx + c)^6 + 2*(3*a^4*b^3 - 2*a^3*b^4 + 3*a^2*b^5)*d*\cosh(dx + c)^4 + 8*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*\cosh(dx + c)^3 + 3*(a^4*b^3 - a^2*b^5)*d*\cosh(dx + c))*\sinh(dx + c)^5 + 2*(35*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*\cosh(dx + c)^4 + 30*(a^4*b^3 - a^2*b^5)*d*\cosh(dx + c)^2 + (3*a^4*b^3 - 2*a^3*b^4 + 3*a^2*b^5)*d)*\sinh(dx + c)^4 + 4*(a^4*b^3 - a^2*b^5)*d*\cosh(dx + c)^2 + 8*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*\cosh(dx + c)^5 + 10*(a^4*b^3 - a^2*b^5)*d*\cosh(dx + c)^3 + (3*a^4*b^3 - 2*a^3*b^4 + 3*a^2*b^5)*d*\cosh(dx + c))*\sinh(dx + c)^3 + 4*(7*(a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*\cosh(dx + c)^6 + 15*(a^4*b^3 - a^2*b^5)*d*\cosh(dx + c)^4 + 3*(3*a^4*b^3 - 2*a^3*b^4 + 3*a^2*b^5)*d*\cosh(dx + c)^2 + (a^4*b^3 - a^2*b^5)*d)*\sinh(dx + c)^2 + (a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d + 8*((a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*d*\cosh(dx + c)^7 + 3*(a^4*b^3 - a^2*b^5)*d*\cosh(dx + c)^5 + (3*a^4*b^3 - 2*a^3*b^4 + 3*a^2*b^5)*d*\cosh(dx + c)^3 + (a^4*b^3 - a^2*b^5)*d*\cosh(dx + c))*\sinh(dx + c)]$$

giac [B] time = 0.93, size = 1040, normalized size = 6.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)^7/(a+b*tanh(dx+c)^2)^3,x, algorithm="giac")

[Out] $\frac{1}{8} * ((2*(8*a^3 - 4*a^2*b + 3*a*b^2)*\sqrt{a^2 - b^2 + 2*\sqrt{-a*b}}*(a + b)) * b^2 * \text{abs}(a*e^{(2*c)} + b*e^{(2*c)}) + (8*a^3 + 4*a^2*b - a*b^2 + 3*b^3)*\sqrt{a^2 - b^2 + 2*\sqrt{-a*b}}*(a + b)) * \sqrt{-a*b} * \text{abs}(a*e^{(2*c)} + b*e^{(2*c)}) * \text{abs}(b - (8*a^3*b^2 - 12*a^2*b^3 + 7*a*b^4 - 3*b^5)*\sqrt{a^2 - b^2 + 2*\sqrt{-a*b}}*(a + b)) * \text{abs}(a*e^{(2*c)} + b*e^{(2*c)})) * \arctan(e^{(d*x)}/\sqrt{(a^3*b^3*e^{(2*c)} - a^2*b^4*e^{(2*c)} + \sqrt{(a^3*b^3*e^{(2*c)} - a^2*b^4*e^{(2*c)})^2 - (a^3*b^3*e^{(4*c)} + a^2*b^4*e^{(4*c}))}) * (a^3*b^3 + a^2*b^4)}) / (a^3*b^3*e^{(4*c)} + a^2*b^4*e^{(4*c)})) * e^{(-2*c)} / ((a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*\sqrt{-a*b} * \text{abs}(b) - (2*(8*a^3 - 4*a^2*b + 3*a*b^2)*\sqrt{a^2 - b^2 - 2*\sqrt{-a*b}}*(a + b)) * b^2 * \text{abs}(a*e^{(2*c)} + b*e^{(2*c)}) - (8*a^3 + 4*a^2*b - a*b^2 + 3*b^3)*\sqrt{a^2 - b^2 - 2*\sqrt{-a*b}}*(a + b)) * \sqrt{-a*b} * \text{abs}(a*e^{(2*c)} + b*e^{(2*c)}) * \text{abs}(b) - (8*a^3*b^2 - 12*a^2*b^3 + 7*a*b^4 - 3*b^5)*\sqrt{a^2 - b^2 - 2*\sqrt{-a*b}}*(a + b)) * \text{abs}(a*e^{(2*c)} + b*e^{(2*c)})) * \arctan(e^{(d*x)}/\sqrt{(a^3*b^3*e^{(2*c)} - a^2*b^4*e^{(2*c)} - \sqrt{(a^3*b^3*e^{(2*c)} - a^2*b^4*e^{(2*c)})^2 - (a^3*b^3*e^{(4*c)} + a^2*b^4*e^{(4*c}))}) * (a^3*b^3 + a^2*b^4)}) / (a^3*b^3*e^{(4*c)} + a^2*b^4*e^{(4*c)})) * e^{(-2*c)} / ((a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 + a^2*b^6)*\sqrt{-a*b} * \text{abs}(b) - 16*\arctan(e^{(d*x + c)})/b^3 - 2*(4*a^3*e^{(7*d*x + 7*c)} + 5*a^2*b*e^{(7*d*x + 7*c)} - 2*a*b^2*e^{(7*d*x + 7*c)} - 3*b^3*e^{(7*d*x + 7*c)} + 4*a^3*e^{(5*d*x + 5*c)} - 19*a^2*b*e^{(5*d*x + 5*c)} - 14*a*b^2*e^{(5*d*x + 5*c)} + 9*b^3*e^{(5*d*x + 5*c)} - 4*a^3*e^{(3*d*x + 3*c)} + 19*a^2*b*e^{(3*d*x + 3*c)} + 14*a*b^2*e^{(3*d*x + 3*c)} - 9*b^3*e^{(3*d*x + 3*c)} - 4*a^3*e^{(d*x + c)} - 5*a^2*b*e^{(d*x + c)} + 2*a*b^2*e^{(d*x + c)} + 3*b^3*e^{(d*x + c)}) / ((a*e^{(4*d*x + 4*c)} + b*e^{(4*d*x + 4*c)} + 2*a*e^{(2*d*x + 2*c)} - 2*b*e^{(2*d*x + 2*c)} + a + b)^2 * a^2 * b^2) / d$

maple [B] time = 0.40, size = 1907, normalized size = 12.22

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(dx+c)^7/(a+b*tanh(dx+c)^2)^3,x)

[Out] $\frac{1}{d} * \frac{1}{b^3 * a} * ((2*(b*(a+b))^{(1/2)} + a + 2*b) * a)^{(1/2)} * \arctan(a * \tanh(1/2 * d * x + 1/2 * c)) / ((2*(b*(a+b))^{(1/2)} + a + 2*b) * a)^{(1/2)} - 1/8 * d / (b * (a+b))^{(1/2)} / a / ((2*(b*(a+b))^{(1/2)} - a - 2*b) * a)^{(1/2)} * \operatorname{arctanh}(a * \tanh(1/2 * d * x + 1/2 * c)) / ((2*(b*(a+b))^{(1/2)} - a - 2*b) * a)^{(1/2)} - 1/8 * d / (b * (a+b))^{(1/2)} / a / ((2*(b*(a+b))^{(1/2)} + a + 2*b) * a)^{(1/2)} * \operatorname{arctan}(a * \tanh(1/2 * d * x + 1/2 * c)) / ((2*(b*(a+b))^{(1/2)} + a + 2*b) * a)^{(1/2)} + 1/d * a / b^2 /$

$$\begin{aligned} & (b*(a+b))^{(1/2)} / ((2*(b*(a+b))^{(1/2)} - a - 2*b)*a)^{(1/2)} * \operatorname{arctanh}(a*\tanh(1/2*d*x + \\ & 1/2*c)) / ((2*(b*(a+b))^{(1/2)} - a - 2*b)*a)^{(1/2)} + 1/d*a/b^2 / (b*(a+b))^{(1/2)} / ((2*(\\ & b*(a+b))^{(1/2)} + a + 2*b)*a)^{(1/2)} * \operatorname{arctan}(a*\tanh(1/2*d*x + 1/2*c)) / ((2*(b*(a+b))^{(1/2)} + \\ & a + 2*b)*a)^{(1/2)} - 1/4/d / (\tanh(1/2*d*x + 1/2*c))^{4*a + 2*\tanh(1/2*d*x + 1/2*c)^{2*a + 4*\tanh(1/2*d*x + 1/2*c)^{2*b + a}} \\ & ^{2/b*\tanh(1/2*d*x + 1/2*c)^{7 + 23/4/d} / (\tanh(1/2*d*x + 1/2*c))^{4*a + 2*\tanh(1/2*d*x + 1/2*c)^{2*b + a}} \\ & ^{2/b*\tanh(1/2*d*x + 1/2*c)^{5 - 23/4/d} / (\tanh(1/2*d*x + 1/2*c))^{4*a + 2*\tanh(1/2*d*x + 1/2*c)^{2*b + a}} \\ & ^{2/b*\tanh(1/2*d*x + 1/2*c)^{3 + 1/4/d} / (\tanh(1/2*d*x + 1/2*c))^{4*a + 2*\tanh(1/2*d*x + 1/2*c)^{2*b + a}} \\ & ^{2/b*\tanh(1/2*d*x + 1/2*c)^{-2/d/b^3} * \operatorname{arctan}(\tanh(1/2*d*x + 1/2*c)) - 1/d/b^3*a / ((2*(b*(a+b))^{(1/2)} - a - 2*b)*a)^{(1/2)} \\ & * \operatorname{arctanh}(a*\tanh(1/2*d*x + 1/2*c)) / ((2*(b*(a+b))^{(1/2)} - a - 2*b)*a)^{(1/2)} + 3/8/d/a^2*b / (b*(a+b))^{(1/2)} / ((2*(b*(a+b))^{(1/2)} + a + 2*b)*a)^{(1/2)} \\ & * \operatorname{arctan}(a*\tanh(1/2*d*x + 1/2*c)) / ((2*(b*(a+b))^{(1/2)} + a + 2*b)*a)^{(1/2)} + 3/8/d/a^2*b / (b*(a+b))^{(1/2)} / ((2*(b*(a+b))^{(1/2)} - a - 2*b)*a)^{(1/2)} \\ & * \operatorname{arctanh}(a*\tanh(1/2*d*x + 1/2*c)) / ((2*(b*(a+b))^{(1/2)} - a - 2*b)*a)^{(1/2)} - 1/2/d/b^2 / ((2*(b*(a+b))^{(1/2)} - a - 2*b)*a)^{(1/2)} \\ & + 1/2/d/b^2 / ((2*(b*(a+b))^{(1/2)} + a + 2*b)*a)^{(1/2)} * \operatorname{arctan}(a*\tanh(1/2*d*x + 1/2*c)) / ((2*(b*(a+b))^{(1/2)} + a + 2*b)*a)^{(1/2)} \\ & - 5/4/d / (\tanh(1/2*d*x + 1/2*c))^{4*a + 2*\tanh(1/2*d*x + 1/2*c)^{2*b + a}} / a*\tanh(1/2*d*x + 1/2*c)^{7 + 7/4/d} / (\tanh(1/2*d*x + 1/2*c))^{4*a + 2*\tanh(1/2*d*x + 1/2*c)^{2*b + a}} \\ & ^{2/a*\tanh(1/2*d*x + 1/2*c)^{5 - 7/4/d} / (\tanh(1/2*d*x + 1/2*c))^{4*a + 2*\tanh(1/2*d*x + 1/2*c)^{2*b + a}} \\ & ^{2/a*\tanh(1/2*d*x + 1/2*c)^{3 + 5/4/d} / (\tanh(1/2*d*x + 1/2*c))^{4*a + 2*\tanh(1/2*d*x + 1/2*c)^{2*b + a}} \\ & - 3/8/d/a^2 / ((2*(b*(a+b))^{(1/2)} - a - 2*b)*a)^{(1/2)} * \operatorname{arctanh}(a*\tanh(1/2*d*x + 1/2*c)) / ((2*(b*(a+b))^{(1/2)} - a - 2*b)*a)^{(1/2)} \\ & + 3/8/d/a^2 / ((2*(b*(a+b))^{(1/2)} + a + 2*b)*a)^{(1/2)} * \operatorname{arctan}(a*\tanh(1/2*d*x + 1/2*c)) / ((2*(b*(a+b))^{(1/2)} + a + 2*b)*a)^{(1/2)} \\ & - 1/d / (\tanh(1/2*d*x + 1/2*c))^{4*a + 2*\tanh(1/2*d*x + 1/2*c)^{2*b + a}} / a/b^2*\tanh(1/2*d*x + 1/2*c)^{3 - 3/d/a^2*b} / (\tanh(1/2*d*x + 1/2*c))^{4*a + 2*\tanh(1/2*d*x + 1/2*c)^{2*b + a}} \\ & + 4*\tanh(1/2*d*x + 1/2*c)^{2*b + a}} / a/b^2*\tanh(1/2*d*x + 1/2*c)^{5 + 3/d/a^2*b} / (\tanh(1/2*d*x + 1/2*c))^{4*a + 2*\tanh(1/2*d*x + 1/2*c)^{2*b + a}} \\ & ^{2*\tanh(1/2*d*x + 1/2*c)^{3 - 1/d} / (\tanh(1/2*d*x + 1/2*c))^{4*a + 2*\tanh(1/2*d*x + 1/2*c)^{2*b + a}} / a/b^2*\tanh(1/2*d*x + 1/2*c) + 1/d / (\tanh(1/2*d*x + 1/2*c))^{4*a + 2*\tanh(1/2*d*x + 1/2*c)^{2*b + a}} \\ & / a/b^2*\tanh(1/2*d*x + 1/2*c)^{7 + 1/d} / (\tanh(1/2*d*x + 1/2*c))^{4*a + 2*\tanh(1/2*d*x + 1/2*c)^{2*b + a}} / a/b^2*\tanh(1/2*d*x + 1/2*c)^{5 + 1/2/d/b} / (b*(a+b))^{(1/2)} / ((2*(b*(a+b))^{(1/2)} - a - 2*b)*a)^{(1/2)} \\ & * \operatorname{arctanh}(a*\tanh(1/2*d*x + 1/2*c)) / ((2*(b*(a+b))^{(1/2)} - a - 2*b)*a)^{(1/2)} + 1/8/d/a/b / ((2*(b*(a+b))^{(1/2)} - a - 2*b)*a)^{(1/2)} * \operatorname{arctanh}(a*\tanh(1/2*d*x + 1/2*c)) / ((2*(b*(a+b))^{(1/2)} - a - 2*b)*a)^{(1/2)} \\ & + 1/2/d/b / (b*(a+b))^{(1/2)} / ((2*(b*(a+b))^{(1/2)} + a + 2*b)*a)^{(1/2)} * \operatorname{arctan}(a*\tanh(1/2*d*x + 1/2*c)) / ((2*(b*(a+b))^{(1/2)} + a + 2*b)*a)^{(1/2)} \\ & - 1/8/d/a/b / ((2*(b*(a+b))^{(1/2)} + a + 2*b)*a)^{(1/2)} * \operatorname{arctan}(a*\tanh(1/2*d*x + 1/2*c)) / ((2*(b*(a+b))^{(1/2)} + a + 2*b)*a)^{(1/2)} \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(4a^3e^{7c} + 5a^2be^{7c} - 2ab^2e^{7c} - 3b^3e^{7c})e^{7dx} + (4a^3e^{5c} - 19a^2be^{5c} - 14ab^2e^{5c} + 9b^3e^{5c})e^{5dx} - (4a^3e^{3c} - 19a^2be^{3c} - 14ab^2e^{3c} + 9b^3e^{3c})e^{3dx} - (4a^3e^c + 5a^2be^c - 2ab^2e^c - 3b^3e^c)e^c}{4(a^4b^2d + 2a^3b^3d + a^2b^4d + (a^4b^2de^{8c} + 2a^3b^3de^{8c} + a^2b^4de^{8c})e^{8dx} + 4(a^4b^2de^{6c} - a^2b^4de^{6c})e^{6dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/4*((4*a^3*e^{(7*c)} + 5*a^2*b*e^{(7*c)} - 2*a*b^2*e^{(7*c)} - 3*b^3*e^{(7*c)})*e^{(7*d*x)} \\ & + (4*a^3*e^{(5*c)} - 19*a^2*b*e^{(5*c)} - 14*a*b^2*e^{(5*c)} + 9*b^3*e^{(5*c)})*e^{(5*d*x)} - (4*a^3*e^{(3*c)} - 19*a^2*b*e^{(3*c)} - 14*a*b^2*e^{(3*c)} + 9*b^3*e^{(3*c)})*e^{(3*d*x)} \\ & - (4*a^3*e^c + 5*a^2*b*e^c - 2*a*b^2*e^c - 3*b^3*e^c)*e^{(d*x)}) / (a^4*b^2*d + 2*a^3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^{(8*c)} + 2*a^3*b^3*d*e^{(8*c)} + a^2*b^4*d*e^{(8*c)})*e^{(8*d*x)} \\ & + 4*(a^4*b^2*d*e^{(6*c)} - a^2*b^4*d*e^{(6*c)})*e^{(6*d*x)} + 2*(3*a^4*b^2*d*e^{(4*c)} - 2*a^3*b^3*d*e^{(4*c)} + \end{aligned}$$

```
3*a^2*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^4*b^2*d*e^(2*c) - a^2*b^4*d*e^(2*c))*
e^(2*d*x)) - 2*arctan(e^(d*x + c))/(b^3*d) + 128*integrate(1/512*((8*a^3*e^
(3*c) + 4*a^2*b*e^(3*c) - a*b^2*e^(3*c) + 3*b^3*e^(3*c))*e^(3*d*x) + (8*a^3
*e^c + 4*a^2*b*e^c - a*b^2*e^c + 3*b^3*e^c)*e^(d*x))/(a^3*b^3 + a^2*b^4 + (
a^3*b^3*e^(4*c) + a^2*b^4*e^(4*c))*e^(4*d*x) + 2*(a^3*b^3*e^(2*c) - a^2*b^4
*e^(2*c))*e^(2*d*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cosh(c + dx)^7 (b \tanh(c + dx)^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cosh(c + d*x)^7*(a + b*tanh(c + d*x)^2)^3), x)

[Out] int(1/(cosh(c + d*x)^7*(a + b*tanh(c + d*x)^2)^3), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**7/(a+b*tanh(d*x+c)**2)**3, x)

[Out] Timed out

3.134 $\int \tanh^4(c + dx) (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=54

$$-\frac{(a+b)\tanh^3(c+dx)}{3d} - \frac{(a+b)\tanh(c+dx)}{d} + x(a+b) - \frac{b\tanh^5(c+dx)}{5d}$$

[Out] (a+b)*x-(a+b)*tanh(d*x+c)/d-1/3*(a+b)*tanh(d*x+c)^3/d-1/5*b*tanh(d*x+c)^5/d

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3631, 3473, 8}

$$-\frac{(a+b)\tanh^3(c+dx)}{3d} - \frac{(a+b)\tanh(c+dx)}{d} + x(a+b) - \frac{b\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2), x]

[Out] (a + b)*x - ((a + b)*Tanh[c + d*x])/d - ((a + b)*Tanh[c + d*x]^3)/(3*d) - (b*Tanh[c + d*x]^5)/(5*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3631

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)^2]), x_Symbol] := Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] && !LeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \tanh^4(c + dx) (a + b \tanh^2(c + dx)) dx &= -\frac{b \tanh^5(c + dx)}{5d} + (a + b) \int \tanh^4(c + dx) dx \\ &= -\frac{(a + b) \tanh^3(c + dx)}{3d} - \frac{b \tanh^5(c + dx)}{5d} + (a + b) \int \tanh^2(c + dx) dx \\ &= -\frac{(a + b) \tanh(c + dx)}{d} - \frac{(a + b) \tanh^3(c + dx)}{3d} - \frac{b \tanh^5(c + dx)}{5d} + \\ &= (a + b)x - \frac{(a + b) \tanh(c + dx)}{d} - \frac{(a + b) \tanh^3(c + dx)}{3d} - \frac{b \tanh^5(c + dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.04, size = 97, normalized size = 1.80

$$\frac{a \tanh^{-1}(\tanh(c + dx))}{d} - \frac{a \tanh^3(c + dx)}{3d} - \frac{a \tanh(c + dx)}{d} + \frac{b \tanh^{-1}(\tanh(c + dx))}{d} - \frac{b \tanh^5(c + dx)}{5d} - \frac{b \tanh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2), x]

[Out] (a*ArcTanh[Tanh[c + d*x]])/d + (b*ArcTanh[Tanh[c + d*x]])/d - (a*Tanh[c + d*x])/d - (b*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3*d) - (b*Tanh[c + d*x]^3)/(3*d) - (b*Tanh[c + d*x]^5)/(5*d)

fricas [B] time = 0.41, size = 339, normalized size = 6.28

$$\frac{(15(a+b)dx + 20a + 23b) \cosh(dx+c)^5 + 5(15(a+b)dx + 20a + 23b) \cosh(dx+c) \sinh(dx+c)^4 - (20a + 23b) \sinh(dx+c)^5}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] 1/15*((15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c)^5 + 5*(15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c)*sinh(d*x + c)^4 - (20*a + 23*b)*sinh(d*x + c)^5 + 5*(15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c)^3 - 5*(2*(20*a + 23*b)*cosh(d*x + c)^2 + 8*a + 5*b)*sinh(d*x + c)^3 + 5*(2*(15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c)^3 + 3*(15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c) - 5*((20*a + 23*b)*cosh(d*x + c)^4 + 3*(8*a + 5*b)*cosh(d*x + c)^2 + 4*a + 10*b)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))

giac [B] time = 0.21, size = 134, normalized size = 2.48

$$\frac{15(dx+c)(a+b) + \frac{2(30ae^{8dx+8c} + 45be^{8dx+8c} + 90ae^{6dx+6c} + 90be^{6dx+6c} + 110ae^{4dx+4c} + 140be^{4dx+4c} + 70ae^{2dx+2c} + 70be^{2dx+2c})}{(e^{2dx+2c}+1)^5}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2), x, algorithm="giac")

[Out] 1/15*(15*(d*x + c)*(a + b) + 2*(30*a*e^(8*d*x + 8*c) + 45*b*e^(8*d*x + 8*c) + 90*a*e^(6*d*x + 6*c) + 90*b*e^(6*d*x + 6*c) + 110*a*e^(4*d*x + 4*c) + 140*b*e^(4*d*x + 4*c) + 70*a*e^(2*d*x + 2*c) + 70*b*e^(2*d*x + 2*c) + 20*a + 23*b)/(e^(2*d*x + 2*c) + 1)^5)/d

maple [B] time = 0.02, size = 128, normalized size = 2.37

$$\frac{b(\tanh^5(dx+c))}{5d} - \frac{a(\tanh^3(dx+c))}{3d} - \frac{b(\tanh^3(dx+c))}{3d} - \frac{a \tanh(dx+c)}{d} - \frac{b \tanh(dx+c)}{d} - \frac{\ln(\tanh(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2), x)

[Out] -1/5*b*tanh(d*x+c)^5/d - 1/3/d*a*tanh(d*x+c)^3 - 1/3*b*tanh(d*x+c)^3/d - a*tanh(d*x+c)/d - b*tanh(d*x+c)/d - 1/2/d*ln(tanh(d*x+c)-1)*a - 1/2/d*ln(tanh(d*x+c)-1)*b + 1/2/d*ln(1+tanh(d*x+c))*a + 1/2/d*ln(1+tanh(d*x+c))*b

maxima [B] time = 0.33, size = 199, normalized size = 3.69

$$\frac{1}{15} b \left(15x + \frac{15c}{d} - \frac{2(70e^{(-2dx-2c)} + 140e^{(-4dx-4c)} + 90e^{(-6dx-6c)} + 45e^{(-8dx-8c)} + 23)}{d(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} \right) + \frac{1}{3} a \left(3x + \frac{3c}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2), x, algorithm="maxima")

```
[Out] 1/15*b*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*
e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^
(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) + 1))) + 1/3*a*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c
) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))
)
```

mupad [B] time = 1.22, size = 50, normalized size = 0.93

$$x(a+b) - \frac{\tanh(c+dx)^3(a+b)}{3d} - \frac{b \tanh(c+dx)^5}{5d} - \frac{\tanh(c+dx)(a+b)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(c + d*x)^4*(a + b*tanh(c + d*x)^2), x)
```

```
[Out] x*(a + b) - (tanh(c + d*x)^3*(a + b))/(3*d) - (b*tanh(c + d*x)^5)/(5*d) - (
tanh(c + d*x)*(a + b))/d
```

sympy [A] time = 0.55, size = 82, normalized size = 1.52

$$\begin{cases} ax - \frac{a \tanh^3(c+dx)}{3d} - \frac{a \tanh(c+dx)}{d} + bx - \frac{b \tanh^5(c+dx)}{5d} - \frac{b \tanh^3(c+dx)}{3d} - \frac{b \tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x(a + b \tanh^2(c)) \tanh^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4*(a+b*tanh(d*x+c)**2), x)
```

```
[Out] Piecewise((a*x - a*tanh(c + d*x)**3/(3*d) - a*tanh(c + d*x)/d + b*x - b*tan
h(c + d*x)**5/(5*d) - b*tanh(c + d*x)**3/(3*d) - b*tanh(c + d*x)/d, Ne(d, 0
)), (x*(a + b*tanh(c)**2)*tanh(c)**4, True))
```

3.135 $\int \tanh^3(c + dx) (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=49

$$-\frac{(a+b)\tanh^2(c+dx)}{2d} + \frac{(a+b)\log(\cosh(c+dx))}{d} - \frac{b\tanh^4(c+dx)}{4d}$$

[Out] (a+b)*ln(cosh(d*x+c))/d-1/2*(a+b)*tanh(d*x+c)^2/d-1/4*b*tanh(d*x+c)^4/d

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3631, 3473, 3475}

$$-\frac{(a+b)\tanh^2(c+dx)}{2d} + \frac{(a+b)\log(\cosh(c+dx))}{d} - \frac{b\tanh^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2), x]

[Out] ((a + b)*Log[Cosh[c + d*x]])/d - ((a + b)*Tanh[c + d*x]^2)/(2*d) - (b*Tanh[c + d*x]^4)/(4*d)

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3631

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] && !LeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \tanh^3(c + dx) (a + b \tanh^2(c + dx)) dx &= -\frac{b \tanh^4(c + dx)}{4d} + (a + b) \int \tanh^3(c + dx) dx \\ &= -\frac{(a + b) \tanh^2(c + dx)}{2d} - \frac{b \tanh^4(c + dx)}{4d} + (a + b) \int \tanh(c + dx) dx \\ &= \frac{(a + b) \log(\cosh(c + dx))}{d} - \frac{(a + b) \tanh^2(c + dx)}{2d} - \frac{b \tanh^4(c + dx)}{4d} \end{aligned}$$

Mathematica [A] time = 0.25, size = 43, normalized size = 0.88

$$-\frac{2(a+b)\tanh^2(c+dx) - 4(a+b)\log(\cosh(c+dx)) + b\tanh^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]

[Out] $-1/4*(-4*(a + b)*\text{Log}[\text{Cosh}[c + d*x]] + 2*(a + b)*\text{Tanh}[c + d*x]^2 + b*\text{Tanh}[c + d*x]^4)/d$

fricas [B] time = 0.44, size = 1205, normalized size = 24.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")

[Out] $-\left((a + b)*d*x*\cosh(d*x + c)^8 + 8*(a + b)*d*x*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a + b)*d*x*\sinh(d*x + c)^8 + 2*(2*(a + b)*d*x - a - 2*b)*\cosh(d*x + c)^6 + 2*(14*(a + b)*d*x*\cosh(d*x + c)^2 + 2*(a + b)*d*x - a - 2*b)*\sinh(d*x + c)^6 + 4*(14*(a + b)*d*x*\cosh(d*x + c)^3 + 3*(2*(a + b)*d*x - a - 2*b)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(3*(a + b)*d*x - 2*a - 2*b)*\cosh(d*x + c)^4 + 2*(35*(a + b)*d*x*\cosh(d*x + c)^4 + 3*(a + b)*d*x + 15*(2*(a + b)*d*x - a - 2*b)*\cosh(d*x + c)^2 - 2*a - 2*b)*\sinh(d*x + c)^4 + 8*(7*(a + b)*d*x*\cosh(d*x + c)^5 + 5*(2*(a + b)*d*x - a - 2*b)*\cosh(d*x + c)^3 + (3*(a + b)*d*x - 2*a - 2*b)*\cosh(d*x + c))*\sinh(d*x + c)^3 + (a + b)*d*x + 2*(2*(a + b)*d*x - a - 2*b)*\cosh(d*x + c)^2 + 2*(14*(a + b)*d*x*\cosh(d*x + c)^6 + 15*(2*(a + b)*d*x - a - 2*b)*\cosh(d*x + c)^4 + 2*(a + b)*d*x + 6*(3*(a + b)*d*x - 2*a - 2*b)*\cosh(d*x + c)^2 - a - 2*b)*\sinh(d*x + c)^2 - ((a + b)*\cosh(d*x + c)^8 + 8*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a + b)*\sinh(d*x + c)^8 + 4*(a + b)*\cosh(d*x + c)^6 + 4*(7*(a + b)*\cosh(d*x + c)^2 + a + b)*\sinh(d*x + c)^6 + 8*(7*(a + b)*\cosh(d*x + c)^3 + 3*(a + b)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 6*(a + b)*\cosh(d*x + c)^4 + 2*(35*(a + b)*\cosh(d*x + c)^4 + 30*(a + b)*\cosh(d*x + c)^2 + 3*a + 3*b)*\sinh(d*x + c)^4 + 8*(7*(a + b)*\cosh(d*x + c)^5 + 10*(a + b)*\cosh(d*x + c)^3 + 3*(a + b)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(a + b)*\cosh(d*x + c)^2 + 4*(7*(a + b)*\cosh(d*x + c)^6 + 15*(a + b)*\cosh(d*x + c)^4 + 9*(a + b)*\cosh(d*x + c)^2 + a + b)*\sinh(d*x + c)^2 + 8*(a + b)*\cosh(d*x + c)^7 + 3*(a + b)*\cosh(d*x + c)^5 + 3*(a + b)*\cosh(d*x + c)^3 + (a + b)*\cosh(d*x + c))*\sinh(d*x + c) + (a + b)*\log(2*\cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*(a + b)*d*x*\cosh(d*x + c)^7 + 3*(2*(a + b)*d*x - a - 2*b)*\cosh(d*x + c)^5 + 2*(3*(a + b)*d*x - 2*a - 2*b)*\cosh(d*x + c)^3 + (2*(a + b)*d*x - a - 2*b)*\cosh(d*x + c))*\sinh(d*x + c))/(d*\cosh(d*x + c)^8 + 8*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + d*\sinh(d*x + c)^8 + 4*d*\cosh(d*x + c)^6 + 4*(7*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^6 + 8*(7*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 6*d*\cosh(d*x + c)^4 + 2*(35*d*\cosh(d*x + c)^4 + 30*d*\cosh(d*x + c)^2 + 3*d)*\sinh(d*x + c)^4 + 8*(7*d*\cosh(d*x + c)^5 + 10*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*d*\cosh(d*x + c)^2 + 4*(7*d*\cosh(d*x + c)^6 + 15*d*\cosh(d*x + c)^4 + 9*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^2 + 8*(d*\cosh(d*x + c)^7 + 3*d*\cosh(d*x + c)^5 + 3*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c) + d)$

giac [B] time = 0.18, size = 92, normalized size = 1.88

$$\frac{(dx + c)(a + b) - (a + b) \log\left(e^{(2dx+2c)} + 1\right) - \frac{2\left((a+2b)e^{(6dx+6c)} + 2(a+b)e^{(4dx+4c)} + (a+2b)e^{(2dx+2c)}\right)}{\left(e^{(2dx+2c)} + 1\right)^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] $-\left((d*x + c)*(a + b) - (a + b)*\log(e^{(2*d*x + 2*c)} + 1) - 2*((a + 2*b)*e^{(6*d*x + 6*c)} + 2*(a + b)*e^{(4*d*x + 4*c)} + (a + 2*b)*e^{(2*d*x + 2*c)})/(e^{(2*d*x + 2*c)} + 1)^4\right)/d$

maple [B] time = 0.01, size = 104, normalized size = 2.12

$$\frac{b \left(\tanh^4(dx+c) \right)}{4d} - \frac{a \left(\tanh^2(dx+c) \right)}{2d} - \frac{b \left(\tanh^2(dx+c) \right)}{2d} - \frac{\ln(\tanh(dx+c)-1) a}{2d} - \frac{\ln(\tanh(dx+c)-1) b}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2), x)

[Out] $-1/4*b*\tanh(d*x+c)^4/d-1/2/d*a*\tanh(d*x+c)^2-1/2*b*\tanh(d*x+c)^2/d-1/2/d*\ln(\tanh(d*x+c)-1)*a-1/2/d*\ln(\tanh(d*x+c)-1)*b-1/2/d*\ln(1+\tanh(d*x+c))*a-1/2/d*\ln(1+\tanh(d*x+c))*b$

maxima [B] time = 0.41, size = 168, normalized size = 3.43

$$b \left(x + \frac{c}{d} + \frac{\log(e^{-2dx-2c} + 1)}{d} + \frac{4(e^{-2dx-2c} + e^{-4dx-4c} + e^{-6dx-6c})}{d(4e^{-2dx-2c} + 6e^{-4dx-4c} + 4e^{-6dx-6c} + e^{-8dx-8c} + 1)} \right) + a \left(x + \frac{c}{d} + \frac{\log(e^{-2dx-2c} + 1)}{d} + \frac{2e^{-2dx-2c}}{d(2e^{-2dx-2c} + e^{-4dx-4c} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2), x, algorithm="maxima")

[Out] $b*(x + c/d + \log(e^{-2*d*x - 2*c} + 1)/d + 4*(e^{-2*d*x - 2*c} + e^{-4*d*x - 4*c} + e^{-6*d*x - 6*c}))/d + 4*(e^{-2*d*x - 2*c} + e^{-4*d*x - 4*c} + 4*e^{-6*d*x - 6*c} + e^{-8*d*x - 8*c} + 1))/d + a*(x + c/d + \log(e^{-2*d*x - 2*c} + 1)/d + 2*e^{-2*d*x - 2*c}/(d*(2*e^{-2*d*x - 2*c} + e^{-4*d*x - 4*c} + 1)))$

mupad [B] time = 0.12, size = 53, normalized size = 1.08

$$x(a+b) - \frac{\tanh(c+dx)^2(a+b)}{2d} - \frac{b \tanh(c+dx)^4}{4d} - \frac{\ln(\tanh(c+dx)+1)(a+b)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c+d*x)^3*(a+b*tanh(c+d*x)^2), x)

[Out] $x*(a+b) - (\tanh(c+d*x)^2*(a+b))/(2*d) - (b*\tanh(c+d*x)^4)/(4*d) - (\log(\tanh(c+d*x)+1)*(a+b))/d$

sympy [A] time = 0.42, size = 88, normalized size = 1.80

$$\begin{cases} ax - \frac{a \log(\tanh(c+dx)+1)}{d} - \frac{a \tanh^2(c+dx)}{2d} + bx - \frac{b \log(\tanh(c+dx)+1)}{d} - \frac{b \tanh^4(c+dx)}{4d} - \frac{b \tanh^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x(a + b \tanh^2(c)) \tanh^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)**3*(a+b*tanh(d*x+c)**2), x)

[Out] $\text{Piecewise}((a*x - a*\log(\tanh(c + d*x) + 1)/d - a*\tanh(c + d*x)**2/(2*d) + b*x - b*\log(\tanh(c + d*x) + 1)/d - b*\tanh(c + d*x)**4/(4*d) - b*\tanh(c + d*x)**2/(2*d), \text{Ne}(d, 0)), (x*(a + b*\tanh(c)**2)*\tanh(c)**3, \text{True}))$

3.136 $\int \tanh^2(c + dx) (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=36

$$-\frac{(a+b)\tanh(c+dx)}{d} + x(a+b) - \frac{b\tanh^3(c+dx)}{3d}$$

[Out] (a+b)*x-(a+b)*tanh(d*x+c)/d-1/3*b*tanh(d*x+c)^3/d

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3631, 3473, 8}

$$-\frac{(a+b)\tanh(c+dx)}{d} + x(a+b) - \frac{b\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2), x]

[Out] (a + b)*x - ((a + b)*Tanh[c + d*x])/d - (b*Tanh[c + d*x]^3)/(3*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3631

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)^2]), x_Symbol] := Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] && !LeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \tanh^2(c + dx) (a + b \tanh^2(c + dx)) dx &= -\frac{b \tanh^3(c + dx)}{3d} - (-a - b) \int \tanh^2(c + dx) dx \\ &= -\frac{(a + b) \tanh(c + dx)}{d} - \frac{b \tanh^3(c + dx)}{3d} - (-a - b) \int 1 dx \\ &= (a + b)x - \frac{(a + b) \tanh(c + dx)}{d} - \frac{b \tanh^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 1.81

$$\frac{a \tanh^{-1}(\tanh(c + dx))}{d} - \frac{a \tanh(c + dx)}{d} + \frac{b \tanh^{-1}(\tanh(c + dx))}{d} - \frac{b \tanh^3(c + dx)}{3d} - \frac{b \tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2), x]

[Out] $(a \cdot \text{ArcTanh}[\text{Tanh}[c + d \cdot x]])/d + (b \cdot \text{ArcTanh}[\text{Tanh}[c + d \cdot x]])/d - (a \cdot \text{Tanh}[c + d \cdot x])/d - (b \cdot \text{Tanh}[c + d \cdot x])/d - (b \cdot \text{Tanh}[c + d \cdot x]^3)/(3 \cdot d)$

fricas [B] time = 0.41, size = 160, normalized size = 4.44

$$\frac{(3(a+b)dx + 3a + 4b) \cosh(dx + c)^3 + 3(3(a+b)dx + 3a + 4b) \cosh(dx + c) \sinh(dx + c)^2 - (3a + 4b) \sinh(dx + c)^3}{3(d \cosh(dx + c)^3 + 3d \cosh(dx + c) \sinh(dx + c)^2 + 3d \sinh(dx + c)^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")`

[Out] $1/3 * ((3 * (a + b) * d * x + 3 * a + 4 * b) * \cosh(d * x + c)^3 + 3 * (3 * (a + b) * d * x + 3 * a + 4 * b) * \cosh(d * x + c) * \sinh(d * x + c)^2 - (3 * a + 4 * b) * \sinh(d * x + c)^3 + 3 * (3 * (a + b) * d * x + 3 * a + 4 * b) * \cosh(d * x + c) - 3 * ((3 * a + 4 * b) * \cosh(d * x + c)^2 + a) * \sinh(d * x + c)) / (d * \cosh(d * x + c)^3 + 3 * d * \cosh(d * x + c) * \sinh(d * x + c)^2 + 3 * d * \cosh(d * x + c))$

giac [B] time = 0.17, size = 86, normalized size = 2.39

$$\frac{3(dx+c)(a+b) + \frac{2(3ae^{4dx+4c} + 6be^{4dx+4c} + 6ae^{2dx+2c} + 6be^{2dx+2c} + 3a+4b)}{(e^{2dx+2c}+1)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")`

[Out] $1/3 * (3 * (d * x + c) * (a + b) + 2 * (3 * a * e^{(4 * d * x + 4 * c)} + 6 * b * e^{(4 * d * x + 4 * c)} + 6 * a * e^{(2 * d * x + 2 * c)} + 6 * b * e^{(2 * d * x + 2 * c)} + 3 * a + 4 * b) / (e^{(2 * d * x + 2 * c)} + 1)^3) / d$

maple [B] time = 0.02, size = 100, normalized size = 2.78

$$\frac{b(\tanh^3(dx+c))}{3d} - \frac{a \tanh(dx+c)}{d} - \frac{b \tanh(dx+c)}{d} - \frac{\ln(\tanh(dx+c)-1)a}{2d} - \frac{\ln(\tanh(dx+c)-1)b}{2d} + \frac{\ln(1+\tanh(dx+c))a}{2d} + \frac{\ln(1+\tanh(dx+c))b}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x)`

[Out] $-1/3 * b * \tanh(d * x + c)^3 / d - a * \tanh(d * x + c) / d - b * \tanh(d * x + c) / d - 1/2 * d * \ln(\tanh(d * x + c) - 1) * a - 1/2 * d * \ln(\tanh(d * x + c) - 1) * b + 1/2 * d * \ln(1 + \tanh(d * x + c)) * a + 1/2 * d * \ln(1 + \tanh(d * x + c)) * b$

maxima [B] time = 0.33, size = 105, normalized size = 2.92

$$\frac{1}{3} b \left(3x + \frac{3c}{d} - \frac{4(3e^{-2dx-2c} + 3e^{-4dx-4c} + 2)}{d(3e^{-2dx-2c} + 3e^{-4dx-4c} + e^{-6dx-6c} + 1)} \right) + a \left(x + \frac{c}{d} - \frac{2}{d(e^{-2dx-2c} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")`

[Out] $1/3 * b * (3 * x + 3 * c / d - 4 * (3 * e^{(-2 * d * x - 2 * c)} + 3 * e^{(-4 * d * x - 4 * c)} + 2) / (d * (3 * e^{(-2 * d * x - 2 * c)} + 3 * e^{(-4 * d * x - 4 * c)} + e^{(-6 * d * x - 6 * c)} + 1))) + a * (x + c / d - 2 / (d * (e^{(-2 * d * x - 2 * c)} + 1)))$

mupad [B] time = 1.17, size = 34, normalized size = 0.94

$$x(a+b) - \frac{b \tanh(c+dx)^3}{3d} - \frac{\tanh(c+dx)(a+b)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(c + d*x)^2*(a + b*tanh(c + d*x)^2), x)`

[Out] `x*(a + b) - (b*tanh(c + d*x)^3)/(3*d) - (tanh(c + d*x)*(a + b))/d`

sympy [A] time = 0.29, size = 54, normalized size = 1.50

$$\begin{cases} ax - \frac{a \tanh(c+dx)}{d} + bx - \frac{b \tanh^3(c+dx)}{3d} - \frac{b \tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x(a + b \tanh^2(c)) \tanh^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(d*x+c)**2*(a+b*tanh(d*x+c)**2), x)`

[Out] `Piecewise((a*x - a*tanh(c + d*x)/d + b*x - b*tanh(c + d*x)**3/(3*d) - b*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)*tanh(c)**2, True))`

3.137 $\int \tanh(c + dx) (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=31

$$\frac{(a + b) \log(\cosh(c + dx))}{d} - \frac{b \tanh^2(c + dx)}{2d}$$

[Out] (a+b)*ln(cosh(d*x+c))/d-1/2*b*tanh(d*x+c)^2/d

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3631, 3475}

$$\frac{(a + b) \log(\cosh(c + dx))}{d} - \frac{b \tanh^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2), x]

[Out] ((a + b)*Log[Cosh[c + d*x]])/d - (b*Tanh[c + d*x]^2)/(2*d)

Rule 3475

Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3631

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(C*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[A - C, Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] && !LeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \tanh(c + dx) (a + b \tanh^2(c + dx)) dx &= -\frac{b \tanh^2(c + dx)}{2d} - (-a - b) \int \tanh(c + dx) dx \\ &= \frac{(a + b) \log(\cosh(c + dx))}{d} - \frac{b \tanh^2(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 1.32

$$\frac{a \log(\cosh(c + dx))}{d} - \frac{b \tanh^2(c + dx)}{2d} + \frac{b \log(\cosh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2), x]

[Out] (a*Log[Cosh[c + d*x]])/d + (b*Log[Cosh[c + d*x]])/d - (b*Tanh[c + d*x]^2)/(2*d)

fricas [B] time = 0.44, size = 399, normalized size = 12.87

$$(a + b)dx \cosh(dx + c)^4 + 4(a + b)dx \cosh(dx + c) \sinh(dx + c)^3 + (a + b)dx \sinh(dx + c)^4 + (a + b)dx + 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")

[Out] $-\left((a+b)d^2x^2\cosh(dx+c)^4 + 4(a+b)d^2x^2\cosh(dx+c)\sinh(dx+c)^3 + (a+b)d^2x^2\sinh(dx+c)^4 + (a+b)d^2x + 2((a+b)d^2x - b)\cosh(dx+c)^2 + 2(3(a+b)d^2x\cosh(dx+c)^2 + (a+b)d^2x - b)\sinh(dx+c)^2 - ((a+b)\cosh(dx+c)^4 + 4(a+b)\cosh(dx+c)\sinh(dx+c)^3 + (a+b)\sinh(dx+c)^4 + 2(a+b)\cosh(dx+c)^2 + 2(3(a+b)\cosh(dx+c)^2 + a+b)\sinh(dx+c)^2 + 4((a+b)\cosh(dx+c)^3 + (a+b)\cosh(dx+c))\sinh(dx+c) + a+b)\log(2\cosh(dx+c)/(\cosh(dx+c) - \sinh(dx+c))) + 4((a+b)d^2x\cosh(dx+c)^3 + ((a+b)d^2x - b)\cosh(dx+c))\sinh(dx+c)\right)/(d^2\cosh(dx+c)^4 + 4d^2\cosh(dx+c)\sinh(dx+c)^3 + d^2\sinh(dx+c)^4 + 2d^2\cosh(dx+c)^2 + 2(3d^2\cosh(dx+c)^2 + d)\sinh(dx+c)^2 + 4(d^2\cosh(dx+c)^3 + d^2\cosh(dx+c))\sinh(dx+c) + d)$

giac [A] time = 0.16, size = 57, normalized size = 1.84

$$\frac{(dx+c)(a+b) - (a+b)\log\left(e^{(2dx+2c)} + 1\right) - \frac{2be^{(2dx+2c)}}{(e^{(2dx+2c)}+1)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] $-\left((dx+c)(a+b) - (a+b)\log(e^{(2dx+2c)} + 1) - 2b\frac{e^{(2dx+2c)}}{(e^{(2dx+2c)}+1)^2}\right)/d$

maple [B] time = 0.02, size = 76, normalized size = 2.45

$$\frac{b\left(\tanh^2(dx+c)\right)}{2d} - \frac{\ln(\tanh(dx+c)-1)a}{2d} - \frac{\ln(\tanh(dx+c)-1)b}{2d} - \frac{\ln(1+\tanh(dx+c))a}{2d} - \frac{\ln(1+\tanh(dx+c))b}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)*(a+b*tanh(d*x+c)^2),x)

[Out] $-1/2*b*\tanh(dx+c)^2/d - 1/2/d*\ln(\tanh(dx+c)-1)*a - 1/2/d*\ln(\tanh(dx+c)-1)*b - 1/2/d*\ln(1+\tanh(dx+c))*a - 1/2/d*\ln(1+\tanh(dx+c))*b$

maxima [B] time = 0.41, size = 76, normalized size = 2.45

$$b\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2dx-2c)} + 1\right)}{d} + \frac{2e^{(-2dx-2c)}}{d\left(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1\right)}\right) + \frac{a\log(\cosh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] $b*(x + c/d + \log(e^{(-2dx-2c)} + 1)/d + 2e^{(-2dx-2c)}/(d*(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1))) + a*\log(\cosh(dx+c))/d$

mupad [B] time = 1.17, size = 37, normalized size = 1.19

$$x(a+b) - \frac{b\tanh(c+dx)^2}{2d} - \frac{\ln(\tanh(c+dx)+1)(a+b)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c+d*x)*(a+b*tanh(c+d*x)^2),x)

[Out] $x*(a+b) - (b*\tanh(c+d*x)^2)/(2*d) - (\log(\tanh(c+d*x)+1)*(a+b))/d$

sympy [A] time = 0.21, size = 60, normalized size = 1.94

$$\begin{cases} ax - \frac{a \log(\tanh(c+dx)+1)}{d} + bx - \frac{b \log(\tanh(c+dx)+1)}{d} - \frac{b \tanh^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x(a + b \tanh^2(c)) \tanh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)**2),x)

[Out] Piecewise((a*x - a*log(tanh(c + d*x) + 1)/d + b*x - b*log(tanh(c + d*x) + 1)/d - b*tanh(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)*tanh(c), True))

3.138 $\int (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=19

$$ax - \frac{b \tanh(c + dx)}{d} + bx$$

[Out] a*x+b*x-b*tanh(d*x+c)/d

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3473, 8}

$$ax - \frac{b \tanh(c + dx)}{d} + bx$$

Antiderivative was successfully verified.

[In] Int[a + b*Tanh[c + d*x]^2,x]

[Out] a*x + b*x - (b*Tanh[c + d*x])/d

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rubi steps

$$\begin{aligned} \int (a + b \tanh^2(c + dx)) dx &= ax + b \int \tanh^2(c + dx) dx \\ &= ax - \frac{b \tanh(c + dx)}{d} + b \int 1 dx \\ &= ax + bx - \frac{b \tanh(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.47

$$ax + \frac{b \tanh^{-1}(\tanh(c + dx))}{d} - \frac{b \tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[a + b*Tanh[c + d*x]^2,x]

[Out] a*x + (b*ArcTanh[Tanh[c + d*x]])/d - (b*Tanh[c + d*x])/d

fricas [A] time = 0.40, size = 37, normalized size = 1.95

$$\frac{((a + b)dx + b) \cosh(dx + c) - b \sinh(dx + c)}{d \cosh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*tanh(d*x+c)^2,x, algorithm="fricas")

[Out] $((a + b)*d*x + b)*\cosh(d*x + c) - b*\sinh(d*x + c)/(d*\cosh(d*x + c))$

giac [A] time = 0.13, size = 29, normalized size = 1.53

$$ax + \frac{\left(dx + c + \frac{2}{e^{2dx+2c}+1}\right)b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*tanh(d*x+c)^2,x, algorithm="giac")`

[Out] $a*x + (d*x + c + 2/(e^{2*d*x} + 2*c) + 1))*b/d$

maple [B] time = 0.02, size = 47, normalized size = 2.47

$$ax - \frac{b \tanh(dx + c)}{d} - \frac{\ln(\tanh(dx + c) - 1)b}{2d} + \frac{\ln(1 + \tanh(dx + c))b}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a+b*tanh(d*x+c)^2,x)`

[Out] $a*x - b*\tanh(d*x+c)/d - 1/2/d*\ln(\tanh(d*x+c)-1)*b + 1/2/d*\ln(1+\tanh(d*x+c))*b$

maxima [A] time = 0.30, size = 31, normalized size = 1.63

$$b\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2dx-2c)} + 1)}\right) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*tanh(d*x+c)^2,x, algorithm="maxima")`

[Out] $b*(x + c/d - 2/(d*(e^{(-2*d*x - 2*c)} + 1))) + a*x$

mupad [B] time = 0.07, size = 18, normalized size = 0.95

$$x(a + b) - \frac{b \tanh(c + dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(a + b*tanh(c + d*x)^2,x)`

[Out] $x*(a + b) - (b*\tanh(c + d*x))/d$

sympy [A] time = 0.16, size = 20, normalized size = 1.05

$$ax + b \left\{ \begin{array}{ll} x - \frac{\tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x \tanh^2(c) & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*tanh(d*x+c)**2,x)`

[Out] $a*x + b*\text{Piecewise}((x - \tanh(c + d*x))/d, \text{Ne}(d, 0)), (x*\tanh(c)**2, \text{True}))$

3.139 $\int \coth(c + dx) (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=25

$$\frac{a \log(\sinh(c + dx))}{d} + \frac{b \log(\cosh(c + dx))}{d}$$

[Out] $b \ln(\cosh(d*x+c))/d + a \ln(\sinh(d*x+c))/d$

Rubi [A] time = 0.04, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3625, 3475}

$$\frac{a \log(\sinh(c + dx))}{d} + \frac{b \log(\cosh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] `Int[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2), x]`

[Out] $(b \cdot \text{Log}[\text{Cosh}[c + d*x]])/d + (a \cdot \text{Log}[\text{Sinh}[c + d*x]])/d$

Rule 3475

`Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

Rule 3625

`Int[((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/tan[(e_.) + (f_.)*(x_)], x_Symbol] := Dist[A, Int[1/Tan[e + f*x], x], x] + Dist[C, Int[Tan[e + f*x], x], x] /; FreeQ[{e, f, A, C}, x] && NeQ[A, C]`

Rubi steps

$$\begin{aligned} \int \coth(c + dx) (a + b \tanh^2(c + dx)) dx &= a \int \coth(c + dx) dx + b \int \tanh(c + dx) dx \\ &= \frac{b \log(\cosh(c + dx))}{d} + \frac{a \log(\sinh(c + dx))}{d} \end{aligned}$$

Mathematica [A] time = 0.04, size = 33, normalized size = 1.32

$$\frac{a(\log(\tanh(c + dx)) + \log(\cosh(c + dx)))}{d} + \frac{b \log(\cosh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] `Integrate[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2), x]`

[Out] $(b \cdot \text{Log}[\text{Cosh}[c + d*x]])/d + (a \cdot (\text{Log}[\text{Cosh}[c + d*x]] + \text{Log}[\text{Tanh}[c + d*x]]))/d$

fricas [B] time = 0.42, size = 70, normalized size = 2.80

$$\frac{(a + b)dx - b \log\left(\frac{2 \cosh(dx+c)}{\cosh(dx+c) - \sinh(dx+c)}\right) - a \log\left(\frac{2 \sinh(dx+c)}{\cosh(dx+c) - \sinh(dx+c)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2), x, algorithm="fricas")`

[Out] $-\left((a+b)d^2x - b \log\left(\frac{2 \cosh(dx+c)}{\cosh(dx+c) - \sinh(dx+c)}\right) - a \log\left(\frac{2 \sinh(dx+c)}{\cosh(dx+c) - \sinh(dx+c)}\right)\right)/d$

giac [A] time = 0.16, size = 46, normalized size = 1.84

$$\frac{(dx+c)(a+b) - b \log\left(e^{(2dx+2c)} + 1\right) - a \log\left(\left|e^{(2dx+2c)} - 1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")`

[Out] $-\left((d^2x+c)(a+b) - b \log\left(e^{(2d^2x+2c)} + 1\right) - a \log\left(\left|e^{(2d^2x+2c)} - 1\right|\right)\right)/d$

maple [A] time = 0.27, size = 26, normalized size = 1.04

$$\frac{b \ln(\cosh(dx+c))}{d} + \frac{a \ln(\sinh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(d*x+c)*(a+b*tanh(d*x+c)^2),x)`

[Out] $b \ln(\cosh(d^2x+c))/d + a \ln(\sinh(d^2x+c))/d$

maxima [A] time = 0.31, size = 35, normalized size = 1.40

$$\frac{b \log\left(e^{(dx+c)} + e^{(-dx-c)}\right)}{d} + \frac{a \log(\sinh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")`

[Out] $b \log\left(e^{(d^2x+c)} + e^{(-d^2x-c)}\right)/d + a \log(\sinh(d^2x+c))/d$

mupad [B] time = 1.31, size = 228, normalized size = 9.12

$$\frac{a \ln\left(8ab - 4a^2 - 4b^2 + 4a^2 e^{4c} e^{4dx} + 4b^2 e^{4c} e^{4dx} - 8ab e^{4c} e^{4dx}\right)}{2d} - bx - \frac{\operatorname{atan}\left(\frac{a e^{2c} e^{2dx} \sqrt{-d^2}}{d \sqrt{a^2 - 2ab + b^2}} - \frac{b e^{2c} e^{2dx} \sqrt{-d^2}}{d \sqrt{a^2 - 2ab + b^2}}\right)}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(c+d*x)*(a+b*tanh(c+d*x)^2),x)`

[Out] $(a \log(8ab - 4a^2 - 4b^2 + 4a^2 \exp(4c) \exp(4dx) + 4b^2 \exp(4c) \exp(4dx) - 8ab \exp(4c) \exp(4dx)) / (2d) - bx - (\operatorname{atan}((a \exp(2c) \exp(2dx) * (-d^2)^{(1/2)}) / (d * (a^2 - 2ab + b^2)^{(1/2)}) - (b \exp(2c) \exp(2dx) * (-d^2)^{(1/2)}) / (d * (a^2 - 2ab + b^2)^{(1/2)})) * (a^2 - 2ab + b^2)^{(1/2)}) / (-d^2)^{(1/2)} - ax + (b \log(8ab - 4a^2 - 4b^2 + 4a^2 \exp(4c) \exp(4dx) + 4b^2 \exp(4c) \exp(4dx) - 8ab \exp(4c) \exp(4dx)) / (2d))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx)) \coth(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)*(a+b*tanh(d*x+c)**2),x)`

[Out] `Integral((a + b*tanh(c + d*x)**2)*coth(c + d*x), x)`

3.140 $\int \coth^2(c + dx) (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=18

$$x(a + b) - \frac{a \coth(c + dx)}{d}$$

[Out] (a+b)*x-a*coth(d*x+c)/d

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3629, 8}

$$x(a + b) - \frac{a \coth(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2), x]

[Out] (a + b)*x - (a*Coth[c + d*x])/d

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rule 3629

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_.)])^(m_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> Simp[((A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned} \int \coth^2(c + dx) (a + b \tanh^2(c + dx)) dx &= -\frac{a \coth(c + dx)}{d} - \int (-a - b) dx \\ &= (a + b)x - \frac{a \coth(c + dx)}{d} \end{aligned}$$

Mathematica [C] time = 0.03, size = 32, normalized size = 1.78

$$bx - \frac{a \coth(c + dx) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \tanh^2(c + dx)\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2), x]

[Out] b*x - (a*Coth[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2])/d

fricas [B] time = 0.40, size = 38, normalized size = 2.11

$$\frac{a \cosh(dx + c) - ((a + b)dx + a) \sinh(dx + c)}{d \sinh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")

[Out] $-(a*\cosh(d*x + c) - ((a + b)*d*x + a)*\sinh(d*x + c))/(d*\sinh(d*x + c))$

giac [A] time = 0.17, size = 30, normalized size = 1.67

$$\frac{(dx + c)(a + b) - \frac{2a}{e^{(2dx+2c)} - 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] $((d*x + c)*(a + b) - 2*a/(e^{(2*d*x + 2*c)} - 1))/d$

maple [A] time = 0.21, size = 28, normalized size = 1.56

$$\frac{a(dx + c - \coth(dx + c)) + (dx + c)b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2),x)

[Out] $1/d*(a*(d*x+c-\coth(d*x+c))+(d*x+c)*b)$

maxima [A] time = 0.32, size = 31, normalized size = 1.72

$$a\left(x + \frac{c}{d} + \frac{2}{d(e^{(-2dx-2c)} - 1)}\right) + bx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] $a*(x + c/d + 2/(d*(e^{(-2*d*x - 2*c)} - 1))) + b*x$

mupad [B] time = 1.27, size = 25, normalized size = 1.39

$$x(a + b) - \frac{2a}{d(e^{2c+2dx} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^2*(a + b*tanh(c + d*x)^2),x)

[Out] $x*(a + b) - (2*a)/(d*(\exp(2*c + 2*d*x) - 1))$

sympy [B] time = 7.56, size = 49, normalized size = 2.72

$$a \left(\begin{cases} \infty x & \text{for } c = \log(-e^{-dx}) \vee c = \log(e^{-dx}) \\ x \coth^2(c) & \text{for } d = 0 \\ x - \frac{1}{d \tanh(c+dx)} & \text{otherwise} \end{cases} \right) + b \left(\begin{cases} x & \text{for } |x| < 1 \\ G_{2,2}^{1,1} \left(\begin{matrix} 1 & 2 \\ 1 & 0 \end{matrix} \middle| x \right) + G_{2,2}^{0,2} \left(\begin{matrix} 2, 1 \\ 1, 0 \end{matrix} \middle| x \right) & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)

[Out] $a*\text{Piecewise}((\text{zoo}*x, \text{Eq}(c, \log(\exp(-d*x))) \mid \text{Eq}(c, \log(-\exp(-d*x))))), (x*\cot h(c)**2, \text{Eq}(d, 0)), (x - 1/(d*\tanh(c + d*x)), \text{True})) + b*\text{Piecewise}((x, \text{Abs}(x) < 1), (\text{meijerg}(((1,), (2,)), ((1,), (0,)), x) + \text{meijerg}(((2, 1), ()), ((), (1, 0)), x), \text{True}))$

3.141 $\int \coth^3(c + dx) (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=31

$$\frac{(a + b) \log(\sinh(c + dx))}{d} - \frac{a \coth^2(c + dx)}{2d}$$

[Out] $-1/2*a*\coth(d*x+c)^2/d+(a+b)*\ln(\sinh(d*x+c))/d$

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3629, 12, 3475}

$$\frac{(a + b) \log(\sinh(c + dx))}{d} - \frac{a \coth^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Coth}[c + d*x]^3*(a + b*\text{Tanh}[c + d*x]^2), x]$

[Out] $-(a*\text{Coth}[c + d*x]^2)/(2*d) + ((a + b)*\text{Log}[\text{Sinh}[c + d*x]])/d$

Rule 12

$\text{Int}[(a_*)(u_), x_Symbol] := \text{Dist}[a, \text{Int}[u, x], x] /; \text{FreeQ}[a, x] \ \&\& \ !\text{MatchQ}[u, (b_*)(v_)] /; \text{FreeQ}[b, x]$

Rule 3475

$\text{Int}[\tan[(c_.) + (d_.)*(x_)], x_Symbol] := -\text{Simp}[\text{Log}[\text{RemoveContent}[\text{Cos}[c + d*x], x]]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 3629

$\text{Int}[((a_.) + (b_.)*\tan[(e_.) + (f_.)*(x_)])^{(m_)}*((A_.) + (C_.)*\tan[(e_.) + (f_.)*(x_)]^2), x_Symbol] := \text{Simp}[(A*b^2 + a^2*C)*(a + b*\text{Tan}[e + f*x])^{(m + 1)})/(b*f*(m + 1)*(a^2 + b^2)), x] + \text{Dist}[1/(a^2 + b^2), \text{Int}[(a + b*\text{Tan}[e + f*x])^{(m + 1)}*\text{Simp}[a*(A - C) - (A*b - b*C)*\text{Tan}[e + f*x], x], x] /; \text{FreeQ}[\{a, b, e, f, A, C\}, x] \ \&\& \ \text{NeQ}[A*b^2 + a^2*C, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{NeQ}[a^2 + b^2, 0]$

Rubi steps

$$\begin{aligned} \int \coth^3(c + dx) (a + b \tanh^2(c + dx)) dx &= -\frac{a \coth^2(c + dx)}{2d} + \int (a + b) \coth(c + dx) dx \\ &= -\frac{a \coth^2(c + dx)}{2d} + (a + b) \int \coth(c + dx) dx \\ &= -\frac{a \coth^2(c + dx)}{2d} + \frac{(a + b) \log(\sinh(c + dx))}{d} \end{aligned}$$

Mathematica [A] time = 0.12, size = 39, normalized size = 1.26

$$\frac{2(a + b)(\log(\tanh(c + dx)) + \log(\cosh(c + dx))) - a \coth^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2), x]

[Out] $(-(a*\text{Coth}[c + d*x]^2) + 2*(a + b)*(\text{Log}[\text{Cosh}[c + d*x]] + \text{Log}[\text{Tanh}[c + d*x]]))/ (2*d)$

fricas [B] time = 0.43, size = 407, normalized size = 13.13

$$(a + b)dx \cosh(dx + c)^4 + 4(a + b)dx \cosh(dx + c) \sinh(dx + c)^3 + (a + b)dx \sinh(dx + c)^4 + (a + b)dx - 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] $-\left((a + b)*d*x*\cosh(d*x + c)^4 + 4*(a + b)*d*x*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a + b)*d*x*\sinh(d*x + c)^4 + (a + b)*d*x - 2*((a + b)*d*x - a)*\cosh(d*x + c)^2 + 2*(3*(a + b)*d*x*\cosh(d*x + c)^2 - (a + b)*d*x + a)*\sinh(d*x + c)^2 - ((a + b)*\cosh(d*x + c)^4 + 4*(a + b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a + b)*\sinh(d*x + c)^4 - 2*(a + b)*\cosh(d*x + c)^2 + 2*(3*(a + b)*\cosh(d*x + c)^2 - a - b)*\sinh(d*x + c)^2 + 4*((a + b)*\cosh(d*x + c)^3 - (a + b)*\cosh(d*x + c))*\sinh(d*x + c) + a + b\right)*\log(2*\sinh(d*x + c)/(\cosh(d*x + c) - \sinh(d*x + c))) + 4*((a + b)*d*x*\cosh(d*x + c)^3 - ((a + b)*d*x - a)*\cosh(d*x + c))*\sinh(d*x + c)/\left(d*\cosh(d*x + c)^4 + 4*d*\cosh(d*x + c)*\sinh(d*x + c)^3 + d*\sinh(d*x + c)^4 - 2*d*\cosh(d*x + c)^2 + 2*(3*d*\cosh(d*x + c)^2 - d)*\sinh(d*x + c)^2 + 4*(d*\cosh(d*x + c)^3 - d*\cosh(d*x + c))*\sinh(d*x + c) + d\right)$

giac [A] time = 0.20, size = 58, normalized size = 1.87

$$\frac{(dx + c)(a + b) - (a + b) \log\left(\left|e^{(2dx+2c)} - 1\right|\right) + \frac{2ae^{(2dx+2c)}}{(e^{(2dx+2c)} - 1)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2), x, algorithm="giac")

[Out] $-\left((d*x + c)*(a + b) - (a + b)*\log(\text{abs}(e^{(2*d*x + 2*c)} - 1)) + 2*a*e^{(2*d*x + 2*c)}/(e^{(2*d*x + 2*c)} - 1)^2\right)/d$

maple [A] time = 0.27, size = 40, normalized size = 1.29

$$\frac{a \ln(\sinh(dx + c))}{d} - \frac{a \left(\coth^2(dx + c)\right)}{2d} + \frac{b \ln(\sinh(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2), x)

[Out] $a*\ln(\sinh(d*x+c))/d - 1/2*a*\coth(d*x+c)^2/d + 1/d*b*\ln(\sinh(d*x+c))$

maxima [B] time = 0.32, size = 106, normalized size = 3.42

$$a\left(x + \frac{c}{d} + \frac{\log(e^{(-dx-c)} + 1)}{d} + \frac{\log(e^{(-dx-c)} - 1)}{d} + \frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)} - e^{(-4dx-4c)} - 1)}\right) + \frac{b \log(e^{(dx+c)} - e^{(-dx-c)})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2), x, algorithm="maxima")

[Out] $a*(x + c/d + \log(e^{(-d*x - c)} + 1)/d + \log(e^{(-d*x - c)} - 1)/d + 2*e^{(-2*d*x - 2*c)}/(d*(2*e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)} - 1))) + b*\log(e^{(d*x + c)} - e^{(-d*x - c)})/d$

mupad [B] time = 1.26, size = 76, normalized size = 2.45

$$\frac{\ln(e^{2c} e^{2dx} - 1) (a + b)}{d} - \frac{2a}{d(e^{2c+2dx} - 1)} - \frac{2a}{d(e^{4c+4dx} - 2e^{2c+2dx} + 1)} - x(a + b)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^3*(a + b*tanh(c + d*x)^2), x)

[Out] (log(exp(2*c)*exp(2*d*x) - 1)*(a + b))/d - (2*a)/(d*(exp(2*c + 2*d*x) - 1)) - (2*a)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - x*(a + b)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx)) \coth^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**3*(a+b*tanh(d*x+c)**2), x)

[Out] Integral((a + b*tanh(c + d*x)**2)*coth(c + d*x)**3, x)

3.142 $\int \coth^4(c + dx) (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=36

$$-\frac{(a+b)\coth(c+dx)}{d} + x(a+b) - \frac{a\coth^3(c+dx)}{3d}$$

[Out] (a+b)*x-(a+b)*coth(d*x+c)/d-1/3*a*coth(d*x+c)^3/d

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3629, 12, 3473, 8}

$$-\frac{(a+b)\coth(c+dx)}{d} + x(a+b) - \frac{a\coth^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2), x]

[Out] (a + b)*x - ((a + b)*Coth[c + d*x])/d - (a*Coth[c + d*x]^3)/(3*d)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 3473

Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]

Rule 3629

Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)^2]), x_Symbol] := Simp[((A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]

Rubi steps

$$\begin{aligned} \int \coth^4(c + dx) (a + b \tanh^2(c + dx)) dx &= -\frac{a \coth^3(c + dx)}{3d} + \int (a + b) \coth^2(c + dx) dx \\ &= -\frac{a \coth^3(c + dx)}{3d} + (a + b) \int \coth^2(c + dx) dx \\ &= -\frac{(a + b) \coth(c + dx)}{d} - \frac{a \coth^3(c + dx)}{3d} + (a + b) \int 1 dx \\ &= (a + b)x - \frac{(a + b) \coth(c + dx)}{d} - \frac{a \coth^3(c + dx)}{3d} \end{aligned}$$

Mathematica [C] time = 0.04, size = 61, normalized size = 1.69

$$\frac{a \operatorname{coth}^3(c + dx) {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; \tanh^2(c + dx)\right)}{3d} - \frac{b \operatorname{coth}(c + dx) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \tanh^2(c + dx)\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2), x]

[Out] -1/3*(a*Coth[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[c + d*x]^2])/d - (b*Coth[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2])/d

fricas [B] time = 0.43, size = 156, normalized size = 4.33

$$\frac{(4a + 3b) \cosh(dx + c)^3 + 3(4a + 3b) \cosh(dx + c) \sinh(dx + c)^2 - (3(a + b)dx + 4a + 3b) \sinh(dx + c)^3 - 3(d \sinh(dx + c)^3 + 3(d \cosh(dx + c) \sinh(dx + c)^2))}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] -1/3*((4*a + 3*b)*cosh(d*x + c)^3 + 3*(4*a + 3*b)*cosh(d*x + c)*sinh(d*x + c)^2 - (3*(a + b)*d*x + 4*a + 3*b)*sinh(d*x + c)^3 - 3*b*cosh(d*x + c) + 3*(3*(a + b)*d*x - (3*(a + b)*d*x + 4*a + 3*b)*cosh(d*x + c)^2 + 4*a + 3*b)*sinh(d*x + c))/(d*sinh(d*x + c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))

giac [B] time = 0.19, size = 86, normalized size = 2.39

$$\frac{3(dx + c)(a + b) - \frac{2(6ae^{4dx+4c} + 3be^{4dx+4c} - 6ae^{2dx+2c} - 6be^{2dx+2c} + 4a + 3b)}{(e^{2dx+2c} - 1)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2), x, algorithm="giac")

[Out] 1/3*(3*(d*x + c)*(a + b) - 2*(6*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) - 6*b*e^(2*d*x + 2*c) + 4*a + 3*b)/(e^(2*d*x + 2*c) - 1)^3)/d

maple [A] time = 0.24, size = 46, normalized size = 1.28

$$\frac{a \left(dx + c - \operatorname{coth}(dx + c) - \frac{(\operatorname{coth}^3(dx + c))}{3} \right) + b(dx + c - \operatorname{coth}(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2), x)

[Out] 1/d*(a*(d*x+c-coth(d*x+c))-1/3*coth(d*x+c)^3)+b*(d*x+c-coth(d*x+c))

maxima [B] time = 0.33, size = 105, normalized size = 2.92

$$\frac{1}{3}a \left(3x + \frac{3c}{d} - \frac{4(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} - 2)}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)} \right) + b \left(x + \frac{c}{d} + \frac{2}{d(e^{(-2dx-2c)} - 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2), x, algorithm="maxima")

[Out] $\frac{1}{3}a(3x + 3c/d - 4(3e^{-2dx} - 2) - 3e^{-4dx} - 4c) - 2)/(d(3e^{-2dx} - 2) - 3e^{-4dx} - 4c) + e^{-6dx} - 6c) - 1)) + b(x + c/d + 2/(d(e^{-2dx} - 2)))$

mupad [B] time = 1.17, size = 162, normalized size = 4.50

$$\frac{\frac{2b}{3d} - \frac{2e^{2c+2dx}(2a+b)}{3d}}{e^{4c+4dx} - 2e^{2c+2dx} + 1} - \frac{\frac{2(2a+b)}{3d} - \frac{4be^{2c+2dx}}{3d} + \frac{2e^{4c+4dx}(2a+b)}{3d}}{3e^{2c+2dx} - 3e^{4c+4dx} + e^{6c+6dx} - 1}}{3d} + x(a+b) - \frac{2(2a+b)}{3d(e^{2c+2dx} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(c + d*x)^4*(a + b*tanh(c + d*x)^2), x)`

[Out] $((2b)/(3d) - (2\exp(2c + 2dx)*(2a + b))/(3d))/(\exp(4c + 4dx) - 2\exp(2c + 2dx) + 1) - ((2*(2a + b))/(3d) - (4b*\exp(2c + 2dx))/(3d) + (2*\exp(4c + 4dx)*(2a + b))/(3d))/(3*\exp(2c + 2dx) - 3*\exp(4c + 4dx) + \exp(6c + 6dx) - 1) + x*(a + b) - (2*(2a + b))/(3d*(\exp(2c + 2dx) - 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx)) \coth^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)**4*(a+b*tanh(d*x+c)**2), x)`

[Out] `Integral((a + b*tanh(c + d*x)**2)*coth(c + d*x)**4, x)`

3.143 $\int \coth^5(c + dx) (a + b \tanh^2(c + dx)) dx$

Optimal. Leaf size=49

$$-\frac{(a+b)\coth^2(c+dx)}{2d} + \frac{(a+b)\log(\sinh(c+dx))}{d} - \frac{a\coth^4(c+dx)}{4d}$$

[Out] $-1/2*(a+b)*\coth(d*x+c)^2/d-1/4*a*\coth(d*x+c)^4/d+(a+b)*\ln(\sinh(d*x+c))/d$

Rubi [A] time = 0.06, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3629, 12, 3473, 3475}

$$-\frac{(a+b)\coth^2(c+dx)}{2d} + \frac{(a+b)\log(\sinh(c+dx))}{d} - \frac{a\coth^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

[In] `Int[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2), x]`

[Out] $-\frac{(a+b)\coth^2(c+dx)}{2d} - \frac{a\coth^4(c+dx)}{4d} + \frac{(a+b)\log(\sinh(c+dx))}{d}$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 3473

`Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(b*(b*Tan[c + d*x])^(n-1))/(d*(n-1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]`

Rule 3475

`Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Log[RemoveContent[Cos[c + d*x], x]]/d, x] /; FreeQ[{c, d}, x]`

Rule 3629

`Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[((A*b^2 + a^2*C)*(a + b*Tan[e + f*x])^(m+1))/(b*f*(m+1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])^(m+1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[a^2 + b^2, 0]`

Rubi steps

$$\begin{aligned} \int \coth^5(c + dx) (a + b \tanh^2(c + dx)) dx &= -\frac{a \coth^4(c + dx)}{4d} + \int (a + b) \coth^3(c + dx) dx \\ &= -\frac{a \coth^4(c + dx)}{4d} + (a + b) \int \coth^3(c + dx) dx \\ &= -\frac{(a + b) \coth^2(c + dx)}{2d} - \frac{a \coth^4(c + dx)}{4d} + (a + b) \int \coth(c + dx) dx \\ &= -\frac{(a + b) \coth^2(c + dx)}{2d} - \frac{a \coth^4(c + dx)}{4d} + \frac{(a + b) \log(\sinh(c + dx))}{d} \end{aligned}$$

Mathematica [A] time = 0.34, size = 51, normalized size = 1.04

$$\frac{2(a+b)\coth^2(c+dx) - 4(a+b)(\log(\tanh(c+dx)) + \log(\cosh(c+dx))) + a\coth^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2), x]

[Out] -1/4*(2*(a + b)*Coth[c + d*x]^2 + a*Coth[c + d*x]^4 - 4*(a + b)*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]]))/d

fricas [B] time = 0.43, size = 1216, normalized size = 24.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] -((a + b)*d*x*cosh(d*x + c)^8 + 8*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + (a + b)*d*x*sinh(d*x + c)^8 - 2*(2*(a + b)*d*x - 2*a - b)*cosh(d*x + c)^6 + 2*(14*(a + b)*d*x*cosh(d*x + c)^2 - 2*(a + b)*d*x + 2*a + b)*sinh(d*x + c)^6 + 4*(14*(a + b)*d*x*cosh(d*x + c)^3 - 3*(2*(a + b)*d*x - 2*a - b)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(3*(a + b)*d*x - 2*a - 2*b)*cosh(d*x + c)^4 + 2*(35*(a + b)*d*x*cosh(d*x + c)^4 + 3*(a + b)*d*x - 15*(2*(a + b)*d*x - 2*a - b)*cosh(d*x + c)^2 - 2*a - 2*b)*sinh(d*x + c)^4 + 8*(7*(a + b)*d*x*cosh(d*x + c)^5 - 5*(2*(a + b)*d*x - 2*a - b)*cosh(d*x + c)^3 + (3*(a + b)*d*x - 2*a - 2*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*d*x - 2*(2*(a + b)*d*x - 2*a - b)*cosh(d*x + c)^2 + 2*(14*(a + b)*d*x*cosh(d*x + c)^6 - 15*(2*(a + b)*d*x - 2*a - b)*cosh(d*x + c)^4 - 2*(a + b)*d*x + 6*(3*(a + b)*d*x - 2*a - 2*b)*cosh(d*x + c)^2 + 2*a + b)*sinh(d*x + c)^2 - ((a + b)*cosh(d*x + c)^8 + 8*(a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a + b)*sinh(d*x + c)^8 - 4*(a + b)*cosh(d*x + c)^6 + 4*(7*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c)^6 + 8*(7*(a + b)*cosh(d*x + c)^3 - 3*(a + b)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(a + b)*cosh(d*x + c)^4 + 2*(35*(a + b)*cosh(d*x + c)^4 - 30*(a + b)*cosh(d*x + c)^2 + 3*a + 3*b)*sinh(d*x + c)^4 + 8*(7*(a + b)*cosh(d*x + c)^5 - 10*(a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(a + b)*cosh(d*x + c)^2 + 4*(7*(a + b)*cosh(d*x + c)^6 - 15*(a + b)*cosh(d*x + c)^4 + 9*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c)^2 + 8*(a + b)*cosh(d*x + c)^7 - 3*(a + b)*cosh(d*x + c)^5 + 3*(a + b)*cosh(d*x + c)^3 - (a + b)*cosh(d*x + c))*sinh(d*x + c) + (a + b)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(2*(a + b)*d*x*cosh(d*x + c)^7 - 3*(2*(a + b)*d*x - 2*a - b)*cosh(d*x + c)^5 + 2*(3*(a + b)*d*x - 2*a - 2*b)*cosh(d*x + c)^3 - (2*(a + b)*d*x - 2*a - b)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 - 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)

giac [B] time = 0.24, size = 93, normalized size = 1.90

$$\frac{(dx + c)(a + b) - (a + b)\log\left(\left|e^{(2dx+2c)} - 1\right|\right) + \frac{2\left((2a+b)e^{(6dx+6c)} - 2(a+b)e^{(4dx+4c)} + (2a+b)e^{(2dx+2c)}\right)}{\left(e^{(2dx+2c)} - 1\right)^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] $-\left(\frac{(d*x + c)*(a + b) - (a + b)*\log(\text{abs}(e^{(2*d*x + 2*c)} - 1)) + 2*((2*a + b)*e^{(6*d*x + 6*c)} - 2*(a + b)*e^{(4*d*x + 4*c)} + (2*a + b)*e^{(2*d*x + 2*c)})}{(e^{(2*d*x + 2*c)} - 1)^4}\right)/d$

maple [A] time = 0.25, size = 68, normalized size = 1.39

$$\frac{a \ln(\sinh(dx + c))}{d} - \frac{a(\coth^2(dx + c))}{2d} - \frac{a(\coth^4(dx + c))}{4d} + \frac{b \ln(\sinh(dx + c))}{d} - \frac{b(\coth^2(dx + c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2),x)

[Out] $a*\ln(\sinh(d*x+c))/d - 1/2*a*\coth(d*x+c)^2/d - 1/4*a*\coth(d*x+c)^4/d + 1/d*b*\ln(\sinh(d*x+c)) - 1/2/d*b*\coth(d*x+c)^2$

maxima [B] time = 0.34, size = 206, normalized size = 4.20

$$a\left(x + \frac{c}{d} + \frac{\log(e^{(-dx-c)} + 1)}{d} + \frac{\log(e^{(-dx-c)} - 1)}{d} + \frac{4(e^{(-2dx-2c)} - e^{(-4dx-4c)} + e^{(-6dx-6c)})}{d(4e^{(-2dx-2c)} - 6e^{(-4dx-4c)} + 4e^{(-6dx-6c)} - e^{(-8dx-8c)} - 1)}\right) + b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] $a*(x + c/d + \log(e^{(-d*x - c)} + 1)/d + \log(e^{(-d*x - c)} - 1)/d + 4*(e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)})/(d*(4*e^{(-2*d*x - 2*c)} - 6*e^{(-4*d*x - 4*c)} + 4*e^{(-6*d*x - 6*c)} - e^{(-8*d*x - 8*c)} - 1))) + b*(x + c/d + \log(e^{(-d*x - c)} + 1)/d + \log(e^{(-d*x - c)} - 1)/d + 2*e^{(-2*d*x - 2*c)}/(d*(2*e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)} - 1)))$

mupad [B] time = 1.25, size = 177, normalized size = 3.61

$$\frac{\ln(e^{2c} e^{2dx} - 1)(a + b)}{d} - \frac{2(2a + b)}{d(e^{2c+2dx} - 1)} - x(a + b) - \frac{2(4a + b)}{d(e^{4c+4dx} - 2e^{2c+2dx} + 1)} - \frac{8a}{d(3e^{2c+2dx} - 3e^{4c+4dx} + e^{6c+6dx} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^5*(a + b*tanh(c + d*x)^2),x)

[Out] $(\log(\exp(2*c)*\exp(2*d*x) - 1)*(a + b))/d - (2*(2*a + b))/(d*(\exp(2*c + 2*d*x) - 1)) - x*(a + b) - (2*(4*a + b))/(d*(\exp(4*c + 4*d*x) - 2*\exp(2*c + 2*d*x) + 1)) - (8*a)/(d*(3*\exp(2*c + 2*d*x) - 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) - 1)) - (4*a)/(d*(6*\exp(4*c + 4*d*x) - 4*\exp(2*c + 2*d*x) - 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx)) \coth^5(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**5*(a+b*tanh(d*x+c)**2),x)

[Out] Integral((a + b*tanh(c + d*x)**2)*coth(c + d*x)**5, x)

3.144 $\int \tanh^4(c + dx) (a + b \tanh^2(c + dx))^2 dx$

Optimal. Leaf size=83

$$\frac{b(2a + b) \tanh^5(c + dx)}{5d} - \frac{(a + b)^2 \tanh^3(c + dx)}{3d} - \frac{(a + b)^2 \tanh(c + dx)}{d} + x(a + b)^2 - \frac{b^2 \tanh^7(c + dx)}{7d}$$

[Out] $(a+b)^2*x - (a+b)^2*\tanh(d*x+c)/d - 1/3*(a+b)^2*\tanh(d*x+c)^3/d - 1/5*b*(2*a+b)*\tanh(d*x+c)^5/d - 1/7*b^2*\tanh(d*x+c)^7/d$

Rubi [A] time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 461, 206}

$$\frac{b(2a + b) \tanh^5(c + dx)}{5d} - \frac{(a + b)^2 \tanh^3(c + dx)}{3d} - \frac{(a + b)^2 \tanh(c + dx)}{d} + x(a + b)^2 - \frac{b^2 \tanh^7(c + dx)}{7d}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2, x]

[Out] $(a + b)^2*x - ((a + b)^2*\text{Tanh}[c + d*x])/d - ((a + b)^2*\text{Tanh}[c + d*x]^3)/(3*d) - (b*(2*a + b)*\text{Tanh}[c + d*x]^5)/(5*d) - (b^2*\text{Tanh}[c + d*x]^7)/(7*d)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 461

Int[(((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f*ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned} \int \tanh^4(c + dx) (a + b \tanh^2(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)^2}{1-x^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(- (a + b)^2 - (a + b)^2 x^2 - b(2a + b)x^4 - b^2 x^6 + \frac{a^2 + 2ab + b^2}{1 - x^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{(a + b)^2 \tanh(c + dx)}{d} - \frac{(a + b)^2 \tanh^3(c + dx)}{3d} - \frac{b(2a + b) \tanh^5(c + dx)}{5d} \\ &= (a + b)^2 x - \frac{(a + b)^2 \tanh(c + dx)}{d} - \frac{(a + b)^2 \tanh^3(c + dx)}{3d} - \frac{b(2a + b) \tanh^5(c + dx)}{5d} \end{aligned}$$

Mathematica [B] time = 0.08, size = 190, normalized size = 2.29

$$\frac{a^2 \tanh^{-1}(\tanh(c + dx))}{d} - \frac{a^2 \tanh^3(c + dx)}{3d} - \frac{a^2 \tanh(c + dx)}{d} + \frac{2ab \tanh^{-1}(\tanh(c + dx))}{d} - \frac{2ab \tanh^5(c + dx)}{5d} - \frac{2a^2 \tanh^7(c + dx)}{7d}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (a^2*ArcTanh[Tanh[c + d*x]])/d + (2*a*b*ArcTanh[Tanh[c + d*x]])/d + (b^2*ArcTanh[Tanh[c + d*x]])/d - (a^2*Tanh[c + d*x])/d - (2*a*b*Tanh[c + d*x])/d - (b^2*Tanh[c + d*x])/d - (a^2*Tanh[c + d*x]^3)/(3*d) - (2*a*b*Tanh[c + d*x]^3)/(3*d) - (b^2*Tanh[c + d*x]^3)/(3*d) - (2*a*b*Tanh[c + d*x]^5)/(5*d) - (b^2*Tanh[c + d*x]^5)/(5*d) - (b^2*Tanh[c + d*x]^7)/(7*d)

fricas [B] time = 0.42, size = 796, normalized size = 9.59

$$\frac{(105(a^2 + 2ab + b^2)dx + 140a^2 + 322ab + 176b^2) \cosh(dx + c)^7 + 7(105(a^2 + 2ab + b^2)dx + 140a^2 + 322ab + 176b^2) \cosh(dx + c)^5 + 7(105(a^2 + 2ab + b^2)dx + 140a^2 + 322ab + 176b^2) \cosh(dx + c)^3 + 7(105(a^2 + 2ab + b^2)dx + 140a^2 + 322ab + 176b^2) \cosh(dx + c)}{(105(a^2 + 2ab + b^2)dx + 140a^2 + 322ab + 176b^2) \cosh(dx + c)^7 + 7(105(a^2 + 2ab + b^2)dx + 140a^2 + 322ab + 176b^2) \cosh(dx + c)^5 + 7(105(a^2 + 2ab + b^2)dx + 140a^2 + 322ab + 176b^2) \cosh(dx + c)^3 + 7(105(a^2 + 2ab + b^2)dx + 140a^2 + 322ab + 176b^2) \cosh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] 1/105*((105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c)^7 + 7*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c)^5 + 7*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c)^3 + 7*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c) - 14*(3*(70*a^2 + 161*a*b + 88*b^2)*cosh(d*x + c)^2 + 40*a^2 + 71*a*b + 28*b^2)*sinh(d*x + c)^5 + 35*((105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c)^3 + (105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 21*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c)^3 - 14*(5*(70*a^2 + 161*a*b + 88*b^2)*cosh(d*x + c)^4 + 10*(40*a^2 + 71*a*b + 28*b^2)*cosh(d*x + c)^2 + 60*a^2 + 123*a*b + 84*b^2)*sinh(d*x + c)^3 + 7*(3*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c)^5 + 10*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c)^3 + 9*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c) - 14*((70*a^2 + 161*a*b + 88*b^2)*cosh(d*x + c)^6 + 5*(40*a^2 + 71*a*b + 28*b^2)*cosh(d*x + c)^4 + 9*(20*a^2 + 41*a*b + 28*b^2)*cosh(d*x + c)^2 + 30*a^2 + 75*a*b)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + 7*d*cosh(d*x + c)^5 + 35*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^4 + 21*d*cosh(d*x + c)^3 + 7*(3*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^2 + 35*d*cosh(d*x + c))

giac [B] time = 0.32, size = 300, normalized size = 3.61

$$105(a^2 + 2ab + b^2)(dx + c) + \frac{4(105a^2e^{(12dx+12c)} + 315abe^{(12dx+12c)} + 210b^2e^{(12dx+12c)} + 525a^2e^{(10dx+10c)} + 1260abe^{(10dx+10c)} + 630b^2e^{(10dx+10c)})}{(105(a^2 + 2ab + b^2)(dx + c) + 4(105a^2e^{(12dx+12c)} + 315abe^{(12dx+12c)} + 210b^2e^{(12dx+12c)} + 525a^2e^{(10dx+10c)} + 1260abe^{(10dx+10c)} + 630b^2e^{(10dx+10c)}) + 1120a^2e^{(8dx+8c)} + 2555a^2e^{(8dx+8c)} + 1540b^2e^{(8dx+8c)} + 1330)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/105*(105*(a^2 + 2*a*b + b^2)*(d*x + c) + 4*(105*a^2*e^(12*d*x + 12*c) + 315*a*b*e^(12*d*x + 12*c) + 210*b^2*e^(12*d*x + 12*c) + 525*a^2*e^(10*d*x + 10*c) + 1260*a*b*e^(10*d*x + 10*c) + 630*b^2*e^(10*d*x + 10*c) + 1120*a^2*e^(8*d*x + 8*c) + 2555*a*b*e^(8*d*x + 8*c) + 1540*b^2*e^(8*d*x + 8*c) + 1330)

$$*a^2*e^{(6*d*x + 6*c)} + 3080*a*b*e^{(6*d*x + 6*c)} + 1540*b^2*e^{(6*d*x + 6*c)} + 945*a^2*e^{(4*d*x + 4*c)} + 2121*a*b*e^{(4*d*x + 4*c)} + 1218*b^2*e^{(4*d*x + 4*c)} + 385*a^2*e^{(2*d*x + 2*c)} + 812*a*b*e^{(2*d*x + 2*c)} + 406*b^2*e^{(2*d*x + 2*c)} + 70*a^2 + 161*a*b + 88*b^2)/(e^{(2*d*x + 2*c)} + 1)^7/d$$

maple [B] time = 0.02, size = 236, normalized size = 2.84

$$\frac{a^2 \tanh(dx + c)}{d} - \frac{\ln(\tanh(dx + c) - 1) a^2}{2d} - \frac{\ln(\tanh(dx + c) - 1) ab}{d} - \frac{\ln(\tanh(dx + c) - 1) b^2}{2d} - \frac{b^2 (\tanh^3(dx + c))}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x)

$$[Out] -a^2*\tanh(d*x+c)/d-1/2/d*\ln(\tanh(d*x+c)-1)*a^2-1/d*\ln(\tanh(d*x+c)-1)*a*b-1/2/d*\ln(\tanh(d*x+c)-1)*b^2-1/3*b^2*\tanh(d*x+c)^3/d-1/3/d*a^2*\tanh(d*x+c)^3-1/5*b^2*\tanh(d*x+c)^5/d-1/7*b^2*\tanh(d*x+c)^7/d+1/2/d*\ln(1+\tanh(d*x+c))*a^2+1/d*\ln(1+\tanh(d*x+c))*a*b+1/2/d*\ln(1+\tanh(d*x+c))*b^2-2/3*a*b*\tanh(d*x+c)^3/d-2/5/d*\tanh(d*x+c)^5*a*b-2*a*b*\tanh(d*x+c)/d-b^2*\tanh(d*x+c)/d$$

maxima [B] time = 0.34, size = 369, normalized size = 4.45

$$\frac{1}{105} b^2 \left(105x + \frac{105c}{d} - \frac{8(203e^{(-2dx-2c)} + 609e^{(-4dx-4c)} + 770e^{(-6dx-6c)} + 770e^{(-8dx-8c)} + 315e^{(-10dx-10c)} + 105e^{(-12dx-12c)} + 44)}{d(7e^{(-2dx-2c)} + 21e^{(-4dx-4c)} + 35e^{(-6dx-6c)} + 35e^{(-8dx-8c)} + 21e^{(-10dx-10c)} + 7e^{(-12dx-12c)} + e^{(-14dx-14c)} + 1)} \right) + \frac{2}{15} a*b*(15*x + 15*c/d - 2*(70*e^{(-2*d*x - 2*c)} + 140*e^{(-4*d*x - 4*c)} + 90*e^{(-6*d*x - 6*c)} + 45*e^{(-8*d*x - 8*c)} + 23)/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1))) + \frac{1}{3} a^2*(3*x + 3*c/d - 4*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + 2)/(d*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} + 1)))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

$$[Out] 1/105*b^2*(105*x + 105*c/d - 8*(203*e^{(-2*d*x - 2*c)} + 609*e^{(-4*d*x - 4*c)} + 770*e^{(-6*d*x - 6*c)} + 770*e^{(-8*d*x - 8*c)} + 315*e^{(-10*d*x - 10*c)} + 105*e^{(-12*d*x - 12*c)} + 44)/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1))) + 2/15*a*b*(15*x + 15*c/d - 2*(70*e^{(-2*d*x - 2*c)} + 140*e^{(-4*d*x - 4*c)} + 90*e^{(-6*d*x - 6*c)} + 45*e^{(-8*d*x - 8*c)} + 23)/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1))) + 1/3*a^2*(3*x + 3*c/d - 4*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + 2)/(d*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} + 1)))$$

mupad [B] time = 0.18, size = 91, normalized size = 1.10

$$x(a^2 + 2ab + b^2) - \frac{\tanh(c + dx)(a + b)^2}{d} - \frac{\tanh(c + dx)^5(b^2 + 2ab)}{5d} - \frac{b^2 \tanh(c + dx)^7}{7d} - \frac{\tanh(c + dx)^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2,x)

$$[Out] x*(2*a*b + a^2 + b^2) - (\tanh(c + d*x)*(a + b)^2)/d - (\tanh(c + d*x)^5*(2*a*b + b^2))/(5*d) - (b^2*\tanh(c + d*x)^7)/(7*d) - (\tanh(c + d*x)^3*(2*a*b + a^2 + b^2))/(3*d)$$

sympy [A] time = 1.11, size = 165, normalized size = 1.99

$$\begin{cases} a^2x - \frac{a^2 \tanh^3(c+dx)}{3d} - \frac{a^2 \tanh(c+dx)}{d} + 2abx - \frac{2ab \tanh^5(c+dx)}{5d} - \frac{2ab \tanh^3(c+dx)}{3d} - \frac{2ab \tanh(c+dx)}{d} + b^2x - \frac{b^2 \tanh^7(c+dx)}{7d} \\ x(a + b \tanh^2(c))^2 \tanh^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x - a**2*tanh(c + d*x)**3/(3*d) - a**2*tanh(c + d*x)/d + 2*  
a*b*x - 2*a*b*tanh(c + d*x)**5/(5*d) - 2*a*b*tanh(c + d*x)**3/(3*d) - 2*a*b  
*tanh(c + d*x)/d + b**2*x - b**2*tanh(c + d*x)**7/(7*d) - b**2*tanh(c + d*x)  
)**5/(5*d) - b**2*tanh(c + d*x)**3/(3*d) - b**2*tanh(c + d*x)/d, Ne(d, 0)),  
(x*(a + b*tanh(c)**2)**2*tanh(c)**4, True))
```

3.145 $\int \tanh^3(c + dx) (a + b \tanh^2(c + dx))^2 dx$

Optimal. Leaf size=76

$$\frac{b(2a + b) \tanh^4(c + dx)}{4d} - \frac{(a + b)^2 \tanh^2(c + dx)}{2d} + \frac{(a + b)^2 \log(\cosh(c + dx))}{d} - \frac{b^2 \tanh^6(c + dx)}{6d}$$

[Out] (a+b)^2*ln(cosh(d*x+c))/d-1/2*(a+b)^2*tanh(d*x+c)^2/d-1/4*b*(2*a+b)*tanh(d*x+c)^4/d-1/6*b^2*tanh(d*x+c)^6/d

Rubi [A] time = 0.11, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 446, 77}

$$\frac{b(2a + b) \tanh^4(c + dx)}{4d} - \frac{(a + b)^2 \tanh^2(c + dx)}{2d} + \frac{(a + b)^2 \log(\cosh(c + dx))}{d} - \frac{b^2 \tanh^6(c + dx)}{6d}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] ((a + b)^2*Log[Cosh[c + d*x]])/d - ((a + b)^2*Tanh[c + d*x]^2)/(2*d) - (b*(2*a + b)*Tanh[c + d*x]^4)/(4*d) - (b^2*Tanh[c + d*x]^6)/(6*d)

Rule 77

Int[((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_.)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
\int \tanh^3(c + dx) (a + b \tanh^2(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \frac{x^3(a+bx^2)^2}{1-x^2} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{x(a+bx)^2}{1-x} dx, x, \tanh^2(c + dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(- (a+b)^2 - \frac{(a+b)^2}{-1+x} - b(2a+b)x - b^2x^2\right) dx, x, \tanh^2(c + dx)\right)}{2d} \\
&= \frac{(a+b)^2 \log(\cosh(c + dx))}{d} - \frac{(a+b)^2 \tanh^2(c + dx)}{2d} - \frac{b(2a+b) \tanh^4(c + dx)}{4d}
\end{aligned}$$

Mathematica [A] time = 0.43, size = 66, normalized size = 0.87

$$\frac{3b(2a + b) \tanh^4(c + dx) + 6(a + b)^2 \tanh^2(c + dx) - 12(a + b)^2 \log(\cosh(c + dx)) + 2b^2 \tanh^6(c + dx)}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] -1/12*(-12*(a + b)^2*Log[Cosh[c + d*x]] + 6*(a + b)^2*Tanh[c + d*x]^2 + 3*b*(2*a + b)*Tanh[c + d*x]^4 + 2*b^2*Tanh[c + d*x]^6)/d

fricas [B] time = 0.48, size = 3441, normalized size = 45.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -1/3*(3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^12 + 36*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^11 + 3*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^12 + 6*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x + c)^10 + 6*(33*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + 3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*sinh(d*x + c)^10 + 60*(11*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 + 3*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c)^8 + 3*(495*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 15*(a^2 + 2*a*b + b^2)*d*x + 90*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x + c)^2 - 8*a^2 - 24*a*b - 12*b^2)*sinh(d*x + c)^8 + 24*(99*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^5 + 30*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x + c)^3 + (15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(15*(a^2 + 2*a*b + b^2)*d*x - 9*a^2 - 24*a*b - 17*b^2)*cosh(d*x + c)^6 + 4*(693*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 315*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x + c)^4 + 15*(a^2 + 2*a*b + b^2)*d*x + 21*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c)^2 - 9*a^2 - 24*a*b - 17*b^2)*sinh(d*x + c)^6 + 24*(99*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^7 + 63*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x + c)^5 + 7*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c)^3 + (15*(a^2 + 2*a*b + b^2)*d*x - 9*a^2 - 24*a*b - 17*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c)^4 + 3*(495*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 420*(3*(a^2 + 2*a*b + b^2)*d*x - a^2 - 4*a*b - 3*b^2)*cosh(d*x + c)^6 + 70*(15*(a^2 + 2*a*b + b^2)*d*x - 8*a^2 - 24*a*b - 12*b^2)*cosh(d*x + c)^4 + 15*(a^2 + 2*a*b + b^2)*d*x + 20*(15*(a^2 + 2*a*b + b^2)*d*x - 9*a^2 - 24*a*b - 17*b^2)*cosh(d*x + c)^

$$\begin{aligned}
& 2 - 8a^2 - 24ab - 12b^2) \sinh(dx + c)^4 + 4(165(a^2 + 2ab + b^2)dx \\
& * \cosh(dx + c)^9 + 180(3(a^2 + 2ab + b^2)dx - a^2 - 4ab - 3b^2) * \\
& \cosh(dx + c)^7 + 42(15(a^2 + 2ab + b^2)dx - 8a^2 - 24ab - 12b^2) * \\
& \cosh(dx + c)^5 + 20(15(a^2 + 2ab + b^2)dx - 9a^2 - 24ab - 17b^2) * \\
& \cosh(dx + c)^3 + 3(15(a^2 + 2ab + b^2)dx - 8a^2 - 24ab - 12b^2) * \\
& \cosh(dx + c) * \sinh(dx + c)^3 + 3(a^2 + 2ab + b^2)dx + 6(3(a^2 + \\
& 2ab + b^2)dx - a^2 - 4ab - 3b^2) * \cosh(dx + c)^2 + 6(33(a^2 + 2ab \\
& + b^2)dx * \cosh(dx + c)^10 + 45(3(a^2 + 2ab + b^2)dx - a^2 - 4ab \\
& - 3b^2) * \cosh(dx + c)^8 + 14(15(a^2 + 2ab + b^2)dx - 8a^2 - 24ab \\
& - 12b^2) * \cosh(dx + c)^6 + 10(15(a^2 + 2ab + b^2)dx - 9a^2 - 24ab \\
& + b^2) * \cosh(dx + c)^4 + 3(a^2 + 2ab + b^2)dx + 3(15(a^2 + 2ab \\
& + b^2)dx - 8a^2 - 24ab - 12b^2) * \cosh(dx + c)^2 - a^2 - 4ab - 3b^2 \\
& ^2) * \sinh(dx + c)^2 - 3((a^2 + 2ab + b^2) * \cosh(dx + c)^12 + 12(a^2 + 2 \\
& * ab + b^2) * \cosh(dx + c) * \sinh(dx + c)^11 + (a^2 + 2ab + b^2) * \sinh(dx + \\
& c)^12 + 6(a^2 + 2ab + b^2) * \cosh(dx + c)^10 + 6(11(a^2 + 2ab + b^2) \\
& * \cosh(dx + c)^2 + a^2 + 2ab + b^2) * \sinh(dx + c)^10 + 20(11(a^2 + 2ab \\
& + b^2) * \cosh(dx + c)^3 + 3(a^2 + 2ab + b^2) * \cosh(dx + c)) * \sinh(dx + \\
& c)^9 + 15(a^2 + 2ab + b^2) * \cosh(dx + c)^8 + 15(33(a^2 + 2ab + b^2) * \\
& \cosh(dx + c)^4 + 18(a^2 + 2ab + b^2) * \cosh(dx + c)^2 + a^2 + 2ab + b^2 \\
& ^2) * \sinh(dx + c)^8 + 24(33(a^2 + 2ab + b^2) * \cosh(dx + c)^5 + 30(a^2 + \\
& 2ab + b^2) * \cosh(dx + c)^3 + 5(a^2 + 2ab + b^2) * \cosh(dx + c)) * \sinh(dx \\
& + c)^7 + 20(a^2 + 2ab + b^2) * \cosh(dx + c)^6 + 4(231(a^2 + 2ab + \\
& b^2) * \cosh(dx + c)^6 + 315(a^2 + 2ab + b^2) * \cosh(dx + c)^4 + 105(a^2 + \\
& 2ab + b^2) * \cosh(dx + c)^2 + 5a^2 + 10ab + 5b^2) * \sinh(dx + c)^6 + 2 \\
& 4(33(a^2 + 2ab + b^2) * \cosh(dx + c)^7 + 63(a^2 + 2ab + b^2) * \cosh(dx \\
& + c)^5 + 35(a^2 + 2ab + b^2) * \cosh(dx + c)^3 + 5(a^2 + 2ab + b^2) * \cosh(dx \\
& + c)) * \sinh(dx + c)^5 + 15(a^2 + 2ab + b^2) * \cosh(dx + c)^4 + 15(\\
& 33(a^2 + 2ab + b^2) * \cosh(dx + c)^8 + 84(a^2 + 2ab + b^2) * \cosh(dx + \\
& c)^6 + 70(a^2 + 2ab + b^2) * \cosh(dx + c)^4 + 20(a^2 + 2ab + b^2) * \cosh \\
& (dx + c)^2 + a^2 + 2ab + b^2) * \sinh(dx + c)^4 + 20(11(a^2 + 2ab + b \\
& ^2) * \cosh(dx + c)^9 + 36(a^2 + 2ab + b^2) * \cosh(dx + c)^7 + 42(a^2 + 2 \\
& ab + b^2) * \cosh(dx + c)^5 + 20(a^2 + 2ab + b^2) * \cosh(dx + c)^3 + 3(a^2 \\
& + 2ab + b^2) * \cosh(dx + c)) * \sinh(dx + c)^3 + 6(a^2 + 2ab + b^2) * \cosh \\
& (dx + c)^2 + 6(11(a^2 + 2ab + b^2) * \cosh(dx + c)^10 + 45(a^2 + 2ab \\
& + b^2) * \cosh(dx + c)^8 + 70(a^2 + 2ab + b^2) * \cosh(dx + c)^6 + 50(a^2 \\
& + 2ab + b^2) * \cosh(dx + c)^4 + 15(a^2 + 2ab + b^2) * \cosh(dx + c)^2 + a \\
& ^2 + 2ab + b^2) * \sinh(dx + c)^2 + a^2 + 2ab + b^2 + 12((a^2 + 2ab + \\
& b^2) * \cosh(dx + c)^11 + 5(a^2 + 2ab + b^2) * \cosh(dx + c)^9 + 10(a^2 + 2 \\
& * ab + b^2) * \cosh(dx + c)^7 + 10(a^2 + 2ab + b^2) * \cosh(dx + c)^5 + 5(a \\
& ^2 + 2ab + b^2) * \cosh(dx + c)^3 + (a^2 + 2ab + b^2) * \cosh(dx + c)) * \sinh \\
& (dx + c)) * \log(2 * \cosh(dx + c) / (\cosh(dx + c) - \sinh(dx + c))) + 12(3(a^2 \\
& + 2ab + b^2) * dx * \cosh(dx + c)^11 + 5(3(a^2 + 2ab + b^2) * dx - a^2 \\
& - 4ab - 3b^2) * \cosh(dx + c)^9 + 2(15(a^2 + 2ab + b^2) * dx - 8a^2 - \\
& 24ab - 12b^2) * \cosh(dx + c)^7 + 2(15(a^2 + 2ab + b^2) * dx - 9a^2 - \\
& 24ab - 17b^2) * \cosh(dx + c)^5 + (15(a^2 + 2ab + b^2) * dx - 8a^2 - 24 \\
& * ab - 12b^2) * \cosh(dx + c)^3 + (3(a^2 + 2ab + b^2) * dx - a^2 - 4ab - \\
& 3b^2) * \cosh(dx + c)) * \sinh(dx + c)) / (d * \cosh(dx + c)^12 + 12d * \cosh(dx + \\
& c) * \sinh(dx + c)^11 + d * \sinh(dx + c)^12 + 6d * \cosh(dx + c)^10 + 6(11d * \\
& \cosh(dx + c)^2 + d) * \sinh(dx + c)^10 + 20(11d * \cosh(dx + c)^3 + 3d * \cosh \\
& (dx + c)) * \sinh(dx + c)^9 + 15d * \cosh(dx + c)^8 + 15(33d * \cosh(dx + c)^ \\
& 4 + 18d * \cosh(dx + c)^2 + d) * \sinh(dx + c)^8 + 24(33d * \cosh(dx + c)^5 + \\
& 30d * \cosh(dx + c)^3 + 5d * \cosh(dx + c)) * \sinh(dx + c)^7 + 20d * \cosh(dx + \\
& c)^6 + 4(231d * \cosh(dx + c)^6 + 315d * \cosh(dx + c)^4 + 105d * \cosh(dx + \\
& c)^2 + 5d) * \sinh(dx + c)^6 + 24(33d * \cosh(dx + c)^7 + 63d * \cosh(dx + c \\
&)^5 + 35d * \cosh(dx + c)^3 + 5d * \cosh(dx + c)) * \sinh(dx + c)^5 + 15d * \cosh \\
& (dx + c)^4 + 15(33d * \cosh(dx + c)^8 + 84d * \cosh(dx + c)^6 + 70d * \cosh(dx \\
& + c)^4 + 20d * \cosh(dx + c)^2 + d) * \sinh(dx + c)^4 + 20(11d * \cosh(dx + \\
& c)^9 + 36d * \cosh(dx + c)^7 + 42d * \cosh(dx + c)^5 + 20d * \cosh(dx + c)^3 \\
& + 3d * \cosh(dx + c)) * \sinh(dx + c)^3 + 6d * \cosh(dx + c)^2 + 6(11d * \cosh(dx
\end{aligned}$$

$x + c)^{10} + 45*d*\cosh(d*x + c)^8 + 70*d*\cosh(d*x + c)^6 + 50*d*\cosh(d*x + c)^4 + 15*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^2 + 12*(d*\cosh(d*x + c)^{11} + 5*d*\cosh(d*x + c)^9 + 10*d*\cosh(d*x + c)^7 + 10*d*\cosh(d*x + c)^5 + 5*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c) + d)$

giac [B] time = 0.29, size = 191, normalized size = 2.51

$$\frac{3(a^2 + 2ab + b^2)(dx + c) - 3(a^2 + 2ab + b^2)\log(e^{2dx+2c} + 1) - \frac{2(3(a^2+4ab+3b^2)e^{10dx+10c} + 6(2a^2+6ab+3b^2)e^{8dx+8c} + 10(3a^2+4ab+3b^2)e^{6dx+6c} + 6(2a^2+6ab+3b^2)e^{4dx+4c} + 3(a^2+4ab+3b^2)e^{2dx+2c} + 1)}{3d}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] $-1/3*(3*(a^2 + 2*a*b + b^2)*(d*x + c) - 3*(a^2 + 2*a*b + b^2)*\log(e^{(2*d*x + 2*c) + 1}) - 2*(3*(a^2 + 4*a*b + 3*b^2)*e^{(10*d*x + 10*c)} + 6*(2*a^2 + 6*a*b + 3*b^2)*e^{(8*d*x + 8*c)} + 2*(9*a^2 + 24*a*b + 17*b^2)*e^{(6*d*x + 6*c)} + 6*(2*a^2 + 6*a*b + 3*b^2)*e^{(4*d*x + 4*c)} + 3*(a^2 + 4*a*b + 3*b^2)*e^{(2*d*x + 2*c)})/(e^{(2*d*x + 2*c) + 1})^6)/d$

maple [B] time = 0.02, size = 196, normalized size = 2.58

$$\frac{\ln(\tanh(dx+c)-1)a^2}{2d} - \frac{\ln(\tanh(dx+c)-1)ab}{d} - \frac{\ln(\tanh(dx+c)-1)b^2}{2d} - \frac{\ln(1+\tanh(dx+c))a^2}{2d} - \frac{\ln(1+\tanh(dx+c))ab}{d} - \frac{\ln(1+\tanh(dx+c))b^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x)

[Out] $-1/2/d*\ln(\tanh(d*x+c)-1)*a^2-1/d*\ln(\tanh(d*x+c)-1)*a*b-1/2/d*\ln(\tanh(d*x+c)-1)*b^2-1/2/d*\ln(1+\tanh(d*x+c))*a^2-1/d*\ln(1+\tanh(d*x+c))*a*b-1/2/d*\ln(1+\tanh(d*x+c))*b^2-1/2*b^2*\tanh(d*x+c)^2/d-1/2/d*a^2*\tanh(d*x+c)^2-1/4/d*\tanh(d*x+c)^4*b^2-1/6*b^2*\tanh(d*x+c)^6/d-1/2/d*\tanh(d*x+c)^4*a*b-a*b*\tanh(d*x+c)^2/d$

maxima [B] time = 0.43, size = 333, normalized size = 4.38

$$\frac{1}{3}b^2\left(3x + \frac{3c}{d} + \frac{3\log(e^{(-2dx-2c)} + 1)}{d} + \frac{2(9e^{(-2dx-2c)} + 18e^{(-4dx-4c)} + 34e^{(-6dx-6c)} + 18e^{(-8dx-8c)} + 9e^{(-10dx-10c)})}{d(6e^{(-2dx-2c)} + 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} + 15e^{(-8dx-8c)} + 6e^{(-10dx-10c)})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $1/3*b^2*(3*x + 3*c/d + 3*\log(e^{(-2*d*x - 2*c) + 1})/d + 2*(9*e^{(-2*d*x - 2*c)} + 18*e^{(-4*d*x - 4*c)} + 34*e^{(-6*d*x - 6*c)} + 18*e^{(-8*d*x - 8*c)} + 9*e^{(-10*d*x - 10*c)})/(d*(6*e^{(-2*d*x - 2*c)} + 15*e^{(-4*d*x - 4*c)} + 20*e^{(-6*d*x - 6*c)} + 15*e^{(-8*d*x - 8*c)} + 6*e^{(-10*d*x - 10*c)} + e^{(-12*d*x - 12*c)} + 1))) + 2*a*b*(x + c/d + \log(e^{(-2*d*x - 2*c) + 1})/d + 4*(e^{(-2*d*x - 2*c)} + e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)})/(d*(4*e^{(-2*d*x - 2*c)} + 6*e^{(-4*d*x - 4*c)} + 4*e^{(-6*d*x - 6*c)} + e^{(-8*d*x - 8*c)} + 1))) + a^2*(x + c/d + \log(e^{(-2*d*x - 2*c) + 1})/d + 2*e^{(-2*d*x - 2*c)})/(d*(2*e^{(-2*d*x - 2*c)} + e^{(-4*d*x - 4*c)} + 1)))$

mupad [B] time = 1.31, size = 100, normalized size = 1.32

$$x(a^2 + 2ab + b^2) - \frac{\tanh(c + dx)^4(b^2 + 2ab)}{4d} - \frac{\ln(\tanh(c + dx) + 1)(a^2 + 2ab + b^2)}{d} - \frac{b^2 \tanh(c + dx)^6}{6d} - \frac{\tanh(c + dx)^8}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2,x)`

[Out] $x*(2*a*b + a^2 + b^2) - (\tanh(c + d*x)^4*(2*a*b + b^2))/(4*d) - (\log(\tanh(c + d*x) + 1)*(2*a*b + a^2 + b^2))/d - (b^2*\tanh(c + d*x)^6)/(6*d) - (\tanh(c + d*x)^2*(2*a*b + a^2 + b^2))/(2*d)$

sympy [A] time = 0.89, size = 170, normalized size = 2.24

$$\left\{ \begin{array}{l} a^2x - \frac{a^2 \log(\tanh(c+dx)+1)}{d} - \frac{a^2 \tanh^2(c+dx)}{2d} + 2abx - \frac{2ab \log(\tanh(c+dx)+1)}{d} - \frac{ab \tanh^4(c+dx)}{2d} - \frac{ab \tanh^2(c+dx)}{d} + b^2x - \frac{b^2 \tanh^6(c+dx)}{6d} - \frac{b^2 \tanh^2(c+dx)}{2d} \\ x(a + b \tanh^2(c))^2 \tanh^3(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)`

[Out] `Piecewise((a**2*x - a**2*log(tanh(c + d*x) + 1)/d - a**2*tanh(c + d*x)**2/(2*d) + 2*a*b*x - 2*a*b*log(tanh(c + d*x) + 1)/d - a*b*tanh(c + d*x)**4/(2*d) - a*b*tanh(c + d*x)**2/d + b**2*x - b**2*log(tanh(c + d*x) + 1)/d - b**2*tanh(c + d*x)**6/(6*d) - b**2*tanh(c + d*x)**4/(4*d) - b**2*tanh(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)**2*tanh(c)**3, True))`

3.146 $\int \tanh^2(c + dx) (a + b \tanh^2(c + dx))^2 dx$

Optimal. Leaf size=63

$$\frac{b(2a + b) \tanh^3(c + dx)}{3d} - \frac{(a + b)^2 \tanh(c + dx)}{d} + x(a + b)^2 - \frac{b^2 \tanh^5(c + dx)}{5d}$$

[Out] (a+b)^2*x - (a+b)^2*tanh(d*x+c)/d - 1/3*b*(2*a+b)*tanh(d*x+c)^3/d - 1/5*b^2*tanh(d*x+c)^5/d

Rubi [A] time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 461, 206}

$$\frac{b(2a + b) \tanh^3(c + dx)}{3d} - \frac{(a + b)^2 \tanh(c + dx)}{d} + x(a + b)^2 - \frac{b^2 \tanh^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (a + b)^2*x - ((a + b)^2*Tanh[c + d*x])/d - (b*(2*a + b)*Tanh[c + d*x]^3)/(3*d) - (b^2*Tanh[c + d*x]^5)/(5*d)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 461

Int[(((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.))/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.))*((a_) + (b_.))*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned} \int \tanh^2(c + dx) (a + b \tanh^2(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^2}{1-x^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(- (a + b)^2 - b(2a + b)x^2 - b^2x^4 + \frac{a^2 + 2ab + b^2}{1-x^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{(a + b)^2 \tanh(c + dx)}{d} - \frac{b(2a + b) \tanh^3(c + dx)}{3d} - \frac{b^2 \tanh^5(c + dx)}{5d} \\ &= (a + b)^2 x - \frac{(a + b)^2 \tanh(c + dx)}{d} - \frac{b(2a + b) \tanh^3(c + dx)}{3d} - \frac{b^2 \tanh^5(c + dx)}{5d} \end{aligned}$$

Mathematica [B] time = 0.05, size = 137, normalized size = 2.17

$$\frac{a^2 \tanh^{-1}(\tanh(c + dx))}{d} - \frac{a^2 \tanh(c + dx)}{d} + \frac{2ab \tanh^{-1}(\tanh(c + dx))}{d} - \frac{2ab \tanh^3(c + dx)}{3d} - \frac{2ab \tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (a^2*ArcTanh[Tanh[c + d*x]])/d + (2*a*b*ArcTanh[Tanh[c + d*x]])/d + (b^2*ArcTanh[Tanh[c + d*x]])/d - (a^2*Tanh[c + d*x])/d - (2*a*b*Tanh[c + d*x])/d - (b^2*Tanh[c + d*x])/d - (2*a*b*Tanh[c + d*x]^3)/(3*d) - (b^2*Tanh[c + d*x]^3)/(3*d) - (b^2*Tanh[c + d*x]^5)/(5*d)

fricas [B] time = 0.43, size = 483, normalized size = 7.67

$$\frac{(15(a^2 + 2ab + b^2)dx + 15a^2 + 40ab + 23b^2) \cosh(dx + c)^5 + 5(15(a^2 + 2ab + b^2)dx + 15a^2 + 40ab + 23b^2) \sinh(dx + c)^4 - (15a^2 + 40ab + 23b^2) \cosh(dx + c)^3 - 5(15(a^2 + 2ab + b^2)dx + 15a^2 + 40ab + 23b^2) \sinh(dx + c)^3 + 5(2(15(a^2 + 2ab + b^2)dx + 15a^2 + 40ab + 23b^2) \cosh(dx + c)^2 + 9a^2 + 16ab + 5b^2) \sinh(dx + c)^2 + 5(2(15(a^2 + 2ab + b^2)dx + 15a^2 + 40ab + 23b^2) \cosh(dx + c) + 3(15(a^2 + 2ab + b^2)dx + 15a^2 + 40ab + 23b^2) \sinh(dx + c)) \sinh(dx + c)^2 + 10(15(a^2 + 2ab + b^2)dx + 15a^2 + 40ab + 23b^2) \cosh(dx + c) - 5((15a^2 + 40ab + 23b^2) \cosh(dx + c)^4 + 3(9a^2 + 16ab + 5b^2) \cosh(dx + c)^2 + 6a^2 + 8ab + 10b^2) \sinh(dx + c)) / (d \cosh(dx + c)^5 + 5d \cosh(dx + c) \sinh(dx + c)^4 + 5d \cosh(dx + c)^3 + 5(2d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^2 + 10d \cosh(dx + c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] 1/15*((15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c)^5 + 5*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*a^2 + 40*a*b + 23*b^2)*sinh(d*x + c)^5 + 5*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c)^3 - 5*(2*(15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c)^2 + 9*a^2 + 16*a*b + 5*b^2)*sinh(d*x + c)^3 + 5*(2*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c)^3 + 3*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c) - 5*((15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c)^4 + 3*(9*a^2 + 16*a*b + 5*b^2)*cosh(d*x + c)^2 + 6*a^2 + 8*a*b + 10*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))

giac [B] time = 0.26, size = 218, normalized size = 3.46

$$\frac{15(a^2 + 2ab + b^2)(dx + c) + \frac{2(15a^2e^{(8dx+8c)} + 60abe^{(8dx+8c)} + 45b^2e^{(8dx+8c)} + 60a^2e^{(6dx+6c)} + 180abe^{(6dx+6c)} + 90b^2e^{(6dx+6c)} + 90a^2e^{(4dx+4c)} + 140abe^{(4dx+4c)} + 140b^2e^{(4dx+4c)} + 60a^2e^{(2dx+2c)} + 140ab^2e^{(2dx+2c)} + 70b^2e^{(2dx+2c)} + 15a^2 + 40ab + 23b^2)}{(e^{(2dx+2c)} + 1)^5}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/15*(15*(a^2 + 2*a*b + b^2)*(d*x + c) + 2*(15*a^2*e^(8*d*x + 8*c) + 60*a*b*e^(8*d*x + 8*c) + 45*b^2*e^(8*d*x + 8*c) + 60*a^2*e^(6*d*x + 6*c) + 180*a*b*e^(6*d*x + 6*c) + 90*b^2*e^(6*d*x + 6*c) + 90*a^2*e^(4*d*x + 4*c) + 220*a*b*e^(4*d*x + 4*c) + 140*b^2*e^(4*d*x + 4*c) + 60*a^2*e^(2*d*x + 2*c) + 140*a*b*e^(2*d*x + 2*c) + 70*b^2*e^(2*d*x + 2*c) + 15*a^2 + 40*a*b + 23*b^2)/(e^(2*d*x + 2*c) + 1)^5)/d

maple [B] time = 0.02, size = 189, normalized size = 3.00

$$\frac{a^2 \tanh(dx + c)}{d} - \frac{\ln(\tanh(dx + c) - 1) a^2}{2d} - \frac{\ln(\tanh(dx + c) - 1) ab}{d} - \frac{\ln(\tanh(dx + c) - 1) b^2}{2d} - \frac{b^2 (\tanh^3(dx + c))}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x)`

[Out] $-a^2 \tanh(d*x+c)/d - 1/2/d * \ln(\tanh(d*x+c)-1) * a^2 - 1/d * \ln(\tanh(d*x+c)-1) * a * b - 1/2/d * \ln(\tanh(d*x+c)-1) * b^2 - 1/3 * b^2 * \tanh(d*x+c)^3/d - 1/5 * b^2 * \tanh(d*x+c)^5/d + 1/2/d * \ln(1+\tanh(d*x+c)) * a^2 + 1/d * \ln(1+\tanh(d*x+c)) * a * b + 1/2/d * \ln(1+\tanh(d*x+c)) * b^2 - 2/3 * a * b * \tanh(d*x+c)^3/d - 2 * a * b * \tanh(d*x+c)/d - b^2 * \tanh(d*x+c)/d$

maxima [B] time = 0.34, size = 231, normalized size = 3.67

$$\frac{1}{15} b^2 \left(15x + \frac{15c}{d} - \frac{2(70e^{(-2dx-2c)} + 140e^{(-4dx-4c)} + 90e^{(-6dx-6c)} + 45e^{(-8dx-8c)} + 23)}{d(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} \right) + \frac{2}{3} ab \left(3x + \frac{3c}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] $1/15 * b^2 * (15 * x + 15 * c/d - 2 * (70 * e^{(-2 * d * x - 2 * c)} + 140 * e^{(-4 * d * x - 4 * c)} + 90 * e^{(-6 * d * x - 6 * c)} + 45 * e^{(-8 * d * x - 8 * c)} + 23) / (d * (5 * e^{(-2 * d * x - 2 * c)} + 10 * e^{(-4 * d * x - 4 * c)} + 10 * e^{(-6 * d * x - 6 * c)} + 5 * e^{(-8 * d * x - 8 * c)} + e^{(-10 * d * x - 10 * c)} + 1))) + 2/3 * a * b * (3 * x + 3 * c/d - 4 * (3 * e^{(-2 * d * x - 2 * c)} + 3 * e^{(-4 * d * x - 4 * c)} + 2) / (d * (3 * e^{(-2 * d * x - 2 * c)} + 3 * e^{(-4 * d * x - 4 * c)} + e^{(-6 * d * x - 6 * c)} + 1))) + a^2 * (x + c/d - 2 / (d * (e^{(-2 * d * x - 2 * c)} + 1)))$

mupad [B] time = 1.30, size = 67, normalized size = 1.06

$$x(a^2 + 2ab + b^2) - \frac{\tanh(c + dx)(a + b)^2}{d} - \frac{\tanh(c + dx)^3(b^2 + 2ab)}{3d} - \frac{b^2 \tanh(c + dx)^5}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2,x)`

[Out] $x * (2 * a * b + a^2 + b^2) - (\tanh(c + d * x) * (a + b)^2) / d - (\tanh(c + d * x)^3 * (2 * a * b + b^2)) / (3 * d) - (b^2 * \tanh(c + d * x)^5) / (5 * d)$

sympy [A] time = 0.64, size = 117, normalized size = 1.86

$$\begin{cases} a^2 x - \frac{a^2 \tanh(c+dx)}{d} + 2abx - \frac{2ab \tanh^3(c+dx)}{3d} - \frac{2ab \tanh(c+dx)}{d} + b^2 x - \frac{b^2 \tanh^5(c+dx)}{5d} - \frac{b^2 \tanh^3(c+dx)}{3d} - \frac{b^2 \tanh(c+dx)}{d} \\ x(a + b \tanh^2(c))^2 \tanh^2(c) \end{cases} \text{ for } \text{oth}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)`

[Out] `Piecewise((a**2*x - a**2*tanh(c + d*x)/d + 2*a*b*x - 2*a*b*tanh(c + d*x)**3/(3*d) - 2*a*b*tanh(c + d*x)/d + b**2*x - b**2*tanh(c + d*x)**5/(5*d) - b**2*tanh(c + d*x)**3/(3*d) - b**2*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**2*tanh(c)**2, True))`

3.147 $\int \tanh(c + dx) \left(a + b \tanh^2(c + dx) \right)^2 dx$

Optimal. Leaf size=57

$$-\frac{b(a+b)\tanh^2(c+dx)}{2d} - \frac{(a+b\tanh^2(c+dx))^2}{4d} + \frac{(a+b)^2 \log(\cosh(c+dx))}{d}$$

[Out] (a+b)^2*ln(cosh(d*x+c))/d-1/2*b*(a+b)*tanh(d*x+c)^2/d-1/4*(a+b*tanh(d*x+c)^2)^2/d

Rubi [A] time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3670, 444, 43}

$$-\frac{b(a+b)\tanh^2(c+dx)}{2d} - \frac{(a+b\tanh^2(c+dx))^2}{4d} + \frac{(a+b)^2 \log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] ((a + b)^2*Log[Cosh[c + d*x]])/d - (b*(a + b)*Tanh[c + d*x]^2)/(2*d) - (a + b*Tanh[c + d*x]^2)^2/(4*d)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff, x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
\int \tanh(c + dx) (a + b \tanh^2(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \frac{x^{(a+bx)^2}}{1-x^2} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{1-x} dx, x, \tanh^2(c + dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-b(a+b) + \frac{(a+b)^2}{1-x} - b(a+bx)\right) dx, x, \tanh^2(c + dx)\right)}{2d} \\
&= \frac{(a+b)^2 \log(\cosh(c + dx))}{d} - \frac{b(a+b) \tanh^2(c + dx)}{2d} - \frac{(a+b \tanh^2(c + dx))^2}{4d}
\end{aligned}$$

Mathematica [A] time = 0.32, size = 50, normalized size = 0.88

$$\frac{2b(2a+b) \tanh^2(c+dx) - 4(a+b)^2 \log(\cosh(c+dx)) + b^2 \tanh^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] -1/4*(-4*(a + b)^2*Log[Cosh[c + d*x]] + 2*b*(2*a + b)*Tanh[c + d*x]^2 + b^2*Tanh[c + d*x]^4)/d

fricas [B] time = 0.43, size = 1638, normalized size = 28.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^8 + 4*((a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + 3*((a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*(a^2 + 2*a*b + b^2)*d*x - 4*a*b - 2*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*x + 30*((a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*cosh(d*x + c)^2 - 4*a*b - 2*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^5 + 10*((a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*cosh(d*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x - 4*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*x + 4*((a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 15*((a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*cosh(d*x + c)^4 + (a^2 + 2*a*b + b^2)*d*x + 3*(3*(a^2 + 2*a*b + b^2)*d*x - 4*a*b - 2*b^2)*cosh(d*x + c)^2 - a*b - b^2)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 30*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*a^2 + 6*a*b + 3*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 10*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x

+ c)^6 + 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 9*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^7 + 3*((a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*cosh(d*x + c)^5 + (3*(a^2 + 2*a*b + b^2)*d*x - 4*a*b - 2*b^2)*cosh(d*x + c)^3 + ((a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 + 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)

giac [B] time = 0.22, size = 116, normalized size = 2.04

$$\frac{(a^2 + 2ab + b^2)(dx + c) - (a^2 + 2ab + b^2) \log(e^{(2dx+2c)} + 1) - \frac{4((ab+b^2)e^{(6dx+6c)} + (2ab+b^2)e^{(4dx+4c)} + (ab+b^2)e^{(2dx+2c)})}{(e^{(2dx+2c)}+1)^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] -((a^2 + 2*a*b + b^2)*(d*x + c) - (a^2 + 2*a*b + b^2)*log(e^(2*d*x + 2*c) + 1) - 4*((a*b + b^2)*e^(6*d*x + 6*c) + (2*a*b + b^2)*e^(4*d*x + 4*c) + (a*b + b^2)*e^(2*d*x + 2*c)))/(e^(2*d*x + 2*c) + 1)^4)/d

maple [B] time = 0.02, size = 149, normalized size = 2.61

$$\frac{(\tanh^4(dx + c))b^2}{4d} - \frac{ab(\tanh^2(dx + c))}{d} - \frac{b^2(\tanh^2(dx + c))}{2d} - \frac{\ln(\tanh(dx + c) - 1)a^2}{2d} - \frac{\ln(\tanh(dx + c) - 1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x)

[Out] -1/4/d*tanh(d*x+c)^4*b^2-a*b*tanh(d*x+c)^2/d-1/2*b^2*tanh(d*x+c)^2/d-1/2/d*ln(tanh(d*x+c)-1)*a^2-1/d*ln(tanh(d*x+c)-1)*a*b-1/2/d*ln(tanh(d*x+c)-1)*b^2-1/2/d*ln(1+tanh(d*x+c))*a^2-1/d*ln(1+tanh(d*x+c))*a*b-1/2/d*ln(1+tanh(d*x+c))*b^2

maxima [B] time = 0.42, size = 186, normalized size = 3.26

$$b^2 \left(x + \frac{c}{d} + \frac{\log(e^{(-2dx-2c)} + 1)}{d} + \frac{4(e^{(-2dx-2c)} + e^{(-4dx-4c)} + e^{(-6dx-6c)})}{d(4e^{(-2dx-2c)} + 6e^{(-4dx-4c)} + 4e^{(-6dx-6c)} + e^{(-8dx-8c)} + 1)} \right) + 2ab \left(x + \frac{c}{d} + \frac{\log(e^{(-2dx-2c)} + 1)}{d} + \frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] b^2*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + 2*a*b*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a^2*log(cosh(d*x + c))/d

mupad [B] time = 1.21, size = 76, normalized size = 1.33

$$x \left(a^2 + 2ab + b^2 \right) - \frac{\tanh(c + dx)^2 (b^2 + 2ab)}{2d} - \frac{\ln(\tanh(c + dx) + 1) (a^2 + 2ab + b^2)}{d} - \frac{b^2 \tanh(c + dx)^4}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)*(a + b*tanh(c + d*x)^2)^2,x)

[Out] x*(2*a*b + a^2 + b^2) - (tanh(c + d*x)^2*(2*a*b + b^2))/(2*d) - (log(tanh(c + d*x) + 1)*(2*a*b + a^2 + b^2))/d - (b^2*tanh(c + d*x)^4)/(4*d)

sympy [A] time = 0.47, size = 122, normalized size = 2.14

$$\left\{ \begin{array}{l} a^2 x - \frac{a^2 \log(\tanh(c+dx)+1)}{d} + 2abx - \frac{2ab \log(\tanh(c+dx)+1)}{d} - \frac{ab \tanh^2(c+dx)}{d} + b^2 x - \frac{b^2 \log(\tanh(c+dx)+1)}{d} - \frac{b^2 \tanh^4(c+dx)}{4d} \\ x \left(a + b \tanh^2(c) \right)^2 \tanh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)

[Out] Piecewise((a**2*x - a**2*log(tanh(c + d*x) + 1)/d + 2*a*b*x - 2*a*b*log(tanh(c + d*x) + 1)/d - a*b*tanh(c + d*x)**2/d + b**2*x - b**2*log(tanh(c + d*x) + 1)/d - b**2*tanh(c + d*x)**4/(4*d) - b**2*tanh(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)**2*tanh(c), True))

3.148 $\int (a + b \tanh^2(c + dx))^2 dx$

Optimal. Leaf size=43

$$-\frac{b(2a+b)\tanh(c+dx)}{d} + x(a+b)^2 - \frac{b^2 \tanh^3(c+dx)}{3d}$$

[Out] (a+b)^2*x-b*(2*a+b)*tanh(d*x+c)/d-1/3*b^2*tanh(d*x+c)^3/d

Rubi [A] time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3661, 390, 206}

$$-\frac{b(2a+b)\tanh(c+dx)}{d} + x(a+b)^2 - \frac{b^2 \tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tanh[c + d*x]^2)^2, x]

[Out] (a + b)^2*x - (b*(2*a + b)*Tanh[c + d*x])/d - (b^2*Tanh[c + d*x]^3)/(3*d)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 3661

Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(ff*x)^n]^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int (a + b \tanh^2(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{1-x^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(-b(2a+b) - b^2x^2 + \frac{(a+b)^2}{1-x^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{b(2a+b)\tanh(c+dx)}{d} - \frac{b^2 \tanh^3(c+dx)}{3d} + \frac{(a+b)^2 \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c+dx)\right)}{d} \\ &= (a+b)^2x - \frac{b(2a+b)\tanh(c+dx)}{d} - \frac{b^2 \tanh^3(c+dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.75, size = 65, normalized size = 1.51

$$\frac{\tanh(c + dx) \left(\frac{3(a+b)^2 \tanh^{-1}(\sqrt{\tanh^2(c+dx)})}{\sqrt{\tanh^2(c+dx)}} - b(6a + b(\tanh^2(c + dx) + 3)) \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Tanh[c + d*x]^2)^2, x]

[Out] (Tanh[c + d*x]*((3*(a + b)^2*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c + d*x]^2] - b*(6*a + b*(3 + Tanh[c + d*x]^2))))/(3*d)

fricas [B] time = 0.40, size = 201, normalized size = 4.67

$$\frac{3(a^2 + 2ab + b^2)dx + 6ab + 4b^2 \cosh(dx + c)^3 + 3(3(a^2 + 2ab + b^2)dx + 6ab + 4b^2) \cosh(dx + c) \sinh(dx + c)}{3(d \cosh(dx + c))^3 + \dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] 1/3*((3*(a^2 + 2*a*b + b^2)*d*x + 6*a*b + 4*b^2)*cosh(d*x + c)^3 + 3*(3*(a^2 + 2*a*b + b^2)*d*x + 6*a*b + 4*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 - 2*(3*a*b + 2*b^2)*sinh(d*x + c)^3 + 3*(3*(a^2 + 2*a*b + b^2)*d*x + 6*a*b + 4*b^2)*cosh(d*x + c) - 6*((3*a*b + 2*b^2)*cosh(d*x + c)^2 + a*b)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))

giac [B] time = 0.14, size = 103, normalized size = 2.40

$$\frac{3(a^2 + 2ab + b^2)(dx + c) + \frac{4(3abe^{4dx+4c} + 3b^2e^{4dx+4c} + 6abe^{2dx+2c} + 3b^2e^{2dx+2c} + 3ab+2b^2)}{(e^{2dx+2c}+1)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/3*(3*(a^2 + 2*a*b + b^2)*(d*x + c) + 4*(3*a*b*e^(4*d*x + 4*c) + 3*b^2*e^(4*d*x + 4*c) + 6*a*b*e^(2*d*x + 2*c) + 3*b^2*e^(2*d*x + 2*c) + 3*a*b + 2*b^2)/(e^(2*d*x + 2*c) + 1)^3)/d

maple [B] time = 0.02, size = 144, normalized size = 3.35

$$\frac{b^2 \tanh^3(dx + c)}{3d} - \frac{2ab \tanh(dx + c)}{d} - \frac{b^2 \tanh(dx + c)}{d} - \frac{\ln(\tanh(dx + c) - 1) a^2}{2d} - \frac{\ln(\tanh(dx + c) - 1) ab}{d} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*tanh(d*x+c)^2)^2,x)

[Out] -1/3*b^2*tanh(d*x+c)^3/d-2*a*b*tanh(d*x+c)/d-b^2*tanh(d*x+c)/d-1/2/d*ln(tanh(d*x+c)-1)*a^2-1/d*ln(tanh(d*x+c)-1)*a*b-1/2/d*ln(tanh(d*x+c)-1)*b^2+1/2/d*ln(1+tanh(d*x+c))*a^2+1/d*ln(1+tanh(d*x+c))*a*b+1/2/d*ln(1+tanh(d*x+c))*b^2

maxima [B] time = 0.33, size = 114, normalized size = 2.65

$$\frac{1}{3} b^2 \left(3x + \frac{3c}{d} - \frac{4(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + 2)}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)} \right) + 2ab \left(x + \frac{c}{d} - \frac{2}{d(e^{(-2dx-2c)} + 1)} \right) + a^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(d*x+c))^2,x, algorithm="maxima")

[Out] $\frac{1}{3}b^2(3x + 3c/d - 4(3e^{(-2dx - 2c)} + 3e^{(-4dx - 4c)} + 2)/(d(3e^{(-2dx - 2c)} + 3e^{(-4dx - 4c)} + e^{(-6dx - 6c)} + 1))) + 2ab(x + c/d - 2/(d(e^{(-2dx - 2c)} + 1))) + a^2x$

mupad [B] time = 1.24, size = 47, normalized size = 1.09

$$x(a^2 + 2ab + b^2) - \frac{b^2 \tanh(c + dx)^3}{3d} - \frac{b \tanh(c + dx)(2a + b)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tanh(c + d*x))^2,x)

[Out] $x(2ab + a^2 + b^2) - (b^2 \tanh(c + dx)^3)/(3d) - (b \tanh(c + dx)(2a + b))/d$

sympy [A] time = 0.32, size = 68, normalized size = 1.58

$$\begin{cases} a^2x + 2abx - \frac{2ab \tanh(c+dx)}{d} + b^2x - \frac{b^2 \tanh^3(c+dx)}{3d} - \frac{b^2 \tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x(a + b \tanh^2(c))^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(d*x+c)**2)**2,x)

[Out] Piecewise((a**2*x + 2*a*b*x - 2*a*b*tanh(c + d*x)/d + b**2*x - b**2*tanh(c + d*x)**3/(3*d) - b**2*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**2, True))

3.149 $\int \coth(c + dx) (a + b \tanh^2(c + dx))^2 dx$

Optimal. Leaf size=49

$$\frac{a^2 \log(\tanh(c + dx))}{d} + \frac{(a + b)^2 \log(\cosh(c + dx))}{d} - \frac{b^2 \tanh^2(c + dx)}{2d}$$

[Out] (a+b)^2*ln(cosh(d*x+c))/d+a^2*ln(tanh(d*x+c))/d-1/2*b^2*tanh(d*x+c)^2/d

Rubi [A] time = 0.08, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3670, 446, 72}

$$\frac{a^2 \log(\tanh(c + dx))}{d} + \frac{(a + b)^2 \log(\cosh(c + dx))}{d} - \frac{b^2 \tanh^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] ((a + b)^2*Log[Cosh[c + d*x]])/d + (a^2*Log[Tanh[c + d*x]])/d - (b^2*Tanh[c + d*x]^2)/(2*d)

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned} \int \coth(c + dx) (a + b \tanh^2(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{x(1-x^2)} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{(1-x)x} dx, x, \tanh^2(c + dx)\right)}{2d} \\ &= \frac{\text{Subst}\left(\int \left(-b^2 - \frac{(a+b)^2}{-1+x} + \frac{a^2}{x}\right) dx, x, \tanh^2(c + dx)\right)}{2d} \\ &= \frac{(a + b)^2 \log(\cosh(c + dx))}{d} + \frac{a^2 \log(\tanh(c + dx))}{d} - \frac{b^2 \tanh^2(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.14, size = 48, normalized size = 0.98

$$\frac{2\left(a^2 \log(\tanh(c + dx)) + (a + b)^2 \log(\cosh(c + dx))\right) - b^2 \tanh^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (2*((a + b)^2*Log[Cosh[c + d*x]] + a^2*Log[Tanh[c + d*x]]) - b^2*Tanh[c + d*x]^2)/(2*d)

fricas [B] time = 0.42, size = 668, normalized size = 13.63

$$\frac{\left(a^2 + 2ab + b^2\right)dx \cosh(dx + c)^4 + 4\left(a^2 + 2ab + b^2\right)dx \cosh(dx + c) \sinh(dx + c)^3 + \left(a^2 + 2ab + b^2\right)dx \sinh(dx + c)^4}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^4 + (a^2 + 2*a*b + b^2)*d*x + 2*((a^2 + 2*a*b + b^2)*d*x - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d*x - b^2)*sinh(d*x + c)^2 - ((2*a*b + b^2)*cosh(d*x + c)^4 + 4*(2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b + b^2)*sinh(d*x + c)^4 + 2*(2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b + b^2)*cosh(d*x + c)^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + 2*a*b + b^2 + 4*((2*a*b + b^2)*cosh(d*x + c)^3 + (2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - (a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*a^2*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + ((a^2 + 2*a*b + b^2)*d*x - b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)

giac [B] time = 0.21, size = 141, normalized size = 2.88

$$\frac{a^2 \log\left(e^{(2dx+2c)} + e^{(-2dx-2c)} - 2\right) + (2ab + b^2) \log\left(e^{(2dx+2c)} + e^{(-2dx-2c)} + 2\right) - \frac{2ab\left(e^{(2dx+2c)} + e^{(-2dx-2c)}\right) + b^2\left(e^{(2dx+2c)} + e^{(-2dx-2c)}\right)}{e^{(2dx+2c)} + e^{(-2dx-2c)}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/2*(a^2*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) + (2*a*b + b^2)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) - (2*a*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))) + 4*a*b - 2*b^2)/(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2))/d

maple [A] time = 0.26, size = 60, normalized size = 1.22

$$\frac{a^2 \ln(\sinh(dx + c))}{d} + \frac{2ab \ln(\cosh(dx + c))}{d} + \frac{b^2 \ln(\cosh(dx + c))}{d} - \frac{b^2 (\tanh^2(dx + c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x)`

[Out] $\frac{1}{d}a^2\ln(\sinh(dx+c))+2ab\ln(\cosh(dx+c))/d+1/d*b^2\ln(\cosh(dx+c))-1/2*b^2*tanh(dx+c)^2/d$

maxima [B] time = 0.42, size = 104, normalized size = 2.12

$$b^2\left(x + \frac{c}{d} + \frac{\log(e^{-2dx-2c} + 1)}{d} + \frac{2e^{-2dx-2c}}{d(2e^{-2dx-2c} + e^{-4dx-4c} + 1)}\right) + \frac{2ab\log(e^{dx+c} + e^{-dx-c})}{d} + \frac{a^2\log(\sinh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] $b^2*(x + c/d + \log(e^{(-2*d*x - 2*c)} + 1)/d + 2*e^{(-2*d*x - 2*c)}/(d*(2*e^{(-2*d*x - 2*c)} + e^{(-4*d*x - 4*c)} + 1))) + 2*a*b*\log(e^{(d*x + c)} + e^{(-d*x - c)})/d + a^2*\log(\sinh(d*x + c))/d$

mupad [B] time = 1.32, size = 210, normalized size = 4.29

$$\frac{2b^2}{d(e^{2c+2dx} + 1)} - x(a+b)^2 - \frac{2b^2}{d(2e^{2c+2dx} + e^{4c+4dx} + 1)} + \frac{\ln(e^{4c+4dx} - 1)(d(b^2 + 2ab) + a^2d)}{2d^2} + \frac{\operatorname{atan}\left(\frac{e^{2c}e^{2dx}}{d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(c + d*x)*(a + b*tanh(c + d*x)^2)^2,x)`

[Out] $(2*b^2)/(d*(\exp(2*c + 2*d*x) + 1)) - x*(a + b)^2 - (2*b^2)/(d*(2*\exp(2*c + 2*d*x) + \exp(4*c + 4*d*x) + 1)) + (\log(\exp(4*c + 4*d*x) - 1)*(d*(2*a*b + b^2) + a^2*d))/(2*d^2) + (\operatorname{atan}((\exp(2*c)*\exp(2*d*x)*(b^2*(-d^2)^{(1/2)} - a^2*(-d^2)^{(1/2)} + 2*a*b*(-d^2)^{(1/2)})))/(d*(4*a*b^3 - 4*a^3*b + a^4 + b^4 + 2*a^2*b^2)^{(1/2)})))/(d*(4*a*b^3 - 4*a^3*b + a^4 + b^4 + 2*a^2*b^2)^{(1/2)})/(-d^2)^{(1/2)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^2 \coth(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)`

[Out] `Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x), x)`

3.150 $\int \coth^2(c + dx) (a + b \tanh^2(c + dx))^2 dx$

Optimal. Leaf size=36

$$-\frac{a^2 \coth(c + dx)}{d} + x(a + b)^2 - \frac{b^2 \tanh(c + dx)}{d}$$

[Out] (a+b)^2*x-a^2*coth(d*x+c)/d-b^2*tanh(d*x+c)/d

Rubi [A] time = 0.07, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 461, 207}

$$-\frac{a^2 \coth(c + dx)}{d} + x(a + b)^2 - \frac{b^2 \tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (a + b)^2*x - (a^2*Coth[c + d*x])/d - (b^2*Tanh[c + d*x])/d

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 461

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f*ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned} \int \coth^2(c + dx) (a + b \tanh^2(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x^2(1-x^2)} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(-b^2 + \frac{a^2}{x^2} - \frac{(a+b)^2}{-1+x^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{a^2 \coth(c + dx)}{d} - \frac{b^2 \tanh(c + dx)}{d} - \frac{(a + b)^2 \text{Subst}\left(\int \frac{1}{-1+x^2} dx\right)}{d} \\ &= (a + b)^2 x - \frac{a^2 \coth(c + dx)}{d} - \frac{b^2 \tanh(c + dx)}{d} \end{aligned}$$

Mathematica [C] time = 0.11, size = 64, normalized size = 1.78

$$\frac{a^2 \coth(c + dx) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \tanh^2(c + dx)\right)}{d} + 2abx + \frac{b^2 \tanh^{-1}(\tanh(c + dx))}{d} - \frac{b^2 \tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2), x]

[Out] 2*a*b*x + (b^2*ArcTanh[Tanh[c + d*x]])/d - (a^2*Coth[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2])/d - (b^2*Tanh[c + d*x])/d

fricas [B] time = 0.39, size = 97, normalized size = 2.69

$$\frac{(a^2 + b^2) \cosh(dx + c)^2 - 2((a^2 + 2ab + b^2)dx + a^2 + b^2) \cosh(dx + c) \sinh(dx + c) + (a^2 + b^2) \sinh(dx + c)}{2d \cosh(dx + c) \sinh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -1/2*((a^2 + b^2)*cosh(d*x + c)^2 - 2*((a^2 + 2*a*b + b^2)*d*x + a^2 + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*sinh(d*x + c)^2 + a^2 - b^2)/(d*cosh(d*x + c)*sinh(d*x + c))

giac [A] time = 0.25, size = 71, normalized size = 1.97

$$\frac{(a^2 + 2ab + b^2)(dx + c) - \frac{2(a^2 e^{2dx+2c} - b^2 e^{2dx+2c} + a^2 + b^2)}{e^{4dx+4c} - 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] ((a^2 + 2*a*b + b^2)*(d*x + c) - 2*(a^2*e^(2*d*x + 2*c) - b^2*e^(2*d*x + 2*c) + a^2 + b^2)/(e^(4*d*x + 4*c) - 1))/d

maple [A] time = 0.21, size = 49, normalized size = 1.36

$$\frac{a^2(dx + c - \coth(dx + c)) + 2ab(dx + c) + b^2(dx + c - \tanh(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x)

[Out] 1/d*(a^2*(d*x+c-coth(d*x+c))+2*a*b*(d*x+c)+b^2*(d*x+c-tanh(d*x+c)))

maxima [A] time = 0.35, size = 64, normalized size = 1.78

$$b^2\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2dx-2c)} + 1)}\right) + a^2\left(x + \frac{c}{d} + \frac{2}{d(e^{(-2dx-2c)} - 1)}\right) + 2abx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] b^2*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^2*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + 2*a*b*x

mupad [B] time = 1.24, size = 59, normalized size = 1.64

$$x(a+b)^2 - \frac{\frac{2(a^2+b^2)}{d} + \frac{2e^{2c+2dx}(a^2-b^2)}{d}}{e^{4c+4dx} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2,x)

[Out] x*(a + b)^2 - ((2*(a^2 + b^2))/d + (2*exp(2*c + 2*d*x)*(a^2 - b^2))/d)/(exp(4*c + 4*d*x) - 1)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^2 \coth^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)

[Out] Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**2, x)

3.151 $\int \coth^3(c + dx) (a + b \tanh^2(c + dx))^2 dx$

Optimal. Leaf size=52

$$-\frac{a^2 \coth^2(c + dx)}{2d} + \frac{a(a + 2b) \log(\tanh(c + dx))}{d} + \frac{(a + b)^2 \log(\cosh(c + dx))}{d}$$

[Out] $-1/2*a^2*\coth(d*x+c)^2/d+(a+b)^2*\ln(\cosh(d*x+c))/d+a*(a+2*b)*\ln(\tanh(d*x+c))/d$

Rubi [A] time = 0.09, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 446, 88}

$$-\frac{a^2 \coth^2(c + dx)}{2d} + \frac{a(a + 2b) \log(\tanh(c + dx))}{d} + \frac{(a + b)^2 \log(\cosh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] `Int[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]`

[Out] $-(a^2*\text{Coth}[c + d*x]^2)/(2*d) + ((a + b)^2*\text{Log}[\text{Cosh}[c + d*x]])/d + (a*(a + 2*b)*\text{Log}[\text{Tanh}[c + d*x]])/d$

Rule 88

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegerQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))`

Rule 446

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]`

Rule 3670

`Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))`

Rubi steps

$$\int \coth^3(c + dx) (a + b \tanh^2(c + dx))^2 dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x^3(1-x^2)} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{(1-x)x^2} dx, x, \tanh^2(c + dx)\right)}{2d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{-1+x} + \frac{a^2}{x^2} + \frac{a(a+2b)}{x}\right) dx, x, \tanh^2(c + dx)\right)}{2d}$$

$$= -\frac{a^2 \coth^2(c + dx)}{2d} + \frac{(a + b)^2 \log(\cosh(c + dx))}{d} + \frac{a(a + 2b) \log(\tanh(c + dx))}{d}$$

Mathematica [A] time = 0.17, size = 50, normalized size = 0.96

$$\frac{-a^2 \coth^2(c + dx) + 2a(a + 2b) \log(\tanh(c + dx)) + 2(a + b)^2 \log(\cosh(c + dx))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] $(-(a^2 \coth^2[c + d*x]^2) + 2*(a + b)^2 \log[\cosh[c + d*x]] + 2*a*(a + 2*b) \log[\tanh[c + d*x]])/(2*d)$

fricas [B] time = 0.43, size = 677, normalized size = 13.02

$$\frac{(a^2 + 2ab + b^2)dx \cosh(dx + c)^4 + 4(a^2 + 2ab + b^2)dx \cosh(dx + c) \sinh(dx + c)^3 + (a^2 + 2ab + b^2)dx \sinh(dx + c)^4}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] $-\left(\left(a^2 + 2ab + b^2\right)d*x*\cosh(d*x + c)^4 + 4*\left(a^2 + 2ab + b^2\right)d*x*\cosh(d*x + c)*\sinh(d*x + c)^3 + \left(a^2 + 2ab + b^2\right)d*x*\sinh(d*x + c)^4 + \left(a^2 + 2ab + b^2\right)d*x - 2*\left(\left(a^2 + 2ab + b^2\right)d*x - a^2\right)*\cosh(d*x + c)^2 + 2*\left(3*\left(a^2 + 2ab + b^2\right)d*x*\cosh(d*x + c)^2 - \left(a^2 + 2ab + b^2\right)d*x + a^2\right)*\sinh(d*x + c)^2 - \left(b^2*\cosh(d*x + c)^4 + 4*b^2*\cosh(d*x + c)*\sinh(d*x + c)^3 + b^2*\sinh(d*x + c)^4 - 2*b^2*\cosh(d*x + c)^2 + 2*\left(3*b^2*\cosh(d*x + c)^2 - b^2\right)*\sinh(d*x + c)^2 + b^2 + 4*\left(b^2*\cosh(d*x + c)^3 - b^2*\cosh(d*x + c)\right)*\sinh(d*x + c)\right)*\log(2*\cosh(d*x + c)/(\cosh(d*x + c) - \sinh(d*x + c))) - \left(\left(a^2 + 2ab\right)*\cosh(d*x + c)^4 + 4*\left(a^2 + 2ab\right)*\cosh(d*x + c)*\sinh(d*x + c)^3 + \left(a^2 + 2ab\right)*\sinh(d*x + c)^4 - 2*\left(a^2 + 2ab\right)*\cosh(d*x + c)^2 + 2*\left(3*\left(a^2 + 2ab\right)*\cosh(d*x + c)^2 - a^2 - 2ab\right)*\sinh(d*x + c)^2 + a^2 + 2ab + 4*\left(\left(a^2 + 2ab\right)*\cosh(d*x + c)^3 - \left(a^2 + 2ab\right)*\cosh(d*x + c)\right)*\sinh(d*x + c)\right)*\log(2*\sinh(d*x + c)/(\cosh(d*x + c) - \sinh(d*x + c))) + 4*\left(\left(a^2 + 2ab + b^2\right)d*x*\cosh(d*x + c)^3 - \left(\left(a^2 + 2ab + b^2\right)d*x - a^2\right)*\cosh(d*x + c)\right)*\sinh(d*x + c)\right)/\left(d*\cosh(d*x + c)^4 + 4*d*\cosh(d*x + c)*\sinh(d*x + c)^3 + d*\sinh(d*x + c)^4 - 2*d*\cosh(d*x + c)^2 + 2*\left(3*d*\cosh(d*x + c)^2 - d\right)*\sinh(d*x + c)^2 + 4*\left(d*\cosh(d*x + c)^3 - d*\cosh(d*x + c)\right)*\sinh(d*x + c) + d\right)$

giac [B] time = 0.27, size = 141, normalized size = 2.71

$$\frac{b^2 \log\left(e^{(2dx+2c)} + e^{(-2dx-2c)} + 2\right) + (a^2 + 2ab) \log\left(e^{(2dx+2c)} + e^{(-2dx-2c)} - 2\right) - \frac{a^2\left(e^{(2dx+2c)} + e^{(-2dx-2c)}\right) + 2ab\left(e^{(2dx+2c)} - e^{(-2dx-2c)}\right)}{e^{(2dx+2c)} + e^{(-2dx-2c)}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] $\frac{1}{2}*(b^2*\log(e^{(2*d*x + 2*c)} + e^{(-2*d*x - 2*c)} + 2) + (a^2 + 2*a*b)*\log(e^{(2*d*x + 2*c)} + e^{(-2*d*x - 2*c)} - 2) - (a^2*(e^{(2*d*x + 2*c)} + e^{(-2*d*x - 2*c)}) + 2*a*b*(e^{(2*d*x + 2*c)} + e^{(-2*d*x - 2*c)})) + 2*a^2 - 4*a*b)/(e^{(2*d*x + 2*c)} + e^{(-2*d*x - 2*c)} - 2))/d$

maple [A] time = 0.27, size = 60, normalized size = 1.15

$$\frac{a^2 \ln(\sinh(dx+c))}{d} - \frac{a^2 (\coth^2(dx+c))}{2d} + \frac{2ab \ln(\sinh(dx+c))}{d} + \frac{b^2 \ln(\cosh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x)

[Out] $\frac{1}{d*a^2*\ln(\sinh(d*x+c))-1/2*a^2*\coth(d*x+c)^2/d+2/d*a*b*\ln(\sinh(d*x+c))+1/d*b^2*\ln(\cosh(d*x+c))$

maxima [B] time = 0.33, size = 134, normalized size = 2.58

$$a^2 \left(x + \frac{c}{d} + \frac{\log(e^{-dx-c} + 1)}{d} + \frac{\log(e^{-dx-c} - 1)}{d} + \frac{2e^{-2dx-2c}}{d(2e^{-2dx-2c} - e^{-4dx-4c} - 1)} \right) + \frac{b^2 \log(e^{dx+c} + e^{-dx-c})}{d} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $a^2*(x + c/d + \log(e^{-d*x - c} + 1)/d + \log(e^{-d*x - c} - 1)/d + 2*e^{(-2*d*x - 2*c)}/(d*(2*e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)} - 1))) + b^2*\log(e^{(d*x + c)} + e^{(-d*x - c)})/d + 2*a*b*\log(e^{(d*x + c)} - e^{(-d*x - c)})/d$

mupad [B] time = 1.41, size = 211, normalized size = 4.06

$$\frac{\ln(e^{4c+4dx} - 1) (d(a^2 + 2ba) + b^2 d)}{2d^2} - \frac{2a^2}{d(e^{2c+2dx} - 1)} - \frac{2a^2}{d(e^{4c+4dx} - 2e^{2c+2dx} + 1)} - x(a+b)^2 - \operatorname{atan}\left(\frac{e^{2c}e^{2dx}}{d\sqrt{\dots}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2,x)

[Out] $(\log(\exp(4*c + 4*d*x) - 1)*(d*(2*a*b + a^2) + b^2*d))/(2*d^2) - (2*a^2)/(d*(\exp(2*c + 2*d*x) - 1)) - (2*a^2)/(d*(\exp(4*c + 4*d*x) - 2*\exp(2*c + 2*d*x) + 1)) - x*(a + b)^2 - (\operatorname{atan}((\exp(2*c)*\exp(2*d*x)*(a^2*(-d^2)^{(1/2)} - b^2*(-d^2)^{(1/2)} + 2*a*b*(-d^2)^{(1/2)})))/(d*(4*a^3*b - 4*a*b^3 + a^4 + b^4 + 2*a^2*b^2)^{(1/2)}))*(4*a^3*b - 4*a*b^3 + a^4 + b^4 + 2*a^2*b^2)^{(1/2)})/(-d^2)^{(1/2)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^2 \coth^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)

[Out] Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**3, x)

3.152 $\int \coth^4(c + dx) (a + b \tanh^2(c + dx))^2 dx$

Optimal. Leaf size=43

$$-\frac{a^2 \coth^3(c + dx)}{3d} - \frac{a(a + 2b) \coth(c + dx)}{d} + x(a + b)^2$$

[Out] (a+b)^2*x-a*(a+2*b)*coth(d*x+c)/d-1/3*a^2*coth(d*x+c)^3/d

Rubi [A] time = 0.07, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 461, 207}

$$-\frac{a^2 \coth^3(c + dx)}{3d} - \frac{a(a + 2b) \coth(c + dx)}{d} + x(a + b)^2$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (a + b)^2*x - (a*(a + 2*b)*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d)

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 461

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned} \int \coth^4(c + dx) (a + b \tanh^2(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x^4(1-x^2)} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^4} + \frac{a(a+2b)}{x^2} - \frac{(a+b)^2}{-1+x^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{a(a + 2b) \coth(c + dx)}{d} - \frac{a^2 \coth^3(c + dx)}{3d} - \frac{(a + b)^2 \text{Subst}\left(\int \frac{1}{x^4} dx, x, \tanh(c + dx)\right)}{d} \\ &= (a + b)^2 x - \frac{a(a + 2b) \coth(c + dx)}{d} - \frac{a^2 \coth^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.62, size = 65, normalized size = 1.51

$$\frac{\coth(c + dx) \left(a \left(a \coth^2(c + dx) + 3a + 6b \right) - 3(a + b)^2 \tanh^{-1} \left(\sqrt{\tanh^2(c + dx)} \right) \sqrt{\tanh^2(c + dx)} \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] -1/3*(Coth[c + d*x]*(a*(3*a + 6*b + a*Coth[c + d*x]^2) - 3*(a + b)^2*ArcTanh[Sqrt[Tanh[c + d*x]^2]]*Sqrt[Tanh[c + d*x]^2]))/d

fricas [B] time = 0.40, size = 197, normalized size = 4.58

$$\frac{2(2a^2 + 3ab) \cosh(dx + c)^3 + 6(2a^2 + 3ab) \cosh(dx + c) \sinh(dx + c)^2 - (3(a^2 + 2ab + b^2)dx + 4a^2 + 6ab)}{3(d \sinh(dx + c) \cosh(dx + c)^3 + 3(a^2 + 2ab + b^2) \cosh(dx + c) \sinh(dx + c)^2 - (3(a^2 + 2ab + b^2)dx + 4a^2 + 6ab))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -1/3*(2*(2*a^2 + 3*a*b)*cosh(d*x + c)^3 + 6*(2*a^2 + 3*a*b)*cosh(d*x + c)*sinh(d*x + c)^2 - (3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*sinh(d*x + c)^3 - 6*a*b*cosh(d*x + c) + 3*(3*(a^2 + 2*a*b + b^2)*d*x - (3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c)^2 + 4*a^2 + 6*a*b)*sinh(d*x + c))/(d*sinh(d*x + c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))

giac [B] time = 0.28, size = 103, normalized size = 2.40

$$\frac{3(a^2 + 2ab + b^2)(dx + c) - \frac{4(3a^2e^{4dx+4c} + 3abe^{4dx+4c} - 3a^2e^{2dx+2c} - 6abe^{2dx+2c} + 2a^2 + 3ab)}{(e^{2dx+2c} - 1)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/3*(3*(a^2 + 2*a*b + b^2)*(d*x + c) - 4*(3*a^2*e^(4*d*x + 4*c) + 3*a*b*e^(4*d*x + 4*c) - 3*a^2*e^(2*d*x + 2*c) - 6*a*b*e^(2*d*x + 2*c) + 2*a^2 + 3*a*b)/(e^(2*d*x + 2*c) - 1)^3)/d

maple [A] time = 0.23, size = 59, normalized size = 1.37

$$\frac{a^2 \left(dx + c - \coth(dx + c) - \frac{(\coth^3(dx + c))}{3} \right) + 2ab(dx + c - \coth(dx + c)) + b^2(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x)

[Out] 1/d*(a^2*(d*x+c-coth(d*x+c))-1/3*coth(d*x+c)^3)+2*a*b*(d*x+c-coth(d*x+c))+b^2*(d*x+c)

maxima [B] time = 0.33, size = 114, normalized size = 2.65

$$\frac{1}{3}a^2 \left(3x + \frac{3c}{d} - \frac{4(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} - 2)}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)} \right) + 2ab \left(x + \frac{c}{d} + \frac{2}{d(e^{(-2dx-2c)} - 1)} \right) + b^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $\frac{1}{3}a^2(3x + 3c/d - 4(3e^{(-2dx - 2c)} - 3e^{(-4dx - 4c)} - 2)/(d(3e^{(-2dx - 2c)} - 3e^{(-4dx - 4c)} + e^{(-6dx - 6c)} - 1))) + 2ab(x + c/d + 2/(d(e^{(-2dx - 2c)} - 1))) + b^2x$

mupad [B] time = 0.17, size = 175, normalized size = 4.07

$$x(a+b)^2 - \frac{\frac{4e^{2c+2dx}(a^2+ba)}{3d} - \frac{4ab}{3d}}{e^{4c+4dx} - 2e^{2c+2dx} + 1} - \frac{\frac{4(a^2+ba)}{3d} + \frac{4e^{4c+4dx}(a^2+ba)}{3d} - \frac{8abe^{2c+2dx}}{3d}}{3e^{2c+2dx} - 3e^{4c+4dx} + e^{6c+6dx} - 1} - \frac{4(a^2+ba)}{3d(e^{2c+2dx} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2,x)

[Out] $x(a+b)^2 - ((4\exp(2c+2dx)(ab+a^2))/(3d) - (4ab)/(3d))/(\exp(4c+4dx) - 2\exp(2c+2dx) + 1) - ((4(ab+a^2))/(3d) + (4\exp(4c+4dx)(ab+a^2))/(3d) - (8ab\exp(2c+2dx))/(3d))/((3\exp(2c+2dx) - 3\exp(4c+4dx) + \exp(6c+6dx) - 1) - (4(ab+a^2))/(3d(\exp(2c+2dx) - 1)))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^2 \coth^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)

[Out] Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**4, x)

3.153 $\int \coth^5(c + dx) (a + b \tanh^2(c + dx))^2 dx$

Optimal. Leaf size=72

$$-\frac{a^2 \coth^4(c + dx)}{4d} - \frac{a(a + 2b) \coth^2(c + dx)}{2d} + \frac{(a + b)^2 \log(\tanh(c + dx))}{d} + \frac{(a + b)^2 \log(\cosh(c + dx))}{d}$$

[Out] $-1/2*a*(a+2*b)*\coth(d*x+c)^2/d-1/4*a^2*\coth(d*x+c)^4/d+(a+b)^2*\ln(\cosh(d*x+c))/d+(a+b)^2*\ln(\tanh(d*x+c))/d$

Rubi [A] time = 0.10, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 446, 88}

$$-\frac{a^2 \coth^4(c + dx)}{4d} - \frac{a(a + 2b) \coth^2(c + dx)}{2d} + \frac{(a + b)^2 \log(\tanh(c + dx))}{d} + \frac{(a + b)^2 \log(\cosh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] `Int[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2)^2,x]`

[Out] $-(a*(a + 2*b)*\text{Coth}[c + d*x]^2)/(2*d) - (a^2*\text{Coth}[c + d*x]^4)/(4*d) + ((a + b)^2*\text{Log}[\text{Cosh}[c + d*x]])/d + ((a + b)^2*\text{Log}[\text{Tanh}[c + d*x]])/d$

Rule 88

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegerQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))`

Rule 446

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]`

Rule 3670

`Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))`

Rubi steps

$$\begin{aligned}
\int \coth^5(c + dx) (a + b \tanh^2(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{x^5(1-x^2)} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{(1-x)x^3} dx, x, \tanh^2(c + dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{-1+x} + \frac{a^2}{x^3} + \frac{a(a+2b)}{x^2} + \frac{(a+b)^2}{x}\right) dx, x, \tanh^2(c + dx)\right)}{2d} \\
&= -\frac{a(a+2b) \coth^2(c + dx)}{2d} - \frac{a^2 \coth^4(c + dx)}{4d} + \frac{(a+b)^2 \log(\cosh(c + dx))}{d}
\end{aligned}$$

Mathematica [A] time = 0.43, size = 58, normalized size = 0.81

$$\frac{a^2 \coth^4(c + dx) + 2a(a + 2b) \coth^2(c + dx) - 4(a + b)^2(\log(\tanh(c + dx)) + \log(\cosh(c + dx)))}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] -1/4*(2*a*(a + 2*b)*Coth[c + d*x]^2 + a^2*Coth[c + d*x]^4 - 4*(a + b)^2*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]]))/d

fricas [B] time = 0.46, size = 1649, normalized size = 22.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^8 - 4*((a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 - (a^2 + 2*a*b + b^2)*d*x + a^2 + a*b)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 - 3*((a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*(a^2 + 2*a*b + b^2)*d*x - 2*a^2 - 4*a*b)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*x - 30*((a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^2 - 2*a^2 - 4*a*b)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^5 - 10*((a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x - 2*a^2 - 4*a*b)*cosh(d*x + c))*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*x - 4*((a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 - 15*((a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^4 - (a^2 + 2*a*b + b^2)*d*x + 3*(3*(a^2 + 2*a*b + b^2)*d*x - 2*a^2 - 4*a*b)*cosh(d*x + c)^2 + a^2 + a*b)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 - 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - a^2 - 2*a*b - b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 - 30*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*a^2 + 6*a*b + 3*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 - 10*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 - 15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 9*(a^2 + 2*a*b + b^2)*co

$$\begin{aligned} & \text{sh}(d*x + c)^2 - a^2 - 2*a*b - b^2) * \sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 8* \\ & ((a^2 + 2*a*b + b^2) * \cosh(d*x + c)^7 - 3*(a^2 + 2*a*b + b^2) * \cosh(d*x + c)^5 + 3*(a^2 + 2*a*b + b^2) * \cosh(d*x + c)^3 - (a^2 + 2*a*b + b^2) * \cosh(d*x + c)) * \sinh(d*x + c) * \log(2 * \sinh(d*x + c) / (\cosh(d*x + c) - \sinh(d*x + c))) + 8 * \\ & ((a^2 + 2*a*b + b^2) * d*x * \cosh(d*x + c)^7 - 3*((a^2 + 2*a*b + b^2) * d*x - a^2 - a*b) * \cosh(d*x + c)^5 + (3*(a^2 + 2*a*b + b^2) * d*x - 2*a^2 - 4*a*b) * \cosh(d*x + c)^3 - ((a^2 + 2*a*b + b^2) * d*x - a^2 - a*b) * \cosh(d*x + c)) * \sinh(d*x + c)) / (d * \cosh(d*x + c)^8 + 8*d * \cosh(d*x + c) * \sinh(d*x + c)^7 + d * \sinh(d*x + c)^8 - 4*d * \cosh(d*x + c)^6 + 4*(7*d * \cosh(d*x + c)^2 - d) * \sinh(d*x + c)^6 + 8*(7*d * \cosh(d*x + c)^3 - 3*d * \cosh(d*x + c)) * \sinh(d*x + c)^5 + 6*d * \cosh(d*x + c)^4 + 2*(35*d * \cosh(d*x + c)^4 - 30*d * \cosh(d*x + c)^2 + 3*d) * \sinh(d*x + c)^4 + 8*(7*d * \cosh(d*x + c)^5 - 10*d * \cosh(d*x + c)^3 + 3*d * \cosh(d*x + c)) * \sinh(d*x + c)^3 - 4*d * \cosh(d*x + c)^2 + 4*(7*d * \cosh(d*x + c)^6 - 15*d * \cosh(d*x + c)^4 + 9*d * \cosh(d*x + c)^2 - d) * \sinh(d*x + c)^2 + 8*(d * \cosh(d*x + c)^7 - 3*d * \cosh(d*x + c)^5 + 3*d * \cosh(d*x + c)^3 - d * \cosh(d*x + c)) * \sinh(d*x + c) + d) \end{aligned}$$

giac [A] time = 0.34, size = 118, normalized size = 1.64

$$\frac{(a^2 + 2ab + b^2)(dx + c) - (a^2 + 2ab + b^2) \log(|e^{2dx+2c} - 1|) + \frac{4((a^2+ab)e^{6dx+6c} - (a^2+2ab)e^{4dx+4c} + (a^2+ab)e^{2dx+2c})}{(e^{2dx+2c}-1)^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] -((a^2 + 2*a*b + b^2)*(d*x + c) - (a^2 + 2*a*b + b^2)*log(abs(e^(2*d*x + 2*c) - 1))) + 4*((a^2 + a*b)*e^(6*d*x + 6*c) - (a^2 + 2*a*b)*e^(4*d*x + 4*c) + (a^2 + a*b)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c) - 1)^4/d

maple [A] time = 0.29, size = 91, normalized size = 1.26

$$\frac{a^2 \ln(\sinh(dx + c))}{d} - \frac{a^2 (\coth^2(dx + c))}{2d} - \frac{a^2 (\coth^4(dx + c))}{4d} - \frac{ab (\coth^2(dx + c))}{d} + \frac{2ab \ln(\sinh(dx + c))}{d} + \frac{b^2 \ln(\sinh(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x)

[Out] 1/d*a^2*ln(sinh(d*x+c))-1/2*a^2*coth(d*x+c)^2/d-1/4*a^2*coth(d*x+c)^4/d-1/d*a*b*coth(d*x+c)^2+2/d*a*b*ln(sinh(d*x+c))+1/d*b^2*ln(sinh(d*x+c))

maxima [B] time = 0.34, size = 236, normalized size = 3.28

$$a^2 \left(x + \frac{c}{d} + \frac{\log(e^{-dx-c} + 1)}{d} + \frac{\log(e^{-dx-c} - 1)}{d} + \frac{4(e^{-2dx-2c} - e^{-4dx-4c} + e^{-6dx-6c})}{d(4e^{-2dx-2c} - 6e^{-4dx-4c} + 4e^{-6dx-6c} - e^{-8dx-8c} - 1)} \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] a^2*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 2*a*b*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b^2*log(e^(d*x + c) - e^(-d*x - c))/d

mupad [B] time = 1.27, size = 197, normalized size = 2.74

$$\frac{\ln(e^{2c} e^{2dx} - 1) (a^2 + 2ab + b^2)}{d} - x(a + b)^2 - \frac{4(2a^2 + ba)}{d(e^{4c+4dx} - 2e^{2c+2dx} + 1)} - \frac{8a^2}{d(3e^{2c+2dx} - 3e^{4c+4dx} + e^{6c+6dx})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^5*(a + b*tanh(c + d*x)^2)^2, x)
```

```
[Out] (log(exp(2*c)*exp(2*d*x) - 1)*(2*a*b + a^2 + b^2))/d - x*(a + b)^2 - (4*(a*b + 2*a^2))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*a^2)/(d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (4*a^2)/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (4*(a*b + a^2))/(d*(exp(2*c + 2*d*x) - 1))
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int (a + b \tanh^2(c + dx))^2 \coth^5(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**5*(a+b*tanh(d*x+c)**2)**2, x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**5, x)
```

3.154 $\int \coth^6(c + dx) (a + b \tanh^2(c + dx))^2 dx$

Optimal. Leaf size=63

$$-\frac{a^2 \coth^5(c + dx)}{5d} - \frac{a(a + 2b) \coth^3(c + dx)}{3d} - \frac{(a + b)^2 \coth(c + dx)}{d} + x(a + b)^2$$

[Out] (a+b)^2*x-(a+b)^2*coth(d*x+c)/d-1/3*a*(a+2*b)*coth(d*x+c)^3/d-1/5*a^2*coth(d*x+c)^5/d

Rubi [A] time = 0.08, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 461, 207}

$$-\frac{a^2 \coth^5(c + dx)}{5d} - \frac{a(a + 2b) \coth^3(c + dx)}{3d} - \frac{(a + b)^2 \coth(c + dx)}{d} + x(a + b)^2$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^6*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (a + b)^2*x - ((a + b)^2*Coth[c + d*x])/d - (a*(a + 2*b)*Coth[c + d*x]^3)/(3*d) - (a^2*Coth[c + d*x]^5)/(5*d)

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 461

Int[(((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_))/((c_) + (d_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])

Rule 3670

Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_))*((a_) + (b_))*((c_)*tan[(e_) + (f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned} \int \coth^6(c + dx) (a + b \tanh^2(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x^6(1-x^2)} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^6} + \frac{a(a+2b)}{x^4} + \frac{(a+b)^2}{x^2} - \frac{(a+b)^2}{-1+x^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{(a + b)^2 \coth(c + dx)}{d} - \frac{a(a + 2b) \coth^3(c + dx)}{3d} - \frac{a^2 \coth^5(c + dx)}{5d} \\ &= (a + b)^2 x - \frac{(a + b)^2 \coth(c + dx)}{d} - \frac{a(a + 2b) \coth^3(c + dx)}{3d} - \frac{a^2 \coth^5(c + dx)}{5d} \end{aligned}$$

Mathematica [C] time = 0.09, size = 98, normalized size = 1.56

$$\frac{a^2 \coth^5(c + dx) {}_2F_1\left(-\frac{5}{2}, 1; -\frac{3}{2}; \tanh^2(c + dx)\right)}{5d} - \frac{2ab \coth^3(c + dx) {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; \tanh^2(c + dx)\right)}{3d} + b^2 \coth(c + dx)$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^6*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] -1/5*(a^2*Coth[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, Tanh[c + d*x]^2])/d - (2*a*b*Coth[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[c + d*x]^2])/(3*d) - (b^2*Coth[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2])/d

fricas [B] time = 0.41, size = 473, normalized size = 7.51

$$\frac{(23a^2 + 40ab + 15b^2) \cosh(dx + c)^5 + 5(23a^2 + 40ab + 15b^2) \cosh(dx + c) \sinh(dx + c)^4 - (15(a^2 + 2ab + b^2) \cosh(dx + c)^3 \sinh(dx + c)^2 + 5(15(a^2 + 2ab + b^2)d \cosh(dx + c)^2 + 23a^2 + 40ab + 15b^2) \sinh(dx + c)^3 + 5(2(23a^2 + 40ab + 15b^2) \cosh(dx + c)^3 - 3(5a^2 + 16ab + 9b^2) \cosh(dx + c)) \sinh(dx + c)^2 + 10(5a^2 + 4ab + 3b^2) \cosh(dx + c) - 5((15(a^2 + 2ab + b^2)d \cosh(dx + c) + 23a^2 + 40ab + 15b^2) \cosh(dx + c)^4 + 30(a^2 + 2ab + b^2)d \cosh(dx + c)^2 + 46a^2 + 80ab + 30b^2) \sinh(dx + c)) / (d \sinh(dx + c)^5 + 5(2d \cosh(dx + c)^2 - d) \sinh(dx + c)^3 + 5(d \cosh(dx + c)^4 - 3d \cosh(dx + c)^2 + 2d) \sinh(dx + c))}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -1/15*((23*a^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^5 + 5*(23*a^2 + 40*a*b + 15*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*(a^2 + 2*a*b + b^2)*d*x + 23*a^2 + 40*a*b + 15*b^2)*sinh(d*x + c)^5 - 5*(5*a^2 + 16*a*b + 9*b^2)*cosh(d*x + c)^3 + 5*(15*(a^2 + 2*a*b + b^2)*d*x - 2*(15*(a^2 + 2*a*b + b^2)*d*x + 23*a^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^2 + 23*a^2 + 40*a*b + 15*b^2)*sinh(d*x + c)^3 + 5*(2*(23*a^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^3 - 3*(5*a^2 + 16*a*b + 9*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^2 + 4*a*b + 3*b^2)*cosh(d*x + c) - 5*((15*(a^2 + 2*a*b + b^2)*d*x + 23*a^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^4 + 30*(a^2 + 2*a*b + b^2)*d*x - 3*(15*(a^2 + 2*a*b + b^2)*d*x + 23*a^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^2 + 46*a^2 + 80*a*b + 30*b^2)*sinh(d*x + c))/(d*sinh(d*x + c)^5 + 5*(2*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*(d*cosh(d*x + c)^4 - 3*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))

giac [B] time = 0.36, size = 218, normalized size = 3.46

$$15(a^2 + 2ab + b^2)(dx + c) - \frac{2(45a^2e^{(8dx+8c)} + 60abe^{(8dx+8c)} + 15b^2e^{(8dx+8c)} - 90a^2e^{(6dx+6c)} - 180abe^{(6dx+6c)} - 60b^2e^{(6dx+6c)} + 140a^2e^{(4dx+4c)} + 220ab^2e^{(4dx+4c)} + 90b^2e^{(4dx+4c)} - 70a^2e^{(2dx+2c)} - 140ab^2e^{(2dx+2c)} - 60b^2e^{(2dx+2c)} + 23a^2 + 40ab + 15b^2)}{(e^{(2dx+2c)} - 1)^5} + \frac{15d}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/15*(15*(a^2 + 2*a*b + b^2)*(d*x + c) - 2*(45*a^2*e^(8*d*x + 8*c) + 60*a*b*e^(8*d*x + 8*c) + 15*b^2*e^(8*d*x + 8*c) - 90*a^2*e^(6*d*x + 6*c) - 180*a*b*e^(6*d*x + 6*c) - 60*b^2*e^(6*d*x + 6*c) + 140*a^2*e^(4*d*x + 4*c) + 220*a*b*e^(4*d*x + 4*c) + 90*b^2*e^(4*d*x + 4*c) - 70*a^2*e^(2*d*x + 2*c) - 140*a*b*e^(2*d*x + 2*c) - 60*b^2*e^(2*d*x + 2*c) + 23*a^2 + 40*a*b + 15*b^2)/(e^(2*d*x + 2*c) - 1)^5)/d

maple [A] time = 0.24, size = 87, normalized size = 1.38

$$\frac{a^2 \left(dx + c - \coth(dx + c) - \frac{(\coth^3(dx+c))}{3} - \frac{(\coth^5(dx+c))}{5} \right) + 2ab \left(dx + c - \coth(dx + c) - \frac{(\coth^3(dx+c))}{3} \right) + b^2 (dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x)

[Out] $1/d*(a^2*(d*x+c-\coth(d*x+c)-1/3*\coth(d*x+c)^3-1/5*\coth(d*x+c)^5)+2*a*b*(d*x+c-\coth(d*x+c)-1/3*\coth(d*x+c)^3)+b^2*(d*x+c-\coth(d*x+c)))$

maxima [B] time = 0.35, size = 231, normalized size = 3.67

$$\frac{1}{15} a^2 \left(15x + \frac{15c}{d} - \frac{2(70e^{(-2dx-2c)} - 140e^{(-4dx-4c)} + 90e^{(-6dx-6c)} - 45e^{(-8dx-8c)} - 23)}{d(5e^{(-2dx-2c)} - 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} - 5e^{(-8dx-8c)} + e^{(-10dx-10c)} - 1)} \right) + \frac{2}{3} ab \left(3x + \frac{3c}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $1/15*a^2*(15*x + 15*c/d - 2*(70*e^{(-2*d*x - 2*c)} - 140*e^{(-4*d*x - 4*c)} + 90*e^{(-6*d*x - 6*c)} - 45*e^{(-8*d*x - 8*c)} - 23)/(d*(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1))) + 2/3*a*b*(3*x + 3*c/d - 4*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} - 2)/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} - 1))) + b^2*(x + c/d + 2/(d*(e^{(-2*d*x - 2*c)} - 1)))$

mupad [B] time = 0.20, size = 529, normalized size = 8.40

$$\frac{\frac{2(b^2+2ab)}{5d} - \frac{2e^{2c+2dx}(3a^2+4ab+b^2)}{5d}}{e^{4c+4dx} - 2e^{2c+2dx} + 1} + \frac{\frac{2(b^2+2ab)}{5d} + \frac{6e^{4c+4dx}(b^2+2ab)}{5d} - \frac{2e^{6c+6dx}(3a^2+4ab+b^2)}{5d} - \frac{2e^{2c+2dx}(5a^2+4ab+3b^2)}{5d}}{6e^{4c+4dx} - 4e^{2c+2dx} - 4e^{6c+6dx} + e^{8c+8dx} + 1} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^6*(a + b*tanh(c + d*x)^2)^2,x)

[Out] $((2*(2*a*b + b^2))/(5*d) - (2*\exp(2*c + 2*d*x)*(4*a*b + 3*a^2 + b^2))/(5*d))/(\exp(4*c + 4*d*x) - 2*\exp(2*c + 2*d*x) + 1) + ((2*(2*a*b + b^2))/(5*d) + (6*\exp(4*c + 4*d*x)*(2*a*b + b^2))/(5*d) - (2*\exp(6*c + 6*d*x)*(4*a*b + 3*a^2 + b^2))/(5*d) - (2*\exp(2*c + 2*d*x)*(4*a*b + 5*a^2 + 3*b^2))/(5*d))/((6*\exp(4*c + 4*d*x) - 4*\exp(2*c + 2*d*x) - 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1) + x*(a + b)^2 - ((2*(4*a*b + 3*a^2 + b^2))/(5*d) - (8*\exp(2*c + 2*d*x)*(2*a*b + b^2))/(5*d) - (8*\exp(6*c + 6*d*x)*(2*a*b + b^2))/(5*d) + (2*\exp(8*c + 8*d*x)*(4*a*b + 3*a^2 + b^2))/(5*d) + (4*\exp(4*c + 4*d*x)*(4*a*b + 5*a^2 + 3*b^2))/(5*d))/((5*\exp(2*c + 2*d*x) - 10*\exp(4*c + 4*d*x) + 10*\exp(6*c + 6*d*x) - 5*\exp(8*c + 8*d*x) + \exp(10*c + 10*d*x) - 1) - ((2*(4*a*b + 5*a^2 + 3*b^2))/(15*d) - (4*\exp(2*c + 2*d*x)*(2*a*b + b^2))/(5*d) + (2*\exp(4*c + 4*d*x)*(4*a*b + 3*a^2 + b^2))/(5*d))/((3*\exp(2*c + 2*d*x) - 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) - 1) - (2*(4*a*b + 3*a^2 + b^2))/(5*d*(\exp(2*c + 2*d*x) - 1)))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^2 \coth^6(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**6*(a+b*tanh(d*x+c)**2)**2,x)

[Out] Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**6, x)

3.155 $\int \coth^7(c + dx) (a + b \tanh^2(c + dx))^2 dx$

Optimal. Leaf size=92

$$\frac{a^2 \coth^6(c + dx)}{6d} - \frac{a(a + 2b) \coth^4(c + dx)}{4d} - \frac{(a + b)^2 \coth^2(c + dx)}{2d} + \frac{(a + b)^2 \log(\tanh(c + dx))}{d} + \frac{(a + b)^2 \log(\tanh(c + dx))}{d}$$

[Out] $-1/2*(a+b)^2*\coth(d*x+c)^2/d-1/4*a*(a+2*b)*\coth(d*x+c)^4/d-1/6*a^2*\coth(d*x+c)^6/d+(a+b)^2*\ln(\cosh(d*x+c))/d+(a+b)^2*\ln(\tanh(d*x+c))/d$

Rubi [A] time = 0.11, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 446, 88}

$$\frac{a^2 \coth^6(c + dx)}{6d} - \frac{a(a + 2b) \coth^4(c + dx)}{4d} - \frac{(a + b)^2 \coth^2(c + dx)}{2d} + \frac{(a + b)^2 \log(\tanh(c + dx))}{d} + \frac{(a + b)^2 \log(\tanh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^7*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] $-((a + b)^2*\text{Coth}[c + d*x]^2)/(2*d) - (a*(a + 2*b)*\text{Coth}[c + d*x]^4)/(4*d) - (a^2*\text{Coth}[c + d*x]^6)/(6*d) + ((a + b)^2*\text{Log}[\text{Cosh}[c + d*x]])/d + ((a + b)^2*\text{Log}[\text{Tanh}[c + d*x]])/d$

Rule 88

Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_.)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff, x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
\int \coth^7(c+dx) (a+b \tanh^2(c+dx))^2 dx &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{x^7(1-x^2)} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^2}{(1-x)x^4} dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^2}{-1+x} + \frac{a^2}{x^4} + \frac{a(a+2b)}{x^3} + \frac{(a+b)^2}{x^2} + \frac{(a+b)^2}{x}\right) dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= -\frac{(a+b)^2 \coth^2(c+dx)}{2d} - \frac{a(a+2b) \coth^4(c+dx)}{4d} - \frac{a^2 \coth^6(c+dx)}{6d}
\end{aligned}$$

Mathematica [A] time = 0.52, size = 74, normalized size = 0.80

$$\frac{2a^2 \coth^6(c+dx) + 3a(a+2b) \coth^4(c+dx) + 6(a+b)^2 \coth^2(c+dx) - 12(a+b)^2(\log(\tanh(c+dx)) + \log(c))}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^7*(a + b*Tanh[c + d*x]^2)^2,x]

[Out] -1/12*(6*(a + b)^2*Coth[c + d*x]^2 + 3*a*(a + 2*b)*Coth[c + d*x]^4 + 2*a^2*Coth[c + d*x]^6 - 12*(a + b)^2*(Log[Cosh[c + d*x]] + Log[Tanh[c + d*x]]))/d

fricas [B] time = 0.47, size = 3454, normalized size = 37.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -1/3*(3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^12 + 36*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^11 + 3*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^12 - 6*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^10 + 6*(33*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 - 3*(a^2 + 2*a*b + b^2)*d*x + 3*a^2 + 4*a*b + b^2)*sinh(d*x + c)^10 + 60*(11*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 - (3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^9 + 3*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c)^8 + 3*(495*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 15*(a^2 + 2*a*b + b^2)*d*x - 90*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^2 - 12*a^2 - 24*a*b - 8*b^2)*sinh(d*x + c)^8 + 24*(99*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^5 - 30*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^3 + (15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(15*(a^2 + 2*a*b + b^2)*d*x - 17*a^2 - 24*a*b - 9*b^2)*cosh(d*x + c)^6 + 4*(693*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 - 315*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^4 - 15*(a^2 + 2*a*b + b^2)*d*x + 21*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c)^2 + 17*a^2 + 24*a*b + 9*b^2)*sinh(d*x + c)^6 + 24*(99*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^7 - 63*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^5 + 7*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c)^3 - (15*(a^2 + 2*a*b + b^2)*d*x - 17*a^2 - 24*a*b - 9*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c)^4 + 3*(495*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 - 420*(3*(a^2 + 2*a*b + b^2)*d*x - 3*a^2 - 4*a*b - b^2)*cosh(d*x + c)^6 + 70*(15*(a^2 + 2*a*b + b^2)*d*x - 12*a^2 - 24*a*b - 8*b^2)*cosh(d*x + c)^4 + 15*(a^2 + 2*a*b + b^2)*d*x - 20*(15*(a^2 + 2*a*b + b^2)*d*x - 17*a^2 - 24*a*b - 9*b^2)*cosh(d*x + c)^

[In] `int(coth(c + d*x)^7*(a + b*tanh(c + d*x)^2)^2,x)`

[Out] $(\log(\exp(2c)\exp(2dx) - 1)(2ab + a^2 + b^2))/d - (2(4ab + 3a^2 + b^2))/(d(\exp(2c + 2dx) - 1)) - (32a^2)/(3d(15\exp(4c + 4dx) - 6\exp(2c + 2dx) - 20\exp(6c + 6dx) + 15\exp(8c + 8dx) - 6\exp(10c + 10dx) + \exp(12c + 12dx) + 1)) - x(a + b)^2 - (2(8ab + 9a^2 + b^2))/(d(\exp(4c + 4dx) - 2\exp(2c + 2dx) + 1)) - (8(6ab + 13a^2))/(3d(3\exp(2c + 2dx) - 3\exp(4c + 4dx) + \exp(6c + 6dx) - 1)) - (4(2ab + 11a^2))/(d(6\exp(4c + 4dx) - 4\exp(2c + 2dx) - 4\exp(6c + 6dx) + \exp(8c + 8dx) + 1)) - (32a^2)/(d(5\exp(2c + 2dx) - 10\exp(4c + 4dx) + 10\exp(6c + 6dx) - 5\exp(8c + 8dx) + \exp(10c + 10dx) - 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^2 \coth^7(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)**7*(a+b*tanh(d*x+c)**2)**2,x)`

[Out] `Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**7, x)`

$$3.156 \quad \int \tanh^4(c + dx) \left(a + b \tanh^2(c + dx) \right)^3 dx$$

Optimal. Leaf size=114

$$\frac{b(3a^2 + 3ab + b^2) \tanh^5(c + dx)}{5d} - \frac{b^2(3a + b) \tanh^7(c + dx)}{7d} - \frac{(a + b)^3 \tanh^3(c + dx)}{3d} - \frac{(a + b)^3 \tanh(c + dx)}{d} + x(a + b)^3 \tanh^3(c + dx)$$

[Out] (a+b)^3*x - (a+b)^3*tanh(d*x+c)/d - 1/3*(a+b)^3*tanh(d*x+c)^3/d - 1/5*b*(3*a^2+3*a*b+b^2)*tanh(d*x+c)^5/d - 1/7*b^2*(3*a+b)*tanh(d*x+c)^7/d - 1/9*b^3*tanh(d*x+c)^9/d

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 461, 206}

$$\frac{b(3a^2 + 3ab + b^2) \tanh^5(c + dx)}{5d} - \frac{b^2(3a + b) \tanh^7(c + dx)}{7d} - \frac{(a + b)^3 \tanh^3(c + dx)}{3d} - \frac{(a + b)^3 \tanh(c + dx)}{d} + x(a + b)^3 \tanh^3(c + dx)$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (a + b)^3*x - ((a + b)^3*Tanh[c + d*x])/d - ((a + b)^3*Tanh[c + d*x]^3)/(3*d) - (b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^5)/(5*d) - (b^2*(3*a + b)*Tanh[c + d*x]^7)/(7*d) - (b^3*Tanh[c + d*x]^9)/(9*d)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 461

Int[(((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_))/((c_) + (d_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])

Rule 3670

Int[(((d_)*tan[(e_) + (f_)*(x_)])^(m_))*((a_) + (b_))*((c_)*tan[(e_) + (f_)*(x_)])^(n_)]^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned} \int \tanh^4(c+dx) (a+b \tanh^2(c+dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{x^4(a+bx^2)^3}{1-x^2} dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(-(a+b)^3 - (a+b)^3x^2 - b(3a^2+3ab+b^2)x^4 - b^2(3a^2+3ab+b^2)x^6\right) dx, x, \tanh(c+dx)\right)}{d} \\ &= -\frac{(a+b)^3 \tanh(c+dx)}{d} - \frac{(a+b)^3 \tanh^3(c+dx)}{3d} - \frac{b(3a^2+3ab+b^2) \tanh^5(c+dx)}{5d} - \frac{b^2(3a^2+3ab+b^2) \tanh^7(c+dx)}{7d} \\ &= (a+b)^3x - \frac{(a+b)^3 \tanh(c+dx)}{d} - \frac{(a+b)^3 \tanh^3(c+dx)}{3d} - \frac{b(3a^2+3ab+b^2) \tanh^5(c+dx)}{5d} - \frac{b^2(3a^2+3ab+b^2) \tanh^7(c+dx)}{7d} \end{aligned}$$

Mathematica [A] time = 1.52, size = 123, normalized size = 1.08

$$\frac{\tanh(c+dx) \left(-63b(3a^2+3ab+b^2) \tanh^4(c+dx) - 45b^2(3a+b) \tanh^6(c+dx) - 105(a+b)^3 \tanh^2(c+dx) \right)}{315d}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (Tanh[c + d*x]*(-315*(a + b)^3 - 105*(a + b)^3*Tanh[c + d*x]^2 - 63*b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^4 - 45*b^2*(3*a + b)*Tanh[c + d*x]^6 - 35*b^3*Tanh[c + d*x]^8 + (315*(a + b)^3*ArcTanh[Sqrt[Tanh[c + d*x]^2]]))/Sqrt[Tanh[c + d*x]^2])/(315*d)

fricas [B] time = 0.45, size = 1563, normalized size = 13.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/315*((420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^9 + 9*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^8 - (420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3)*sinh(d*x + c)^9 + 9*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 - 9*(280*a^3 + 819*a^2*b + 744*a*b^2 + 213*b^3 + 4*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 21*(4*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + 3*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^6 + 36*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 9*(14*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3)*cosh(d*x + c)^4 + 700*a^3 + 2016*a^2*b + 2136*a*b^2 + 852*b^3 + 21*(280*a^3 + 819*a^2*b + 744*a*b^2 + 213*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 9*(14*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + 20*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^4 + 84*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - 3*(28*(420*a^3 + 1449*a^2*b + 1584*a*b^2 +

$$563b^3 \cosh(dx + c)^6 + 105(280a^3 + 819a^2b + 744ab^2 + 213b^3) \cosh(dx + c)^4 + 2660a^3 + 8232a^2b + 8232ab^2 + 1764b^3 + 120(175a^3 + 504a^2b + 534ab^2 + 213b^3) \cosh(dx + c)^2 \sinh(dx + c)^3 + 9(4(420a^3 + 1449a^2b + 1584ab^2 + 563b^3 + 315(a^3 + 3a^2b + 3ab^2 + b^3)d^2x) \cosh(dx + c)^7 + 21(420a^3 + 1449a^2b + 1584ab^2 + 563b^3 + 315(a^3 + 3a^2b + 3ab^2 + b^3)d^2x) \cosh(dx + c)^5 + 40(420a^3 + 1449a^2b + 1584ab^2 + 563b^3 + 315(a^3 + 3a^2b + 3ab^2 + b^3)d^2x) \cosh(dx + c)^3 + 28(420a^3 + 1449a^2b + 1584ab^2 + 563b^3 + 315(a^3 + 3a^2b + 3ab^2 + b^3)d^2x) \cosh(dx + c)) \sinh(dx + c)^2 + 126(420a^3 + 1449a^2b + 1584ab^2 + 563b^3 + 315(a^3 + 3a^2b + 3ab^2 + b^3)d^2x) \cosh(dx + c) - 9((420a^3 + 1449a^2b + 1584ab^2 + 563b^3) \cosh(dx + c)^8 + 7(280a^3 + 819a^2b + 744ab^2 + 213b^3) \cosh(dx + c)^6 + 20(175a^3 + 504a^2b + 534ab^2 + 213b^3) \cosh(dx + c)^4 + 420a^3 + 1386a^2b + 1176ab^2 + 882b^3 + 28(95a^3 + 294a^2b + 294ab^2 + 63b^3) \cosh(dx + c)^2) \sinh(dx + c)) / (d \cosh(dx + c)^9 + 9d \cosh(dx + c) \sinh(dx + c)^8 + 9d \cosh(dx + c)^7 + 21(4d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^6 + 36d \cosh(dx + c)^5 + 9(14d \cosh(dx + c)^5 + 35d \cosh(dx + c)^3 + 20d \cosh(dx + c)) \sinh(dx + c)^4 + 84d \cosh(dx + c)^3 + 9(4d \cosh(dx + c)^7 + 21d \cosh(dx + c)^5 + 40d \cosh(dx + c)^3 + 28d \cosh(dx + c)) \sinh(dx + c)^2 + 126d \cosh(dx + c))$$

giac [B] time = 0.54, size = 534, normalized size = 4.68

$$315(a^3 + 3a^2b + 3ab^2 + b^3)(dx + c) + \frac{2(630a^3e^{(16dx+16c)} + 2835a^2be^{(16dx+16c)} + 3780ab^2e^{(16dx+16c)} + 1575b^3e^{(16dx+16c)} + 4410a^3e^{(14dx+14c)} + 17010a^2be^{(14dx+14c)} + 18900ab^2e^{(14dx+14c)} + 6300b^3e^{(14dx+14c)} + 13650a^3e^{(12dx+12c)} + 48510a^2be^{(12dx+12c)} + 54180ab^2e^{(12dx+12c)} + 21000b^3e^{(12dx+12c)} + 24570a^3e^{(10dx+10c)} + 85050a^2be^{(10dx+10c)} + 94500ab^2e^{(10dx+10c)} + 31500b^3e^{(10dx+10c)} + 28350a^3e^{(8dx+8c)} + 97524a^2be^{(8dx+8c)} + 105084ab^2e^{(8dx+8c)} + 39438b^3e^{(8dx+8c)} + 21630a^3e^{(6dx+6c)} + 73206a^2be^{(6dx+6c)} + 78876ab^2e^{(6dx+6c)} + 26292b^3e^{(6dx+6c)} + 10710a^3e^{(4dx+4c)} + 35154a^2be^{(4dx+4c)} + 38124ab^2e^{(4dx+4c)} + 13968b^3e^{(4dx+4c)} + 3150a^3e^{(2dx+2c)} + 10206a^2be^{(2dx+2c)} + 10476ab^2e^{(2dx+2c)} + 3492b^3e^{(2dx+2c)} + 420a^3 + 1449a^2b + 1584ab^2 + 563b^3) / (e^{(2dx+2c)} + 1)^9 / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(dx+c)^4*(a+b*tanh(dx+c)^2)^3,x, algorithm="giac")

[Out] $\frac{1}{315} (315(a^3 + 3a^2b + 3ab^2 + b^3)(dx + c) + 2(630a^3e^{(16dx+16c)} + 2835a^2be^{(16dx+16c)} + 3780ab^2e^{(16dx+16c)} + 1575b^3e^{(16dx+16c)} + 4410a^3e^{(14dx+14c)} + 17010a^2be^{(14dx+14c)} + 18900ab^2e^{(14dx+14c)} + 6300b^3e^{(14dx+14c)} + 13650a^3e^{(12dx+12c)} + 48510a^2be^{(12dx+12c)} + 54180ab^2e^{(12dx+12c)} + 21000b^3e^{(12dx+12c)} + 24570a^3e^{(10dx+10c)} + 85050a^2be^{(10dx+10c)} + 94500ab^2e^{(10dx+10c)} + 31500b^3e^{(10dx+10c)} + 28350a^3e^{(8dx+8c)} + 97524a^2be^{(8dx+8c)} + 105084ab^2e^{(8dx+8c)} + 39438b^3e^{(8dx+8c)} + 21630a^3e^{(6dx+6c)} + 73206a^2be^{(6dx+6c)} + 78876ab^2e^{(6dx+6c)} + 26292b^3e^{(6dx+6c)} + 10710a^3e^{(4dx+4c)} + 35154a^2be^{(4dx+4c)} + 38124ab^2e^{(4dx+4c)} + 13968b^3e^{(4dx+4c)} + 3150a^3e^{(2dx+2c)} + 10206a^2be^{(2dx+2c)} + 10476ab^2e^{(2dx+2c)} + 3492b^3e^{(2dx+2c)} + 420a^3 + 1449a^2b + 1584ab^2 + 563b^3) / (e^{(2dx+2c)} + 1)^9) / d$

maple [B] time = 0.02, size = 365, normalized size = 3.20

$$\frac{\ln(1 + \tanh(dx + c)) a^3}{2d} + \frac{3 \ln(1 + \tanh(dx + c)) a^2 b}{2d} + \frac{3 \ln(1 + \tanh(dx + c)) a b^2}{2d} + \frac{\ln(1 + \tanh(dx + c)) b^3}{2d} - \frac{a^3}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(dx+c)^4*(a+b*tanh(dx+c)^2)^3,x)

[Out] $\frac{1}{2} d \ln(1 + \tanh(dx + c)) a^3 + \frac{3}{2} d \ln(1 + \tanh(dx + c)) a^2 b + \frac{3}{2} d \ln(1 + \tanh(dx + c)) a b^2 + \frac{1}{2} d \ln(1 + \tanh(dx + c)) b^3 - \frac{1}{3} d a^3 \tanh(dx + c)^3 - b^3 \tanh(dx + c) / d - \frac{1}{9} b^3 \tanh(dx + c)^9 / d - a^3 \tanh(dx + c) / d - \frac{1}{2} d \ln(\tanh(dx + c) - 1) a^3 - \frac{3}{2} d \ln(\tanh(dx + c) - 1) a^2 b - \frac{3}{2} d \ln(\tanh(dx + c) - 1) a b^2 - \frac{1}{2} d \ln(\tanh(dx + c) - 1) b^3$

$$\frac{h(dx+c)-1}{d} \cdot b^3 - \frac{1}{7} b^3 \tanh(dx+c)^7/d - \frac{1}{5} b^3 \tanh(dx+c)^5/d - \frac{1}{3} b^3 \tanh(dx+c)^3/d - \frac{3}{5} \frac{a^2 b^3}{d} \tanh(dx+c)^5 + \frac{a^2 b^3}{5} \frac{a^2 b^3 \tanh(dx+c)^5/d - a^2 b^3 \tanh(dx+c)^3/d - a^2 b^3 \tanh(dx+c)}{d} + \frac{3}{7} \frac{a^2 b^3 \tanh(dx+c)^7}{d} + \frac{3}{7} \frac{a^2 b^3 \tanh(dx+c)^5/d - a^2 b^3 \tanh(dx+c)^3/d - a^2 b^3 \tanh(dx+c)}{d}$$

maxima [B] time = 0.37, size = 583, normalized size = 5.11

$$\frac{1}{315} b^3 \left(315x + \frac{315c}{d} - \frac{2(3492e^{(-2dx-2c)} + 13968e^{(-4dx-4c)} + 26292e^{(-6dx-6c)} + 39438e^{(-8dx-8c)} + 31500e^{(-10dx-10c)} + 21000e^{(-12dx-12c)} + 6300e^{(-14dx-14c)} + 1575e^{(-16dx-16c)} + 563)}{d(9e^{(-2dx-2c)} + 36e^{(-4dx-4c)} + 84e^{(-6dx-6c)} + 126e^{(-8dx-8c)} + 126e^{(-10dx-10c)} + 84e^{(-12dx-12c)} + 36e^{(-14dx-14c)} + 9e^{(-16dx-16c)} + e^{(-18dx-18c)} + 1)} \right) + \frac{1}{35} \frac{a^2 b^2 (105x + 105c/d - 8(203e^{(-2dx-2c)} + 609e^{(-4dx-4c)} + 770e^{(-6dx-6c)} + 770e^{(-8dx-8c)} + 315e^{(-10dx-10c)} + 105e^{(-12dx-12c)} + 44)}{d(7e^{(-2dx-2c)} + 21e^{(-4dx-4c)} + 35e^{(-6dx-6c)} + 35e^{(-8dx-8c)} + 21e^{(-10dx-10c)} + 7e^{(-12dx-12c)} + e^{(-14dx-14c)} + 1)} \right) + \frac{1}{5} \frac{a^2 b (15x + 15c/d - 2(70e^{(-2dx-2c)} + 140e^{(-4dx-4c)} + 90e^{(-6dx-6c)} + 45e^{(-8dx-8c)} + 23)}{d(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} \right) + \frac{1}{3} \frac{a^3 (3x + 3c/d - 4(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + 2)}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(dx+c)^4*(a+b*tanh(dx+c))^2)^3,x, algorithm="maxima")

[Out] 1/315*b^3*(315*x + 315*c/d - 2*(3492*e^(-2*d*x - 2*c) + 13968*e^(-4*d*x - 4*c) + 26292*e^(-6*d*x - 6*c) + 39438*e^(-8*d*x - 8*c) + 31500*e^(-10*d*x - 10*c) + 21000*e^(-12*d*x - 12*c) + 6300*e^(-14*d*x - 14*c) + 1575*e^(-16*d*x - 16*c) + 563)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1))) + 1/35*a*b^2*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*e^(-4*d*x - 4*c) + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-10*d*x - 10*c) + 105*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 1/5*a^2*b*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 1/3*a^3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))

mupad [B] time = 0.26, size = 138, normalized size = 1.21

$$x(a^3 + 3a^2b + 3ab^2 + b^3) - \frac{\tanh(c+dx)(a+b)^3}{d} - \frac{\tanh(c+dx)^5(3a^2b + 3ab^2 + b^3)}{5d} - \frac{\tanh(c+dx)^7(b^3)}{7d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c+dx)^4*(a+b*tanh(c+dx))^2)^3,x

[Out] x*(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c+dx)*(a+b)^3)/d - (tanh(c+dx)^5*(3*a*b^2 + 3*a^2*b + b^3))/(5*d) - (tanh(c+dx)^7*(3*a*b^2 + b^3))/(7*d) - (b^3*tanh(c+dx)^9)/(9*d) - (tanh(c+dx)^3*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(3*d)

sympy [A] time = 2.09, size = 260, normalized size = 2.28

$$\left\{ \begin{array}{l} a^3x - \frac{a^3 \tanh^3(c+dx)}{3d} - \frac{a^3 \tanh(c+dx)}{d} + 3a^2bx - \frac{3a^2b \tanh^5(c+dx)}{5d} - \frac{a^2b \tanh^3(c+dx)}{d} - \frac{3a^2b \tanh(c+dx)}{d} + 3ab^2x - \frac{3ab^2 \tanh^7(c+dx)}{7d} \\ x(a + b \tanh^2(c))^3 \tanh^4(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(dx+c)**4*(a+b*tanh(dx+c)**2)**3,x)

[Out] Piecewise((a**3*x - a**3*tanh(c + d*x)**3/(3*d) - a**3*tanh(c + d*x)/d + 3*a**2*b*x - 3*a**2*b*tanh(c + d*x)**5/(5*d) - a**2*b*tanh(c + d*x)**3/d - 3*a**2*b*tanh(c + d*x)/d + 3*a*b**2*x - 3*a*b**2*tanh(c + d*x)**7/(7*d) - 3*a*b**2*tanh(c + d*x)**5/(5*d) - a*b**2*tanh(c + d*x)**3/d - 3*a*b**2*tanh(c + d*x)/d + b**3*x - b**3*tanh(c + d*x)**9/(9*d) - b**3*tanh(c + d*x)**7/(7*d) - b**3*tanh(c + d*x)**5/(5*d) - b**3*tanh(c + d*x)**3/(3*d) - b**3*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**3*tanh(c)**4, True))

3.157 $\int \tanh^3(c + dx) \left(a + b \tanh^2(c + dx) \right)^3 dx$

Optimal. Leaf size=107

$$\frac{b(3a^2 + 3ab + b^2) \tanh^4(c + dx)}{4d} - \frac{b^2(3a + b) \tanh^6(c + dx)}{6d} - \frac{(a + b)^3 \tanh^2(c + dx)}{2d} + \frac{(a + b)^3 \log(\cosh(c + dx))}{d}$$

[Out] $(a+b)^3 \ln(\cosh(dx+c))/d - 1/2*(a+b)^3 \tanh(dx+c)^2/d - 1/4*b*(3*a^2+3*a*b+b^2)*\tanh(dx+c)^4/d - 1/6*b^2*(3*a+b)*\tanh(dx+c)^6/d - 1/8*b^3*\tanh(dx+c)^8/d$

Rubi [A] time = 0.15, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 446, 77}

$$\frac{b(3a^2 + 3ab + b^2) \tanh^4(c + dx)}{4d} - \frac{b^2(3a + b) \tanh^6(c + dx)}{6d} - \frac{(a + b)^3 \tanh^2(c + dx)}{2d} + \frac{(a + b)^3 \log(\cosh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] $((a + b)^3 \text{Log}[\text{Cosh}[c + d*x]])/d - ((a + b)^3 \text{Tanh}[c + d*x]^2)/(2*d) - (b*(3*a^2 + 3*a*b + b^2)*\text{Tanh}[c + d*x]^4)/(4*d) - (b^2*(3*a + b)*\text{Tanh}[c + d*x]^6)/(6*d) - (b^3*\text{Tanh}[c + d*x]^8)/(8*d)$

Rule 77

Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
\int \tanh^3(c + dx) (a + b \tanh^2(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{x^3(a+bx^2)^3}{1-x^2} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{x(a+bx)^3}{1-x} dx, x, \tanh^2(c + dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(- (a + b)^3 - \frac{(a+b)^3}{-1+x} - b(3a^2 + 3ab + b^2)x - b^2(3a + b)\right) dx, x, \tanh(c + dx)\right)}{2d} \\
&= \frac{(a + b)^3 \log(\cosh(c + dx))}{d} - \frac{(a + b)^3 \tanh^2(c + dx)}{2d} - \frac{b(3a^2 + 3ab + b^2) \tanh^4(c + dx) - \frac{1}{3}b^2(3a + b) \tanh^6(c + dx) - (a + b)^3 \tanh^2(c + dx) + 2(a + b)^3 \log(\cosh(c + dx))}{2d}
\end{aligned}$$

Mathematica [A] time = 0.31, size = 98, normalized size = 0.92

$$\frac{-\frac{1}{2}b(3a^2 + 3ab + b^2) \tanh^4(c + dx) - \frac{1}{3}b^2(3a + b) \tanh^6(c + dx) - (a + b)^3 \tanh^2(c + dx) + 2(a + b)^3 \log(\cosh(c + dx))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (2*(a + b)^3*Log[Cosh[c + d*x]] - (a + b)^3*Tanh[c + d*x]^2 - (b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^4)/2 - (b^2*(3*a + b)*Tanh[c + d*x]^6)/3 - (b^3*Tanh[c + d*x]^8)/4)/(2*d)

fricas [B] time = 0.56, size = 7502, normalized size = 70.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^16 + 48*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^15 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*sinh(d*x + c)^16 - 6*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^14 + 6*(60*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 - a^3 - 6*a^2*b - 9*a*b^2 - 4*b^3 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*sinh(d*x + c)^14 + 84*(20*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^3 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^13 - 12*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^12 + 6*(910*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^4 - 6*a^3 - 30*a^2*b - 36*a*b^2 - 12*b^3 + 14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 91*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 24*(546*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^5 - 91*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - 6*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^11 - 2*(45*a^3 + 198*a^2*b + 237*a*b^2 + 100*b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^10 + 2*(12012*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^6 - 3003*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 45*a^3 - 198*a^2*b - 237*a*b^2 - 100*b^3 + 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 396*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 4*(8580*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^7 - 3003*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3

$$\begin{aligned}
& - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c)^5 - 660*(3*a^3 + 15* \\
& a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x \\
& + c)^3 - 5*(45*a^3 + 198*a^2*b + 237*a*b^2 + 100*b^3 - 84*(a^3 + 3*a^2*b + \\
& 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^9 - 2*(60*a^3 + 252*a^2*b \\
& + 312*a*b^2 + 104*b^3 - 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + \\
& c)^8 + 2*(19305*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*\cosh(d*x + c)^8 - 9009 \\
& *(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)* \\
& \cosh(d*x + c)^6 - 2970*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^ \\
& 2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c)^4 - 60*a^3 - 252*a^2*b - 312*a*b^2 \\
& - 104*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 45*(45*a^3 + 198*a^2* \\
& b + 237*a*b^2 + 100*b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x \\
& + c)^2)*\sinh(d*x + c)^8 + 16*(2145*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*\cosh \\
& (d*x + c)^9 - 1287*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3* \\
& a*b^2 + b^3)*d*x)*\cosh(d*x + c)^7 - 594*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^ \\
& 3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c)^5 - 15*(45*a^3 + 1 \\
& 98*a^2*b + 237*a*b^2 + 100*b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\co \\
& sh(d*x + c)^3 - (60*a^3 + 252*a^2*b + 312*a*b^2 + 104*b^3 - 105*(a^3 + 3*a^ \\
& 2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^7 - 2*(45*a^3 + 198* \\
& a^2*b + 237*a*b^2 + 100*b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(\\
& d*x + c)^6 + 2*(12012*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*\cosh(d*x + c)^10 \\
& - 9009*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) \\
& *d*x)*\cosh(d*x + c)^8 - 5544*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 \\
& + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c)^6 - 210*(45*a^3 + 198*a^2*b + \\
& 237*a*b^2 + 100*b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c \\
&)^4 - 45*a^3 - 198*a^2*b - 237*a*b^2 - 100*b^3 + 84*(a^3 + 3*a^2*b + 3*a*b^ \\
& 2 + b^3)*d*x - 28*(60*a^3 + 252*a^2*b + 312*a*b^2 + 104*b^3 - 105*(a^3 + 3* \\
& a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 4*(3276*(a^3 \\
& + 3*a^2*b + 3*a*b^2 + b^3)*d*x*\cosh(d*x + c)^11 - 3003*(a^3 + 6*a^2*b + 9* \\
& a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c)^9 - 23 \\
& 76*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) \\
& *d*x)*\cosh(d*x + c)^7 - 126*(45*a^3 + 198*a^2*b + 237*a*b^2 + 100*b^3 - 84* \\
& (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c)^5 - 28*(60*a^3 + 252*a^2 \\
& *b + 312*a*b^2 + 104*b^3 - 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d* \\
& x + c)^3 - 3*(45*a^3 + 198*a^2*b + 237*a*b^2 + 100*b^3 - 84*(a^3 + 3*a^2*b \\
& + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 12*(3*a^3 + 15*a^2*b \\
& + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c)^ \\
& 4 + 2*(2730*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*\cosh(d*x + c)^12 - 3003*(a^ \\
& 3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh \\
& (d*x + c)^10 - 2970*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b \\
& + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c)^8 - 210*(45*a^3 + 198*a^2*b + 237*a*b^ \\
& 2 + 100*b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c)^6 - 70* \\
& (60*a^3 + 252*a^2*b + 312*a*b^2 + 104*b^3 - 105*(a^3 + 3*a^2*b + 3*a*b^2 + \\
& b^3)*d*x)*\cosh(d*x + c)^4 - 18*a^3 - 90*a^2*b - 108*a*b^2 - 36*b^3 + 42*(a^ \\
& 3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 15*(45*a^3 + 198*a^2*b + 237*a*b^2 + 100 \\
& *b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + \\
& c)^4 + 8*(210*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*\cosh(d*x + c)^13 - 273*(a \\
& ^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cos \\
& h(d*x + c)^11 - 330*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b \\
& + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c)^9 - 30*(45*a^3 + 198*a^2*b + 237*a*b^2 \\
& + 100*b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c)^7 - 14*(\\
& 60*a^3 + 252*a^2*b + 312*a*b^2 + 104*b^3 - 105*(a^3 + 3*a^2*b + 3*a*b^2 + b \\
& ^3)*d*x)*\cosh(d*x + c)^5 - 5*(45*a^3 + 198*a^2*b + 237*a*b^2 + 100*b^3 - 84 \\
& *(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c)^3 - 6*(3*a^3 + 15*a^2*b \\
& + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c) \\
&)*\sinh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 6*(a^3 + 6*a^2*b \\
& + 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c)^2 \\
& + 2*(180*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*\cosh(d*x + c)^14 - 273*(a^3 + \\
& 6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d* \\
& x + c)^12 - 396*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3
\end{aligned}$$

$$\begin{aligned}
& *a*b^2 + b^3)*d*x)*\cosh(d*x + c)^{10} - 45*(45*a^3 + 198*a^2*b + 237*a*b^2 + \\
& 100*b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c)^8 - 28*(60* \\
& a^3 + 252*a^2*b + 312*a*b^2 + 104*b^3 - 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) \\
& *d*x)*\cosh(d*x + c)^6 - 15*(45*a^3 + 198*a^2*b + 237*a*b^2 + 100*b^3 - 84*(\\
& a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c)^4 - 3*a^3 - 18*a^2*b - 27 \\
& *a*b^2 - 12*b^3 + 12*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 36*(3*a^3 + 15*a \\
& ^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + \\
& c)^2)*\sinh(d*x + c)^2 - 3*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^{1 \\
& 6 + 16*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)*\sinh(d*x + c)^{15} + (a^ \\
& 3 + 3*a^2*b + 3*a*b^2 + b^3)*\sinh(d*x + c)^{16} + 8*(a^3 + 3*a^2*b + 3*a*b^2 \\
& + b^3)*\cosh(d*x + c)^{14} + 8*(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 15*(a^3 + 3*a^ \\
& 2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{14} + 112*(5*(a^3 + 3*a^ \\
& 2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh \\
& (d*x + c))*\sinh(d*x + c)^{13} + 28*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + \\
& c)^{12} + 28*(65*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^4 + a^3 + 3*a \\
& ^2*b + 3*a*b^2 + b^3 + 26*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2)* \\
& \sinh(d*x + c)^{12} + 112*(39*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^5 \\
& + 26*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3 \\
& *a*b^2 + b^3)*\cosh(d*x + c))*\sinh(d*x + c)^{11} + 56*(a^3 + 3*a^2*b + 3*a*b^2 \\
& + b^3)*\cosh(d*x + c)^{10} + 56*(143*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x \\
& + c)^6 + 143*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^4 + a^3 + 3*a^2 \\
& *b + 3*a*b^2 + b^3 + 33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2)*\si \\
& nh(d*x + c)^{10} + 16*(715*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^7 + \\
& 1001*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^5 + 385*(a^3 + 3*a^2*b + \\
& 3*a*b^2 + b^3)*\cosh(d*x + c)^3 + 35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d \\
& *x + c))*\sinh(d*x + c)^9 + 70*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c) \\
& ^8 + 2*(6435*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^8 + 12012*(a^3 + \\
& 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^6 + 6930*(a^3 + 3*a^2*b + 3*a*b^2 + \\
& b^3)*\cosh(d*x + c)^4 + 35*a^3 + 105*a^2*b + 105*a*b^2 + 35*b^3 + 1260*(a^3 \\
& + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 16*(715*(a^3 \\
& + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^9 + 1716*(a^3 + 3*a^2*b + 3*a*b^2 \\
& + b^3)*\cosh(d*x + c)^7 + 1386*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c \\
&)^5 + 420*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^3 + 35*(a^3 + 3*a^2 \\
& *b + 3*a*b^2 + b^3)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 56*(a^3 + 3*a^2*b + 3* \\
& a*b^2 + b^3)*\cosh(d*x + c)^6 + 56*(143*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh \\
& (d*x + c)^{10} + 429*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^8 + 462*(a \\
& ^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^6 + 210*(a^3 + 3*a^2*b + 3*a*b^ \\
& 2 + b^3)*\cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 35*(a^3 + 3*a^2* \\
& b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 112*(39*(a^3 + 3*a^2* \\
& b + 3*a*b^2 + b^3)*\cosh(d*x + c)^{11} + 143*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c \\
& osh(d*x + c)^9 + 198*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^7 + 126* \\
& (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^5 + 35*(a^3 + 3*a^2*b + 3*a*b \\
& ^2 + b^3)*\cosh(d*x + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c) \\
&)*\sinh(d*x + c)^5 + 28*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^4 + 28 \\
& *(65*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^{12} + 286*(a^3 + 3*a^2*b \\
& + 3*a*b^2 + b^3)*\cosh(d*x + c)^{10} + 495*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos \\
& h(d*x + c)^8 + 420*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^6 + 175*(a \\
& ^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b \\
& ^3 + 30*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + \\
& 112*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^{13} + 26*(a^3 + 3*a^2*b \\
& + 3*a*b^2 + b^3)*\cosh(d*x + c)^{11} + 55*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos \\
& h(d*x + c)^9 + 60*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^7 + 35*(a^3 \\
& + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^5 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + \\
& b^3)*\cosh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c))*\sinh \\
& (d*x + c)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 8*(a^3 + 3*a^2*b + 3*a*b^2 + \\
& b^3)*\cosh(d*x + c)^2 + 8*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^ \\
& 14 + 91*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^{12} + 231*(a^3 + 3*a^2 \\
& *b + 3*a*b^2 + b^3)*\cosh(d*x + c)^{10} + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)* \\
& cosh(d*x + c)^8 + 245*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(d*x + c)^6 + 105
\end{aligned}$$

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*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2
+ b^3 + 21*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 16*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^15 + 7*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*cosh(d*x + c)^13 + 21*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh
(d*x + c)^11 + 35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 35*(a^3
+ 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 21*(a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*cosh(d*x + c)^5 + 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 +
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d
*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(12*(a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*d*x*cosh(d*x + c)^15 - 21*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4*(a^3 +
3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^13 - 36*(3*a^3 + 15*a^2*b + 18
*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^11 -
5*(45*a^3 + 198*a^2*b + 237*a*b^2 + 100*b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*d*x)*cosh(d*x + c)^9 - 4*(60*a^3 + 252*a^2*b + 312*a*b^2 + 104*b^3 -
105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 - 3*(45*a^3 + 198*
a^2*b + 237*a*b^2 + 100*b^3 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(
d*x + c)^5 - 12*(3*a^3 + 15*a^2*b + 18*a*b^2 + 6*b^3 - 7*(a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - 3*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3 - 4
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh
(d*x + c)^16 + 16*d*cosh(d*x + c)*sinh(d*x + c)^15 + d*sinh(d*x + c)^16 + 8
*d*cosh(d*x + c)^14 + 8*(15*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^14 + 112*(
5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^13 + 28*d*cosh(d*x + c
)^12 + 28*(65*d*cosh(d*x + c)^4 + 26*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^1
2 + 112*(39*d*cosh(d*x + c)^5 + 26*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*s
inh(d*x + c)^11 + 56*d*cosh(d*x + c)^10 + 56*(143*d*cosh(d*x + c)^6 + 143*d
*cosh(d*x + c)^4 + 33*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^10 + 16*(715*d*c
osh(d*x + c)^7 + 1001*d*cosh(d*x + c)^5 + 385*d*cosh(d*x + c)^3 + 35*d*cosh
(d*x + c))*sinh(d*x + c)^9 + 70*d*cosh(d*x + c)^8 + 2*(6435*d*cosh(d*x + c)
^8 + 12012*d*cosh(d*x + c)^6 + 6930*d*cosh(d*x + c)^4 + 1260*d*cosh(d*x + c
)^2 + 35*d)*sinh(d*x + c)^8 + 16*(715*d*cosh(d*x + c)^9 + 1716*d*cosh(d*x +
c)^7 + 1386*d*cosh(d*x + c)^5 + 420*d*cosh(d*x + c)^3 + 35*d*cosh(d*x + c)
)*sinh(d*x + c)^7 + 56*d*cosh(d*x + c)^6 + 56*(143*d*cosh(d*x + c)^10 + 429
*d*cosh(d*x + c)^8 + 462*d*cosh(d*x + c)^6 + 210*d*cosh(d*x + c)^4 + 35*d*c
osh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 112*(39*d*cosh(d*x + c)^11 + 143*d*co
sh(d*x + c)^9 + 198*d*cosh(d*x + c)^7 + 126*d*cosh(d*x + c)^5 + 35*d*cosh(d
*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^4 + 28*
(65*d*cosh(d*x + c)^12 + 286*d*cosh(d*x + c)^10 + 495*d*cosh(d*x + c)^8 + 4
20*d*cosh(d*x + c)^6 + 175*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + d)*si
nh(d*x + c)^4 + 112*(5*d*cosh(d*x + c)^13 + 26*d*cosh(d*x + c)^11 + 55*d*co
sh(d*x + c)^9 + 60*d*cosh(d*x + c)^7 + 35*d*cosh(d*x + c)^5 + 10*d*cosh(d*x
+ c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^3 + 8*d*cosh(d*x + c)^2 + 8*(15*d*
cosh(d*x + c)^14 + 91*d*cosh(d*x + c)^12 + 231*d*cosh(d*x + c)^10 + 315*d*c
osh(d*x + c)^8 + 245*d*cosh(d*x + c)^6 + 105*d*cosh(d*x + c)^4 + 21*d*cosh(
d*x + c)^2 + d)*sinh(d*x + c)^2 + 16*(d*cosh(d*x + c)^15 + 7*d*cosh(d*x + c
)^13 + 21*d*cosh(d*x + c)^11 + 35*d*cosh(d*x + c)^9 + 35*d*cosh(d*x + c)^7
+ 21*d*cosh(d*x + c)^5 + 7*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x +
c) + d)

```

giac [B] time = 0.47, size = 309, normalized size = 2.89

$$3(a^3 + 3a^2b + 3ab^2 + b^3)(dx + c) - 3(a^3 + 3a^2b + 3ab^2 + b^3) \log(e^{(2dx+2c)} + 1) - \frac{2(3(a^3+6a^2b+9ab^2+4b^3)e^{(14dx+14c)} + 1)}{e^{(14dx+14c)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c))^2)^3,x, algorithm="giac")

[Out] -1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(e^(2*d*x + 2*c) + 1) - 2*(3*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)

) $e^{(14dx + 14c)} + 18(a^3 + 5a^2b + 6ab^2 + 2b^3)e^{(12dx + 12c)}$
 $+ (45a^3 + 198a^2b + 237ab^2 + 100b^3)e^{(10dx + 10c)} + 4(15a^3 + 63a^2b + 78ab^2 + 26b^3)e^{(8dx + 8c)} + (45a^3 + 198a^2b + 237ab^2 + 100b^3)e^{(6dx + 6c)} + 18(a^3 + 5a^2b + 6ab^2 + 2b^3)e^{(4dx + 4c)} + 3(a^3 + 6a^2b + 9ab^2 + 4b^3)e^{(2dx + 2c)}/(e^{(2dx + 2c)} + 1)^8/d$

maple [B] time = 0.02, size = 307, normalized size = 2.87

$$\frac{\ln(1 + \tanh(dx + c))a^3}{2d} - \frac{3\ln(1 + \tanh(dx + c))a^2b}{2d} - \frac{3\ln(1 + \tanh(dx + c))ab^2}{2d} - \frac{\ln(1 + \tanh(dx + c))b^3}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\tanh(dx+c)^3(a+b*\tanh(dx+c))^2, x)$

[Out] $-1/2/d*\ln(1+\tanh(dx+c))*a^3-3/2/d*\ln(1+\tanh(dx+c))*a^2*b-3/2/d*\ln(1+\tanh(dx+c))*a*b^2-1/2/d*\ln(1+\tanh(dx+c))*b^3-1/2/d*a^3*\tanh(dx+c)^2-1/8*b^3*\tanh(dx+c)^8/d-1/2/d*\ln(\tanh(dx+c)-1)*a^3-3/2/d*\ln(\tanh(dx+c)-1)*a^2*b-3/2/d*\ln(\tanh(dx+c)-1)*a*b^2-1/2/d*\ln(\tanh(dx+c)-1)*b^3-1/6*b^3*\tanh(dx+c)^6/d-1/4*b^3*\tanh(dx+c)^4/d-1/2*b^3*\tanh(dx+c)^2/d-3/4/d*\tanh(dx+c)^4*a^2*b-3/4/d*\tanh(dx+c)^4*a*b^2-3/2*a^2*b*\tanh(dx+c)^2/d-3/2/d*\tanh(dx+c)^2*a*b^2-1/2/d*\tanh(dx+c)^6*a*b^2$

maxima [B] time = 0.45, size = 540, normalized size = 5.05

$$ab^2\left(3x + \frac{3c}{d} + \frac{3 \log(e^{(-2dx-2c)} + 1)}{d} + \frac{2(9e^{(-2dx-2c)} + 18e^{(-4dx-4c)} + 34e^{(-6dx-6c)} + 18e^{(-8dx-8c)} + 6e^{(-10dx-10c)})}{d(6e^{(-2dx-2c)} + 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} + 15e^{(-8dx-8c)} + 6e^{(-10dx-10c)})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\tanh(dx+c)^3(a+b*\tanh(dx+c))^2, x, \text{algorithm}="maxima")$

[Out] $a*b^2*(3*x + 3*c/d + 3*\log(e^{(-2*d*x - 2*c)} + 1)/d + 2*(9*e^{(-2*d*x - 2*c)} + 18*e^{(-4*d*x - 4*c)} + 34*e^{(-6*d*x - 6*c)} + 18*e^{(-8*d*x - 8*c)} + 9*e^{(-10*d*x - 10*c)})/(d*(6*e^{(-2*d*x - 2*c)} + 15*e^{(-4*d*x - 4*c)} + 20*e^{(-6*d*x - 6*c)} + 15*e^{(-8*d*x - 8*c)} + 6*e^{(-10*d*x - 10*c)} + e^{(-12*d*x - 12*c)} + 1))) + 1/3*b^3*(3*x + 3*c/d + 3*\log(e^{(-2*d*x - 2*c)} + 1)/d + 8*(3*e^{(-2*d*x - 2*c)} + 9*e^{(-4*d*x - 4*c)} + 25*e^{(-6*d*x - 6*c)} + 26*e^{(-8*d*x - 8*c)} + 25*e^{(-10*d*x - 10*c)} + 9*e^{(-12*d*x - 12*c)} + 3*e^{(-14*d*x - 14*c)})/(d*(8*e^{(-2*d*x - 2*c)} + 28*e^{(-4*d*x - 4*c)} + 56*e^{(-6*d*x - 6*c)} + 70*e^{(-8*d*x - 8*c)} + 56*e^{(-10*d*x - 10*c)} + 28*e^{(-12*d*x - 12*c)} + 8*e^{(-14*d*x - 14*c)} + e^{(-16*d*x - 16*c)} + 1))) + 3*a^2*b*(x + c/d + \log(e^{(-2*d*x - 2*c)} + 1)/d + 4*(e^{(-2*d*x - 2*c)} + e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)})/(d*(4*e^{(-2*d*x - 2*c)} + 6*e^{(-4*d*x - 4*c)} + 4*e^{(-6*d*x - 6*c)} + e^{(-8*d*x - 8*c)} + 1))) + a^3*(x + c/d + \log(e^{(-2*d*x - 2*c)} + 1)/d + 2*e^{(-2*d*x - 2*c)}/(d*(2*e^{(-2*d*x - 2*c)} + e^{(-4*d*x - 4*c)} + 1)))$

mupad [B] time = 1.24, size = 155, normalized size = 1.45

$$x(a^3 + 3a^2b + 3ab^2 + b^3) - \frac{\tanh(c + dx)^4(3a^2b + 3ab^2 + b^3)}{4d} - \frac{\ln(\tanh(c + dx) + 1)(a^3 + 3a^2b + 3ab^2 + b^3)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\tanh(c + d*x)^3(a + b*\tanh(c + d*x))^2, x)$

[Out] $x*(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (\tanh(c + d*x)^4*(3*a*b^2 + 3*a^2*b + b^3))/(4*d) - (\log(\tanh(c + d*x) + 1)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/d - (\tanh(c + d*x)^6*(3*a*b^2 + b^3))/(6*d) - (b^3*\tanh(c + d*x)^8)/(8*d) - (\tanh(c + d*x)^2*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(2*d)$

sympy [A] time = 1.68, size = 279, normalized size = 2.61

$$\begin{cases} a^3x - \frac{a^3 \log(\tanh(c+dx)+1)}{d} - \frac{a^3 \tanh^2(c+dx)}{2d} + 3a^2bx - \frac{3a^2b \log(\tanh(c+dx)+1)}{d} - \frac{3a^2b \tanh^4(c+dx)}{4d} - \frac{3a^2b \tanh^2(c+dx)}{2d} + 3ab^2x \\ x(a + b \tanh^2(c))^3 \tanh^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)

[Out] Piecewise((a**3*x - a**3*log(tanh(c + d*x) + 1)/d - a**3*tanh(c + d*x)**2/(2*d) + 3*a**2*b*x - 3*a**2*b*log(tanh(c + d*x) + 1)/d - 3*a**2*b*tanh(c + d*x)**4/(4*d) - 3*a**2*b*tanh(c + d*x)**2/(2*d) + 3*a*b**2*x - 3*a*b**2*log(tanh(c + d*x) + 1)/d - a*b**2*tanh(c + d*x)**6/(2*d) - 3*a*b**2*tanh(c + d*x)**4/(4*d) - 3*a*b**2*tanh(c + d*x)**2/(2*d) + b**3*x - b**3*log(tanh(c + d*x) + 1)/d - b**3*tanh(c + d*x)**8/(8*d) - b**3*tanh(c + d*x)**6/(6*d) - b**3*tanh(c + d*x)**4/(4*d) - b**3*tanh(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)**3*tanh(c)**3, True))

3.158 $\int \tanh^2(c + dx) \left(a + b \tanh^2(c + dx) \right)^3 dx$

Optimal. Leaf size=94

$$\frac{b(3a^2 + 3ab + b^2) \tanh^3(c + dx)}{3d} - \frac{b^2(3a + b) \tanh^5(c + dx)}{5d} - \frac{(a + b)^3 \tanh(c + dx)}{d} + x(a + b)^3 - \frac{b^3 \tanh^7(c + dx)}{7d}$$

[Out] $(a+b)^3 x - (a+b)^3 \tanh(dx+c)/d - 1/3 b (3a^2 + 3ab + b^2) \tanh(dx+c)^3/d - 1/5 b^2 (3a+b) \tanh(dx+c)^5/d - 1/7 b^3 \tanh(dx+c)^7/d$

Rubi [A] time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 461, 206}

$$\frac{b(3a^2 + 3ab + b^2) \tanh^3(c + dx)}{3d} - \frac{b^2(3a + b) \tanh^5(c + dx)}{5d} - \frac{(a + b)^3 \tanh(c + dx)}{d} + x(a + b)^3 - \frac{b^3 \tanh^7(c + dx)}{7d}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] $(a + b)^3 x - ((a + b)^3 \tanh[c + d*x])/d - (b(3a^2 + 3ab + b^2) \tanh[c + d*x]^3)/(3*d) - (b^2(3a + b) \tanh[c + d*x]^5)/(5*d) - (b^3 \tanh[c + d*x]^7)/(7*d)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 461

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
\int \tanh^2(c + dx) (a + b \tanh^2(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{x^2(a+bx^2)^3}{1-x^2} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(- (a+b)^3 - b(3a^2 + 3ab + b^2)x^2 - b^2(3a+b)x^4 - b^3x^6\right) dx, x, \tanh(c + dx)\right)}{d} \\
&= -\frac{(a+b)^3 \tanh(c + dx)}{d} - \frac{b(3a^2 + 3ab + b^2) \tanh^3(c + dx)}{3d} - \frac{b^2(3a+b) \tanh^5(c + dx)}{5d} - \frac{b^3 \tanh^7(c + dx)}{7d} \\
&= (a+b)^3 x - \frac{(a+b)^3 \tanh(c + dx)}{d} - \frac{b(3a^2 + 3ab + b^2) \tanh^3(c + dx)}{3d} - \frac{b^2(3a+b) \tanh^5(c + dx)}{5d} - \frac{b^3 \tanh^7(c + dx)}{7d}
\end{aligned}$$

Mathematica [A] time = 1.72, size = 108, normalized size = 1.15

$$\frac{\tanh(c + dx) \left(-35b(3a^2 + 3ab + b^2) \tanh^2(c + dx) - 21b^2(3a + b) \tanh^4(c + dx) + \frac{105(a+b)^3 \tanh^{-1}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}} \right)}{105d}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (Tanh[c + d*x]*(-105*(a + b)^3 - 35*b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^2 - 21*b^2*(3*a + b)*Tanh[c + d*x]^4 - 15*b^3*Tanh[c + d*x]^6 + (105*(a + b)^3*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c + d*x]^2]))/(105*d)

fricas [B] time = 0.43, size = 1036, normalized size = 11.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/105*((105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 + 7*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^6 - (105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3)*sinh(d*x + c)^7 + 7*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 7*(75*a^3 + 240*a^2*b + 213*a*b^2 + 56*b^3 + 3*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 35*((105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + (105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^4 + 21*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - 7*(5*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3)*cosh(d*x + c)^4 + 135*a^3 + 360*a^2*b + 369*a*b^2 + 168*b^3 + 10*(75*a^3 + 240*a^2*b + 213*a*b^2 + 56*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 7*(3*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 + 10*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + 9*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c) - 7*((105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3)*cosh(d*x + c)^6 + 5*(75*a^3 + 240*a^2*b + 213*a*b^2 + 56*b^3)*cosh(d*x + c)^4 + 75*a^3 + 180*a^2*b + 225*a*b^2

$$+ 9*(45*a^3 + 120*a^2*b + 123*a*b^2 + 56*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c) / (d*\cosh(d*x + c)^7 + 7*d*\cosh(d*x + c)*\sinh(d*x + c)^6 + 7*d*\cosh(d*x + c)^5 + 35*(d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c)^4 + 21*d*\cosh(d*x + c)^3 + 7*(3*d*\cosh(d*x + c)^5 + 10*d*\cosh(d*x + c)^3 + 9*d*\cosh(d*x + c))*\sinh(d*x + c)^2 + 35*d*\cosh(d*x + c))$$

giac [B] time = 0.39, size = 418, normalized size = 4.45

$$105(a^3 + 3a^2b + 3ab^2 + b^3)(dx + c) + \frac{2(105a^3e^{(12dx+12c)} + 630a^2be^{(12dx+12c)} + 945ab^2e^{(12dx+12c)} + 420b^3e^{(12dx+12c)} + 630a^3e^{(10dx+10c)} + 3150a^2b^2e^{(10dx+10c)} + 3780a^2b^2e^{(10dx+10c)} + 1260b^3e^{(10dx+10c)} + 1575a^3e^{(8dx+8c)} + 6720a^2b^2e^{(8dx+8c)} + 7665a^2b^2e^{(8dx+8c)} + 3080b^3e^{(8dx+8c)} + 2100a^3e^{(6dx+6c)} + 7980a^2b^2e^{(6dx+6c)} + 9240a^2b^2e^{(6dx+6c)} + 3080b^3e^{(6dx+6c)} + 1575a^3e^{(4dx+4c)} + 5670a^2b^2e^{(4dx+4c)} + 6363a^2b^2e^{(4dx+4c)} + 2436b^3e^{(4dx+4c)} + 630a^3e^{(2dx+2c)} + 2310a^2b^2e^{(2dx+2c)} + 2436a^2b^2e^{(2dx+2c)} + 812b^3e^{(2dx+2c)} + 105a^3 + 420a^2b + 483a^2b^2 + 176b^3)/(e^{(2dx+2c)} + 1)^7/d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/105*(105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) + 2*(105*a^3*e^(12*d*x + 12*c) + 630*a^2*b*e^(12*d*x + 12*c) + 945*a*b^2*e^(12*d*x + 12*c) + 420*b^3*e^(12*d*x + 12*c) + 630*a^3*e^(10*d*x + 10*c) + 3150*a^2*b*e^(10*d*x + 10*c) + 3780*a*b^2*e^(10*d*x + 10*c) + 1260*b^3*e^(10*d*x + 10*c) + 1575*a^3*e^(8*d*x + 8*c) + 6720*a^2*b*e^(8*d*x + 8*c) + 7665*a*b^2*e^(8*d*x + 8*c) + 3080*b^3*e^(8*d*x + 8*c) + 2100*a^3*e^(6*d*x + 6*c) + 7980*a^2*b*e^(6*d*x + 6*c) + 9240*a*b^2*e^(6*d*x + 6*c) + 3080*b^3*e^(6*d*x + 6*c) + 1575*a^3*e^(4*d*x + 4*c) + 5670*a^2*b*e^(4*d*x + 4*c) + 6363*a*b^2*e^(4*d*x + 4*c) + 2436*b^3*e^(4*d*x + 4*c) + 630*a^3*e^(2*d*x + 2*c) + 2310*a^2*b*e^(2*d*x + 2*c) + 2436*a*b^2*e^(2*d*x + 2*c) + 812*b^3*e^(2*d*x + 2*c) + 105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3)/(e^(2*d*x + 2*c) + 1)^7/d

maple [B] time = 0.02, size = 299, normalized size = 3.18

$$\frac{\ln(1 + \tanh(dx + c))a^3}{2d} + \frac{3 \ln(1 + \tanh(dx + c))a^2b}{2d} + \frac{3 \ln(1 + \tanh(dx + c))ab^2}{2d} + \frac{\ln(1 + \tanh(dx + c))b^3}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x)

[Out] 1/2/d*ln(1+tanh(d*x+c))*a^3+3/2/d*ln(1+tanh(d*x+c))*a^2*b+3/2/d*ln(1+tanh(d*x+c))*a*b^2+1/2/d*ln(1+tanh(d*x+c))*b^3-b^3*tanh(d*x+c)/d-a^3*tanh(d*x+c)/d-1/2/d*ln(tanh(d*x+c)-1)*a^3-3/2/d*ln(tanh(d*x+c)-1)*a^2*b-3/2/d*ln(tanh(d*x+c)-1)*a*b^2-1/2/d*ln(tanh(d*x+c)-1)*b^3-1/7*b^3*tanh(d*x+c)^7/d-1/5*b^3*tanh(d*x+c)^5/d-1/3*b^3*tanh(d*x+c)^3/d-3/5*a*b^2*tanh(d*x+c)^5/d-a^2*b*tanh(d*x+c)^3/d-a*b^2*tanh(d*x+c)^3/d-3*a^2*b*tanh(d*x+c)/d-3*a*b^2*tanh(d*x+c)/d

maxima [B] time = 0.35, size = 400, normalized size = 4.26

$$\frac{1}{105}b^3\left(105x + \frac{105c}{d} - \frac{8(203e^{(-2dx-2c)} + 609e^{(-4dx-4c)} + 770e^{(-6dx-6c)} + 770e^{(-8dx-8c)} + 315e^{(-10dx-10c)} + 7e^{(-12dx-12c)})}{d(7e^{(-2dx-2c)} + 21e^{(-4dx-4c)} + 35e^{(-6dx-6c)} + 35e^{(-8dx-8c)} + 21e^{(-10dx-10c)} + 7e^{(-12dx-12c)})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 1/105*b^3*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*e^(-4*d*x - 4*c) + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-10*d*x - 10*c) + 105*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 1/5*a*b^2*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 10*e^(-8*d*x - 8*c) + 5)))

$x - 6*c) + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1))) + a^2*b*(3*x + 3*c/d - 4*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + 2)/(d*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} + 1))) + a^3*(x + c/d - 2/(d*(e^{(-2*d*x - 2*c)} + 1)))$

mupad [B] time = 1.22, size = 106, normalized size = 1.13

$$x \left(a^3 + 3 a^2 b + 3 a b^2 + b^3 \right) - \frac{\tanh(c + dx) (a + b)^3}{d} - \frac{\tanh(c + dx)^3 (3 a^2 b + 3 a b^2 + b^3)}{3 d} - \frac{\tanh(c + dx)^5 (b^3 + a^3)}{5 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)^2*(a + b*tanh(c + d*x))^3,x)

[Out] $x*(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (\tanh(c + d*x)*(a + b)^3)/d - (\tanh(c + d*x)^3*(3*a*b^2 + 3*a^2*b + b^3))/(3*d) - (\tanh(c + d*x)^5*(3*a*b^2 + b^3))/(5*d) - (b^3*tanh(c + d*x)^7)/(7*d)$

sympy [A] time = 1.20, size = 192, normalized size = 2.04

$$\left\{ \begin{array}{l} a^3 x - \frac{a^3 \tanh(c+dx)}{d} + 3a^2 b x - \frac{a^2 b \tanh^3(c+dx)}{d} - \frac{3a^2 b \tanh(c+dx)}{d} + 3ab^2 x - \frac{3ab^2 \tanh^5(c+dx)}{5d} - \frac{ab^2 \tanh^3(c+dx)}{d} - \frac{3ab^2 \tanh(c+dx)}{d} \\ x (a + b \tanh^2(c))^3 \tanh^2(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)

[Out] Piecewise((a**3*x - a**3*tanh(c + d*x)/d + 3*a**2*b*x - a**2*b*tanh(c + d*x)**3/d - 3*a**2*b*tanh(c + d*x)/d + 3*a*b**2*x - 3*a*b**2*tanh(c + d*x)**5/(5*d) - a*b**2*tanh(c + d*x)**3/d - 3*a*b**2*tanh(c + d*x)/d + b**3*x - b**3*tanh(c + d*x)**7/(7*d) - b**3*tanh(c + d*x)**5/(5*d) - b**3*tanh(c + d*x)**3/(3*d) - b**3*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**3*tanh(c)**2, True))

3.159 $\int \tanh(c + dx) (a + b \tanh^2(c + dx))^3 dx$

Optimal. Leaf size=83

$$\frac{b(a+b)^2 \tanh^2(c+dx)}{2d} - \frac{(a+b)(a+b \tanh^2(c+dx))^2}{4d} - \frac{(a+b \tanh^2(c+dx))^3}{6d} + \frac{(a+b)^3 \log(\cosh(c+dx))}{d}$$

[Out] (a+b)^3*ln(cosh(d*x+c))/d-1/2*b*(a+b)^2*tanh(d*x+c)^2/d-1/4*(a+b)*(a+b*tanh(d*x+c)^2)^2/d-1/6*(a+b*tanh(d*x+c)^2)^3/d

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3670, 444, 43}

$$\frac{b(a+b)^2 \tanh^2(c+dx)}{2d} - \frac{(a+b)(a+b \tanh^2(c+dx))^2}{4d} - \frac{(a+b \tanh^2(c+dx))^3}{6d} + \frac{(a+b)^3 \log(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] ((a + b)^3*Log[Cosh[c + d*x]])/d - (b*(a + b)^2*Tanh[c + d*x]^2)/(2*d) - ((a + b)*(a + b*Tanh[c + d*x]^2)^2)/(4*d) - (a + b*Tanh[c + d*x]^2)^3/(6*d)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
\int \tanh(c + dx) (a + b \tanh^2(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{x(a+bx^2)^3}{1-x^2} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{1-x} dx, x, \tanh^2(c + dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-b(a+b)^2 + \frac{(a+b)^3}{1-x} - b(a+b)(a+bx) - b(a+bx)^2\right) dx, x\right)}{2d} \\
&= \frac{(a+b)^3 \log(\cosh(c + dx))}{d} - \frac{b(a+b)^2 \tanh^2(c + dx)}{2d} - \frac{(a+b)(a+b \tanh^2(c + dx))^2}{2d}
\end{aligned}$$

Mathematica [A] time = 0.25, size = 76, normalized size = 0.92

$$\frac{b(a+b)^2 \tanh^2(c + dx) + \frac{1}{2}(a+b)(a+b \tanh^2(c + dx))^2 + \frac{1}{3}(a+b \tanh^2(c + dx))^3 - 2(a+b)^3 \log(\cosh(c + dx))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] -1/2*(-2*(a + b)^3*Log[Cosh[c + d*x]] + b*(a + b)^2*Tanh[c + d*x]^2 + ((a + b)*(a + b*Tanh[c + d*x]^2)^2)/2 + (a + b*Tanh[c + d*x]^2)^3/3)/d

fricas [B] time = 0.49, size = 4298, normalized size = 51.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^12 + 36*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^11 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*sinh(d*x + c)^12 - 18*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^10 + 18*(11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 - a^2*b - 2*a*b^2 - b^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*sinh(d*x + c)^10 + 60*(11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^3 - 3*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 - 9*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^8 + 9*(165*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^4 - 8*a^2*b - 12*a*b^2 - 4*b^3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 90*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 72*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^5 - 30*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - (8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(27*a^2*b + 36*a*b^2 + 17*b^3 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 + 4*(693*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^6 - 945*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 27*a^2*b - 36*a*b^2 - 17*b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 63*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 24*(99*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^7 - 189*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 21*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - (27*a^2*b + 36*a*b^2 + 17*b^3 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)

$$\begin{aligned}
&)) * \sinh(dx + c)^5 - 9*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)^4 + 3*(495*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx * \cosh(dx + c)^8 - 1260*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)^6 - 210*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)^4 - 24*a^2*b - 36*a*b^2 - 12*b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx - 20*(27*a^2*b + 36*a*b^2 + 17*b^3 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)^2) * \sinh(dx + c)^4 + 4*(165*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx * \cosh(dx + c)^9 - 540*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)^7 - 126*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)^5 - 20*(27*a^2*b + 36*a*b^2 + 17*b^3 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)^3 - 9*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)) * \sinh(dx + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx - 18*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)^2 + 6*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx * \cosh(dx + c)^10 - 135*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)^8 - 42*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)^6 - 10*(27*a^2*b + 36*a*b^2 + 17*b^3 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)^4 - 3*a^2*b - 6*a*b^2 - 3*b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx - 9*(8*a^2*b + 12*a*b^2 + 4*b^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)^2) * \sinh(dx + c)^2 - 3*((a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^12 + 12*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c) * \sinh(dx + c)^11 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \sinh(dx + c)^12 + 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^10 + 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^2) * \sinh(dx + c)^10 + 20*(11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)) * \sinh(dx + c)^9 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^8 + 15*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 18*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^2) * \sinh(dx + c)^8 + 24*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^5 + 30*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)) * \sinh(dx + c)^7 + 20*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^6 + 4*(231*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^6 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^4 + 5*a^3 + 15*a^2*b + 15*a*b^2 + 5*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^2) * \sinh(dx + c)^6 + 24*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^7 + 63*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^5 + 35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)) * \sinh(dx + c)^5 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^4 + 15*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^8 + 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^6 + 70*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 20*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^2) * \sinh(dx + c)^4 + 20*(11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^9 + 36*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^7 + 42*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^5 + 20*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)) * \sinh(dx + c)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^2 + 6*(11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^10 + 45*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^8 + 70*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^6 + 50*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^2) * \sinh(dx + c)^2 + 12*((a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^11 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^9 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^7 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)) * \sinh(dx + c)) * \log(2 * \cosh(dx + c) / (\cosh(dx + c) - \sinh(dx + c))) + 12*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * dx * \cosh(dx + c)^11 - 15*(a^2*b + 2*a*b^2 + b^3 - (a^3 + 3*a^2*b +
\end{aligned}$$

$3a^3b^2 + b^3)d*x)*\cosh(d*x + c)^9 - 6*(8a^2b + 12a*b^2 + 4b^3 - 5*(a^3 + 3a^2b + 3a*b^2 + b^3)d*x)*\cosh(d*x + c)^7 - 2*(27a^2b + 36a*b^2 + 17b^3 - 15*(a^3 + 3a^2b + 3a*b^2 + b^3)d*x)*\cosh(d*x + c)^5 - 3*(8a^2b + 12a*b^2 + 4b^3 - 5*(a^3 + 3a^2b + 3a*b^2 + b^3)d*x)*\cosh(d*x + c)^3 - 3*(a^2b + 2a*b^2 + b^3 - (a^3 + 3a^2b + 3a*b^2 + b^3)d*x)*\cosh(d*x + c))*\sinh(d*x + c))/(d*\cosh(d*x + c)^12 + 12*d*\cosh(d*x + c)*\sinh(d*x + c)^11 + d*\sinh(d*x + c)^12 + 6*d*\cosh(d*x + c)^10 + 6*(11*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^10 + 20*(11*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^9 + 15*d*\cosh(d*x + c)^8 + 15*(33*d*\cosh(d*x + c)^4 + 18*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^8 + 24*(33*d*\cosh(d*x + c)^5 + 30*d*\cosh(d*x + c)^3 + 5*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 20*d*\cosh(d*x + c)^6 + 4*(231*d*\cosh(d*x + c)^6 + 315*d*\cosh(d*x + c)^4 + 105*d*\cosh(d*x + c)^2 + 5*d)*\sinh(d*x + c)^6 + 24*(33*d*\cosh(d*x + c)^7 + 63*d*\cosh(d*x + c)^5 + 35*d*\cosh(d*x + c)^3 + 5*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 15*d*\cosh(d*x + c)^4 + 15*(33*d*\cosh(d*x + c)^8 + 84*d*\cosh(d*x + c)^6 + 70*d*\cosh(d*x + c)^4 + 20*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^4 + 20*(11*d*\cosh(d*x + c)^9 + 36*d*\cosh(d*x + c)^7 + 42*d*\cosh(d*x + c)^5 + 20*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 6*d*\cosh(d*x + c)^2 + 6*(11*d*\cosh(d*x + c)^10 + 45*d*\cosh(d*x + c)^8 + 70*d*\cosh(d*x + c)^6 + 50*d*\cosh(d*x + c)^4 + 15*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^2 + 12*(d*\cosh(d*x + c)^11 + 5*d*\cosh(d*x + c)^9 + 10*d*\cosh(d*x + c)^7 + 10*d*\cosh(d*x + c)^5 + 5*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c) + d)$

giac [B] time = 0.35, size = 216, normalized size = 2.60

$$3(a^3 + 3a^2b + 3ab^2 + b^3)(dx + c) - 3(a^3 + 3a^2b + 3ab^2 + b^3) \log(e^{(2dx+2c)} + 1) - \frac{2(9(a^2b+2ab^2+b^3)e^{(10dx+10c)}+18a^2b^2+17b^3)e^{(6dx+6c)}+18(2a^2b+3ab^2+b^3)e^{(4dx+4c)}+9(a^2b+2ab^2+b^3)e^{(2dx+2c)})}{(e^{(2dx+2c)} + 1)^6} / d$$

3d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(dx+c)*(a+b*tanh(dx+c)^2)^3,x, algorithm="giac")

[Out] $-1/3*(3*(a^3 + 3a^2b + 3a*b^2 + b^3)*(dx + c) - 3*(a^3 + 3a^2b + 3a*b^2 + b^3)*\log(e^{(2*d*x + 2*c)} + 1) - 2*(9*(a^2*b + 2*a*b^2 + b^3)*e^{(10*d*x + 10*c)} + 18*(2*a^2*b + 3*a*b^2 + b^3)*e^{(8*d*x + 8*c)} + 2*(27*a^2*b + 36*a*b^2 + 17*b^3)*e^{(6*d*x + 6*c)} + 18*(2*a^2*b + 3*a*b^2 + b^3)*e^{(4*d*x + 4*c)} + 9*(a^2*b + 2*a*b^2 + b^3)*e^{(2*d*x + 2*c)})/(e^{(2*d*x + 2*c)} + 1)^6)/d$

maple [B] time = 0.02, size = 241, normalized size = 2.90

$$\frac{\ln(1 + \tanh(dx + c)) a^3}{2d} - \frac{3 \ln(1 + \tanh(dx + c)) a^2 b}{2d} - \frac{3 \ln(1 + \tanh(dx + c)) a b^2}{2d} - \frac{\ln(1 + \tanh(dx + c)) b^3}{2d} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(dx+c)*(a+b*tanh(dx+c)^2)^3,x)

[Out] $-1/2/d*\ln(1+\tanh(dx+c))*a^3-3/2/d*\ln(1+\tanh(dx+c))*a^2b-3/2/d*\ln(1+\tanh(dx+c))*ab^2-1/2/d*\ln(1+\tanh(dx+c))*b^3-1/2/d*\ln(\tanh(dx+c)-1)*a^3-3/2/d*\ln(\tanh(dx+c)-1)*a^2b-3/2/d*\ln(\tanh(dx+c)-1)*ab^2-1/2/d*\ln(\tanh(dx+c)-1)*b^3-1/6*b^3*\tanh(dx+c)^6/d-1/4*b^3*\tanh(dx+c)^4/d-1/2*b^3*\tanh(dx+c)^2/d-3/4/d*\tanh(dx+c)^4*a*b^2-3/2*a^2*b*\tanh(dx+c)^2/d-3/2/d*\tanh(dx+c)^2*a*b^2$

maxima [B] time = 0.44, size = 351, normalized size = 4.23

$$\frac{1}{3}b^3\left(3x + \frac{3c}{d} + \frac{3 \log(e^{(-2dx-2c)} + 1)}{d} + \frac{2(9e^{(-2dx-2c)} + 18e^{(-4dx-4c)} + 34e^{(-6dx-6c)} + 18e^{(-8dx-8c)} + 9e^{(-10dx-10c)})}{d(6e^{(-2dx-2c)} + 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} + 15e^{(-8dx-8c)} + 6e^{(-10dx-10c)})}\right) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $\frac{1}{3}b^3(3x + 3c/d + 3\log(e^{-2dx-2c} + 1)/d + 2(9e^{-2dx-2c} + 18e^{-4dx-4c} + 34e^{-6dx-6c} + 18e^{-8dx-8c} + 9e^{-10dx-10c}))/d + 2(9e^{-2dx-2c} + 18e^{-4dx-4c} + 34e^{-6dx-6c} + 18e^{-8dx-8c} + 9e^{-10dx-10c}))/d + 20e^{-6dx-6c} + 15e^{-8dx-8c} + 6e^{-10dx-10c} + e^{-12dx-12c} + 1)) + 3ab^2(x + c/d + \log(e^{-2dx-2c} + 1)/d + 4(e^{-2dx-2c} + e^{-4dx-4c} + e^{-6dx-6c}))/d + 4(e^{-2dx-2c} + e^{-4dx-4c} + e^{-6dx-6c}))/d + 6e^{-4dx-4c} + 4e^{-6dx-6c} + e^{-8dx-8c} + 1)) + 3a^2b(x + c/d + \log(e^{-2dx-2c} + 1)/d + 2e^{-2dx-2c}/(d(2e^{-2dx-2c} + e^{-4dx-4c} + 1))) + a^3\log(\cosh(dx + c))/d$

mupad [B] time = 1.25, size = 123, normalized size = 1.48

$$x(a^3 + 3a^2b + 3ab^2 + b^3) - \frac{\tanh(c + dx)^2 (3a^2b + 3ab^2 + b^3)}{2d} - \frac{\ln(\tanh(c + dx) + 1) (a^3 + 3a^2b + 3ab^2)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)*(a + b*tanh(c + d*x)^2)^3,x)

[Out] $x(3ab^2 + 3a^2b + a^3 + b^3) - (\tanh(c + dx))^2(3ab^2 + 3a^2b + b^3)/(2d) - (\log(\tanh(c + dx) + 1)(3ab^2 + 3a^2b + a^3 + b^3))/d - (\tanh(c + dx))^4(3ab^2 + b^3)/(4d) - (b^3\tanh(c + dx)^6)/(6d)$

sympy [A] time = 0.97, size = 211, normalized size = 2.54

$$\left\{ \begin{array}{l} a^3x - \frac{a^3 \log(\tanh(c+dx)+1)}{d} + 3a^2bx - \frac{3a^2b \log(\tanh(c+dx)+1)}{d} - \frac{3a^2b \tanh^2(c+dx)}{2d} + 3ab^2x - \frac{3ab^2 \log(\tanh(c+dx)+1)}{d} - \frac{3ab^2}{d} \\ x(a + b \tanh^2(c))^3 \tanh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)

[Out] Piecewise((a**3*x - a**3*log(tanh(c + d*x) + 1)/d + 3*a**2*b*x - 3*a**2*b*log(tanh(c + d*x) + 1)/d - 3*a**2*b*tanh(c + d*x)**2/(2*d) + 3*a*b**2*x - 3*a*b**2*log(tanh(c + d*x) + 1)/d - 3*a*b**2*tanh(c + d*x)**4/(4*d) - 3*a*b**2*tanh(c + d*x)**2/(2*d) + b**3*x - b**3*log(tanh(c + d*x) + 1)/d - b**3*tanh(c + d*x)**6/(6*d) - b**3*tanh(c + d*x)**4/(4*d) - b**3*tanh(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)**3*tanh(c), True))

3.160 $\int (a + b \tanh^2(c + dx))^3 dx$

Optimal. Leaf size=74

$$-\frac{b(3a^2 + 3ab + b^2) \tanh(c + dx)}{d} - \frac{b^2(3a + b) \tanh^3(c + dx)}{3d} + x(a + b)^3 - \frac{b^3 \tanh^5(c + dx)}{5d}$$

[Out] (a+b)^3*x-b*(3*a^2+3*a*b+b^2)*tanh(d*x+c)/d-1/3*b^2*(3*a+b)*tanh(d*x+c)^3/d-1/5*b^3*tanh(d*x+c)^5/d

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3661, 390, 206}

$$-\frac{b(3a^2 + 3ab + b^2) \tanh(c + dx)}{d} - \frac{b^2(3a + b) \tanh^3(c + dx)}{3d} + x(a + b)^3 - \frac{b^3 \tanh^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (a + b)^3*x - (b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x])/d - (b^2*(3*a + b)*Tanh[c + d*x]^3)/(3*d) - (b^3*Tanh[c + d*x]^5)/(5*d)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 3661

Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(ff*x)^n]^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int (a + b \tanh^2(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{1-x^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(-b(3a^2 + 3ab + b^2) - b^2(3a + b)x^2 - b^3x^4 + \frac{(a+b)^3}{1-x^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{b(3a^2 + 3ab + b^2) \tanh(c + dx)}{d} - \frac{b^2(3a + b) \tanh^3(c + dx)}{3d} - \frac{b^3 \tanh^5(c + dx)}{5d} \\ &= (a + b)^3x - \frac{b(3a^2 + 3ab + b^2) \tanh(c + dx)}{d} - \frac{b^2(3a + b) \tanh^3(c + dx)}{3d} - \frac{b^3 \tanh^5(c + dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.61, size = 95, normalized size = 1.28

$$\tanh(c + dx) \left(\frac{15(a+b)^3 \tanh^{-1}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}} - b(45a^2 + 15ab(\tanh^2(c+dx) + 3) + b^2(3 \tanh^4(c+dx) + 5 \tanh^2(c+dx))) \right)$$

15d

Antiderivative was successfully verified.

[In] Integrate[(a + b*Tanh[c + d*x]^2)^3, x]

[Out] (Tanh[c + d*x]*((15*(a + b)^3*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c + d*x]^2] - b*(45*a^2 + 15*a*b*(3 + Tanh[c + d*x]^2) + b^2*(15 + 5*Tanh[c + d*x]^2 + 3*Tanh[c + d*x]^4))))/(15*d)

fricas [B] time = 0.41, size = 567, normalized size = 7.66

$$(45 a^2 b + 60 a b^2 + 23 b^3 + 15 (a^3 + 3 a^2 b + 3 a b^2 + b^3) dx) \cosh(dx + c)^5 + 5 (45 a^2 b + 60 a b^2 + 23 b^3 + 15 (a^3 + 3 a^2 b + 3 a b^2 + b^3) dx) \sinh(dx + c)^4 - (45 a^2 b + 60 a b^2 + 23 b^3) \sinh(dx + c)^5 + 5 (45 a^2 b + 60 a b^2 + 23 b^3 + 15 (a^3 + 3 a^2 b + 3 a b^2 + b^3) dx) \cosh(dx + c)^3 - 5 (27 a^2 b + 24 a b^2 + 5 b^3 + 2 (45 a^2 b + 60 a b^2 + 23 b^3) \cosh(dx + c)^2) \sinh(dx + c)^3 + 5 (2 (45 a^2 b + 60 a b^2 + 23 b^3 + 15 (a^3 + 3 a^2 b + 3 a b^2 + b^3) dx) \cosh(dx + c)^3 + 3 (45 a^2 b + 60 a b^2 + 23 b^3 + 15 (a^3 + 3 a^2 b + 3 a b^2 + b^3) dx) \cosh(dx + c)) \sinh(dx + c)^2 + 10 (45 a^2 b + 60 a b^2 + 23 b^3 + 15 (a^3 + 3 a^2 b + 3 a b^2 + b^3) dx) \cosh(dx + c) - 5 ((45 a^2 b + 60 a b^2 + 23 b^3) \cosh(dx + c)^4 + 18 a^2 b + 12 a b^2 + 10 b^3 + 3 (27 a^2 b + 24 a b^2 + 5 b^3) \cosh(dx + c)^2) \sinh(dx + c) / (d \cosh(dx + c)^5 + 5 d \cosh(dx + c) \sinh(dx + c)^4 + 5 d \cosh(dx + c)^3 + 5 (2 d \cosh(dx + c)^3 + 3 d \cosh(dx + c)) \sinh(dx + c)^2 + 10 d \cosh(dx + c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/15*((45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 + 5*(45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^4 - (45*a^2*b + 60*a*b^2 + 23*b^3)*sinh(d*x + c)^5 + 5*(45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - 5*(27*a^2*b + 24*a*b^2 + 5*b^3 + 2*(45*a^2*b + 60*a*b^2 + 23*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 5*(2*(45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(dx + c)^3 + 3*(45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(dx + c))*sinh(dx + c)^2 + 10*(45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(dx + c) - 5*((45*a^2*b + 60*a*b^2 + 23*b^3)*cosh(dx + c)^4 + 18*a^2*b + 12*a*b^2 + 10*b^3 + 3*(27*a^2*b + 24*a*b^2 + 5*b^3)*cosh(dx + c)^2)*sinh(dx + c))/(d*cosh(dx + c)^5 + 5*d*cosh(dx + c)*sinh(dx + c)^4 + 5*d*cosh(dx + c)^3 + 5*(2*d*cosh(dx + c)^3 + 3*d*cosh(dx + c))*sinh(dx + c)^2 + 10*d*cosh(dx + c))

giac [B] time = 0.18, size = 241, normalized size = 3.26

$$15 (a^3 + 3 a^2 b + 3 a b^2 + b^3) (dx + c) + \frac{2 (45 a^2 b e^{(8 dx + 8 c)} + 90 a b^2 e^{(8 dx + 8 c)} + 45 b^3 e^{(8 dx + 8 c)} + 180 a^2 b e^{(6 dx + 6 c)} + 270 a b^2 e^{(6 dx + 6 c)} + 90 b^3 e^{(6 dx + 6 c)})}{(e^{(2 dx + 2 c)} + 1)^5} / d$$

15d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/15*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) + 2*(45*a^2*b*e^(8*d*x + 8*c) + 90*a*b^2*e^(8*d*x + 8*c) + 45*b^3*e^(8*d*x + 8*c) + 180*a^2*b*e^(6*d*x + 6*c) + 270*a*b^2*e^(6*d*x + 6*c) + 90*b^3*e^(6*d*x + 6*c) + 270*a^2*b*e^(4*d*x + 4*c) + 330*a*b^2*e^(4*d*x + 4*c) + 140*b^3*e^(4*d*x + 4*c) + 180*a^2*b*e^(2*d*x + 2*c) + 210*a*b^2*e^(2*d*x + 2*c) + 70*b^3*e^(2*d*x + 2*c) + 45*a^2*b + 60*a*b^2 + 23*b^3)/(e^(2*d*x + 2*c) + 1)^5)/d

maple [B] time = 0.02, size = 235, normalized size = 3.18

$$\frac{\ln(1 + \tanh(dx + c)) a^3}{2d} + \frac{3 \ln(1 + \tanh(dx + c)) a^2 b}{2d} + \frac{3 \ln(1 + \tanh(dx + c)) a b^2}{2d} + \frac{\ln(1 + \tanh(dx + c)) b^3}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tanh(d*x+c)^2)^3,x)`

[Out] $\frac{1}{2}d \ln(1+\tanh(dx+c))a^3 + \frac{3}{2}d \ln(1+\tanh(dx+c))a^2b + \frac{3}{2}d \ln(1+\tanh(dx+c))ab^2 + \frac{1}{2}d \ln(1+\tanh(dx+c))b^3 - \frac{b^3 \tanh(dx+c)}{d} - \frac{1}{2}d \ln(\tanh(dx+c)-1)a^3 - \frac{3}{2}d \ln(\tanh(dx+c)-1)a^2b - \frac{3}{2}d \ln(\tanh(dx+c)-1)ab^2 - \frac{1}{2}d \ln(\tanh(dx+c)-1)b^3 - \frac{1}{5}b^3 \tanh(dx+c)^5/d - \frac{1}{3}b^3 \tanh(dx+c)^3/d - ab^2 \tanh(dx+c)^3/d - 3a^2b \tanh(dx+c)/d - 3ab^2 \tanh(dx+c)/d$

maxima [B] time = 0.35, size = 239, normalized size = 3.23

$$\frac{1}{15}b^3 \left(15x + \frac{15c}{d} - \frac{2(70e^{(-2dx-2c)} + 140e^{(-4dx-4c)} + 90e^{(-6dx-6c)} + 45e^{(-8dx-8c)} + 23)}{d(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} \right) + ab^2 \left(3x + \frac{3c}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")`

[Out] $\frac{1}{15}b^3(15x + 15c/d - 2(70e^{(-2dx-2c)} + 140e^{(-4dx-4c)} + 90e^{(-6dx-6c)} + 45e^{(-8dx-8c)} + 23)/(d(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1))) + ab^2(3x + 3c/d - 4(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + 2)/(d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1))) + 3a^2b(x + c/d - 2/(d(e^{(-2dx-2c)} + 1))) + a^3x$

mupad [B] time = 1.28, size = 86, normalized size = 1.16

$$x(a^3 + 3a^2b + 3ab^2 + b^3) - \frac{\tanh(c+dx)^3(b^3 + 3ab^2)}{3d} - \frac{b^3 \tanh(c+dx)^5}{5d} - \frac{b \tanh(c+dx)(3a^2 + 3ab + b^2)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*tanh(c + d*x)^2)^3,x)`

[Out] $x(3a^2b + 3a^2b + a^3 + b^3) - (\tanh(c+dx)^3(3a^2b + b^3))/(3d) - (b^3 \tanh(c+dx)^5)/(5d) - (b \tanh(c+dx)(3a^2b + 3a^2 + b^2))/d$

sympy [A] time = 0.68, size = 126, normalized size = 1.70

$$\left\{ \begin{array}{l} a^3x + 3a^2bx - \frac{3a^2b \tanh(c+dx)}{d} + 3ab^2x - \frac{ab^2 \tanh^3(c+dx)}{d} - \frac{3ab^2 \tanh(c+dx)}{d} + b^3x - \frac{b^3 \tanh^5(c+dx)}{5d} - \frac{b^3 \tanh^3(c+dx)}{3d} - \frac{b^3 \tanh(c+dx)}{d} \\ x(a + b \tanh^2(c))^3 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tanh(d*x+c)**2)**3,x)`

[Out] `Piecewise((a**3*x + 3*a**2*b*x - 3*a**2*b*tanh(c + d*x)/d + 3*a*b**2*x - a*b**2*tanh(c + d*x)**3/d - 3*a*b**2*tanh(c + d*x)/d + b**3*x - b**3*tanh(c + d*x)**5/(5*d) - b**3*tanh(c + d*x)**3/(3*d) - b**3*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**3, True))`

3.161 $\int \coth(c + dx) (a + b \tanh^2(c + dx))^3 dx$

Optimal. Leaf size=72

$$\frac{a^3 \log(\tanh(c + dx))}{d} - \frac{b^2(3a + b) \tanh^2(c + dx)}{2d} + \frac{(a + b)^3 \log(\cosh(c + dx))}{d} - \frac{b^3 \tanh^4(c + dx)}{4d}$$

[Out] (a+b)^3*ln(cosh(d*x+c))/d+a^3*ln(tanh(d*x+c))/d-1/2*b^2*(3*a+b)*tanh(d*x+c)^2/d-1/4*b^3*tanh(d*x+c)^4/d

Rubi [A] time = 0.10, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3670, 446, 72}

$$\frac{a^3 \log(\tanh(c + dx))}{d} - \frac{b^2(3a + b) \tanh^2(c + dx)}{2d} + \frac{(a + b)^3 \log(\cosh(c + dx))}{d} - \frac{b^3 \tanh^4(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] ((a + b)^3*Log[Cosh[c + d*x]])/d + (a^3*Log[Tanh[c + d*x]])/d - (b^2*(3*a + b)*Tanh[c + d*x]^2)/(2*d) - (b^3*Tanh[c + d*x]^4)/(4*d)

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f*f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned} \int \coth(c + dx) (a + b \tanh^2(c + dx))^3 dx &= \frac{\text{Subst} \left(\int \frac{(a+bx^2)^3}{x(1-x^2)} dx, x, \tanh(c + dx) \right)}{d} \\ &= \frac{\text{Subst} \left(\int \frac{(a+bx)^3}{(1-x)x} dx, x, \tanh^2(c + dx) \right)}{2d} \\ &= \frac{\text{Subst} \left(\int \left(-b^2(3a + b) - \frac{(a+b)^3}{-1+x} + \frac{a^3}{x} - b^3x \right) dx, x, \tanh^2(c + dx) \right)}{2d} \\ &= \frac{(a + b)^3 \log(\cosh(c + dx))}{d} + \frac{a^3 \log(\tanh(c + dx))}{d} - \frac{b^2(3a + b) \tanh^2(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.55, size = 67, normalized size = 0.93

$$\frac{2a^3 \log(\tanh(c + dx)) - b^2(3a + b) \tanh^2(c + dx) + 2(a + b)^3 \log(\cosh(c + dx)) - \frac{1}{2}b^3 \tanh^4(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (2*(a + b)^3*Log[Cosh[c + d*x]] + 2*a^3*Log[Tanh[c + d*x]] - b^2*(3*a + b)*Tanh[c + d*x]^2 - (b^3*Tanh[c + d*x]^4)/2)/(2*d)

fricas [B] time = 0.47, size = 2381, normalized size = 33.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^8 + 8*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*sinh(d*x + c)^8 - 2*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 + 2*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 - 3*a*b^2 - 2*b^3 + 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*sinh(d*x + c)^6 + 4*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^3 - 3*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(6*a*b^2 + 2*b^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 2*(35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^4 - 6*a*b^2 - 2*b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 15*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^5 - 5*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - (6*a*b^2 + 2*b^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 2*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2 + 2*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^6 - 15*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 3*a*b^2 - 2*b^3 + 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 6*(6*a*b^2 + 2*b^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 8*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^8 + 4*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 4*(3*a^2*b + 3*a*b^2 + b^3 + 7*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 3*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*(35*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 9*a^2*b + 9*a*b^2 + 3*b^3 + 30*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 10*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + 3*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a^2*b + 3*a*b^2 + b^3 + 4*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2 + 4*(7*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 15*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 3*a^2*b + 3*a*b^2 + b^3 + 9*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 3*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 + 3*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - (a^3*cosh(d*x + c)^8 + 8*a^3*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*sinh(d*x + c)^8 + 4*a^3*cosh(d*x + c)^6 + 6*a^3*cosh(d*x + c)^4 + 4*(7*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^6 + 8*(7*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c))*sinh(d*x + c)^5 + 4*a^3*cosh(d*x + c)^2 + 2*(35*a^3*cosh(d*x + c)^4 + 30*a^3*cosh(d*x + c)^2 + 3*a^3)*sinh(d*x + c)^4 + 8*(

$$7a^3 \cosh(dx + c)^5 + 10a^3 \cosh(dx + c)^3 + 3a^3 \cosh(dx + c) \sinh(dx + c)^3 + a^3 + 4(7a^3 \cosh(dx + c)^6 + 15a^3 \cosh(dx + c)^4 + 9a^3 \cosh(dx + c)^2 + a^3) \sinh(dx + c)^2 + 8(a^3 \cosh(dx + c)^7 + 3a^3 \cosh(dx + c)^5 + 3a^3 \cosh(dx + c)^3 + a^3 \cosh(dx + c)) \sinh(dx + c) \log(2 \sinh(dx + c) / (\cosh(dx + c) - \sinh(dx + c))) + 4(2(a^3 + 3a^2b + 3ab^2 + b^3) dx \cosh(dx + c)^7 - 3(3a^2b^2 + 2b^3 - 2(a^3 + 3a^2b + 3ab^2 + b^3) dx) \cosh(dx + c)^5 - 2(6a^2b^2 + 2b^3 - 3(a^3 + 3a^2b + 3ab^2 + b^3) dx) \cosh(dx + c)^3 - (3a^2b^2 + 2b^3 - 2(a^3 + 3a^2b + 3ab^2 + b^3) dx) \cosh(dx + c) \sinh(dx + c)) / (d \cosh(dx + c)^8 + 8d \cosh(dx + c) \sinh(dx + c)^7 + d \sinh(dx + c)^8 + 4d \cosh(dx + c)^6 + 4(7d \cosh(dx + c)^2 + d) \sinh(dx + c)^6 + 8(7d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^5 + 6d \cosh(dx + c)^4 + 2(35d \cosh(dx + c)^4 + 30d \cosh(dx + c)^2 + 3d) \sinh(dx + c)^4 + 8(7d \cosh(dx + c)^5 + 10d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^3 + 4d \cosh(dx + c)^2 + 4(7d \cosh(dx + c)^6 + 15d \cosh(dx + c)^4 + 9d \cosh(dx + c)^2 + d) \sinh(dx + c)^2 + 8(d \cosh(dx + c)^7 + 3d \cosh(dx + c)^5 + 3d \cosh(dx + c)^3 + d \cosh(dx + c)) \sinh(dx + c) + d)$$

giac [B] time = 0.33, size = 267, normalized size = 3.71

$$2a^3 \log(e^{(2dx+2c)} + e^{(-2dx-2c)} - 2) + 2(3a^2b + 3ab^2 + b^3) \log(e^{(2dx+2c)} + e^{(-2dx-2c)} + 2) - \frac{9a^2b(e^{(2dx+2c)} + e^{(-2dx-2c)})}{e^{(2dx+2c)} + e^{(-2dx-2c)} + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(dx+c)*(a+b*tanh(dx+c)^2)^3,x, algorithm="giac")

[Out] 1/4*(2*a^3*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) + 2*(3*a^2*b + 3*a*b^2 + b^3)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) - (9*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 9*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 3*b^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 36*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 12*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 4*b^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 36*a^2*b - 12*a*b^2 - 4*b^3)/(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2)^2)/d

maple [A] time = 0.29, size = 111, normalized size = 1.54

$$\frac{a^3 \ln(\sinh(dx + c))}{d} + \frac{3a^2b \ln(\cosh(dx + c))}{d} + \frac{3ab^2 \ln(\cosh(dx + c))}{d} - \frac{3(\tanh^2(dx + c))ab^2}{2d} + \frac{b^3 \ln(\cosh(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(dx+c)*(a+b*tanh(dx+c)^2)^3,x)

[Out] 1/d*a^3*ln(sinh(dx+c))+3/d*a^2*b*ln(cosh(dx+c))+3/d*a*b^2*ln(cosh(dx+c))-3/2/d*tanh(dx+c)^2*a*b^2+1/d*b^3*ln(cosh(dx+c))-1/2*b^3*tanh(dx+c)^2/d-1/4*b^3*tanh(dx+c)^4/d

maxima [B] time = 0.44, size = 214, normalized size = 2.97

$$b^3 \left(x + \frac{c}{d} + \frac{\log(e^{(-2dx-2c)} + 1)}{d} + \frac{4(e^{(-2dx-2c)} + e^{(-4dx-4c)} + e^{(-6dx-6c)})}{d(4e^{(-2dx-2c)} + 6e^{(-4dx-4c)} + 4e^{(-6dx-6c)} + e^{(-8dx-8c)} + 1)} \right) + 3ab^2 \left(x + \frac{c}{d} + \frac{\log(e^{(-2dx-2c)} + 1)}{d} + \frac{4(e^{(-2dx-2c)} + e^{(-4dx-4c)} + e^{(-6dx-6c)})}{d(4e^{(-2dx-2c)} + 6e^{(-4dx-4c)} + 4e^{(-6dx-6c)} + e^{(-8dx-8c)} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(dx+c)*(a+b*tanh(dx+c)^2)^3,x, algorithm="maxima")

[Out] b^3*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + 3*a*b^2*(x + c/d + log(e^(-2

$(d*x - 2*c) + 1)/d + 2*e^{(-2*d*x - 2*c)}/(d*(2*e^{(-2*d*x - 2*c)} + e^{(-4*d*x - 4*c)} + 1))) + 3*a^2*b*log(e^{(d*x + c)} + e^{(-d*x - c)})/d + a^3*log(\sinh(d*x + c))/d$

mupad [B] time = 0.40, size = 380, normalized size = 5.28

$$\frac{\ln(e^{4c+4dx} - 1) (a^3 d + d (3 a^2 b + 3 a b^2 + b^3))}{2 d^2} - x (a + b)^3 + \frac{2 (2 b^3 + 3 a b^2)}{d (e^{2c+2dx} + 1)} + \frac{8 b^3}{d (3 e^{2c+2dx} + 3 e^{4c+4dx} + e^{6c+6dx})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)*(a + b*tanh(c + d*x)^2)^3,x)

[Out] (log(exp(4*c + 4*d*x) - 1)*(a^3*d + d*(3*a*b^2 + 3*a^2*b + b^3)))/(2*d^2) - x*(a + b)^3 + (2*(3*a*b^2 + 2*b^3))/(d*(exp(2*c + 2*d*x) + 1)) + (8*b^3)/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (atan((exp(2*c)*exp(2*d*x)*(b^3*(-d^2)^(1/2) - a^3*(-d^2)^(1/2) + 3*a*b^2*(-d^2)^(1/2) + 3*a^2*b*(-d^2)^(1/2)))/(d*(6*a*b^5 - 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 16*a^3*b^3 + 3*a^4*b^2)^(1/2)))*(6*a*b^5 - 6*a^5*b + a^6 + b^6 + 15*a^2*b^4 + 16*a^3*b^3 + 3*a^4*b^2)^(1/2))/(-d^2)^(1/2) - (2*(3*a*b^2 + 4*b^3))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (4*b^3)/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^3 \coth(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)

[Out] Integral((a + b*tanh(c + d*x)**2)**3*coth(c + d*x), x)

3.162 $\int \coth^2(c + dx) \left(a + b \tanh^2(c + dx) \right)^3 dx$

Optimal. Leaf size=59

$$-\frac{a^3 \coth(c + dx)}{d} - \frac{b^2(3a + b) \tanh(c + dx)}{d} + x(a + b)^3 - \frac{b^3 \tanh^3(c + dx)}{3d}$$

[Out] $(a+b)^3*x - a^3*\coth(d*x+c)/d - b^2*(3*a+b)*\tanh(d*x+c)/d - 1/3*b^3*\tanh(d*x+c)^3/d$

Rubi [A] time = 0.08, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 461, 207}

$$-\frac{a^3 \coth(c + dx)}{d} - \frac{b^2(3a + b) \tanh(c + dx)}{d} + x(a + b)^3 - \frac{b^3 \tanh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3, x]

[Out] $(a + b)^3*x - (a^3*\coth[c + d*x])/d - (b^2*(3*a + b)*\tanh[c + d*x])/d - (b^3*\tanh[c + d*x]^3)/(3*d)$

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 461

Int[(((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f*f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned} \int \coth^2(c + dx) \left(a + b \tanh^2(c + dx) \right)^3 dx &= \frac{\text{Subst} \left(\int \frac{(a+bx^2)^3}{x^2(1-x^2)} dx, x, \tanh(c + dx) \right)}{d} \\ &= \frac{\text{Subst} \left(\int \left(-b^2(3a + b) + \frac{a^3}{x^2} - b^3x^2 - \frac{(a+b)^3}{-1+x^2} \right) dx, x, \tanh(c + dx) \right)}{d} \\ &= -\frac{a^3 \coth(c + dx)}{d} - \frac{b^2(3a + b) \tanh(c + dx)}{d} - \frac{b^3 \tanh^3(c + dx)}{3d} \\ &= (a + b)^3x - \frac{a^3 \coth(c + dx)}{d} - \frac{b^2(3a + b) \tanh(c + dx)}{d} - \frac{b^3 \tanh^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 2.31, size = 81, normalized size = 1.37

$$\frac{\tanh(c + dx) \left(-3a^3 \coth^2(c + dx) - b^2 (9a + b \tanh^2(c + dx) + 3b) + 3(a + b)^3 \sqrt{\coth^2(c + dx)} \tanh^{-1} \left(\sqrt{\coth^2(c + dx)} \right) \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (Tanh[c + d*x]*(-3*a^3*Coth[c + d*x]^2 + 3*(a + b)^3*ArcTanh[Sqrt[Coth[c + d*x]^2]])*Sqrt[Coth[c + d*x]^2] - b^2*(9*a + 3*b + b*Tanh[c + d*x]^2))/(3*d)

fricas [B] time = 0.43, size = 341, normalized size = 5.78

$$\frac{(3a^3 + 9ab^2 + 4b^3) \cosh(dx + c)^4 - 4(3a^3 + 9ab^2 + 4b^3 + 3(a^3 + 3a^2b + 3ab^2 + b^3)dx) \cosh(dx + c) \sinh(dx + c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -1/12*((3*a^3 + 9*a*b^2 + 4*b^3)*cosh(d*x + c)^4 - 4*(3*a^3 + 9*a*b^2 + 4*b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^3 + 9*a*b^2 + 4*b^3)*sinh(d*x + c)^4 + 9*a^3 - 9*a*b^2 + 4*(3*a^3 - b^3)*cosh(d*x + c)^2 + 2*(6*a^3 - 2*b^3 + 3*(3*a^3 + 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 4*((3*a^3 + 9*a*b^2 + 4*b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + (3*a^3 + 9*a*b^2 + 4*b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)*sinh(d*x + c)^3 + (d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c))

giac [B] time = 0.41, size = 135, normalized size = 2.29

$$\frac{3(a^3 + 3a^2b + 3ab^2 + b^3)(dx + c) - \frac{6a^3}{e^{(2dx+2c)} - 1} + \frac{2(9ab^2e^{(4dx+4c)} + 6b^3e^{(4dx+4c)} + 18ab^2e^{(2dx+2c)} + 6b^3e^{(2dx+2c)} + 9ab^2 + 4b^3)}{(e^{(2dx+2c)} + 1)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) - 6*a^3/(e^(2*d*x + 2*c) - 1) + 2*(9*a*b^2*e^(4*d*x + 4*c) + 6*b^3*e^(4*d*x + 4*c) + 18*a*b^2*e^(2*d*x + 2*c) + 6*b^3*e^(2*d*x + 2*c) + 9*a*b^2 + 4*b^3)/(e^(2*d*x + 2*c) + 1)^3)/d

maple [A] time = 0.25, size = 80, normalized size = 1.36

$$\frac{a^3(dx + c - \coth(dx + c)) + 3a^2b(dx + c) + 3ab^2(dx + c - \tanh(dx + c)) + b^3 \left(dx + c - \tanh(dx + c) - \frac{(\tanh^3(dx + c) - \tanh(dx + c))}{3} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x)

[Out] 1/d*(a^3*(d*x+c-coth(d*x+c))+3*a^2*b*(d*x+c)+3*a*b^2*(d*x+c-tanh(d*x+c))+b^3*(d*x+c-tanh(d*x+c)-1/3*tanh(d*x+c)^3))

maxima [B] time = 0.34, size = 147, normalized size = 2.49

$$\frac{1}{3}b^3 \left(3x + \frac{3c}{d} - \frac{4(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + 2)}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)} \right) + 3ab^2 \left(x + \frac{c}{d} - \frac{2}{d(e^{(-2dx-2c)} + 1)} \right) + a^3 \left(x + \frac{c}{d} + \frac{1}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $\frac{1}{3}b^3(3x + 3c/d - 4(3e^{(-2dx - 2c)} + 3e^{(-4dx - 4c)} + 2)/(d(3e^{(-2dx - 2c)} + 3e^{(-4dx - 4c)} + e^{(-6dx - 6c)} + 1))) + 3ab^2(x + c/d - 2/(d(e^{(-2dx - 2c)} + 1))) + a^3(x + c/d + 2/(d(e^{(-2dx - 2c)} - 1))) + 3a^2bx$

mupad [B] time = 1.31, size = 218, normalized size = 3.69

$$x(a+b)^3 + \frac{\frac{2ab^2}{d} + \frac{2e^{2c+2dx}(2b^3+3ab^2)}{3d}}{2e^{2c+2dx} + e^{4c+4dx} + 1} + \frac{\frac{2(2b^3+3ab^2)}{3d} + \frac{2e^{4c+4dx}(2b^3+3ab^2)}{3d} + \frac{4ab^2e^{2c+2dx}}{d}}{3e^{2c+2dx} + 3e^{4c+4dx} + e^{6c+6dx} + 1} - \frac{2a^3}{d(e^{2c+2dx} - 1)} + \frac{2(2b^3+3ab^2)}{3d(e^{2c+2dx} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3,x)

[Out] $x(a+b)^3 + ((2ab^2)/d + (2\exp(2c + 2dx)(3ab^2 + 2b^3))/(3d))/(2\exp(2c + 2dx) + \exp(4c + 4dx) + 1) + ((2(3ab^2 + 2b^3))/(3d) + (2\exp(4c + 4dx)(3ab^2 + 2b^3))/(3d) + (4ab^2\exp(2c + 2dx))/d)/(3\exp(2c + 2dx) + 3\exp(4c + 4dx) + \exp(6c + 6dx) + 1) - (2a^3)/(d(\exp(2c + 2dx) - 1)) + (2(3ab^2 + 2b^3))/(3d(\exp(2c + 2dx) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^3 \coth^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)

[Out] Integral((a + b*tanh(c + d*x)**2)**3*coth(c + d*x)**2, x)

$$3.163 \quad \int \coth^3(c + dx) \left(a + b \tanh^2(c + dx) \right)^3 dx$$

Optimal. Leaf size=72

$$-\frac{a^3 \coth^2(c + dx)}{2d} + \frac{a^2(a + 3b) \log(\tanh(c + dx))}{d} + \frac{(a + b)^3 \log(\cosh(c + dx))}{d} - \frac{b^3 \tanh^2(c + dx)}{2d}$$

[Out] $-1/2*a^3*\coth(d*x+c)^2/d+(a+b)^3*\ln(\cosh(d*x+c))/d+a^2*(a+3*b)*\ln(\tanh(d*x+c))/d-1/2*b^3*\tanh(d*x+c)^2/d$

Rubi [A] time = 0.11, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 446, 88}

$$\frac{a^2(a + 3b) \log(\tanh(c + dx))}{d} - \frac{a^3 \coth^2(c + dx)}{2d} + \frac{(a + b)^3 \log(\cosh(c + dx))}{d} - \frac{b^3 \tanh^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] $-(a^3*\text{Coth}[c + d*x]^2)/(2*d) + ((a + b)^3*\text{Log}[\text{Cosh}[c + d*x]])/d + (a^2*(a + 3*b)*\text{Log}[\text{Tanh}[c + d*x]])/d - (b^3*\text{Tanh}[c + d*x]^2)/(2*d)$

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegerQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p/(c^2 + f*f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
\int \coth^3(c + dx) (a + b \tanh^2(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{x^3(1-x^2)} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{(1-x)x^2} dx, x, \tanh^2(c + dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-b^3 - \frac{(a+b)^3}{-1+x} + \frac{a^3}{x^2} + \frac{a^2(a+3b)}{x}\right) dx, x, \tanh^2(c + dx)\right)}{2d} \\
&= -\frac{a^3 \coth^2(c + dx)}{2d} + \frac{(a + b)^3 \log(\cosh(c + dx))}{d} + \frac{a^2(a + 3b) \log(\tanh(c + dx))}{d}
\end{aligned}$$

Mathematica [A] time = 0.46, size = 63, normalized size = 0.88

$$\frac{a^3 \coth^2(c + dx) - 2a^2(a + 3b) \log(\tanh(c + dx)) - 2(a + b)^3 \log(\cosh(c + dx)) + b^3 \tanh^2(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] -1/2*(a^3*Coth[c + d*x]^2 - 2*(a + b)^3*Log[Cosh[c + d*x]] - 2*a^2*(a + 3*b)*Log[Tanh[c + d*x]] + b^3*Tanh[c + d*x]^2)/d

fricas [B] time = 0.45, size = 1686, normalized size = 23.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^8 + 8*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*sinh(d*x + c)^8 + 2*(a^3 - b^3)*cosh(d*x + c)^6 + 2*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 + a^3 - b^3)*sinh(d*x + c)^6 + 4*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^3 + 3*(a^3 - b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(2*a^3 + 2*b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 2*(35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^4 + 2*a^3 + 2*b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 15*(a^3 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^5 + 5*(a^3 - b^3)*cosh(d*x + c)^3 + (2*a^3 + 2*b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 2*(a^3 - b^3)*cosh(d*x + c)^2 + 2*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^6 + 15*(a^3 - b^3)*cosh(d*x + c)^4 + a^3 - b^3 + 6*(2*a^3 + 2*b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a*b^2 + b^3)*cosh(d*x + c)^8 + 56*(3*a*b^2 + b^3)*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*(3*a*b^2 + b^3)*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*(3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a*b^2 + b^3)*sinh(d*x + c)^8 - 2*(3*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*(35*(3*a*b^2 + b^3)*cosh(d*x + c)^4 - 3*a*b^2 - b^3)*sinh(d*x + c)^4 + 8*(7*(3*a*b^2 + b^3)*cosh(d*x + c)^5 - (3*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a*b^2 + b^3 + 4*(7*(3*a*b^2 + b^3)*cosh(d*x + c)^6 - 3*(3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a*b^2 + b^3)*cosh(d*x + c)^7 - (3*a*b^2 + b^3)*cosh(d*x + c)^3)*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - ((a^3 + 3*a^2*b)*cosh(d*x + c)^8 + 56*(a^3 + 3*a^2*b)*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*(a^3 + 3*a^2*b)*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*(a^3 + 3*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^7 +

$$(a^3 + 3a^2b) \sinh(dx + c)^8 - 2(a^3 + 3a^2b) \cosh(dx + c)^4 + 2(35(a^3 + 3a^2b) \cosh(dx + c)^4 - a^3 - 3a^2b) \sinh(dx + c)^4 + 8(7(a^3 + 3a^2b) \cosh(dx + c)^5 - (a^3 + 3a^2b) \cosh(dx + c)) \sinh(dx + c)^3 + a^3 + 3a^2b + 4(7(a^3 + 3a^2b) \cosh(dx + c)^6 - 3(a^3 + 3a^2b) \cosh(dx + c)^2) \sinh(dx + c)^2 + 8((a^3 + 3a^2b) \cosh(dx + c)^7 - (a^3 + 3a^2b) \cosh(dx + c)^3) \sinh(dx + c) \log(2 \sinh(dx + c) / (\cosh(dx + c) - \sinh(dx + c))) + 4(2(a^3 + 3a^2b + 3ab^2 + b^3) dx \cosh(dx + c)^7 + 3(a^3 - b^3) \cosh(dx + c)^5 + 2(2a^3 + 2b^3 - (a^3 + 3a^2b + 3ab^2 + b^3) dx) \cosh(dx + c)^3 + (a^3 - b^3) \cosh(dx + c)) \sinh(dx + c) / (d \cosh(dx + c)^8 + 56d \cosh(dx + c)^3 \sinh(dx + c)^5 + 28d \cosh(dx + c)^2 \sinh(dx + c)^6 + 8d \cosh(dx + c) \sinh(dx + c)^7 + d \sinh(dx + c)^8 - 2d \cosh(dx + c)^4 + 2(35d \cosh(dx + c)^4 - d) \sinh(dx + c)^4 + 8(7d \cosh(dx + c)^5 - d \cosh(dx + c)) \sinh(dx + c)^3 + 4(7d \cosh(dx + c)^6 - 3d \cosh(dx + c)^2) \sinh(dx + c)^2 + 8(d \cosh(dx + c)^7 - d \cosh(dx + c)^3) \sinh(dx + c) + d)$$

giac [B] time = 0.44, size = 274, normalized size = 3.81

$$2(3ab^2 + b^3) \log(e^{(2dx+2c)} + e^{(-2dx-2c)} + 2) + 2(a^3 + 3a^2b) \log(e^{(2dx+2c)} + e^{(-2dx-2c)} - 2) - \frac{a^3(e^{(2dx+2c)} + e^{(-2dx-2c)})}{e^{(2dx+2c)} + e^{(-2dx-2c)} - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(dx+c)^3*(a+b*tanh(dx+c)^2)^3,x, algorithm="giac")

[Out] $\frac{1}{4} * (2 * (3 * a * b^2 + b^3) * \log(e^{(2 * dx + 2 * c)} + e^{(-2 * dx - 2 * c)} + 2) + 2 * (a^3 + 3 * a^2 * b) * \log(e^{(2 * dx + 2 * c)} + e^{(-2 * dx - 2 * c)} - 2) - (a^3 * (e^{(2 * dx + 2 * c)} + e^{(-2 * dx - 2 * c)})^2 + 3 * a^2 * b * (e^{(2 * dx + 2 * c)} + e^{(-2 * dx - 2 * c)})^2 + 3 * a * b^2 * (e^{(2 * dx + 2 * c)} + e^{(-2 * dx - 2 * c)})^2 + b^3 * (e^{(2 * dx + 2 * c)} + e^{(-2 * dx - 2 * c)})^2 + 8 * a^3 * (e^{(2 * dx + 2 * c)} + e^{(-2 * dx - 2 * c)}) - 8 * b^3 * (e^{(2 * dx + 2 * c)} + e^{(-2 * dx - 2 * c)}) + 12 * a^3 - 12 * a^2 * b - 12 * a * b^2 + 12 * b^3) / ((e^{(2 * dx + 2 * c)} + e^{(-2 * dx - 2 * c)})^2 - 4)) / d$

maple [A] time = 0.28, size = 94, normalized size = 1.31

$$\frac{a^3 \ln(\sinh(dx + c))}{d} - \frac{a^3 (\coth^2(dx + c))}{2d} + \frac{3a^2b \ln(\sinh(dx + c))}{d} + \frac{3ab^2 \ln(\cosh(dx + c))}{d} + \frac{b^3 \ln(\cosh(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(dx+c)^3*(a+b*tanh(dx+c)^2)^3,x)

[Out] $\frac{1}{d} * a^3 * \ln(\sinh(dx+c)) - \frac{1}{2} * a^3 * \coth(dx+c)^2 / d + \frac{3}{d} * a^2 * b * \ln(\sinh(dx+c)) + \frac{3}{d} * a * b^2 * \ln(\cosh(dx+c)) + \frac{1}{d} * b^3 * \ln(\cosh(dx+c)) - \frac{1}{2} * b^3 * \tanh(dx+c)^2 / d$

maxima [B] time = 0.44, size = 203, normalized size = 2.82

$$a^3 \left(x + \frac{c}{d} + \frac{\log(e^{(-dx-c)} + 1)}{d} + \frac{\log(e^{(-dx-c)} - 1)}{d} + \frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)} - e^{(-4dx-4c)} - 1)} \right) + b^3 \left(x + \frac{c}{d} + \frac{\log(e^{(-2dx-2c)})}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(dx+c)^3*(a+b*tanh(dx+c)^2)^3,x, algorithm="maxima")

[Out] $a^3 * (x + c/d + \log(e^{(-dx-c)} + 1)/d + \log(e^{(-dx-c)} - 1)/d + 2 * e^{(-2 * dx - 2 * c)} / (d * (2 * e^{(-2 * dx - 2 * c)} - e^{(-4 * dx - 4 * c)} - 1))) + b^3 * (x + c/d + \log(e^{(-2 * dx - 2 * c)} + 1)/d + 2 * e^{(-2 * dx - 2 * c)} / (d * (2 * e^{(-2 * dx - 2 * c)} + e^{(-4 * dx - 4 * c)} + 1))) + 3 * a * b^2 * \log(e^{(dx+c)} + e^{(-dx-c)})/d + 3 * a^2 * b * \log(e^{(dx+c)} - e^{(-dx-c)})/d$

mupad [B] time = 2.48, size = 327, normalized size = 4.54

$$\frac{\ln\left(e^{4c+4dx} - 1\right) \left(d a^3 + 3 d a^2 b + 3 d a b^2 + d b^3\right)}{2 d^2} - \frac{\frac{4\left(a^3+b^3\right)}{d} + \frac{2 e^{2c+2dx}\left(a^3-b^3\right)}{d}}{e^{4c+4dx} - 1} - \frac{\frac{4\left(a^3+b^3\right)}{d} + \frac{4 e^{2c+2dx}\left(a^3-b^3\right)}{d}}{e^{8c+8dx} - 2 e^{4c+4dx} + 1} - \operatorname{atan}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3,x)

[Out] (log(exp(4*c + 4*d*x) - 1)*(a^3*d + b^3*d + 3*a*b^2*d + 3*a^2*b*d))/(2*d^2) - ((4*(a^3 + b^3))/d + (2*exp(2*c + 2*d*x)*(a^3 - b^3))/d)/(exp(4*c + 4*d*x) - 1) - ((4*(a^3 + b^3))/d + (4*exp(2*c + 2*d*x)*(a^3 - b^3))/d)/(exp(8*c + 8*d*x) - 2*exp(4*c + 4*d*x) + 1) - (atan((exp(2*c)*exp(2*d*x)*(a^3*(-d^2)^(1/2) - b^3*(-d^2)^(1/2) - 3*a*b^2*(-d^2)^(1/2) + 3*a^2*b*(-d^2)^(1/2)))/(d*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 3*a^2*b^4 - 20*a^3*b^3 + 3*a^4*b^2)^(1/2)))*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 3*a^2*b^4 - 20*a^3*b^3 + 3*a^4*b^2)^(1/2))/(-d^2)^(1/2) - x*(a + b)^3

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^3 \operatorname{coth}^3(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)

[Out] Integral((a + b*tanh(c + d*x)**2)**3*coth(c + d*x)**3, x)

3.164 $\int \coth^4(c + dx) (a + b \tanh^2(c + dx))^3 dx$

Optimal. Leaf size=59

$$-\frac{a^3 \coth^3(c + dx)}{3d} - \frac{a^2(a + 3b) \coth(c + dx)}{d} + x(a + b)^3 - \frac{b^3 \tanh(c + dx)}{d}$$

[Out] (a+b)^3*x-a^2*(a+3*b)*coth(d*x+c)/d-1/3*a^3*coth(d*x+c)^3/d-b^3*tanh(d*x+c)/d

Rubi [A] time = 0.08, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 461, 207}

$$-\frac{a^2(a + 3b) \coth(c + dx)}{d} - \frac{a^3 \coth^3(c + dx)}{3d} + x(a + b)^3 - \frac{b^3 \tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (a + b)^3*x - (a^2*(a + 3*b)*Coth[c + d*x])/d - (a^3*Coth[c + d*x]^3)/(3*d) - (b^3*Tanh[c + d*x])/d

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 461

Int[(((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.))/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.))*((a_) + (b_.))*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned} \int \coth^4(c + dx) (a + b \tanh^2(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{x^4(1-x^2)} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(-b^3 + \frac{a^3}{x^4} + \frac{a^2(a+3b)}{x^2} - \frac{(a+b)^3}{-1+x^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{a^2(a + 3b) \coth(c + dx)}{d} - \frac{a^3 \coth^3(c + dx)}{3d} - \frac{b^3 \tanh(c + dx)}{d} - \frac{(a+b)^3 x}{d} \\ &= (a + b)^3 x - \frac{a^2(a + 3b) \coth(c + dx)}{d} - \frac{a^3 \coth^3(c + dx)}{3d} - \frac{b^3 \tanh(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 1.28, size = 82, normalized size = 1.39

$$\frac{\tanh(c + dx) \left(-a^3 \coth^4(c + dx) - 3a^2(a + 3b) \coth^2(c + dx) + 3(a + b)^3 \sqrt{\coth^2(c + dx)} \tanh^{-1} \left(\sqrt{\coth^2(c + dx)} \right) \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] ((-3*b^3 - 3*a^2*(a + 3*b)*Coth[c + d*x]^2 - a^3*Coth[c + d*x]^4 + 3*(a + b)^3*ArcTanh[Sqrt[Coth[c + d*x]^2]]*Sqrt[Coth[c + d*x]^2])*Tanh[c + d*x])/(3*d)

fricas [B] time = 0.44, size = 341, normalized size = 5.78

$$\frac{(4a^3 + 9a^2b + 3b^3) \cosh(dx + c)^4 - 4(4a^3 + 9a^2b + 3b^3 + 3(a^3 + 3a^2b + 3ab^2 + b^3)dx) \cosh(dx + c) \sinh(dx + c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -1/12*((4*a^3 + 9*a^2*b + 3*b^3)*cosh(d*x + c)^4 - 4*(4*a^3 + 9*a^2*b + 3*b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^3 + (4*a^3 + 9*a^2*b + 3*b^3)*sinh(d*x + c)^4 - 9*a^2*b + 9*b^3 + 4*(a^3 - 3*b^3)*cosh(d*x + c)^2 + 2*(2*a^3 - 6*b^3 + 3*(4*a^3 + 9*a^2*b + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 4*((4*a^3 + 9*a^2*b + 3*b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - (4*a^3 + 9*a^2*b + 3*b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)*sinh(d*x + c)^3 + (d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c))

giac [B] time = 0.52, size = 135, normalized size = 2.29

$$\frac{3(a^3 + 3a^2b + 3ab^2 + b^3)(dx + c) + \frac{6b^3}{e^{2dx+2c}+1} - \frac{2(6a^3e^{4dx+4c}+9a^2be^{4dx+4c}-6a^3e^{2dx+2c}-18a^2be^{2dx+2c}+4a^3+9a^2b)}{(e^{2dx+2c}-1)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) + 6*b^3/(e^(2*d*x + 2*c) + 1) - 2*(6*a^3*e^(4*d*x + 4*c) + 9*a^2*b*e^(4*d*x + 4*c) - 6*a^3*e^(2*d*x + 2*c) - 18*a^2*b*e^(2*d*x + 2*c) + 4*a^3 + 9*a^2*b)/(e^(2*d*x + 2*c) - 1)^3)/d

maple [A] time = 0.25, size = 80, normalized size = 1.36

$$\frac{a^3 \left(dx + c - \coth(dx + c) - \frac{(\coth^3(dx + c))}{3} \right) + 3a^2b(dx + c - \coth(dx + c)) + 3ab^2(dx + c) + b^3(dx + c - \tanh(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x)

[Out] 1/d*(a^3*(d*x+c-coth(d*x+c))-1/3*coth(d*x+c)^3)+3*a^2*b*(d*x+c-coth(d*x+c))+3*a*b^2*(d*x+c)+b^3*(d*x+c-tanh(d*x+c))

maxima [B] time = 0.34, size = 147, normalized size = 2.49

$$\frac{1}{3}a^3 \left(3x + \frac{3c}{d} - \frac{4(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} - 2)}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)} \right) + b^3 \left(x + \frac{c}{d} - \frac{2}{d(e^{(-2dx-2c)} + 1)} \right) + 3a^2b \left(x + \frac{c}{d} + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 1/3*a^3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + b^3*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + 3*a^2*b*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + 3*a*b^2*x

mupad [B] time = 1.29, size = 219, normalized size = 3.71

$$x(a+b)^3 + \frac{\frac{2a^2b}{d} - \frac{2e^{2c+2dx}(2a^3+3ba^2)}{3d}}{e^{4c+4dx} - 2e^{2c+2dx} + 1} - \frac{\frac{2(2a^3+3ba^2)}{3d}}{3e^{2c+2dx} - 3e^{4c+4dx} + e^{6c+6dx} - 1} + \frac{\frac{2e^{4c+4dx}(2a^3+3ba^2)}{3d} - \frac{4a^2be^{2c+2dx}}{d}}{d(e^{2c+2dx} + 1)} + \frac{2b^3}{3d(e^{2c+2dx} + 1)} - \frac{2(2a^3 + 3ba^2)}{3d(e^{2c+2dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3,x)

[Out] x*(a + b)^3 + ((2*a^2*b)/d - (2*exp(2*c + 2*d*x)*(3*a^2*b + 2*a^3))/(3*d))/ (exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - ((2*(3*a^2*b + 2*a^3))/(3*d) + (2*exp(4*c + 4*d*x)*(3*a^2*b + 2*a^3))/(3*d) - (4*a^2*b*exp(2*c + 2*d*x))/d)/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) + (2*b^3)/(d*(exp(2*c + 2*d*x) + 1)) - (2*(3*a^2*b + 2*a^3))/(3*d*(exp(2*c + 2*d*x) - 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^3 \coth^4(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)

[Out] Integral((a + b*tanh(c + d*x)**2)**3*coth(c + d*x)**4, x)

3.165 $\int \coth^5(c + dx) (a + b \tanh^2(c + dx))^3 dx$

Optimal. Leaf size=83

$$-\frac{a^3 \coth^4(c + dx)}{4d} + \frac{a(a^2 + 3ab + 3b^2) \log(\tanh(c + dx))}{d} - \frac{a^2(a + 3b) \coth^2(c + dx)}{2d} + \frac{(a + b)^3 \log(\cosh(c + dx))}{d}$$

[Out] $-1/2*a^2*(a+3*b)*\coth(d*x+c)^2/d-1/4*a^3*\coth(d*x+c)^4/d+(a+b)^3*\ln(\cosh(d*x+c))/d+a*(a^2+3*a*b+3*b^2)*\ln(\tanh(d*x+c))/d$

Rubi [A] time = 0.12, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 446, 88}

$$\frac{a(a^2 + 3ab + 3b^2) \log(\tanh(c + dx))}{d} - \frac{a^2(a + 3b) \coth^2(c + dx)}{2d} - \frac{a^3 \coth^4(c + dx)}{4d} + \frac{(a + b)^3 \log(\cosh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] $-(a^2*(a + 3*b)*\text{Coth}[c + d*x]^2)/(2*d) - (a^3*\text{Coth}[c + d*x]^4)/(4*d) + ((a + b)^3*\text{Log}[\text{Cosh}[c + d*x]])/d + (a*(a^2 + 3*a*b + 3*b^2)*\text{Log}[\text{Tanh}[c + d*x]])/d$

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
\int \coth^5(c + dx) (a + b \tanh^2(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{x^5(1-x^2)} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{(a+bx)^3}{(1-x)x^3} dx, x, \tanh^2(c + dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{(a+b)^3}{-1+x} + \frac{a^3}{x^3} + \frac{a^2(a+3b)}{x^2} + \frac{a(a^2+3ab+3b^2)}{x}\right) dx, x, \tanh^2(c + dx)\right)}{2d} \\
&= -\frac{a^2(a+3b) \coth^2(c + dx)}{2d} - \frac{a^3 \coth^4(c + dx)}{4d} + \frac{(a+b)^3 \log(\cosh(c + dx))}{d}
\end{aligned}$$

Mathematica [A] time = 0.58, size = 67, normalized size = 0.81

$$\frac{-\frac{1}{2}a^3 \coth^4(c + dx) - a^2(a + 3b) \coth^2(c + dx) + 2(a + b)^3 \log(\sinh(c + dx)) - 2b^3 \log(\tanh(c + dx))}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] $(-(a^2(a + 3b) \coth[c + d*x]^2) - (a^3 \coth[c + d*x]^4)/2 + 2(a + b)^3 \log[\sinh[c + d*x]] - 2b^3 \log[\tanh[c + d*x]])/(2d)$

fricas [B] time = 0.46, size = 2393, normalized size = 28.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] $-(a^3 + 3a^2b + 3ab^2 + b^3)d*x*\cosh(d*x + c)^8 + 8(a^3 + 3a^2b + 3ab^2 + b^3)d*x*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^3 + 3a^2b + 3ab^2 + b^3)d*x*\sinh(d*x + c)^8 + 2(2a^3 + 3a^2b - 2(a^3 + 3a^2b + 3ab^2 + b^3)d*x)*\cosh(d*x + c)^6 + 2(14(a^3 + 3a^2b + 3ab^2 + b^3)d*x*\cosh(d*x + c)^2 + 2a^3 + 3a^2b - 2(a^3 + 3a^2b + 3ab^2 + b^3)d*x)*\sinh(d*x + c)^6 + 4(14(a^3 + 3a^2b + 3ab^2 + b^3)d*x*\cosh(d*x + c)^3 + 3(2a^3 + 3a^2b - 2(a^3 + 3a^2b + 3ab^2 + b^3)d*x)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2(2a^3 + 6a^2b - 3(a^3 + 3a^2b + 3ab^2 + b^3)d*x)*\cosh(d*x + c)^4 + 2(35(a^3 + 3a^2b + 3ab^2 + b^3)d*x*\cosh(d*x + c)^4 - 2a^3 - 6a^2b + 3(a^3 + 3a^2b + 3ab^2 + b^3)d*x + 15(2a^3 + 3a^2b - 2(a^3 + 3a^2b + 3ab^2 + b^3)d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8(7(a^3 + 3a^2b + 3ab^2 + b^3)d*x*\cosh(d*x + c)^5 + 5(2a^3 + 3a^2b - 2(a^3 + 3a^2b + 3ab^2 + b^3)d*x)*\cosh(d*x + c)^3 - (2a^3 + 6a^2b - 3(a^3 + 3a^2b + 3ab^2 + b^3)d*x)*\cosh(d*x + c))*\sinh(d*x + c)^3 + (a^3 + 3a^2b + 3ab^2 + b^3)d*x + 2(2a^3 + 3a^2b - 2(a^3 + 3a^2b + 3ab^2 + b^3)d*x)*\cosh(d*x + c)^2 + 2(14(a^3 + 3a^2b + 3ab^2 + b^3)d*x*\cosh(d*x + c)^6 + 15(2a^3 + 3a^2b - 2(a^3 + 3a^2b + 3ab^2 + b^3)d*x)*\cosh(d*x + c)^4 + 2a^3 + 3a^2b - 2(a^3 + 3a^2b + 3ab^2 + b^3)d*x - 6(2a^3 + 6a^2b - 3(a^3 + 3a^2b + 3ab^2 + b^3)d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 - (b^3*\cosh(d*x + c)^8 + 8b^3*\cosh(d*x + c)*\sinh(d*x + c)^7 + b^3*\sinh(d*x + c)^8 - 4b^3*\cosh(d*x + c)^6 + 6b^3*\cosh(d*x + c)^4 + 4(7b^3*\cosh(d*x + c)^2 - b^3)*\sinh(d*x + c)^6 + 8(7b^3*\cosh(d*x + c)^3 - 3b^3*\cosh(d*x + c))*\sinh(d*x + c)^5 - 4b^3*\cosh(d*x + c)^2 + 2(35b^3*\cosh(d*x + c)^4 - 30b^3*\cosh(d*x + c)^2 + 3b^3)*\sinh(d*x + c)^4 + 8(7b^3*\cosh(d*x + c)^5 - 10b^3*\cosh(d*x + c)^3 +$

$$\begin{aligned}
& 3b^3 \cosh(dx + c) \sinh(dx + c)^3 + b^3 + 4(7b^3 \cosh(dx + c)^6 - 15b^3 \cosh(dx + c)^4 + 9b^3 \cosh(dx + c)^2 - b^3) \sinh(dx + c)^2 + 8(b^3 \cosh(dx + c)^7 - 3b^3 \cosh(dx + c)^5 + 3b^3 \cosh(dx + c)^3 - b^3 \cosh(dx + c)) \sinh(dx + c) \\
& \log(2 \cosh(dx + c) / (\cosh(dx + c) - \sinh(dx + c))) - ((a^3 + 3a^2b + 3ab^2) \cosh(dx + c)^8 + 8(a^3 + 3a^2b + 3ab^2) \cosh(dx + c) \sinh(dx + c)^7 + (a^3 + 3a^2b + 3ab^2) \sinh(dx + c)^8 - 4(a^3 + 3a^2b + 3ab^2) \cosh(dx + c)^6 - 4(a^3 + 3a^2b + 3ab^2) \cosh(dx + c)^2 \sinh(dx + c)^6 + 8(7(a^3 + 3a^2b + 3ab^2) \cosh(dx + c)^3 - 3(a^3 + 3a^2b + 3ab^2) \cosh(dx + c)) \sinh(dx + c)^5 + 6(a^3 + 3a^2b + 3ab^2) \cosh(dx + c)^4 + 2(35(a^3 + 3a^2b + 3ab^2) \cosh(dx + c)^4 + 3a^3 + 9a^2b + 9ab^2 - 30(a^3 + 3a^2b + 3ab^2) \cosh(dx + c)^2) \sinh(dx + c)^4 + 8(7(a^3 + 3a^2b + 3ab^2) \cosh(dx + c)^5 - 10(a^3 + 3a^2b + 3ab^2) \cosh(dx + c)^3 + 3(a^3 + 3a^2b + 3ab^2) \cosh(dx + c)) \sinh(dx + c)^3 + a^3 + 3a^2b + 3ab^2 - 4(a^3 + 3a^2b + 3ab^2) \cosh(dx + c)^2 + 4(7(a^3 + 3a^2b + 3ab^2) \cosh(dx + c)^6 - 15(a^3 + 3a^2b + 3ab^2) \cosh(dx + c)^4 - a^3 - 3a^2b - 3ab^2 + 9(a^3 + 3a^2b + 3ab^2) \cosh(dx + c)^2) \sinh(dx + c)^2 + 8((a^3 + 3a^2b + 3ab^2) \cosh(dx + c)^7 - 3(a^3 + 3a^2b + 3ab^2) \cosh(dx + c)^5 + 3(a^3 + 3a^2b + 3ab^2) \cosh(dx + c)^3 - (a^3 + 3a^2b + 3ab^2) \cosh(dx + c)) \sinh(dx + c)) \log(2 \sinh(dx + c) / (\cosh(dx + c) - \sinh(dx + c))) + 4(2(a^3 + 3a^2b + 3ab^2 + b^3) dx \cosh(dx + c)^7 + 3(2a^3 + 3a^2b - 2(a^3 + 3a^2b + 3ab^2 + b^3) dx) \cosh(dx + c)^5 - 2(2a^3 + 6a^2b - 3(a^3 + 3a^2b + 3ab^2 + b^3) dx) \cosh(dx + c)^3 + (2a^3 + 3a^2b - 2(a^3 + 3a^2b + 3ab^2 + b^3) dx) \cosh(dx + c)) \sinh(dx + c)) / (d \cosh(dx + c)^8 + 8d \cosh(dx + c) \sinh(dx + c)^7 + d \sinh(dx + c)^8 - 4d \cosh(dx + c)^6 + 4(7d \cosh(dx + c)^2 - d) \sinh(dx + c)^6 + 8(7d \cosh(dx + c)^3 - 3d \cosh(dx + c)) \sinh(dx + c)^5 + 6d \cosh(dx + c)^4 + 2(35d \cosh(dx + c)^4 - 30d \cosh(dx + c)^2 + 3d) \sinh(dx + c)^4 + 8(7d \cosh(dx + c)^5 - 10d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^3 - 4d \cosh(dx + c)^2 + 4(7d \cosh(dx + c)^6 - 15d \cosh(dx + c)^4 + 9d \cosh(dx + c)^2 - d) \sinh(dx + c)^2 + 8(d \cosh(dx + c)^7 - 3d \cosh(dx + c)^5 + 3d \cosh(dx + c)^3 - d \cosh(dx + c)) \sinh(dx + c) + d)
\end{aligned}$$

giac [B] time = 0.58, size = 267, normalized size = 3.22

$$2b^3 \log(e^{(2dx+2c)} + e^{(-2dx-2c)} + 2) + 2(a^3 + 3a^2b + 3ab^2) \log(e^{(2dx+2c)} + e^{(-2dx-2c)} - 2) - \frac{3a^3(e^{(2dx+2c)} + e^{(-2dx-2c)})}{e^{(2dx+2c)} + e^{(-2dx-2c)} - 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(dx+c)^5*(a+b*tanh(dx+c)^2)^3,x, algorithm="giac")

[Out] $\frac{1}{4}(2b^3 \log(e^{(2dx+2c)} + e^{(-2dx-2c)} + 2) + 2(a^3 + 3a^2b + 3ab^2) \log(e^{(2dx+2c)} + e^{(-2dx-2c)} - 2) - (3a^3(e^{(2dx+2c)} + e^{(-2dx-2c)})^2 + 9a^2b(e^{(2dx+2c)} + e^{(-2dx-2c)})^2 + 9ab^2(e^{(2dx+2c)} + e^{(-2dx-2c)})^2 + 4a^3(e^{(2dx+2c)} + e^{(-2dx-2c)}) - 12a^2b(e^{(2dx+2c)} + e^{(-2dx-2c)}) - 36ab^2(e^{(2dx+2c)} + e^{(-2dx-2c)}) - 4a^3 - 12a^2b + 36ab^2) / (e^{(2dx+2c)} + e^{(-2dx-2c)} - 2)^2) / d$

maple [A] time = 0.27, size = 111, normalized size = 1.34

$$\frac{a^3 \ln(\sinh(dx+c))}{d} - \frac{a^3 (\coth^2(dx+c))}{2d} - \frac{a^3 (\coth^4(dx+c))}{4d} + \frac{3a^2b \ln(\sinh(dx+c))}{d} - \frac{3a^2b (\coth^2(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(dx+c)^5*(a+b*tanh(dx+c)^2)^3,x)

[Out] $1/d*a^3*\ln(\sinh(d*x+c))-1/2*a^3*\coth(d*x+c)^2/d-1/4*a^3*\coth(d*x+c)^4/d+3/d*a^2*b*\ln(\sinh(d*x+c))-3/2/d*a^2*b*\coth(d*x+c)^2+3/d*a*b^2*\ln(\sinh(d*x+c))+1/d*b^3*\ln(\cosh(d*x+c))$

maxima [B] time = 0.35, size = 264, normalized size = 3.18

$$a^3 \left(x + \frac{c}{d} + \frac{\log(e^{-dx-c} + 1)}{d} + \frac{\log(e^{-dx-c} - 1)}{d} + \frac{4(e^{-2dx-2c} - e^{-4dx-4c} + e^{-6dx-6c})}{d(4e^{-2dx-2c} - 6e^{-4dx-4c} + 4e^{-6dx-6c} - e^{-8dx-8c} - 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $a^3*(x + c/d + \log(e^{-dx-c} + 1)/d + \log(e^{-dx-c} - 1)/d + 4*(e^{-2dx-2c} - e^{-4dx-4c} + e^{-6dx-6c})/(d*(4e^{-2dx-2c} - 6e^{-4dx-4c} + 4e^{-6dx-6c} - e^{-8dx-8c} - 1))) + 3*a^2*b*(x + c/d + \log(e^{-dx-c} + 1)/d + \log(e^{-dx-c} - 1)/d + 2*e^{-2dx-2c}/(d*(2e^{-2dx-2c} - e^{-4dx-4c} - 1))) + b^3*\log(e^{dx+c} + e^{-dx-c})/d + 3*a*b^2*\log(e^{dx+c} - e^{-dx-c})/d$

mupad [B] time = 0.40, size = 381, normalized size = 4.59

$$\frac{\ln(e^{4c+4dx} - 1) (b^3 d + d (a^3 + 3 a^2 b + 3 a b^2))}{2 d^2} - x (a + b)^3 - \frac{2 (2 a^3 + 3 b a^2)}{d (e^{2c+2dx} - 1)} - \frac{8 a^3}{d (3 e^{2c+2dx} - 3 e^{4c+4dx} + e^{6c+6dx})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^5*(a + b*tanh(c + d*x)^2)^3,x)

[Out] $(\log(\exp(4*c + 4*d*x) - 1)*(b^3*d + d*(3*a*b^2 + 3*a^2*b + a^3)))/(2*d^2) - x*(a + b)^3 - (2*(3*a^2*b + 2*a^3))/(d*(\exp(2*c + 2*d*x) - 1)) - (8*a^3)/(d*(3*\exp(2*c + 2*d*x) - 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) - 1)) - (\operatorname{atan}((\exp(2*c)*\exp(2*d*x)*(a^3*(-d^2)^{(1/2)} - b^3*(-d^2)^{(1/2)} + 3*a*b^2*(-d^2)^{(1/2)} + 3*a^2*b*(-d^2)^{(1/2)})))/(d*(6*a^5*b - 6*a*b^5 + a^6 + b^6 + 3*a^2*b^4 + 16*a^3*b^3 + 15*a^4*b^2)^{(1/2)}))*(6*a^5*b - 6*a*b^5 + a^6 + b^6 + 3*a^2*b^4 + 16*a^3*b^3 + 15*a^4*b^2)^{(1/2)})/(-d^2)^{(1/2)} - (2*(3*a^2*b + 4*a^3))/(d*(\exp(4*c + 4*d*x) - 2*\exp(2*c + 2*d*x) + 1)) - (4*a^3)/(d*(6*\exp(4*c + 4*d*x) - 4*\exp(2*c + 2*d*x) - 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(c + dx))^3 \coth^5(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**5*(a+b*tanh(d*x+c)**2)**3,x)

[Out] Integral((a + b*tanh(c + d*x)**2)**3*coth(c + d*x)**5, x)

3.166 $\int \coth^6(c + dx) (a + b \tanh^2(c + dx))^3 dx$

Optimal. Leaf size=74

$$\frac{a^3 \coth^5(c + dx)}{5d} - \frac{a(a^2 + 3ab + 3b^2) \coth(c + dx)}{d} - \frac{a^2(a + 3b) \coth^3(c + dx)}{3d} + x(a + b)^3$$

[Out] $(a+b)^3*x - a*(a^2+3*a*b+3*b^2)*\coth(d*x+c)/d - 1/3*a^2*(a+3*b)*\coth(d*x+c)^3/d - 1/5*a^3*\coth(d*x+c)^5/d$

Rubi [A] time = 0.09, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 461, 207}

$$-\frac{a(a^2 + 3ab + 3b^2) \coth(c + dx)}{d} - \frac{a^2(a + 3b) \coth^3(c + dx)}{3d} - \frac{a^3 \coth^5(c + dx)}{5d} + x(a + b)^3$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^6*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] $(a + b)^3*x - (a*(a^2 + 3*a*b + 3*b^2)*\text{Coth}[c + d*x])/d - (a^2*(a + 3*b)*\text{Coth}[c + d*x]^3)/(3*d) - (a^3*\text{Coth}[c + d*x]^5)/(5*d)$

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 461

Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
\int \coth^6(c+dx) (a+b \tanh^2(c+dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{(a+bx^2)^3}{x^6(1-x^2)} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a^3}{x^6} + \frac{a^2(a+3b)}{x^4} + \frac{a(a^2+3ab+3b^2)}{x^2} - \frac{(a+b)^3}{-1+x^2}\right) dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{a(a^2+3ab+3b^2) \coth(c+dx)}{d} - \frac{a^2(a+3b) \coth^3(c+dx)}{3d} - \frac{a^3}{3d} \\
&= (a+b)^3 x - \frac{a(a^2+3ab+3b^2) \coth(c+dx)}{d} - \frac{a^2(a+3b) \coth^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 1.78, size = 100, normalized size = 1.35

$$\frac{(a+b)^3 \tanh^{-1}\left(\sqrt{\tanh^2(c+dx)}\right) \tanh(c+dx)}{d\sqrt{\tanh^2(c+dx)}} - \frac{a \coth(c+dx) (15(a^2+3ab+3b^2) + 3a^2 \coth^4(c+dx) + 5a^3)}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^6*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] -1/15*(a*Coth[c + d*x]*(15*(a^2 + 3*a*b + 3*b^2) + 5*a*(a + 3*b)*Coth[c + d*x]^2 + 3*a^2*Coth[c + d*x]^4))/d + ((a + b)^3*ArcTanh[Sqrt[Tanh[c + d*x]^2]]*Tanh[c + d*x])/(d*Sqrt[Tanh[c + d*x]^2])

fricas [B] time = 0.44, size = 557, normalized size = 7.53

$$\frac{(23a^3 + 60a^2b + 45ab^2) \cosh(dx+c)^5 + 5(23a^3 + 60a^2b + 45ab^2) \cosh(dx+c) \sinh(dx+c)^4 - (23a^3 + 60a^2b + 45ab^2) \sinh(dx+c)^5}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -1/15*((23*a^3 + 60*a^2*b + 45*a*b^2)*cosh(d*x + c)^5 + 5*(23*a^3 + 60*a^2*b + 45*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 - (23*a^3 + 60*a^2*b + 45*a*b^2 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*sinh(d*x + c)^5 - 5*(5*a^3 + 24*a^2*b + 27*a*b^2)*cosh(d*x + c)^3 + 5*(23*a^3 + 60*a^2*b + 45*a*b^2 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 2*(23*a^3 + 60*a^2*b + 45*a*b^2 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 5*(2*(23*a^3 + 60*a^2*b + 45*a*b^2)*cosh(d*x + c)^3 - 3*(5*a^3 + 24*a^2*b + 27*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^3 + 6*a^2*b + 9*a*b^2)*cosh(d*x + c) - 5*((23*a^3 + 60*a^2*b + 45*a*b^2 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 46*a^3 + 120*a^2*b + 90*a*b^2 + 30*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 3*(23*a^3 + 60*a^2*b + 45*a*b^2 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^5 + 5*(2*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*(d*cosh(d*x + c)^4 - 3*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))

giac [B] time = 0.56, size = 241, normalized size = 3.26

$$15(a^3 + 3a^2b + 3ab^2 + b^3)(dx+c) - \frac{2(45a^3e^{(8dx+8c)} + 90a^2be^{(8dx+8c)} + 45ab^2e^{(8dx+8c)} - 90a^3e^{(6dx+6c)} - 270a^2be^{(6dx+6c)} - 180ab^2e^{(6dx+6c)})}{15d}$$


```
*c + 8*d*x) + 1) + ((6*(a*b^2 + a^2*b))/(5*d) - (6*exp(2*c + 2*d*x)*(a*b^2
+ 2*a^2*b + a^3))/(5*d))/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - (6*(
a*b^2 + 2*a^2*b + a^3))/(5*d*(exp(2*c + 2*d*x) - 1))
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int (a + b \tanh^2(c + dx))^3 \coth^6(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**6*(a+b*tanh(d*x+c)**2)**3,x)
```

```
[Out] Integral((a + b*tanh(c + d*x)**2)**3*coth(c + d*x)**6, x)
```

3.167 $\int \coth^7(c + dx) (a + b \tanh^2(c + dx))^3 dx$

Optimal. Leaf size=103

$$\frac{a^3 \coth^6(c + dx)}{6d} - \frac{a(a^2 + 3ab + 3b^2) \coth^2(c + dx)}{2d} - \frac{a^2(a + 3b) \coth^4(c + dx)}{4d} + \frac{(a + b)^3 \log(\tanh(c + dx))}{d}$$

[Out] $-1/2*a*(a^2+3*a*b+3*b^2)*\coth(d*x+c)^2/d-1/4*a^2*(a+3*b)*\coth(d*x+c)^4/d-1/6*a^3*\coth(d*x+c)^6/d+(a+b)^3*\ln(\cosh(d*x+c))/d+(a+b)^3*\ln(\tanh(d*x+c))/d$

Rubi [A] time = 0.13, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 446, 88}

$$\frac{a(a^2 + 3ab + 3b^2) \coth^2(c + dx)}{2d} - \frac{a^2(a + 3b) \coth^4(c + dx)}{4d} - \frac{a^3 \coth^6(c + dx)}{6d} + \frac{(a + b)^3 \log(\tanh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^7*(a + b*Tanh[c + d*x]^2)^3,x]

[Out] $-(a*(a^2 + 3*a*b + 3*b^2)*\text{Coth}[c + d*x]^2)/(2*d) - (a^2*(a + 3*b)*\text{Coth}[c + d*x]^4)/(4*d) - (a^3*\text{Coth}[c + d*x]^6)/(6*d) + ((a + b)^3*\text{Log}[\text{Cosh}[c + d*x]])/d + ((a + b)^3*\text{Log}[\text{Tanh}[c + d*x]])/d$

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f*f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
&)) * \sinh(dx + c)^5 - 9*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)^4 + 3*(495*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx * \cosh(dx + c)^8 + 1260*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)^6 - 210*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)^4 - 12*a^3 - 36*a^2*b - 24*a*b^2 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx + 20*(17*a^3 + 36*a^2*b + 27*a*b^2 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)^2) * \sinh(dx + c)^4 + 4*(165*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx * \cosh(dx + c)^9 + 540*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)^7 - 126*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)^5 + 20*(17*a^3 + 36*a^2*b + 27*a*b^2 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)^3 - 9*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)) * \sinh(dx + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx + 18*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)^2 + 6*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx * \cosh(dx + c)^10 + 135*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)^8 - 42*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)^6 + 10*(17*a^3 + 36*a^2*b + 27*a*b^2 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)^4 + 3*a^3 + 6*a^2*b + 3*a*b^2 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx - 9*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*dx) * \cosh(dx + c)^2) * \sinh(dx + c)^2 - 3*((a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^12 + 12*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c) * \sinh(dx + c)^11 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \sinh(dx + c)^12 - 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^10 - 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^2) * \sinh(dx + c)^10 + 20*(11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)) * \sinh(dx + c)^9 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^8 + 15*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 18*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^2) * \sinh(dx + c)^8 + 24*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^5 - 30*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^3 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)) * \sinh(dx + c)^7 - 20*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^6 + 4*(231*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^6 - 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^4 - 5*a^3 - 15*a^2*b - 15*a*b^2 - 5*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^2) * \sinh(dx + c)^6 + 24*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^7 - 63*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^5 + 35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^3 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)) * \sinh(dx + c)^5 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^4 + 15*(33*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^8 - 84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^6 + 70*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 20*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^2) * \sinh(dx + c)^4 + 20*(11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^9 - 36*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^7 + 42*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^5 - 20*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)) * \sinh(dx + c)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^2 + 6*(11*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^10 - 45*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^8 + 70*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^6 - 50*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^2) * \sinh(dx + c)^2 + 12*((a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^11 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^9 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^7 - 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^5 + 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3) * \cosh(dx + c)) * \sinh(dx + c)) * \log(2 * \sinh(dx + c) / (\cosh(dx + c) - \sinh(dx + c))) + 12*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3) * dx * \cosh(dx + c)^11 + 15*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b +
\end{aligned}$$

$3*a*b^2 + b^3)*d*x)*\cosh(d*x + c)^9 - 6*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c)^7 + 2*(17*a^3 + 36*a^2*b + 27*a*b^2 - 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c)^5 - 3*(4*a^3 + 12*a^2*b + 8*a*b^2 - 5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c)^3 + 3*(a^3 + 2*a^2*b + a*b^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*\cosh(d*x + c))*\sinh(d*x + c))/(d*\cosh(d*x + c)^12 + 12*d*\cosh(d*x + c)*\sinh(d*x + c)^11 + d*\sinh(d*x + c)^12 - 6*d*\cosh(d*x + c)^10 + 6*(11*d*\cosh(d*x + c)^2 - d)*\sinh(d*x + c)^10 + 20*(11*d*\cosh(d*x + c)^3 - 3*d*\cosh(d*x + c))*\sinh(d*x + c)^9 + 15*d*\cosh(d*x + c)^8 + 15*(33*d*\cosh(d*x + c)^4 - 18*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^8 + 24*(33*d*\cosh(d*x + c)^5 - 30*d*\cosh(d*x + c)^3 + 5*d*\cosh(d*x + c))*\sinh(d*x + c)^7 - 20*d*\cosh(d*x + c)^6 + 4*(231*d*\cosh(d*x + c)^6 - 315*d*\cosh(d*x + c)^4 + 105*d*\cosh(d*x + c)^2 - 5*d)*\sinh(d*x + c)^6 + 24*(33*d*\cosh(d*x + c)^7 - 63*d*\cosh(d*x + c)^5 + 35*d*\cosh(d*x + c)^3 - 5*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 15*d*\cosh(d*x + c)^4 + 15*(33*d*\cosh(d*x + c)^8 - 84*d*\cosh(d*x + c)^6 + 70*d*\cosh(d*x + c)^4 - 20*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^4 + 20*(11*d*\cosh(d*x + c)^9 - 36*d*\cosh(d*x + c)^7 + 42*d*\cosh(d*x + c)^5 - 20*d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c))*\sinh(d*x + c)^3 - 6*d*\cosh(d*x + c)^2 + 6*(11*d*\cosh(d*x + c)^10 - 45*d*\cosh(d*x + c)^8 + 70*d*\cosh(d*x + c)^6 - 50*d*\cosh(d*x + c)^4 + 15*d*\cosh(d*x + c)^2 - d)*\sinh(d*x + c)^2 + 12*(d*\cosh(d*x + c)^11 - 5*d*\cosh(d*x + c)^9 + 10*d*\cosh(d*x + c)^7 - 10*d*\cosh(d*x + c)^5 + 5*d*\cosh(d*x + c)^3 - d*\cosh(d*x + c))*\sinh(d*x + c) + d)$

giac [B] time = 0.78, size = 217, normalized size = 2.11

$$3(a^3 + 3a^2b + 3ab^2 + b^3)(dx + c) - 3(a^3 + 3a^2b + 3ab^2 + b^3) \log(|e^{2dx+2c} - 1|) + \frac{2(9(a^3+2a^2b+ab^2)e^{10dx+10c}-1)}{3d}$$

3d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out] $-1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\log(\text{abs}(e^{(2*d*x + 2*c)} - 1)) + 2*(9*(a^3 + 2*a^2*b + a*b^2)*e^{(10*d*x + 10*c)} - 18*(a^3 + 3*a^2*b + 2*a*b^2)*e^{(8*d*x + 8*c)} + 2*(17*a^3 + 36*a^2*b + 27*a*b^2)*e^{(6*d*x + 6*c)} - 18*(a^3 + 3*a^2*b + 2*a*b^2)*e^{(4*d*x + 4*c)} + 9*(a^3 + 2*a^2*b + a*b^2)*e^{(2*d*x + 2*c)})/(e^{(2*d*x + 2*c)} - 1)^6)/d$

maple [A] time = 0.27, size = 161, normalized size = 1.56

$$\frac{a^3 \ln(\sinh(dx + c))}{d} - \frac{a^3 (\coth^2(dx + c))}{2d} - \frac{a^3 (\coth^4(dx + c))}{4d} - \frac{a^3 (\coth^6(dx + c))}{6d} + \frac{3a^2b \ln(\sinh(dx + c))}{d} - \frac{3a^2b \coth^2(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x)

[Out] $1/d*a^3*\ln(\sinh(d*x+c))-1/2*a^3*\coth(d*x+c)^2/d-1/4*a^3*\coth(d*x+c)^4/d-1/6*a^3*\coth(d*x+c)^6/d+3/d*a^2*b*\ln(\sinh(d*x+c))-3/2/d*a^2*b*\coth(d*x+c)^2-3/4/d*a^2*b*\coth(d*x+c)^4+3/d*a*b^2*\ln(\sinh(d*x+c))-3/2/d*a*b^2*\coth(d*x+c)^2+1/d*b^3*\ln(\sinh(d*x+c))$

maxima [B] time = 0.35, size = 420, normalized size = 4.08

$$\frac{1}{3}a^3\left(3x + \frac{3c}{d} + \frac{3 \log(e^{-dx-c} + 1)}{d} + \frac{3 \log(e^{-dx-c} - 1)}{d} + \frac{2(9e^{(-2dx-2c)} - 18e^{(-4dx-4c)} + 34e^{(-6dx-6c)} - 15e^{(-8dx-8c)} + 9e^{(-10dx-10c)} - 1)}{d(6e^{(-2dx-2c)} - 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} - 15e^{(-8dx-8c)} + 6e^{(-10dx-10c)} - 1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $\frac{1}{3}a^3(3x + 3c/d + 3\log(e^{-d*x - c} + 1)/d + 3\log(e^{-d*x - c} - 1)/d + 2(9e^{-2*d*x - 2*c} - 18e^{-4*d*x - 4*c} + 34e^{-6*d*x - 6*c} - 18e^{-8*d*x - 8*c} + 9e^{-10*d*x - 10*c}))/d + 3a^2b(x + c/d + \log(e^{-d*x - c} + 1)/d + \log(e^{-d*x - c} - 1)/d + 4(e^{-2*d*x - 2*c} - e^{-4*d*x - 4*c} + e^{-6*d*x - 6*c}))/d + 3ab^2(x + c/d + \log(e^{-d*x - c} + 1)/d + \log(e^{-d*x - c} - 1)/d + 2e^{-2*d*x - 2*c})/d + b^3\log(e^{d*x + c} - e^{-d*x - c})/d$

mupad [B] time = 0.32, size = 380, normalized size = 3.69

$$\frac{\ln(e^{2c} e^{2dx} - 1) (a^3 + 3a^2b + 3ab^2 + b^3)}{d} - \frac{4(11a^3 + 3ba^2)}{d(6e^{4c+4dx} - 4e^{2c+2dx} - 4e^{6c+6dx} + e^{8c+8dx} + 1)} - \frac{1}{3d(15e^{4c+4dx} - 4e^{2c+2dx} - 4e^{6c+6dx} + e^{8c+8dx} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^7*(a + b*tanh(c + d*x)^2)^3,x)

[Out] $(\log(\exp(2*c)*\exp(2*d*x) - 1)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/d - (4*(3*a^2*b + 11*a^3))/(d*(6*\exp(4*c + 4*d*x) - 4*\exp(2*c + 2*d*x) - 4*\exp(6*c + 6*d*x) + \exp(8*c + 8*d*x) + 1)) - (32*a^3)/(3*d*(15*\exp(4*c + 4*d*x) - 6*\exp(2*c + 2*d*x) - 20*\exp(6*c + 6*d*x) + 15*\exp(8*c + 8*d*x) - 6*\exp(10*c + 10*d*x) + \exp(12*c + 12*d*x) + 1)) - (6*(a*b^2 + 4*a^2*b + 3*a^3))/(d*(\exp(4*c + 4*d*x) - 2*\exp(2*c + 2*d*x) + 1)) - (6*(a*b^2 + 2*a^2*b + a^3))/(d*(\exp(2*c + 2*d*x) - 1)) - (8*(9*a^2*b + 13*a^3))/(3*d*(3*\exp(2*c + 2*d*x) - 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) - 1)) - (32*a^3)/(d*(5*\exp(2*c + 2*d*x) - 10*\exp(4*c + 4*d*x) + 10*\exp(6*c + 6*d*x) - 5*\exp(8*c + 8*d*x) + \exp(10*c + 10*d*x) - 1)) - x*(a + b)^3$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**7*(a+b*tanh(d*x+c)**2)**3,x)

[Out] Timed out

3.168 $\int (a + b \tanh^2(c + dx))^4 dx$

Optimal. Leaf size=110

$$\frac{b^2(6a^2 + 4ab + b^2) \tanh^3(c + dx)}{3d} - \frac{b(2a + b)(2a^2 + 2ab + b^2) \tanh(c + dx)}{d} - \frac{b^3(4a + b) \tanh^5(c + dx)}{5d} + x(a + b)$$

[Out] (a+b)^4*x-b*(2*a+b)*(2*a^2+2*a*b+b^2)*tanh(d*x+c)/d-1/3*b^2*(6*a^2+4*a*b+b^2)*tanh(d*x+c)^3/d-1/5*b^3*(4*a+b)*tanh(d*x+c)^5/d-1/7*b^4*tanh(d*x+c)^7/d

Rubi [A] time = 0.07, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3661, 390, 206}

$$\frac{b^2(6a^2 + 4ab + b^2) \tanh^3(c + dx)}{3d} - \frac{b(2a + b)(2a^2 + 2ab + b^2) \tanh(c + dx)}{d} - \frac{b^3(4a + b) \tanh^5(c + dx)}{5d} + x(a + b)$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tanh[c + d*x]^2)^4, x]

[Out] (a + b)^4*x - (b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tanh[c + d*x])/d - (b^2*(6*a^2 + 4*a*b + b^2)*Tanh[c + d*x]^3)/(3*d) - (b^3*(4*a + b)*Tanh[c + d*x]^5)/(5*d) - (b^4*Tanh[c + d*x]^7)/(7*d)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 3661

Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(ff*x)^n]^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$2*b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*\cosh(d*x + c) - 28*((105*a^3*b + 210*a^2*b^2 + 161*a*b^3 + 44*b^4)*\cosh(d*x + c)^6 + 5*(75*a^3*b + 120*a^2*b^2 + 71*a*b^3 + 14*b^4)*\cosh(d*x + c)^4 + 75*a^3*b + 90*a^2*b^2 + 75*a*b^3 + 9*(45*a^3*b + 60*a^2*b^2 + 41*a*b^3 + 14*b^4)*\cosh(d*x + c)^2*\sinh(d*x + c))/(d*\cosh(d*x + c)^7 + 7*d*\cosh(d*x + c)*\sinh(d*x + c)^6 + 7*d*\cosh(d*x + c)^5 + 35*(d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c)^4 + 21*d*\cosh(d*x + c)^3 + 7*(3*d*\cosh(d*x + c)^5 + 10*d*\cosh(d*x + c)^3 + 9*d*\cosh(d*x + c))*\sinh(d*x + c)^2 + 35*d*\cosh(d*x + c))$$

giac [B] time = 0.22, size = 447, normalized size = 4.06

$$105 \left(a^4 + 4 a^3 b + 6 a^2 b^2 + 4 a b^3 + b^4 \right) (d x + c) + \frac{8 \left(105 a^3 b e^{(12 d x + 12 c)} + 315 a^2 b^2 e^{(12 d x + 12 c)} + 315 a b^3 e^{(12 d x + 12 c)} + 105 b^4 e^{(12 d x + 12 c)} + 630 a^3 b e^{(10 d x + 10 c)} + 1575 a^2 b^2 e^{(10 d x + 10 c)} + 1260 a b^3 e^{(10 d x + 10 c)} + 315 b^4 e^{(10 d x + 10 c)} + 1575 a^3 b e^{(8 d x + 8 c)} + 3360 a^2 b^2 e^{(8 d x + 8 c)} + 255 5 a b^3 e^{(8 d x + 8 c)} + 770 b^4 e^{(8 d x + 8 c)} + 2100 a^3 b e^{(6 d x + 6 c)} + 3990 a^2 b^2 e^{(6 d x + 6 c)} + 3080 a b^3 e^{(6 d x + 6 c)} + 770 b^4 e^{(6 d x + 6 c)} + 1575 a^3 b e^{(4 d x + 4 c)} + 2835 a^2 b^2 e^{(4 d x + 4 c)} + 2121 a b^3 e^{(4 d x + 4 c)} + 609 b^4 e^{(4 d x + 4 c)} + 630 a^3 b e^{(2 d x + 2 c)} + 1155 a^2 b^2 e^{(2 d x + 2 c)} + 812 a b^3 e^{(2 d x + 2 c)} + 203 b^4 e^{(2 d x + 2 c)} + 105 a^3 b + 210 a^2 b^2 + 161 a b^3 + 44 b^4 \right) / \left(e^{(2 d x + 2 c)} + 1 \right)^7 / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(d*x+c)^2)^4,x, algorithm="giac")

[Out] 1/105*(105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(d*x + c) + 8*(105*a^3*b*e^(12*d*x + 12*c) + 315*a^2*b^2*e^(12*d*x + 12*c) + 315*a*b^3*e^(12*d*x + 12*c) + 105*b^4*e^(12*d*x + 12*c) + 630*a^3*b*e^(10*d*x + 10*c) + 1575*a^2*b^2*e^(10*d*x + 10*c) + 1260*a*b^3*e^(10*d*x + 10*c) + 315*b^4*e^(10*d*x + 10*c) + 1575*a^3*b*e^(8*d*x + 8*c) + 3360*a^2*b^2*e^(8*d*x + 8*c) + 255 5*a*b^3*e^(8*d*x + 8*c) + 770*b^4*e^(8*d*x + 8*c) + 2100*a^3*b*e^(6*d*x + 6*c) + 3990*a^2*b^2*e^(6*d*x + 6*c) + 3080*a*b^3*e^(6*d*x + 6*c) + 770*b^4*e^(6*d*x + 6*c) + 1575*a^3*b*e^(4*d*x + 4*c) + 2835*a^2*b^2*e^(4*d*x + 4*c) + 2121*a*b^3*e^(4*d*x + 4*c) + 609*b^4*e^(4*d*x + 4*c) + 630*a^3*b*e^(2*d*x + 2*c) + 1155*a^2*b^2*e^(2*d*x + 2*c) + 812*a*b^3*e^(2*d*x + 2*c) + 203*b^4*e^(2*d*x + 2*c) + 105*a^3*b + 210*a^2*b^2 + 161*a*b^3 + 44*b^4)/(e^(2*d*x + 2*c) + 1)^7)/d

maple [B] time = 0.02, size = 344, normalized size = 3.13

$$\frac{\ln(1 + \tanh(dx + c)) a^4}{2d} + \frac{2 \ln(1 + \tanh(dx + c)) a^3 b}{d} + \frac{3 \ln(1 + \tanh(dx + c)) a^2 b^2}{d} + \frac{2 \ln(1 + \tanh(dx + c)) a b^3}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*tanh(d*x+c)^2)^4,x)

[Out] 1/2/d*ln(1+tanh(d*x+c))*a^4+2/d*ln(1+tanh(d*x+c))*a^3*b+3/d*ln(1+tanh(d*x+c))*a^2*b^2+2/d*ln(1+tanh(d*x+c))*a*b^3+1/2/d*ln(1+tanh(d*x+c))*b^4-1/2/d*ln(tanh(d*x+c)-1)*a^4-2/d*ln(tanh(d*x+c)-1)*a^3*b-3/d*ln(tanh(d*x+c)-1)*a^2*b^2-2/d*ln(tanh(d*x+c)-1)*a*b^3-1/2/d*ln(tanh(d*x+c)-1)*b^4-1/5/d*tanh(d*x+c)^5*b^4-1/d*b^4*tanh(d*x+c)-1/3/d*tanh(d*x+c)^3*b^4-1/7*b^4*tanh(d*x+c)^7/d-2/d*tanh(d*x+c)^3*a^2*b^2-4/3/d*tanh(d*x+c)^3*a*b^3-4/5/d*tanh(d*x+c)^5*a*b^3-4/d*a*b^3*tanh(d*x+c)-4/d*a^3*b*tanh(d*x+c)-6/d*a^2*b^2*tanh(d*x+c)

maxima [B] time = 0.35, size = 410, normalized size = 3.73

$$\frac{1}{105} b^4 \left(105 x + \frac{105 c}{d} - \frac{8 \left(203 e^{(-2 d x - 2 c)} + 609 e^{(-4 d x - 4 c)} + 770 e^{(-6 d x - 6 c)} + 770 e^{(-8 d x - 8 c)} + 315 e^{(-10 d x - 10 c)} + 105 e^{(-12 d x - 12 c)} \right)}{d \left(7 e^{(-2 d x - 2 c)} + 21 e^{(-4 d x - 4 c)} + 35 e^{(-6 d x - 6 c)} + 35 e^{(-8 d x - 8 c)} + 21 e^{(-10 d x - 10 c)} + 7 e^{(-12 d x - 12 c)} \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(d*x+c)^2)^4,x, algorithm="maxima")

[Out] 1/105*b^4*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*e^(-4*d*x - 4*c) + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-10*d*x - 10*c) + 105*e^(-12*d*x - 12*c)) / (7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c)))

$$05e^{(-12dx - 12c)} + 44) / (d(7e^{(-2dx - 2c)} + 21e^{(-4dx - 4c)} + 35e^{(-6dx - 6c)} + 35e^{(-8dx - 8c)} + 21e^{(-10dx - 10c)} + 7e^{(-12dx - 12c)} + e^{(-14dx - 14c)} + 1))) + 4/15ab^3(15x + 15c/d - 2(70e^{(-2dx - 2c)} + 140e^{(-4dx - 4c)} + 90e^{(-6dx - 6c)} + 45e^{(-8dx - 8c)} + 23) / (d(5e^{(-2dx - 2c)} + 10e^{(-4dx - 4c)} + 10e^{(-6dx - 6c)} + 5e^{(-8dx - 8c)} + e^{(-10dx - 10c)} + 1))) + 2a^2b^2(3x + 3c/d - 4(3e^{(-2dx - 2c)} + 3e^{(-4dx - 4c)} + 2) / (d(3e^{(-2dx - 2c)} + 3e^{(-4dx - 4c)} + e^{(-6dx - 6c)} + 1))) + 4a^3b(x + c/d - 2 / (d(e^{(-2dx - 2c)} + 1))) + a^4x$$

mupad [B] time = 0.20, size = 133, normalized size = 1.21

$$x(a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4) - \frac{\tanh(c + dx)^3(6a^2b^2 + 4ab^3 + b^4)}{3d} - \frac{\tanh(c + dx)^5(b^4 + 4ab^3)}{5d} - \frac{b^4}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tanh(c + d*x))^2)^4, x)

[Out] x*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2) - (tanh(c + d*x)^3*(4*a*b^3 + b^4 + 6*a^2*b^2))/(3*d) - (tanh(c + d*x)^5*(4*a*b^3 + b^4))/(5*d) - (b^4*tanh(c + d*x)^7)/(7*d) - (b*tanh(c + d*x)*(4*a*b^2 + 6*a^2*b + 4*a^3 + b^3))/d

sympy [A] time = 1.28, size = 209, normalized size = 1.90

$$\left\{ \begin{array}{l} a^4x + 4a^3bx - \frac{4a^3b \tanh(c+dx)}{d} + 6a^2b^2x - \frac{2a^2b^2 \tanh^3(c+dx)}{d} - \frac{6a^2b^2 \tanh(c+dx)}{d} + 4ab^3x - \frac{4ab^3 \tanh^5(c+dx)}{5d} - \frac{4ab^3 \tanh^3(c+dx)}{3d} \\ x(a + b \tanh^2(c))^4 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(d*x+c)**2)**4, x)

[Out] Piecewise((a**4*x + 4*a**3*b*x - 4*a**3*b*tanh(c + d*x)/d + 6*a**2*b**2*x - 2*a**2*b**2*tanh(c + d*x)**3/d - 6*a**2*b**2*tanh(c + d*x)/d + 4*a*b**3*x - 4*a*b**3*tanh(c + d*x)**5/(5*d) - 4*a*b**3*tanh(c + d*x)**3/(3*d) - 4*a*b**3*tanh(c + d*x)/d + b**4*x - b**4*tanh(c + d*x)**7/(7*d) - b**4*tanh(c + d*x)**5/(5*d) - b**4*tanh(c + d*x)**3/(3*d) - b**4*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**4, True))

3.169 $\int (a + b \tanh^2(c + dx))^5 dx$

Optimal. Leaf size=160

$$\frac{b^3 (10a^2 + 5ab + b^2) \tanh^5(c + dx)}{5d} - \frac{b^2 (10a^3 + 10a^2b + 5ab^2 + b^3) \tanh^3(c + dx)}{3d} - \frac{b (5a^4 + 10a^3b + 10a^2b^2 + 5a^2b + 5ab^2 + b^3) \tanh(c + dx)}{d}$$

[Out] (a+b)^5*x-b*(5*a^4+10*a^3*b+10*a^2*b^2+5*a*b^3+b^4)*tanh(d*x+c)/d-1/3*b^2*(10*a^3+10*a^2*b+5*a*b^2+b^3)*tanh(d*x+c)^3/d-1/5*b^3*(10*a^2+5*a*b+b^2)*tanh(d*x+c)^5/d-1/7*b^4*(5*a+b)*tanh(d*x+c)^7/d-1/9*b^5*tanh(d*x+c)^9/d

Rubi [A] time = 0.09, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3661, 390, 206}

$$\frac{b^3 (10a^2 + 5ab + b^2) \tanh^5(c + dx)}{5d} - \frac{b^2 (10a^2b + 10a^3 + 5ab^2 + b^3) \tanh^3(c + dx)}{3d} - \frac{b (10a^2b^2 + 10a^3b + 5a^4 + 5a^2b + 5ab^2 + b^3) \tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tanh[c + d*x]^2)^5, x]

[Out] (a + b)^5*x - (b*(5*a^4 + 10*a^3*b + 10*a^2*b^2 + 5*a*b^3 + b^4)*Tanh[c + d*x])/d - (b^2*(10*a^3 + 10*a^2*b + 5*a*b^2 + b^3)*Tanh[c + d*x]^3)/(3*d) - (b^3*(10*a^2 + 5*a*b + b^2)*Tanh[c + d*x]^5)/(5*d) - (b^4*(5*a + b)*Tanh[c + d*x]^7)/(7*d) - (b^5*Tanh[c + d*x]^9)/(9*d)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 390

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 3661

Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(ff*x)^n]^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$200a^3b^2 + 4830a^2b^3 + 2640ab^4 + 563b^5 + 315(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5)dx) \cosh(dx + c)^5 + 35(1575a^4b + 4200a^3b^2 + 4830a^2b^3 + 2640ab^4 + 563b^5 + 315(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5)dx) \cosh(dx + c)^3 + 20(1575a^4b + 4200a^3b^2 + 4830a^2b^3 + 2640ab^4 + 563b^5 + 315(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5)dx) \cosh(dx + c) \sinh(dx + c)^4 + 84(1575a^4b + 4200a^3b^2 + 4830a^2b^3 + 2640ab^4 + 563b^5 + 315(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5)dx) \cosh(dx + c)^3 - 3(28(1575a^4b + 4200a^3b^2 + 4830a^2b^3 + 2640ab^4 + 563b^5) \cosh(dx + c)^6 + 14700a^4b + 26600a^3b^2 + 27440a^2b^3 + 13720ab^4 + 1764b^5 + 105(1225a^4b + 2800a^3b^2 + 2730a^2b^3 + 1240ab^4 + 213b^5) \cosh(dx + c)^4 + 120(875a^4b + 1750a^3b^2 + 1680a^2b^3 + 890ab^4 + 213b^5) \cosh(dx + c)^2) \sinh(dx + c)^3 + 9(4(1575a^4b + 4200a^3b^2 + 4830a^2b^3 + 2640ab^4 + 563b^5 + 315(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5)dx) \cosh(dx + c)^7 + 21(1575a^4b + 4200a^3b^2 + 4830a^2b^3 + 2640ab^4 + 563b^5 + 315(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5)dx) \cosh(dx + c)^5 + 40(1575a^4b + 4200a^3b^2 + 4830a^2b^3 + 2640ab^4 + 563b^5 + 315(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5)dx) \cosh(dx + c)^3 + 28(1575a^4b + 4200a^3b^2 + 4830a^2b^3 + 2640ab^4 + 563b^5 + 315(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5)dx) \cosh(dx + c)) \sinh(dx + c)^2 + 126(1575a^4b + 4200a^3b^2 + 4830a^2b^3 + 2640ab^4 + 563b^5 + 315(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5)dx) \cosh(dx + c) - 9((1575a^4b + 4200a^3b^2 + 4830a^2b^3 + 2640ab^4 + 563b^5) \cosh(dx + c)^8 + 7(1225a^4b + 2800a^3b^2 + 2730a^2b^3 + 1240ab^4 + 213b^5) \cosh(dx + c)^6 + 2450a^4b + 4200a^3b^2 + 4620a^2b^3 + 1960ab^4 + 882b^5 + 20(875a^4b + 1750a^3b^2 + 1680a^2b^3 + 890ab^4 + 213b^5) \cosh(dx + c)^4 + 28(525a^4b + 950a^3b^2 + 980a^2b^3 + 490ab^4 + 63b^5) \cosh(dx + c)^2) \sinh(dx + c)) / (d \cosh(dx + c)^9 + 9d \cosh(dx + c) \sinh(dx + c)^8 + 9d \cosh(dx + c)^7 + 21(4d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^6 + 36d \cosh(dx + c)^5 + 9(14d \cosh(dx + c)^5 + 35d \cosh(dx + c)^3 + 20d \cosh(dx + c)) \sinh(dx + c)^4 + 84d \cosh(dx + c)^3 + 9(4d \cosh(dx + c)^7 + 21d \cosh(dx + c)^5 + 40d \cosh(dx + c)^3 + 28d \cosh(dx + c)) \sinh(dx + c)^2 + 126d \cosh(dx + c))$$

giac [B] time = 0.31, size = 721, normalized size = 4.51

$$315(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5)(dx + c) + \frac{2(1575a^4be^{16dx+16c}) + 6300a^3b^2e^{16dx+16c} + 9450a^2b^3e^{16dx+16c} + 6300ab^4e^{16dx+16c} + 1575b^5e^{16dx+16c}}{d \cosh(dx + c)^9 + 9d \cosh(dx + c) \sinh(dx + c)^8 + 9d \cosh(dx + c)^7 + 21(4d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^6 + 36d \cosh(dx + c)^5 + 9(14d \cosh(dx + c)^5 + 35d \cosh(dx + c)^3 + 20d \cosh(dx + c)) \sinh(dx + c)^4 + 84d \cosh(dx + c)^3 + 9(4d \cosh(dx + c)^7 + 21d \cosh(dx + c)^5 + 40d \cosh(dx + c)^3 + 28d \cosh(dx + c)) \sinh(dx + c)^2 + 126d \cosh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(dx+c)^2)^5,x, algorithm="giac")

[Out] 1/315*(315*(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5)*(dx + c) + 2*(1575a^4b*e^(16*dx + 16*c) + 6300a^3b^2*e^(16*dx + 16*c) + 9450a^2b^3*e^(16*dx + 16*c) + 6300a*b^4*e^(16*dx + 16*c) + 1575b^5*e^(16*dx + 16*c) + 12600a^4b*e^(14*dx + 14*c) + 44100a^3b^2*e^(14*dx + 14*c) + 56700a^2b^3*e^(14*dx + 14*c) + 31500a*b^4*e^(14*dx + 14*c) + 6300b^5*e^(14*dx + 14*c) + 44100a^4b*e^(12*dx + 12*c) + 136500a^3b^2*e^(12*dx + 12*c) + 161700a^2b^3*e^(12*dx + 12*c) + 90300a*b^4*e^(12*dx + 12*c) + 21000b^5*e^(12*dx + 12*c) + 88200a^4b*e^(10*dx + 10*c) + 245700a^3b^2*e^(10*dx + 10*c) + 283500a^2b^3*e^(10*dx + 10*c) + 157500a*b^4*e^(10*dx + 10*c) + 31500b^5*e^(10*dx + 10*c) + 110250a^4b*e^(8*dx + 8*c) + 283500a^3b^2*e^(8*dx + 8*c) + 325080a^2b^3*e^(8*dx + 8*c) + 175140a*b^4*e^(8*dx + 8*c) + 39438b^5*e^(8*dx + 8*c) + 88200a^4b*e^(6*dx + 6*c) + 216300a^3b^2*e^(6*dx + 6*c) + 244020a^2b^3*e^(6*dx + 6*c) + 131460a*b^4*e^(6*dx + 6*c) + 26292b^5*e^(6*dx + 6*c) + 44100a^

$$4*b*e^{(4*d*x + 4*c)} + 107100*a^3*b^2*e^{(4*d*x + 4*c)} + 117180*a^2*b^3*e^{(4*d*x + 4*c)} + 63540*a*b^4*e^{(4*d*x + 4*c)} + 13968*b^5*e^{(4*d*x + 4*c)} + 12600*a^4*b*e^{(2*d*x + 2*c)} + 31500*a^3*b^2*e^{(2*d*x + 2*c)} + 34020*a^2*b^3*e^{(2*d*x + 2*c)} + 17460*a*b^4*e^{(2*d*x + 2*c)} + 3492*b^5*e^{(2*d*x + 2*c)} + 1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5)/(e^{(2*d*x + 2*c)} + 1)^9/d$$

maple [B] time = 0.02, size = 472, normalized size = 2.95

$$\frac{5 \ln(1 + \tanh(dx + c)) a^3 b^2}{d} + \frac{5 \ln(1 + \tanh(dx + c)) a^2 b^3}{d} + \frac{5 \ln(1 + \tanh(dx + c)) a b^4}{2d} - \frac{5 \ln(\tanh(dx + c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*tanh(d*x+c)^2)^5,x)

[Out] $-2/d*\tanh(d*x+c)^5*a^2*b^3+5/d*\ln(1+\tanh(d*x+c))*a^3*b^2+5/d*\ln(1+\tanh(d*x+c))*a^2*b^3+5/2/d*\ln(1+\tanh(d*x+c))*a*b^4-5/2/d*\ln(\tanh(d*x+c)-1)*a^4*b-5/7/d*\tanh(d*x+c)^7*a*b^4-10/d*a^2*b^3*\tanh(d*x+c)-5/d*a*b^4*\tanh(d*x+c)-10/d*a^3*b^2*\tanh(d*x+c)+5/2/d*\ln(1+\tanh(d*x+c))*a^4*b-5/d*a^4*b*\tanh(d*x+c)-5/d*\ln(\tanh(d*x+c)-1)*a^3*b^2-5/d*\ln(\tanh(d*x+c)-1)*a^2*b^3-5/2/d*\ln(\tanh(d*x+c)-1)*a*b^4-5/3/d*\tanh(d*x+c)^3*a*b^4-10/3/d*\tanh(d*x+c)^3*a^2*b^3-10/3/d*\tanh(d*x+c)^3*a^3*b^2-1/d*\tanh(d*x+c)^5*a*b^4-1/9*b^5*\tanh(d*x+c)^9/d+1/2/d*\ln(1+\tanh(d*x+c))*a^5+1/2/d*\ln(1+\tanh(d*x+c))*b^5-1/2/d*\ln(\tanh(d*x+c)-1)*a^5-1/d*b^5*\tanh(d*x+c)-1/5/d*\tanh(d*x+c)^5*b^5-1/3/d*\tanh(d*x+c)^3*b^5-1/7/d*\tanh(d*x+c)^7*b^5-1/2/d*\ln(\tanh(d*x+c)-1)*b^5$

maxima [B] time = 0.36, size = 624, normalized size = 3.90

$$\frac{1}{315} b^5 \left(315x + \frac{315c}{d} - \frac{2(3492e^{(-2dx-2c)} + 13968e^{(-4dx-4c)} + 26292e^{(-6dx-6c)} + 39438e^{(-8dx-8c)} + 31500e^{(-10dx-10c)})}{d(9e^{(-2dx-2c)} + 36e^{(-4dx-4c)} + 84e^{(-6dx-6c)} + 126e^{(-8dx-8c)} + 126e^{(-10dx-10c)})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(d*x+c)^2)^5,x, algorithm="maxima")

[Out] $1/315*b^5*(315*x + 315*c/d - 2*(3492*e^{(-2*d*x - 2*c)} + 13968*e^{(-4*d*x - 4*c)} + 26292*e^{(-6*d*x - 6*c)} + 39438*e^{(-8*d*x - 8*c)} + 31500*e^{(-10*d*x - 10*c)} + 21000*e^{(-12*d*x - 12*c)} + 6300*e^{(-14*d*x - 14*c)} + 1575*e^{(-16*d*x - 16*c)} + 563)/(d*(9*e^{(-2*d*x - 2*c)} + 36*e^{(-4*d*x - 4*c)} + 84*e^{(-6*d*x - 6*c)} + 126*e^{(-8*d*x - 8*c)} + 126*e^{(-10*d*x - 10*c)} + 84*e^{(-12*d*x - 12*c)} + 36*e^{(-14*d*x - 14*c)} + 9*e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} + 1))) + 1/21*a*b^4*(105*x + 105*c/d - 8*(203*e^{(-2*d*x - 2*c)} + 609*e^{(-4*d*x - 4*c)} + 770*e^{(-6*d*x - 6*c)} + 770*e^{(-8*d*x - 8*c)} + 315*e^{(-10*d*x - 10*c)} + 105*e^{(-12*d*x - 12*c)} + 44)/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1))) + 2/3*a^2*b^3*(15*x + 15*c/d - 2*(70*e^{(-2*d*x - 2*c)} + 140*e^{(-4*d*x - 4*c)} + 90*e^{(-6*d*x - 6*c)} + 45*e^{(-8*d*x - 8*c)} + 23)/(d*(5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} + 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} + 1))) + 10/3*a^3*b^2*(3*x + 3*c/d - 4*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + 2)/(d*(3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} + 1))) + 5*a^4*b*(x + c/d - 2/(d*(e^{(-2*d*x - 2*c)} + 1))) + a^5*x$

mupad [B] time = 1.32, size = 188, normalized size = 1.18

$$x(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5) - \frac{\tanh(c + dx)^3(10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5)}{3d} - \frac{\tanh(c + dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(c + d*x)^2)^5,x)
```

```
[Out] x*(5*a*b^4 + 5*a^4*b + a^5 + b^5 + 10*a^2*b^3 + 10*a^3*b^2) - (tanh(c + d*x)
)^3*(5*a*b^4 + b^5 + 10*a^2*b^3 + 10*a^3*b^2))/(3*d) - (tanh(c + d*x)^5*(5*
a*b^4 + b^5 + 10*a^2*b^3))/(5*d) - (tanh(c + d*x)^7*(5*a*b^4 + b^5))/(7*d)
- (b^5*tanh(c + d*x)^9)/(9*d) - (b*tanh(c + d*x)*(5*a*b^3 + 10*a^3*b + 5*a^
4 + b^4 + 10*a^2*b^2))/d
```

sympy [A] time = 2.32, size = 308, normalized size = 1.92

$$\left\{ \begin{array}{l} a^5x + 5a^4bx - \frac{5a^4b \tanh(c+dx)}{d} + 10a^3b^2x - \frac{10a^3b^2 \tanh^3(c+dx)}{3d} - \frac{10a^3b^2 \tanh(c+dx)}{d} + 10a^2b^3x - \frac{2a^2b^3 \tanh^5(c+dx)}{d} - \frac{10a^2b^3}{d} \\ x(a + b \tanh^2(c))^5 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(d*x+c)**2)**5,x)
```

```
[Out] Piecewise((a**5*x + 5*a**4*b*x - 5*a**4*b*tanh(c + d*x)/d + 10*a**3*b**2*x
- 10*a**3*b**2*tanh(c + d*x)**3/(3*d) - 10*a**3*b**2*tanh(c + d*x)/d + 10*a
**2*b**3*x - 2*a**2*b**3*tanh(c + d*x)**5/d - 10*a**2*b**3*tanh(c + d*x)**3
/(3*d) - 10*a**2*b**3*tanh(c + d*x)/d + 5*a*b**4*x - 5*a*b**4*tanh(c + d*x)
**7/(7*d) - a*b**4*tanh(c + d*x)**5/d - 5*a*b**4*tanh(c + d*x)**3/(3*d) - 5
*a*b**4*tanh(c + d*x)/d + b**5*x - b**5*tanh(c + d*x)**9/(9*d) - b**5*tanh(
c + d*x)**7/(7*d) - b**5*tanh(c + d*x)**5/(5*d) - b**5*tanh(c + d*x)**3/(3*
d) - b**5*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**5, True))
```

$$3.170 \quad \int \frac{\tanh^5(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=66

$$\frac{a^2 \log(a + b \tanh^2(c + dx))}{2b^2 d(a + b)} + \frac{\log(\cosh(c + dx))}{d(a + b)} - \frac{\tanh^2(c + dx)}{2bd}$$

[Out] $\ln(\cosh(d*x+c))/(a+b)/d+1/2*a^2*\ln(a+b*\tanh(d*x+c)^2)/b^2/(a+b)/d-1/2*\tanh(d*x+c)^2/b/d$

Rubi [A] time = 0.11, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 446, 72}

$$\frac{a^2 \log(a + b \tanh^2(c + dx))}{2b^2 d(a + b)} + \frac{\log(\cosh(c + dx))}{d(a + b)} - \frac{\tanh^2(c + dx)}{2bd}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2), x]

[Out] $\text{Log}[\text{Cosh}[c + d*x]]/((a + b)*d) + (a^2*\text{Log}[a + b*\text{Tanh}[c + d*x]^2])/((2*b^2*(a + b)*d) - \text{Tanh}[c + d*x]^2/(2*b*d)$

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{x^5}{(1-x^2)(a+bx^2)} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)(a+bx)} dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{b} - \frac{1}{(a+b)(-1+x)} + \frac{a^2}{b(a+b)(a+bx)}\right) dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\log(\cosh(c+dx))}{(a+b)d} + \frac{a^2 \log(a+b\tanh^2(c+dx))}{2b^2(a+b)d} - \frac{\tanh^2(c+dx)}{2bd}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 60, normalized size = 0.91

$$\frac{-\frac{a^2 \log(a+b \tanh^2(c+dx))}{b^2(a+b)} - \frac{2 \log(\cosh(c+dx))}{a+b} + \frac{\tanh^2(c+dx)}{b}}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2), x]

[Out] -1/2*((-2*Log[Cosh[c + d*x]])/(a + b) - (a^2*Log[a + b*Tanh[c + d*x]^2])/(b^2*(a + b)) + Tanh[c + d*x]^2/b)/d

fricas [B] time = 0.52, size = 742, normalized size = 11.24

$$2b^2 dx \cosh(dx+c)^4 + 8b^2 dx \cosh(dx+c) \sinh(dx+c)^3 + 2b^2 dx \sinh(dx+c)^4 + 2b^2 dx + 4(b^2 dx - ab - b^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] -1/2*(2*b^2*d*x*cosh(d*x + c)^4 + 8*b^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*b^2*d*x*sinh(d*x + c)^4 + 2*b^2*d*x + 4*(b^2*d*x - a*b - b^2)*cosh(d*x + c)^2 + 4*(3*b^2*d*x*cosh(d*x + c)^2 + b^2*d*x - a*b - b^2)*sinh(d*x + c)^2 - (a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4 + 2*a^2*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x + c))*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 2*((a^2 - b^2)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(b^2*d*x*cosh(d*x + c)^3 + (b^2*d*x - a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))/((a*b^2 + b^3)*d*cosh(d*x + c)^4 + 4*(a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b^2 + b^3)*d*sinh(d*x + c)^4 + 2*(a*b^2 + b^3)*d*cosh(d*x + c)^2 + 2*(3*(a*b^2 + b^3)*d*cosh(d*x + c)^2 + (a*b^2 + b^3)*d)*sinh(d*x + c)^2 + (a*b^2 + b^3)*d + 4*((a*b^2 + b^3)*d*cosh(d*x + c)^3 + (a*b^2 + b^3)*d*cosh(d*x + c))*sinh(d*x + c))

giac [B] time = 0.26, size = 132, normalized size = 2.00

$$\frac{a^2 \log(ae^{(4dx+4c)} + be^{(4dx+4c)} + 2ae^{(2dx+2c)} - 2be^{(2dx+2c)} + a + b)}{ab^2 + b^3} - \frac{2(dx+c)}{a+b} - \frac{2(a-b) \log(e^{(2dx+2c)} + 1)}{b^2} + \frac{4e^{(2dx+2c)}}{b(e^{(2dx+2c)} + 1)^2}$$

2d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] $\frac{1}{2}*(a^2*\log(a*e^{(4*d*x + 4*c)} + b*e^{(4*d*x + 4*c)} + 2*a*e^{(2*d*x + 2*c)} - 2*b*e^{(2*d*x + 2*c)} + a + b)/(a*b^2 + b^3) - 2*(d*x + c)/(a + b) - 2*(a - b)*\log(e^{(2*d*x + 2*c)} + 1)/b^2 + 4*e^{(2*d*x + 2*c)}/(b*(e^{(2*d*x + 2*c)} + 1)^2))/d$

maple [A] time = 0.09, size = 93, normalized size = 1.41

$$\frac{\tanh^2(dx+c)}{2bd} - \frac{\ln(\tanh(dx+c)-1)}{d(2b+2a)} - \frac{\ln(1+\tanh(dx+c))}{d(2b+2a)} + \frac{a^2 \ln(a+b(\tanh^2(dx+c)))}{2b^2(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2),x)

[Out] $-1/2*\tanh(d*x+c)^2/b/d - 1/d/(2*b+2*a)*\ln(\tanh(d*x+c)-1) - 1/d/(2*b+2*a)*\ln(1+\tanh(d*x+c)) + 1/2*a^2*\ln(a+b*\tanh(d*x+c)^2)/b^2/(a+b)/d$

maxima [B] time = 0.43, size = 133, normalized size = 2.02

$$\frac{a^2 \log(2(a-b)e^{(-2dx-2c)} + (a+b)e^{(-4dx-4c)} + a+b)}{2(ab^2+b^3)d} + \frac{dx+c}{(a+b)d} + \frac{2e^{(-2dx-2c)}}{(2be^{(-2dx-2c)} + be^{(-4dx-4c)} + b)d} - \frac{(a-b)\log(\dots)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] $\frac{1}{2}*a^2*\log(2*(a-b)*e^{(-2*d*x-2*c)} + (a+b)*e^{(-4*d*x-4*c)} + a+b)/((a*b^2+b^3)*d) + (d*x+c)/((a+b)*d) + 2*e^{(-2*d*x-2*c)}/((2*b*e^{(-2*d*x-2*c)} + b*e^{(-4*d*x-4*c)} + b)*d) - (a-b)*\log(e^{(-2*d*x-2*c)} + 1)/(b^2*d)$

mupad [B] time = 0.27, size = 72, normalized size = 1.09

$$\frac{b^2 \left(\ln(\tanh(c+dx)+1) - dx + \frac{\tanh(c+dx)^2}{2} \right) - \frac{a^2 \ln(b \tanh(c+dx)^2 + a)}{2} + \frac{ab \tanh(c+dx)^2}{2}}{b^2 d (a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c+d*x)^5/(a+b*tanh(c+d*x)^2),x)

[Out] $-(b^2*(\log(\tanh(c+d*x)+1) - d*x + \tanh(c+d*x)^2/2) - (a^2*\log(a+b*\tanh(c+d*x)^2))/2 + (a*b*\tanh(c+d*x)^2)/2)/(b^2*d*(a+b))$

sympy [A] time = 13.46, size = 425, normalized size = 6.44

$$\left\{ \begin{array}{l} \infty x \tanh^3(c) \\ x \frac{\log(\tanh(c+dx)+1) - \frac{\tanh^4(c+dx)}{4d} - \frac{\tanh^2(c+dx)}{2d}}{d} \\ \frac{4dx \tanh^2(c+dx)}{2bd \tanh^2(c+dx) - 2bd} - \frac{4dx}{2bd \tanh^2(c+dx) - 2bd} - \frac{4 \log(\tanh(c+dx)+1) \tanh^2(c+dx)}{2bd \tanh^2(c+dx) - 2bd} + \frac{4 \log(\tanh(c+dx)+1)}{2bd \tanh^2(c+dx) - 2bd} - \frac{\tanh^4(c+dx)}{2bd \tanh^2(c+dx) - 2bd} \\ \frac{x \tanh^5(c)}{a+b \tanh^2(c)} \\ \frac{a^2 \log\left(-i\sqrt{a}\sqrt{\frac{1}{b}} + \tanh(c+dx)\right)}{2ab^2d+2b^3d} + \frac{a^2 \log\left(i\sqrt{a}\sqrt{\frac{1}{b}} + \tanh(c+dx)\right)}{2ab^2d+2b^3d} - \frac{ab \tanh^2(c+dx)}{2ab^2d+2b^3d} + \frac{2b^2 dx}{2ab^2d+2b^3d} - \frac{2b^2 \log(\tanh(c+dx)+1)}{2ab^2d+2b^3d} - \frac{b^2 \tanh^2(c+dx)}{2ab^2d+2b^3d} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**5/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x*tanh(c)**3, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - log(tanh(c + d*x) + 1)/d - tanh(c + d*x)**4/(4*d) - tanh(c + d*x)**2/(2*d))/a, Eq(b, 0)), (4*d*x*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 4*d*x/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 4*log(tanh(c + d*x) + 1)*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + 4*log(tanh(c + d*x) + 1)/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - tanh(c + d*x)**4/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + 2/(2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x*tanh(c)**5/(a + b*tanh(c)**2), Eq(d, 0)), (a**2*log(-I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))/(2*a*b**2*d + 2*b**3*d) + a**2*log(I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))/(2*a*b**2*d + 2*b**3*d) - a*b*tanh(c + d*x)**2/(2*a*b**2*d + 2*b**3*d) + 2*b**2*d*x/(2*a*b**2*d + 2*b**3*d) - 2*b**2*log(tanh(c + d*x) + 1)/(2*a*b**2*d + 2*b**3*d) - b**2*tanh(c + d*x)**2/(2*a*b**2*d + 2*b**3*d), True))
```


$$3.171 \quad \int \frac{\tanh^4(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=59

$$\frac{a^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}d(a+b)} + \frac{x}{a+b} - \frac{\tanh(c+dx)}{bd}$$

[Out] x/(a+b)+a^(3/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/b^(3/2)/(a+b)/d-tanh(d*x+c)/b/d

Rubi [A] time = 0.11, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3670, 479, 522, 206, 205}

$$\frac{a^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}d(a+b)} + \frac{x}{a+b} - \frac{\tanh(c+dx)}{bd}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]

[Out] x/(a + b) + (a^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(b^(3/2)*(a + b)*d) - Tanh[c + d*x]/(b*d)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 479

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n

, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned} \int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a+bx^2)} dx, x, \tanh(c+dx)\right)}{d} \\ &= -\frac{\tanh(c+dx)}{bd} + \frac{\text{Subst}\left(\int \frac{a+(-a+b)x^2}{(1-x^2)(a+bx^2)} dx, x, \tanh(c+dx)\right)}{bd} \\ &= -\frac{\tanh(c+dx)}{bd} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c+dx)\right)}{(a+b)d} + \frac{a^2 \text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \tanh(c+dx)\right)}{b(a+b)d} \\ &= \frac{x}{a+b} + \frac{a^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}(a+b)d} - \frac{\tanh(c+dx)}{bd} \end{aligned}$$

Mathematica [A] time = 0.20, size = 66, normalized size = 1.12

$$\frac{a^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}d(a+b)} + \frac{c+dx}{d(a+b)} - \frac{\tanh(c+dx)}{bd}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2), x]

[Out] (c + d*x)/((a + b)*d) + (a^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(b^(3/2)*(a + b)*d) - Tanh[c + d*x]/(b*d)

fricas [B] time = 0.46, size = 777, normalized size = 13.17

$$\left[\frac{2 b d x \cosh (d x + c)^2 + 4 b d x \cosh (d x + c) \sinh (d x + c) + 2 b d x \sinh (d x + c)^2 + 2 b d x + (a \cosh (d x + c)^2 + 2 a \sinh (d x + c)^2 + a^2) \sqrt{-a / b} \log \left(\frac{(a^2 + 2 a b + b^2) \cosh (d x + c)^4 + 4 (a^2 + 2 a b + b^2) \cosh (d x + c) \sinh (d x + c)^3 + (a^2 + 2 a b + b^2) \sinh (d x + c)^4 + 2 (a^2 - b^2) \cosh (d x + c)^2 + 2 (3 (a^2 + 2 a b + b^2) \cosh (d x + c)^2 + a^2 - b^2) \sinh (d x + c)^2 + a^2 - 6 a b + b^2 + 4 ((a^2 + 2 a b + b^2) \cosh (d x + c)^3 + (a^2 - b^2) \cosh (d x + c) \sinh (d x + c) + 4 ((a b + b^2) \cosh (d x + c)^2 + 2 (a b + b^2) \cosh (d x + c) \sinh (d x + c) + (a b + b^2) \sinh (d x + c)^2 + a b - b^2) \sqrt{-a / b}}{(a + b) \cosh (d x + c)^4 + 4 (a + b) \cosh (d x + c) \sinh (d x + c)^3 + (a + b) \sinh (d x + c)^4 + 2 (a - b) \cosh (d x + c)^2 + 2 (3 (a + b) \cosh (d x + c)^2 + a - b) \sinh (d x + c)^2 + 4 ((a + b) \cosh (d x + c)^3 + (a - b) \cosh (d x + c) \sinh (d x + c) + a + b)} + 4 a + 4 b \right] / ((a b + b^2) d \cosh (d x + c)^2 + 2 (a b + b^2) d \cosh (d x + c) \sinh (d x + c) + (a b + b^2) d \sinh (d x + c)^2 + (a^2 + 2 a b + b^2) \cosh (d x + c)^2 + 2 (a^2 - b^2) \cosh (d x + c) \sinh (d x + c) + a^2 - b^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] [1/2*(2*b*d*x*cosh(d*x + c)^2 + 4*b*d*x*cosh(d*x + c)*sinh(d*x + c) + 2*b*d*x*sinh(d*x + c)^2 + 2*b*d*x + (a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a)*sqrt(-a/b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*((a*b + b^2)*cosh(d*x + c)^2 + 2*(a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b + b^2)*sinh(d*x + c)^2 + a*b - b^2)*sqrt(-a/b)))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c)*sinh(d*x + c) + a + b)) + 4*a + 4*b]/((a*b + b^2)*d*cosh(d*x + c)^2 + 2*(a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a*b + b^2)*d*sinh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + a^2 - b^2)

$b + b^2)d$, $(b*d*x*cosh(d*x + c)^2 + 2*b*d*x*cosh(d*x + c)*sinh(d*x + c) + b*d*x*sinh(d*x + c)^2 + b*d*x + (a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a)*sqrt(a/b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a/b)/a) + 2*a + 2*b)/((a*b + b^2)*d*cosh(d*x + c)^2 + 2*(a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a*b + b^2)*d*sinh(d*x + c)^2 + (a*b + b^2)*d]$

giac [A] time = 0.23, size = 87, normalized size = 1.47

$$\frac{a^2 \arctan\left(\frac{ae^{(2dx+2c)+be^{(2dx+2c)+a-b}}}{2\sqrt{ab}}\right)}{(ab+b^2)\sqrt{ab}} + \frac{dx+c}{a+b} + \frac{2}{b(e^{(2dx+2c)+1})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(dx+c)^4/(a+b*tanh(dx+c)^2),x, algorithm="giac")

[Out] $(a^2 \arctan(1/2*(a*e^{(2*d*x + 2*c)} + b*e^{(2*d*x + 2*c)} + a - b)/\sqrt{a*b}))/((a*b + b^2)*\sqrt{a*b}) + (d*x + c)/(a + b) + 2/(b*(e^{(2*d*x + 2*c)} + 1)))/d$

maple [A] time = 0.10, size = 95, normalized size = 1.61

$$-\frac{\tanh(dx+c)}{bd} - \frac{\ln(\tanh(dx+c)-1)}{d(2b+2a)} + \frac{\ln(1+\tanh(dx+c))}{d(2b+2a)} + \frac{a^2 \arctan\left(\frac{\tanh(dx+c)b}{\sqrt{ab}}\right)}{db(a+b)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(dx+c)^4/(a+b*tanh(dx+c)^2),x)

[Out] $-\tanh(dx+c)/b/d - 1/d/(2*b+2*a)*\ln(\tanh(dx+c)-1) + 1/d/(2*b+2*a)*\ln(1+\tanh(dx+c)) + 1/d/b/(a+b)*a^2/(a*b)^{(1/2)}*\arctan(\tanh(dx+c)*b/(a*b)^{(1/2)})$

maxima [B] time = 0.60, size = 509, normalized size = 8.63

$$-\frac{(a-b)\log((a+b)e^{(4dx+4c)} + 2(a-b)e^{(2dx+2c)} + a+b)}{8(ab+b^2)d} + \frac{(a-b)\log(2(a-b)e^{(-2dx-2c)} + (a+b)e^{(-4dx-4c)})}{8(ab+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(dx+c)^4/(a+b*tanh(dx+c)^2),x, algorithm="maxima")

[Out] $-1/8*(a-b)*\log((a+b)*e^{(4*d*x + 4*c)} + 2*(a-b)*e^{(2*d*x + 2*c)} + a + b)/((a*b + b^2)*d) + 1/8*(a-b)*\log(2*(a-b)*e^{(-2*d*x - 2*c)} + (a+b)*e^{(-4*d*x - 4*c)} + a + b)/((a*b + b^2)*d) + 1/16*(a^2 - 6*a*b + b^2)*\arctan(1/2*((a+b)*e^{(2*d*x + 2*c)} + a - b)/\sqrt{a*b})/((a*b + b^2)*\sqrt{a*b}*d) + 1/4*(a-b)*\arctan(1/2*((a+b)*e^{(2*d*x + 2*c)} + a - b)/\sqrt{a*b})/(\sqrt{a*b}*b*d) - 1/16*(a^2 - 6*a*b + b^2)*\arctan(1/2*((a+b)*e^{(-2*d*x - 2*c)} + a - b)/\sqrt{a*b})/((a*b + b^2)*\sqrt{a*b}*d) - 3/8*(a+b)*\arctan(1/2*((a+b)*e^{(-2*d*x - 2*c)} + a - b)/\sqrt{a*b})/(\sqrt{a*b}*b*d) - 1/4*(a-b)*\arctan(1/2*((a+b)*e^{(-2*d*x - 2*c)} + a - b)/\sqrt{a*b})/(\sqrt{a*b}*b*d) - 1/4*\log((a+b)*e^{(4*d*x + 4*c)} + 2*(a-b)*e^{(2*d*x + 2*c)} + a + b)/(b*d) + 1/4*\log(2*(a-b)*e^{(-2*d*x - 2*c)} + (a+b)*e^{(-4*d*x - 4*c)} + a + b)/(b*d) + 3/4*\log(e^{(2*d*x + 2*c)} + 1)/(b*d) - 3/4*\log(e^{(-2*d*x - 2*c)} + 1)/(b*d) + 5/8/((b*e^{(2*d*x + 2*c)} + b)*d) - 11/8/((b*e^{(-2*d*x - 2*c)} + b)*d)$

mupad [B] time = 1.23, size = 56, normalized size = 0.95

$$\frac{x}{a+b} - \frac{\tanh(c+dx)}{bd} + \frac{a^2 \operatorname{atan}\left(\frac{b \tanh(c+dx)}{\sqrt{ab}}\right)}{bd \sqrt{ab} (a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(c + d*x)^4/(a + b*tanh(c + d*x)^2), x)`

[Out] $x/(a + b) - \tanh(c + d*x)/(b*d) + (a^2*\operatorname{atan}((b*\tanh(c + d*x))/(a*b)^{(1/2)}))/ (b*d*(a*b)^{(1/2)*(a + b)})$

sympy [A] time = 11.28, size = 495, normalized size = 8.39

$$\left\{ \begin{array}{l} \infty x \tanh^2(c) \\ x - \frac{\frac{\tanh^3(c+dx)}{3d} - \frac{\tanh(c+dx)}{d}}{a} \\ x - \frac{\tanh(c+dx)}{b} \\ \frac{3dx \tanh^2(c+dx)}{2bd \tanh^2(c+dx) - 2bd} - \frac{3dx}{2bd \tanh^2(c+dx) - 2bd} - \frac{2 \tanh^3(c+dx)}{2bd \tanh^2(c+dx) - 2bd} + \frac{3 \tanh(c+dx)}{2bd \tanh^2(c+dx) - 2bd} \\ \frac{x \tanh^4(c)}{a + b \tanh^2(c)} \\ \frac{2ia^{\frac{3}{2}}b\sqrt{\frac{1}{b}} \tanh(c+dx)}{2ia^{\frac{3}{2}}b^2d\sqrt{\frac{1}{b}} + 2i\sqrt{a}b^3d\sqrt{\frac{1}{b}}} + \frac{2i\sqrt{a}b^2d\sqrt{\frac{1}{b}}}{2ia^{\frac{3}{2}}b^2d\sqrt{\frac{1}{b}} + 2i\sqrt{a}b^3d\sqrt{\frac{1}{b}}} - \frac{2i\sqrt{a}b^2\sqrt{\frac{1}{b}} \tanh(c+dx)}{2ia^{\frac{3}{2}}b^2d\sqrt{\frac{1}{b}} + 2i\sqrt{a}b^3d\sqrt{\frac{1}{b}}} + \frac{a^2 \log\left(-i\sqrt{a}\sqrt{\frac{1}{b}} + \tanh(c+dx)\right)}{2ia^{\frac{3}{2}}b^2d\sqrt{\frac{1}{b}} + 2i\sqrt{a}b^3d\sqrt{\frac{1}{b}}} - \frac{a^2 \log\left(i\sqrt{a}\sqrt{\frac{1}{b}} + \tanh(c+dx)\right)}{2ia^{\frac{3}{2}}b^2d\sqrt{\frac{1}{b}} + 2i\sqrt{a}b^3d\sqrt{\frac{1}{b}}} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(d*x+c)**4/(a+b*tanh(d*x+c)**2), x)`

[Out] `Piecewise((zoo*x*tanh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - tanh(c + d*x)**3/(3*d) - tanh(c + d*x)/d)/a, Eq(b, 0)), ((x - tanh(c + d*x)/d)/b, Eq(a, 0)), (3*d*x*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 3*d*x/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 2*tanh(c + d*x)**3/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + 3*tanh(c + d*x)/(2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x*tanh(c)**4/(a + b*tanh(c)**2), Eq(d, 0)), (-2*I*a**(3/2)*b*sqrt(1/b)*tanh(c + d*x)/(2*I*a**(3/2)*b**2*d*sqrt(1/b) + 2*I*sqrt(a)*b**3*d*sqrt(1/b)) + 2*I*sqrt(a)*b**2*d*x*sqrt(1/b)/(2*I*a**(3/2)*b**2*d*sqrt(1/b) + 2*I*sqrt(a)*b**3*d*sqrt(1/b)) - 2*I*sqrt(a)*b**2*sqrt(1/b)*tanh(c + d*x)/(2*I*a**(3/2)*b**2*d*sqrt(1/b) + 2*I*sqrt(a)*b**3*d*sqrt(1/b)) + a**2*log(-I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))/(2*I*a**(3/2)*b**2*d*sqrt(1/b) + 2*I*sqrt(a)*b**3*d*sqrt(1/b)) - a**2*log(I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))/(2*I*a**(3/2)*b**2*d*sqrt(1/b) + 2*I*sqrt(a)*b**3*d*sqrt(1/b)), True))`

$$3.172 \quad \int \frac{\tanh^3(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=46

$$\frac{\log(\cosh(c+dx))}{d(a+b)} - \frac{a \log(a+b \tanh^2(c+dx))}{2bd(a+b)}$$

[Out] ln(cosh(d*x+c))/(a+b)/d-1/2*a*ln(a+b*tanh(d*x+c)^2)/b/(a+b)/d

Rubi [A] time = 0.10, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 446, 72}

$$\frac{\log(\cosh(c+dx))}{d(a+b)} - \frac{a \log(a+b \tanh^2(c+dx))}{2bd(a+b)}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]

[Out] Log[Cosh[c + d*x]]/((a + b)*d) - (a*Log[a + b*Tanh[c + d*x]^2])/(2*b*(a + b)*d)

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned} \int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(1-x^2)(a+bx^2)} dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \frac{x}{(1-x)(a+bx)} dx, x, \tanh^2(c+dx)\right)}{2d} \\ &= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)(-1+x)} - \frac{a}{(a+b)(a+bx)}\right) dx, x, \tanh^2(c+dx)\right)}{2d} \\ &= \frac{\log(\cosh(c+dx))}{(a+b)d} - \frac{a \log(a+b\tanh^2(c+dx))}{2b(a+b)d} \end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 0.91

$$\frac{2b \log(\cosh(c+dx)) - a \log(a+b\tanh^2(c+dx))}{2abd + 2b^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2), x]

[Out] (2*b*Log[Cosh[c + d*x]] - a*Log[a + b*Tanh[c + d*x]^2])/(2*a*b*d + 2*b^2*d)

fricas [B] time = 0.46, size = 118, normalized size = 2.57

$$\frac{2bdx + a \log\left(\frac{2((a+b)\cosh(dx+c)^2 + (a+b)\sinh(dx+c)^2 + a-b)}{\cosh(dx+c)^2 - 2\cosh(dx+c)\sinh(dx+c) + \sinh(dx+c)^2}\right) - 2(a+b) \log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c) - \sinh(dx+c)}\right)}{2(ab + b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] -1/2*(2*b*d*x + a*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*(a + b)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/(a*b + b^2)*d)

giac [B] time = 0.23, size = 96, normalized size = 2.09

$$\frac{a \log(ae^{4dx+4c} + be^{4dx+4c}) + 2ae^{2dx+2c} - 2be^{2dx+2c} + a + b}{ab+b^2} + \frac{2(dx+c)}{a+b} - \frac{2 \log(e^{2dx+2c} + 1)}{b}$$

$2d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2), x, algorithm="giac")

[Out] -1/2*(a*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a*b + b^2) + 2*(d*x + c)/(a + b) - 2*log(e^(2*d*x + 2*c) + 1)/b)/d

maple [A] time = 0.11, size = 75, normalized size = 1.63

$$\frac{\ln(\tanh(dx+c)-1)}{d(2b+2a)} - \frac{\ln(1+\tanh(dx+c))}{d(2b+2a)} - \frac{a \ln(a+b(\tanh^2(dx+c)))}{2b(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x)

[Out] $-1/d/(2*b+2*a)*\ln(\tanh(d*x+c)-1)-1/d/(2*b+2*a)*\ln(1+\tanh(d*x+c))-1/2*a*\ln(a+b*\tanh(d*x+c)^2)/b/(a+b)/d$

maxima [A] time = 0.41, size = 82, normalized size = 1.78

$$-\frac{a \log\left(2(a-b)e^{-2dx-2c} + (a+b)e^{-4dx-4c} + a+b\right)}{2(ab+b^2)d} + \frac{dx+c}{(a+b)d} + \frac{\log\left(e^{-2dx-2c} + 1\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] $-1/2*a*\log(2*(a-b)*e^{-2*d*x-2*c} + (a+b)*e^{-4*d*x-4*c} + a+b)/((a*b+b^2)*d) + (d*x+c)/((a+b)*d) + \log(e^{-2*d*x-2*c} + 1)/(b*d)$

mupad [B] time = 1.24, size = 46, normalized size = 1.00

$$-\frac{\frac{a \ln(b \tanh(c+dx)^2+a)}{2} + b (\ln(\tanh(c+dx) + 1) - dx)}{bd(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c+d*x)^3/(a+b*tanh(c+d*x)^2),x)

[Out] $-((a*\log(a+b*\tanh(c+d*x)^2))/2 + b*(\log(\tanh(c+d*x) + 1) - d*x))/(b*d*(a+b))$

sympy [A] time = 9.65, size = 316, normalized size = 6.87

$$\left\{ \begin{array}{l} \infty x \tanh(c) \\ \frac{2dx \tanh^2(c+dx)}{2bd \tanh^2(c+dx)-2bd} - \frac{2dx}{2bd \tanh^2(c+dx)-2bd} - \frac{2 \log(\tanh(c+dx)+1) \tanh^2(c+dx)}{2bd \tanh^2(c+dx)-2bd} + \frac{2 \log(\tanh(c+dx)+1)}{2bd \tanh^2(c+dx)-2bd} + \frac{1}{2bd \tanh^2(c+dx)-2bd} \\ x - \frac{\log(\tanh(c+dx)+1)}{d} - \frac{\tanh^2(c+dx)}{2d} \\ \frac{x \tanh^3(c)}{a+b \tanh^2(c)} \\ - \frac{a \log\left(-i\sqrt{a}\sqrt{\frac{1}{b}} + \tanh(c+dx)\right)}{2abd+2b^2d} - \frac{a \log\left(i\sqrt{a}\sqrt{\frac{1}{b}} + \tanh(c+dx)\right)}{2abd+2b^2d} + \frac{2bdx}{2abd+2b^2d} - \frac{2b \log(\tanh(c+dx)+1)}{2abd+2b^2d} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)

[Out] $\text{Piecewise}((\text{zoo}*x*\tanh(c), \text{Eq}(a, 0) \& \text{Eq}(b, 0) \& \text{Eq}(d, 0)), (2*d*x*\tanh(c+d*x)**2/(2*b*d*\tanh(c+d*x)**2 - 2*b*d) - 2*d*x/(2*b*d*\tanh(c+d*x)**2 - 2*b*d) - 2*\log(\tanh(c+d*x) + 1)*\tanh(c+d*x)**2/(2*b*d*\tanh(c+d*x)**2 - 2*b*d) + 2*\log(\tanh(c+d*x) + 1)/(2*b*d*\tanh(c+d*x)**2 - 2*b*d) + 1/(2*b*d*\tanh(c+d*x)**2 - 2*b*d), \text{Eq}(a, -b)), ((x - \log(\tanh(c+d*x) + 1))/d - \tanh(c+d*x)**2/(2*d))/a, \text{Eq}(b, 0)), (x*\tanh(c)**3/(a+b*\tanh(c)**2), \text{Eq}(d, 0)), (-a*\log(-I*\sqrt{a}*\sqrt{1/b} + \tanh(c+d*x))/(2*a*b*d + 2*b**2*d) - a*\log(I*\sqrt{a}*\sqrt{1/b} + \tanh(c+d*x))/(2*a*b*d + 2*b**2*d) + 2*b*d*x/(2*a*b*d + 2*b**2*d) - 2*b*\log(\tanh(c+d*x) + 1)/(2*a*b*d + 2*b**2*d), \text{True}))$

$$3.173 \quad \int \frac{\tanh^2(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=46

$$\frac{x}{a+b} - \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b} d(a+b)}$$

[Out] x/(a+b)-arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*a^(1/2)/(a+b)/d/b^(1/2)

Rubi [A] time = 0.08, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3670, 481, 206, 205}

$$\frac{x}{a+b} - \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b} d(a+b)}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2), x]

[Out] x/(a + b) - (Sqrt[a]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[b]*(a + b)*d)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 481

Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x], x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m, 2*n - 1]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)} dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)(a+bx^2)} dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c+dx)\right)}{(a+b)d} - \frac{a \text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \tanh(c+dx)\right)}{(a+b)d}$$

$$= \frac{x}{a+b} - \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)d}$$

Mathematica [A] time = 0.03, size = 47, normalized size = 1.02

$$\frac{\tanh^{-1}(\tanh(c+dx)) - \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{d(a+b)}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2), x]

[Out] (-(Sqrt[a]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[b]) + ArcTanh[Tanh[c + d*x]]/((a + b)*d)

fricas [B] time = 0.49, size = 486, normalized size = 10.57

$$\left[\frac{2dx + \sqrt{-\frac{a}{b}} \log\left(\frac{(a^2+2ab+b^2)\cosh(dx+c)^4 + 4(a^2+2ab+b^2)\cosh(dx+c)\sinh(dx+c)^3 + (a^2+2ab+b^2)\sinh(dx+c)^4 + 2(a^2-b^2)\cosh(dx+c)^2}{(a+b)\cosh(dx+c)^4 + 4(a+b)\cosh(dx+c)\sinh(dx+c)^3 + (a+b)\sinh(dx+c)^4}\right)}{(a+b)d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] [1/2*(2*d*x + sqrt(-a/b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a*b + b^2)*cosh(d*x + c)^2 + 2*(a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b + b^2)*sinh(d*x + c)^2 + a*b - b^2)*sqrt(-a/b)))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)))/((a + b)*d), (d*x - sqrt(a/b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a/b/a)))/((a + b)*d)]

giac [A] time = 0.18, size = 65, normalized size = 1.41

$$\frac{\frac{a \arctan\left(\frac{ae^{(2dx+2c)+be^{(2dx+2c)+a-b}}}{2\sqrt{ab}}\right)}{\sqrt{ab}(a+b)} - \frac{dx+c}{a+b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] $-(a*\arctan(1/2*(a*e^{(2*d*x + 2*c)} + b*e^{(2*d*x + 2*c)} + a - b)/\sqrt{a*b}))/(\sqrt{a*b}*(a + b)) - (d*x + c)/(a + b))/d$

maple [A] time = 0.07, size = 77, normalized size = 1.67

$$-\frac{\ln(\tanh(dx+c)-1)}{d(2b+2a)} + \frac{\ln(1+\tanh(dx+c))}{d(2b+2a)} - \frac{a \arctan\left(\frac{\tanh(dx+c)b}{\sqrt{ab}}\right)}{d(a+b)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x)

[Out] $-1/d/(2*b+2*a)*\ln(\tanh(d*x+c)-1)+1/d/(2*b+2*a)*\ln(1+\tanh(d*x+c))-1/d/(a+b)*a/(a*b)^{(1/2)}*\arctan(\tanh(d*x+c)*b/(a*b)^{(1/2)})$

maxima [B] time = 0.45, size = 215, normalized size = 4.67

$$-\frac{(a-b)\arctan\left(\frac{(a+b)e^{(2dx+2c)+a-b}}{2\sqrt{ab}}\right)}{4\sqrt{ab}(a+b)d} + \frac{\arctan\left(\frac{(a+b)e^{(-2dx-2c)+a-b}}{2\sqrt{ab}}\right)}{2\sqrt{ab}d} + \frac{(a-b)\arctan\left(\frac{(a+b)e^{(-2dx-2c)+a-b}}{2\sqrt{ab}}\right)}{4\sqrt{ab}(a+b)d} + \frac{\log((a+b)e^{4c})}{4\sqrt{ab}(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] $-1/4*(a-b)*\arctan(1/2*((a+b)*e^{(2*d*x+2*c)}+a-b)/\sqrt{a*b})/(\sqrt{a*b}*(a+b)*d) + 1/2*\arctan(1/2*((a+b)*e^{(-2*d*x-2*c)}+a-b)/\sqrt{a*b})/(\sqrt{a*b}*d) + 1/4*(a-b)*\arctan(1/2*((a+b)*e^{(-2*d*x-2*c)}+a-b)/\sqrt{a*b})/(\sqrt{a*b}*(a+b)*d) + 1/4*\log((a+b)*e^{(4*d*x+4*c)}+2*(a-b)*e^{(2*d*x+2*c)}+a+b)/((a+b)*d) - 1/4*\log(2*(a-b)*e^{(-2*d*x-2*c)}+(a+b)*e^{(-4*d*x-4*c)}+a+b)/((a+b)*d)$

mupad [B] time = 0.11, size = 38, normalized size = 0.83

$$\frac{x}{a+b} - \frac{a \operatorname{atan}\left(\frac{b \tanh(c+dx)}{\sqrt{ab}}\right)}{d \sqrt{ab} (a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c+d*x)^2/(a+b*tanh(c+d*x)^2),x)

[Out] $x/(a+b) - (a*\operatorname{atan}((b*\tanh(c+d*x))/\sqrt{a*b}))/((d*(a*b)^{(1/2)}*(a+b))$

sympy [A] time = 8.97, size = 294, normalized size = 6.39

$$\left\{ \begin{array}{ll} \infty x & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{x}{b} & \text{for } a = 0 \\ \frac{dx \tanh^2(c+dx)}{2bd \tanh^2(c+dx)-2bd} - \frac{dx}{2bd \tanh^2(c+dx)-2bd} + \frac{\tanh(c+dx)}{2bd \tanh^2(c+dx)-2bd} & \text{for } a = -b \\ \frac{x \tanh^2(c)}{a+b \tanh^2(c)} & \text{for } d = 0 \\ x - \frac{\tanh(c+dx)}{d} & \text{for } b = 0 \\ \frac{2i\sqrt{a}bdx\sqrt{\frac{1}{b}}}{2ia^2bd\sqrt{\frac{1}{b}}+2i\sqrt{a}b^2d\sqrt{\frac{1}{b}}} - \frac{a \log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\tanh(c+dx)\right)}{2ia^2bd\sqrt{\frac{1}{b}}+2i\sqrt{a}b^2d\sqrt{\frac{1}{b}}} + \frac{a \log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\tanh(c+dx)\right)}{2ia^2bd\sqrt{\frac{1}{b}}+2i\sqrt{a}b^2d\sqrt{\frac{1}{b}}} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/b, Eq(a, 0)), (d*x*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - d*x/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + tanh(c + d*x)/(2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x*tanh(c)**2/(a + b*tanh(c)**2), Eq(d, 0)), ((x - tanh(c + d*x)/d)/a, Eq(b, 0)), (2*I*sqrt(a)*b*d*x*sqrt(1/b)/(2*I*a**(3/2)*b*d*sqrt(1/b) + 2*I*sqrt(a)*b**2*d*sqrt(1/b)) - a*log(-I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))/(2*I*a**(3/2)*b*d*sqrt(1/b) + 2*I*sqrt(a)*b**2*d*sqrt(1/b)) + a*log(I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))/(2*I*a**(3/2)*b*d*sqrt(1/b) + 2*I*sqrt(a)*b**2*d*sqrt(1/b)), True))
```

$$3.174 \quad \int \frac{\tanh(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=42

$$\frac{\log(a + b \tanh^2(c + dx))}{2d(a + b)} + \frac{\log(\cosh(c + dx))}{d(a + b)}$$

[Out] ln(cosh(d*x+c))/(a+b)/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)/d

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3670, 444, 36, 31}

$$\frac{\log(a + b \tanh^2(c + dx))}{2d(a + b)} + \frac{\log(\cosh(c + dx))}{d(a + b)}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]

[Out] Log[Cosh[c + d*x]]/((a + b)*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)*d)

Rule 31

Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 444

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 3670

Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*((c_)*tan[(e_) + (f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(c+dx)}{a+b \tanh^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{x}{(1-x^2)(a+bx^2)} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)} dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{1-x} dx, x, \tanh^2(c+dx)\right)}{2(a+b)d} + \frac{b \text{Subst}\left(\int \frac{1}{a+bx} dx, x, \tanh^2(c+dx)\right)}{2(a+b)d} \\
&= \frac{\log(\cosh(c+dx))}{(a+b)d} + \frac{\log(a+b \tanh^2(c+dx))}{2(a+b)d}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 35, normalized size = 0.83

$$\frac{\log(a+b \tanh^2(c+dx)) + 2 \log(\cosh(c+dx))}{2ad + 2bd}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2), x]

[Out] (2*Log[Cosh[c + d*x]] + Log[a + b*Tanh[c + d*x]^2])/(2*a*d + 2*b*d)

fricas [B] time = 0.41, size = 82, normalized size = 1.95

$$\frac{2 dx - \log\left(\frac{2((a+b)\cosh(dx+c)^2 + (a+b)\sinh(dx+c)^2 + a-b)}{\cosh(dx+c)^2 - 2\cosh(dx+c)\sinh(dx+c) + \sinh(dx+c)^2}\right)}{2(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] -1/2*(2*d*x - log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)))/((a + b)*d)

giac [A] time = 0.17, size = 61, normalized size = 1.45

$$\frac{\log\left(\left|a(e^{2dx+2c}) + e^{(-2dx-2c)}\right| + b(e^{2dx+2c}) + e^{(-2dx-2c)}\right) + 2a - 2b}{2(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2), x, algorithm="giac")

[Out] 1/2*log(abs(a*(e^(2*d*x + 2*c)) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2*b)/((a + b)*d)

maple [A] time = 0.09, size = 71, normalized size = 1.69

$$\frac{\ln(\tanh(dx+c)-1)}{d(2b+2a)} - \frac{\ln(1+\tanh(dx+c))}{d(2b+2a)} + \frac{\ln(a+b(\tanh^2(dx+c)))}{2(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)/(a+b*tanh(d*x+c)^2), x)

[Out] $-1/d/(2*b+2*a)*\ln(\tanh(d*x+c)-1)-1/d/(2*b+2*a)*\ln(1+\tanh(d*x+c))+1/2*\ln(a+b*\tanh(d*x+c)^2)/(a+b)/d$

maxima [A] time = 0.33, size = 58, normalized size = 1.38

$$\frac{dx + c}{(a + b)d} + \frac{\log(2(a - b)e^{(-2dx-2c)} + (a + b)e^{(-4dx-4c)} + a + b)}{2(a + b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] $(d*x + c)/((a + b)*d) + 1/2*\log(2*(a - b)*e^{(-2*d*x - 2*c)} + (a + b)*e^{(-4*d*x - 4*c)} + a + b)/((a + b)*d)$

mupad [B] time = 1.17, size = 43, normalized size = 1.02

$$\frac{x}{a + b} - \frac{\ln(\tanh(c + dx) + 1) - \frac{\ln(b \tanh(c+dx)^2 + a)}{2}}{d(a + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)/(a + b*tanh(c + d*x)^2),x)

[Out] $x/(a + b) - (\log(\tanh(c + d*x) + 1) - \log(a + b*\tanh(c + d*x)^2)/2)/(d*(a + b))$

sympy [A] time = 8.83, size = 156, normalized size = 3.71

$\left\{ \begin{array}{l} \frac{\infty x}{\tanh(c)} \\ \frac{1}{2bd \tanh^2(c+dx) - 2bd} \\ \frac{x \tanh(c)}{a+b \tanh^2(c)} \\ x - \frac{\log(\tanh(c+dx)+1)}{d} \\ \frac{2dx}{2ad+2bd} + \frac{\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}} + \tanh(c+dx)\right)}{2ad+2bd} + \frac{\log\left(i\sqrt{a}\sqrt{\frac{1}{b}} + \tanh(c+dx)\right)}{2ad+2bd} - \frac{2\log(\tanh(c+dx)+1)}{2ad+2bd} \end{array} \right.$	for $a = 0 \wedge b = 0 \wedge d = 0$
	for $a = -b$
	for $d = 0$
	for $b = 0$
	otherwise

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)**2),x)

[Out] Piecewise((zoo*x/tanh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (1/(2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x*tanh(c)/(a + b*tanh(c)**2), Eq(d, 0)), ((x - log(tanh(c + d*x) + 1)/d)/a, Eq(b, 0)), (2*d*x/(2*a*d + 2*b*d) + log(-I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))/(2*a*d + 2*b*d) + log(I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))/(2*a*d + 2*b*d) - 2*log(tanh(c + d*x) + 1)/(2*a*d + 2*b*d), True))

$$3.175 \quad \int \frac{1}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=45

$$\frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} d(a+b)} + \frac{x}{a+b}$$

[Out] $x/(a+b)+\arctan(b^{(1/2)}*\tanh(d*x+c)/a^{(1/2)})*b^{(1/2)}/(a+b)/d/a^{(1/2)}$

Rubi [A] time = 0.07, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3660, 3675, 205}

$$\frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} d(a+b)} + \frac{x}{a+b}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tanh[c + d*x]^2)^(-1), x]

[Out] $x/(a + b) + (\text{Sqrt}[b]*\text{ArcTan}[(\text{Sqrt}[b]*\text{Tanh}[c + d*x])/\text{Sqrt}[a]])/(\text{Sqrt}[a]*(a + b)*d)$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 3660

Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/(a - b), x] - Dist[b/(a - b), Int[Sec[e + f*x]^2/(a + b*Tan[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a, b]

Rule 3675

Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)]))^(n_)]^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/(c^(m - 1)*f), Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^p, x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \frac{1}{a+b \tanh^2(c+dx)} dx &= \frac{x}{a+b} + \frac{b \int \frac{\text{sech}^2(c+dx)}{a+b \tanh^2(c+dx)} dx}{a+b} \\ &= \frac{x}{a+b} + \frac{b \text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \tanh(c+dx)\right)}{(a+b)d} \\ &= \frac{x}{a+b} + \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)d} \end{aligned}$$

Mathematica [A] time = 0.08, size = 65, normalized size = 1.44

$$\frac{2\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\log(1 - \tanh(c + dx)) + \log(\tanh(c + dx) + 1)}{2ad + 2bd}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Tanh[c + d*x]^2)^(-1), x]

[Out] ((2*sqrt[b]*ArcTan[(sqrt[b]*Tanh[c + d*x])/sqrt[a]])/sqrt[a] - Log[1 - Tanh[c + d*x]] + Log[1 + Tanh[c + d*x]])/(2*a*d + 2*b*d)

fricas [B] time = 0.46, size = 484, normalized size = 10.76

$$\left[2dx + \sqrt{\frac{b}{a}} \log\left(\frac{(a^2+2ab+b^2)\cosh(dx+c)^4+4(a^2+2ab+b^2)\cosh(dx+c)\sinh(dx+c)^3+(a^2+2ab+b^2)\sinh(dx+c)^4+2(a^2-b^2)\cosh(dx+c)^2+2(3(a+b)\cosh(dx+c)^4+4(a+b)\cosh(dx+c)\sinh(dx+c)\sinh(dx+c)^2)}{(a+b)\cosh(dx+c)^4+4(a+b)\cosh(dx+c)\sinh(dx+c)\sinh(dx+c)^2}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] [1/2*(2*d*x + sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a)))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)))/((a + b)*d), (d*x + sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b))/((a + b)*d)]

giac [A] time = 0.15, size = 63, normalized size = 1.40

$$\frac{b \arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)+a-b}}{2\sqrt{ab}}\right)}{\sqrt{ab}(a+b)} + \frac{dx+c}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tanh(d*x+c)^2), x, algorithm="giac")

[Out] (b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b)))/(sqrt(a*b)*(a + b)) + (d*x + c)/(a + b))/d

maple [B] time = 0.14, size = 76, normalized size = 1.69

$$-\frac{\ln(\tanh(dx + c) - 1)}{d(2b + 2a)} + \frac{\ln(1 + \tanh(dx + c))}{d(2b + 2a)} + \frac{b \arctan\left(\frac{\tanh(dx+c)b}{\sqrt{ab}}\right)}{d(a + b)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*tanh(d*x+c)^2), x)

[Out] $-1/d/(2*b+2*a)*\ln(\tanh(d*x+c)-1)+1/d/(2*b+2*a)*\ln(1+\tanh(d*x+c))+1/d*b/(a+b)/(a*b)^{(1/2)}*\arctan(\tanh(d*x+c)*b/(a*b)^{(1/2)})$

maxima [A] time = 0.42, size = 57, normalized size = 1.27

$$-\frac{b \arctan\left(\frac{(a+b)e^{(-2dx-2c)+a-b}}{2\sqrt{ab}}\right)}{\sqrt{ab}(a+b)d} + \frac{dx+c}{(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*tanh(d*x+c)^2), x, algorithm="maxima")`

[Out] $-b*\arctan(1/2*((a+b)*e^{(-2*d*x-2*c)+a-b})/\sqrt{a*b})/(\sqrt{a*b}*(a+b)*d) + (d*x+c)/((a+b)*d)$

mupad [B] time = 0.09, size = 37, normalized size = 0.82

$$\frac{x}{a+b} + \frac{b \operatorname{atan}\left(\frac{b \tanh(c+dx)}{\sqrt{ab}}\right)}{d \sqrt{ab} (a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*tanh(c+d*x)^2), x)`

[Out] $x/(a+b) + (b*\operatorname{atan}((b*\tanh(c+d*x))/(a*b)^{(1/2)}))/((d*(a*b)^{(1/2)}*(a+b))$

sympy [A] time = 8.96, size = 280, normalized size = 6.22

$$\left\{ \begin{array}{ll} \frac{\infty x}{\tanh^2(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{x}{a} & \text{for } b = 0 \\ x - \frac{1}{d \tanh(c+dx)} & \text{for } a = 0 \\ -\frac{dx \tanh^2(c+dx)}{2bd \tanh^2(c+dx) - 2bd} + \frac{dx}{2bd \tanh^2(c+dx) - 2bd} + \frac{\tanh(c+dx)}{2bd \tanh^2(c+dx) - 2bd} & \text{for } a = -b \\ \frac{x}{a+b \tanh^2(c)} & \text{for } d = 0 \\ \frac{2i\sqrt{a} dx \sqrt{\frac{1}{b}}}{2ia^{\frac{3}{2}} d \sqrt{\frac{1}{b}} + 2i\sqrt{a} bd \sqrt{\frac{1}{b}}} + \frac{\log\left(-i\sqrt{a} \sqrt{\frac{1}{b}} + \tanh(c+dx)\right)}{2ia^{\frac{3}{2}} d \sqrt{\frac{1}{b}} + 2i\sqrt{a} bd \sqrt{\frac{1}{b}}} - \frac{\log\left(i\sqrt{a} \sqrt{\frac{1}{b}} + \tanh(c+dx)\right)}{2ia^{\frac{3}{2}} d \sqrt{\frac{1}{b}} + 2i\sqrt{a} bd \sqrt{\frac{1}{b}}} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*tanh(d*x+c)**2), x)`

[Out] `Piecewise((zoo*x/tanh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a, Eq(b, 0)), ((x - 1/(d*tanh(c + d*x)))/b, Eq(a, 0)), (-d*x*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + d*x/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + tanh(c + d*x)/(2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x/(a + b*tanh(c)**2), Eq(d, 0)), (2*I*sqrt(a)*d*x*sqrt(1/b)/(2*I*a**(3/2)*d*sqrt(1/b) + 2*I*sqrt(a)*b*d*sqrt(1/b)) + log(-I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))/(2*I*a**(3/2)*d*sqrt(1/b) + 2*I*sqrt(a)*b*d*sqrt(1/b)) - log(I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))/(2*I*a**(3/2)*d*sqrt(1/b) + 2*I*sqrt(a)*b*d*sqrt(1/b)), True))`

$$3.176 \quad \int \frac{\coth(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=60

$$-\frac{b \log(a + b \tanh^2(c + dx))}{2ad(a + b)} + \frac{\log(\cosh(c + dx))}{d(a + b)} + \frac{\log(\tanh(c + dx))}{ad}$$

[Out] ln(cosh(d*x+c))/(a+b)/d+ln(tanh(d*x+c))/a/d-1/2*b*ln(a+b*tanh(d*x+c)^2)/a/(a+b)/d

Rubi [A] time = 0.10, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3670, 446, 72}

$$-\frac{b \log(a + b \tanh^2(c + dx))}{2ad(a + b)} + \frac{\log(\cosh(c + dx))}{d(a + b)} + \frac{\log(\tanh(c + dx))}{ad}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2), x]

[Out] Log[Cosh[c + d*x]]/((a + b)*d) + Log[Tanh[c + d*x]]/(a*d) - (b*Log[a + b*Tanh[c + d*x]^2])/(2*a*(a + b)*d)

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
\int \frac{\coth(c+dx)}{a+b \tanh^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(1-x^2)(a+bx^2)} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{(1-x)x(a+bx)} dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)(-1+x)} + \frac{1}{ax} - \frac{b^2}{a(a+b)(a+bx)}\right) dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\log(\cosh(c+dx))}{(a+b)d} + \frac{\log(\tanh(c+dx))}{ad} - \frac{b \log(a+b \tanh^2(c+dx))}{2a(a+b)d}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 54, normalized size = 0.90

$$\frac{-b \log(a+b \tanh^2(c+dx)) + 2(a+b) \log(\tanh(c+dx)) + 2a \log(\cosh(c+dx))}{2ad(a+b)}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2), x]

[Out] (2*a*Log[Cosh[c + d*x]] + 2*(a + b)*Log[Tanh[c + d*x]] - b*Log[a + b*Tanh[c + d*x]^2])/(2*a*(a + b)*d)

fricas [B] time = 0.45, size = 118, normalized size = 1.97

$$\frac{2adx + b \log\left(\frac{2((a+b)\cosh(dx+c)^2 + (a+b)\sinh(dx+c)^2 + a-b)}{\cosh(dx+c)^2 - 2\cosh(dx+c)\sinh(dx+c) + \sinh(dx+c)^2}\right) - 2(a+b) \log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c) - \sinh(dx+c)}\right)}{2(a^2 + ab)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] -1/2*(2*a*d*x + b*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*(a + b)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c)))/((a^2 + a*b)*d)

giac [A] time = 0.18, size = 97, normalized size = 1.62

$$\frac{\frac{b \log(ae^{4dx+4c} + be^{4dx+4c}) + 2ae^{2dx+2c} - 2be^{2dx+2c} + a + b}{a^2 + ab} + \frac{2(dx+c)}{a+b} - \frac{2 \log(|e^{2dx+2c} - 1|)}{a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2), x, algorithm="giac")

[Out] -1/2*(b*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^2 + a*b) + 2*(d*x + c)/(a + b) - 2*log(abs(e^(2*d*x + 2*c) - 1))/a)/d

maple [B] time = 0.40, size = 121, normalized size = 2.02

$$\frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{d(a+b)} - \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{d(a+b)} - \frac{b \ln\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a + 4\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a + 4\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a + 4\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a}{2da(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(d*x+c)/(a+b*tanh(d*x+c)^2),x)`

[Out] $-1/d/(a+b)*\ln(\tanh(1/2*d*x+1/2*c)-1)-1/d/(a+b)*\ln(\tanh(1/2*d*x+1/2*c)+1)-1/2/d*b/a/(a+b)*\ln(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)+1/d/a*\ln(\tanh(1/2*d*x+1/2*c))$

maxima [A] time = 0.33, size = 101, normalized size = 1.68

$$-\frac{b \log\left(2(a-b)e^{(-2dx-2c)} + (a+b)e^{(-4dx-4c)} + a+b\right)}{2(a^2+ab)d} + \frac{dx+c}{(a+b)d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{ad} + \frac{\log\left(e^{(-dx-c)}-1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")`

[Out] $-1/2*b*\log(2*(a-b)*e^{(-2*d*x-2*c)} + (a+b)*e^{(-4*d*x-4*c)} + a+b)/(a^2+a*b)*d + (d*x+c)/((a+b)*d) + \log(e^{(-d*x-c)}+1)/(a*d) + \log(e^{(-d*x-c)}-1)/(a*d)$

mupad [B] time = 1.47, size = 194, normalized size = 3.23

$$\frac{\ln\left(12ab^2 + 4a^2b + 9b^3 - 9b^3e^{2c}e^{2dx} - 12a^2be^{2c}e^{2dx} - 4a^2be^{2c}e^{2dx}\right)}{ad} + \frac{b \ln\left(5ab + 2a^2 + 3b^2 + 4a^2e^{2c}e^{2dx}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(c+d*x)/(a+b*tanh(c+d*x)^2),x)`

[Out] $\log(12*a*b^2 + 4*a^2*b + 9*b^3 - 9*b^3*\exp(2*c)*\exp(2*d*x) - 12*a*b^2*\exp(2*c)*\exp(2*d*x) - 4*a^2*b*\exp(2*c)*\exp(2*d*x))/(a*d) - (b*\log(5*a*b + 2*a^2 + 3*b^2 + 4*a^2*\exp(2*c)*\exp(2*d*x) + 2*a^2*\exp(4*c)*\exp(4*d*x) - 6*b^2*\exp(2*c)*\exp(2*d*x) + 3*b^2*\exp(4*c)*\exp(4*d*x) + 2*a*b*\exp(2*c)*\exp(2*d*x) + 5*a*b*\exp(4*c)*\exp(4*d*x)))/(2*a^2*d + 2*a*b*d) - x/(a+b)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)/(a+b*tanh(d*x+c)**2),x)`

[Out] `Integral(coth(c+d*x)/(a+b*tanh(c+d*x)**2),x)`

$$3.177 \quad \int \frac{\coth^2(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=60

$$-\frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d(a+b)} + \frac{x}{a+b} - \frac{\coth(c+dx)}{ad}$$

[Out] $x/(a+b) - b^{(3/2)} * \arctan(b^{(1/2)} * \tanh(d*x+c)/a^{(1/2)}) / a^{(3/2)} / (a+b) / d - \coth(d*x+c) / a / d$

Rubi [A] time = 0.11, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3670, 480, 522, 206, 205}

$$-\frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d(a+b)} + \frac{x}{a+b} - \frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2), x]

[Out] $x/(a+b) - (b^{(3/2)} * \text{ArcTan}[(\text{Sqrt}[b] * \text{Tanh}[c + d*x]) / \text{Sqrt}[a]]) / (a^{(3/2)} * (a + b) * d) - \text{Coth}[c + d*x] / (a * d)$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 480

Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[((e*x)^(m+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(a*c*e*(m+1)), x] - Dist[1/(a*c*e^n*(m+1)), Int[(e*x)^(m+n)*(a+b*x^n)^p*(c+d*x^n)^q*Simp[(b*c+a*d)*(m+n+1)+n*(b*c*p+a*d*q)+b*d*(m+n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n}

, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned} \int \frac{\coth^2(c+dx)}{a+b \tanh^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+bx^2)} dx, x, \tanh(c+dx)\right)}{d} \\ &= -\frac{\coth(c+dx)}{ad} + \frac{\text{Subst}\left(\int \frac{a-b+bx^2}{(1-x^2)(a+bx^2)} dx, x, \tanh(c+dx)\right)}{ad} \\ &= -\frac{\coth(c+dx)}{ad} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c+dx)\right)}{(a+b)d} - \frac{b^2 \text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \tanh(c+dx)\right)}{a(a+b)d} \\ &= \frac{x}{a+b} - \frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a+b)d} - \frac{\coth(c+dx)}{ad} \end{aligned}$$

Mathematica [A] time = 0.19, size = 67, normalized size = 1.12

$$-\frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d(a+b)} + \frac{c+dx}{d(a+b)} - \frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2), x]

[Out] (c + d*x)/((a + b)*d) - (b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a + b)*d) - Coth[c + d*x]/(a*d)

fricas [B] time = 0.45, size = 784, normalized size = 13.07

$$\left[\frac{2 \, adx \cosh(dx + c)^2 + 4 \, adx \cosh(dx + c) \sinh(dx + c) + 2 \, adx \sinh(dx + c)^2 - 2 \, adx + (b \cosh(dx + c)^2 + 2 \, b \sinh(dx + c)^2 - b)}{a^2 d^2 (a + b)^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] [1/2*(2*a*d*x*cosh(d*x + c)^2 + 4*a*d*x*cosh(d*x + c)*sinh(d*x + c) + 2*a*d*x*sinh(d*x + c)^2 - 2*a*d*x + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b) - 4*a - 4*b)/((a^2 + a*b)*d*cosh(d*x + c)^2 + 2*(a^2 + a*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*d*sinh(d*x + c)^2 - (a^2 + a*b)*d)

$2 + a*b)*d)$, $(a*d*x*\cosh(d*x + c)^2 + 2*a*d*x*\cosh(d*x + c)*\sinh(d*x + c) + a*d*x*\sinh(d*x + c)^2 - a*d*x - (b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 - b)*\sqrt{b/a}*\arctan(1/2*((a + b)*\cosh(d*x + c)^2 + 2*(a + b)*\cosh(d*x + c)*\sinh(d*x + c) + (a + b)*\sinh(d*x + c)^2 + a - b)*\sqrt{b/a}/b) - 2*a - 2*b)/((a^2 + a*b)*d*\cosh(d*x + c)^2 + 2*(a^2 + a*b)*d*\cosh(d*x + c)*\sinh(d*x + c) + (a^2 + a*b)*d*\sinh(d*x + c)^2 - (a^2 + a*b)*d)]$

giac [A] time = 0.21, size = 89, normalized size = 1.48

$$\frac{b^2 \arctan\left(\frac{ae^{2dx+2c} + be^{2dx+2c} + a - b}{2\sqrt{ab}}\right)}{(a^2 + ab)\sqrt{ab}} - \frac{dx+c}{a+b} + \frac{2}{a(e^{2dx+2c} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] $-(b^2*\arctan(1/2*(a*e^{(2*d*x + 2*c)} + b*e^{(2*d*x + 2*c)} + a - b)/\sqrt{a*b}))/((a^2 + a*b)*\sqrt{a*b}) - (d*x + c)/(a + b) + 2/(a*(e^{(2*d*x + 2*c)} - 1))$
/d

maple [B] time = 0.40, size = 494, normalized size = 8.23

$$\frac{\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{2da} - \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{d(a+b)} + \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{d(a+b)} - \frac{1}{2da \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)} + \frac{b^2 \operatorname{arctanh}\left(\frac{1}{\sqrt{b(a+b)}}\right)}{d(a+b)\sqrt{b(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2),x)

[Out] $-1/2/d/a*\tanh(1/2*d*x+1/2*c)-1/d/(a+b)*\ln(\tanh(1/2*d*x+1/2*c)-1)+1/d/(a+b)*\ln(\tanh(1/2*d*x+1/2*c)+1)-1/2/d/a/\tanh(1/2*d*x+1/2*c)+1/d*b^2/(a+b)/(b*(a+b))^{(1/2)}/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)})-1/d*b^2/(a+b)/a/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)})+1/d*b^3/(a+b)/(b*(a+b))^{(1/2)}/a/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)})+1/d*b^2/(a+b)/(b*(a+b))^{(1/2)}/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)})+1/d*b^3/(a+b)/(b*(a+b))^{(1/2)}/a/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)})$

maxima [B] time = 0.47, size = 329, normalized size = 5.48

$$\frac{b \log\left((a + b)e^{4dx+4c} + 2(a - b)e^{2dx+2c} + a + b\right)}{4(a^2 + ab)d} + \frac{b \log\left(2(a - b)e^{(-2dx-2c)} + (a + b)e^{(-4dx-4c)} + a + b\right)}{4(a^2 + ab)d} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] $-1/4*b*\log((a + b)*e^{(4*d*x + 4*c)} + 2*(a - b)*e^{(2*d*x + 2*c)} + a + b)/((a^2 + a*b)*d) + 1/4*b*\log(2*(a - b)*e^{(-2*d*x - 2*c)} + (a + b)*e^{(-4*d*x - 4*c)} + a + b)/((a^2 + a*b)*d) + 1/4*(a*b - b^2)*\arctan(1/2*((a + b)*e^{(2*d*x + 2*c)} + a - b)/\sqrt{a*b})/((a^2 + a*b)*\sqrt{a*b}*d) - 1/4*(a*b - b^2)*\arctan(1/2*((a + b)*e^{(-2*d*x - 2*c)} + a - b)/\sqrt{a*b})/((a^2 + a*b)*\sqrt{a*b}$

) * d) + 1/2 * b * arctan(1/2 * ((a + b) * e^(-2 * d * x - 2 * c) + a - b) / sqrt(a * b)) / (sqrt(a * b) * a * d) + 1/2 * log(e^(2 * d * x + 2 * c) - 1) / (a * d) - 1/2 * log(e^(-2 * d * x - 2 * c) - 1) / (a * d) - 1/2 / ((a * e^(2 * d * x + 2 * c) - a) * d) + 3/2 / ((a * e^(-2 * d * x - 2 * c) - a) * d)

mupad [B] time = 1.62, size = 402, normalized size = 6.70

$$\frac{x}{a+b} \operatorname{atan} \left(\left(e^{2c} e^{2dx} \left(\frac{4b^2}{ad(a+b)^3(a^2+ba)\sqrt{b^3}} + \frac{(a^3d\sqrt{b^3}-ab^2d\sqrt{b^3})(a-b)}{b^2(a+b)^2(a^2+ba)\sqrt{a^5d^2+2a^4bd^2+a^3b^2d^2}\sqrt{a^3d^2(a+b)^2}} \right) \right) + \frac{(a-b)(a^3d\sqrt{b^3}}{b^2(a+b)^2(a^2+ba)\sqrt{a^5d^2+2a^4bd^2+a^3b^2d^2}\sqrt{a^3d^2(a+b)^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2), x)

[Out] x/(a + b) - (atan((exp(2*c)*exp(2*d*x))*((4*b^2)/(a*d*(a + b)^3*(a*b + a^2)*(b^3)^(1/2)) + ((a^3*d*(b^3)^(1/2) - a*b^2*d*(b^3)^(1/2))*(a - b))/(b^2*(a + b)^2*(a*b + a^2)*(a^5*d^2 + 2*a^4*b*d^2 + a^3*b^2*d^2)^(1/2)*(a^3*d^2*(a + b)^2)^(1/2))) + ((a - b)*(a^3*d*(b^3)^(1/2) + a*b^2*d*(b^3)^(1/2) + 2*a^2*b*d*(b^3)^(1/2)))/(b^2*(a + b)^2*(a*b + a^2)*(a^5*d^2 + 2*a^4*b*d^2 + a^3*b^2*d^2)^(1/2)*(a^3*d^2*(a + b)^2)^(1/2))) * ((a^3*(a^5*d^2 + 2*a^4*b*d^2 + a^3*b^2*d^2)^(1/2))/2 + (a*b^2*(a^5*d^2 + 2*a^4*b*d^2 + a^3*b^2*d^2)^(1/2))/2 + a^2*b*(a^5*d^2 + 2*a^4*b*d^2 + a^3*b^2*d^2)^(1/2)) * (b^3)^(1/2)) / (a^5*d^2 + 2*a^4*b*d^2 + a^3*b^2*d^2)^(1/2) - 2/(a*d*(exp(2*c + 2*d*x) - 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{coth}^2(c + dx)}{a + b \operatorname{tanh}^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**2/(a+b*tanh(d*x+c)**2), x)

[Out] Integral(coth(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)

$$3.178 \quad \int \frac{\coth^3(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=85

$$\frac{b^2 \log(a + b \tanh^2(c + dx))}{2a^2 d(a + b)} + \frac{(a - b) \log(\tanh(c + dx))}{a^2 d} + \frac{\log(\cosh(c + dx))}{d(a + b)} - \frac{\coth^2(c + dx)}{2ad}$$

[Out] $-1/2*\coth(d*x+c)^2/a/d+\ln(\cosh(d*x+c))/(a+b)/d+(a-b)*\ln(\tanh(d*x+c))/a^2/d+1/2*b^2*\ln(a+b*\tanh(d*x+c)^2)/a^2/(a+b)/d$

Rubi [A] time = 0.14, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 446, 72}

$$\frac{b^2 \log(a + b \tanh^2(c + dx))}{2a^2 d(a + b)} + \frac{(a - b) \log(\tanh(c + dx))}{a^2 d} + \frac{\log(\cosh(c + dx))}{d(a + b)} - \frac{\coth^2(c + dx)}{2ad}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2), x]

[Out] $-\text{Coth}[c + d*x]^2/(2*a*d) + \text{Log}[\text{Cosh}[c + d*x]]/((a + b)*d) + ((a - b)*\text{Log}[\text{Tanh}[c + d*x]])/(a^2*d) + (b^2*\text{Log}[a + b*\text{Tanh}[c + d*x]^2])/(2*a^2*(a + b)*d)$

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff, x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(c+dx)}{a+b \tanh^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3(1-x^2)(a+bx^2)} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{(1-x)x^2(a+bx)} dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)(-1+x)} + \frac{1}{ax^2} + \frac{a-b}{a^2x} + \frac{b^3}{a^2(a+b)(a+bx)}\right) dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= -\frac{\coth^2(c+dx)}{2ad} + \frac{\log(\cosh(c+dx))}{(a+b)d} + \frac{(a-b)\log(\tanh(c+dx))}{a^2d} + \frac{b^2 \log(a+b \tanh(c+dx))}{2a^2(a+b)d}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 60, normalized size = 0.71

$$-\frac{b^2 \log(a \coth^2(c+dx)+b)}{a^2(a+b)} - \frac{2 \log(\sinh(c+dx))}{a+b} + \frac{\coth^2(c+dx)}{a}$$

$$\frac{\hspace{10em}}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2), x]

[Out] -1/2*(Coth[c + d*x]^2/a - (b^2*Log[b + a*Coth[c + d*x]^2])/(a^2*(a + b)) - (2*Log[Sinh[c + d*x]])/(a + b))/d

fricas [B] time = 0.52, size = 747, normalized size = 8.79

$$2 a^2 dx \cosh(dx + c)^4 + 8 a^2 dx \cosh(dx + c) \sinh(dx + c)^3 + 2 a^2 dx \sinh(dx + c)^4 + 2 a^2 dx - 4 (a^2 dx - a^2 - ab)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] -1/2*(2*a^2*d*x*cosh(d*x + c)^4 + 8*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*a^2*d*x*sinh(d*x + c)^4 + 2*a^2*d*x - 4*(a^2*d*x - a^2 - a*b)*cosh(d*x + c)^2 + 4*(3*a^2*d*x*cosh(d*x + c)^2 - a^2*d*x + a^2 + a*b)*sinh(d*x + c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 - 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x + c))*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*((a^2 - b^2)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - b^2)*sinh(d*x + c)^4 - 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - b^2)*cosh(d*x + c)^2 - a^2 + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(d*x + c)^3 - (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(a^2*d*x*cosh(d*x + c)^3 - (a^2*d*x - a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + a^2*b)*d*cosh(d*x + c)^4 + 4*(a^3 + a^2*b)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + a^2*b)*d*sinh(d*x + c)^4 - 2*(a^3 + a^2*b)*d*cosh(d*x + c)^2 + 2*(3*(a^3 + a^2*b)*d*cosh(d*x + c)^2 - (a^3 + a^2*b)*d)*sinh(d*x + c)^2 + (a^3 + a^2*b)*d + 4*((a^3 + a^2*b)*d*cosh(d*x + c)^3 - (a^3 + a^2*b)*d*cosh(d*x + c))*sinh(d*x + c))

giac [A] time = 0.25, size = 133, normalized size = 1.56

$$\frac{b^2 \log(ae^{4dx+4c}+be^{4dx+4c})+2ae^{2dx+2c}-2be^{2dx+2c}+a+b}{a^3+a^2b} - \frac{2(dx+c)}{a+b} + \frac{2(a-b)\log(|e^{2dx+2c}-1|)}{a^2} - \frac{4e^{2dx+2c}}{a(e^{2dx+2c}-1)^2}$$

$$\frac{\hspace{10em}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3/(a+b*tanh(d*x+c)**2), x)
```

```
[Out] Integral(coth(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

$$3.179 \quad \int \frac{\coth^4(c+dx)}{a+b \tanh^2(c+dx)} dx$$

Optimal. Leaf size=82

$$\frac{b^{5/2} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d(a+b)} - \frac{(a-b) \coth(c+dx)}{a^2d} + \frac{x}{a+b} - \frac{\coth^3(c+dx)}{3ad}$$

[Out] x/(a+b)+b^(5/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/(a+b)/d-(a-b)*coth(d*x+c)/a^2/d-1/3*coth(d*x+c)^3/a/d

Rubi [A] time = 0.18, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3670, 480, 583, 522, 206, 205}

$$\frac{b^{5/2} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d(a+b)} - \frac{(a-b) \coth(c+dx)}{a^2d} + \frac{x}{a+b} - \frac{\coth^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]

[Out] x/(a + b) + (b^(5/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*(a + b)*d) - ((a - b)*Coth[c + d*x])/(a^2*d) - Coth[c + d*x]^3/(3*a*d)

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 480

Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q) + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 583

Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -

$e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[\{a, b, c, d, e, f, g, p, q\}, x] \&\& IGtQ[n, 0] \&\& LtQ[m, -1]$

Rule 3670

$Int[((d_.)*tan[(e_.) + (f_.)*(x_.)])^(m_.)*((a_.) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.))^(p_.), x_Symbol] := With[\{ff = FreeFactors[Tan[e + f*x], x]\}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[\{a, b, c, d, e, f, m, n, p\}, x] \&\& (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] \&\& RationalQ[n]))$

Rubi steps

$$\begin{aligned} \int \frac{\coth^4(c+dx)}{a+b \tanh^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+bx^2)} dx, x, \tanh(c+dx)\right)}{d} \\ &= -\frac{\coth^3(c+dx)}{3ad} + \frac{\text{Subst}\left(\int \frac{3(a-b)+3bx^2}{x^2(1-x^2)(a+bx^2)} dx, x, \tanh(c+dx)\right)}{3ad} \\ &= -\frac{(a-b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad} - \frac{\text{Subst}\left(\int \frac{-3(a^2-ab+b^2)-3(a-b)bx^2}{(1-x^2)(a+bx^2)} dx, x, \tanh(c+dx)\right)}{3a^2d} \\ &= -\frac{(a-b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c+dx)\right)}{(a+b)d} + \frac{b^3 \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c+dx)\right)}{(a+b)d} \\ &= \frac{x}{a+b} + \frac{b^{5/2} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a+b)d} - \frac{(a-b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad} \end{aligned}$$

Mathematica [A] time = 0.71, size = 91, normalized size = 1.11

$$\frac{6 \left(\frac{b^{5/2} \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}} + c+dx \right)}{a+b} - \frac{\coth(c+dx) \text{csch}^2(c+dx) ((4a-3b) \cosh(2(c+dx)) - 2a+3b)}{a^2}}{6d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2), x]

[Out] ((6*(c + d*x + (b^(5/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(5/2)))/(a + b) - ((-2*a + 3*b + (4*a - 3*b)*Cosh[2*(c + d*x)])*Coth[c + d*x]*Csch[c + d*x]^2)/a^2)/(6*d)

fricas [B] time = 0.46, size = 2368, normalized size = 28.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")

[Out] [1/6*(6*a^2*d*x*cosh(d*x + c)^6 + 36*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^5 + 6*a^2*d*x*sinh(d*x + c)^6 - 6*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*

$$\begin{aligned}
& x + c)^4 + 6*(15*a^2*d*x*cosh(d*x + c)^2 - 3*a^2*d*x - 4*a^2 - 2*a*b + 2*b^2)*sinh(d*x + c)^4 - 6*a^2*d*x + 24*(5*a^2*d*x*cosh(d*x + c)^3 - (3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(3*a^2*d*x + 4*a^2 - 4*b^2)*cosh(d*x + c)^2 + 6*(15*a^2*d*x*cosh(d*x + c)^4 + 3*a^2*d*x - 6*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^2 + 4*a^2 - 4*b^2)*sinh(d*x + c)^2 + 3*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^6 - 3*b^2*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^4 + 3*b^2*cosh(d*x + c)^2 + 4*(5*b^2*cosh(d*x + c)^3 - 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^2*cosh(d*x + c)^4 - 6*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 - b^2 + 6*(b^2*cosh(d*x + c)^5 - 2*b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 16*a^2 - 4*a*b + 12*b^2 + 12*(3*a^2*d*x*cosh(d*x + c)^5 - 2*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^3 + (3*a^2*d*x + 4*a^2 - 4*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + a^2*b)*d*cosh(d*x + c)^6 + 6*(a^3 + a^2*b)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^3 + a^2*b)*d*sinh(d*x + c)^6 - 3*(a^3 + a^2*b)*d*cosh(d*x + c)^4 + 3*(5*(a^3 + a^2*b)*d*cosh(d*x + c)^2 - (a^3 + a^2*b)*d)*sinh(d*x + c)^4 + 3*(a^3 + a^2*b)*d*cosh(d*x + c)^2 + 4*(5*(a^3 + a^2*b)*d*cosh(d*x + c)^3 - 3*(a^3 + a^2*b)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*(a^3 + a^2*b)*d*cosh(d*x + c)^4 - 6*(a^3 + a^2*b)*d*cosh(d*x + c)^2 + (a^3 + a^2*b)*d)*sinh(d*x + c)^2 - (a^3 + a^2*b)*d + 6*((a^3 + a^2*b)*d*cosh(d*x + c)^5 - 2*(a^3 + a^2*b)*d*cosh(d*x + c)^3 + (a^3 + a^2*b)*d*cosh(d*x + c))*sinh(d*x + c)), 1/3*(3*a^2*d*x*cosh(d*x + c)^6 + 18*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^5 + 3*a^2*d*x*sinh(d*x + c)^6 - 3*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^4 + 3*(15*a^2*d*x*cosh(d*x + c)^2 - 3*a^2*d*x - 4*a^2 - 2*a*b + 2*b^2)*sinh(d*x + c)^4 - 3*a^2*d*x + 12*(5*a^2*d*x*cosh(d*x + c)^3 - (3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(3*a^2*d*x + 4*a^2 - 4*b^2)*cosh(d*x + c)^2 + 3*(15*a^2*d*x*cosh(d*x + c)^4 + 3*a^2*d*x - 6*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^2 + 4*a^2 - 4*b^2)*sinh(d*x + c)^2 + 3*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^6 - 3*b^2*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^4 + 3*b^2*cosh(d*x + c)^2 + 4*(5*b^2*cosh(d*x + c)^3 - 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^2*cosh(d*x + c)^4 - 6*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 - b^2 + 6*(b^2*cosh(d*x + c)^5 - 2*b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) - 8*a^2 - 2*a*b + 6*b^2 + 6*(3*a^2*d*x*cosh(d*x + c)^5 - 2*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^3 + (3*a^2*d*x + 4*a^2 - 4*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + a^2*b)*d*cosh(d*x + c)^6 + 6*(a^3 + a^2*b)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^3 + a^2*b)*d*sinh(d*x + c)^6 - 3*(a^3 + a^2*b)*d*cosh(d*x + c)^4 + 3*(5*(a^3 + a^2*b)*d*cosh(d*x + c)^2 - (a^3 + a^2*b)*d)*sinh(d*x + c)^4 + 3*(a^3 + a^2*b)*d*cosh(d*x + c)^2 + 4*(5*(a^3 + a^2*b)*d*cosh(d*x + c)^3 - 3*(a^3 + a^2*b)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*(a^3 + a^2*b)*d*cosh(d*x + c)^4 - 6*(a^3 + a^2*b)*d*cosh(d*x + c)^2 + (a^3 + a^2*b)*d)*sinh(d*x + c)^2 - (a^3 + a^2*b)*d + 6*((a^3 + a^2*b)*d*cosh(d*x + c)^5 - 2*(a^3 + a^2*b)*d*cosh(d*x + c)^3 + (a^3 + a^2*b)*d*cosh(d*x + c))*sinh(d*x + c))]
\end{aligned}$$

giac [B] time = 0.25, size = 147, normalized size = 1.79

$$\frac{3b^3 \arctan\left(\frac{ae^{2dx+2c} + be^{2dx+2c} + a - b}{2\sqrt{ab}}\right)}{(a^3 + a^2b)\sqrt{ab}} + \frac{3(dx+c)}{a+b} - \frac{2(6ae^{4dx+4c} - 3be^{4dx+4c} - 6ae^{2dx+2c} + 6be^{2dx+2c} + 4a - 3b)}{a^2(e^{2dx+2c} - 1)^3}$$

$$3d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")

[Out] $\frac{1}{3} \cdot (3b^3 \arctan(1/2 \cdot (ae^{2dx+2c} + be^{2dx+2c} + a - b) / \sqrt{ab})) / ((a^3 + a^2b) \sqrt{ab}) + 3(dx+c)/(a+b) - 2(6ae^{4dx+4c} - 3be^{4dx+4c} - 6ae^{2dx+2c} + 6be^{2dx+2c} + 4a - 3b) / (a^2(e^{2dx+2c} - 1)^3) / d$

maple [B] time = 0.48, size = 580, normalized size = 7.07

$$\frac{\tanh^3\left(\frac{dx}{2} + \frac{c}{2}\right)}{24da} - \frac{5 \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)}{8da} + \frac{\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)b}{2da^2} - \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{d(a+b)} + \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{d(a+b)} - \frac{1}{d(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2),x)

[Out] $-1/24/d/a \cdot \tanh(1/2 \cdot dx + 1/2 \cdot c)^3 - 5/8/d/a \cdot \tanh(1/2 \cdot dx + 1/2 \cdot c) + 1/2/d/a^2 \cdot \tanh(1/2 \cdot dx + 1/2 \cdot c) \cdot b - 1/d/(a+b) \cdot \ln(\tanh(1/2 \cdot dx + 1/2 \cdot c) - 1) + 1/d/(a+b) \cdot \ln(\tanh(1/2 \cdot dx + 1/2 \cdot c) + 1) - 1/d \cdot b^3/(a+b) / (b \cdot (a+b))^{1/2} / a / ((2 \cdot (b \cdot (a+b))^{1/2} - a - 2 \cdot b) \cdot a)^{1/2} \cdot \operatorname{arctanh}(a \cdot \tanh(1/2 \cdot dx + 1/2 \cdot c) / ((2 \cdot (b \cdot (a+b))^{1/2} - a - 2 \cdot b) \cdot a)^{1/2}) + 1/d \cdot b^3/(a+b) / a^2 / ((2 \cdot (b \cdot (a+b))^{1/2} - a - 2 \cdot b) \cdot a)^{1/2} \cdot \operatorname{arctanh}(a \cdot \tanh(1/2 \cdot dx + 1/2 \cdot c) / ((2 \cdot (b \cdot (a+b))^{1/2} - a - 2 \cdot b) \cdot a)^{1/2}) - 1/d \cdot b^4/(a+b) / a^2 / (b \cdot (a+b))^{1/2} / ((2 \cdot (b \cdot (a+b))^{1/2} - a - 2 \cdot b) \cdot a)^{1/2} \cdot \operatorname{arctanh}(a \cdot \tanh(1/2 \cdot dx + 1/2 \cdot c) / ((2 \cdot (b \cdot (a+b))^{1/2} - a - 2 \cdot b) \cdot a)^{1/2}) - 1/d \cdot b^3/(a+b) / a^2 / ((2 \cdot (b \cdot (a+b))^{1/2} + a + 2 \cdot b) \cdot a)^{1/2} \cdot \operatorname{arctan}(a \cdot \tanh(1/2 \cdot dx + 1/2 \cdot c) / ((2 \cdot (b \cdot (a+b))^{1/2} + a + 2 \cdot b) \cdot a)^{1/2}) - 1/d \cdot b^4/(a+b) / a^2 / (b \cdot (a+b))^{1/2} / ((2 \cdot (b \cdot (a+b))^{1/2} + a + 2 \cdot b) \cdot a)^{1/2} \cdot \operatorname{arctan}(a \cdot \tanh(1/2 \cdot dx + 1/2 \cdot c) / ((2 \cdot (b \cdot (a+b))^{1/2} + a + 2 \cdot b) \cdot a)^{1/2}) - 1/24/d/a \cdot \tanh(1/2 \cdot dx + 1/2 \cdot c)^3 - 5/8/d/a \cdot \tanh(1/2 \cdot dx + 1/2 \cdot c) + 1/2/d \cdot b/a^2 \cdot \tanh(1/2 \cdot dx + 1/2 \cdot c)$

maxima [B] time = 0.61, size = 1038, normalized size = 12.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")

[Out] $-1/8 \cdot (a \cdot b - b^2) \cdot \log((a+b) \cdot e^{4dx+4c} + 2 \cdot (a-b) \cdot e^{2dx+2c} + a+b) / ((a^3 + a^2b) \cdot d) + 1/8 \cdot (a \cdot b - b^2) \cdot \log(2 \cdot (a-b) \cdot e^{-2dx-2c} + (a+b) \cdot e^{-4dx-4c} + a+b) / ((a^3 + a^2b) \cdot d) + 1/16 \cdot (a^2b - 6a \cdot b^2 + b^3) \cdot \arctan(1/2 \cdot ((a+b) \cdot e^{2dx+2c} + a-b) / \sqrt{ab}) / ((a^3 + a^2b) \cdot \sqrt{ab} \cdot d) - 1/16 \cdot (a^2b - 6a \cdot b^2 + b^3) \cdot \arctan(1/2 \cdot ((a+b) \cdot e^{-2dx-2c} + a-b) / \sqrt{ab}) / ((a^3 + a^2b) \cdot \sqrt{ab} \cdot d) - 1/24 \cdot (3 \cdot (12a - b) \cdot e^{4dx+4c} - 6 \cdot (9a - b) \cdot e^{2dx+2c} + 22a - 3b) / ((a^2 \cdot e^{6dx+6c} - 3a^2 \cdot e^{4dx+4c} + 3a^2 \cdot e^{2dx+2c} - a^2) \cdot d) - 1/6 \cdot (3 \cdot (4a - b) \cdot e^{4dx+4c} - 6 \cdot (2a - b) \cdot e^{2dx+2c} + 4a - 3b) / ((a^2 \cdot e^{6dx+6c} - 3a^2 \cdot e^{4dx+4c} + 3a^2 \cdot e^{2dx+2c} - a^2) \cdot d) - 1/24 \cdot (6 \cdot (9a - b) \cdot e^{-2dx-2c} - 3 \cdot (12a - b) \cdot e^{-4dx-4c} - 2$

$$\frac{2*a + 3*b}{((3*a^2*e^{(-2*d*x - 2*c)} - 3*a^2*e^{(-4*d*x - 4*c)} + a^2*e^{(-6*d*x - 6*c)} - a^2)*d) - 1/6*(6*(2*a - b)*e^{(-2*d*x - 2*c)} - 3*(4*a - b)*e^{(-4*d*x - 4*c)} - 4*a + 3*b)/((3*a^2*e^{(-2*d*x - 2*c)} - 3*a^2*e^{(-4*d*x - 4*c)} + a^2*e^{(-6*d*x - 6*c)} - a^2)*d) + 1/4*(6*(a + b)*e^{(-2*d*x - 2*c)} - 3*b*e^{(-4*d*x - 4*c)} - 2*a - 3*b)/((3*a^2*e^{(-2*d*x - 2*c)} - 3*a^2*e^{(-4*d*x - 4*c)} + a^2*e^{(-6*d*x - 6*c)} - a^2)*d) + 1/4*b*log((a + b)*e^{(4*d*x + 4*c)} + 2*(a - b)*e^{(2*d*x + 2*c)} + a + b)/(a^2*d) - 1/4*b*log(2*(a - b)*e^{(-2*d*x - 2*c)} + (a + b)*e^{(-4*d*x - 4*c)} + a + b)/(a^2*d) + 1/4*(2*a - b)*log(e^{(2*d*x + 2*c)} - 1)/(a^2*d) - 1/2*b*log(e^{(2*d*x + 2*c)} - 1)/(a^2*d) - 1/4*(2*a - b)*log(e^{(-2*d*x - 2*c)} - 1)/(a^2*d) + 1/2*b*log(e^{(-2*d*x - 2*c)} - 1)/(a^2*d) - 1/4*(a*b - b^2)*arctan(1/2*((a + b)*e^{(2*d*x + 2*c)} + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*d) - 3/8*(a*b + b^2)*arctan(1/2*((a + b)*e^{(-2*d*x - 2*c)} + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*d) + 1/4*(a*b - b^2)*arctan(1/2*((a + b)*e^{(-2*d*x - 2*c)} + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*d)$$

mupad [B] time = 1.61, size = 519, normalized size = 6.33

$$\frac{x}{a+b} + \frac{\operatorname{atan}\left(\left(e^{2c} e^{2dx} \left(\frac{4b^3}{a^2 d (a+b)^3 (a^3 + b a^2) \sqrt{b^5}} + \frac{(a^4 d \sqrt{b^5} - a^2 b^2 d \sqrt{b^5})^{(a-b)}}{b^3 (a+b)^2 (a^3 + b a^2) \sqrt{a^7 d^2 + 2 a^6 b d^2 + a^5 b^2 d^2} \sqrt{a^5 d^2 (a+b)^2}}\right)\right)}{b^3 (a+b)^2 (a^3 + b a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2), x)

[Out] $x/(a + b) + (\operatorname{atan}((\exp(2*c)*\exp(2*d*x)*((4*b^3)/(a^2*d*(a + b)^3*(a^2*b + a^3)*(b^5)^{(1/2)})) + ((a^4*d*(b^5)^{(1/2)} - a^2*b^2*d*(b^5)^{(1/2)})*(a - b))/(b^3*(a + b)^2*(a^2*b + a^3)*(a^7*d^2 + 2*a^6*b*d^2 + a^5*b^2*d^2)^{(1/2)}*(a^5*d^2*(a + b)^2)^{(1/2)})) + ((a - b)*(a^4*d*(b^5)^{(1/2)} + 2*a^3*b*d*(b^5)^{(1/2)} + a^2*b^2*d*(b^5)^{(1/2)}))/(b^3*(a + b)^2*(a^2*b + a^3)*(a^7*d^2 + 2*a^6*b*d^2 + a^5*b^2*d^2)^{(1/2)}*(a^5*d^2*(a + b)^2)^{(1/2)})))*((a^4*(a^7*d^2 + 2*a^6*b*d^2 + a^5*b^2*d^2)^{(1/2)})/2 + a^3*b*(a^7*d^2 + 2*a^6*b*d^2 + a^5*b^2*d^2)^{(1/2)} + (a^2*b^2*(a^7*d^2 + 2*a^6*b*d^2 + a^5*b^2*d^2)^{(1/2)})/2))*(b^5)^{(1/2))/(a^7*d^2 + 2*a^6*b*d^2 + a^5*b^2*d^2)^{(1/2)} - 8/(3*a*d*(3*\exp(2*c + 2*d*x) - 3*\exp(4*c + 4*d*x) + \exp(6*c + 6*d*x) - 1)) - (4*(a*b + a^2))/(a^2*d*(a + b)*(exp(4*c + 4*d*x) - 2*\exp(2*c + 2*d*x) + 1)) - (2*(a*b + 2*a^2 - b^2))/(a^2*d*(exp(2*c + 2*d*x) - 1)*(a + b))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(c + dx)}{a + b \tanh^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**4/(a+b*tanh(d*x+c)**2), x)

[Out] Integral(coth(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)

$$3.180 \quad \int \frac{\tanh^5(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=83

$$\frac{a^2}{2b^2d(a+b)(a+b \tanh^2(c+dx))} - \frac{a(a+2b) \log(a+b \tanh^2(c+dx))}{2b^2d(a+b)^2} + \frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

[Out] $\ln(\cosh(d*x+c))/(a+b)^2/d-1/2*a*(a+2*b)*\ln(a+b*\tanh(d*x+c)^2)/b^2/(a+b)^2/d-1/2*a^2/b^2/(a+b)/d/(a+b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.15, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 446, 88}

$$\frac{a^2}{2b^2d(a+b)(a+b \tanh^2(c+dx))} - \frac{a(a+2b) \log(a+b \tanh^2(c+dx))}{2b^2d(a+b)^2} + \frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^2,x]

[Out] $\text{Log}[\text{Cosh}[c + d*x]]/((a + b)^2*d) - (a*(a + 2*b)*\text{Log}[a + b*\text{Tanh}[c + d*x]^2])/(2*b^2*(a + b)^2*d) - a^2/(2*b^2*(a + b)*d*(a + b*\text{Tanh}[c + d*x]^2))$

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegerQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^5(c+dx)}{(a+b\tanh^2(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{x^5}{(1-x^2)(a+bx^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)(a+bx)^2} dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)} + \frac{a^2}{b(a+b)(a+bx)^2} - \frac{a(a+2b)}{b(a+b)^2(a+bx)}\right) dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\log(\cosh(c+dx))}{(a+b)^2 d} - \frac{a(a+2b)\log(a+b\tanh^2(c+dx))}{2b^2(a+b)^2 d} - \frac{a^2}{2b^2(a+b)d(a+b)}
\end{aligned}$$

Mathematica [A] time = 0.56, size = 69, normalized size = 0.83

$$\frac{\frac{a^2(a+b)}{b^2(a+b\tanh^2(c+dx))} + \frac{a(a+2b)\log(a+b\tanh^2(c+dx))}{b^2} - 2\log(\cosh(c+dx))}{2d(a+b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^2,x]

[Out] -1/2*(-2*Log[Cosh[c + d*x]] + (a*(a + 2*b)*Log[a + b*Tanh[c + d*x]^2])/b^2 + (a^2*(a + b))/(b^2*(a + b*Tanh[c + d*x]^2)))/((a + b)^2*d)

fricas [B] time = 0.55, size = 1141, normalized size = 13.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -1/2*(2*(a*b^2 + b^3)*d*x*cosh(d*x + c)^4 + 8*(a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a*b^2 + b^3)*d*x*sinh(d*x + c)^4 + 2*(a*b^2 + b^3)*d*x + 4*(a^2*b + (a*b^2 - b^3)*d*x)*cosh(d*x + c)^2 + 4*(3*(a*b^2 + b^3)*d*x*cosh(d*x + c)^2 + a^2*b + (a*b^2 - b^3)*d*x)*sinh(d*x + c)^2 + ((a^3 + 3*a^2*b + 2*a*b^2)*cosh(d*x + c)^4 + 4*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 2*a*b^2)*sinh(d*x + c)^4 + a^3 + 3*a^2*b + 2*a*b^2 + 2*(a^3 + a^2*b - 2*a*b^2)*cosh(d*x + c)^2 + 2*(a^3 + a^2*b - 2*a*b^2 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3 + 3*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + (a^3 + a^2*b - 2*a*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*((a*b^2 + b^3)*d*x*cosh(d*x + c)^3 + (a^2*b + (a*b^2 - b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*d*sinh(d*x + c)^4 + 2*(a^3*b^2 + a^2*b^3 - a*b^4 - b^5)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 + 3*a^2*b^3 + 3

$a^2b^4 + b^5)d \cosh(dx + c)^2 + (a^3b^2 + a^2b^3 - ab^4 - b^5)d \sinh(dx + c)^2 + (a^3b^2 + 3a^2b^3 + 3ab^4 + b^5)d + 4((a^3b^2 + 3a^2b^3 + 3ab^4 + b^5)d \cosh(dx + c)^3 + (a^3b^2 + a^2b^3 - ab^4 - b^5)d \cosh(dx + c)) \sinh(dx + c)$

giac [B] time = 0.45, size = 194, normalized size = 2.34

$$\frac{(a^2+2ab)\log(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b)}{a^2b^2+2ab^3+b^4} + \frac{2(dx+c)}{a^2+2ab+b^2} + \frac{4a^2e^{(2dx+2c)}}{(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b)}$$

$$2d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(dx+c)^5/(a+b*tanh(dx+c)^2)^2,x, algorithm="giac")

[Out] $-1/2((a^2 + 2ab) \log(ae^{(4dx + 4c)} + be^{(4dx + 4c)} + 2ae^{(2dx + 2c)} - 2be^{(2dx + 2c)} + a + b) / (a^2b^2 + 2ab^3 + b^4) + 2(dx + c) / (a^2 + 2ab + b^2) + 4a^2e^{(2dx + 2c)} / ((ae^{(4dx + 4c)} + be^{(4dx + 4c)} + 2ae^{(2dx + 2c)} - 2be^{(2dx + 2c)} + a + b)) - 2b \log(e^{(2dx + 2c)} + 1) / b^2) / d$

maple [A] time = 0.10, size = 156, normalized size = 1.88

$$\frac{\ln(\tanh(dx+c)-1)}{2d(a+b)^2} - \frac{\ln(1+\tanh(dx+c))}{2d(a+b)^2} - \frac{a^2 \ln(a+b(\tanh^2(dx+c)))}{2d(a+b)^2 b^2} - \frac{a \ln(a+b(\tanh^2(dx+c)))}{d(a+b)^2 b} - \frac{1}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(dx+c)^5/(a+b*tanh(dx+c)^2)^2,x)

[Out] $-1/2/d/(a+b)^2 \ln(\tanh(dx+c)-1) - 1/2/d/(a+b)^2 \ln(1+\tanh(dx+c)) - 1/2/d*a^2/(a+b)^2/b^2 \ln(a+b*tanh(dx+c)^2) - 1/d*a/(a+b)^2/b \ln(a+b*tanh(dx+c)^2) - 1/2/d*a^3/(a+b)^2/b^2/(a+b*tanh(dx+c)^2) - 1/2/d*a^2/(a+b)^2/b/(a+b*tanh(dx+c)^2)$

maxima [B] time = 0.44, size = 217, normalized size = 2.61

$$\frac{2a^2e^{(-2dx-2c)}}{(a^3b + 3a^2b^2 + 3ab^3 + b^4 + 2(a^3b + a^2b^2 - ab^3 - b^4)e^{(-2dx-2c)} + (a^3b + 3a^2b^2 + 3ab^3 + b^4)e^{(-4dx-4c)})d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(dx+c)^5/(a+b*tanh(dx+c)^2)^2,x, algorithm="maxima")

[Out] $-2a^2e^{(-2dx - 2c)} / ((a^3b + 3a^2b^2 + 3ab^3 + b^4 + 2(a^3b + a^2b^2 - ab^3 - b^4)e^{(-2dx - 2c)} + (a^3b + 3a^2b^2 + 3ab^3 + b^4)e^{(-4dx - 4c)})d) - 1/2(a^2 + 2ab) \log(2(a - b)e^{(-2dx - 2c)} + (a + b)e^{(-4dx - 4c)} + a + b) / ((a^2b^2 + 2ab^3 + b^4)d) + (dx + c) / ((a^2 + 2ab + b^2)d) + \log(e^{(-2dx - 2c)} + 1) / (b^2d)$

mupad [B] time = 1.61, size = 170, normalized size = 2.05

$$\frac{a^2}{2(d a^2 b^2 + d a b^3 \tanh(c + dx)^2 + d a b^3 + d b^4 \tanh(c + dx)^2)} - \frac{\ln(\tanh(c + dx)^2 - 1)}{2(d a^2 + 2 d a b + d b^2)} - \frac{a^2 \ln(b \tanh(c + dx))}{2(d a^2 b^2 + 2 d a b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + dx)^5/(a + b*tanh(c + dx)^2)^2,x)

[Out] $-a^2/(2(a^2b^2d + b^4d \tanh(c + dx)^2 + ab^3d + ab^3d \tanh(c + dx)^2)) - \log(\tanh(c + dx)^2 - 1)/(2(a^2d + b^2d + 2ab^3d)) - (a^2 \log(\tanh(c + dx)^2 - 1) / (2(a^2d + b^2d + 2ab^3d)))$

$$\frac{a + b \tanh(c + dx)^2}{2(b^4d + a^2b^2d + 2ab^3d)} - (ab \log(a + b \tanh(c + dx)^2)) / (b^4d + a^2b^2d + 2ab^3d)$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(dx+c)**5/(a+b*tanh(dx+c)**2)**2,x)

[Out] Timed out

$$3.181 \quad \int \frac{\tanh^4(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=89

$$-\frac{\sqrt{a}(a+3b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2b^{3/2}d(a+b)^2} + \frac{a \tanh(c+dx)}{2bd(a+b)(a+b \tanh^2(c+dx))} + \frac{x}{(a+b)^2}$$

[Out] x/(a+b)^2-1/2*(a+3*b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*a^(1/2)/b^(3/2)/(a+b)^2/d+1/2*a*tanh(d*x+c)/b/(a+b)/d/(a+b*tanh(d*x+c)^2)

Rubi [A] time = 0.12, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3670, 470, 522, 206, 205}

$$-\frac{\sqrt{a}(a+3b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2b^{3/2}d(a+b)^2} + \frac{a \tanh(c+dx)}{2bd(a+b)(a+b \tanh^2(c+dx))} + \frac{x}{(a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2), x]

[Out] x/(a + b)^2 - (Sqrt[a]*(a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*b^(3/2)*(a + b)^2*d) + (a*Tanh[c + d*x])/(2*b*(a + b)*d*(a + b*Tanh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_.) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned} \int \frac{\tanh^4(c+dx)}{(a+b \tanh^2(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a+bx^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{a \tanh(c+dx)}{2b(a+b)d(a+b \tanh^2(c+dx))} - \frac{\text{Subst}\left(\int \frac{a+(-a-2b)x^2}{(1-x^2)(a+bx^2)} dx, x, \tanh(c+dx)\right)}{2b(a+b)d} \\ &= \frac{a \tanh(c+dx)}{2b(a+b)d(a+b \tanh^2(c+dx))} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c+dx)\right)}{(a+b)^2d} - \frac{a(a+b)}{(a+b)^2d} \\ &= \frac{x}{(a+b)^2} - \frac{\sqrt{a}(a+3b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2b^{3/2}(a+b)^2d} + \frac{a \tanh(c+dx)}{2b(a+b)d(a+b \tanh^2(c+dx))} \end{aligned}$$

Mathematica [A] time = 0.60, size = 90, normalized size = 1.01

$$\frac{\frac{\sqrt{a}(a+3b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}} + \frac{a(a+b) \sinh(2(c+dx))}{b((a+b) \cosh(2(c+dx))+a-b)} + 2(c+dx)}{2d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2), x]
```

```
[Out] (2*(c + d*x) - (Sqrt[a]*(a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/
b^(3/2) + (a*(a + b)*Sinh[2*(c + d*x)]/(b*(a - b + (a + b)*Cosh[2*(c + d*x
)])))/(2*(a + b)^2*d)
```

fricas [B] time = 0.48, size = 1950, normalized size = 21.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2), x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a*b + b^2)*d*x*cosh(d*x + c)^4 + 16*(a*b + b^2)*d*x*cosh(d*x + c)*
sinh(d*x + c)^3 + 4*(a*b + b^2)*d*x*sinh(d*x + c)^4 + 4*(a*b + b^2)*d*x + 4
*(2*(a*b - b^2)*d*x - a^2 + a*b)*cosh(d*x + c)^2 + 4*(6*(a*b + b^2)*d*x*cos
h(d*x + c)^2 + 2*(a*b - b^2)*d*x - a^2 + a*b)*sinh(d*x + c)^2 + ((a^2 + 4*a
*b + 3*b^2)*cosh(d*x + c)^4 + 4*(a^2 + 4*a*b + 3*b^2)*cosh(d*x + c)*sinh(d*
x + c)^3 + (a^2 + 4*a*b + 3*b^2)*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b - 3*b^2)*
cosh(d*x + c)^2 + 2*(3*(a^2 + 4*a*b + 3*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b
- 3*b^2)*sinh(d*x + c)^2 + a^2 + 4*a*b + 3*b^2 + 4*((a^2 + 4*a*b + 3*b^2)*c
osh(d*x + c)^3 + (a^2 + 2*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-
a/b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b
```

$$\begin{aligned} &^2) * \cosh(dx + c)^2 + 2 * (3 * (a^2 + 2 * a * b + b^2) * \cosh(dx + c)^2 + a^2 - b^2) \\ & * \sinh(dx + c)^2 + a^2 - 6 * a * b + b^2 + 4 * ((a^2 + 2 * a * b + b^2) * \cosh(dx + c) \\ & ^3 + (a^2 - b^2) * \cosh(dx + c)) * \sinh(dx + c) - 4 * ((a * b + b^2) * \cosh(dx + c) \\ &)^2 + 2 * (a * b + b^2) * \cosh(dx + c) * \sinh(dx + c) + (a * b + b^2) * \sinh(dx + c) \\ & ^2 + a * b - b^2) * \sqrt{-a/b}) / ((a + b) * \cosh(dx + c)^4 + 4 * (a + b) * \cosh(dx + c) \\ &) * \sinh(dx + c)^3 + (a + b) * \sinh(dx + c)^4 + 2 * (a - b) * \cosh(dx + c)^2 + \\ & 2 * (3 * (a + b) * \cosh(dx + c)^2 + a - b) * \sinh(dx + c)^2 + 4 * ((a + b) * \cosh(dx + c)^3 + \\ & (a - b) * \cosh(dx + c)) * \sinh(dx + c) + a + b) - 4 * a^2 - 4 * a * b + \\ & 8 * (2 * (a * b + b^2) * dx * \cosh(dx + c)^3 + (2 * (a * b - b^2) * dx - a^2 + a * b) * \cosh(dx + c)) * \sinh(dx + c)) / ((a^3 * b + 3 * a^2 * b^2 + 3 * a * b^3 + b^4) * d * \cosh(dx + c)^4 + 4 * (a^3 * b + 3 * a^2 * b^2 + 3 * a * b^3 + b^4) * d * \cosh(dx + c) * \sinh(dx + c)^3 + (a^3 * b + 3 * a^2 * b^2 + 3 * a * b^3 + b^4) * d * \sinh(dx + c)^4 + 2 * (a^3 * b + a^2 * b^2 - a * b^3 - b^4) * d * \cosh(dx + c)^2 + 2 * (3 * (a^3 * b + 3 * a^2 * b^2 + 3 * a * b^3 + b^4) * d * \cosh(dx + c)^2 + (a^3 * b + a^2 * b^2 - a * b^3 - b^4) * d) * \sinh(dx + c)^2 + (a^3 * b + 3 * a^2 * b^2 + 3 * a * b^3 + b^4) * d + 4 * ((a^3 * b + 3 * a^2 * b^2 + 3 * a * b^3 + b^4) * d * \cosh(dx + c)^3 + (a^3 * b + a^2 * b^2 - a * b^3 - b^4) * d * \cosh(dx + c)) * \sinh(dx + c)), 1/2 * (2 * (a * b + b^2) * dx * \cosh(dx + c)^4 + 8 * (a * b + b^2) * dx * \cosh(dx + c) * \sinh(dx + c)^3 + 2 * (a * b + b^2) * dx * \sinh(dx + c)^4 + 2 * (a * b + b^2) * dx + 2 * (2 * (a * b - b^2) * dx - a^2 + a * b) * \cosh(dx + c)^2 + 2 * (6 * (a * b + b^2) * dx * \cosh(dx + c)^2 + 2 * (a * b - b^2) * dx - a^2 + a * b) * \sinh(dx + c)^2 - ((a^2 + 4 * a * b + 3 * b^2) * \cosh(dx + c)^4 + 4 * (a^2 + 4 * a * b + 3 * b^2) * \cosh(dx + c) * \sinh(dx + c)^3 + (a^2 + 4 * a * b + 3 * b^2) * \sinh(dx + c)^4 + 2 * (a^2 + 2 * a * b - 3 * b^2) * \cosh(dx + c)^2 + 2 * (3 * (a^2 + 4 * a * b + 3 * b^2) * \cosh(dx + c)^2 + a^2 + 2 * a * b - 3 * b^2) * \sinh(dx + c)^2 + a^2 + 4 * a * b + 3 * b^2 + 4 * ((a^2 + 4 * a * b + 3 * b^2) * \cosh(dx + c)^3 + (a^2 + 2 * a * b - 3 * b^2) * \cosh(dx + c)) * \sinh(dx + c)) * \sqrt{a/b} * \arctan(1/2 * ((a + b) * \cosh(dx + c)^2 + 2 * (a + b) * \cosh(dx + c) * \sinh(dx + c) + (a + b) * \sinh(dx + c)^2 + a - b) * \sqrt{a/b}) / a) - 2 * a^2 - 2 * a * b + 4 * (2 * (a * b + b^2) * dx * \cosh(dx + c)^3 + (2 * (a * b - b^2) * dx - a^2 + a * b) * \cosh(dx + c)) * \sinh(dx + c)) / ((a^3 * b + 3 * a^2 * b^2 + 3 * a * b^3 + b^4) * d * \cosh(dx + c)^4 + 4 * (a^3 * b + 3 * a^2 * b^2 + 3 * a * b^3 + b^4) * d * \cosh(dx + c) * \sinh(dx + c)^3 + (a^3 * b + 3 * a^2 * b^2 + 3 * a * b^3 + b^4) * d * \sinh(dx + c)^4 + 2 * (a^3 * b + a^2 * b^2 - a * b^3 - b^4) * d * \cosh(dx + c)^2 + 2 * (3 * (a^3 * b + 3 * a^2 * b^2 + 3 * a * b^3 + b^4) * d * \cosh(dx + c)^2 + (a^3 * b + a^2 * b^2 - a * b^3 - b^4) * d) * \sinh(dx + c)^2 + (a^3 * b + 3 * a^2 * b^2 + 3 * a * b^3 + b^4) * d + 4 * ((a^3 * b + 3 * a^2 * b^2 + 3 * a * b^3 + b^4) * d * \cosh(dx + c)^3 + (a^3 * b + a^2 * b^2 - a * b^3 - b^4) * d * \cosh(dx + c)) * \sinh(dx + c))] \end{aligned}$$

giac [B] time = 0.34, size = 195, normalized size = 2.19

$$\frac{(a^2+3ab) \arctan\left(\frac{ae^{2dx+2c}+be^{2dx+2c}+a-b}{2\sqrt{ab}}\right)}{(a^2b+2ab^2+b^3)\sqrt{ab}} - \frac{2(dx+c)}{a^2+2ab+b^2} + \frac{2(a^2e^{2dx+2c}-abe^{2dx+2c}+a^2+ab)}{(a^2b+2ab^2+b^3)(ae^{4dx+4c}+be^{4dx+4c}+2ae^{2dx+2c}-2be^{2dx+2c}+a+b)}$$

$$2d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(dx+c)^4/(a+b*tanh(dx+c)^2)^2,x, algorithm="giac")

[Out] -1/2*((a^2 + 3*a*b)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b)))/((a^2*b + 2*a*b^2 + b^3)*sqrt(a*b)) - 2*(d*x + c)/(a^2 + 2*a*b + b^2) + 2*(a^2*e^(2*d*x + 2*c) - a*b*e^(2*d*x + 2*c) + a^2 + a*b)/((a^2*b + 2*a*b^2 + b^3)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b))/d

maple [B] time = 0.10, size = 172, normalized size = 1.93

$$-\frac{\ln(\tanh(dx+c)-1)}{2d(a+b)^2} + \frac{\ln(1+\tanh(dx+c))}{2d(a+b)^2} + \frac{a^2 \tanh(dx+c)}{2d(a+b)^2 b(a+b(\tanh^2(dx+c)))} + \frac{a \tanh(dx+c)}{2d(a+b)^2(a+b(\tanh^2(dx+c)))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^4/(a+b*tanh(d*x+c))^2,x)

[Out] $-1/2/d/(a+b)^2*\ln(\tanh(d*x+c)-1)+1/2/d/(a+b)^2*\ln(1+\tanh(d*x+c))+1/2/d*a^2/(a+b)^2/b*\tanh(d*x+c)/(a+b*tanh(d*x+c))^2+1/2/d*a/(a+b)^2*tanh(d*x+c)/(a+b*tanh(d*x+c)^2)-1/2/d*a^2/(a+b)^2/b/(a*b)^{(1/2)}*\arctan(\tanh(d*x+c)*b/(a*b)^{(1/2)})-3/2/d*a/(a+b)^2/(a*b)^{(1/2)}*\arctan(\tanh(d*x+c)*b/(a*b)^{(1/2)})$

maxima [B] time = 0.79, size = 1010, normalized size = 11.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c))^2,x, algorithm="maxima")

[Out] $-1/32*(a^3 + 9*a^2*b - 9*a*b^2 - b^3)*\arctan(1/2*((a + b)*e^{(2*d*x + 2*c)} + a - b)/\sqrt{a*b})/((a^3*b + 2*a^2*b^2 + a*b^3)*\sqrt{a*b}*d) + 1/32*(a^3 + 9*a^2*b - 9*a*b^2 - b^3)*\arctan(1/2*((a + b)*e^{(-2*d*x - 2*c)} + a - b)/\sqrt{a*b})/((a^3*b + 2*a^2*b^2 + a*b^3)*\sqrt{a*b}*d) - 1/16*(a^3 - 5*a^2*b - 5*a*b^2 + b^3 + (a^3 - 15*a^2*b + 15*a*b^2 - b^3)*e^{(2*d*x + 2*c)})/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + (a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*e^{(4*d*x + 4*c)} + 2*(a^4*b + a^3*b^2 - a^2*b^3 - a*b^4)*e^{(2*d*x + 2*c)})*d) + 1/16*(a^3 - 5*a^2*b - 5*a*b^2 + b^3 + (a^3 - 15*a^2*b + 15*a*b^2 - b^3)*e^{(-2*d*x - 2*c)})/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + 2*(a^4*b + a^3*b^2 - a^2*b^3 - a*b^4)*e^{(-2*d*x - 2*c)} + (a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*e^{(-4*d*x - 4*c)})*d) - 1/4*(a^2 - b^2 + (a^2 - 6*a*b + b^2)*e^{(2*d*x + 2*c)})/((a^3*b + 2*a^2*b^2 + a*b^3 + (a^3*b + 2*a^2*b^2 + a*b^3)*e^{(4*d*x + 4*c)} + 2*(a^3*b - a*b^3)*e^{(2*d*x + 2*c)})*d) + 1/4*(a^2 - b^2 + (a^2 - 6*a*b + b^2)*e^{(-2*d*x - 2*c)})/((a^3*b + 2*a^2*b^2 + a*b^3 + 2*(a^3*b - a*b^3)*e^{(-2*d*x - 2*c)} + (a^3*b + 2*a^2*b^2 + a*b^3)*e^{(-4*d*x - 4*c)})*d) + 3/8*(a - b)*e^{(-2*d*x - 2*c)} + a + b)/((a^2*b + a*b^2 + 2*(a^2*b - a*b^2)*e^{(-2*d*x - 2*c)} + (a^2*b + a*b^2)*e^{(-4*d*x - 4*c)})*d) + 1/4*log((a + b)*e^{(4*d*x + 4*c)} + 2*(a - b)*e^{(2*d*x + 2*c)} + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/4*log(2*(a - b)*e^{(-2*d*x - 2*c)} + (a + b)*e^{(-4*d*x - 4*c)} + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/8*(a + b)*\arctan(1/2*((a + b)*e^{(2*d*x + 2*c)} + a - b)/\sqrt{a*b})/(\sqrt{a*b}*a*b*d) + 1/8*(a + b)*\arctan(1/2*((a + b)*e^{(-2*d*x - 2*c)} + a - b)/\sqrt{a*b})/(\sqrt{a*b}*a*b*d) + 3/16*(a - b)*\arctan(1/2*((a + b)*e^{(-2*d*x - 2*c)} + a - b)/\sqrt{a*b})/(\sqrt{a*b}*a*b*d)$

mupad [B] time = 1.69, size = 1655, normalized size = 18.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)^4/(a + b*tanh(c + d*x))^2,x)

[Out] $\log(\tanh(c + d*x) + 1)/(2*a^2*d + 2*b^2*d + 4*a*b*d) - \log(\tanh(c + d*x) - 1)/(2*d*(a + b)^2) - (\operatorname{atan}(\frac{((a + 3*b)*(-a*b^3)^{(1/2)}*((\tanh(c + d*x))*(6*a^3*b + a^4 + 4*b^4 + 9*a^2*b^2))}{(2*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2))} + ((a + 3*b)*(-a*b^3)^{(1/2)}*((2*a*b^6*d^2 + 8*a^2*b^5*d^2 + 12*a^3*b^4*d^2 + 8*a^4*b^3*d^2 + 2*a^5*b^2*d^2))/(b^4*d^3 + 3*a*b^3*d^3 + a^3*b*d^3 + 3*a^2*b^2*d^3) - (\tanh(c + d*x)*(a + 3*b)*(-a*b^3)^{(1/2)}*(16*b^8*d^2 + 48*a*b^7*d^2 + 32*a^2*b^6*d^2 - 32*a^3*b^5*d^2 - 48*a^4*b^4*d^2 - 16*a^5*b^3*d^2)))/(8*(b^5*d + a^2*b^3*d + 2*a*b^4*d)*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2)))))/(4*(b^5*d + a^2*b^3*d + 2*a*b^4*d))*1i)/(4*(b^5*d + a^2*b^3*d + 2*a*b^4*d)) + ((a + 3*b)*(-a*b^3)^{(1/2)}*((\tanh(c + d*x))*(6*a^3*b + a^4 + 4*b^4 + 9*a^2*b^2)))/(2*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2)) - ((a + 3*b)*(-a*b^3)^{(1/2)}*((2*a*b^6*d^2 + 8*a^2*b^5*d^2 + 12*a^3*b^4*d^2 + 8*a^4*b^3*d^2 + 2*a^5*b^2*d^2))/(b^4*d^3 + 3*a*b^3*d^3 + a^3*b*d^3 + 3*a^2*b^2*d^3) + (\tanh(c + d*x)*(a + 3*b)*(-a*b^3)^{(1/2)}*(16*b^8*d^2 + 48*a*b^7*d^2 + 32*a^2*b^6*d^2 - 32*a^3*b^5*d^2 - 48*a^4*b^4*d^2 - 16*a^5*b^3*d^2)))/(8*(b^5*d + a^2*b^3*d + 2*a*b^4*d)$

```

)*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2)))/(4*(b^5*d + a^2*b^3*d + 2*a*b^4*d
))*1i)/(4*(b^5*d + a^2*b^3*d + 2*a*b^4*d))/((3*a*b^2 + (5*a^2*b)/2 + a^3/2
)/(b^4*d^3 + 3*a*b^3*d^3 + a^3*b*d^3 + 3*a^2*b^2*d^3) - ((a + 3*b)*(-a*b^3)
^(1/2))*((tanh(c + d*x))*(6*a^3*b + a^4 + 4*b^4 + 9*a^2*b^2))/(2*(b^3*d^2 + 2
*a*b^2*d^2 + a^2*b*d^2)) + ((a + 3*b)*(-a*b^3)^(1/2))*((2*a*b^6*d^2 + 8*a^2*
b^5*d^2 + 12*a^3*b^4*d^2 + 8*a^4*b^3*d^2 + 2*a^5*b^2*d^2)/(b^4*d^3 + 3*a*b^
3*d^3 + a^3*b*d^3 + 3*a^2*b^2*d^3) - (tanh(c + d*x)*(a + 3*b)*(-a*b^3)^(1/2
))*(16*b^8*d^2 + 48*a*b^7*d^2 + 32*a^2*b^6*d^2 - 32*a^3*b^5*d^2 - 48*a^4*b^4
*d^2 - 16*a^5*b^3*d^2))/(8*(b^5*d + a^2*b^3*d + 2*a*b^4*d)*(b^3*d^2 + 2*a*b
^2*d^2 + a^2*b*d^2)))/(4*(b^5*d + a^2*b^3*d + 2*a*b^4*d)))/(4*(b^5*d + a^
2*b^3*d + 2*a*b^4*d)) + ((a + 3*b)*(-a*b^3)^(1/2))*((tanh(c + d*x))*(6*a^3*b
+ a^4 + 4*b^4 + 9*a^2*b^2))/(2*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2)) - ((a +
3*b)*(-a*b^3)^(1/2))*((2*a*b^6*d^2 + 8*a^2*b^5*d^2 + 12*a^3*b^4*d^2 + 8*a^4
*b^3*d^2 + 2*a^5*b^2*d^2)/(b^4*d^3 + 3*a*b^3*d^3 + a^3*b*d^3 + 3*a^2*b^2*d^
3) + (tanh(c + d*x)*(a + 3*b)*(-a*b^3)^(1/2))*(16*b^8*d^2 + 48*a*b^7*d^2 + 3
2*a^2*b^6*d^2 - 32*a^3*b^5*d^2 - 48*a^4*b^4*d^2 - 16*a^5*b^3*d^2))/(8*(b^5*
d + a^2*b^3*d + 2*a*b^4*d)*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2)))/(4*(b^5*d
+ a^2*b^3*d + 2*a*b^4*d)))/(4*(b^5*d + a^2*b^3*d + 2*a*b^4*d)))*(a + 3*b
)*(-a*b^3)^(1/2)*1i)/(2*(b^5*d + a^2*b^3*d + 2*a*b^4*d)) + (a*tanh(c + d*x)
)/(2*b*(a + b)*(a*d + b*d*tanh(c + d*x)^2))

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)

[Out] Timed out

$$3.182 \quad \int \frac{\tanh^3(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=72

$$\frac{a}{2bd(a+b)(a+b \tanh^2(c+dx))} + \frac{\log(a+b \tanh^2(c+dx))}{2d(a+b)^2} + \frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

[Out] $\ln(\cosh(d*x+c))/(a+b)^2/d+1/2*\ln(a+b*\tanh(d*x+c)^2)/(a+b)^2/d+1/2*a/b/(a+b)/d/(a+b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.12, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 446, 77}

$$\frac{a}{2bd(a+b)(a+b \tanh^2(c+dx))} + \frac{\log(a+b \tanh^2(c+dx))}{2d(a+b)^2} + \frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Tanh}[c + d*x]^3/(a + b*\text{Tanh}[c + d*x]^2)^2, x]$

[Out] $\text{Log}[\text{Cosh}[c + d*x]]/((a + b)^2*d) + \text{Log}[a + b*\text{Tanh}[c + d*x]^2]/(2*(a + b)^2*d) + a/(2*b*(a + b)*d*(a + b*\text{Tanh}[c + d*x]^2))$

Rule 77

$\text{Int}[(a_. + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, n\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ ((\text{ILtQ}[n, 0] \ \&\& \ \text{ILtQ}[p, 0]) \ || \ \text{EqQ}[p, 1] \ || \ (\text{IGtQ}[p, 0] \ \&\& \ (!\text{IntegerQ}[n] \ || \ \text{LeQ}[9*p + 5*(n + 2), 0] \ || \ \text{GeQ}[n + p + 1, 0] \ || \ (\text{GeQ}[n + p + 2, 0] \ \&\& \ \text{RationalQ}[a, b, c, d, e, f])))$

Rule 446

$\text{Int}[(x_)^{(m_.)}*((a_.) + (b_.)*(x_)^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_)^{(n_.)})^{(q_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n) - 1}*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /;$ $\text{FreeQ}\{a, b, c, d, m, n, p, q\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 3670

$\text{Int}[(d_.)*\text{tan}[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*((c_.)*\text{tan}[(e_.) + (f_.)*(x_.)])^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{With}\{\text{ff} = \text{FreeFactors}[\text{Tan}[e + f*x], x]\}, \text{Dist}[(c*ff)/f, \text{Subst}[\text{Int}[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*\text{Tan}[e + f*x])/ff], x] /;$ $\text{FreeQ}\{a, b, c, d, e, f, m, n, p\}, x\} \ \&\& \ (\text{IGtQ}[p, 0] \ || \ \text{EqQ}[n, 2] \ || \ \text{EqQ}[n, 4] \ || \ (\text{IntegerQ}[p] \ \&\& \ \text{RationalQ}[n]))$

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(c+dx)}{(a+b\tanh^2(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(1-x^2)(a+bx^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{x}{(1-x)(a+bx)^2} dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)} - \frac{a}{(a+b)(a+bx)^2} + \frac{b}{(a+b)^2(a+bx)}\right) dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\log(\cosh(c+dx))}{(a+b)^2d} + \frac{\log(a+b\tanh^2(c+dx))}{2(a+b)^2d} + \frac{a}{2b(a+b)d(a+b\tanh^2(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.53, size = 57, normalized size = 0.79

$$\frac{\frac{a(a+b)}{b(a+b\tanh^2(c+dx))} + \log(a+b\tanh^2(c+dx)) + 2\log(\cosh(c+dx))}{2d(a+b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]

[Out] (2*Log[Cosh[c + d*x]] + Log[a + b*Tanh[c + d*x]^2] + (a*(a + b))/(b*(a + b*Tanh[c + d*x]^2)))/(2*(a + b)^2*d)

fricas [B] time = 0.47, size = 629, normalized size = 8.74

$$\frac{2(a+b)dx \cosh(dx+c)^4 + 8(a+b)dx \cosh(dx+c) \sinh(dx+c)^3 + 2(a+b)dx \sinh(dx+c)^4 + 2(a+b)dx + 2((a^3 + 3a^2b + 3ab^2 + b^3)d \cosh(dx+c)^4 + 4(a+b)dx \cosh(dx+c)^3 \sinh(dx+c) + 2(a+b)dx \sinh(dx+c)^4 + 2(a-b)\cosh(dx+c)^2 + 4(3(a+b)dx \cosh(dx+c)^2 + (a-b)dx - a)\sinh(dx+c)^2 - ((a+b)\cosh(dx+c)^4 + 4(a+b)\cosh(dx+c)\sinh(dx+c)^3 + (a+b)\sinh(dx+c)^4 + 2(a-b)\cosh(dx+c)^2 + 2(3(a+b)\cosh(dx+c)^2 + a-b)\sinh(dx+c)^2 + 4((a+b)\cosh(dx+c)^3 + (a-b)\cosh(dx+c))\sinh(dx+c) + a+b)\log(2((a+b)\cosh(dx+c)^2 + (a+b)\sinh(dx+c)^2 + a-b)/(\cosh(dx+c)^2 - 2\cosh(dx+c)\sinh(dx+c) + \sinh(dx+c)^2)) + 8((a+b)dx \cosh(dx+c)^3 + ((a-b)dx - a)\cosh(dx+c))\sinh(dx+c)}{(a^3 + 3a^2b + 3ab^2 + b^3)d \cosh(dx+c)^4 + 4(a^3 + 3a^2b + 3ab^2 + b^3)d \cosh(dx+c)\sinh(dx+c)^3 + (a^3 + 3a^2b + 3ab^2 + b^3)d \sinh(dx+c)^4 + 2(a^3 + a^2b - ab^2 - b^3)d \cosh(dx+c)^2 + 2(3(a^3 + 3a^2b + 3ab^2 + b^3)d \cosh(dx+c)^2 + (a^3 + a^2b - ab^2 - b^3)d)\sinh(dx+c)^2 + (a^3 + 3a^2b + 3ab^2 + b^3)d + 4((a^3 + 3a^2b + 3ab^2 + b^3)d \cosh(dx+c)^3 + (a^3 + a^2b - ab^2 - b^3)d \cosh(dx+c))\sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -1/2*(2*(a + b)*d*x*cosh(d*x + c)^4 + 8*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a + b)*d*x*sinh(d*x + c)^4 + 2*(a + b)*d*x + 4*((a - b)*d*x - a)*cosh(d*x + c)^2 + 4*(3*(a + b)*d*x*cosh(d*x + c)^2 + (a - b)*d*x - a)*sinh(d*x + c)^2 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(\cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + \sinh(d*x + c)^2)) + 8*((a + b)*d*x*cosh(d*x + c)^3 + ((a - b)*d*x - a)*cosh(d*x + c))*sinh(d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*sinh(d*x + c)^4 + 2*(a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^2 + (a^3 + a^2*b - a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^3 + (a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)

giac [B] time = 0.32, size = 149, normalized size = 2.07

$$\frac{\log\left(\frac{a(e^{2dx+2c})+e^{(-2dx-2c)}+b(e^{2dx+2c})+e^{(-2dx-2c)}+2a-2b}{a^2+2ab+b^2}\right)}{2d} - \frac{e^{(2dx+2c)}+e^{(-2dx-2c)}-2}{(a(e^{2dx+2c})+e^{(-2dx-2c)})+b(e^{2dx+2c})+e^{(-2dx-2c)}+2a-2b)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] $\frac{1}{2} * (\log(\text{abs}(a * (e^{(2*d*x + 2*c)} + e^{(-2*d*x - 2*c)}) + b * (e^{(2*d*x + 2*c)} + e^{(-2*d*x - 2*c)}) + 2*a - 2*b)) / (a^2 + 2*a*b + b^2) - (e^{(2*d*x + 2*c)} + e^{(-2*d*x - 2*c)} - 2) / ((a * (e^{(2*d*x + 2*c)} + e^{(-2*d*x - 2*c)}) + b * (e^{(2*d*x + 2*c)} + e^{(-2*d*x - 2*c)}) + 2*a - 2*b) * (a + b))) / d$

maple [A] time = 0.10, size = 118, normalized size = 1.64

$$\frac{\ln(\tanh(dx+c)-1)}{2d(a+b)^2} - \frac{\ln(1+\tanh(dx+c))}{2d(a+b)^2} + \frac{\ln(a+b(\tanh^2(dx+c)))}{2(a+b)^2 d} + \frac{a^2}{2d(a+b)^2 b(a+b(\tanh^2(dx+c)))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x)

[Out] $-1/2/d/(a+b)^2 * \ln(\tanh(d*x+c)-1) - 1/2/d/(a+b)^2 * \ln(1+\tanh(d*x+c)) + 1/2 * \ln(a+b * \tanh(d*x+c)^2) / (a+b)^2/d + 1/2/d * a^2 / (a+b)^2/b / (a+b * \tanh(d*x+c)^2) + 1/2/d / (a+b)^2 * a / (a+b * \tanh(d*x+c)^2)$

maxima [B] time = 0.34, size = 170, normalized size = 2.36

$$\frac{2ae^{(-2dx-2c)}}{a^3 + 3a^2b + 3ab^2 + b^3 + 2(a^3 + a^2b - ab^2 - b^3)e^{(-2dx-2c)} + (a^3 + 3a^2b + 3ab^2 + b^3)e^{(-4dx-4c)}}d + \frac{dx + c}{a^2 + 2ab + b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $2*a*e^{(-2*d*x - 2*c)} / ((a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2*b - a*b^2 - b^3)*e^{(-2*d*x - 2*c)} + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^{(-4*d*x - 4*c)}) * d + (d*x + c) / ((a^2 + 2*a*b + b^2) * d) + 1/2 * \log(2*(a - b)*e^{(-2*d*x - 2*c)} + (a + b)*e^{(-4*d*x - 4*c)} + a + b) / ((a^2 + 2*a*b + b^2) * d)$

mupad [B] time = 0.43, size = 210, normalized size = 2.92

$$\frac{-a^2 + ab \left(-1 + \operatorname{atan} \left(\frac{a \tanh(c+dx)^2 + b \tanh(c+dx)^2}{2a - a \tanh(c+dx)^2 + b \tanh(c+dx)^2} \right) \right) + b^2 \tanh(c+dx)^2 \operatorname{atan} \left(\frac{a \tanh(c+dx)^2 + b \tanh(c+dx)^2}{2a - a \tanh(c+dx)^2 + b \tanh(c+dx)^2} \right)}{2da^3b + 2da^2b^2 \tanh(c+dx)^2 + 4da^2b^2 + 4dab^3 \tanh(c+dx)^2 + 2dab^3 + 2db^4 \tanh(c+dx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c+d*x)^3/(a+b*tanh(c+d*x)^2)^2,x)

[Out] $-(a*b*(\operatorname{atan}((a*\tanh(c+d*x)^2 + b*\tanh(c+d*x)^2)/(2*a - a*\tanh(c+d*x)^2 + b*\tanh(c+d*x)^2))*2i - 1) - a^2 + b^2*\tanh(c+d*x)^2*\operatorname{atan}((a*\tanh(c+d*x)^2 + b*\tanh(c+d*x)^2)/(2*a - a*\tanh(c+d*x)^2 + b*\tanh(c+d*x)^2))*2i) / (4*a^2*b^2*d + 2*b^4*d*\tanh(c+d*x)^2 + 2*a*b^3*d + 2*a^3*b*d + 2*a^2*b^2*d*\tanh(c+d*x)^2 + 4*a*b^3*d*\tanh(c+d*x)^2)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)

[Out] Timed out

$$3.183 \quad \int \frac{\tanh^2(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=85

$$-\frac{\tanh(c+dx)}{2d(a+b)(a+b \tanh^2(c+dx))} - \frac{(a-b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d(a+b)^2} + \frac{x}{(a+b)^2}$$

[Out] $x/(a+b)^2 - 1/2*(a-b)*\arctan(b^{(1/2)}*\tanh(d*x+c)/a^{(1/2)})/(a+b)^2/d/a^{(1/2)}/b^{(1/2)} - 1/2*\tanh(d*x+c)/(a+b)/d/(a+b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.11, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3670, 471, 522, 206, 205}

$$-\frac{\tanh(c+dx)}{2d(a+b)(a+b \tanh^2(c+dx))} - \frac{(a-b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}d(a+b)^2} + \frac{x}{(a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2), x]

[Out] $x/(a+b)^2 - ((a-b)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Tanh}[c+d*x])/(\text{Sqrt}[a])])/(2*\text{Sqrt}[a]*\text{Sqrt}[b]*(a+b)^2*d) - \text{Tanh}[c+d*x]/(2*(a+b)*d*(a+b*\text{Tanh}[c+d*x]^2))$

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 471

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n-1)*(e*x)^(m-n+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(n*(b*c-a*d)*(p+1)), x] - Dist[e^n/(n*(b*c-a*d)*(p+1)), Int[(e*x)^(m-n)*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[c*(m-n+1)+d*(m+n*(p+q+1)+1]*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m-n+1] && GtQ[m-n+1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_))), x_Symbol] := Dist[(b*e-a*f)/(b*c-a*d), Int[1/(a+b*x^n), x], x] - Dist[(d*e-c*f)/(b*c-a*d), Int[1/(c+d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 3670

Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_))*((c_)*tan[(e_) + (f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a+b*(ff*x)^n)^p]/(c^2+f

$f^2x^2), x], x, (c*\text{Tan}[e + f*x])/ff], x]] /; \text{FreeQ}\{a, b, c, d, e, f, m, n, p\}, x] \&\& (\text{IGtQ}[p, 0] \|\ \text{EqQ}[n, 2] \|\ \text{EqQ}[n, 4] \|\ (\text{IntegerQ}[p] \&\& \text{RationalQ}[n]))$

Rubi steps

$$\begin{aligned} \int \frac{\tanh^2(c+dx)}{(a+b\tanh^2(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)(a+bx^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\ &= -\frac{\tanh(c+dx)}{2(a+b)d(a+b\tanh^2(c+dx))} + \frac{\text{Subst}\left(\int \frac{1+x^2}{(1-x^2)(a+bx^2)} dx, x, \tanh(c+dx)\right)}{2(a+b)d} \\ &= -\frac{\tanh(c+dx)}{2(a+b)d(a+b\tanh^2(c+dx))} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c+dx)\right)}{(a+b)^2d} - \frac{(a-b)}{(a+b)^2} \\ &= \frac{x}{(a+b)^2} - \frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}(a+b)^2d} - \frac{\tanh(c+dx)}{2(a+b)d(a+b\tanh^2(c+dx))} \end{aligned}$$

Mathematica [A] time = 0.47, size = 86, normalized size = 1.01

$$\frac{(b-a)\tan^{-1}\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} - \frac{(a+b)\sinh(2(c+dx))}{(a+b)\cosh(2(c+dx))+a-b} + 2(c+dx)}{2d(a+b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]

[Out] $(2*(c + d*x) + ((-a + b)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Tanh}[c + d*x])/(\text{Sqrt}[a])]) / (\text{Sqrt}[a]*\text{Sqrt}[b]) - ((a + b)*\text{Sinh}[2*(c + d*x)]) / (a - b + (a + b)*\text{Cosh}[2*(c + d*x)])) / (2*(a + b)^2*d)$

fricas [B] time = 0.46, size = 2025, normalized size = 23.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] $[1/4*(4*(a^2*b + a*b^2)*d*x*\cosh(d*x + c)^4 + 16*(a^2*b + a*b^2)*d*x*\cosh(d*x + c)*\sinh(d*x + c)^3 + 4*(a^2*b + a*b^2)*d*x*\sinh(d*x + c)^4 + 4*a^2*b + 4*a*b^2 + 4*(a^2*b + a*b^2)*d*x + 4*(a^2*b - a*b^2 + 2*(a^2*b - a*b^2)*d*x)*\cosh(d*x + c)^2 + 4*(6*(a^2*b + a*b^2)*d*x*\cosh(d*x + c)^2 + a^2*b - a*b^2 + 2*(a^2*b - a*b^2)*d*x)*\sinh(d*x + c)^2 + ((a^2 - b^2)*\cosh(d*x + c)^4 + 4*(a^2 - b^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^2 - b^2)*\sinh(d*x + c)^4 + 2*(a^2 - 2*a*b + b^2)*\cosh(d*x + c)^2 + 2*(3*(a^2 - b^2)*\cosh(d*x + c)^2 + a^2 - 2*a*b + b^2)*\sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 - b^2)*\cosh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-a*b}*\log(((a^2 + 2*a*b + b^2)*\cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*\sinh(d*x + c)^4 + 2*(a^2 - b^2)*\cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)^2 + a^2 - b^2)*\sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*\cosh(d*x + c)^3 + (a^2 - b^2)*\cosh(d*x + c))*\sinh(d*x + c) - 4*((a + b)*\cosh(d*x + c)^2 + 2*(a + b$

```

)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b)
)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a +
b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c
)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*
x + c))*sinh(d*x + c) + a + b)) + 8*(2*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^3
+ (a^2*b - a*b^2 + 2*(a^2*b - a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a
^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 4*(a^4*b + 3*a^3*
b^2 + 3*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b + 3*a^3*b
^2 + 3*a^2*b^3 + a*b^4)*d*sinh(d*x + c)^4 + 2*(a^4*b + a^3*b^2 - a^2*b^3 -
a*b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d*c
osh(d*x + c)^2 + (a^4*b + a^3*b^2 - a^2*b^3 - a*b^4)*d)*sinh(d*x + c)^2 + (
a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d + 4*((a^4*b + 3*a^3*b^2 + 3*a^2*b^
3 + a*b^4)*d*cosh(d*x + c)^3 + (a^4*b + a^3*b^2 - a^2*b^3 - a*b^4)*d*cosh(d
*x + c))*sinh(d*x + c)), 1/2*(2*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^
2*b + a*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a^2*b + a*b^2)*d*x*sinh
(d*x + c)^4 + 2*a^2*b + 2*a*b^2 + 2*(a^2*b + a*b^2)*d*x + 2*(a^2*b - a*b^2
+ 2*(a^2*b - a*b^2)*d*x)*cosh(d*x + c)^2 + 2*(6*(a^2*b + a*b^2)*d*x*cosh(d*
x + c)^2 + a^2*b - a*b^2 + 2*(a^2*b - a*b^2)*d*x)*sinh(d*x + c)^2 - ((a^2 -
b^2)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2
- b^2)*sinh(d*x + c)^4 + 2*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2
- b^2)*cosh(d*x + c)^2 + a^2 - 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4
*((a^2 - b^2)*cosh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x
+ c))*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x +
c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b)) + 4*(
2*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^3 + (a^2*b - a*b^2 + 2*(a^2*b - a*b^2)*
d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)
*d*cosh(d*x + c)^4 + 4*(a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d*cosh(d*x +
c)*sinh(d*x + c)^3 + (a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d*sinh(d*x +
c)^4 + 2*(a^4*b + a^3*b^2 - a^2*b^3 - a*b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^4*
b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^4*b + a^3*b^2 - a
^2*b^3 - a*b^4)*d)*sinh(d*x + c)^2 + (a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4
)*d + 4*((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + (a^4*b
+ a^3*b^2 - a^2*b^3 - a*b^4)*d*cosh(d*x + c))*sinh(d*x + c))]

```

giac [B] time = 0.31, size = 177, normalized size = 2.08

$$\frac{(a-b) \arctan\left(\frac{ae^{(2dx+2c)} + be^{(2dx+2c)} + a - b}{2\sqrt{ab}}\right)}{(a^2 + 2ab + b^2)\sqrt{ab}} - \frac{2(dx+c)}{a^2 + 2ab + b^2} - \frac{2(ae^{(2dx+2c)} - be^{(2dx+2c)} + a + b)}{(a^2 + 2ab + b^2)(ae^{(4dx+4c)} + be^{(4dx+4c)} + 2ae^{(2dx+2c)} - 2be^{(2dx+2c)} + a + b)}$$

2d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] -1/2*((a - b)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b)))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) - 2*(d*x + c)/(a^2 + 2*a*b + b^2) - 2*(a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + a + b)/((a^2 + 2*a*b + b^2)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b))/d

maple [B] time = 0.10, size = 162, normalized size = 1.91

$$-\frac{\ln(\tanh(dx+c)-1)}{2d(a+b)^2} + \frac{\ln(1+\tanh(dx+c))}{2d(a+b)^2} - \frac{a \tanh(dx+c)}{2d(a+b)^2(a+b(\tanh^2(dx+c)))} - \frac{b \tanh(dx+c)}{2(a+b)^2 d(a+b(\tanh^2(dx+c)))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x)

[Out] $-1/2/d/(a+b)^2 \ln(\tanh(dx+c)-1) + 1/2/d/(a+b)^2 \ln(1+\tanh(dx+c)) - 1/2/d*a/(a+b)^2 \tanh(dx+c)/(a+b \tanh(dx+c)^2) - 1/2*b \tanh(dx+c)/(a+b)^2/d/(a+b \tanh(dx+c)^2) - 1/2/d*a/(a+b)^2/(a*b)^{(1/2)} \arctan(\tanh(dx+c)*b/(a*b)^{(1/2)}) + 1/2/d/(a+b)^2/(a*b)^{(1/2)} \arctan(\tanh(dx+c)*b/(a*b)^{(1/2)}) * b$

maxima [B] time = 0.61, size = 614, normalized size = 7.22

$$\frac{(a^2 - 4ab - b^2) \arctan\left(\frac{(a+b)e^{(2dx+2c)+a-b}}{2\sqrt{ab}}\right)}{8(a^3 + 2a^2b + ab^2)\sqrt{abd}} + \frac{(a^2 - 4ab - b^2) \arctan\left(\frac{(a+b)e^{(-2dx-2c)+a-b}}{2\sqrt{ab}}\right)}{8(a^3 + 2a^2b + ab^2)\sqrt{abd}} + \frac{1}{4(a^4 + 3a^3b + 3a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(dx+c)^2/(a+b*tanh(dx+c)^2)^2,x, algorithm="maxima")`

[Out] $-1/8*(a^2 - 4*a*b - b^2)*\arctan(1/2*((a + b)*e^{(2*d*x + 2*c)} + a - b)/\sqrt{a*b})/((a^3 + 2*a^2*b + a*b^2)*\sqrt{a*b}*d) + 1/8*(a^2 - 4*a*b - b^2)*\arctan(1/2*((a + b)*e^{(-2*d*x - 2*c)} + a - b)/\sqrt{a*b})/((a^3 + 2*a^2*b + a*b^2)*\sqrt{a*b}*d) + 1/4*(a^2 - b^2 + (a^2 - 6*a*b + b^2)*e^{(2*d*x + 2*c)})/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*e^{(4*d*x + 4*c)} + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*e^{(2*d*x + 2*c)})*d) - 1/4*(a^2 - b^2 + (a^2 - 6*a*b + b^2)*e^{(-2*d*x - 2*c)})/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*e^{(-2*d*x - 2*c)} + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*e^{(-4*d*x - 4*c)})*d) - 1/2*((a - b)*e^{(-2*d*x - 2*c)} + a + b)/((a^3 + 2*a^2*b + a*b^2 + 2*(a^3 - a*b^2)*e^{(-2*d*x - 2*c)} + (a^3 + 2*a^2*b + a*b^2)*e^{(-4*d*x - 4*c)})*d) + 1/4*\log((a + b)*e^{(4*d*x + 4*c)} + 2*(a - b)*e^{(2*d*x + 2*c)} + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/4*\log(2*(a - b)*e^{(-2*d*x - 2*c)} + (a + b)*e^{(-4*d*x - 4*c)} + a + b)/((a^2 + 2*a*b + b^2)*d) + 1/4*\arctan(1/2*((a + b)*e^{(-2*d*x - 2*c)} + a - b)/\sqrt{a*b})/(\sqrt{a*b}*a*d)$

mupad [B] time = 0.66, size = 106, normalized size = 1.25

$$\frac{\frac{ax}{(a+b)^2} - \frac{\tanh(c+dx)}{2ad+2bd} + \frac{bx \tanh(c+dx)^2}{(a+b)^2}}{b \tanh(c+dx)^2 + a} - \frac{\operatorname{atan}\left(\frac{b \tanh(c+dx)}{\sqrt{ab}}\right) (a-b)}{\sqrt{ab} (2da^2 + 4dab + 2db^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(c + dx)^2/(a + b*tanh(c + dx)^2)^2,x)`

[Out] $((a*x)/(a + b)^2 - \tanh(c + dx)/(2*a*d + 2*b*d) + (b*x*\tanh(c + dx)^2)/(a + b)^2)/(a + b*\tanh(c + dx)^2) - (\operatorname{atan}((b*\tanh(c + dx))/\sqrt{a*b}))/((a*b)^{(1/2)})*(a - b))/((a*b)^{(1/2)}*(2*a^2*d + 2*b^2*d + 4*a*b*d))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(dx+c)**2/(a+b*tanh(dx+c)**2)**2,x)`

[Out] Timed out

$$3.184 \quad \int \frac{\tanh(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=68

$$-\frac{1}{2d(a+b)(a+b \tanh^2(c+dx))} + \frac{\log(a+b \tanh^2(c+dx))}{2d(a+b)^2} + \frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

[Out] $\ln(\cosh(d*x+c))/(a+b)^2/d+1/2*\ln(a+b*\tanh(d*x+c)^2)/(a+b)^2/d-1/2/(a+b)/d/(a+b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.09, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3670, 444, 44}

$$-\frac{1}{2d(a+b)(a+b \tanh^2(c+dx))} + \frac{\log(a+b \tanh^2(c+dx))}{2d(a+b)^2} + \frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2), x]

[Out] $\text{Log}[\text{Cosh}[c + d*x]]/(a + b)^2*d + \text{Log}[a + b*\text{Tanh}[c + d*x]^2]/(2*(a + b)^2*d) - 1/(2*(a + b)*d*(a + b*\text{Tanh}[c + d*x]^2))$

Rule 44

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])]

Rule 444

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 3670

Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*((c_) * tan[(e_) + (f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff, x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(c+dx)}{(a+b \tanh^2(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{x}{(1-x^2)(a+bx^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^2} dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)} + \frac{b}{(a+b)(a+bx)^2} + \frac{b}{(a+b)^2(a+bx)}\right) dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\log(\cosh(c+dx))}{(a+b)^2 d} + \frac{\log(a+b \tanh^2(c+dx))}{2(a+b)^2 d} - \frac{1}{2(a+b)d(a+b \tanh^2(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.47, size = 55, normalized size = 0.81

$$\frac{\frac{a+b}{a+b \tanh^2(c+dx)} - \log(a+b \tanh^2(c+dx)) - 2 \log(\cosh(c+dx))}{2d(a+b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2, x]

[Out] -1/2*(-2*Log[Cosh[c + d*x]] - Log[a + b*Tanh[c + d*x]^2] + (a + b)/(a + b*Tanh[c + d*x]^2))/((a + b)^2*d)

fricas [B] time = 0.44, size = 623, normalized size = 9.16

$$\frac{2(a+b)dx \cosh(dx+c)^4 + 8(a+b)dx \cosh(dx+c) \sinh(dx+c)^3 + 2(a+b)dx \sinh(dx+c)^4 + 2(a+b)d}{2((a^3 + 3a^2b + 3ab^2 + b^3))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] -1/2*(2*(a + b)*d*x*cosh(d*x + c)^4 + 8*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a + b)*d*x*sinh(d*x + c)^4 + 2*(a + b)*d*x + 4*((a - b)*d*x + b)*cosh(d*x + c)^2 + 4*(3*(a + b)*d*x*cosh(d*x + c)^2 + (a - b)*d*x + b)*sinh(d*x + c)^2 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c)*sinh(d*x + c) + a + b)*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 8*((a + b)*d*x*cosh(d*x + c)^3 + ((a - b)*d*x + b)*cosh(d*x + c)*sinh(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*sinh(d*x + c)^4 + 2*(a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^2 + (a^3 + a^2*b - a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)^3 + (a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))

giac [B] time = 0.27, size = 149, normalized size = 2.19

$$\frac{\log\left(\frac{a(e^{2dx+2c})+e^{(-2dx-2c)}+b(e^{2dx+2c})+e^{(-2dx-2c)}+2a-2b}{a^2+2ab+b^2}\right)}{2d} - \frac{e^{(2dx+2c)}+e^{(-2dx-2c)}+2}{(a(e^{2dx+2c})+e^{(-2dx-2c)})+b(e^{2dx+2c})+e^{(-2dx-2c)}+2a-2b)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] $\frac{1}{2} * (\log(\text{abs}(a * (e^{(2*d*x + 2*c)} + e^{(-2*d*x - 2*c)}) + b * (e^{(2*d*x + 2*c)} + e^{(-2*d*x - 2*c)})) + 2*a - 2*b)) / (a^2 + 2*a*b + b^2) - (e^{(2*d*x + 2*c)} + e^{(-2*d*x - 2*c)} + 2) / ((a * (e^{(2*d*x + 2*c)} + e^{(-2*d*x - 2*c)}) + b * (e^{(2*d*x + 2*c)} + e^{(-2*d*x - 2*c)})) + 2*a - 2*b) * (a + b)) / d$

maple [A] time = 0.12, size = 113, normalized size = 1.66

$$\frac{\ln(\tanh(dx+c)-1)}{2d(a+b)^2} - \frac{\ln(1+\tanh(dx+c))}{2d(a+b)^2} + \frac{\ln(a+b(\tanh^2(dx+c)))}{2(a+b)^2 d} - \frac{a}{2d(a+b)^2(a+b(\tanh^2(dx+c)))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x)

[Out] $-1/2/d/(a+b)^2 * \ln(\tanh(d*x+c)-1) - 1/2/d/(a+b)^2 * \ln(1+\tanh(d*x+c)) + 1/2 * \ln(a+b * \tanh(d*x+c)^2) / (a+b)^2/d - 1/2/d/(a+b)^2 * a / (a+b * \tanh(d*x+c)^2) - 1/2/d * b / (a+b)^2 / (a+b * \tanh(d*x+c)^2)$

maxima [B] time = 0.34, size = 170, normalized size = 2.50

$$\frac{2be^{(-2dx-2c)}}{(a^3 + 3a^2b + 3ab^2 + b^3 + 2(a^3 + a^2b - ab^2 - b^3)e^{(-2dx-2c)} + (a^3 + 3a^2b + 3ab^2 + b^3)e^{(-4dx-4c)})d} + \frac{dx+c}{(a^2 + 2ab)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $-2*b*e^{(-2*d*x - 2*c)} / ((a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2*b - a*b^2 - b^3)*e^{(-2*d*x - 2*c)} + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^{(-4*d*x - 4*c)}) * d) + (d*x + c) / ((a^2 + 2*a*b + b^2) * d) + 1/2 * \log(2*(a - b)*e^{(-2*d*x - 2*c)} + (a + b)*e^{(-4*d*x - 4*c)} + a + b) / ((a^2 + 2*a*b + b^2) * d)$

mupad [B] time = 1.47, size = 129, normalized size = 1.90

$$\frac{\frac{ax}{a^2+2ab+b^2} + \frac{bx \tanh(c+dx)^2}{a^2+2ab+b^2} + \frac{b \tanh(c+dx)^2}{2ad(a+b)}}{b \tanh(c+dx)^2 + a} + \frac{\ln(b \tanh(c+dx)^2 + a)}{2d(a^2 + 2ab + b^2)} - \frac{\ln(\tanh(c+dx) + 1)}{d(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c+d*x)/(a+b*tanh(c+d*x)^2)^2,x)

[Out] $((a*x)/(2*a*b + a^2 + b^2) + (b*x*tanh(c+d*x)^2)/(2*a*b + a^2 + b^2) + (b * tanh(c+d*x)^2)/(2*a*d*(a+b)))/(a+b*tanh(c+d*x)^2) + \log(a+b*tanh(c+d*x)^2)/(2*d*(2*a*b + a^2 + b^2)) - \log(\tanh(c+d*x) + 1)/(d*(a+b)^2)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)

[Out] Timed out

$$3.185 \quad \int \frac{1}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=89

$$\frac{\sqrt{b}(3a+b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^2} + \frac{b \tanh(c+dx)}{2ad(a+b)(a+b \tanh^2(c+dx))} + \frac{x}{(a+b)^2}$$

[Out] $x/(a+b)^2 + 1/2*(3*a+b)*\arctan(b^{(1/2)*\tanh(d*x+c)/a^{(1/2)})*b^{(1/2)/a^{(3/2)/(a+b)^2/d + 1/2*b*\tanh(d*x+c)/a/(a+b)/d/(a+b*\tanh(d*x+c)^2}$

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3661, 414, 522, 206, 205}

$$\frac{\sqrt{b}(3a+b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^2} + \frac{b \tanh(c+dx)}{2ad(a+b)(a+b \tanh^2(c+dx))} + \frac{x}{(a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tanh[c + d*x]^2)^(-2), x]

[Out] $x/(a+b)^2 + (\text{Sqrt}[b]*(3*a+b)*\text{ArcTan}[(\text{Sqrt}[b]*\text{Tanh}[c+d*x])/\text{Sqrt}[a]])/(2*a^{(3/2)*(a+b)^2*d} + (b*\text{Tanh}[c+d*x])/(2*a*(a+b)*d*(a+b*\text{Tanh}[c+d*x]^2))$

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 3661

Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(

$\text{ff} \cdot x)^n)^p / (c^2 + \text{ff}^2 \cdot x^2), x], x, (c \cdot \text{Tan}[e + f \cdot x]) / \text{ff}], x]] /;$ FreeQ[{a, b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + b \tanh^2(c + dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{b \tanh(c + dx)}{2a(a + b)d (a + b \tanh^2(c + dx))} - \frac{\text{Subst}\left(\int \frac{b-2(a+b)+bx^2}{(1-x^2)(a+bx^2)} dx, x, \tanh(c + dx)\right)}{2a(a + b)d} \\ &= \frac{b \tanh(c + dx)}{2a(a + b)d (a + b \tanh^2(c + dx))} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{(a + b)^2 d} + \frac{(b(3a + b))}{2a(a + b)d (a + b \tanh^2(c + dx))} \\ &= \frac{x}{(a + b)^2} + \frac{\sqrt{b}(3a + b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a + b)^2 d} + \frac{b \tanh(c + dx)}{2a(a + b)d (a + b \tanh^2(c + dx))} \end{aligned}$$

Mathematica [A] time = 0.58, size = 97, normalized size = 1.09

$$\frac{\frac{\sqrt{b}(3a+b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{b(a+b) \tanh(c+dx)}{a(a+b \tanh^2(c+dx))} - \log(1 - \tanh(c + dx)) + \log(\tanh(c + dx) + 1)}{2d(a + b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Tanh[c + d*x]^2)^(-2), x]

[Out] ((Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2) - Log[1 - Tanh[c + d*x]] + Log[1 + Tanh[c + d*x]] + (b*(a + b)*Tanh[c + d*x])/(a*(a + b*Tanh[c + d*x]^2)))/(2*(a + b)^2*d)

fricas [B] time = 0.47, size = 1942, normalized size = 21.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/4*(4*(a^2 + a*b)*d*x*cosh(d*x + c)^4 + 16*(a^2 + a*b)*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(a^2 + a*b)*d*x*sinh(d*x + c)^4 + 4*(a^2 + a*b)*d*x + 4*(2*(a^2 - a*b)*d*x - a*b + b^2)*cosh(d*x + c)^2 + 4*(6*(a^2 + a*b)*d*x*cosh(d*x + c)^2 + 2*(a^2 - a*b)*d*x - a*b + b^2)*sinh(d*x + c)^2 + ((3*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 4*(3*a^2 + 4*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 + 4*a*b + b^2)*sinh(d*x + c)^4 + 2*(3*a^2 - 2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + 4*a*b + b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a*b - b^2)*sinh(d*x + c)^2 + 3*a^2 + 4*a*b + b^2 + 4*((3*a^2 + 4*a*b + b^2)*cosh(d*x + c)^3 + (3*a^2 - 2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)

$$\begin{aligned} &^3 + (a^2 - b^2) \cosh(dx + c) \sinh(dx + c) + 4((a^2 + ab) \cosh(dx + c) \\ &)^2 + 2(a^2 + ab) \cosh(dx + c) \sinh(dx + c) + (a^2 + ab) \sinh(dx + c) \\ &^2 + a^2 - ab) \sqrt{-b/a} / ((a + b) \cosh(dx + c)^4 + 4(a + b) \cosh(dx + c) \\ & \sinh(dx + c)^3 + (a + b) \sinh(dx + c)^4 + 2(a - b) \cosh(dx + c)^2 + \\ & 2(3(a + b) \cosh(dx + c)^2 + a - b) \sinh(dx + c)^2 + 4((a + b) \cosh(dx + c) \\ &^3 + (a - b) \cosh(dx + c)) \sinh(dx + c) + a + b) - 4ab - 4b^2 + \\ & 8(2(a^2 + ab) dx \cosh(dx + c)^3 + (2(a^2 - ab) dx - ab + b^2) \cosh(dx + c) \sinh(dx + c)) / ((a^4 + 3a^3b + 3a^2b^2 + ab^3) dx \cosh(dx + c)^4 + 4(a^4 + 3a^3b + 3a^2b^2 + ab^3) dx \cosh(dx + c) \sinh(dx + c)^3 + (a^4 + 3a^3b + 3a^2b^2 + ab^3) dx \sinh(dx + c)^4 + 2(a^4 + a^3b - a^2b^2 - ab^3) dx \cosh(dx + c)^2 + 2(3(a^4 + 3a^3b + 3a^2b^2 + ab^3) dx \cosh(dx + c)^2 + (a^4 + a^3b - a^2b^2 - ab^3) dx) \sinh(dx + c)^2 + (a^4 + 3a^3b + 3a^2b^2 + ab^3) dx + 4((a^4 + 3a^3b + 3a^2b^2 + ab^3) dx \cosh(dx + c)^3 + (a^4 + a^3b - a^2b^2 - ab^3) dx \cosh(dx + c)) \sinh(dx + c)), 1/2(2(a^2 + ab) dx \cosh(dx + c)^4 + 8(a^2 + ab) dx \cosh(dx + c) \sinh(dx + c)^3 + 2(a^2 + ab) dx \sinh(dx + c)^4 + 2(a^2 + ab) dx + 2(2(a^2 - ab) dx - ab + b^2) \cosh(dx + c)^2 + 2(6(a^2 + ab) dx \cosh(dx + c)^2 + 2(a^2 - ab) dx - ab + b^2) \sinh(dx + c)^2 + ((3a^2 + 4ab + b^2) \cosh(dx + c)^4 + 4(3a^2 + 4ab + b^2) \cosh(dx + c) \sinh(dx + c)^3 + (3a^2 + 4ab + b^2) \sinh(dx + c)^4 + 2(3a^2 - 2ab - b^2) \cosh(dx + c)^2 + 2(3(3a^2 + 4ab + b^2) \cosh(dx + c)^2 + 3a^2 - 2ab - b^2) \sinh(dx + c)^2 + 3a^2 + 4ab + b^2 + 4((3a^2 + 4ab + b^2) \cosh(dx + c)^3 + (3a^2 - 2ab - b^2) \cosh(dx + c)) \sinh(dx + c)) \sqrt{b/a} \arctan(1/2((a + b) \cosh(dx + c)^2 + 2(a + b) \cosh(dx + c) \sinh(dx + c) + (a + b) \sinh(dx + c)^2 + a - b) \sqrt{b/a}/b) - 2ab - 2b^2 + 4(2(a^2 + ab) dx \cosh(dx + c)^3 + (2(a^2 - ab) dx - ab + b^2) \cosh(dx + c)) \sinh(dx + c)) / ((a^4 + 3a^3b + 3a^2b^2 + ab^3) dx \cosh(dx + c)^4 + 4(a^4 + 3a^3b + 3a^2b^2 + ab^3) dx \cosh(dx + c) \sinh(dx + c)^3 + (a^4 + 3a^3b + 3a^2b^2 + ab^3) dx \sinh(dx + c)^4 + 2(a^4 + a^3b - a^2b^2 - ab^3) dx \cosh(dx + c)^2 + 2(3(a^4 + 3a^3b + 3a^2b^2 + ab^3) dx \cosh(dx + c)^2 + (a^4 + a^3b - a^2b^2 - ab^3) dx) \sinh(dx + c)^2 + (a^4 + 3a^3b + 3a^2b^2 + ab^3) dx + 4((a^4 + 3a^3b + 3a^2b^2 + ab^3) dx \cosh(dx + c)^3 + (a^4 + a^3b - a^2b^2 - ab^3) dx \cosh(dx + c)) \sinh(dx + c))] \end{aligned}$$

giac [B] time = 0.18, size = 195, normalized size = 2.19

$$\frac{(3ab+b^2) \arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{(a^3+2a^2b+ab^2)\sqrt{ab}} + \frac{2(dx+c)}{a^2+2ab+b^2} - \frac{2(abe^{(2dx+2c)}-b^2e^{(2dx+2c)}+ab+b^2)}{(a^3+2a^2b+ab^2)(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b)}$$

2d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tanh(dx+c))^2,x, algorithm="giac")

[Out] 1/2*((3*a*b + b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b)))/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b)) + 2*(d*x + c)/(a^2 + 2*a*b + b^2) - 2*(a*b*e^(2*d*x + 2*c) - b^2*e^(2*d*x + 2*c) + a*b + b^2)/((a^3 + 2*a^2*b + a*b^2)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b))/d

maple [B] time = 0.15, size = 172, normalized size = 1.93

$$\frac{\ln(\tanh(dx + c) - 1)}{2d(a + b)^2} + \frac{\ln(1 + \tanh(dx + c))}{2d(a + b)^2} + \frac{b \tanh(dx + c)}{2(a + b)^2 d (a + b (\tanh^2(dx + c)))} + \frac{b^2 \tanh(dx + c)}{2d(a + b)^2 a (a + b (\tanh^2(dx + c)))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*tanh(dx+c))^2,x)

[Out] $-1/2/d/(a+b)^2*\ln(\tanh(d*x+c)-1)+1/2/d/(a+b)^2*\ln(1+\tanh(d*x+c))+1/2*b*\tanh(d*x+c)/(a+b)^2/d/(a+b*\tanh(d*x+c)^2)+1/2/d*b^2/(a+b)^2/a*\tanh(d*x+c)/(a+b*\tanh(d*x+c)^2)+3/2/d/(a+b)^2/(a*b)^{(1/2)}*\arctan(\tanh(d*x+c)*b/(a*b)^{(1/2)})*b+1/2/d*b^2/(a+b)^2/a/(a*b)^{(1/2)}*\arctan(\tanh(d*x+c)*b/(a*b)^{(1/2)})$

maxima [B] time = 0.46, size = 206, normalized size = 2.31

$$\frac{(3ab + b^2) \arctan\left(\frac{(a+b)e^{(-2dx-2c)+a-b}}{2\sqrt{ab}}\right)}{2(a^3 + 2a^2b + ab^2)\sqrt{abd}} + \frac{ab + b^2 + (ab - b^2)e^{(-2dx-2c)}}{(a^4 + 3a^3b + 3a^2b^2 + ab^3 + 2(a^4 + a^3b - a^2b^2 - ab^3)e^{(-2dx-2c)} + (a^4 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tanh(d*x+c))^2,x, algorithm="maxima")

[Out] $-1/2*(3*a*b + b^2)*\arctan(1/2*((a + b)*e^{(-2*d*x - 2*c)} + a - b)/\text{sqrt}(a*b))/((a^3 + 2*a^2*b + a*b^2)*\text{sqrt}(a*b)*d) + (a*b + b^2 + (a*b - b^2)*e^{(-2*d*x - 2*c)})/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*e^{(-2*d*x - 2*c)} + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*e^{(-4*d*x - 4*c)})*d) + (d*x + c)/((a^2 + 2*a*b + b^2)*d)$

mupad [B] time = 1.54, size = 110, normalized size = 1.24

$$\frac{\frac{ax}{(a+b)^2} + \frac{bx \tanh(c+dx)^2}{(a+b)^2} + \frac{b \tanh(c+dx)}{2ad(a+b)}}{b \tanh(c+dx)^2 + a} + \frac{\text{atan}\left(\frac{b \tanh(c+dx)}{\sqrt{ab}}\right) (b^2 + 3ab)}{\sqrt{ab} (2a^3d + ab(4ad + 2bd))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*tanh(c + d*x))^2,x)

[Out] $((a*x)/(a + b)^2 + (b*x*\tanh(c + d*x)^2)/(a + b)^2 + (b*\tanh(c + d*x))/(2*a*d*(a + b)))/(a + b*\tanh(c + d*x)^2) + (\text{atan}((b*\tanh(c + d*x))/(a*b)^{(1/2)})*(3*a*b + b^2)))/((a*b)^{(1/2)}*(2*a^3*d + a*b*(4*a*d + 2*b*d)))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \tanh^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tanh(d*x+c)**2)**2,x)

[Out] Integral((a + b*tanh(c + d*x)**2)**(-2), x)

$$3.186 \quad \int \frac{\coth(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=95

$$-\frac{b(2a+b) \log(a+b \tanh^2(c+dx))}{2a^2d(a+b)^2} + \frac{\log(\tanh(c+dx))}{a^2d} + \frac{b}{2ad(a+b)(a+b \tanh^2(c+dx))} + \frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

[Out] $\ln(\cosh(d*x+c))/(a+b)^{2/d} + \ln(\tanh(d*x+c))/a^{2/d} - 1/2*b*(2*a+b)*\ln(a+b*\tanh(d*x+c)^2)/a^{2/d} + 1/2*b/a/(a+b)/d/(a+b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.15, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3670, 446, 72}

$$-\frac{b(2a+b) \log(a+b \tanh^2(c+dx))}{2a^2d(a+b)^2} + \frac{\log(\tanh(c+dx))}{a^2d} + \frac{b}{2ad(a+b)(a+b \tanh^2(c+dx))} + \frac{\log(\cosh(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

[In] `Int[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2), x]`

[Out] `Log[Cosh[c + d*x]]/(a + b)^2*d + Log[Tanh[c + d*x]]/(a^2*d) - (b*(2*a + b)*Log[a + b*Tanh[c + d*x]^2])/(2*a^2*(a + b)^2*d + b/(2*a*(a + b)*d*(a + b*Tanh[c + d*x]^2))`

Rule 72

`Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]`

Rule 446

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]`

Rule 3670

`Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f*f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))`

Rubi steps

$c)^3 + (a^5 + 3a^4b + 3a^3b^2 + a^2b^3) * d * \sinh(dx + c)^4 + 2 * (a^5 + a^4b - a^3b^2 - a^2b^3) * d * \cosh(dx + c)^2 + 2 * (3 * (a^5 + 3a^4b + 3a^3b^2 + a^2b^3) * d * \cosh(dx + c)^2 + (a^5 + a^4b - a^3b^2 - a^2b^3) * d) * \sinh(dx + c)^2 + (a^5 + 3a^4b + 3a^3b^2 + a^2b^3) * d + 4 * ((a^5 + 3a^4b + 3a^3b^2 + a^2b^3) * d * \cosh(dx + c)^3 + (a^5 + a^4b - a^3b^2 - a^2b^3) * d * \cosh(dx + c)) * \sinh(dx + c)$

giac [B] time = 0.31, size = 195, normalized size = 2.05

$$\frac{(2ab+b^2)\log(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b)}{a^4+2a^3b+a^2b^2} + \frac{2(dx+c)}{a^2+2ab+b^2} - \frac{4b^2e^{(2dx+2c)}}{(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b)}$$

$$2d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")

[Out] $-1/2 * ((2*a*b + b^2) * \log(a * e^{(4*d*x + 4*c)} + b * e^{(4*d*x + 4*c)} + 2*a * e^{(2*d*x + 2*c)} - 2*b * e^{(2*d*x + 2*c)} + a + b) / (a^4 + 2*a^3*b + a^2*b^2) + 2 * (d*x + c) / (a^2 + 2*a*b + b^2) - 4*b^2 * e^{(2*d*x + 2*c)} / ((a * e^{(4*d*x + 4*c)} + b * e^{(4*d*x + 4*c)} + 2*a * e^{(2*d*x + 2*c)} - 2*b * e^{(2*d*x + 2*c)} + a + b)) * (a + b)^{-2} * a - 2 * \log(\text{abs}(e^{(2*d*x + 2*c)} - 1)) / a^2) / d$

maple [B] time = 0.46, size = 325, normalized size = 3.42

$$\frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{d(a+b)^2} - \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{d(a+b)^2} - \frac{2b^2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d(a+b)^2 a\left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right) a + 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x)

[Out] $-1/d / (a+b)^2 * \ln(\tanh(1/2*d*x+1/2*c)-1) - 1/d / (a+b)^2 * \ln(\tanh(1/2*d*x+1/2*c)+1) - 2/d * b^2 / (a+b)^2 / a * \tanh(1/2*d*x+1/2*c)^2 / (\tanh(1/2*d*x+1/2*c)^4 * a + 2 * \tanh(1/2*d*x+1/2*c)^2 * a + 4 * \tanh(1/2*d*x+1/2*c)^2 * b + a) - 2/d * b^3 / (a+b)^2 / a^2 * \tanh(1/2*d*x+1/2*c)^2 / (\tanh(1/2*d*x+1/2*c)^4 * a + 2 * \tanh(1/2*d*x+1/2*c)^2 * a + 4 * \tanh(1/2*d*x+1/2*c)^2 * b + a) - 1/d * b / (a+b)^2 / a * \ln(\tanh(1/2*d*x+1/2*c)^4 * a + 2 * \tanh(1/2*d*x+1/2*c)^2 * a + 4 * \tanh(1/2*d*x+1/2*c)^2 * b + a) - 1/2 / d * b^2 / (a+b)^2 / a^2 * \ln(\tanh(1/2*d*x+1/2*c)^4 * a + 2 * \tanh(1/2*d*x+1/2*c)^2 * a + 4 * \tanh(1/2*d*x+1/2*c)^2 * b + a) + 1/d / a^2 * \ln(\tanh(1/2*d*x+1/2*c))$

maxima [B] time = 0.33, size = 235, normalized size = 2.47

$$\frac{2b^2e^{(-2dx-2c)}}{(a^4 + 3a^3b + 3a^2b^2 + ab^3 + 2(a^4 + a^3b - a^2b^2 - ab^3)e^{(-2dx-2c)} + (a^4 + 3a^3b + 3a^2b^2 + ab^3)e^{(-4dx-4c)})d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $2 * b^2 * e^{(-2 * d * x - 2 * c)} / ((a^4 + 3 * a^3 * b + 3 * a^2 * b^2 + a * b^3 + 2 * (a^4 + a^3 * b - a^2 * b^2 - a * b^3) * e^{(-2 * d * x - 2 * c)} + (a^4 + 3 * a^3 * b + 3 * a^2 * b^2 + a * b^3) * e^{(-4 * d * x - 4 * c)}) * d) - 1/2 * (2 * a * b + b^2) * \log(2 * (a - b) * e^{(-2 * d * x - 2 * c)} + (a + b) * e^{(-4 * d * x - 4 * c)} + a + b) / ((a^4 + 2 * a^3 * b + a^2 * b^2) * d) + (d * x + c) / ((a^2 + 2 * a * b + b^2) * d) + \log(e^{(-d * x - c)} + 1) / (a^2 * d) + \log(e^{(-d * x - c)} - 1) / (a^2 * d)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(c + dx)}{(b \tanh(c + dx)^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(c + d*x)/(a + b*tanh(c + d*x)^2)^2, x)`

[Out] `int(coth(c + d*x)/(a + b*tanh(c + d*x)^2)^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)/(a+b*tanh(d*x+c)**2)**2, x)`

[Out] `Integral(coth(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)`

$$3.187 \quad \int \frac{\coth^2(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=119

$$-\frac{b^{3/2}(5a+3b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d(a+b)^2} - \frac{(2a+3b) \coth(c+dx)}{2a^2d(a+b)} + \frac{b \coth(c+dx)}{2ad(a+b)(a+b \tanh^2(c+dx))} + \frac{x}{(a+b)^2}$$

[Out] x/(a+b)^2-1/2*b^(3/2)*(5*a+3*b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/(a+b)^2/d-1/2*(2*a+3*b)*coth(d*x+c)/a^2/(a+b)/d+1/2*b*coth(d*x+c)/a/(a+b)/d/(a+b*tanh(d*x+c)^2)

Rubi [A] time = 0.19, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3670, 472, 583, 522, 206, 205}

$$-\frac{b^{3/2}(5a+3b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d(a+b)^2} - \frac{(2a+3b) \coth(c+dx)}{2a^2d(a+b)} + \frac{b \coth(c+dx)}{2ad(a+b)(a+b \tanh^2(c+dx))} + \frac{x}{(a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2), x]

[Out] x/(a + b)^2 - (b^(3/2)*(5*a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(5/2)*(a + b)^2*d) - ((2*a + 3*b)*Coth[c + d*x])/(2*a^2*(a + b)*d) + (b*Coth[c + d*x])/(2*a*(a + b)*d*(a + b*Tanh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 472

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(a*e*n*(b*c-a*d)*(p+1)), x] + Dist[1/(a*n*(b*c-a*d)*(p+1)), Int[(e*x)^m*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[c*b*(m+1)+n*(b*c-a*d)*(p+1)+d*b*(m+n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))
```

Rubi steps

$$\int \frac{\coth^2(c + dx)}{(a + b \tanh^2(c + dx))^2} dx = \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+bx^2)^2} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{b \coth(c + dx)}{2a(a + b)d(a + b \tanh^2(c + dx))} - \frac{\text{Subst}\left(\int \frac{-2a-3b+3bx^2}{x^2(1-x^2)(a+bx^2)} dx, x, \tanh(c + dx)\right)}{2a(a + b)d}$$

$$= -\frac{(2a + 3b) \coth(c + dx)}{2a^2(a + b)d} + \frac{b \coth(c + dx)}{2a(a + b)d(a + b \tanh^2(c + dx))} + \frac{\text{Subst}\left(\int \frac{2a^2-2ab-3b^2}{(1-x^2)} dx, x, \tanh(c + dx)\right)}{2a(a + b)d}$$

$$= -\frac{(2a + 3b) \coth(c + dx)}{2a^2(a + b)d} + \frac{b \coth(c + dx)}{2a(a + b)d(a + b \tanh^2(c + dx))} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{(a + b)d}$$

$$= \frac{x}{(a + b)^2} - \frac{b^{3/2}(5a + 3b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}(a + b)^2d} - \frac{(2a + 3b) \coth(c + dx)}{2a^2(a + b)d} + \frac{1}{2a(a + b)d}$$

Mathematica [A] time = 1.86, size = 111, normalized size = 0.93

$$\frac{\frac{b^{3/2}(5a+3b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a+b)^2} + \frac{b^2 \sinh(2(c+dx))}{a^2(a+b)((a+b) \cosh(2(c+dx))+a-b)} + \frac{2 \coth(c+dx)}{a^2} - \frac{2(c+dx)}{(a+b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2), x]
[Out] -1/2*((-2*(c + d*x))/(a + b)^2 + (b^(3/2)*(5*a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*(a + b)^2) + (2*Coth[c + d*x])/a^2 + (b^2*Sinh[2*(c + d*x)]/(a^2*(a + b)*(a - b + (a + b)*Cosh[2*(c + d*x)])))/d
```

fricas [B] time = 0.50, size = 3725, normalized size = 31.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

$$2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)-1/d*b^3/(a+b)^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)+5/2/d*b^2/(a+b)^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-5/2/d*b^2/(a+b)^2/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+4/d*b^3/(a+b)^2/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+5/2/d*b^2/(a+b)^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+5/2/d*b^2/(a+b)^2/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+4/d*b^3/(a+b)^2/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-3/2/d*b^3/(a+b)^2/a^2/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/2/d*b^4/(a+b)^2/a^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+3/2/d*b^3/(a+b)^2/a^2/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+3/2/d*b^4/(a+b)^2/a^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/2/d/a^2/tanh(1/2*d*x+1/2*c)$$

maxima [B] time = 0.64, size = 976, normalized size = 8.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")

[Out]
$$-1/4*(2*a*b + b^2)*\log((a + b)*e^{(4*d*x + 4*c)} + 2*(a - b)*e^{(2*d*x + 2*c)} + a + b)/((a^4 + 2*a^3*b + a^2*b^2)*d) + 1/4*(2*a*b + b^2)*\log(2*(a - b)*e^{(-2*d*x - 2*c)} + (a + b)*e^{(-4*d*x - 4*c)} + a + b)/((a^4 + 2*a^3*b + a^2*b^2)*d) + 1/8*(3*a^2*b - 4*a*b^2 - 3*b^3)*\arctan(1/2*((a + b)*e^{(2*d*x + 2*c)} + a - b)/\sqrt{a*b})/((a^4 + 2*a^3*b + a^2*b^2)*\sqrt{a*b}*d) - 1/8*(3*a^2*b - 4*a*b^2 - 3*b^3)*\arctan(1/2*((a + b)*e^{(-2*d*x - 2*c)} + a - b)/\sqrt{a*b})/((a^4 + 2*a^3*b + a^2*b^2)*\sqrt{a*b}*d) + 1/4*(2*a^3 + 5*a^2*b + 6*a*b^2 + 3*b^3 + (2*a^3 + 7*a^2*b + 3*b^3)*e^{(4*d*x + 4*c)} + 2*(2*a^3 + 2*a^2*b + a*b^2 - 3*b^3)*e^{(2*d*x + 2*c)})/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 - (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*e^{(6*d*x + 6*c)} - (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*e^{(4*d*x + 4*c)} + (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*e^{(2*d*x + 2*c)})*d) - 1/4*(2*a^3 + 5*a^2*b + 6*a*b^2 + 3*b^3 + 2*(2*a^3 + 2*a^2*b + a*b^2 - 3*b^3)*e^{(-2*d*x - 2*c)} + (2*a^3 + 7*a^2*b + 3*b^3)*e^{(-4*d*x - 4*c)})/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*e^{(-2*d*x - 2*c)} - (a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*e^{(-4*d*x - 4*c)} - (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*e^{(-6*d*x - 6*c)})*d) - 1/2*(2*a^2 + 5*a*b + 3*b^2 + 2*(2*a^2 - 3*b^2)*e^{(-2*d*x - 2*c)} + (2*a^2 + 3*a*b + 3*b^2)*e^{(-4*d*x - 4*c)})/((a^4 + 2*a^3*b + a^2*b^2 + (a^4 - 2*a^3*b - 3*a^2*b^2)*e^{(-2*d*x - 2*c)} - (a^4 - 2*a^3*b - 3*a^2*b^2)*e^{(-4*d*x - 4*c)} - (a^4 + 2*a^3*b + a^2*b^2)*e^{(-6*d*x - 6*c)})*d) + 3/4*b*\arctan(1/2*((a + b)*e^{(-2*d*x - 2*c)} + a - b)/\sqrt{a*b})/(\sqrt{a*b}*a^2*d) + 1/2*\log(e^{(2*d*x + 2*c)} - 1)/(a^2*d) - 1/2*\log(e^{(-2*d*x - 2*c)} - 1)/(a^2*d)$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(c + dx)^2}{(b \tanh(c + dx)^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2,x)

[Out] `int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)`

[Out] `Integral(coth(c + d*x)**2/(a + b*tanh(c + d*x)**2)**2, x)`

$$3.188 \quad \int \frac{\coth^3(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=124

$$\frac{b^2(3a+2b) \log(a+b \tanh^2(c+dx))}{2a^3d(a+b)^2} + \frac{(a-2b) \log(\tanh(c+dx))}{a^3d} - \frac{b^2}{2a^2d(a+b)(a+b \tanh^2(c+dx))} - \frac{\coth^2(c+dx)}{2a^2d}$$

[Out] $-1/2*\coth(d*x+c)^2/a^2/d+\ln(\cosh(d*x+c))/(a+b)^2/d+(a-2*b)*\ln(\tanh(d*x+c))/a^3/d+1/2*b^2*(3*a+2*b)*\ln(a+b*\tanh(d*x+c)^2)/a^3/(a+b)^2/d-1/2*b^2/a^2/(a+b)/d/(a+b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.19, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 446, 88}

$$-\frac{b^2}{2a^2d(a+b)(a+b \tanh^2(c+dx))} + \frac{b^2(3a+2b) \log(a+b \tanh^2(c+dx))}{2a^3d(a+b)^2} + \frac{(a-2b) \log(\tanh(c+dx))}{a^3d} - \frac{\coth^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]

[Out] $-\text{Coth}[c + d*x]^2/(2*a^2*d) + \text{Log}[\text{Cosh}[c + d*x]]/((a + b)^2*d) + ((a - 2*b)*\text{Log}[\text{Tanh}[c + d*x]])/(a^3*d) + (b^2*(3*a + 2*b)*\text{Log}[a + b*\text{Tanh}[c + d*x]^2])/(2*a^3*(a + b)^2*d) - b^2/(2*a^2*(a + b)*d*(a + b*\text{Tanh}[c + d*x]^2))$

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(c+dx)}{(a+b\tanh^2(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3(1-x^2)(a+bx^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{(1-x)x^2(a+bx)^2} dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^2(-1+x)} + \frac{1}{a^2x^2} + \frac{a-2b}{a^3x} + \frac{b^3}{a^2(a+b)(a+bx)^2} + \frac{b^3(3a+2b)}{a^3(a+b)^2(a+bx)}\right) dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= -\frac{\coth^2(c+dx)}{2a^2d} + \frac{\log(\cosh(c+dx))}{(a+b)^2d} + \frac{(a-2b)\log(\tanh(c+dx))}{a^3d} + \frac{b^2(3a+2b)}{a^3d}
\end{aligned}$$

Mathematica [A] time = 0.92, size = 93, normalized size = 0.75

$$\frac{\frac{b^3}{a^3(a+b)(a\coth^2(c+dx)+b)} + \frac{b^2(3a+2b)\log(a\coth^2(c+dx)+b)}{a^3(a+b)^2} - \frac{\coth^2(c+dx)}{a^2} + \frac{2\log(\sinh(c+dx))}{(a+b)^2}}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2), x]

[Out] $(-\text{Coth}[c + d*x]^2/a^2) + b^3/(a^3*(a + b)*(b + a*\text{Coth}[c + d*x]^2)) + (b^2*(3*a + 2*b)*\text{Log}[b + a*\text{Coth}[c + d*x]^2])/(a^3*(a + b)^2) + (2*\text{Log}[\text{Sinh}[c + d*x]])/(a + b)^2/(2*d)$

fricas [B] time = 0.67, size = 3468, normalized size = 27.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] $-1/2*(2*(a^4 + a^3*b)*d*x*\cosh(d*x + c)^8 + 16*(a^4 + a^3*b)*d*x*\cosh(d*x + c)*\sinh(d*x + c)^7 + 2*(a^4 + a^3*b)*d*x*\sinh(d*x + c)^8 - 4*(2*a^3*b*d*x - a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*\cosh(d*x + c)^6 - 4*(2*a^3*b*d*x - 14*(a^4 + a^3*b)*d*x*\cosh(d*x + c)^2 - a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*\sinh(d*x + c)^6 + 8*(14*(a^4 + a^3*b)*d*x*\cosh(d*x + c)^3 - 3*(2*a^3*b*d*x - a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 4*(2*a^4 + 2*a^3*b - 2*a^2*b^2 - 4*a*b^3 - (a^4 - 3*a^3*b)*d*x)*\cosh(d*x + c)^4 + 4*(35*(a^4 + a^3*b)*d*x*\cosh(d*x + c)^4 + 2*a^4 + 2*a^3*b - 2*a^2*b^2 - 4*a*b^3 - (a^4 - 3*a^3*b)*d*x - 15*(2*a^3*b*d*x - a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 16*(7*(a^4 + a^3*b)*d*x*\cosh(d*x + c)^5 - 5*(2*a^3*b*d*x - a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*\cosh(d*x + c)^3 + (2*a^4 + 2*a^3*b - 2*a^2*b^2 - 4*a*b^3 - (a^4 - 3*a^3*b)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 2*(a^4 + a^3*b)*d*x - 4*(2*a^3*b*d*x - a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*\cosh(d*x + c)^2 + 4*(14*(a^4 + a^3*b)*d*x*\cosh(d*x + c)^6 - 2*a^3*b*d*x - 15*(2*a^3*b*d*x - a^4 - 3*a^3*b - 3*a^2*b^2 - 2*a*b^3)*\cosh(d*x + c)^4 + a^4 + 3*a^3*b + 3*a^2*b^2 + 2*a*b^3 + 6*(2*a^4 + 2*a^3*b - 2*a^2*b^2 - 4*a*b^3 - (a^4 - 3*a^3*b)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 - ((3*a^2*b^2 + 5*a*b^3 + 2*b^4)*\cosh(d*x + c)^8 + 8*(3*a^2*b^2 + 5*a*b^3 + 2*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (3*a^2*b^2 + 5*a*b^3 + 2*b^4)*\sinh(d*x + c)^8 - 4*(3*a*b^3 + 2*b^4)*\cosh(d*x + c)^6 - 4*(3*a*b^3 + 2*b^4 - 7*(3*a^2*b^2 + 5*a*b^3 + 2*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(3*a^2*b^2 + 5*a*b^3 + 2*b^4)*\cosh(d*x + c)^3 - 3*(3*a*b^3 + 2*b^4$

$$\begin{aligned}
&) * \cosh(dx + c) * \sinh(dx + c)^5 - 2 * (3 * a^2 * b^2 - 7 * a * b^3 - 6 * b^4) * \cosh(dx + c)^4 + 2 * (35 * (3 * a^2 * b^2 + 5 * a * b^3 + 2 * b^4) * \cosh(dx + c)^4 - 3 * a^2 * b^2 + 7 * a * b^3 + 6 * b^4 - 30 * (3 * a * b^3 + 2 * b^4) * \cosh(dx + c)^2) * \sinh(dx + c)^4 + \\
& 3 * a^2 * b^2 + 5 * a * b^3 + 2 * b^4 + 8 * (7 * (3 * a^2 * b^2 + 5 * a * b^3 + 2 * b^4) * \cosh(dx + c)^5 - 10 * (3 * a * b^3 + 2 * b^4) * \cosh(dx + c)^3 - (3 * a^2 * b^2 - 7 * a * b^3 - 6 * b^4) \\
&) * \cosh(dx + c) * \sinh(dx + c)^3 - 4 * (3 * a * b^3 + 2 * b^4) * \cosh(dx + c)^2 + 4 * (7 * (3 * a^2 * b^2 + 5 * a * b^3 + 2 * b^4) * \cosh(dx + c)^6 - 15 * (3 * a * b^3 + 2 * b^4) * \cosh(dx + c)^4 - 3 * a * b^3 - 2 * b^4 - 3 * (3 * a^2 * b^2 - 7 * a * b^3 - 6 * b^4) * \cosh(dx + c)^2) * \sinh(dx + c)^2 + 8 * ((3 * a^2 * b^2 + 5 * a * b^3 + 2 * b^4) * \cosh(dx + c)^7 - 3 * (3 * a * b^3 + 2 * b^4) * \cosh(dx + c)^5 - (3 * a^2 * b^2 - 7 * a * b^3 - 6 * b^4) * \cosh(dx + c)^3 - (3 * a * b^3 + 2 * b^4) * \cosh(dx + c)) * \sinh(dx + c) * \log(2 * ((a + b) * \cosh(dx + c)^2 + (a + b) * \sinh(dx + c)^2 + a - b) / (\cosh(dx + c)^2 - 2 * \cosh(dx + c) * \sinh(dx + c) + \sinh(dx + c)^2)) - 2 * ((a^4 + a^3 * b - 3 * a^2 * b^2 - 5 * a * b^3 - 2 * b^4) * \cosh(dx + c)^8 + 8 * (a^4 + a^3 * b - 3 * a^2 * b^2 - 5 * a * b^3 - 2 * b^4) * \cosh(dx + c) * \sinh(dx + c)^7 + (a^4 + a^3 * b - 3 * a^2 * b^2 - 5 * a * b^3 - 2 * b^4) * \sinh(dx + c)^8 - 4 * (a^3 * b - 3 * a * b^3 - 2 * b^4) * \cosh(dx + c)^6 - 4 * (a^3 * b - 3 * a * b^3 - 2 * b^4 - 7 * (a^4 + a^3 * b - 3 * a^2 * b^2 - 5 * a * b^3 - 2 * b^4) * \cosh(dx + c)^2) * \sinh(dx + c)^6 + 8 * (7 * (a^4 + a^3 * b - 3 * a^2 * b^2 - 5 * a * b^3 - 2 * b^4) * \cosh(dx + c)^3 - 3 * (a^3 * b - 3 * a * b^3 - 2 * b^4) * \cosh(dx + c)) * \sinh(dx + c)^5 - 2 * (a^4 - 3 * a^3 * b - 3 * a^2 * b^2 + 7 * a * b^3 + 6 * b^4) * \cosh(dx + c)^4 + 2 * (35 * (a^4 + a^3 * b - 3 * a^2 * b^2 - 5 * a * b^3 - 2 * b^4) * \cosh(dx + c)^4 - a^4 + 3 * a^3 * b + 3 * a^2 * b^2 - 7 * a * b^3 - 6 * b^4 - 30 * (a^3 * b - 3 * a * b^3 - 2 * b^4) * \cosh(dx + c)^2) * \sinh(dx + c)^4 + a^4 + a^3 * b - 3 * a^2 * b^2 - 5 * a * b^3 - 2 * b^4 + 8 * (7 * (a^4 + a^3 * b - 3 * a^2 * b^2 - 5 * a * b^3 - 2 * b^4) * \cosh(dx + c)^5 - 10 * (a^3 * b - 3 * a * b^3 - 2 * b^4) * \cosh(dx + c)^3 - (a^4 - 3 * a^3 * b - 3 * a^2 * b^2 + 7 * a * b^3 + 6 * b^4) * \cosh(dx + c)) * \sinh(dx + c)^3 - 4 * (a^3 * b - 3 * a * b^3 - 2 * b^4) * \cosh(dx + c)^2 + 4 * (7 * (a^4 + a^3 * b - 3 * a^2 * b^2 - 5 * a * b^3 - 2 * b^4) * \cosh(dx + c)^6 - 15 * (a^3 * b - 3 * a * b^3 - 2 * b^4) * \cosh(dx + c)^4 - a^3 * b + 3 * a * b^3 + 2 * b^4 - 3 * (a^4 - 3 * a^3 * b - 3 * a^2 * b^2 + 7 * a * b^3 + 6 * b^4) * \cosh(dx + c)^2) * \sinh(dx + c)^2 + 8 * ((a^4 + a^3 * b - 3 * a^2 * b^2 - 5 * a * b^3 - 2 * b^4) * \cosh(dx + c)^7 - 3 * (a^3 * b - 3 * a * b^3 - 2 * b^4) * \cosh(dx + c)^5 - (a^4 - 3 * a^3 * b - 3 * a^2 * b^2 + 7 * a * b^3 + 6 * b^4) * \cosh(dx + c)^3 - (a^3 * b - 3 * a * b^3 - 2 * b^4) * \cosh(dx + c)) * \sinh(dx + c) * \log(2 * \sinh(dx + c) / (\cosh(dx + c) - \sinh(dx + c))) + 8 * (2 * (a^4 + a^3 * b) * dx * \cosh(dx + c)^7 - 3 * (2 * a^3 * b * dx - a^4 - 3 * a^3 * b - 3 * a^2 * b^2 - 2 * a * b^3) * \cosh(dx + c)^5 + 2 * (2 * a^4 + 2 * a^3 * b - 2 * a^2 * b^2 - 4 * a * b^3 - (a^4 - 3 * a^3 * b) * dx) * \cosh(dx + c)^3 - (2 * a^3 * b * dx - a^4 - 3 * a^3 * b - 3 * a^2 * b^2 - 2 * a * b^3) * \cosh(dx + c)) * \sinh(dx + c)) / ((a^6 + 3 * a^5 * b + 3 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c)^8 + 8 * (a^6 + 3 * a^5 * b + 3 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c) * \sinh(dx + c)^7 + (a^6 + 3 * a^5 * b + 3 * a^4 * b^2 + a^3 * b^3) * d * \sinh(dx + c)^8 - 4 * (a^5 * b + 2 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c)^6 + 4 * (7 * (a^6 + 3 * a^5 * b + 3 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c)^2 - (a^5 * b + 2 * a^4 * b^2 + a^3 * b^3) * d) * \sinh(dx + c)^6 - 2 * (a^6 - a^5 * b - 5 * a^4 * b^2 - 3 * a^3 * b^3) * d * \cosh(dx + c)^4 + 8 * (7 * (a^6 + 3 * a^5 * b + 3 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c)^3 - 3 * (a^5 * b + 2 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c)) * \sinh(dx + c)^5 + 2 * (35 * (a^6 + 3 * a^5 * b + 3 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c)^4 - 30 * (a^5 * b + 2 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c)^2 - (a^6 - a^5 * b - 5 * a^4 * b^2 - 3 * a^3 * b^3) * d) * \sinh(dx + c)^4 - 4 * (a^5 * b + 2 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c)^2 + 8 * (7 * (a^6 + 3 * a^5 * b + 3 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c)^5 - 10 * (a^5 * b + 2 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c)^3 - (a^6 - a^5 * b - 5 * a^4 * b^2 - 3 * a^3 * b^3) * d * \cosh(dx + c)) * \sinh(dx + c)^3 + 4 * (7 * (a^6 + 3 * a^5 * b + 3 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c)^6 - 15 * (a^5 * b + 2 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c)^4 - 3 * (a^6 - a^5 * b - 5 * a^4 * b^2 - 3 * a^3 * b^3) * d * \cosh(dx + c)^2 - (a^5 * b + 2 * a^4 * b^2 + a^3 * b^3) * d) * \sinh(dx + c)^2 + (a^6 + 3 * a^5 * b + 3 * a^4 * b^2 + a^3 * b^3) * d + 8 * ((a^6 + 3 * a^5 * b + 3 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c)^7 - 3 * (a^5 * b + 2 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c)^5 - (a^6 - a^5 * b - 5 * a^4 * b^2 - 3 * a^3 * b^3) * d * \cosh(dx + c)^3 - (a^5 * b + 2 * a^4 * b^2 + a^3 * b^3) * d * \cosh(dx + c)) * \sinh(dx + c))
\end{aligned}$$

giac [B] time = 0.38, size = 323, normalized size = 2.60

$$\frac{(3ab^2+2b^3)\log(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b)}{a^5+2a^4b+a^3b^2} - \frac{2(dx+c)}{a^2+2ab+b^2} + \frac{2(a-2b)\log(|e^{(2dx+2c)}-1|)}{a^3} - \frac{4\left(\frac{(a^4+3a^3b+3a^2b^2+2ab^3)e}{a+b}\right)}{(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b)}$$

$2d$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((3*a*b^2 + 2*b^3)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^5 + 2*a^4*b + a^3*b^2) - 2*(d*x + c)/(a^2 + 2*a*b + b^2) + 2*(a - 2*b)*log(abs(e^(2*d*x + 2*c) - 1))/a^3 - 4*((a^4 + 3*a^3*b + 3*a^2*b^2 + 2*a*b^3)*e^(6*d*x + 6*c)/(a + b) + 2*(a^4 + a^3*b - a^2*b^2 - 2*a*b^3)*e^(4*d*x + 4*c)/(a + b) + (a^4 + 3*a^3*b + 3*a^2*b^2 + 2*a*b^3)*e^(2*d*x + 2*c)/(a + b))/((a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*(a + b)*a^3*(e^(2*d*x + 2*c) - 1)^2))/d
```

maple [B] time = 0.48, size = 383, normalized size = 3.09

$$\frac{\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)}{8da^2} - \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)}{d(a+b)^2} - \frac{\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)}{d(a+b)^2} + \frac{2b^3\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d(a+b)^2 a^2 \left(\left(\tanh^4\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + 2\left(\tanh^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)a + a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x)
```

```
[Out] -1/8/d*tanh(1/2*d*x+1/2*c)^2/a^2-1/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)-1)-1/d/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)+1)+2/d*b^3/(a+b)^2/a^2*tanh(1/2*d*x+1/2*c)^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)+2/d*b^4/a^3/(a+b)^2*tanh(1/2*d*x+1/2*c)^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)+3/2/d*b^2/(a+b)^2/a^2*ln(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)+1/d*b^3/a^3/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)-1/8/d/a^2/tanh(1/2*d*x+1/2*c)^2+1/d/a^2*ln(tanh(1/2*d*x+1/2*c))-2/d/a^3*ln(tanh(1/2*d*x+1/2*c))*b
```

maxima [B] time = 0.35, size = 402, normalized size = 3.24

$$\frac{(3ab^2 + 2b^3)\log(2(a-b)e^{(-2dx-2c)} + (a+b)e^{(-4dx-4c)} + a+b)}{2(a^5 + 2a^4b + a^3b^2)d} + \frac{dx+c}{(a^2 + 2ab + b^2)d} - \frac{4\left(\frac{(a^4+3a^3b+3a^2b^2+2ab^3)e}{a+b}\right)}{(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(3*a*b^2 + 2*b^3)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^5 + 2*a^4*b + a^3*b^2)*d) + (d*x + c)/((a^2 + 2*a*b + b^2)*d) - 2*((a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*e^(-2*d*x - 2*c) + 2*(a^3 + a^2*b - a*b^2 - 2*b^3)*e^(-4*d*x - 4*c) + (a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*e^(-6*d*x - 6*c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 - 4*(a^4*b + 2*a^3*b^2 + a^2*b^3)*e^(-2*d*x - 2*c) - 2*(a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^3)*e^(-4*d*x - 4*c) - 4*(a^4*b + 2*a^3*b^2 + a^2*b^3)*e^(-6*d*x - 6*c) + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*e^(-8*d*x - 8*c))*d) + (a - 2*b)*log(e^(-d*x - c) + 1)/(a^3*d) + (a - 2*b)*log(e^(-d*x - c) - 1)/(a^3*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(c + dx)^3}{(b \tanh(c + dx)^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2, x)

[Out] int(coth(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2, x)

[Out] Integral(coth(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)

$$3.189 \quad \int \frac{\coth^4(c+dx)}{(a+b \tanh^2(c+dx))^2} dx$$

Optimal. Leaf size=159

$$\frac{b^{5/2}(7a+5b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d(a+b)^2} - \frac{(2a+5b) \coth^3(c+dx)}{6a^2d(a+b)} - \frac{(2a^2-2ab-5b^2) \coth(c+dx)}{2a^3d(a+b)} + \frac{b \coth^3(c+dx)}{2ad(a+b)(a+b)}$$

[Out] x/(a+b)^2+1/2*b^(5/2)*(7*a+5*b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(7/2)
/(a+b)^2/d-1/2*(2*a^2-2*a*b-5*b^2)*coth(d*x+c)/a^3/(a+b)/d-1/6*(2*a+5*b)*co
th(d*x+c)^3/a^2/(a+b)/d+1/2*b*coth(d*x+c)^3/a/(a+b)/d/(a+b*tanh(d*x+c)^2)

Rubi [A] time = 0.28, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3670, 472, 583, 522, 206, 205}

$$-\frac{(2a^2-2ab-5b^2) \coth(c+dx)}{2a^3d(a+b)} + \frac{b^{5/2}(7a+5b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d(a+b)^2} - \frac{(2a+5b) \coth^3(c+dx)}{6a^2d(a+b)} + \frac{b \coth^3(c+dx)}{2ad(a+b)(a+b)}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2, x]

[Out] x/(a + b)^2 + (b^(5/2)*(7*a + 5*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(7/2)*(a + b)^2*d) - ((2*a^2 - 2*a*b - 5*b^2)*Coth[c + d*x])/(2*a^3*(a + b)*d) - ((2*a + 5*b)*Coth[c + d*x]^3)/(6*a^2*(a + b)*d) + (b*Coth[c + d*x]^3)/(2*a*(a + b)*d*(a + b*Tanh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 472

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*(e*x)^(m+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(a*e*n*(b*c-a*d)*(p+1)), x] + Dist[1/(a*n*(b*c-a*d)*(p+1)), Int[(e*x)^m*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[c*b*(m+1)+n*(b*c-a*d)*(p+1)+d*b*(m+n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 583


```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f*f^2*x^2), x], x, (c*Tan[e + f*x])/ff, x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))
```

Rubi steps

$$\begin{aligned} \int \frac{\coth^4(c + dx)}{(a + b \tanh^2(c + dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+bx^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{b \coth^3(c + dx)}{2a(a + b)d(a + b \tanh^2(c + dx))} - \frac{\text{Subst}\left(\int \frac{-2a-5b+5bx^2}{x^4(1-x^2)(a+bx^2)} dx, x, \tanh(c + dx)\right)}{2a(a + b)d} \\ &= -\frac{(2a + 5b) \coth^3(c + dx)}{6a^2(a + b)d} + \frac{b \coth^3(c + dx)}{2a(a + b)d(a + b \tanh^2(c + dx))} + \frac{\text{Subst}\left(\int \frac{3(2a^2 - 2ab - 5b^2)}{x^4(1-x^2)(a+bx^2)} dx, x, \tanh(c + dx)\right)}{2a(a + b)d} \\ &= -\frac{(2a^2 - 2ab - 5b^2) \coth(c + dx)}{2a^3(a + b)d} - \frac{(2a + 5b) \coth^3(c + dx)}{6a^2(a + b)d} + \frac{b \coth^3(c + dx)}{2a(a + b)d(a + b \tanh^2(c + dx))} \\ &= -\frac{(2a^2 - 2ab - 5b^2) \coth(c + dx)}{2a^3(a + b)d} - \frac{(2a + 5b) \coth^3(c + dx)}{6a^2(a + b)d} + \frac{b \coth^3(c + dx)}{2a(a + b)d(a + b \tanh^2(c + dx))} \\ &= \frac{x}{(a + b)^2} + \frac{b^{5/2}(7a + 5b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}(a + b)^2d} - \frac{(2a^2 - 2ab - 5b^2) \coth(c + dx)}{2a^3(a + b)d} \end{aligned}$$

Mathematica [A] time = 1.56, size = 139, normalized size = 0.87

$$\frac{3b^{5/2}(7a+5b) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}(a+b)^2} + \frac{3b^3 \sinh(2(c+dx))}{a^3(a+b)((a+b) \cosh(2(c+dx))+a-b)} + \frac{4(3b-2a) \coth(c+dx)}{a^3} - \frac{2 \coth(c+dx) \text{csch}^2(c+dx)}{a^2} + \frac{6(c+dx)}{(a+b)^2}$$

6d

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2, x]
```

```
[Out] ((6*(c + d*x))/(a + b)^2 + (3*b^(5/2)*(7*a + 5*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(7/2)*(a + b)^2) + (4*(-2*a + 3*b)*Coth[c + d*x])/a^3 - (2*Coth[c + d*x]*Csch[c + d*x]^2)/a^2 + (3*b^3*Sinh[2*(c + d*x)]/(a^3*(a + b)*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(6*d)
```

fricas [B] time = 0.59, size = 8482, normalized size = 53.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/12*(12*(a^4 + a^3*b)*d*x*cosh(d*x + c)^10 + 120*(a^4 + a^3*b)*d*x*cosh(d*x + c)*sinh(d*x + c)^9 + 12*(a^4 + a^3*b)*d*x*sinh(d*x + c)^10 - 12*(4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d*x)*cosh(d*x + c)^8 + 12*(4*5*(a^4 + a^3*b)*d*x*cosh(d*x + c)^2 - 4*a^4 - 8*a^3*b + 7*a*b^3 + 5*b^4 - (a^4 + 5*a^3*b)*d*x)*sinh(d*x + c)^8 + 96*(15*(a^4 + a^3*b)*d*x*cosh(d*x + c)^3 - (4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 24*(2*a^4 - 2*a^3*b - 2*a^2*b^2 + 9*a*b^3 + 10*b^4 + (a^4 - 5*a^3*b)*d*x)*cosh(d*x + c)^6 + 24*(105*(a^4 + a^3*b)*d*x*cosh(d*x + c)^4 - 2*a^4 + 2*a^3*b + 2*a^2*b^2 - 9*a*b^3 - 10*b^4 - (a^4 - 5*a^3*b)*d*x - 14*(4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 48*(63*(a^4 + a^3*b)*d*x*cosh(d*x + c)^5 - 14*(4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d*x)*cosh(d*x + c)^3 - 3*(2*a^4 - 2*a^3*b - 2*a^2*b^2 + 9*a*b^3 + 10*b^4 + (a^4 - 5*a^3*b)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 8*(2*a^4 - 30*a^3*b - 30*a^2*b^2 + 38*a*b^3 + 4*5*b^4 + 3*(a^4 - 5*a^3*b)*d*x)*cosh(d*x + c)^4 + 8*(315*(a^4 + a^3*b)*d*x*cosh(d*x + c)^6 - 105*(4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d*x)*cosh(d*x + c)^4 + 2*a^4 - 30*a^3*b - 30*a^2*b^2 + 38*a*b^3 + 45*b^4 + 3*(a^4 - 5*a^3*b)*d*x - 45*(2*a^4 - 2*a^3*b - 2*a^2*b^2 + 9*a*b^3 + 10*b^4 + (a^4 - 5*a^3*b)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 32*a^4 - 48*a^3*b + 48*a^2*b^2 + 124*a*b^3 + 60*b^4 + 32*(45*(a^4 + a^3*b)*d*x*cosh(d*x + c)^7 - 21*(4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d*x)*cosh(d*x + c)^5 - 15*(2*a^4 - 2*a^3*b - 2*a^2*b^2 + 9*a*b^3 + 10*b^4 + (a^4 - 5*a^3*b)*d*x)*cosh(d*x + c)^3 + (2*a^4 - 30*a^3*b - 30*a^2*b^2 + 38*a*b^3 + 45*b^4 + 3*(a^4 - 5*a^3*b)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 12*(a^4 + a^3*b)*d*x - 4*(4*a^4 - 20*a^3*b - 4*a^2*b^2 + 74*a*b^3 + 60*b^4 - 3*(a^4 + 5*a^3*b)*d*x)*cosh(d*x + c)^2 + 4*(135*(a^4 + a^3*b)*d*x*cosh(d*x + c)^8 - 84*(4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d*x)*cosh(d*x + c)^6 - 9*0*(2*a^4 - 2*a^3*b - 2*a^2*b^2 + 9*a*b^3 + 10*b^4 + (a^4 - 5*a^3*b)*d*x)*cosh(d*x + c)^4 - 4*a^4 + 20*a^3*b + 4*a^2*b^2 - 74*a*b^3 - 60*b^4 + 3*(a^4 + 5*a^3*b)*d*x + 12*(2*a^4 - 30*a^3*b - 30*a^2*b^2 + 38*a*b^3 + 45*b^4 + 3*(a^4 - 5*a^3*b)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*((7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x + c)^10 + 10*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x + c)*sinh(d*x + c)^9 + (7*a^2*b^2 + 12*a*b^3 + 5*b^4)*sinh(d*x + c)^10 - (7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x + c)^8 - (7*a^2*b^2 + 40*a*b^3 + 25*b^4 - 45*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x + c)^3 - (7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c)^6 + 2*(105*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x + c)^4 - 7*a^2*b^2 + 30*a*b^3 + 25*b^4 - 14*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x + c)^5 - 14*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x + c)^3 - 3*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c)^4 + 2*(105*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x + c)^6 - 35*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x + c)^4 + 7*a^2*b^2 - 30*a*b^3 - 25*b^4 - 15*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 7*a^2*b^2 - 12*a*b^3 - 5*b^4 + 8*(15*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x + c)^7 - 7*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x + c)^5 - 5*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c)^3 + (7*a^2*b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + (7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x + c)^2 + (45*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*cosh(d*x + c)^8 - 28*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x + c)^6 - 30*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*cosh(d*x + c)^4 + 7*a^2*b^2 + 40*a*b^3 + 25*b^4 + 12*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*cosh(d*x + c)^2 + 12*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*sinh(d*x + c)^2 + 12*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*sinh(d*x + c)^3 + 12*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*sinh(d*x + c)^4 + 12*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*sinh(d*x + c)^5 + 12*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*sinh(d*x + c)^6 + 12*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*sinh(d*x + c)^7 + 12*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*sinh(d*x + c)^8 + 12*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*sinh(d*x + c)^9 + 12*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*sinh(d*x + c)^10

$$\begin{aligned}
& b^2 - 30ab^3 - 25b^4) \cosh(dx + c)^2) \sinh(dx + c)^2 + 2(5(7a^2b^2 \\
& + 12ab^3 + 5b^4) \cosh(dx + c)^9 - 4(7a^2b^2 + 40ab^3 + 25b^4) \cosh(dx + c)^7 - 6(7a^2b^2 - 30ab^3 - 25b^4) \cosh(dx + c)^5 + 4(7a^2 \\
& 2b^2 - 30ab^3 - 25b^4) \cosh(dx + c)^3 + (7a^2b^2 + 40ab^3 + 25b^4) \cosh(dx + c) \sinh(dx + c)) \sqrt{-b/a} \log(((a^2 + 2ab + b^2) \cosh(dx \\
& x + c)^4 + 4(a^2 + 2ab + b^2) \cosh(dx + c) \sinh(dx + c)^3 + (a^2 + 2a \\
& *b + b^2) \sinh(dx + c)^4 + 2(a^2 - b^2) \cosh(dx + c)^2 + 2(3(a^2 + 2a \\
& *b + b^2) \cosh(dx + c)^2 + a^2 - b^2) \sinh(dx + c)^2 + a^2 - 6ab + b^2 \\
& + 4((a^2 + 2ab + b^2) \cosh(dx + c)^3 + (a^2 - b^2) \cosh(dx + c)) \sinh(dx \\
& dx + c) + 4((a^2 + ab) \cosh(dx + c)^2 + 2(a^2 + ab) \cosh(dx + c) \sinh(dx + c) + (a^2 + ab) \sinh(dx + c)^2 + a^2 - ab) \sqrt{-b/a}) / ((a + b) * \\
& cosh(dx + c)^4 + 4(a + b) \cosh(dx + c) \sinh(dx + c)^3 + (a + b) \sinh(dx \\
& x + c)^4 + 2(a - b) \cosh(dx + c)^2 + 2(3(a + b) \cosh(dx + c)^2 + a - b \\
&) \sinh(dx + c)^2 + 4((a + b) \cosh(dx + c)^3 + (a - b) \cosh(dx + c)) \sinh(dx \\
& dx + c) + a + b)) + 8(15(a^4 + a^3b) dx \cosh(dx + c)^9 - 12(4a^4 \\
& + 8a^3b - 7ab^3 - 5b^4 + (a^4 + 5a^3b) dx) \cosh(dx + c)^7 - 18(2a^4 \\
& - 2a^3b - 2a^2b^2 + 9ab^3 + 10b^4 + (a^4 - 5a^3b) dx) \cosh(dx \\
& x + c)^5 + 4(2a^4 - 30a^3b - 30a^2b^2 + 38ab^3 + 45b^4 + 3(a^4 - \\
& 5a^3b) dx) \cosh(dx + c)^3 - (4a^4 - 20a^3b - 4a^2b^2 + 74ab^3 + \\
& 60b^4 - 3(a^4 + 5a^3b) dx) \cosh(dx + c)) \sinh(dx + c)) / ((a^6 + 3a^5 \\
& *b + 3a^4b^2 + a^3b^3) dx \cosh(dx + c)^10 + 10(a^6 + 3a^5b + 3a^4b^2 \\
& + a^3b^3) dx \cosh(dx + c) \sinh(dx + c)^9 + (a^6 + 3a^5b + 3a^4b^2 + \\
& a^3b^3) dx \sinh(dx + c)^10 - (a^6 + 7a^5b + 11a^4b^2 + 5a^3b^3) dx \cosh(dx + c)^8 + (45(a^6 + 3a^5b + 3a^4b^2 + a^3b^3) dx \cosh(dx + c)^2 \\
& - (a^6 + 7a^5b + 11a^4b^2 + 5a^3b^3) dx) \sinh(dx + c)^8 - 2(a^6 - \\
& 3a^5b - 9a^4b^2 - 5a^3b^3) dx \cosh(dx + c)^6 + 8(15(a^6 + 3a^5b + \\
& 3a^4b^2 + a^3b^3) dx \cosh(dx + c)^3 - (a^6 + 7a^5b + 11a^4b^2 + 5a^3 \\
& b^3) dx \cosh(dx + c)) \sinh(dx + c)^7 + 2(105(a^6 + 3a^5b + 3a^4b^2 \\
& + a^3b^3) dx \cosh(dx + c)^4 - 14(a^6 + 7a^5b + 11a^4b^2 + 5a^3b^3) \\
&) dx \cosh(dx + c)^2 - (a^6 - 3a^5b - 9a^4b^2 - 5a^3b^3) dx) \sinh(dx + \\
& c)^6 + 2(a^6 - 3a^5b - 9a^4b^2 - 5a^3b^3) dx \cosh(dx + c)^4 + 4(63 \\
& * (a^6 + 3a^5b + 3a^4b^2 + a^3b^3) dx \cosh(dx + c)^5 - 14(a^6 + 7a^5b \\
& b + 11a^4b^2 + 5a^3b^3) dx \cosh(dx + c)^3 - 3(a^6 - 3a^5b - 9a^4b^2 \\
& - 5a^3b^3) dx \cosh(dx + c)) \sinh(dx + c)^5 + 2(105(a^6 + 3a^5b + 3 \\
& *a^4b^2 + a^3b^3) dx \cosh(dx + c)^6 - 35(a^6 + 7a^5b + 11a^4b^2 + 5 \\
& a^3b^3) dx \cosh(dx + c)^4 - 15(a^6 - 3a^5b - 9a^4b^2 - 5a^3b^3) dx \cosh(dx + c)^2 + (a^6 - 3a^5b - 9a^4b^2 - 5a^3b^3) dx) \sinh(dx + c)^4 \\
& + (a^6 + 7a^5b + 11a^4b^2 + 5a^3b^3) dx \cosh(dx + c)^2 + 8(15(a^6 \\
& + 3a^5b + 3a^4b^2 + a^3b^3) dx \cosh(dx + c)^7 - 7(a^6 + 7a^5b + 11 \\
& a^4b^2 + 5a^3b^3) dx \cosh(dx + c)^5 - 5(a^6 - 3a^5b - 9a^4b^2 - 5a^3 \\
& b^3) dx \cosh(dx + c)^3 + (a^6 - 3a^5b - 9a^4b^2 - 5a^3b^3) dx \cosh(dx + c)) \sinh(dx + c)^3 + (45(a^6 + 3a^5b + 3a^4b^2 + a^3b^3) dx \cosh(dx + c)^8 - 28(a^6 + 7a^5b + 11a^4b^2 + 5a^3b^3) dx \cosh(dx + c)^6 - 30(a^6 - 3a^5b - 9a^4b^2 - 5a^3b^3) dx \cosh(dx + c)^4 + 12(a^6 - 3a^5b - 9a^4b^2 - 5a^3b^3) dx \cosh(dx + c)^2 + (a^6 + 7a^5b + 11a^4b^2 + 5a^3b^3) dx) \sinh(dx + c)^2 - (a^6 + 3a^5b + 3a^4b^2 + a^3b^3) dx + 2(5(a^6 + 3a^5b + 3a^4b^2 + a^3b^3) dx \cosh(dx + c)^9 - 4(a^6 + 7a^5b + 11a^4b^2 + 5a^3b^3) dx \cosh(dx + c)^7 - 6(a^6 - 3a^5b - 9a^4b^2 - 5a^3b^3) dx \cosh(dx + c)^5 + 4(a^6 - 3a^5b - 9a^4b^2 - 5a^3b^3) dx \cosh(dx + c)^3 + (a^6 + 7a^5b + 11a^4b^2 + 5a^3b^3) dx \cosh(dx + c) \sinh(dx + c)), 1/6(6(a^4 + a^3b) dx \cosh(dx + c)^10 + 60(a^4 + a^3b) dx \cosh(dx + c) \sinh(dx + c)^9 + 6(a^4 + a^3b) dx \sinh(dx + c)^10 - 6(4a^4 + 8a^3b - 7ab^3 - 5b^4 + (a^4 + 5a^3b) dx) \cosh(dx + c)^8 + 6(45(a^4 + a^3b) dx \cosh(dx + c)^2 - 4a^4 - 8a^3b + 7ab^3 + 5b^4 - (a^4 + 5a^3b) dx) \sinh(dx + c)^8 + 48(15(a^4 + a^3b) dx \cosh(dx + c)^3 - (4a^4 + 8a^3b - 7ab^3 - 5b^4 + (a^4 + 5a^3b) dx) \cosh(dx + c)) \sinh(dx + c)^7 - 12(2a^4 - 2a^3b - 2a^2b^2 + 9ab^3 + 10b^4 + (a^4 - 5a^3b) dx) \cosh(dx + c)^6 + 12(105(a^4 + a^3b) dx \cosh(dx + c)^4 - 2a^4 + 2a^3b + 2a^2b^2 - 9ab^3 - 1
\end{aligned}$$

$$\begin{aligned}
& 0*b^4 - (a^4 - 5*a^3*b)*d*x - 14*(4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 \\
& + 5*a^3*b)*d*x)*\cosh(d*x + c)^2*\sinh(d*x + c)^6 + 24*(63*(a^4 + a^3*b)*d*x \\
& *\cosh(d*x + c)^5 - 14*(4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)* \\
& d*x)*\cosh(d*x + c)^3 - 3*(2*a^4 - 2*a^3*b - 2*a^2*b^2 + 9*a*b^3 + 10*b^4 + \\
& (a^4 - 5*a^3*b)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 4*(2*a^4 - 30*a^3*b - \\
& 30*a^2*b^2 + 38*a*b^3 + 45*b^4 + 3*(a^4 - 5*a^3*b)*d*x)*\cosh(d*x + c)^4 + \\
& 4*(315*(a^4 + a^3*b)*d*x*\cosh(d*x + c)^6 - 105*(4*a^4 + 8*a^3*b - 7*a*b^3 - \\
& 5*b^4 + (a^4 + 5*a^3*b)*d*x)*\cosh(d*x + c)^4 + 2*a^4 - 30*a^3*b - 30*a^2*b \\
& ^2 + 38*a*b^3 + 45*b^4 + 3*(a^4 - 5*a^3*b)*d*x - 45*(2*a^4 - 2*a^3*b - 2*a^ \\
& 2*b^2 + 9*a*b^3 + 10*b^4 + (a^4 - 5*a^3*b)*d*x)*\cosh(d*x + c)^2*\sinh(d*x + \\
& c)^4 - 16*a^4 - 24*a^3*b + 24*a^2*b^2 + 62*a*b^3 + 30*b^4 + 16*(45*(a^4 + \\
& a^3*b)*d*x*\cosh(d*x + c)^7 - 21*(4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + \\
& 5*a^3*b)*d*x)*\cosh(d*x + c)^5 - 15*(2*a^4 - 2*a^3*b - 2*a^2*b^2 + 9*a*b^3 \\
& + 10*b^4 + (a^4 - 5*a^3*b)*d*x)*\cosh(d*x + c)^3 + (2*a^4 - 30*a^3*b - 30*a^ \\
& 2*b^2 + 38*a*b^3 + 45*b^4 + 3*(a^4 - 5*a^3*b)*d*x)*\cosh(d*x + c))*\sinh(d*x \\
& + c)^3 - 6*(a^4 + a^3*b)*d*x - 2*(4*a^4 - 20*a^3*b - 4*a^2*b^2 + 74*a*b^3 + \\
& 60*b^4 - 3*(a^4 + 5*a^3*b)*d*x)*\cosh(d*x + c)^2 + 2*(135*(a^4 + a^3*b)*d*x \\
& *\cosh(d*x + c)^8 - 84*(4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)* \\
& d*x)*\cosh(d*x + c)^6 - 90*(2*a^4 - 2*a^3*b - 2*a^2*b^2 + 9*a*b^3 + 10*b^4 + \\
& (a^4 - 5*a^3*b)*d*x)*\cosh(d*x + c)^4 - 4*a^4 + 20*a^3*b + 4*a^2*b^2 - 74*a \\
& *b^3 - 60*b^4 + 3*(a^4 + 5*a^3*b)*d*x + 12*(2*a^4 - 30*a^3*b - 30*a^2*b^2 + \\
& 38*a*b^3 + 45*b^4 + 3*(a^4 - 5*a^3*b)*d*x)*\cosh(d*x + c)^2*\sinh(d*x + c)^ \\
& 2 + 3*((7*a^2*b^2 + 12*a*b^3 + 5*b^4)*\cosh(d*x + c)^10 + 10*(7*a^2*b^2 + 12 \\
& *a*b^3 + 5*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^9 + (7*a^2*b^2 + 12*a*b^3 + 5*b \\
& ^4)*\sinh(d*x + c)^10 - (7*a^2*b^2 + 40*a*b^3 + 25*b^4)*\cosh(d*x + c)^8 - (7 \\
& *a^2*b^2 + 40*a*b^3 + 25*b^4 - 45*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*\cosh(d*x + \\
& c)^2)*\sinh(d*x + c)^8 + 8*(15*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*\cosh(d*x + c) \\
& ^3 - (7*a^2*b^2 + 40*a*b^3 + 25*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^7 - 2*(7* \\
& a^2*b^2 - 30*a*b^3 - 25*b^4)*\cosh(d*x + c)^6 + 2*(105*(7*a^2*b^2 + 12*a*b^3 \\
& + 5*b^4)*\cosh(d*x + c)^4 - 7*a^2*b^2 + 30*a*b^3 + 25*b^4 - 14*(7*a^2*b^2 + \\
& 40*a*b^3 + 25*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 4*(63*(7*a^2*b^2 + 1 \\
& 2*a*b^3 + 5*b^4)*\cosh(d*x + c)^5 - 14*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*\cosh(\\
& d*x + c)^3 - 3*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*\cosh(d*x + c))*\sinh(d*x + c) \\
& ^5 + 2*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*\cosh(d*x + c)^4 + 2*(105*(7*a^2*b^2 \\
& + 12*a*b^3 + 5*b^4)*\cosh(d*x + c)^6 - 35*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*\c \\
& osh(d*x + c)^4 + 7*a^2*b^2 - 30*a*b^3 - 25*b^4 - 15*(7*a^2*b^2 - 30*a*b^3 - \\
& 25*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 - 7*a^2*b^2 - 12*a*b^3 - 5*b^4 + 8 \\
& *(15*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*\cosh(d*x + c)^7 - 7*(7*a^2*b^2 + 40*a*b \\
& ^3 + 25*b^4)*\cosh(d*x + c)^5 - 5*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*\cosh(d*x + \\
& c)^3 + (7*a^2*b^2 - 30*a*b^3 - 25*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + (7 \\
& *a^2*b^2 + 40*a*b^3 + 25*b^4)*\cosh(d*x + c)^2 + (45*(7*a^2*b^2 + 12*a*b^3 + \\
& 5*b^4)*\cosh(d*x + c)^8 - 28*(7*a^2*b^2 + 40*a*b^3 + 25*b^4)*\cosh(d*x + c)^ \\
& 6 - 30*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*\cosh(d*x + c)^4 + 7*a^2*b^2 + 40*a*b \\
& ^3 + 25*b^4 + 12*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*\cosh(d*x + c)^2)*\sinh(d*x \\
& + c)^2 + 2*(5*(7*a^2*b^2 + 12*a*b^3 + 5*b^4)*\cosh(d*x + c)^9 - 4*(7*a^2*b^2 \\
& + 40*a*b^3 + 25*b^4)*\cosh(d*x + c)^7 - 6*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*\c \\
& osh(d*x + c)^5 + 4*(7*a^2*b^2 - 30*a*b^3 - 25*b^4)*\cosh(d*x + c)^3 + (7*a^2 \\
& *b^2 + 40*a*b^3 + 25*b^4)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{b/a}*\arctan(1/ \\
& 2*((a + b)*\cosh(d*x + c)^2 + 2*(a + b)*\cosh(d*x + c)*\sinh(d*x + c) + (a + b) \\
&)*\sinh(d*x + c)^2 + a - b)*\sqrt{b/a}/b) + 4*(15*(a^4 + a^3*b)*d*x*\cosh(d*x \\
& + c)^9 - 12*(4*a^4 + 8*a^3*b - 7*a*b^3 - 5*b^4 + (a^4 + 5*a^3*b)*d*x)*\cosh(\\
& d*x + c)^7 - 18*(2*a^4 - 2*a^3*b - 2*a^2*b^2 + 9*a*b^3 + 10*b^4 + (a^4 - 5* \\
& a^3*b)*d*x)*\cosh(d*x + c)^5 + 4*(2*a^4 - 30*a^3*b - 30*a^2*b^2 + 38*a*b^3 + \\
& 45*b^4 + 3*(a^4 - 5*a^3*b)*d*x)*\cosh(d*x + c)^3 - (4*a^4 - 20*a^3*b - 4*a^ \\
& 2*b^2 + 74*a*b^3 + 60*b^4 - 3*(a^4 + 5*a^3*b)*d*x)*\cosh(d*x + c))*\sinh(d*x \\
& + c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\cosh(d*x + c)^10 + 10*(a^6 + \\
& 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^9 + (a^6 + 3* \\
& a^5*b + 3*a^4*b^2 + a^3*b^3)*d*\sinh(d*x + c)^10 - (a^6 + 7*a^5*b + 11*a^4*b \\
& ^2 + 5*a^3*b^3)*d*\cosh(d*x + c)^8 + (45*(a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^
\end{aligned}$$

$$\begin{aligned}
& 3) * d * \cosh(dx + c)^2 - (a^6 + 7a^5b + 11a^4b^2 + 5a^3b^3) * d * \sinh(dx + c) \\
& + c)^8 - 2 * (a^6 - 3a^5b - 9a^4b^2 - 5a^3b^3) * d * \cosh(dx + c)^6 + 8 * (\\
& 15 * (a^6 + 3a^5b + 3a^4b^2 + a^3b^3) * d * \cosh(dx + c)^3 - (a^6 + 7a^5b \\
& + 11a^4b^2 + 5a^3b^3) * d * \cosh(dx + c)) * \sinh(dx + c)^7 + 2 * (105 * (a^6 + \\
& 3a^5b + 3a^4b^2 + a^3b^3) * d * \cosh(dx + c)^4 - 14 * (a^6 + 7a^5b + 11 * \\
& a^4b^2 + 5a^3b^3) * d * \cosh(dx + c)^2 - (a^6 - 3a^5b - 9a^4b^2 - 5a^3 \\
& * b^3) * d) * \sinh(dx + c)^6 + 2 * (a^6 - 3a^5b - 9a^4b^2 - 5a^3b^3) * d * \cosh \\
& (dx + c)^4 + 4 * (63 * (a^6 + 3a^5b + 3a^4b^2 + a^3b^3) * d * \cosh(dx + c)^5 \\
& - 14 * (a^6 + 7a^5b + 11a^4b^2 + 5a^3b^3) * d * \cosh(dx + c)^3 - 3 * (a^6 - \\
& 3a^5b - 9a^4b^2 - 5a^3b^3) * d * \cosh(dx + c)) * \sinh(dx + c)^5 + 2 * (105 \\
& * (a^6 + 3a^5b + 3a^4b^2 + a^3b^3) * d * \cosh(dx + c)^6 - 35 * (a^6 + 7a^5 * \\
& b + 11a^4b^2 + 5a^3b^3) * d * \cosh(dx + c)^4 - 15 * (a^6 - 3a^5b - 9a^4b \\
& ^2 - 5a^3b^3) * d * \cosh(dx + c)^2 + (a^6 - 3a^5b - 9a^4b^2 - 5a^3b^3) \\
& * d) * \sinh(dx + c)^4 + (a^6 + 7a^5b + 11a^4b^2 + 5a^3b^3) * d * \cosh(dx + \\
& c)^2 + 8 * (15 * (a^6 + 3a^5b + 3a^4b^2 + a^3b^3) * d * \cosh(dx + c)^7 - 7 * (\\
& a^6 + 7a^5b + 11a^4b^2 + 5a^3b^3) * d * \cosh(dx + c)^5 - 5 * (a^6 - 3a^5 * \\
& b - 9a^4b^2 - 5a^3b^3) * d * \cosh(dx + c)^3 + (a^6 - 3a^5b - 9a^4b^2 - \\
& 5a^3b^3) * d * \cosh(dx + c)) * \sinh(dx + c)^3 + (45 * (a^6 + 3a^5b + 3a^4b \\
& ^2 + a^3b^3) * d * \cosh(dx + c)^8 - 28 * (a^6 + 7a^5b + 11a^4b^2 + 5a^3b^ \\
& 3) * d * \cosh(dx + c)^6 - 30 * (a^6 - 3a^5b - 9a^4b^2 - 5a^3b^3) * d * \cosh(dx \\
& x + c)^4 + 12 * (a^6 - 3a^5b - 9a^4b^2 - 5a^3b^3) * d * \cosh(dx + c)^2 + (\\
& a^6 + 7a^5b + 11a^4b^2 + 5a^3b^3) * d) * \sinh(dx + c)^2 - (a^6 + 3a^5b \\
& + 3a^4b^2 + a^3b^3) * d + 2 * (5 * (a^6 + 3a^5b + 3a^4b^2 + a^3b^3) * d * \c \\
& sh(dx + c)^9 - 4 * (a^6 + 7a^5b + 11a^4b^2 + 5a^3b^3) * d * \cosh(dx + c)^ \\
& 7 - 6 * (a^6 - 3a^5b - 9a^4b^2 - 5a^3b^3) * d * \cosh(dx + c)^5 + 4 * (a^6 - \\
& 3a^5b - 9a^4b^2 - 5a^3b^3) * d * \cosh(dx + c)^3 + (a^6 + 7a^5b + 11a^ \\
& 4b^2 + 5a^3b^3) * d * \cosh(dx + c)) * \sinh(dx + c)]
\end{aligned}$$

giac [A] time = 0.44, size = 281, normalized size = 1.77

$$\frac{3(7ab^3 + 5b^4) \arctan\left(\frac{ae^{2dx+2c} + be^{2dx+2c} + a - b}{2\sqrt{ab}}\right)}{(a^5 + 2a^4b + a^3b^2)\sqrt{ab}} + \frac{6(dx+c)}{a^2 + 2ab + b^2} - \frac{6(ab^3e^{2dx+2c} - b^4e^{2dx+2c} + ab^3 + b^4)}{(a^5 + 2a^4b + a^3b^2)(ae^{4dx+4c} + be^{4dx+4c} + 2ae^{2dx+2c} - 2be^{2dx+2c} + a + b)}$$

$6d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(dx+c)^4/(a+b*tanh(dx+c)^2)^2,x, algorithm="giac")

[Out]
$$\begin{aligned}
& 1/6 * (3 * (7 * a * b^3 + 5 * b^4) * \arctan(1/2 * (a * e^{(2 * dx + 2 * c)} + b * e^{(2 * dx + 2 * c)} \\
& + a - b) / \sqrt{a * b})) / ((a^5 + 2 * a^4 * b + a^3 * b^2) * \sqrt{a * b}) + 6 * (dx + c) / (a^ \\
& 2 + 2 * a * b + b^2) - 6 * (a * b^3 * e^{(2 * dx + 2 * c)} - b^4 * e^{(2 * dx + 2 * c)} + a * b^3 + \\
& b^4) / ((a^5 + 2 * a^4 * b + a^3 * b^2) * (a * e^{(4 * dx + 4 * c)} + b * e^{(4 * dx + 4 * c)} + 2 \\
& * a * e^{(2 * dx + 2 * c)} - 2 * b * e^{(2 * dx + 2 * c)} + a + b)) - 8 * (3 * a * e^{(4 * dx + 4 * c)} \\
& - 3 * b * e^{(4 * dx + 4 * c)} - 3 * a * e^{(2 * dx + 2 * c)} + 6 * b * e^{(2 * dx + 2 * c)} + 2 * a - \\
& 3 * b) / (a^3 * (e^{(2 * dx + 2 * c)} - 1)^3) / d
\end{aligned}$$

maple [B] time = 0.47, size = 1137, normalized size = 7.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(dx+c)^4/(a+b*tanh(dx+c)^2)^2,x)

[Out]
$$\begin{aligned}
& -1/24/d/a^2 * \tanh(1/2 * dx + 1/2 * c)^3 - 5/8/d/a^2 * \tanh(1/2 * dx + 1/2 * c) + 1/d/a^3 * \tan \\
& h(1/2 * dx + 1/2 * c) * b - 1/d/(a+b)^2 * \ln(\tanh(1/2 * dx + 1/2 * c) - 1) + 1/d/(a+b)^2 * \ln(\tan \\
& h(1/2 * dx + 1/2 * c) + 1) + 1/d * b^3/(a+b)^2/a^2/(\tanh(1/2 * dx + 1/2 * c)^4 * a + 2 * \tanh(1/2 \\
& * dx + 1/2 * c)^2 * a + 4 * \tanh(1/2 * dx + 1/2 * c)^2 * b + a) * \tanh(1/2 * dx + 1/2 * c)^3 + 1/d * b^4/ \\
& (a+b)^2/a^3/(\tanh(1/2 * dx + 1/2 * c)^4 * a + 2 * \tanh(1/2 * dx + 1/2 * c)^2 * a + 4 * \tanh(1/2 * d \\
& * x + 1/2 * c)^2 * b + a) * \tanh(1/2 * dx + 1/2 * c)^3 + 1/d * b^3/(a+b)^2/a^2/(\tanh(1/2 * dx + 1/ \\
& 2 * c)^4 * a + 2 * \tanh(1/2 * dx + 1/2 * c)^2 * a + 4 * \tanh(1/2 * dx + 1/2 * c)^2 * b + a) * \tanh(1/2 * d
\end{aligned}$$

$$\begin{aligned}
& x+1/2*c)+1/d*b^4/(a+b)^2/a^3/(\tanh(1/2*d*x+1/2*c))^4*a+2*\tanh(1/2*d*x+1/2*c) \\
& ^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)*\tanh(1/2*d*x+1/2*c)-7/2/d*b^3/(a+b)^2/a/ \\
& (b*(a+b))^{(1/2)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1 \\
& /2*c)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)})+7/2/d*b^3/(a+b)^2/a^2/((2*(b*(a+ \\
& b))^{(1/2)}-a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)} \\
& -a-2*b)*a)^{(1/2)})-6/d*b^4/(a+b)^2/a^2/(b*(a+b))^{(1/2)/((2*(b*(a+b))^{(1/2)}-a \\
& -2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}) \\
& -7/2/d*b^3/(a+b)^2/a/(b*(a+b))^{(1/2)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+ \\
& 1/2*c)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)})-7/2/d \\
& *b^3/(a+b)^2/a^2/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+ \\
& 1/2*c)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)})-6/d*b^4/(a+b)^2/a^2/(b*(a+b))^{(\\
& 1/2)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(\\
& b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)})+5/2/d*b^4/(a+b)^2/a^3/((2*(b*(a+b))^{(1/2)}-a \\
& -2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}) \\
& -5/2/d*b^5/(a+b)^2/a^3/(b*(a+b))^{(1/2)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)}-a-2*b)*a)^{(1/2)}) \\
& -5/2/d*b^4/(a+b)^2/a^3/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d \\
& *x+1/2*c)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)})-5/2/d*b^5/(a+b)^2/a^3/(b*(a+ \\
& b))^{(1/2)/((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/ \\
& ((2*(b*(a+b))^{(1/2)}+a+2*b)*a)^{(1/2)})-1/24/d/a^2/\tanh(1/2*d*x+1/2*c)^3-5/8/d \\
& /a^2/\tanh(1/2*d*x+1/2*c)+1/d/a^3/\tanh(1/2*d*x+1/2*c)*b
\end{aligned}$$

maxima [B] time = 1.08, size = 2345, normalized size = 14.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c))^2,x, algorithm="maxima")

[Out]
$$\begin{aligned}
& -1/4*(a^2*b - a*b^2 - b^3)*\log((a + b)*e^{(4*d*x + 4*c)} + 2*(a - b)*e^{(2*d*x \\
& + 2*c)} + a + b)/((a^5 + 2*a^4*b + a^3*b^2)*d) + 1/4*(a^2*b - a*b^2 - b^3)* \\
& \log(2*(a - b)*e^{(-2*d*x - 2*c)} + (a + b)*e^{(-4*d*x - 4*c)} + a + b)/((a^5 + \\
& 2*a^4*b + a^3*b^2)*d) + 1/32*(3*a^3*b - 29*a^2*b^2 - 11*a*b^3 + 5*b^4)*\arct \\
& \text{an}(1/2*((a + b)*e^{(2*d*x + 2*c)} + a - b)/\text{sqrt}(a*b))/((a^5 + 2*a^4*b + a^3*b \\
& ^2)*\text{sqrt}(a*b)*d) - 1/32*(3*a^3*b - 29*a^2*b^2 - 11*a*b^3 + 5*b^4)*\arctan(1/ \\
& 2*((a + b)*e^{(-2*d*x - 2*c)} + a - b)/\text{sqrt}(a*b))/((a^5 + 2*a^4*b + a^3*b^2)* \\
& \text{sqrt}(a*b)*d) + 1/48*(44*a^4 + 117*a^3*b + 111*a^2*b^2 + 23*a*b^3 - 15*b^4 + \\
& 3*(24*a^4 + 69*a^3*b + 45*a^2*b^2 + 27*a*b^3 - 5*b^4)*e^{(8*d*x + 8*c)} + 6* \\
& (6*a^4 - 31*a^3*b - 50*a^2*b^2 - 51*a*b^3 + 10*b^4)*e^{(6*d*x + 6*c)} - 2*(50 \\
& *a^4 - 78*a^3*b - 225*a^2*b^2 - 196*a*b^3 + 45*b^4)*e^{(4*d*x + 4*c)} - 2*(10 \\
& *a^4 + 115*a^3*b + 182*a^2*b^2 + 95*a*b^3 - 30*b^4)*e^{(2*d*x + 2*c)})/((a^6 \\
& + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 - (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*e^{(\\
& 10*d*x + 10*c)} + (a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a^3*b^3)*e^{(8*d*x + 8*c)} + \\
& 2*(a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*e^{(6*d*x + 6*c)} - 2*(a^6 - 3*a^5 \\
& *b - 9*a^4*b^2 - 5*a^3*b^3)*e^{(4*d*x + 4*c)} - (a^6 + 7*a^5*b + 11*a^4*b^2 + \\
& 5*a^3*b^3)*e^{(2*d*x + 2*c)})*d) - 1/48*(44*a^4 + 117*a^3*b + 111*a^2*b^2 + \\
& 23*a*b^3 - 15*b^4 - 2*(10*a^4 + 115*a^3*b + 182*a^2*b^2 + 95*a*b^3 - 30*b^4 \\
&)*e^{(-2*d*x - 2*c)} - 2*(50*a^4 - 78*a^3*b - 225*a^2*b^2 - 196*a*b^3 + 45*b^4 \\
&)*e^{(-4*d*x - 4*c)} + 6*(6*a^4 - 31*a^3*b - 50*a^2*b^2 - 51*a*b^3 + 10*b^4) \\
& *e^{(-6*d*x - 6*c)} + 3*(24*a^4 + 69*a^3*b + 45*a^2*b^2 + 27*a*b^3 - 5*b^4)*e \\
& ^{(-8*d*x - 8*c)})/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 - (a^6 + 7*a^5*b + 1 \\
& 1*a^4*b^2 + 5*a^3*b^3)*e^{(-2*d*x - 2*c)} - 2*(a^6 - 3*a^5*b - 9*a^4*b^2 - 5* \\
& a^3*b^3)*e^{(-4*d*x - 4*c)} + 2*(a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*e^{(-6 \\
& *d*x - 6*c)} + (a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a^3*b^3)*e^{(-8*d*x - 8*c)} - (\\
& a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*e^{(-10*d*x - 10*c)})*d) + 1/12*(8*a^3 + \\
& 7*a^2*b - 16*a*b^2 - 15*b^3 + 3*(8*a^3 + 11*a^2*b + 6*a*b^2 - 5*b^3)*e^{(8* \\
& d*x + 8*c)} + 6*(4*a^3 - 7*a^2*b - 13*a*b^2 + 10*b^3)*e^{(6*d*x + 6*c)} - 2*(8 \\
& *a^3 - 44*a^2*b - 43*a*b^2 + 45*b^3)*e^{(4*d*x + 4*c)} - 2*(4*a^3 + 27*a^2*b \\
& + 5*a*b^2 - 30*b^3)*e^{(2*d*x + 2*c)})/((a^5 + 2*a^4*b + a^3*b^2 - (a^5 + 2*a \\
& ^4*b + a^3*b^2)*e^{(10*d*x + 10*c)} + (a^5 + 6*a^4*b + 5*a^3*b^2)*e^{(8*d*x +
\end{aligned}$$

$8*c) + 2*(a^5 - 4*a^4*b - 5*a^3*b^2)*e^{(6*d*x + 6*c)} - 2*(a^5 - 4*a^4*b - 5*a^3*b^2)*e^{(4*d*x + 4*c)} - (a^5 + 6*a^4*b + 5*a^3*b^2)*e^{(2*d*x + 2*c))*d$
 $- 1/12*(8*a^3 + 7*a^2*b - 16*a*b^2 - 15*b^3 - 2*(4*a^3 + 27*a^2*b + 5*a*b^2 - 30*b^3))*e^{(-2*d*x - 2*c)} - 2*(8*a^3 - 44*a^2*b - 43*a*b^2 + 45*b^3)*e^{(-4*d*x - 4*c)} + 6*(4*a^3 - 7*a^2*b - 13*a*b^2 + 10*b^3)*e^{(-6*d*x - 6*c)} + 3*(8*a^3 + 11*a^2*b + 6*a*b^2 - 5*b^3)*e^{(-8*d*x - 8*c))/((a^5 + 2*a^4*b + a^3*b^2 - (a^5 + 6*a^4*b + 5*a^3*b^2)*e^{(-2*d*x - 2*c)} - 2*(a^5 - 4*a^4*b - 5*a^3*b^2)*e^{(-4*d*x - 4*c)} + 2*(a^5 - 4*a^4*b - 5*a^3*b^2)*e^{(-6*d*x - 6*c)} + (a^5 + 6*a^4*b + 5*a^3*b^2)*e^{(-8*d*x - 8*c)} - (a^5 + 2*a^4*b + a^3*b^2)*e^{(-10*d*x - 10*c)))*d) + 1/8*(4*a^2 + 19*a*b + 15*b^2 - 2*(2*a^2 + 13*a*b + 30*b^2))*e^{(-2*d*x - 2*c)} - 2*(10*a^2 - 2*a*b - 45*b^2)*e^{(-4*d*x - 4*c)} - 6*(2*a^2 + a*b + 10*b^2)*e^{(-6*d*x - 6*c)} + 3*(3*a*b + 5*b^2)*e^{(-8*d*x - 8*c))/((a^4 + a^3*b - (a^4 + 5*a^3*b)*e^{(-2*d*x - 2*c)} - 2*(a^4 - 5*a^3*b)*e^{(-4*d*x - 4*c)} + 2*(a^4 - 5*a^3*b)*e^{(-6*d*x - 6*c)} + (a^4 + 5*a^3*b)*e^{(-8*d*x - 8*c)} - (a^4 + a^3*b)*e^{(-10*d*x - 10*c)))*d) + 1/2*b*log((a + b)*e^{(4*d*x + 4*c)} + 2*(a - b)*e^{(2*d*x + 2*c)} + a + b)/(a^3*d) - 1/2*b*log(2*(a - b)*e^{(-2*d*x - 2*c)} + (a + b)*e^{(-4*d*x - 4*c)} + a + b)/(a^3*d) + 1/2*(a - b)*log(e^{(2*d*x + 2*c)} - 1)/(a^3*d) - b*log(e^{(2*d*x + 2*c)} - 1)/(a^3*d) - 1/2*(a - b)*log(e^{(-2*d*x - 2*c)} - 1)/(a^3*d) + b*log(e^{(-2*d*x - 2*c)} - 1)/(a^3*d) - 1/8*(3*a*b - 5*b^2)*arctan(1/2*((a + b)*e^{(2*d*x + 2*c)} + a - b)/sqrt(a*b))/(sqrt(a*b)*a^3*d) - 3/16*(3*a*b + 5*b^2)*arctan(1/2*((a + b)*e^{(-2*d*x - 2*c)} + a - b)/sqrt(a*b))/(sqrt(a*b)*a^3*d) + 1/8*(3*a*b - 5*b^2)*arctan(1/2*((a + b)*e^{(-2*d*x - 2*c)} + a - b)/sqrt(a*b))/(sqrt(a*b)*a^3*d)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(c + dx)^4}{(b \tanh(c + dx)^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2)^2, x)

[Out] int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(c + dx)}{(a + b \tanh^2(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2, x)

[Out] Integral(coth(c + d*x)**4/(a + b*tanh(c + d*x)**2)**2, x)

$$3.190 \quad \int \frac{\tanh^6(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=144

$$\frac{\sqrt{a} (3a^2 + 10ab + 15b^2) \tan^{-1} \left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}} \right)}{8b^{5/2}d(a+b)^3} + \frac{a(3a+7b) \tanh(c+dx)}{8b^2d(a+b)^2(a+b \tanh^2(c+dx))} + \frac{a \tanh^3(c+dx)}{4bd(a+b)(a+b \tanh^2(c+dx))}$$

[Out] x/(a+b)^3-1/8*(3*a^2+10*a*b+15*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*a^(1/2)/b^(5/2)/(a+b)^3/d+1/4*a*tanh(d*x+c)^3/b/(a+b)/d/(a+b*tanh(d*x+c)^2)^2+1/8*a*(3*a+7*b)*tanh(d*x+c)/b^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)

Rubi [A] time = 0.21, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3670, 470, 578, 522, 206, 205}

$$\frac{\sqrt{a} (3a^2 + 10ab + 15b^2) \tan^{-1} \left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}} \right)}{8b^{5/2}d(a+b)^3} + \frac{a(3a+7b) \tanh(c+dx)}{8b^2d(a+b)^2(a+b \tanh^2(c+dx))} + \frac{a \tanh^3(c+dx)}{4bd(a+b)(a+b \tanh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] x/(a + b)^3 - (Sqrt[a]*(3*a^2 + 10*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*b^(5/2)*(a + b)^3*d) + (a*Tanh[c + d*x]^3)/(4*b*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) + (a*(3*a + 7*b)*Tanh[c + d*x])/(8*b^2*(a + b)^2*d*(a + b*Tanh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 578

```
Int[((g_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.)*((e_.) + (f_.)*(x_.)^(n_.)), x_Symbol] := Simp[(g^(n - 1)*(b*e - a*f)*(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_.)])^(m_.)*((a_.) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))
```

Rubi steps

$$\begin{aligned} \int \frac{\tanh^6(c + dx)}{(a + b \tanh^2(c + dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)(a+bx^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{a \tanh^3(c + dx)}{4b(a + b)d (a + b \tanh^2(c + dx))^2} - \frac{\text{Subst}\left(\int \frac{x^2(3a+(-3a-4b)x^2)}{(1-x^2)(a+bx^2)^2} dx, x, \tanh(c + dx)\right)}{4b(a + b)d} \\ &= \frac{a \tanh^3(c + dx)}{4b(a + b)d (a + b \tanh^2(c + dx))^2} + \frac{a(3a + 7b) \tanh(c + dx)}{8b^2(a + b)^2d (a + b \tanh^2(c + dx))} - \frac{a \tanh^3(c + dx)}{4b(a + b)d (a + b \tanh^2(c + dx))^2} \\ &= \frac{a \tanh^3(c + dx)}{4b(a + b)d (a + b \tanh^2(c + dx))^2} + \frac{a(3a + 7b) \tanh(c + dx)}{8b^2(a + b)^2d (a + b \tanh^2(c + dx))} + \frac{a \tanh^3(c + dx)}{4b(a + b)d (a + b \tanh^2(c + dx))^2} \\ &= \frac{x}{(a + b)^3} - \frac{\sqrt{a} (3a^2 + 10ab + 15b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8b^{5/2}(a + b)^3d} + \frac{a \tanh^3(c + dx)}{4b(a + b)d (a + b \tanh^2(c + dx))^2} \end{aligned}$$

Mathematica [A] time = 1.29, size = 144, normalized size = 1.00

$$\frac{\sqrt{a}(3a^2+10ab+15b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right) - \frac{4a^2(a+b) \sinh(2(c+dx))}{b((a+b) \cosh(2(c+dx))+a-b)^2} + \frac{3a(a+b)(a+3b) \sinh(2(c+dx))}{b^2((a+b) \cosh(2(c+dx))+a-b)} + 8(c + dx)}{8d(a + b)^3}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (8*(c + d*x) - (Sqrt[a]*(3*a^2 + 10*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/b^(5/2) - (4*a^2*(a + b)*Sinh[2*(c + d*x)]/(b*(a - b + (a + b)*Cosh[2*(c + d*x)]^2) + (3*a*(a + b)*(a + 3*b)*Sinh[2*(c + d*x)]/(b^2*(a - b + (a + b)*Cosh[2*(c + d*x)]))))/(8*(a + b)^3*d)

fricas [B] time = 0.58, size = 7528, normalized size = 52.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/16*(16*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh(d*x + c)^8 + 128*(a^2*b^2 + 2* \\ & a*b^3 + b^4)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 16*(a^2*b^2 + 2*a*b^3 + b^4) \\ & *d*x*sinh(d*x + c)^8 - 4*(3*a^4 + 13*a^3*b + a^2*b^2 - 9*a*b^3 - 16*(a^2* \\ & b^2 - b^4)*d*x)*cosh(d*x + c)^6 + 4*(112*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh \\ & (d*x + c)^2 - 3*a^4 - 13*a^3*b - a^2*b^2 + 9*a*b^3 + 16*(a^2*b^2 - b^4)*d*x \\ &)*sinh(d*x + c)^6 + 8*(112*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh(d*x + c)^3 - \\ & 3*(3*a^4 + 13*a^3*b + a^2*b^2 - 9*a*b^3 - 16*(a^2*b^2 - b^4)*d*x)*cosh(d*x \\ & + c))*sinh(d*x + c)^5 - 4*(9*a^4 + 21*a^3*b - 9*a^2*b^2 + 27*a*b^3 - 8*(3*a^2*b^2 \\ & - 2*a*b^3 + 3*b^4)*d*x)*cosh(d*x + c)^4 + 4*(280*(a^2*b^2 + 2*a*b^3 \\ & + b^4)*d*x*cosh(d*x + c)^4 - 9*a^4 - 21*a^3*b + 9*a^2*b^2 - 27*a*b^3 + 8*(3 \\ & *a^2*b^2 - 2*a*b^3 + 3*b^4)*d*x - 15*(3*a^4 + 13*a^3*b + a^2*b^2 - 9*a*b^3 \\ & - 16*(a^2*b^2 - b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 12*a^4 - 60*a^3*b \\ & - 84*a^2*b^2 - 36*a*b^3 + 16*(56*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh(d*x \\ & + c)^5 - 5*(3*a^4 + 13*a^3*b + a^2*b^2 - 9*a*b^3 - 16*(a^2*b^2 - b^4)*d*x) \\ & *cosh(d*x + c)^3 - (9*a^4 + 21*a^3*b - 9*a^2*b^2 + 27*a*b^3 - 8*(3*a^2*b^2 \\ & - 2*a*b^3 + 3*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 16*(a^2*b^2 + 2*a* \\ & b^3 + b^4)*d*x - 4*(9*a^4 + 23*a^3*b - 13*a^2*b^2 - 27*a*b^3 - 16*(a^2*b^2 \\ & - b^4)*d*x)*cosh(d*x + c)^2 + 4*(112*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cosh(d*x \\ & + c)^6 - 15*(3*a^4 + 13*a^3*b + a^2*b^2 - 9*a*b^3 - 16*(a^2*b^2 - b^4)*d*x) \\ &)*cosh(d*x + c)^4 - 9*a^4 - 23*a^3*b + 13*a^2*b^2 + 27*a*b^3 + 16*(a^2*b^2 \\ & - b^4)*d*x - 6*(9*a^4 + 21*a^3*b - 9*a^2*b^2 + 27*a*b^3 - 8*(3*a^2*b^2 - 2* \\ & a*b^3 + 3*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^4 + 16*a^3*b + \\ & 38*a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)^8 + 8*(3*a^4 + 16*a^3*b + 38 \\ & *a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4 + 16*a^3*b \\ & + 38*a^2*b^2 + 40*a*b^3 + 15*b^4)*sinh(d*x + c)^8 + 4*(3*a^4 + 10*a^3*b \\ & b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*x + c)^6 + 4*(3*a^4 + 10*a^3*b + \\ & 12*a^2*b^2 - 10*a*b^3 - 15*b^4 + 7*(3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 \\ & + 15*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4 + 16*a^3*b + 38 \\ & *a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)^3 + 3*(3*a^4 + 10*a^3*b + 12*a^2*b^2 \\ & - 10*a*b^3 - 15*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(9*a^4 + 24*a^3*b \\ & + 34*a^2*b^2 + 45*b^4)*cosh(d*x + c)^4 + 2*(35*(3*a^4 + 16*a^3*b + 38* \\ & a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)^4 + 9*a^4 + 24*a^3*b + 34*a^2*b^2 \\ & + 45*b^4 + 30*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*x \\ & + c)^2)*sinh(d*x + c)^4 + 3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4 \\ & + 8*(7*(3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c) \\ & ^5 + 10*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*x + c)^3 \\ & + (9*a^4 + 24*a^3*b + 34*a^2*b^2 + 45*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 \\ & + 4*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*x + c)^2 + 4 \\ & *(7*(3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)^6 + 1 \\ & 5*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*x + c)^4 + 3*a^4 \\ & + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4 + 3*(9*a^4 + 24*a^3*b + 34*a^2*b^2 \\ & + 45*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^4 + 16*a^3*b + \\ & 38*a^2*b^2 + 40*a*b^3 + 15*b^4)*cosh(d*x + c)^7 + 3*(3*a^4 + 10*a^3*b + 12* \\ & a^2*b^2 - 10*a*b^3 - 15*b^4)*cosh(d*x + c)^5 + (9*a^4 + 24*a^3*b + 34*a^2*b^2 \\ & + 45*b^4)*cosh(d*x + c)^3 + (3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - \\ & 15*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a/b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 \\ & + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 \\ & + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 \\ & + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c)) \\ & *sinh(d*x + c) - 4*((a*b + b^2)*cosh(d*x + c)^2 + 2*(a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) \\ & + (a*b + b^2)*sinh(d*x + c)^2 + a*b - b^2)*sqrt(-a/b))/((a \end{aligned}$$

$$\begin{aligned}
& 2*b^2 + 40*a*b^3 + 15*b^4)*\cosh(d*x + c)^8 + 8*(3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4)*\sinh(d*x + c)^8 + 4*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*\cosh(d*x + c)^6 + 4*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4 + 7*(3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4))*\cosh(d*x + c)^2*\sinh(d*x + c)^6 + 8*(7*(3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4))*\cosh(d*x + c)^3 + 3*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(9*a^4 + 24*a^3*b + 34*a^2*b^2 + 45*b^4)*\cosh(d*x + c)^4 + 2*(35*(3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4))*\cosh(d*x + c)^4 + 9*a^4 + 24*a^3*b + 34*a^2*b^2 + 45*b^4 + 30*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4 + 8*(7*(3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4))*\cosh(d*x + c)^5 + 10*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4)*\cosh(d*x + c)^3 + (9*a^4 + 24*a^3*b + 34*a^2*b^2 + 45*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4))*\cosh(d*x + c)^2 + 4*(7*(3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4))*\cosh(d*x + c)^6 + 15*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4))*\cosh(d*x + c)^4 + 3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4 + 3*(9*a^4 + 24*a^3*b + 34*a^2*b^2 + 45*b^4))*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((3*a^4 + 16*a^3*b + 38*a^2*b^2 + 40*a*b^3 + 15*b^4))*\cosh(d*x + c)^7 + 3*(3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4))*\cosh(d*x + c)^5 + (9*a^4 + 24*a^3*b + 34*a^2*b^2 + 45*b^4))*\cosh(d*x + c)^3 + (3*a^4 + 10*a^3*b + 12*a^2*b^2 - 10*a*b^3 - 15*b^4))*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{a/b)*\arctan(1/2*((a + b)*\cosh(d*x + c)^2 + 2*(a + b)*\cosh(d*x + c)*\sinh(d*x + c) + (a + b)*\sinh(d*x + c)^2 + a - b)*\sqrt{a/b)/a) + 4*(16*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*\cosh(d*x + c)^7 - 3*(3*a^4 + 13*a^3*b + a^2*b^2 - 9*a*b^3 - 16*(a^2*b^2 - b^4)*d*x)*\cosh(d*x + c)^5 - 2*(9*a^4 + 21*a^3*b - 9*a^2*b^2 + 27*a*b^3 - 8*(3*a^2*b^2 - 2*a*b^3 + 3*b^4)*d*x)*\cosh(d*x + c)^3 - (9*a^4 + 23*a^3*b - 13*a^2*b^2 - 27*a*b^3 - 16*(a^2*b^2 - b^4)*d*x)*\cosh(d*x + c))*\sinh(d*x + c))/((a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*\cosh(d*x + c)^8 + 8*(a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*\sinh(d*x + c)^8 + 4*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d*\cosh(d*x + c)^6 + 4*(7*(a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*\cosh(d*x + c)^2 + (a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d)*\sinh(d*x + c)^6 + 2*(3*a^5*b^2 + 7*a^4*b^3 + 6*a^3*b^4 + 6*a^2*b^5 + 7*a*b^6 + 3*b^7)*d*\cosh(d*x + c)^4 + 8*(7*(a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*\cosh(d*x + c)^3 + 3*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*\cosh(d*x + c)^4 + 30*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d*\cosh(d*x + c)^2 + (3*a^5*b^2 + 7*a^4*b^3 + 6*a^3*b^4 + 6*a^2*b^5 + 7*a*b^6 + 3*b^7)*d)*\sinh(d*x + c)^4 + 4*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d*\cosh(d*x + c)^2 + 8*(7*(a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*\cosh(d*x + c)^5 + 10*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d*\cosh(d*x + c)^3 + (3*a^5*b^2 + 7*a^4*b^3 + 6*a^3*b^4 + 6*a^2*b^5 + 7*a*b^6 + 3*b^7)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*\cosh(d*x + c)^6 + 15*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d*\cosh(d*x + c)^4 + 3*(3*a^5*b^2 + 7*a^4*b^3 + 6*a^3*b^4 + 6*a^2*b^5 + 7*a*b^6 + 3*b^7)*d*\cosh(d*x + c)^2 + (a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d)*\sinh(d*x + c)^2 + (a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d + 8*((a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*d*\cosh(d*x + c)^7 + 3*(a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d*\cosh(d*x + c)^5 + (3*a^5*b^2 + 7*a^4*b^3 + 6*a^3*b^4 + 6*a^2*b^5 + 7*a*b^6 + 3*b^7)*d*\cosh(d*x + c)^3 + (a^5*b^2 + 3*a^4*b^3 + 2*a^3*b^4 - 2*a^2*b^5 - 3*a*b^6 - b^7)*d*\cosh(d*x + c))*\sinh(d*x + c))]
\end{aligned}$$

giac [B] time = 0.60, size = 408, normalized size = 2.83

$$\frac{(3a^3+10a^2b+15ab^2)\arctan\left(\frac{ae^{(2dx+2c)+be^{(2dx+2c)+a-b}}}{2\sqrt{ab}}\right)}{(a^3b^2+3a^2b^3+3ab^4+b^5)\sqrt{ab}} - \frac{8(dx+c)}{a^3+3a^2b+3ab^2+b^3} + \frac{2(3a^4e^{(6dx+6c)}+13a^3be^{(6dx+6c)}+a^2b^2e^{(6dx+6c)}-9ab^3e^{(6dx+6c)})}{(a^3b^2+3a^2b^3+3ab^4+b^5)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out]
$$-1/8*((3a^3 + 10a^2b + 15ab^2)*\arctan(1/2*(a*e^{(2*d*x + 2*c)} + b*e^{(2*d*x + 2*c)} + a - b)/\sqrt{a*b}))/((a^3*b^2 + 3a^2*b^3 + 3a*b^4 + b^5)*\sqrt{a*b}) - 8*(d*x + c)/(a^3 + 3a^2*b + 3a*b^2 + b^3) + 2*(3a^4*e^{(6*d*x + 6*c)} + 13a^3*b*e^{(6*d*x + 6*c)} + a^2*b^2*e^{(6*d*x + 6*c)} - 9a*b^3*e^{(6*d*x + 6*c)} + 9a^4*e^{(4*d*x + 4*c)} + 21a^3*b*e^{(4*d*x + 4*c)} - 9a^2*b^2*e^{(4*d*x + 4*c)} + 27a*b^3*e^{(4*d*x + 4*c)} + 9a^4*e^{(2*d*x + 2*c)} + 23a^3*b*e^{(2*d*x + 2*c)} - 13a^2*b^2*e^{(2*d*x + 2*c)} - 27a*b^3*e^{(2*d*x + 2*c)} + 3a^4 + 15a^3*b + 21a^2*b^2 + 9a*b^3)/((a^3*b^2 + 3a^2*b^3 + 3a*b^4 + b^5)*(a*e^{(4*d*x + 4*c)} + b*e^{(4*d*x + 4*c)} + 2a*e^{(2*d*x + 2*c)} - 2b*e^{(2*d*x + 2*c)} + a + b)^2))/d$$

maple [B] time = 0.10, size = 352, normalized size = 2.44

$$-\frac{\ln(\tanh(dx+c)-1)}{2d(a+b)^3} + \frac{\ln(1+\tanh(dx+c))}{2d(a+b)^3} + \frac{5a^3(\tanh^3(dx+c))}{8d(a+b)^3(a+b(\tanh^2(dx+c)))^2} + \frac{7a^2(\tanh^3(dx+c))}{4d(a+b)^3(a+b(\tanh^2(dx+c)))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x)

[Out]
$$-1/2/d/(a+b)^3*\ln(\tanh(d*x+c)-1)+1/2/d/(a+b)^3*\ln(1+\tanh(d*x+c))+5/8/d*a^3/(a+b)^3/(a+b*tanh(d*x+c)^2)^2/b*tanh(d*x+c)^3+7/4/d*a^2/(a+b)^3/(a+b*tanh(d*x+c)^2)^2*tanh(d*x+c)^3+9/8/d*a/(a+b)^3/(a+b*tanh(d*x+c)^2)^2*b*tanh(d*x+c)^3+3/8/d*a^4/(a+b)^3/(a+b*tanh(d*x+c)^2)^2/b^2*tanh(d*x+c)+5/4/d*a^3/(a+b)^3/(a+b*tanh(d*x+c)^2)^2/b*tanh(d*x+c)+7/8/d*a^2/(a+b)^3/(a+b*tanh(d*x+c)^2)^2*tanh(d*x+c)-3/8/d*a^3/(a+b)^3/b^2/(a*b)^(1/2)*arctan(tanh(d*x+c)*b/(a*b)^(1/2))-5/4/d*a^2/(a+b)^3/b/(a*b)^(1/2)*arctan(tanh(d*x+c)*b/(a*b)^(1/2))-15/8/d*a/(a+b)^3/(a*b)^(1/2)*arctan(tanh(d*x+c)*b/(a*b)^(1/2))$$

maxima [B] time = 1.89, size = 3354, normalized size = 23.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out]
$$-1/512*(3a^5 + 25a^4b + 150a^3b^2 - 150a^2b^3 - 25ab^4 - 3b^5)*\arctan(1/2*((a+b)*e^{(2*d*x + 2*c)} + a - b)/\sqrt{a*b}))/((a^5*b^2 + 3a^4*b^3 + 3a^3*b^4 + a^2*b^5)*\sqrt{a*b}*d) + 1/512*(3a^5 + 25a^4b + 150a^3b^2 - 150a^2b^3 - 25ab^4 - 3b^5)*\arctan(1/2*((a+b)*e^{(-2*d*x - 2*c)} + a - b)/\sqrt{a*b}))/((a^5*b^2 + 3a^4*b^3 + 3a^3*b^4 + a^2*b^5)*\sqrt{a*b}*d) - 1/256*(3a^6 + 30a^5b - 99a^4b^2 - 252a^3b^3 - 99a^2b^4 + 30ab^5 + 3b^6 + (3a^6 + 28a^5b - 465a^4b^2 + 465a^2b^4 - 28ab^5 - 3b^6)*e^{(6*d*x + 6*c)} + (9a^6 + 66a^5b - 905a^4b^2 + 1148a^3b^3 - 905a^2b^4 + 66ab^5 + 9b^6)*e^{(4*d*x + 4*c)} + (9a^6 + 68a^5b - 659a^4b^2 + 659a^2b^4 - 68ab^5 - 9b^6)*e^{(2*d*x + 2*c)}))/((a^7*b^2 + 5a^6*b^3 + 10a^5*b^4 + 10a^4*b^5 + 5a^3*b^6 + a^2*b^7 + (a^7*b^2 + 5a^6*b^3 + 10a^5*b^4 + 10a^4*b^5 + 5a^3*b^6 + a^2*b^7)*e^{(8*d*x + 8*c)} + 4*(a^7*b^2$$

$$\begin{aligned}
& + 3*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - 3*a^3*b^6 - a^2*b^7)*e^{(6*d*x + 6*c)} \\
& + 2*(3*a^7*b^2 + 7*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 + 7*a^3*b^6 + 3*a^2*b^7) \\
& *e^{(4*d*x + 4*c)} + 4*(a^7*b^2 + 3*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - 3*a^3*b^6 - a^2*b^7)*e^{(2*d*x + 2*c))*d) + 1/256*(3*a^6 + 30*a^5*b - 99*a^4*b^2 - \\
& 252*a^3*b^3 - 99*a^2*b^4 + 30*a*b^5 + 3*b^6 + (9*a^6 + 68*a^5*b - 659*a^4*b^2 + 659*a^2*b^4 - 68*a*b^5 - 9*b^6)*e^{(-2*d*x - 2*c)} + (9*a^6 + 66*a^5*b - \\
& 905*a^4*b^2 + 1148*a^3*b^3 - 905*a^2*b^4 + 66*a*b^5 + 9*b^6)*e^{(-4*d*x - 4*c)} + (3*a^6 + 28*a^5*b - 465*a^4*b^2 + 465*a^2*b^4 - 28*a*b^5 - 3*b^6)*e^{(-6*d*x - 6*c))/((a^7*b^2 + 5*a^6*b^3 + 10*a^5*b^4 + 10*a^4*b^5 + 5*a^3*b^6 \\
& + a^2*b^7 + 4*(a^7*b^2 + 3*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - 3*a^3*b^6 - a^2*b^7)*e^{(-2*d*x - 2*c)} + 2*(3*a^7*b^2 + 7*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 \\
& + 7*a^3*b^6 + 3*a^2*b^7)*e^{(-4*d*x - 4*c)} + 4*(a^7*b^2 + 3*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - 3*a^3*b^6 - a^2*b^7)*e^{(-6*d*x - 6*c)} + (a^7*b^2 + 5*a^6*b^3 \\
& + 10*a^5*b^4 + 10*a^4*b^5 + 5*a^3*b^6 + a^2*b^7)*e^{(-8*d*x - 8*c))*d) - \\
& 3/128*(3*a^5 + 17*a^4*b + 14*a^3*b^2 - 14*a^2*b^3 - 17*a*b^4 - 3*b^5 + (3*a^5 + 15*a^4*b - 98*a^3*b^2 - 98*a^2*b^3 + 15*a*b^4 + 3*b^5)*e^{(6*d*x + 6*c)} \\
&) + (9*a^5 + 27*a^4*b - 110*a^3*b^2 + 110*a^2*b^3 - 27*a*b^4 - 9*b^5)*e^{(4*d*x + 4*c)} + (9*a^5 + 29*a^4*b - 86*a^3*b^2 - 86*a^2*b^3 + 29*a*b^4 + 9*b^5) \\
&)*e^{(2*d*x + 2*c))/((a^6*b^2 + 4*a^5*b^3 + 6*a^4*b^4 + 4*a^3*b^5 + a^2*b^6 + 4*(a^6*b^2 + 4*a^5*b^3 + 6*a^4*b^4 + 4*a^3*b^5 + a^2*b^6)*e^{(8*d*x + 8*c)} + \\
& 4*(a^6*b^2 + 2*a^5*b^3 - 2*a^3*b^5 - a^2*b^6)*e^{(6*d*x + 6*c)} + 2*(3*a^6*b^2 + 4*a^5*b^3 + 2*a^4*b^4 + 4*a^3*b^5 + 3*a^2*b^6)*e^{(4*d*x + 4*c)} + 4*(a^6*b^2 + 2*a^5*b^3 - 2*a^3*b^5 - a^2*b^6)*e^{(2*d*x + 2*c))*d) + 3/128*(3*a^5 \\
& + 17*a^4*b + 14*a^3*b^2 - 14*a^2*b^3 - 17*a*b^4 - 3*b^5 + (9*a^5 + 29*a^4*b - 86*a^3*b^2 - 86*a^2*b^3 + 29*a*b^4 + 9*b^5)*e^{(-2*d*x - 2*c)} + (9*a^5 + 27*a^4*b - 110*a^3*b^2 + 110*a^2*b^3 - 27*a*b^4 - 9*b^5)*e^{(-4*d*x - 4*c)} \\
& + (3*a^5 + 15*a^4*b - 98*a^3*b^2 - 98*a^2*b^3 + 15*a*b^4 + 3*b^5)*e^{(-6*d*x - 6*c))/((a^6*b^2 + 4*a^5*b^3 + 6*a^4*b^4 + 4*a^3*b^5 + a^2*b^6 + 4*(a^6*b^2 + 2*a^5*b^3 - 2*a^3*b^5 - a^2*b^6)*e^{(-2*d*x - 2*c)} + 2*(3*a^6*b^2 + 4*a^5*b^3 + 2*a^4*b^4 + 4*a^3*b^5 + 3*a^2*b^6)*e^{(-4*d*x - 4*c)} + 4*(a^6*b^2 + 2*a^5*b^3 - 2*a^3*b^5 - a^2*b^6)*e^{(-6*d*x - 6*c)} + (a^6*b^2 + 4*a^5*b^3 + 6*a^4*b^4 + 4*a^3*b^5 + a^2*b^6)*e^{(-8*d*x - 8*c))*d) - 15/256*(3*a^4 + 8*a^3*b + 10*a^2*b^2 + 8*a*b^3 + 3*b^4 + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*e^{(6*d*x + 6*c)} + (9*a^4 + 46*a^2*b^2 + 9*b^4)*e^{(4*d*x + 4*c)} + (9*a^4 + 2*a^3*b - 2*a*b^3 - 9*b^4)*e^{(2*d*x + 2*c))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5 + (a^5*b^2 + a^4*b^3 - a^3*b^4 - a^2*b^5)*e^{(6*d*x + 6*c)} + 2*(3*a^5*b^2 + a^4*b^3 + a^3*b^4 + 3*a^2*b^5)*e^{(4*d*x + 4*c)} + 4*(a^5*b^2 + a^4*b^3 - a^3*b^4 - a^2*b^5)*e^{(2*d*x + 2*c))*d) + 15/256*(3*a^4 + 8*a^3*b + 10*a^2*b^2 + 8*a*b^3 + 3*b^4 + (9*a^4 + 2*a^3*b - 2*a*b^3 - 9*b^4)*e^{(-2*d*x - 2*c)} + (9*a^4 + 46*a^2*b^2 + 9*b^4)*e^{(-4*d*x - 4*c)} + 3*(a^4 + 2*a^3*b - 2*a*b^3 - b^4)*e^{(-6*d*x - 6*c))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5 + 4*(a^5*b^2 + a^4*b^3 - a^3*b^4 - a^2*b^5)*e^{(-2*d*x - 2*c)} + 2*(3*a^5*b^2 + a^4*b^3 + a^3*b^4 + 3*a^2*b^5)*e^{(-4*d*x - 4*c)} + 4*(a^5*b^2 + a^4*b^3 - a^3*b^4 - a^2*b^5)*e^{(-6*d*x - 6*c)} + (a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*e^{(-8*d*x - 8*c))*d) + 5/64*(3*a^3 + 3*a^2*b - 3*a*b^2 - 3*b^3 + (9*a^3 - 13*a^2*b - 13*a*b^2 + 9*b^3)*e^{(-2*d*x - 2*c)} + 3*(3*a^3 - 5*a^2*b + 5*a*b^2 - 3*b^3)*e^{(-4*d*x - 4*c)} + (3*a^3 + a^2*b + a*b^2 + 3*b^3)*e^{(-6*d*x - 6*c))/((a^4*b^2 + 2*a^3*b^3 + a^2*b^4 + 4*(a^4*b^2 - a^2*b^4)*e^{(-2*d*x - 2*c)} + 2*(3*a^4*b^2 - 2*a^3*b^3 + 3*a^2*b^4)*e^{(-4*d*x - 4*c)} + 4*(a^4*b^2 - a^2*b^4)*e^{(-6*d*x - 6*c)} + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*e^{(-8*d*x - 8*c))*d) + 1/4*log((a + b)*e^{(4*d*x + 4*c)} + 2*(a - b)*e^{(2*d*x + 2*c)} + a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/4*log(2*(a - b)*e^{(-2*d*x - 2*c)} + (a + b)*e^{(-4*d*x - 4*c)} + a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 9/256*(a^2 + 2*a*b + b^2)*arctan(1/2*((a + b)*e^{(2*d*x + 2*c)} + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*b^2*d) - 45/512*(a^2 - b^2)*arctan(1/2*((a + b)*e^{(2*d*x + 2*c)} + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*b^2*d) + 5/128*(3*a^2 - 2*a*b + 3*b^2)*arctan(1/2*((a + b)*e^{(-2*d*x - 2*c)} + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*b^2*d) + 9/256*(a^2 + 2*a*b + b^2)*arctan(1/2*((a + b)*e^{(-2*d*x - 2*c)} + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*b^2*d)
\end{aligned}$$

c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*b^2*d) + 45/512*(a^2 - b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*b^2*d)

mupad [B] time = 0.92, size = 2669, normalized size = 18.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)^6/(a + b*tanh(c + d*x)^2)^3,x)

[Out] log(tanh(c + d*x) + 1)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) + ((tanh(c + d*x)^3*(9*a*b + 5*a^2))/(8*b*(2*a*b + a^2 + b^2)) + (a*tanh(c + d*x)*(7*a*b + 3*a^2))/(8*b^2*(2*a*b + a^2 + b^2)))/(a^2*d + b^2*d*tanh(c + d*x)^4 + 2*a*b*d*tanh(c + d*x)^2) - log(tanh(c + d*x) - 1)/(2*d*(a + b)^3) - (atan(((-a*b^5)^(1/2)*((tanh(c + d*x)*(60*a^5*b + 9*a^6 + 64*b^6 + 225*a^2*b^4 + 300*a^3*b^3 + 190*a^4*b^2)))/(32*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 + 4*a^3*b^4*d^2 + a^4*b^3*d^2))) + (((224*a*b^10*d^2 + 1440*a^2*b^9*d^2 + 3936*a^3*b^8*d^2 + 5920*a^4*b^7*d^2 + 5280*a^5*b^6*d^2 + 2784*a^6*b^5*d^2 + 800*a^7*b^4*d^2 + 96*a^8*b^3*d^2)/(64*(b^9*d^3 + 6*a*b^8*d^3 + 15*a^2*b^7*d^3 + 20*a^3*b^6*d^3 + 15*a^4*b^5*d^3 + 6*a^5*b^4*d^3 + a^6*b^3*d^3))) - (tanh(c + d*x)*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2)*(256*b^12*d^2 + 1280*a*b^11*d^2 + 2304*a^2*b^10*d^2 + 1280*a^3*b^9*d^2 - 1280*a^4*b^8*d^2 - 2304*a^5*b^7*d^2 - 1280*a^6*b^6*d^2 - 256*a^7*b^5*d^2))/(512*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 + 4*a^3*b^4*d^2 + a^4*b^3*d^2)))*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2))/(16*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)))*(10*a*b + 3*a^2 + 15*b^2)*1i)/(16*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)) + ((-a*b^5)^(1/2)*((tanh(c + d*x)*(60*a^5*b + 9*a^6 + 64*b^6 + 225*a^2*b^4 + 300*a^3*b^3 + 190*a^4*b^2)))/(32*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 + 4*a^3*b^4*d^2 + a^4*b^3*d^2)) - (((224*a*b^10*d^2 + 1440*a^2*b^9*d^2 + 3936*a^3*b^8*d^2 + 5920*a^4*b^7*d^2 + 5280*a^5*b^6*d^2 + 2784*a^6*b^5*d^2 + 800*a^7*b^4*d^2 + 96*a^8*b^3*d^2)/(64*(b^9*d^3 + 6*a*b^8*d^3 + 15*a^2*b^7*d^3 + 20*a^3*b^6*d^3 + 15*a^4*b^5*d^3 + 6*a^5*b^4*d^3 + a^6*b^3*d^3))) + (tanh(c + d*x)*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2)*(256*b^12*d^2 + 1280*a*b^11*d^2 + 2304*a^2*b^10*d^2 + 1280*a^3*b^9*d^2 - 1280*a^4*b^8*d^2 - 2304*a^5*b^7*d^2 - 1280*a^6*b^6*d^2 - 256*a^7*b^5*d^2))/(512*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 + 4*a^3*b^4*d^2 + a^4*b^3*d^2)))*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2))/(16*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)))*(10*a*b + 3*a^2 + 15*b^2)*1i)/(16*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)))/((120*a*b^4 + 51*a^4*b + 9*a^5 + 185*a^2*b^3 + 139*a^3*b^2)/(32*(b^9*d^3 + 6*a*b^8*d^3 + 15*a^2*b^7*d^3 + 20*a^3*b^6*d^3 + 15*a^4*b^5*d^3 + 6*a^5*b^4*d^3 + a^6*b^3*d^3)) - ((-a*b^5)^(1/2)*((tanh(c + d*x)*(60*a^5*b + 9*a^6 + 64*b^6 + 225*a^2*b^4 + 300*a^3*b^3 + 190*a^4*b^2)))/(32*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 + 4*a^3*b^4*d^2 + a^4*b^3*d^2))) + (((224*a*b^10*d^2 + 1440*a^2*b^9*d^2 + 3936*a^3*b^8*d^2 + 5920*a^4*b^7*d^2 + 5280*a^5*b^6*d^2 + 2784*a^6*b^5*d^2 + 800*a^7*b^4*d^2 + 96*a^8*b^3*d^2)/(64*(b^9*d^3 + 6*a*b^8*d^3 + 15*a^2*b^7*d^3 + 20*a^3*b^6*d^3 + 15*a^4*b^5*d^3 + 6*a^5*b^4*d^3 + a^6*b^3*d^3))) - (tanh(c + d*x)*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2)*(256*b^12*d^2 + 1280*a*b^11*d^2 + 2304*a^2*b^10*d^2 + 1280*a^3*b^9*d^2 - 1280*a^4*b^8*d^2 - 2304*a^5*b^7*d^2 - 1280*a^6*b^6*d^2 - 256*a^7*b^5*d^2))/(512*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 + 4*a^3*b^4*d^2 + a^4*b^3*d^2)))*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2))/(16*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)))*(10*a*b + 3*a^2 + 15*b^2))/((120*a*b^4 + 51*a^4*b + 9*a^5 + 185*a^2*b^3 + 139*a^3*b^2)/(32*(b^9*d^3 + 6*a*b^8*d^3 + 15*a^2*b^7*d^3 + 20*a^3*b^6*d^3 + 15*a^4*b^5*d^3 + 6*a^5*b^4*d^3 + a^6*b^3*d^3))) - ((-a*b^5)^(1/2)*((tanh(c + d*x)*(60*a^5*b + 9*a^6 + 64*b^6 + 225*a^2*b^4 + 300*a^3*b^3 + 190*a^4*b^2)))/(32*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 + 4*a^3*b^4*d^2 + a^4*b^3*d^2))) + (((224*a*b^10*d^2 + 1440*a^2*b^9*d^2 + 3936*a^3*b^8*d^2 + 5920*a^4*b^7*d^2 + 5280*a^5*b^6*d^2 + 2784*a^6*b^5*d^2 + 800*a^7*b^4*d^2 + 96*a^8*b^3*d^2)/(64*(b^9*d^3 + 6*a*b^8*d^3 + 15*a^2*b^7*d^3 + 20*a^3*b^6*d^3 + 15*a^4*b^5*d^3 + 6*a^5*b^4*d^3 + a^6*b^3*d^3))) - (tanh(c + d*x)*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2)*(256*b^12*d^2 + 1280*a*b^11*d^2 + 2304*a^2*b^10*d^2 + 1280*a^3*b^9*d^2 - 1280*a^4*b^8*d^2 - 2304*a^5*b^7*d^2 - 1280*a^6*b^6*d^2 - 256*a^7*b^5*d^2))/(512*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 + 4*a^3*b^4*d^2 + a^4*b^3*d^2)))*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2))/(16*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)))*(10*a*b + 3*a^2 + 15*b^2))/((120*a*b^4 + 51*a^4*b + 9*a^5 + 185*a^2*b^3 + 139*a^3*b^2)/(32*(b^9*d^3 + 6*a*b^8*d^3 + 15*a^2*b^7*d^3 + 20*a^3*b^6*d^3 + 15*a^4*b^5*d^3 + 6*a^5*b^4*d^3 + a^6*b^3*d^3))) + (ta

```

nh(c + d*x)*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2)*(256*b^12*d^2 + 1280*a
*b^11*d^2 + 2304*a^2*b^10*d^2 + 1280*a^3*b^9*d^2 - 1280*a^4*b^8*d^2 - 2304*
a^5*b^7*d^2 - 1280*a^6*b^6*d^2 - 256*a^7*b^5*d^2))/(512*(b^8*d + 3*a^2*b^6*
d + a^3*b^5*d + 3*a*b^7*d)*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 + 4*a^3*b
^4*d^2 + a^4*b^3*d^2)))*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2))/(16*(b^8*
d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)))*(10*a*b + 3*a^2 + 15*b^2))/(16*(
b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)))*(-a*b^5)^(1/2)*(10*a*b + 3*
a^2 + 15*b^2)*1i)/(8*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d))

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)**6/(a+b*tanh(d*x+c)**2)**3,x)

[Out] Timed out

$$3.191 \quad \int \frac{\tanh^5(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=109

$$-\frac{a^2}{4b^2d(a+b)(a+b \tanh^2(c+dx))^2} + \frac{a(a+2b)}{2b^2d(a+b)^2(a+b \tanh^2(c+dx))} + \frac{\log(a+b \tanh^2(c+dx))}{2d(a+b)^3} + \frac{\log(\cosh(dx+c))}{d(a+b)}$$

[Out] ln(cosh(d*x+c))/(a+b)^3/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^3/d-1/4*a^2/b^2/(a+b)/d/(a+b*tanh(d*x+c)^2)^2+1/2*a*(a+2*b)/b^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)

Rubi [A] time = 0.17, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 446, 88}

$$-\frac{a^2}{4b^2d(a+b)(a+b \tanh^2(c+dx))^2} + \frac{a(a+2b)}{2b^2d(a+b)^2(a+b \tanh^2(c+dx))} + \frac{\log(a+b \tanh^2(c+dx))}{2d(a+b)^3} + \frac{\log(\cosh(dx+c))}{d(a+b)}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] Log[Cosh[c + d*x]]/((a + b)^3*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)^3*d) - a^2/(4*b^2*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) + (a*(a + 2*b))/(2*b^2*(a + b)^2*d*(a + b*Tanh[c + d*x]^2))

Rule 88

Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegerQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 446

Int[(x_.)^(m_.)*((a_.) + (b_.)*(x_.))^(n_.)*((c_.) + (d_.)*(x_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_.)])^(m_.)*((a_.) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^5(c+dx)}{(a+b\tanh^2(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{x^5}{(1-x^2)(a+bx^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)(a+bx)^3} dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} + \frac{a^2}{b(a+b)(a+bx)^3} - \frac{a(a+2b)}{b(a+b)^2(a+bx)^2} + \frac{b}{(a+b)^3(a+bx)}\right) dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\log(\cosh(c+dx))}{(a+b)^3d} + \frac{\log(a+b\tanh^2(c+dx))}{2(a+b)^3d} - \frac{a^2}{4b^2(a+b)d(a+b\tanh^2(c+dx))}
\end{aligned}$$

Mathematica [A] time = 1.10, size = 91, normalized size = 0.83

$$\frac{\frac{a^2(a+b)^2}{b^2(a+b\tanh^2(c+dx))^2} - \frac{2a(a+2b)(a+b)}{b^2(a+b\tanh^2(c+dx))} - 2\log(a+b\tanh^2(c+dx)) - 4\log(\cosh(c+dx))}{4d(a+b)^3}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] -1/4*(-4*Log[Cosh[c + d*x]] - 2*Log[a + b*Tanh[c + d*x]^2] + (a^2*(a + b)^2)/(b^2*(a + b*Tanh[c + d*x]^2)^2) - (2*a*(a + b)*(a + 2*b))/(b^2*(a + b*Tanh[c + d*x]^2)))/((a + b)^3*d)

fricas [B] time = 0.48, size = 2584, normalized size = 23.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -1/2*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 16*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^8 + 8*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + (a^2 - b^2)*d*x - a^2 - a*b)*sinh(d*x + c)^6 + 16*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + 3*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 - 2*a*b + 3*b^2)*d*x - 2*a^2 + 4*a*b)*cosh(d*x + c)^4 + 4*(35*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + (3*a^2 - 2*a*b + 3*b^2)*d*x + 30*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^2 - 2*a^2 + 4*a*b)*sinh(d*x + c)^4 + 16*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^5 + 10*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^3 + ((3*a^2 - 2*a*b + 3*b^2)*d*x - 2*a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*d*x + 8*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^2 + 8*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 15*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^4 + (a^2 - b^2)*d*x + 3*((3*a^2 - 2*a*b + 3*b^2)*d*x - 2*a^2 + 4*a*b)*cosh(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 4*(a^2 - b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 30*(a^2 - b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a*b + 3*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 10*(a^2 - b^2)*

$$\begin{aligned} & \cosh(dx + c)^3 + (3a^2 - 2ab + 3b^2) \cosh(dx + c) \sinh(dx + c)^3 + \\ & 4(a^2 - b^2) \cosh(dx + c)^2 + 4(7(a^2 + 2ab + b^2) \cosh(dx + c)^6 + \\ & 15(a^2 - b^2) \cosh(dx + c)^4 + 3(3a^2 - 2ab + 3b^2) \cosh(dx + c)^2 + \\ & a^2 - b^2) \sinh(dx + c)^2 + a^2 + 2ab + b^2 + 8((a^2 + 2ab + b^2) \cosh(dx + c)^7 + \\ & 3(a^2 - b^2) \cosh(dx + c)^5 + (3a^2 - 2ab + 3b^2) \cosh(dx + c)^3 + \\ & (a^2 - b^2) \cosh(dx + c) \sinh(dx + c)) \log(2((a + b) \cosh(dx + c)^2 + \\ & (a + b) \sinh(dx + c)^2 + a - b) / (\cosh(dx + c)^2 - 2 \cosh(dx + c) \sinh(dx + c) + \\ & \sinh(dx + c)^2)) + 16((a^2 + 2ab + b^2) d \cosh(dx + c)^7 + 3((a^2 - b^2) d x - \\ & a^2 - ab) \cosh(dx + c)^5 + ((3a^2 - 2ab + 3b^2) d x - 2a^2 + 4ab) \cosh(dx + c)^3 + \\ & ((a^2 - b^2) d x - a^2 - ab) \cosh(dx + c) \sinh(dx + c)) / ((a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + \\ & 5ab^4 + b^5) d \cosh(dx + c)^8 + 8(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5) d \cosh(dx + c) \sinh(dx + c)^7 + \\ & (a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5) d \sinh(dx + c)^8 + 4(a^5 + 3a^4b + 2a^3b^2 - \\ & 2a^2b^3 - 3ab^4 - b^5) d \cosh(dx + c)^6 + 4(7(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5) d \cosh(dx + c)^2 + \\ & (a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5) d) \sinh(dx + c)^6 + 2(3a^5 + 7a^4b + 6a^3b^2 + 6a^2b^3 + 7ab^4 + 3b^5) d \cosh(dx + c)^4 + \\ & 8(7(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5) d \cosh(dx + c)^3 + 3(a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5) d \cosh(dx + c) \sinh(dx + c)^5 + \\ & 2(35(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5) d \cosh(dx + c)^4 + 30(a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5) d \cosh(dx + c)^2 + \\ & (3a^5 + 7a^4b + 6a^3b^2 + 6a^2b^3 + 7ab^4 + 3b^5) d) \sinh(dx + c)^4 + 4(a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5) d \cosh(dx + c)^2 + \\ & 8(7(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5) d \cosh(dx + c)^5 + 10(a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5) d \cosh(dx + c)^3 + \\ & (3a^5 + 7a^4b + 6a^3b^2 + 6a^2b^3 + 7ab^4 + 3b^5) d \cosh(dx + c)) \sinh(dx + c)^3 + 4(7(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5) d \cosh(dx + c)^6 + \\ & 15(a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5) d \cosh(dx + c)^4 + 3(3a^5 + 7a^4b + 6a^3b^2 + 6a^2b^3 + 7ab^4 + 3b^5) d \cosh(dx + c)^2 + \\ & (a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5) d) \sinh(dx + c)^2 + (a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5) d + 8((a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5) d \cosh(dx + c)^7 + \\ & 3(a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5) d \cosh(dx + c)^5 + (3a^5 + 7a^4b + 6a^3b^2 + 6a^2b^3 + 7ab^4 + 3b^5) d \cosh(dx + c)^3 + (a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5) d \cosh(dx + c)) \sinh(dx + c) \end{aligned}$$

giac [B] time = 0.51, size = 245, normalized size = 2.25

$$\frac{2 \log\left(\left|a\left(e^{2dx+2c}+e^{-2dx-2c}\right)+b\left(e^{2dx+2c}+e^{-2dx-2c}\right)+2a-2b\right|\right)}{a^3+3a^2b+3ab^2+b^3} - \frac{3a\left(e^{2dx+2c}+e^{-2dx-2c}\right)^2+3b\left(e^{2dx+2c}+e^{-2dx-2c}\right)^2-4a\left(e^{2dx+2c}+e^{-2dx-2c}\right)}{\left(a^2+2ab+b^2\right)\left(a\left(e^{2dx+2c}+e^{-2dx-2c}\right)+b\left(e^{2dx+2c}+e^{-2dx-2c}\right)\right)} \cdot \frac{1}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(dx+c)^5/(a+b*tanh(dx+c))^2)^3,x, algorithm="giac"

[Out] $\frac{1}{4} \cdot \frac{2 \log\left(\left|a\left(e^{2dx+2c}+e^{-2dx-2c}\right)+b\left(e^{2dx+2c}+e^{-2dx-2c}\right)+2a-2b\right|\right)}{a^3+3a^2b+3ab^2+b^3} - \frac{3a\left(e^{2dx+2c}+e^{-2dx-2c}\right)^2+3b\left(e^{2dx+2c}+e^{-2dx-2c}\right)^2-4a\left(e^{2dx+2c}+e^{-2dx-2c}\right)}{\left(a^2+2ab+b^2\right)\left(a\left(e^{2dx+2c}+e^{-2dx-2c}\right)+b\left(e^{2dx+2c}+e^{-2dx-2c}\right)\right)} \cdot \frac{1}{4d}$

maple [B] time = 0.11, size = 234, normalized size = 2.15

$$\frac{\ln(\tanh(dx+c)-1)}{2d(a+b)^3} - \frac{\ln(1+\tanh(dx+c))}{2d(a+b)^3} - \frac{a^4}{4d(a+b)^3 b^2 \left(a+b\left(\tanh^2(dx+c)\right)\right)^2} - \frac{a^3}{2d(a+b)^3 b \left(a+b\left(\tanh^2(dx+c)\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x)`

[Out]
$$-1/2/d/(a+b)^3 \ln(\tanh(d*x+c)-1) - 1/2/d/(a+b)^3 \ln(1+\tanh(d*x+c)) - 1/4/d/(a+b)^3 a^4/b^2/(a+b \tanh(d*x+c)^2)^2 - 1/2/d/(a+b)^3 a^3/b/(a+b \tanh(d*x+c)^2)^2 - 1/4/d/(a+b)^3 a^2/(a+b \tanh(d*x+c)^2)^2 + 1/2 \ln(a+b \tanh(d*x+c)^2)/(a+b)^3/d + 1/2/d/(a+b)^3 a^3/b^2/(a+b \tanh(d*x+c)^2) + 3/2/d/(a+b)^3 a^2/b/(a+b \tanh(d*x+c)^2) + 1/d/(a+b)^3 a/(a+b \tanh(d*x+c)^2)$$

maxima [B] time = 0.36, size = 376, normalized size = 3.45

$$\frac{dx+c}{(a^3+3a^2b+3ab^2+b^3)d} + \frac{1}{(a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5+4(a^5+3a^4b+2a^3b^2-2a^2b^3-3ab^4))d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")`

[Out]
$$(d*x+c)/((a^3+3a^2b+3ab^2+b^3)*d) + 4*((a^2+a*b)*e^{(-2*d*x-2*c)} + (a^2-2*a*b)*e^{(-4*d*x-4*c)} + (a^2+a*b)*e^{(-6*d*x-6*c)})/((a^5+5*a^4*b+10*a^3*b^2+10*a^2*b^3+5*a*b^4+b^5+4*(a^5+3*a^4*b+2*a^3*b^2-2*a^2*b^3-3*a*b^4-b^5)*e^{(-2*d*x-2*c)} + 2*(3*a^5+7*a^4*b+6*a^3*b^2+6*a^2*b^3+7*a*b^4+3*b^5)*e^{(-4*d*x-4*c)} + 4*(a^5+3*a^4*b+2*a^3*b^2-2*a^2*b^3-3*a*b^4-b^5)*e^{(-6*d*x-6*c)} + (a^5+5*a^4*b+10*a^3*b^2+10*a^2*b^3+5*a*b^4+b^5)*e^{(-8*d*x-8*c)})*d) + 1/2*\log(2*(a-b)*e^{(-2*d*x-2*c)} + (a+b)*e^{(-4*d*x-4*c)} + a+b)/((a^3+3*a^2*b+3*a*b^2+b^3)*d)$$

mupad [B] time = 0.83, size = 416, normalized size = 3.82

$$\frac{a^4+a^3b(2\tanh(c+dx)^2+4)-ab^3(-4\tanh(c+dx)^2+\tanh(c+dx)^2)\operatorname{atan}\left(\frac{a\tanh(c+dx)^2+1+b\tanh(c+dx)^2}{2a-a\tanh(c+dx)^2+b\tanh(c+dx)^2}\right)}{4da^5b^2+8da^4b^3\tanh(c+dx)^2+12da^4b^3+4da^3b^4\tanh(c+dx)^4+24da^3b^4\tanh(c+dx)^2+12da^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(c+d*x)^5/(a+b*tanh(c+d*x)^2)^3,x)`

[Out]
$$(a^4+a^3b*(2*\tanh(c+d*x)^2+4)-a*b^3*(\tanh(c+d*x)^2*\operatorname{atan}((a*\tanh(c+d*x)^2*1i+b*\tanh(c+d*x)^2*1i)/(2*a-a*\tanh(c+d*x)^2+b*\tanh(c+d*x)^2)))*8i-4*\tanh(c+d*x)^2+a^2*b^2*(6*\tanh(c+d*x)^2-\operatorname{atan}((a*\tanh(c+d*x)^2*1i+b*\tanh(c+d*x)^2*1i)/(2*a-a*\tanh(c+d*x)^2+b*\tanh(c+d*x)^2)))*4i+3)-b^4*\tanh(c+d*x)^4*\operatorname{atan}((a*\tanh(c+d*x)^2*1i+b*\tanh(c+d*x)^2*1i)/(2*a-a*\tanh(c+d*x)^2+b*\tanh(c+d*x)^2))*4i)/(4*a^2*b^5*d+12*a^3*b^4*d+12*a^4*b^3*d+4*a^5*b^2*d+4*b^7*d*\tanh(c+d*x)^4+24*a^2*b^5*d*\tanh(c+d*x)^2+24*a^3*b^4*d*\tanh(c+d*x)^2+8*a^4*b^3*d*\tanh(c+d*x)^2+12*a^2*b^5*d*\tanh(c+d*x)^4+4*a^3*b^4*d*\tanh(c+d*x)^4+8*a*b^6*d*\tanh(c+d*x)^2+12*a*b^6*d*\tanh(c+d*x)^4)$$

sympy [A] time = 157.94, size = 3937, normalized size = 36.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(d*x+c)**5/(a+b*tanh(d*x+c)**2)**3,x)`

[Out] `Piecewise((zoo*x/tanh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - log(tanh(c + d*x) + 1)/d - tanh(c + d*x)**4/(4*d) - tanh(c + d*x)**2/(2*d))/a**3, Eq(b, 0)), (3*tanh(c + d*x)**4/(6*b**3*d*tanh(c + d*x)**6 - 18*b**3*d*tanh(c + d*x)**4 + 18*b**3*d*tanh(c + d*x)**2 - 6*b**3*d) - 3*tanh(c + d*x)**2/(6*b`


```

*2*b**5*d*tanh(c + d*x)**2 + 4*a**2*b**5*d + 12*a*b**6*d*tanh(c + d*x)**4 +
  8*a*b**6*d*tanh(c + d*x)**2 + 4*b**7*d*tanh(c + d*x)**4) - 8*a*b**3*log(ta
nh(c + d*x) + 1)*tanh(c + d*x)**2/(4*a**5*b**2*d + 8*a**4*b**3*d*tanh(c + d
*x)**2 + 12*a**4*b**3*d + 4*a**3*b**4*d*tanh(c + d*x)**4 + 24*a**3*b**4*d*t
anh(c + d*x)**2 + 12*a**3*b**4*d + 12*a**2*b**5*d*tanh(c + d*x)**4 + 24*a**
2*b**5*d*tanh(c + d*x)**2 + 4*a**2*b**5*d + 12*a*b**6*d*tanh(c + d*x)**4 +
  8*a*b**6*d*tanh(c + d*x)**2 + 4*b**7*d*tanh(c + d*x)**4) + 4*a*b**3*tanh(c
+ d*x)**2/(4*a**5*b**2*d + 8*a**4*b**3*d*tanh(c + d*x)**2 + 12*a**4*b**3*d
+ 4*a**3*b**4*d*tanh(c + d*x)**4 + 24*a**3*b**4*d*tanh(c + d*x)**2 + 12*a**
3*b**4*d + 12*a**2*b**5*d*tanh(c + d*x)**4 + 24*a**2*b**5*d*tanh(c + d*x)**
2 + 4*a**2*b**5*d + 12*a*b**6*d*tanh(c + d*x)**4 + 8*a*b**6*d*tanh(c + d*x)
**2 + 4*b**7*d*tanh(c + d*x)**4) + 4*b**4*d*x*tanh(c + d*x)**4/(4*a**5*b**2
*d + 8*a**4*b**3*d*tanh(c + d*x)**2 + 12*a**4*b**3*d + 4*a**3*b**4*d*tanh(c
+ d*x)**4 + 24*a**3*b**4*d*tanh(c + d*x)**2 + 12*a**3*b**4*d + 12*a**2*b**
5*d*tanh(c + d*x)**4 + 24*a**2*b**5*d*tanh(c + d*x)**2 + 4*a**2*b**5*d + 12
*a*b**6*d*tanh(c + d*x)**4 + 8*a*b**6*d*tanh(c + d*x)**2 + 4*b**7*d*tanh(c
+ d*x)**4) + 2*b**4*log(-I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))*tanh(c + d*x)
**4/(4*a**5*b**2*d + 8*a**4*b**3*d*tanh(c + d*x)**2 + 12*a**4*b**3*d + 4*a**
3*b**4*d*tanh(c + d*x)**4 + 24*a**3*b**4*d*tanh(c + d*x)**2 + 12*a**3*b**4
*d + 12*a**2*b**5*d*tanh(c + d*x)**4 + 24*a**2*b**5*d*tanh(c + d*x)**2 + 4*
a**2*b**5*d + 12*a*b**6*d*tanh(c + d*x)**4 + 8*a*b**6*d*tanh(c + d*x)**2 +
  4*b**7*d*tanh(c + d*x)**4) + 2*b**4*log(I*sqrt(a)*sqrt(1/b) + tanh(c + d*x)
)*tanh(c + d*x)**4/(4*a**5*b**2*d + 8*a**4*b**3*d*tanh(c + d*x)**2 + 12*a**
4*b**3*d + 4*a**3*b**4*d*tanh(c + d*x)**4 + 24*a**3*b**4*d*tanh(c + d*x)**2
+ 12*a**3*b**4*d + 12*a**2*b**5*d*tanh(c + d*x)**4 + 24*a**2*b**5*d*tanh(c
+ d*x)**2 + 4*a**2*b**5*d + 12*a*b**6*d*tanh(c + d*x)**4 + 8*a*b**6*d*tanh
(c + d*x)**2 + 4*b**7*d*tanh(c + d*x)**4) - 4*b**4*log(tanh(c + d*x) + 1)*t
anh(c + d*x)**4/(4*a**5*b**2*d + 8*a**4*b**3*d*tanh(c + d*x)**2 + 12*a**4*b
**3*d + 4*a**3*b**4*d*tanh(c + d*x)**4 + 24*a**3*b**4*d*tanh(c + d*x)**2 +
  12*a**3*b**4*d + 12*a**2*b**5*d*tanh(c + d*x)**4 + 24*a**2*b**5*d*tanh(c +
d*x)**2 + 4*a**2*b**5*d + 12*a*b**6*d*tanh(c + d*x)**4 + 8*a*b**6*d*tanh(c
+ d*x)**2 + 4*b**7*d*tanh(c + d*x)**4), True))

```

$$3.192 \quad \int \frac{\tanh^4(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=137

$$\frac{(a^2 + 6ab - 3b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a} b^{3/2} d (a+b)^3} - \frac{(a+5b) \tanh(c+dx)}{8bd(a+b)^2 (a+b \tanh^2(c+dx))} + \frac{a \tanh(c+dx)}{4bd(a+b) (a+b \tanh^2(c+dx))^2}$$

[Out] x/(a+b)^3-1/8*(a^2+6*a*b-3*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/b^(3/2)/(a+b)^3/d/a^(1/2)+1/4*a*tanh(d*x+c)/b/(a+b)/d/(a+b*tanh(d*x+c)^2)^2-1/8*(a+5*b)*tanh(d*x+c)/b/(a+b)^2/d/(a+b*tanh(d*x+c)^2)

Rubi [A] time = 0.20, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3670, 470, 527, 522, 206, 205}

$$\frac{(a^2 + 6ab - 3b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a} b^{3/2} d (a+b)^3} - \frac{(a+5b) \tanh(c+dx)}{8bd(a+b)^2 (a+b \tanh^2(c+dx))} + \frac{a \tanh(c+dx)}{4bd(a+b) (a+b \tanh^2(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] x/(a + b)^3 - ((a^2 + 6*a*b - 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*Sqrt[a]*b^(3/2)*(a + b)^3*d) + (a*Tanh[c + d*x])/(4*b*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) - ((a + 5*b)*Tanh[c + d*x])/(8*b*(a + b)^2*d*(a + b*Tanh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3670

```
Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*((c_)*tan[(e_) + (f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))
```

Rubi steps

$$\int \frac{\tanh^4(c + dx)}{(a + b \tanh^2(c + dx))^3} dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a+bx^2)^3} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{a \tanh(c + dx)}{4b(a + b)d (a + b \tanh^2(c + dx))^2} - \frac{\text{Subst}\left(\int \frac{a+(-a-4b)x^2}{(1-x^2)(a+bx^2)^2} dx, x, \tanh(c + dx)\right)}{4b(a + b)d}$$

$$= \frac{a \tanh(c + dx)}{4b(a + b)d (a + b \tanh^2(c + dx))^2} - \frac{(a + 5b) \tanh(c + dx)}{8b(a + b)^2d (a + b \tanh^2(c + dx))} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{4b(a + b)d}$$

$$= \frac{a \tanh(c + dx)}{4b(a + b)d (a + b \tanh^2(c + dx))^2} - \frac{(a + 5b) \tanh(c + dx)}{8b(a + b)^2d (a + b \tanh^2(c + dx))} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{4b(a + b)d}$$

$$= \frac{x}{(a + b)^3} - \frac{(a^2 + 6ab - 3b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a} b^{3/2} (a + b)^3 d} + \frac{a \tanh(c + dx)}{4b(a + b)d (a + b \tanh^2(c + dx))}$$

Mathematica [A] time = 1.15, size = 135, normalized size = 0.99

$$\frac{-\frac{(a^2+6ab-3b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a} b^{3/2}} + \frac{(a-5b)(a+b) \sinh(2(c+dx))}{b((a+b) \cosh(2(c+dx))+a-b)} + \frac{4a(a+b) \sinh(2(c+dx))}{((a+b) \cosh(2(c+dx))+a-b)^2} + 8(c + dx)}{8d(a + b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (8*(c + d*x) - ((a^2 + 6*a*b - 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)) + (4*a*(a + b)*Sinh[2*(c + d*x)]/(a - b + (a + b)*Cosh[2*(c + d*x)])^2 + ((a - 5*b)*(a + b)*Sinh[2*(c + d*x)]/(b*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(8*(a + b)^3*d)
```

fricas [B] time = 0.58, size = 7757, normalized size = 56.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(16*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*cosh(d*x + c)^8 + 128*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 16*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*sinh(d*x + c)^8 - 4*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*cosh(d*x + c)^6 - 4*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 112*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*cosh(d*x + c)^2 - 16*(a^3*b^2 - a*b^4)*d*x)*sinh(d*x + c)^6 + 8*(112*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*cosh(d*x + c)^3 - 3*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 4*a^4*b + 12*a^3*b^2 + 36*a^2*b^3 + 20*a*b^4 - 4*(3*a^4*b - 17*a^3*b^2 + 13*a^2*b^3 - 15*a*b^4 - 8*(3*a^3*b^2 - 2*a^2*b^3 + 3*a*b^4)*d*x)*cosh(d*x + c)^4 + 4*(280*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*cosh(d*x + c)^4 - 3*a^4*b + 17*a^3*b^2 - 13*a^2*b^3 + 15*a*b^4 + 8*(3*a^3*b^2 - 2*a^2*b^3 + 3*a*b^4)*d*x - 15*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(56*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*cosh(d*x + c)^5 - 5*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*cosh(d*x + c)^3 - (3*a^4*b - 17*a^3*b^2 + 13*a^2*b^3 - 15*a*b^4 - 8*(3*a^3*b^2 - 2*a^2*b^3 + 3*a*b^4)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 16*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x - 4*(3*a^4*b - 11*a^3*b^2 + a^2*b^3 + 15*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*cosh(d*x + c)^2 + 4*(112*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*cosh(d*x + c)^6 - 3*a^4*b + 11*a^3*b^2 - a^2*b^3 - 15*a*b^4 - 15*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*cosh(d*x + c)^4 + 16*(a^3*b^2 - a*b^4)*d*x - 6*(3*a^4*b - 17*a^3*b^2 + 13*a^2*b^3 - 15*a*b^4 - 8*(3*a^3*b^2 - 2*a^2*b^3 + 3*a*b^4)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^8 + 8*(a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*sinh(d*x + c)^8 + 4*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 4*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4 + 7*(a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^3 + 3*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^4 + 16*a^3*b - 18*a^2*b^2 + 24*a*b^3 - 9*b^4)*cosh(d*x + c)^4 + 2*(35*(a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^4 + 3*a^4 + 16*a^3*b - 18*a^2*b^2 + 24*a*b^3 - 9*b^4 + 30*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4 + 8*(7*(a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^5 + 10*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (3*a^4 + 16*a^3*b - 18*a^2*b^2 + 24*a*b^3 - 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4)*cosh(d*x + c)^2 + 4*(7*(a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^6 + 15*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4 + 3*(3*a^4 + 16*a^3*b - 18*a^2*b^2 + 24*a*b^3 - 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*cosh(d*x + c)^7 + 3*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + (3*a^4 + 16*a^3*b - 18*a^2*b^2 + 24*a*b^3 - 9*b^4)*cosh(d*x + c)^3 + (a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 8*(16*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*cosh(d*x + c)^7 - 3*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 16*(a^3*b^2

$$\begin{aligned}
& b^2 - a*b^4)*d*x)*\cosh(d*x + c)^5 - 2*(3*a^4*b - 17*a^3*b^2 + 13*a^2*b^3 - \\
& 15*a*b^4 - 8*(3*a^3*b^2 - 2*a^2*b^3 + 3*a*b^4)*d*x)*\cosh(d*x + c)^3 - (3*a^4*b - 11*a^3*b^2 + a^2*b^3 + 15*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*\cosh(d*x \\
& + c))*\sinh(d*x + c))/((a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7)*d*\cosh(d*x + c)^8 + 8*(a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10 \\
& *a^3*b^5 + 5*a^2*b^6 + a*b^7)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7)*d*\sinh(d*x + c)^8 \\
& + 4*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7)*d*\cosh(d*x + c)^6 + 4*(7*(a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7)*d*\cosh(d*x + c)^2 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7)*d)*\sinh(d*x + c)^6 + 2*(3*a^6*b^2 + 7*a^5*b^3 + 6*a^4*b^4 + 6*a^3*b^5 + 7*a^2*b^6 + 3*a*b^7)*d*\cosh(d*x + c)^4 + 8*(7*(a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7)*d*\cosh(d*x + c)^3 + 3*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7)*d*\cosh(d*x + c)^4 + 30*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7)*d*\cosh(d*x + c)^2 + (3*a^6*b^2 + 7*a^5*b^3 + 6*a^4*b^4 + 6*a^3*b^5 + 7*a^2*b^6 + 3*a*b^7)*d)*\sinh(d*x + c)^4 + 4*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7)*d*\cosh(d*x + c)^2 + 8*(7*(a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7)*d*\cosh(d*x + c)^5 + 10*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7)*d*\cosh(d*x + c)^3 + (3*a^6*b^2 + 7*a^5*b^3 + 6*a^4*b^4 + 6*a^3*b^5 + 7*a^2*b^6 + 3*a*b^7)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7)*d*\cosh(d*x + c)^6 + 15*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7)*d*\cosh(d*x + c)^4 + 3*(3*a^6*b^2 + 7*a^5*b^3 + 6*a^4*b^4 + 6*a^3*b^5 + 7*a^2*b^6 + 3*a*b^7)*d*\cosh(d*x + c)^2 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7)*d)*\sinh(d*x + c)^2 + (a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7)*d + 8*((a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7)*d*\cosh(d*x + c)^7 + 3*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7)*d*\cosh(d*x + c)^5 + (3*a^6*b^2 + 7*a^5*b^3 + 6*a^4*b^4 + 6*a^3*b^5 + 7*a^2*b^6 + 3*a*b^7)*d*\cosh(d*x + c)^3 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7)*d*\cosh(d*x + c))*\sinh(d*x + c)), 1/8*(8*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*\cosh(d*x + c)^8 + 64*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*\cosh(d*x + c)*\sinh(d*x + c)^7 + 8*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*\sinh(d*x + c)^8 - 2*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*\cosh(d*x + c)^6 - 2*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 12*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*\cosh(d*x + c)^2 - 16*(a^3*b^2 - a*b^4)*d*x)*\sinh(d*x + c)^6 + 4*(112*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*\cosh(d*x + c)^3 - 3*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2*a^4*b + 6*a^3*b^2 + 18*a^2*b^3 + 10*a*b^4 - 2*(3*a^4*b - 17*a^3*b^2 + 13*a^2*b^3 - 15*a*b^4 - 8*(3*a^3*b^2 - 2*a^2*b^3 + 3*a*b^4)*d*x)*\cosh(d*x + c)^4 + 2*(280*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*\cosh(d*x + c)^4 - 3*a^4*b + 17*a^3*b^2 - 13*a^2*b^3 + 15*a*b^4 + 8*(3*a^3*b^2 - 2*a^2*b^3 + 3*a*b^4)*d*x - 15*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(56*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*\cosh(d*x + c)^5 - 5*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*\cosh(d*x + c)^3 - (3*a^4*b - 17*a^3*b^2 + 13*a^2*b^3 - 15*a*b^4 - 8*(3*a^3*b^2 - 2*a^2*b^3 + 3*a*b^4)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 8*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x - 2*(3*a^4*b - 11*a^3*b^2 + a^2*b^3 + 15*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*\cosh(d*x + c)^2 + 2*(112*(a^3*b^2 + 2*a^2*b^3 + a*b^4)*d*x*\cosh(d*x + c)^6 - 3*a^4*b + 11*a^3*b^2 - a^2*b^3 - 15*a*b^4 - 15*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 16*(a^3*b^2 - a*b^4)*d*x)*\cosh(d*x + c)^4 + 16*(a^3*b^2 - a*b^4)*d*x - 6*(3*a^4*b - 17*a^3*b^2 + 13*a^2*b^3 - 15*a*b^4 - 8*(3*a^3*b^2 - 2*a^2*b^3 + 3*a*b^4)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 - ((a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*\cosh(d*x + c)^8 + 8*(a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^4 + 8*a^3*b + 10*a^2*b^2 - 3*
\end{aligned}$$

$$\begin{aligned}
& b^4) * \sinh(dx + c)^8 + 4*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4) * \cosh \\
& (dx + c)^6 + 4*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4 + 7*(a^4 + 8*a \\
& ^3*b + 10*a^2*b^2 - 3*b^4) * \cosh(dx + c)^2) * \sinh(dx + c)^6 + 8*(7*(a^4 + 8 \\
& *a^3*b + 10*a^2*b^2 - 3*b^4) * \cosh(dx + c)^3 + 3*(a^4 + 6*a^3*b - 4*a^2*b^2 \\
& - 6*a*b^3 + 3*b^4) * \cosh(dx + c)) * \sinh(dx + c)^5 + 2*(3*a^4 + 16*a^3*b - \\
& 18*a^2*b^2 + 24*a*b^3 - 9*b^4) * \cosh(dx + c)^4 + 2*(35*(a^4 + 8*a^3*b + 10* \\
& a^2*b^2 - 3*b^4) * \cosh(dx + c)^4 + 3*a^4 + 16*a^3*b - 18*a^2*b^2 + 24*a*b^3 \\
& - 9*b^4 + 30*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4) * \cosh(dx + c)^2 \\
&) * \sinh(dx + c)^4 + a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4 + 8*(7*(a^4 + 8*a^3* \\
& b + 10*a^2*b^2 - 3*b^4) * \cosh(dx + c)^5 + 10*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6 \\
& *a*b^3 + 3*b^4) * \cosh(dx + c)^3 + (3*a^4 + 16*a^3*b - 18*a^2*b^2 + 24*a*b^3 \\
& - 9*b^4) * \cosh(dx + c)) * \sinh(dx + c)^3 + 4*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6 \\
& *a*b^3 + 3*b^4) * \cosh(dx + c)^2 + 4*(7*(a^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4) \\
& * \cosh(dx + c)^6 + 15*(a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4) * \cosh(dx \\
& x + c)^4 + a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 + 3*b^4 + 3*(3*a^4 + 16*a^3* \\
& b - 18*a^2*b^2 + 24*a*b^3 - 9*b^4) * \cosh(dx + c)^2) * \sinh(dx + c)^2 + 8*((a \\
& ^4 + 8*a^3*b + 10*a^2*b^2 - 3*b^4) * \cosh(dx + c)^7 + 3*(a^4 + 6*a^3*b - 4*a \\
& ^2*b^2 - 6*a*b^3 + 3*b^4) * \cosh(dx + c)^5 + (3*a^4 + 16*a^3*b - 18*a^2*b^2 \\
& + 24*a*b^3 - 9*b^4) * \cosh(dx + c)^3 + (a^4 + 6*a^3*b - 4*a^2*b^2 - 6*a*b^3 \\
& + 3*b^4) * \cosh(dx + c)) * \sinh(dx + c)) * \sqrt{a*b} * \arctan(1/2*((a + b) * \cosh(dx \\
& *x + c)^2 + 2*(a + b) * \cosh(dx + c) * \sinh(dx + c) + (a + b) * \sinh(dx + c)^2 \\
& + a - b) * \sqrt{a*b} / (a*b)) + 4*(16*(a^3*b^2 + 2*a^2*b^3 + a*b^4) * d*x * \cosh(dx \\
& *x + c)^7 - 3*(a^4*b - 9*a^3*b^2 - 5*a^2*b^3 + 5*a*b^4 - 16*(a^3*b^2 - a*b^ \\
& 4) * d*x) * \cosh(dx + c)^5 - 2*(3*a^4*b - 17*a^3*b^2 + 13*a^2*b^3 - 15*a*b^4 - \\
& 8*(3*a^3*b^2 - 2*a^2*b^3 + 3*a*b^4) * d*x) * \cosh(dx + c)^3 - (3*a^4*b - 11*a \\
& ^3*b^2 + a^2*b^3 + 15*a*b^4 - 16*(a^3*b^2 - a*b^4) * d*x) * \cosh(dx + c)) * \sinh \\
& (dx + c)) / ((a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a* \\
& b^7) * d * \cosh(dx + c)^8 + 8*(a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + \\
& 5*a^2*b^6 + a*b^7) * d * \cosh(dx + c) * \sinh(dx + c)^7 + (a^6*b^2 + 5*a^5*b^3 \\
& + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7) * d * \sinh(dx + c)^8 + 4*(a^6*b \\
& ^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7) * d * \cosh(dx + c) \\
& ^6 + 4*(7*(a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^ \\
& 7) * d * \cosh(dx + c)^2 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2 \\
& *b^6 - a*b^7) * d) * \sinh(dx + c)^6 + 2*(3*a^6*b^2 + 7*a^5*b^3 + 6*a^4*b^4 + 6 \\
& *a^3*b^5 + 7*a^2*b^6 + 3*a*b^7) * d * \cosh(dx + c)^4 + 8*(7*(a^6*b^2 + 5*a^5*b \\
& ^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7) * d * \cosh(dx + c)^3 + 3*(a^ \\
& 6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7) * d * \cosh(dx + \\
& c)) * \sinh(dx + c)^5 + 2*(35*(a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 \\
& + 5*a^2*b^6 + a*b^7) * d * \cosh(dx + c)^4 + 30*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b \\
& ^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7) * d * \cosh(dx + c)^2 + (3*a^6*b^2 + 7*a^5* \\
& b^3 + 6*a^4*b^4 + 6*a^3*b^5 + 7*a^2*b^6 + 3*a*b^7) * d) * \sinh(dx + c)^4 + 4*(\\
& a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7) * d * \cosh(dx \\
& + c)^2 + 8*(7*(a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + \\
& a*b^7) * d * \cosh(dx + c)^5 + 10*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 \\
& - 3*a^2*b^6 - a*b^7) * d * \cosh(dx + c)^3 + (3*a^6*b^2 + 7*a^5*b^3 + 6*a^4*b^ \\
& 4 + 6*a^3*b^5 + 7*a^2*b^6 + 3*a*b^7) * d * \cosh(dx + c)) * \sinh(dx + c)^3 + 4*(\\
& 7*(a^6*b^2 + 5*a^5*b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7) * d * \cos \\
& h(dx + c)^6 + 15*(a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 \\
& - a*b^7) * d * \cosh(dx + c)^4 + 3*(3*a^6*b^2 + 7*a^5*b^3 + 6*a^4*b^4 + 6*a^3*b \\
& ^5 + 7*a^2*b^6 + 3*a*b^7) * d * \cosh(dx + c)^2 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4* \\
& b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7) * d) * \sinh(dx + c)^2 + (a^6*b^2 + 5*a^5* \\
& b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7) * d + 8*((a^6*b^2 + 5*a^5* \\
& b^3 + 10*a^4*b^4 + 10*a^3*b^5 + 5*a^2*b^6 + a*b^7) * d * \cosh(dx + c)^7 + 3*(a \\
& ^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^2*b^6 - a*b^7) * d * \cosh(dx \\
& + c)^5 + (3*a^6*b^2 + 7*a^5*b^3 + 6*a^4*b^4 + 6*a^3*b^5 + 7*a^2*b^6 + 3*a*b \\
& ^7) * d * \cosh(dx + c)^3 + (a^6*b^2 + 3*a^5*b^3 + 2*a^4*b^4 - 2*a^3*b^5 - 3*a^ \\
& 2*b^6 - a*b^7) * d * \cosh(dx + c)) * \sinh(dx + c))]
\end{aligned}$$

giac [B] time = 0.49, size = 384, normalized size = 2.80

$$\frac{(a^2+6ab-3b^2)\arctan\left(\frac{ae^{(2dx+2c)+be^{(2dx+2c)+a-b}}}{2\sqrt{ab}}\right)}{(a^3b+3a^2b^2+3ab^3+b^4)\sqrt{ab}} - \frac{8(dx+c)}{a^3+3a^2b+3ab^2+b^3} + \frac{2(a^3e^{(6dx+6c)}-9a^2be^{(6dx+6c)}-5ab^2e^{(6dx+6c)}+5b^3e^{(6dx+6c)}+3a^3e^{(4d} + \dots)}{(a^3b + \dots)} \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out] -1/8*((a^2 + 6*a*b - 3*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*sqrt(a*b)) - 8*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(a^3*e^(6*d*x + 6*c) - 9*a^2*b*e^(6*d*x + 6*c) - 5*a*b^2*e^(6*d*x + 6*c) + 5*b^3*e^(6*d*x + 6*c) + 3*a^3*e^(4*d*x + 4*c) - 17*a^2*b*e^(4*d*x + 4*c) + 13*a*b^2*e^(4*d*x + 4*c) - 15*b^3*e^(4*d*x + 4*c) + 3*a^3*e^(2*d*x + 2*c) - 11*a^2*b*e^(2*d*x + 2*c) + a*b^2*e^(2*d*x + 2*c) + 15*b^3*e^(2*d*x + 2*c) + a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2))/d

maple [B] time = 0.12, size = 340, normalized size = 2.48

$$\frac{\ln(\tanh(dx+c)-1)}{2d(a+b)^3} + \frac{\ln(1+\tanh(dx+c))}{2d(a+b)^3} - \frac{a^2(\tanh^3(dx+c))}{8d(a+b)^3(a+b(\tanh^2(dx+c)))^2} - \frac{3ab(\tanh^3(dx+c))}{4d(a+b)^3(a+b(\tanh^2(dx+c)))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x)

[Out] -1/2/d/(a+b)^3*ln(tanh(d*x+c)-1)+1/2/d/(a+b)^3*ln(1+tanh(d*x+c))-1/8/d*a^2/(a+b)^3/(a+b*tanh(d*x+c)^2)^2*tanh(d*x+c)^3-3/4/d*a/(a+b)^3/(a+b*tanh(d*x+c)^2)^2*b*tanh(d*x+c)^3-5/8/d/(a+b)^3/(a+b*tanh(d*x+c)^2)^2*tanh(d*x+c)^3*b^2+1/8/d*a^3/(a+b)^3/(a+b*tanh(d*x+c)^2)^2/b*tanh(d*x+c)-1/4/d*a^2/(a+b)^3/(a+b*tanh(d*x+c)^2)^2*tanh(d*x+c)-3/8/d/(a+b)^3/(a+b*tanh(d*x+c)^2)^2*a*b*tanh(d*x+c)-1/8/d*a^2/(a+b)^3/b/(a*b)^(1/2)*arctan(tanh(d*x+c)*b/(a*b)^(1/2))-3/4/d*a/(a+b)^3/(a*b)^(1/2)*arctan(tanh(d*x+c)*b/(a*b)^(1/2))+3/8/d/(a+b)^3*b/(a*b)^(1/2)*arctan(tanh(d*x+c)*b/(a*b)^(1/2))

maxima [B] time = 1.35, size = 2432, normalized size = 17.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] -1/128*(a^4 + 24*a^3*b - 54*a^2*b^2 - 16*a*b^3 - 3*b^4)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) + 1/128*(a^4 + 24*a^3*b - 54*a^2*b^2 - 16*a*b^3 - 3*b^4)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) - 1/64*(a^5 - 33*a^4*b - 54*a^3*b^2 - 2*a^2*b^3 + 21*a*b^4 + 3*b^5 + (a^5 - 71*a^4*b + 98*a^3*b^2 + 154*a^2*b^3 - 19*a*b^4 - 3*b^5)*e^(6*d*x + 6*c) + (3*a^5 - 171*a^4*b + 310*a^3*b^2 - 254*a^2*b^3 + 39*a*b^4 + 9*b^5)*e^(4*d*x + 4*c) + (3*a^5 - 133*a^4*b + 86*a^3*b^2 + 190*a^2*b^3 - 41*a*b^4 - 9*b^5)*e^(2*d*x + 2*c))/((a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6 + (a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6)*e^(8*d*x + 8*c) + 4*(a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*e^(6*d*x + 6*c) + 2*(3*a^7*b + 7*a^6*b^2 + 6*a^5*b^3 + 6*a^4*b^4 + 7*a^3*b^5 + 3*a^2*b^6)*e^(4*

$$\begin{aligned}
& d*x + 4*c) + 4*(a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*e^{(2*d*x + 2*c)}*d) + 1/64*(a^5 - 33*a^4*b - 54*a^3*b^2 - 2*a^2*b^3 + \\
& 21*a*b^4 + 3*b^5 + (3*a^5 - 133*a^4*b + 86*a^3*b^2 + 190*a^2*b^3 - 41*a*b^4 - 9*b^5)*e^{(-2*d*x - 2*c)} + (3*a^5 - 171*a^4*b + 310*a^3*b^2 - 254*a^2*b^3 + \\
& 39*a*b^4 + 9*b^5)*e^{(-4*d*x - 4*c)} + (a^5 - 71*a^4*b + 98*a^3*b^2 + 154*a^2*b^3 - 19*a*b^4 - 3*b^5)*e^{(-6*d*x - 6*c)})/((a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + \\
& 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6 + 4*(a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*e^{(-2*d*x - 2*c)} + 2*(3*a^7*b + 7*a^6*b^2 + \\
& 6*a^5*b^3 + 6*a^4*b^4 + 7*a^3*b^5 + 3*a^2*b^6)*e^{(-4*d*x - 4*c)} + 4*(a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*e^{(-6*d*x - \\
& 6*c)} + (a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6)*e^{(-8*d*x - 8*c)})*d) - 1/16*(a^4 - 4*a^3*b - 14*a^2*b^2 - 12*a*b^3 - 3*b^4 + \\
& (a^4 - 26*a^3*b - 20*a^2*b^2 + 10*a*b^3 + 3*b^4)*e^{(6*d*x + 6*c)} + (3*a^4 - 52*a^3*b + 6*a^2*b^2 - 12*a*b^3 - 9*b^4)*e^{(4*d*x + 4*c)} + (3*a^4 - 30*a^3*b - \\
& 28*a^2*b^2 + 14*a*b^3 + 9*b^4)*e^{(2*d*x + 2*c)})/((a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*e^{(8*d*x + 8*c)} + 4*(a^6*b + 2*a^5*b^2 - 2*a^3*b^4 - a^2*b^5) * \\
& e^{(6*d*x + 6*c)} + 2*(3*a^6*b + 4*a^5*b^2 + 2*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5)*e^{(4*d*x + 4*c)} + 4*(a^6*b + 2*a^5*b^2 - 2*a^3*b^4 - a^2*b^5)*e^{(2*d*x + \\
& 2*c)})*d) + 1/16*(a^4 - 4*a^3*b - 14*a^2*b^2 - 12*a*b^3 - 3*b^4 + (3*a^4 - 30*a^3*b - 28*a^2*b^2 + 14*a*b^3 + 9*b^4)*e^{(-2*d*x - 2*c)} + (3*a^4 - 52*a^3*b + \\
& 6*a^2*b^2 - 12*a*b^3 - 9*b^4)*e^{(-4*d*x - 4*c)} + (a^4 - 26*a^3*b - 20*a^2*b^2 + 10*a*b^3 + 3*b^4)*e^{(-6*d*x - 6*c)})/((a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + \\
& a^2*b^5 + 4*(a^6*b + 2*a^5*b^2 - 2*a^3*b^4 - a^2*b^5)*e^{(-2*d*x - 2*c)} + 2*(3*a^6*b + 4*a^5*b^2 + 2*a^4*b^3 + 4*a^3*b^4 + 3*a^2*b^5)*e^{(-4*d*x - 4*c)} + 4*(a^6*b + 2*a^5*b^2 - \\
& 2*a^3*b^4 - a^2*b^5)*e^{(-6*d*x - 6*c)} + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*e^{(-8*d*x - 8*c)})*d) + 3/32*(a^3 + 5*a^2*b + 7*a*b^2 + 3*b^3 + (3*a^3 + 13*a^2*b + a*b^2 - \\
& 9*b^3)*e^{(-2*d*x - 2*c)} + (3*a^3 + 7*a^2*b - 3*a*b^2 + 9*b^3)*e^{(-4*d*x - 4*c)} + (a^3 - a^2*b - 5*a*b^2 - 3*b^3)*e^{(-6*d*x - 6*c)})/((a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + \\
& a^2*b^4 + 4*(a^5*b + a^4*b^2 - a^3*b^3 - a^2*b^4)*e^{(-2*d*x - 2*c)} + 2*(3*a^5*b + a^4*b^2 + a^3*b^3 + 3*a^2*b^4)*e^{(-4*d*x - 4*c)} + 4*(a^5*b + a^4*b^2 - a^3*b^3 - a^2*b^4)*e^{(-6*d*x - 6*c)} + \\
& (a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*e^{(-8*d*x - 8*c)})*d) + 1/4*log((a + b)*e^{(4*d*x + 4*c)} + 2*(a - b)*e^{(2*d*x + 2*c)} + a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - \\
& 1/4*log(2*(a - b)*e^{(-2*d*x - 2*c)} + (a + b)*e^{(-4*d*x - 4*c)} + a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/32*(a + 3*b)*arctan(1/2*((a + b)*e^{(2*d*x + 2*c)} + a - b)/sqrt(a*b))/ \\
& (sqrt(a*b)*a^2*b*d) + 1/32*(a + 3*b)*arctan(1/2*((a + b)*e^{(-2*d*x - 2*c)} + a - b)/sqrt(a*b))/sqrt(a*b)*a^2*b*d) + 3/64*(a - 3*b)*arctan(1/2*((a + b)*e^{(-2*d*x - 2*c)} + a - b)/sqrt(a*b)) / \\
& (sqrt(a*b)*a^2*b*d)
\end{aligned}$$

mupad [B] time = 1.86, size = 2574, normalized size = 18.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\tanh(c + d*x)^4/(a + b*\tanh(c + d*x)^2)^3, x)$

[Out] $\log(\tanh(c + d*x) + 1)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) - ((\tanh(c + d*x)^3*(a + 5*b))/(8*(2*a*b + a^2 + b^2)) - (a*\tanh(c + d*x)*(a - 3*b))/(8*b*(2*a*b + a^2 + b^2)))/(a^2*d + b^2*d*\tanh(c + d*x)^4 + 2*a*b*d*\tanh(c + d*x)^2) - \log(\tanh(c + d*x) - 1)/(2*d*(a + b)^3) - (\text{atan}(\frac{((\tanh(c + d*x))^3*(12*a^3*b - 36*a*b^3 + a^4 + 73*b^4 + 30*a^2*b^2))}{(32*(b^5*d^2 + 4*a*b^4*d^2 + a^4*b*d^2 + 6*a^2*b^3*d^2 + 4*a^3*b^2*d^2))} + ((96*b^9*d^2 + 544*a*b^8*d^2 + 1248*a^2*b^7*d^2 + 1440*a^3*b^6*d^2 + 800*a^4*b^5*d^2 + 96*a^5*b^4*d^2 - 96*a^6*b^3*d^2 - 32*a^7*b^2*d^2)/(64*(b^7*d^3 + 6*a*b^6*d^3 + a^6*b*d^3 + 15*a^2*b^5*d^3 + 20*a^3*b^4*d^3 + 15*a^4*b^3*d^3 + 6*a^5*b^2*d^3)) - (\tanh(c + d*x)*(-a*b^3)^{1/2}*(6*a*b + a^2 - 3*b^2)*(256*b^10*d^2 + 1280*a*b^9*d^2 + 2304*a^2*b^8*d^2 + 1280*a^3*b^7*d^2 - 1280*a^4*b^6*d^2 - 2304*$

$$\begin{aligned}
& a^5 b^5 d^2 - 1280 a^6 b^4 d^2 - 256 a^7 b^3 d^2) / (512 (3 a^2 b^5 d + 3 a^3 b^4 d + a^4 b^3 d + a b^6 d) * (b^5 d^2 + 4 a b^4 d^2 + a^4 b d^2 + 6 a^2 b^3 d^2 + 4 a^3 b^2 d^2)) * (-a b^3)^{(1/2)} * (6 a b + a^2 - 3 b^2) / (16 (3 a^2 b^5 d + 3 a^3 b^4 d + a^4 b^3 d + a b^6 d)) * (-a b^3)^{(1/2)} * (6 a b + a^2 - 3 b^2) * i) / (16 (3 a^2 b^5 d + 3 a^3 b^4 d + a^4 b^3 d + a b^6 d)) + (((\tanh(c + d x) * (12 a^3 b - 36 a b^3 + a^4 + 73 b^4 + 30 a^2 b^2)) / (32 (b^5 d^2 + 4 a b^4 d^2 + a^4 b d^2 + 6 a^2 b^3 d^2 + 4 a^3 b^2 d^2)) - (((96 b^9 d^2 + 544 a b^8 d^2 + 1248 a^2 b^7 d^2 + 1440 a^3 b^6 d^2 + 800 a^4 b^5 d^2 + 96 a^5 b^4 d^2 - 96 a^6 b^3 d^2 - 32 a^7 b^2 d^2) / (64 (b^7 d^3 + 6 a b^6 d^3 + a^6 b d^3 + 15 a^2 b^5 d^3 + 20 a^3 b^4 d^3 + 15 a^4 b^3 d^3 + 6 a^5 b^2 d^3)) + (\tanh(c + d x) * (-a b^3)^{(1/2)} * (6 a b + a^2 - 3 b^2) * (256 b^{10} d^2 + 1280 a b^9 d^2 + 2304 a^2 b^8 d^2 + 1280 a^3 b^7 d^2 - 1280 a^4 b^6 d^2 - 2304 a^5 b^5 d^2 - 1280 a^6 b^4 d^2 - 256 a^7 b^3 d^2)) / (512 (3 a^2 b^5 d + 3 a^3 b^4 d + a^4 b^3 d + a b^6 d) * (b^5 d^2 + 4 a b^4 d^2 + a^4 b d^2 + 6 a^2 b^3 d^2 + 4 a^3 b^2 d^2))) * (-a b^3)^{(1/2)} * (6 a b + a^2 - 3 b^2)) / (16 (3 a^2 b^5 d + 3 a^3 b^4 d + a^4 b^3 d + a b^6 d)) * (-a b^3)^{(1/2)} * (6 a b + a^2 - 3 b^2) * i) / (16 (3 a^2 b^5 d + 3 a^3 b^4 d + a^4 b^3 d + a b^6 d)) / ((27 a b^2 + 11 a^2 b + a^3 - 15 b^3) / (32 (b^7 d^3 + 6 a b^6 d^3 + a^6 b d^3 + 15 a^2 b^5 d^3 + 20 a^3 b^4 d^3 + 15 a^4 b^3 d^3 + 6 a^5 b^2 d^3)) + (((\tanh(c + d x) * (12 a^3 b - 36 a b^3 + a^4 + 73 b^4 + 30 a^2 b^2)) / (32 (b^5 d^2 + 4 a b^4 d^2 + a^4 b d^2 + 6 a^2 b^3 d^2 + 4 a^3 b^2 d^2)) + (((96 b^9 d^2 + 544 a b^8 d^2 + 1248 a^2 b^7 d^2 + 1440 a^3 b^6 d^2 + 800 a^4 b^5 d^2 + 96 a^5 b^4 d^2 - 96 a^6 b^3 d^2 - 32 a^7 b^2 d^2) / (64 (b^7 d^3 + 6 a b^6 d^3 + a^6 b d^3 + 15 a^2 b^5 d^3 + 20 a^3 b^4 d^3 + 15 a^4 b^3 d^3 + 6 a^5 b^2 d^3)) - (\tanh(c + d x) * (-a b^3)^{(1/2)} * (6 a b + a^2 - 3 b^2) * (256 b^{10} d^2 + 1280 a b^9 d^2 + 2304 a^2 b^8 d^2 + 1280 a^3 b^7 d^2 - 1280 a^4 b^6 d^2 - 2304 a^5 b^5 d^2 - 1280 a^6 b^4 d^2 - 256 a^7 b^3 d^2)) / (512 (3 a^2 b^5 d + 3 a^3 b^4 d + a^4 b^3 d + a b^6 d) * (b^5 d^2 + 4 a b^4 d^2 + a^4 b d^2 + 6 a^2 b^3 d^2 + 4 a^3 b^2 d^2))) * (-a b^3)^{(1/2)} * (6 a b + a^2 - 3 b^2)) / (16 (3 a^2 b^5 d + 3 a^3 b^4 d + a^4 b^3 d + a b^6 d)) * (-a b^3)^{(1/2)} * (6 a b + a^2 - 3 b^2) * i) / (16 (3 a^2 b^5 d + 3 a^3 b^4 d + a^4 b^3 d + a b^6 d)) - ((\tanh(c + d x) * (12 a^3 b - 36 a b^3 + a^4 + 73 b^4 + 30 a^2 b^2)) / (32 (b^5 d^2 + 4 a b^4 d^2 + a^4 b d^2 + 6 a^2 b^3 d^2 + 4 a^3 b^2 d^2)) - (((96 b^9 d^2 + 544 a b^8 d^2 + 1248 a^2 b^7 d^2 + 1440 a^3 b^6 d^2 + 800 a^4 b^5 d^2 + 96 a^5 b^4 d^2 - 96 a^6 b^3 d^2 - 32 a^7 b^2 d^2) / (64 (b^7 d^3 + 6 a b^6 d^3 + a^6 b d^3 + 15 a^2 b^5 d^3 + 20 a^3 b^4 d^3 + 15 a^4 b^3 d^3 + 6 a^5 b^2 d^3)) + (\tanh(c + d x) * (-a b^3)^{(1/2)} * (6 a b + a^2 - 3 b^2) * (256 b^{10} d^2 + 1280 a b^9 d^2 + 2304 a^2 b^8 d^2 + 1280 a^3 b^7 d^2 - 1280 a^4 b^6 d^2 - 2304 a^5 b^5 d^2 - 1280 a^6 b^4 d^2 - 256 a^7 b^3 d^2)) / (512 (3 a^2 b^5 d + 3 a^3 b^4 d + a^4 b^3 d + a b^6 d) * (b^5 d^2 + 4 a b^4 d^2 + a^4 b d^2 + 6 a^2 b^3 d^2 + 4 a^3 b^2 d^2))) * (-a b^3)^{(1/2)} * (6 a b + a^2 - 3 b^2)) / (16 (3 a^2 b^5 d + 3 a^3 b^4 d + a^4 b^3 d + a b^6 d)) * (-a b^3)^{(1/2)} * (6 a b + a^2 - 3 b^2) * i) / (8 (3 a^2 b^5 d + 3 a^3 b^4 d + a^4 b^3 d + a b^6 d))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)

[Out] Timed out

$$3.193 \quad \int \frac{\tanh^3(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=98

$$\frac{a}{4bd(a+b)(a+b \tanh^2(c+dx))^2} - \frac{1}{2d(a+b)^2(a+b \tanh^2(c+dx))} + \frac{\log(a+b \tanh^2(c+dx))}{2d(a+b)^3} + \frac{\log(\cosh(c+dx))}{d(a+b)}$$

[Out] $\ln(\cosh(d*x+c))/(a+b)^{3/d+1/2} * \ln(a+b*\tanh(d*x+c)^2)/(a+b)^{3/d+1/4} * a/b/(a+b) / d / (a+b*\tanh(d*x+c)^2)^{-1/2} / (a+b)^{2/d} / (a+b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.14, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 446, 77}

$$\frac{a}{4bd(a+b)(a+b \tanh^2(c+dx))^2} - \frac{1}{2d(a+b)^2(a+b \tanh^2(c+dx))} + \frac{\log(a+b \tanh^2(c+dx))}{2d(a+b)^3} + \frac{\log(\cosh(c+dx))}{d(a+b)}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3, x]

[Out] $\text{Log}[\text{Cosh}[c + d*x]] / ((a + b)^{3*d}) + \text{Log}[a + b*\text{Tanh}[c + d*x]^2] / (2*(a + b)^{3*d} + a/(4*b*(a + b)*d*(a + b*\text{Tanh}[c + d*x]^2)^2) - 1/(2*(a + b)^{2*d}*(a + b*\text{Tanh}[c + d*x]^2))$

Rule 77

Int[((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_.)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f*f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(c+dx)}{(a+b\tanh^2(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(1-x^2)(a+bx^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{x}{(1-x)(a+bx)^3} dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} - \frac{a}{(a+b)(a+bx)^3} + \frac{b}{(a+b)^2(a+bx)^2} + \frac{b}{(a+b)^3(a+bx)}\right) dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\log(\cosh(c+dx))}{(a+b)^3d} + \frac{\log(a+b\tanh^2(c+dx))}{2(a+b)^3d} + \frac{a}{4b(a+b)d(a+b\tanh^2(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.50, size = 80, normalized size = 0.82

$$\frac{\frac{a(a+b)^2}{b(a+b\tanh^2(c+dx))^2} - \frac{2(a+b)}{a+b\tanh^2(c+dx)} + 2\log(a+b\tanh^2(c+dx)) + 4\log(\cosh(c+dx))}{4d(a+b)^3}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] (4*Log[Cosh[c + d*x]] + 2*Log[a + b*Tanh[c + d*x]^2] + (a*(a + b)^2)/(b*(a + b*Tanh[c + d*x]^2)^2) - (2*(a + b))/(a + b*Tanh[c + d*x]^2))/(4*(a + b)^3*d)

fricas [B] time = 0.48, size = 2611, normalized size = 26.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -1/2*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 16*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^8 + 4*(2*(a^2 - b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^6 + 4*(14*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d*x - a^2 + b^2)*sinh(d*x + c)^6 + 8*(14*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + 3*(2*(a^2 - b^2)*d*x - a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 - 2*a*b + 3*b^2)*d*x - 2*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^4 + 4*(35*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + (3*a^2 - 2*a*b + 3*b^2)*d*x + 15*(2*(a^2 - b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^2 - 2*a^2 + 2*a*b - 2*b^2)*sinh(d*x + c)^4 + 16*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^5 + 5*(2*(a^2 - b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^3 + ((3*a^2 - 2*a*b + 3*b^2)*d*x - 2*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*d*x + 4*(2*(a^2 - b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^2 + 4*(14*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 15*(2*(a^2 - b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^4 + 2*(a^2 - b^2)*d*x + 6*((3*a^2 - 2*a*b + 3*b^2)*d*x - 2*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^2 - a^2 + b^2)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 4*(a^2 - b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 30*(a^2 - b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a*b + 3*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2

$$\begin{aligned}
& + 2*a*b + b^2)*\cosh(d*x + c)^5 + 10*(a^2 - b^2)*\cosh(d*x + c)^3 + (3*a^2 - \\
& 2*a*b + 3*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(a^2 - b^2)*\cosh(d*x + c \\
&)^2 + 4*(7*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)^6 + 15*(a^2 - b^2)*\cosh(d*x + \\
& c)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*\cosh(d*x + c)^2 + a^2 - b^2)*\sinh(d*x + c) \\
& ^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*\cosh(d*x + c)^7 + 3*(a^2 - \\
& b^2)*\cosh(d*x + c)^5 + (3*a^2 - 2*a*b + 3*b^2)*\cosh(d*x + c)^3 + (a^2 - b^2 \\
&)*\cosh(d*x + c))*\sinh(d*x + c))*\log(2*((a + b)*\cosh(d*x + c)^2 + (a + b)*\si \\
& nh(d*x + c)^2 + a - b)/(\cosh(d*x + c)^2 - 2*\cosh(d*x + c)*\sinh(d*x + c) + s \\
& inh(d*x + c)^2)) + 8*(2*(a^2 + 2*a*b + b^2)*d*x*\cosh(d*x + c)^7 + 3*(2*(a^2 \\
& - b^2)*d*x - a^2 + b^2)*\cosh(d*x + c)^5 + 2*((3*a^2 - 2*a*b + 3*b^2)*d*x - \\
& 2*a^2 + 2*a*b - 2*b^2)*\cosh(d*x + c)^3 + (2*(a^2 - b^2)*d*x - a^2 + b^2)*c \\
& osh(d*x + c))*\sinh(d*x + c))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5* \\
& a*b^4 + b^5)*d*\cosh(d*x + c)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 \\
& + 5*a*b^4 + b^5)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^5 + 5*a^4*b + 10*a^3 \\
& *b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\sinh(d*x + c)^8 + 4*(a^5 + 3*a^4*b + 2 \\
& *a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*\cosh(d*x + c)^6 + 4*(7*(a^5 + 5*a^4 \\
& *b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(d*x + c)^2 + (a^5 + 3* \\
& a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d)*\sinh(d*x + c)^6 + 2*(3*a^ \\
& 5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*\cosh(d*x + c)^4 + \\
& 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(d*x + \\
& c)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*\cosh(d* \\
& x + c))*\sinh(d*x + c)^5 + 2*(35*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + \\
& 5*a*b^4 + b^5)*d*\cosh(d*x + c)^4 + 30*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^ \\
& 3 - 3*a*b^4 - b^5)*d*\cosh(d*x + c)^2 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2 \\
& *b^3 + 7*a*b^4 + 3*b^5)*d)*\sinh(d*x + c)^4 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - \\
& 2*a^2*b^3 - 3*a*b^4 - b^5)*d*\cosh(d*x + c)^2 + 8*(7*(a^5 + 5*a^4*b + 10*a^ \\
& 3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(d*x + c)^5 + 10*(a^5 + 3*a^4*b + \\
& 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*\cosh(d*x + c)^3 + (3*a^5 + 7*a^4* \\
& b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c) \\
& ^3 + 4*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*\cosh(\\
& d*x + c)^6 + 15*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*c \\
& osh(d*x + c)^4 + 3*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b \\
& ^5)*d*\cosh(d*x + c)^2 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - \\
& b^5)*d)*\sinh(d*x + c)^2 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^ \\
& 4 + b^5)*d + 8*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d \\
& *\cosh(d*x + c)^7 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 \\
&)*d*\cosh(d*x + c)^5 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + \\
& 3*b^5)*d*\cosh(d*x + c)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 \\
& - b^5)*d*\cosh(d*x + c))*\sinh(d*x + c))
\end{aligned}$$

giac [B] time = 0.48, size = 245, normalized size = 2.50

$$\frac{2 \log\left(\left|a(e^{2dx+2c})+e^{(-2dx-2c)}\right|+b(e^{2dx+2c})+e^{(-2dx-2c)}\right)+2a-2b}{a^3+3a^2b+3ab^2+b^3} - \frac{3a(e^{2dx+2c})^2+3b(e^{2dx+2c})^2+4a(e^{2dx+2c})+e^{(-2dx-2c)}}{(a^2+2ab+b^2)\left(a(e^{2dx+2c})+e^{(-2dx-2c)}\right)+b(e^{2dx+2c})+e^{(-2dx-2c)}}$$

$4d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out] $\frac{1}{4}*(2*\log(\text{abs}(a*(e^{(2*d*x + 2*c)} + e^{(-2*d*x - 2*c)})) + b*(e^{(2*d*x + 2*c)} + e^{(-2*d*x - 2*c)})) + 2*a - 2*b))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a*(e^{(2*d*x + 2*c)} + e^{(-2*d*x - 2*c)})^2 + 3*b*(e^{(2*d*x + 2*c)} + e^{(-2*d*x - 2*c)})^2 + 4*a*(e^{(2*d*x + 2*c)} + e^{(-2*d*x - 2*c)}) - 4*b*(e^{(2*d*x + 2*c)} + e^{(-2*d*x - 2*c)}) - 4*a - 4*b)/((a^2 + 2*a*b + b^2)*(a*(e^{(2*d*x + 2*c)} + e^{(-2*d*x - 2*c)}) + b*(e^{(2*d*x + 2*c)} + e^{(-2*d*x - 2*c)}) + 2*a - 2*b)^2))/d$

maple [B] time = 0.10, size = 196, normalized size = 2.00

$$\frac{\ln(\tanh(dx+c)-1)}{2d(a+b)^3} - \frac{\ln(1+\tanh(dx+c))}{2d(a+b)^3} + \frac{a^3}{4d(a+b)^3 b (a+b(\tanh^2(dx+c)))^2} + \frac{a^2}{2d(a+b)^3 (a+b(\tanh^2(dx+c)))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x)

[Out] -1/2/d/(a+b)^3*ln(tanh(d*x+c)-1)-1/2/d/(a+b)^3*ln(1+tanh(d*x+c))+1/4/d/(a+b)^3*a^3/b/(a+b*tanh(d*x+c)^2)^2+1/2/d/(a+b)^3*a^2/(a+b*tanh(d*x+c)^2)^2+1/4/d/(a+b)^3*a*b/(a+b*tanh(d*x+c)^2)^2+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^3/d-1/2/d/(a+b)^3*a/(a+b*tanh(d*x+c)^2)-1/2/d/(a+b)^3/(a+b*tanh(d*x+c)^2)*b

maxima [B] time = 0.37, size = 384, normalized size = 3.92

$$\frac{dx+c}{(a^3+3a^2b+3ab^2+b^3)d} + \frac{1}{(a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5+4(a^5+3a^4b+2a^3b^2-2a^2b^3-3ab^4))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] (d*x+c)/((a^3+3*a^2*b+3*a*b^2+b^3)*d)+2*((a^2-b^2)*e^(-2*d*x-2*c)+2*(a^2-a*b+b^2)*e^(-4*d*x-4*c)+(a^2-b^2)*e^(-6*d*x-6*c))/((a^5+5*a^4*b+10*a^3*b^2+10*a^2*b^3+5*a*b^4+b^5+4*(a^5+3*a^4*b+2*a^3*b^2-2*a^2*b^3-3*a*b^4-b^5)*e^(-2*d*x-2*c)+2*(3*a^5+7*a^4*b+6*a^3*b^2+6*a^2*b^3+7*a*b^4+3*b^5)*e^(-4*d*x-4*c)+4*(a^5+3*a^4*b+2*a^3*b^2-2*a^2*b^3-3*a*b^4-b^5)*e^(-6*d*x-6*c)+(a^5+5*a^4*b+10*a^3*b^2+10*a^2*b^3+5*a*b^4+b^5)*e^(-8*d*x-8*c))*d)+1/2*log(2*(a-b)*e^(-2*d*x-2*c)+(a+b)*e^(-4*d*x-4*c)+a+b)/((a^3+3*a^2*b+3*a*b^2+b^3)*d)

mupad [B] time = 1.81, size = 397, normalized size = 4.05

$$\frac{-a^3+b^3 \left(2 \tanh(c+dx)^2 + \tanh(c+dx)^4 \operatorname{atan} \left(\frac{a \tanh(c+dx)^2 + b \tanh(c+dx)^2}{2a - a \tanh(c+dx)^2 + b \tanh(c+dx)^2} \right) 4i \right) + a b^2}{4 d a^5 b + 8 d a^4 b^2 \tanh(c+dx)^2 + 12 d a^4 b^2 + 4 d a^3 b^3 \tanh(c+dx)^4 + 24 d a^3 b^3 \tanh(c+dx)^2 + 12 d a^3 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c+d*x)^3/(a+b*tanh(c+d*x)^2)^3,x)

[Out] -(b^3*(tanh(c+d*x)^4*atan((a*tanh(c+d*x)^2*1i+b*tanh(c+d*x)^2*1i)/(2*a-a*tanh(c+d*x)^2+b*tanh(c+d*x)^2))*4i+2*tanh(c+d*x)^2)-a^3+a*b^2*(tanh(c+d*x)^2*atan((a*tanh(c+d*x)^2*1i+b*tanh(c+d*x)^2*1i)/(2*a-a*tanh(c+d*x)^2+b*tanh(c+d*x)^2))*8i+2*tanh(c+d*x)^2+1)/(2*a-a*tanh(c+d*x)^2+b*tanh(c+d*x)^2)*4i)/(4*a^2*b^4*d+12*a^3*b^3*d+12*a^4*b^2*d+4*b^6*d*tanh(c+d*x)^4+4*a^5*b*d+24*a^2*b^4*d*tanh(c+d*x)^2+24*a^3*b^3*d*tanh(c+d*x)^2+8*a^4*b^2*d*tanh(c+d*x)^2+12*a^2*b^4*d*tanh(c+d*x)^4+4*a^3*b^3*d*tanh(c+d*x)^4+8*a*b^5*d*tanh(c+d*x)^2+12*a*b^5*d*tanh(c+d*x)^4)

sympy [A] time = 157.41, size = 3434, normalized size = 35.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)

```
[Out] Piecewise((zoo*x/tanh(c)**3, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - log(tan
h(c + d*x) + 1)/d - tanh(c + d*x)**2/(2*d))/a**3, Eq(b, 0)), (3*tanh(c + d*
x)**2/(12*b**3*d*tanh(c + d*x)**6 - 36*b**3*d*tanh(c + d*x)**4 + 36*b**3*d*
tanh(c + d*x)**2 - 12*b**3*d) - 1/(12*b**3*d*tanh(c + d*x)**6 - 36*b**3*d*t
anh(c + d*x)**4 + 36*b**3*d*tanh(c + d*x)**2 - 12*b**3*d), Eq(a, -b)), (x*t
anh(c)**3/(a + b*tanh(c)**2)**3, Eq(d, 0)), (a**3/(4*a**5*b*d + 8*a**4*b**2
*d*tanh(c + d*x)**2 + 12*a**4*b**2*d + 4*a**3*b**3*d*tanh(c + d*x)**4 + 24*
a**3*b**3*d*tanh(c + d*x)**2 + 12*a**3*b**3*d + 12*a**2*b**4*d*tanh(c + d*x
)**4 + 24*a**2*b**4*d*tanh(c + d*x)**2 + 4*a**2*b**4*d + 12*a*b**5*d*tanh(c
 + d*x)**4 + 8*a*b**5*d*tanh(c + d*x)**2 + 4*b**6*d*tanh(c + d*x)**4) + 4*a
**2*b*d*x/(4*a**5*b*d + 8*a**4*b**2*d*tanh(c + d*x)**2 + 12*a**4*b**2*d + 4
*a**3*b**3*d*tanh(c + d*x)**4 + 24*a**3*b**3*d*tanh(c + d*x)**2 + 12*a**3*b
**3*d + 12*a**2*b**4*d*tanh(c + d*x)**4 + 24*a**2*b**4*d*tanh(c + d*x)**2 +
4*a**2*b**4*d + 12*a*b**5*d*tanh(c + d*x)**4 + 8*a*b**5*d*tanh(c + d*x)**2
 + 4*b**6*d*tanh(c + d*x)**4) + 2*a**2*b*log(-I*sqrt(a)*sqrt(1/b) + tanh(c
 + d*x))/(4*a**5*b*d + 8*a**4*b**2*d*tanh(c + d*x)**2 + 12*a**4*b**2*d + 4*a
**3*b**3*d*tanh(c + d*x)**4 + 24*a**3*b**3*d*tanh(c + d*x)**2 + 12*a**3*b**
3*d + 12*a**2*b**4*d*tanh(c + d*x)**4 + 24*a**2*b**4*d*tanh(c + d*x)**2 + 4
*a**2*b**4*d + 12*a*b**5*d*tanh(c + d*x)**4 + 8*a*b**5*d*tanh(c + d*x)**2 +
4*b**6*d*tanh(c + d*x)**4) + 2*a**2*b*log(I*sqrt(a)*sqrt(1/b) + tanh(c + d
*x))/(4*a**5*b*d + 8*a**4*b**2*d*tanh(c + d*x)**2 + 12*a**4*b**2*d + 4*a**3
*b**3*d*tanh(c + d*x)**4 + 24*a**3*b**3*d*tanh(c + d*x)**2 + 12*a**3*b**3*d
 + 12*a**2*b**4*d*tanh(c + d*x)**4 + 24*a**2*b**4*d*tanh(c + d*x)**2 + 4*a*
**2*b**4*d + 12*a*b**5*d*tanh(c + d*x)**4 + 8*a*b**5*d*tanh(c + d*x)**2 + 4*
b**6*d*tanh(c + d*x)**4) - 4*a**2*b*log(tanh(c + d*x) + 1)/(4*a**5*b*d + 8*
a**4*b**2*d*tanh(c + d*x)**2 + 12*a**4*b**2*d + 4*a**3*b**3*d*tanh(c + d*x)
**4 + 24*a**3*b**3*d*tanh(c + d*x)**2 + 12*a**3*b**3*d + 12*a**2*b**4*d*tan
h(c + d*x)**4 + 24*a**2*b**4*d*tanh(c + d*x)**2 + 4*a**2*b**4*d + 12*a*b**5
*d*tanh(c + d*x)**4 + 8*a*b**5*d*tanh(c + d*x)**2 + 4*b**6*d*tanh(c + d*x)*
**4) + 8*a*b**2*d*x*tanh(c + d*x)**2/(4*a**5*b*d + 8*a**4*b**2*d*tanh(c + d*
x)**2 + 12*a**4*b**2*d + 4*a**3*b**3*d*tanh(c + d*x)**4 + 24*a**3*b**3*d*ta
nh(c + d*x)**2 + 12*a**3*b**3*d + 12*a**2*b**4*d*tanh(c + d*x)**4 + 24*a**2
*b**4*d*tanh(c + d*x)**2 + 4*a**2*b**4*d + 12*a*b**5*d*tanh(c + d*x)**4 + 8
*a*b**5*d*tanh(c + d*x)**2 + 4*b**6*d*tanh(c + d*x)**4) + 4*a*b**2*log(-I*s
qrt(a)*sqrt(1/b) + tanh(c + d*x))*tanh(c + d*x)**2/(4*a**5*b*d + 8*a**4*b**
2*d*tanh(c + d*x)**2 + 12*a**4*b**2*d + 4*a**3*b**3*d*tanh(c + d*x)**4 + 24
*a**3*b**3*d*tanh(c + d*x)**2 + 12*a**3*b**3*d + 12*a**2*b**4*d*tanh(c + d*
x)**4 + 24*a**2*b**4*d*tanh(c + d*x)**2 + 4*a**2*b**4*d + 12*a*b**5*d*tanh(
c + d*x)**4 + 8*a*b**5*d*tanh(c + d*x)**2 + 4*b**6*d*tanh(c + d*x)**4) + 4*
a*b**2*log(I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))*tanh(c + d*x)**2/(4*a**5*b*
d + 8*a**4*b**2*d*tanh(c + d*x)**2 + 12*a**4*b**2*d + 4*a**3*b**3*d*tanh(c
 + d*x)**4 + 24*a**3*b**3*d*tanh(c + d*x)**2 + 12*a**3*b**3*d + 12*a**2*b**4
*d*tanh(c + d*x)**4 + 24*a**2*b**4*d*tanh(c + d*x)**2 + 4*a**2*b**4*d + 12*
a*b**5*d*tanh(c + d*x)**4 + 8*a*b**5*d*tanh(c + d*x)**2 + 4*b**6*d*tanh(c +
d*x)**4) - 8*a*b**2*log(tanh(c + d*x) + 1)*tanh(c + d*x)**2/(4*a**5*b*d +
8*a**4*b**2*d*tanh(c + d*x)**2 + 12*a**4*b**2*d + 4*a**3*b**3*d*tanh(c + d*
x)**4 + 24*a**3*b**3*d*tanh(c + d*x)**2 + 12*a**3*b**3*d + 12*a**2*b**4*d*t
anh(c + d*x)**4 + 24*a**2*b**4*d*tanh(c + d*x)**2 + 4*a**2*b**4*d + 12*a*b*
**5*d*tanh(c + d*x)**4 + 8*a*b**5*d*tanh(c + d*x)**2 + 4*b**6*d*tanh(c + d*x
)**4) - 2*a*b**2*tanh(c + d*x)**2/(4*a**5*b*d + 8*a**4*b**2*d*tanh(c + d*x)
**2 + 12*a**4*b**2*d + 4*a**3*b**3*d*tanh(c + d*x)**4 + 24*a**3*b**3*d*tanh
(c + d*x)**2 + 12*a**3*b**3*d + 12*a**2*b**4*d*tanh(c + d*x)**4 + 24*a**2*b
**4*d*tanh(c + d*x)**2 + 4*a**2*b**4*d + 12*a*b**5*d*tanh(c + d*x)**4 + 8*a
*b**5*d*tanh(c + d*x)**2 + 4*b**6*d*tanh(c + d*x)**4) - a*b**2/(4*a**5*b*d
 + 8*a**4*b**2*d*tanh(c + d*x)**2 + 12*a**4*b**2*d + 4*a**3*b**3*d*tanh(c +
d*x)**4 + 24*a**3*b**3*d*tanh(c + d*x)**2 + 12*a**3*b**3*d + 12*a**2*b**4*d
*tanh(c + d*x)**4 + 24*a**2*b**4*d*tanh(c + d*x)**2 + 4*a**2*b**4*d + 12*a*
b**5*d*tanh(c + d*x)**4 + 8*a*b**5*d*tanh(c + d*x)**2 + 4*b**6*d*tanh(c + d
*x)**4) + 4*b**3*d*x*tanh(c + d*x)**4/(4*a**5*b*d + 8*a**4*b**2*d*tanh(c +
```

```

d*x)**2 + 12*a**4*b**2*d + 4*a**3*b**3*d*tanh(c + d*x)**4 + 24*a**3*b**3*d*
tanh(c + d*x)**2 + 12*a**3*b**3*d + 12*a**2*b**4*d*tanh(c + d*x)**4 + 24*a*
*2*b**4*d*tanh(c + d*x)**2 + 4*a**2*b**4*d + 12*a*b**5*d*tanh(c + d*x)**4 +
8*a*b**5*d*tanh(c + d*x)**2 + 4*b**6*d*tanh(c + d*x)**4) + 2*b**3*log(-I*s
qrt(a)*sqrt(1/b) + tanh(c + d*x))*tanh(c + d*x)**4/(4*a**5*b*d + 8*a**4*b**
2*d*tanh(c + d*x)**2 + 12*a**4*b**2*d + 4*a**3*b**3*d*tanh(c + d*x)**4 + 24
*a**3*b**3*d*tanh(c + d*x)**2 + 12*a**3*b**3*d + 12*a**2*b**4*d*tanh(c + d*
x)**4 + 24*a**2*b**4*d*tanh(c + d*x)**2 + 4*a**2*b**4*d + 12*a*b**5*d*tanh(
c + d*x)**4 + 8*a*b**5*d*tanh(c + d*x)**2 + 4*b**6*d*tanh(c + d*x)**4) + 2*
b**3*log(I*sqrt(a)*sqrt(1/b) + tanh(c + d*x))*tanh(c + d*x)**4/(4*a**5*b*d
+ 8*a**4*b**2*d*tanh(c + d*x)**2 + 12*a**4*b**2*d + 4*a**3*b**3*d*tanh(c +
d*x)**4 + 24*a**3*b**3*d*tanh(c + d*x)**2 + 12*a**3*b**3*d + 12*a**2*b**4*d
*tanh(c + d*x)**4 + 24*a**2*b**4*d*tanh(c + d*x)**2 + 4*a**2*b**4*d + 12*a*
b**5*d*tanh(c + d*x)**4 + 8*a*b**5*d*tanh(c + d*x)**2 + 4*b**6*d*tanh(c + d
*x)**4) - 4*b**3*log(tanh(c + d*x) + 1)*tanh(c + d*x)**4/(4*a**5*b*d + 8*a*
*4*b**2*d*tanh(c + d*x)**2 + 12*a**4*b**2*d + 4*a**3*b**3*d*tanh(c + d*x)**
4 + 24*a**3*b**3*d*tanh(c + d*x)**2 + 12*a**3*b**3*d + 12*a**2*b**4*d*tanh(
c + d*x)**4 + 24*a**2*b**4*d*tanh(c + d*x)**2 + 4*a**2*b**4*d + 12*a*b**5*d
*tanh(c + d*x)**4 + 8*a*b**5*d*tanh(c + d*x)**2 + 4*b**6*d*tanh(c + d*x)**4
) - 2*b**3*tanh(c + d*x)**2/(4*a**5*b*d + 8*a**4*b**2*d*tanh(c + d*x)**2 +
12*a**4*b**2*d + 4*a**3*b**3*d*tanh(c + d*x)**4 + 24*a**3*b**3*d*tanh(c + d
*x)**2 + 12*a**3*b**3*d + 12*a**2*b**4*d*tanh(c + d*x)**4 + 24*a**2*b**4*d*
tanh(c + d*x)**2 + 4*a**2*b**4*d + 12*a*b**5*d*tanh(c + d*x)**4 + 8*a*b**5*
d*tanh(c + d*x)**2 + 4*b**6*d*tanh(c + d*x)**4), True))

```

$$3.194 \quad \int \frac{\tanh^2(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=137

$$\frac{(3a^2 - 6ab - b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}\sqrt{b}d(a+b)^3} - \frac{(3a-b) \tanh(c+dx)}{8ad(a+b)^2(a+b \tanh^2(c+dx))} - \frac{\tanh(c+dx)}{4d(a+b)(a+b \tanh^2(c+dx))^2}$$

[Out] x/(a+b)^3-1/8*(3*a^2-6*a*b-b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)
/(a+b)^3/d/b^(1/2)-1/4*tanh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c)^2)^2-1/8*(3*a-b
)*tanh(d*x+c)/a/(a+b)^2/d/(a+b*tanh(d*x+c)^2)

Rubi [A] time = 0.16, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3670, 471, 527, 522, 206, 205}

$$\frac{(3a^2 - 6ab - b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}\sqrt{b}d(a+b)^3} - \frac{(3a-b) \tanh(c+dx)}{8ad(a+b)^2(a+b \tanh^2(c+dx))} - \frac{\tanh(c+dx)}{4d(a+b)(a+b \tanh^2(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Int[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] x/(a + b)^3 - ((3*a^2 - 6*a*b - b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(3/2)*Sqrt[b]*(a + b)^3*d - Tanh[c + d*x]/(4*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) - ((3*a - b)*Tanh[c + d*x])/(8*a*(a + b)^2*d*(a + b*Tanh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 471

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n-1)*(e*x)^(m-n+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(n*(b*c-a*d)*(p+1)), x] - Dist[e^n/(n*(b*c-a*d)*(p+1)), Int[(e*x)^(m-n)*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[c*(m-n+1)+d*(m+n*(p+q+1)+1]*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m-n+1] && GtQ[m-n+1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3670

```
Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*((c_)*tan[(e_) + (f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p/(c^2 + f*f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))
```

Rubi steps

$$\int \frac{\tanh^2(c + dx)}{(a + b \tanh^2(c + dx))^3} dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)(a+bx^2)^3} dx, x, \tanh(c + dx)\right)}{d}$$

$$= -\frac{\tanh(c + dx)}{4(a + b)d(a + b \tanh^2(c + dx))^2} + \frac{\text{Subst}\left(\int \frac{1+3x^2}{(1-x^2)(a+bx^2)^2} dx, x, \tanh(c + dx)\right)}{4(a + b)d}$$

$$= -\frac{\tanh(c + dx)}{4(a + b)d(a + b \tanh^2(c + dx))^2} - \frac{(3a - b) \tanh(c + dx)}{8a(a + b)^2d(a + b \tanh^2(c + dx))} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{4(a + b)d}$$

$$= -\frac{\tanh(c + dx)}{4(a + b)d(a + b \tanh^2(c + dx))^2} - \frac{(3a - b) \tanh(c + dx)}{8a(a + b)^2d(a + b \tanh^2(c + dx))} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{4(a + b)d}$$

$$= \frac{x}{(a + b)^3} - \frac{(3a^2 - 6ab - b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}\sqrt{b}(a + b)^3d} - \frac{\tanh(c + dx)}{4(a + b)d(a + b \tanh^2(c + dx))}$$

Mathematica [A] time = 1.17, size = 137, normalized size = 1.00

$$\frac{(-3a^2+6ab+b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2} \sqrt{b}} - \frac{(5a-b)(a+b) \sinh(2(c+dx))}{a((a+b) \cosh(2(c+dx))+a-b)} - \frac{4b(a+b) \sinh(2(c+dx))}{((a+b) \cosh(2(c+dx))+a-b)^2} + 8(c + dx)}{8d(a + b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

```
[Out] (8*(c + d*x) + ((-3*a^2 + 6*a*b + b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*Sqrt[b]) - (4*b*(a + b)*Sinh[2*(c + d*x)]/(a - b + (a + b)*Cosh[2*(c + d*x)])^2 - ((5*a - b)*(a + b)*Sinh[2*(c + d*x)]/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(8*(a + b)^3*d)
```

fricas [B] time = 0.58, size = 7791, normalized size = 56.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c))^3,x, algorithm="fricas")

[Out] [1/16*(16*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^8 + 128*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 16*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*sinh(d*x + c)^8 + 4*(5*a^4*b - 5*a^3*b^2 - 9*a^2*b^3 + a*b^4 + 16*(a^4*b - a^2*b^3)*d*x)*cosh(d*x + c)^6 + 4*(5*a^4*b - 5*a^3*b^2 - 9*a^2*b^3 + a*b^4 + 112*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^2 + 16*(a^4*b - a^2*b^3)*d*x)*sinh(d*x + c)^6 + 8*(112*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^3 + 3*(5*a^4*b - 5*a^3*b^2 - 9*a^2*b^3 + a*b^4 + 16*(a^4*b - a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 20*a^4*b + 36*a^3*b^2 + 12*a^2*b^3 - 4*a*b^4 + 4*(15*a^4*b - 13*a^3*b^2 + 17*a^2*b^3 - 3*a*b^4 + 8*(3*a^4*b - 2*a^3*b^2 + 3*a^2*b^3)*d*x)*cosh(d*x + c)^4 + 4*(280*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^4 + 15*a^4*b - 13*a^3*b^2 + 17*a^2*b^3 - 3*a*b^4 + 8*(3*a^4*b - 2*a^3*b^2 + 3*a^2*b^3)*d*x + 15*(5*a^4*b - 5*a^3*b^2 - 9*a^2*b^3 + a*b^4 + 16*(a^4*b - a^2*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(56*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^5 + 5*(5*a^4*b - 5*a^3*b^2 - 9*a^2*b^3 + a*b^4 + 16*(a^4*b - a^2*b^3)*d*x)*cosh(d*x + c)^3 + (15*a^4*b - 13*a^3*b^2 + 17*a^2*b^3 - 3*a*b^4 + 8*(3*a^4*b - 2*a^3*b^2 + 3*a^2*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 16*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x + 4*(15*a^4*b + a^3*b^2 - 11*a^2*b^3 + 3*a*b^4 + 16*(a^4*b - a^2*b^3)*d*x)*cosh(d*x + c)^2 + 4*(112*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^6 + 15*a^4*b + a^3*b^2 - 11*a^2*b^3 + 3*a*b^4 + 15*(5*a^4*b - 5*a^3*b^2 - 9*a^2*b^3 + a*b^4 + 16*(a^4*b - a^2*b^3)*d*x)*cosh(d*x + c)^4 + 16*(a^4*b - a^2*b^3)*d*x + 6*(15*a^4*b - 13*a^3*b^2 + 17*a^2*b^3 - 3*a*b^4 + 8*(3*a^4*b - 2*a^3*b^2 + 3*a^2*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^8 + 8*(3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*sinh(d*x + c)^8 + 4*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4)*cosh(d*x + c)^6 + 4*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4 + 7*(3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^3 + 3*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(9*a^4 - 24*a^3*b + 18*a^2*b^2 - 16*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 2*(35*(3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^4 + 9*a^4 - 24*a^3*b + 18*a^2*b^2 - 16*a*b^3 - 3*b^4 + 30*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4 + 8*(7*(3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^5 + 10*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4)*cosh(d*x + c)^3 + (9*a^4 - 24*a^3*b + 18*a^2*b^2 - 16*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4)*cosh(d*x + c)^2 + 4*(7*(3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^6 + 15*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4)*cosh(d*x + c)^4 + 3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4 + 3*(9*a^4 - 24*a^3*b + 18*a^2*b^2 - 16*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4)*cosh(d*x + c)^7 + 3*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4)*cosh(d*x + c)^5 + (9*a^4 - 24*a^3*b + 18*a^2*b^2 - 16*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 8*(16*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*x*cosh(d*x + c)^7 + 3*(5*a^4*b - 5*a^3*b^2 - 9*a^2*b^3 + a*b^4 + 16*(a^4*b

$$\begin{aligned}
& b - a^2 b^3) d x) \cosh(d x + c)^5 + 2*(15 a^4 b - 13 a^3 b^2 + 17 a^2 b^3 - \\
& 3 a b^4 + 8*(3 a^4 b - 2 a^3 b^2 + 3 a^2 b^3) d x) \cosh(d x + c)^3 + (15 a \\
& ^4 b + a^3 b^2 - 11 a^2 b^3 + 3 a b^4 + 16*(a^4 b - a^2 b^3) d x) \cosh(d x \\
& + c)) \sinh(d x + c) / ((a^7 b + 5 a^6 b^2 + 10 a^5 b^3 + 10 a^4 b^4 + 5 a^3 \\
& b^5 + a^2 b^6) d \cosh(d x + c)^8 + 8*(a^7 b + 5 a^6 b^2 + 10 a^5 b^3 + 10 a \\
& ^4 b^4 + 5 a^3 b^5 + a^2 b^6) d \cosh(d x + c) \sinh(d x + c)^7 + (a^7 b + 5 \\
& a^6 b^2 + 10 a^5 b^3 + 10 a^4 b^4 + 5 a^3 b^5 + a^2 b^6) d \sinh(d x + c)^8 \\
& + 4*(a^7 b + 3 a^6 b^2 + 2 a^5 b^3 - 2 a^4 b^4 - 3 a^3 b^5 - a^2 b^6) d \cos \\
& h(d x + c)^6 + 4*(7*(a^7 b + 5 a^6 b^2 + 10 a^5 b^3 + 10 a^4 b^4 + 5 a^3 b^ \\
& 5 + a^2 b^6) d \cosh(d x + c)^2 + (a^7 b + 3 a^6 b^2 + 2 a^5 b^3 - 2 a^4 b^4 \\
& - 3 a^3 b^5 - a^2 b^6) d) \sinh(d x + c)^6 + 2*(3 a^7 b + 7 a^6 b^2 + 6 a^5 \\
& b^3 + 6 a^4 b^4 + 7 a^3 b^5 + 3 a^2 b^6) d \cosh(d x + c)^4 + 8*(7*(a^7 b + \\
& 5 a^6 b^2 + 10 a^5 b^3 + 10 a^4 b^4 + 5 a^3 b^5 + a^2 b^6) d \cosh(d x + c) \\
& ^3 + 3*(a^7 b + 3 a^6 b^2 + 2 a^5 b^3 - 2 a^4 b^4 - 3 a^3 b^5 - a^2 b^6) d * \\
& \cosh(d x + c)) \sinh(d x + c)^5 + 2*(35*(a^7 b + 5 a^6 b^2 + 10 a^5 b^3 + 10 \\
& a^4 b^4 + 5 a^3 b^5 + a^2 b^6) d \cosh(d x + c)^4 + 30*(a^7 b + 3 a^6 b^2 + \\
& 2 a^5 b^3 - 2 a^4 b^4 - 3 a^3 b^5 - a^2 b^6) d \cosh(d x + c)^2 + (3 a^7 b \\
& + 7 a^6 b^2 + 6 a^5 b^3 + 6 a^4 b^4 + 7 a^3 b^5 + 3 a^2 b^6) d) \sinh(d x + \\
& c)^4 + 4*(a^7 b + 3 a^6 b^2 + 2 a^5 b^3 - 2 a^4 b^4 - 3 a^3 b^5 - a^2 b^6) * \\
& d \cosh(d x + c)^2 + 8*(7*(a^7 b + 5 a^6 b^2 + 10 a^5 b^3 + 10 a^4 b^4 + 5 a \\
& ^3 b^5 + a^2 b^6) d \cosh(d x + c)^5 + 10*(a^7 b + 3 a^6 b^2 + 2 a^5 b^3 - 2 \\
& a^4 b^4 - 3 a^3 b^5 - a^2 b^6) d \cosh(d x + c)^3 + (3 a^7 b + 7 a^6 b^2 + \\
& 6 a^5 b^3 + 6 a^4 b^4 + 7 a^3 b^5 + 3 a^2 b^6) d \cosh(d x + c)) \sinh(d x + \\
& c)^3 + 4*(7*(a^7 b + 5 a^6 b^2 + 10 a^5 b^3 + 10 a^4 b^4 + 5 a^3 b^5 + a^2 \\
& b^6) d \cosh(d x + c)^6 + 15*(a^7 b + 3 a^6 b^2 + 2 a^5 b^3 - 2 a^4 b^4 - 3 \\
& a^3 b^5 - a^2 b^6) d \cosh(d x + c)^4 + 3*(3 a^7 b + 7 a^6 b^2 + 6 a^5 b^3 + \\
& 6 a^4 b^4 + 7 a^3 b^5 + 3 a^2 b^6) d \cosh(d x + c)^2 + (a^7 b + 3 a^6 b^2 \\
& + 2 a^5 b^3 - 2 a^4 b^4 - 3 a^3 b^5 - a^2 b^6) d) \sinh(d x + c)^2 + (a^7 b \\
& + 5 a^6 b^2 + 10 a^5 b^3 + 10 a^4 b^4 + 5 a^3 b^5 + a^2 b^6) d + 8*((a^7 b \\
& + 5 a^6 b^2 + 10 a^5 b^3 + 10 a^4 b^4 + 5 a^3 b^5 + a^2 b^6) d \cosh(d x + c \\
&)^7 + 3*(a^7 b + 3 a^6 b^2 + 2 a^5 b^3 - 2 a^4 b^4 - 3 a^3 b^5 - a^2 b^6) d \\
& * \cosh(d x + c)^5 + (3 a^7 b + 7 a^6 b^2 + 6 a^5 b^3 + 6 a^4 b^4 + 7 a^3 b^5 \\
& + 3 a^2 b^6) d \cosh(d x + c)^3 + (a^7 b + 3 a^6 b^2 + 2 a^5 b^3 - 2 a^4 b^4 \\
& - 3 a^3 b^5 - a^2 b^6) d \cosh(d x + c)) \sinh(d x + c)), 1/8*(8*(a^4 b + 2 \\
& a^3 b^2 + a^2 b^3) d x \cosh(d x + c)^8 + 64*(a^4 b + 2 a^3 b^2 + a^2 b^3) * \\
& d x \cosh(d x + c) \sinh(d x + c)^7 + 8*(a^4 b + 2 a^3 b^2 + a^2 b^3) d x \sin \\
& h(d x + c)^8 + 2*(5 a^4 b - 5 a^3 b^2 - 9 a^2 b^3 + a b^4 + 16*(a^4 b - a^2 \\
& b^3) d x) \cosh(d x + c)^6 + 2*(5 a^4 b - 5 a^3 b^2 - 9 a^2 b^3 + a b^4 + 1 \\
& 12*(a^4 b + 2 a^3 b^2 + a^2 b^3) d x \cosh(d x + c)^2 + 16*(a^4 b - a^2 b^3) \\
& * d x) \sinh(d x + c)^6 + 4*(112*(a^4 b + 2 a^3 b^2 + a^2 b^3) d x \cosh(d x + \\
& c)^3 + 3*(5 a^4 b - 5 a^3 b^2 - 9 a^2 b^3 + a b^4 + 16*(a^4 b - a^2 b^3) d \\
& x) \cosh(d x + c)) \sinh(d x + c)^5 + 10 a^4 b + 18 a^3 b^2 + 6 a^2 b^3 - 2 \\
& a b^4 + 2*(15 a^4 b - 13 a^3 b^2 + 17 a^2 b^3 - 3 a b^4 + 8*(3 a^4 b - 2 a^ \\
& 3 b^2 + 3 a^2 b^3) d x) \cosh(d x + c)^4 + 2*(280*(a^4 b + 2 a^3 b^2 + a^2 b \\
& ^3) d x \cosh(d x + c)^4 + 15 a^4 b - 13 a^3 b^2 + 17 a^2 b^3 - 3 a b^4 + 8 * \\
& (3 a^4 b - 2 a^3 b^2 + 3 a^2 b^3) d x + 15*(5 a^4 b - 5 a^3 b^2 - 9 a^2 b^3 \\
& + a b^4 + 16*(a^4 b - a^2 b^3) d x) \cosh(d x + c)^2) \sinh(d x + c)^4 + 8*(\\
& 56*(a^4 b + 2 a^3 b^2 + a^2 b^3) d x \cosh(d x + c)^5 + 5*(5 a^4 b - 5 a^3 b \\
& ^2 - 9 a^2 b^3 + a b^4 + 16*(a^4 b - a^2 b^3) d x) \cosh(d x + c)^3 + (15 a^ \\
& 4 b - 13 a^3 b^2 + 17 a^2 b^3 - 3 a b^4 + 8*(3 a^4 b - 2 a^3 b^2 + 3 a^2 b^ \\
& 3) d x) \cosh(d x + c)) \sinh(d x + c)^3 + 8*(a^4 b + 2 a^3 b^2 + a^2 b^3) d * \\
& x + 2*(15 a^4 b + a^3 b^2 - 11 a^2 b^3 + 3 a b^4 + 16*(a^4 b - a^2 b^3) d x) \\
&) \cosh(d x + c)^2 + 2*(112*(a^4 b + 2 a^3 b^2 + a^2 b^3) d x \cosh(d x + c)^ \\
& 6 + 15 a^4 b + a^3 b^2 - 11 a^2 b^3 + 3 a b^4 + 15*(5 a^4 b - 5 a^3 b^2 - 9 \\
& a^2 b^3 + a b^4 + 16*(a^4 b - a^2 b^3) d x) \cosh(d x + c)^4 + 16*(a^4 b - \\
& a^2 b^3) d x + 6*(15 a^4 b - 13 a^3 b^2 + 17 a^2 b^3 - 3 a b^4 + 8*(3 a^4 b \\
& - 2 a^3 b^2 + 3 a^2 b^3) d x) \cosh(d x + c)^2) \sinh(d x + c)^2 - ((3 a^4 - \\
& 10 a^2 b^2 - 8 a b^3 - b^4) \cosh(d x + c)^8 + 8*(3 a^4 - 10 a^2 b^2 - 8 a * \\
& b^3 - b^4) \cosh(d x + c) \sinh(d x + c)^7 + (3 a^4 - 10 a^2 b^2 - 8 a b^3 -
\end{aligned}$$

$$\begin{aligned}
& b^4) * \sinh(dx + c)^8 + 4*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4) * \cosh \\
& (dx + c)^6 + 4*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4 + 7*(3*a^4 - 1 \\
& 0*a^2*b^2 - 8*a*b^3 - b^4) * \cosh(dx + c)^2) * \sinh(dx + c)^6 + 8*(7*(3*a^4 - \\
& 10*a^2*b^2 - 8*a*b^3 - b^4) * \cosh(dx + c)^3 + 3*(3*a^4 - 6*a^3*b - 4*a^2*b \\
& ^2 + 6*a*b^3 + b^4) * \cosh(dx + c)) * \sinh(dx + c)^5 + 2*(9*a^4 - 24*a^3*b + \\
& 18*a^2*b^2 - 16*a*b^3 - 3*b^4) * \cosh(dx + c)^4 + 2*(35*(3*a^4 - 10*a^2*b^2 \\
& - 8*a*b^3 - b^4) * \cosh(dx + c)^4 + 9*a^4 - 24*a^3*b + 18*a^2*b^2 - 16*a*b^3 \\
& - 3*b^4 + 30*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4) * \cosh(dx + c)^2 \\
&) * \sinh(dx + c)^4 + 3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4 + 8*(7*(3*a^4 - 10*a \\
& ^2*b^2 - 8*a*b^3 - b^4) * \cosh(dx + c)^5 + 10*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + \\
& 6*a*b^3 + b^4) * \cosh(dx + c)^3 + (9*a^4 - 24*a^3*b + 18*a^2*b^2 - 16*a*b^3 \\
& - 3*b^4) * \cosh(dx + c)) * \sinh(dx + c)^3 + 4*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + \\
& 6*a*b^3 + b^4) * \cosh(dx + c)^2 + 4*(7*(3*a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4) \\
& * \cosh(dx + c)^6 + 15*(3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4) * \cosh(dx \\
& x + c)^4 + 3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^3 + b^4 + 3*(9*a^4 - 24*a^3* \\
& b + 18*a^2*b^2 - 16*a*b^3 - 3*b^4) * \cosh(dx + c)^2) * \sinh(dx + c)^2 + 8*((3 \\
& *a^4 - 10*a^2*b^2 - 8*a*b^3 - b^4) * \cosh(dx + c)^7 + 3*(3*a^4 - 6*a^3*b - 4 \\
& *a^2*b^2 + 6*a*b^3 + b^4) * \cosh(dx + c)^5 + (9*a^4 - 24*a^3*b + 18*a^2*b^2 \\
& - 16*a*b^3 - 3*b^4) * \cosh(dx + c)^3 + (3*a^4 - 6*a^3*b - 4*a^2*b^2 + 6*a*b^ \\
& 3 + b^4) * \cosh(dx + c)) * \sinh(dx + c)) * \sqrt{a*b} * \arctan(1/2*((a + b) * \cosh(dx \\
& *x + c)^2 + 2*(a + b) * \cosh(dx + c) * \sinh(dx + c) + (a + b) * \sinh(dx + c)^2 \\
& + a - b) * \sqrt{a*b} / (a*b)) + 4*(16*(a^4*b + 2*a^3*b^2 + a^2*b^3) * dx * \cosh(dx \\
& *x + c)^7 + 3*(5*a^4*b - 5*a^3*b^2 - 9*a^2*b^3 + a*b^4 + 16*(a^4*b - a^2*b^ \\
& 3) * dx) * \cosh(dx + c)^5 + 2*(15*a^4*b - 13*a^3*b^2 + 17*a^2*b^3 - 3*a*b^4 + \\
& 8*(3*a^4*b - 2*a^3*b^2 + 3*a^2*b^3) * dx) * \cosh(dx + c)^3 + (15*a^4*b + a^3 \\
& *b^2 - 11*a^2*b^3 + 3*a*b^4 + 16*(a^4*b - a^2*b^3) * dx) * \cosh(dx + c)) * \sinh \\
& (dx + c)) / ((a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2* \\
& b^6) * dx * \cosh(dx + c)^8 + 8*(a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5 \\
& *a^3*b^5 + a^2*b^6) * dx * \cosh(dx + c) * \sinh(dx + c)^7 + (a^7*b + 5*a^6*b^2 + \\
& 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6) * dx * \sinh(dx + c)^8 + 4*(a^7*b \\
& + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6) * dx * \cosh(dx + c) \\
& ^6 + 4*(7*(a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^ \\
& 6) * dx * \cosh(dx + c)^2 + (a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b \\
& ^5 - a^2*b^6) * dx) * \sinh(dx + c)^6 + 2*(3*a^7*b + 7*a^6*b^2 + 6*a^5*b^3 + 6*a \\
& ^4*b^4 + 7*a^3*b^5 + 3*a^2*b^6) * dx * \cosh(dx + c)^4 + 8*(7*(a^7*b + 5*a^6*b^2 \\
& + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6) * dx * \cosh(dx + c)^3 + 3*(a^ \\
& 7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6) * dx * \cosh(dx + \\
& c)) * \sinh(dx + c)^5 + 2*(35*(a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + \\
& 5*a^3*b^5 + a^2*b^6) * dx * \cosh(dx + c)^4 + 30*(a^7*b + 3*a^6*b^2 + 2*a^5*b^3 \\
& - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6) * dx * \cosh(dx + c)^2 + (3*a^7*b + 7*a^6*b^ \\
& 2 + 6*a^5*b^3 + 6*a^4*b^4 + 7*a^3*b^5 + 3*a^2*b^6) * dx) * \sinh(dx + c)^4 + 4*(\\
& a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6) * dx * \cosh(dx \\
& + c)^2 + 8*(7*(a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a \\
& ^2*b^6) * dx * \cosh(dx + c)^5 + 10*(a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - \\
& 3*a^3*b^5 - a^2*b^6) * dx * \cosh(dx + c)^3 + (3*a^7*b + 7*a^6*b^2 + 6*a^5*b^3 \\
& + 6*a^4*b^4 + 7*a^3*b^5 + 3*a^2*b^6) * dx * \cosh(dx + c)) * \sinh(dx + c)^3 + 4*(\\
& 7*(a^7*b + 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6) * dx * \cos \\
& h(dx + c)^6 + 15*(a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - \\
& a^2*b^6) * dx * \cosh(dx + c)^4 + 3*(3*a^7*b + 7*a^6*b^2 + 6*a^5*b^3 + 6*a^4*b^4 \\
& + 7*a^3*b^5 + 3*a^2*b^6) * dx * \cosh(dx + c)^2 + (a^7*b + 3*a^6*b^2 + 2*a^5*b^ \\
& 3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6) * dx) * \sinh(dx + c)^2 + (a^7*b + 5*a^6*b^ \\
& 2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6) * dx + 8*((a^7*b + 5*a^6*b^ \\
& 2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6) * dx * \cosh(dx + c)^7 + 3*(a \\
& ^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6) * dx * \cosh(dx \\
& + c)^5 + (3*a^7*b + 7*a^6*b^2 + 6*a^5*b^3 + 6*a^4*b^4 + 7*a^3*b^5 + 3*a^2*b \\
& ^6) * dx * \cosh(dx + c)^3 + (a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3* \\
& b^5 - a^2*b^6) * dx * \cosh(dx + c)) * \sinh(dx + c))]
\end{aligned}$$

$$4 - 18a^2b^2 - 16ab^3 - 3b^4 + (15a^4 - 58a^3b - 56a^2b^2 + 26ab^3 + 9b^4)e^{-2dx - 2c} + (15a^4 - 104a^3b + 58a^2b^2 - 24ab^3 - 9b^4)e^{-4dx - 4c} + (5a^4 - 46a^3b - 40a^2b^2 + 14ab^3 + 3b^4)e^{-6dx - 6c} / ((a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 + 5a^3b^4 + a^2b^5 + 4(a^7 + 3a^6b + 2a^5b^2 - 2a^4b^3 - 3a^3b^4 - a^2b^5))e^{-2dx - 2c} + 2(3a^7 + 7a^6b + 6a^5b^2 + 6a^4b^3 + 7a^3b^4 + 3a^2b^5)e^{-4dx - 4c} + 4(a^7 + 3a^6b + 2a^5b^2 - 2a^4b^3 - 3a^3b^4 - a^2b^5)e^{-6dx - 6c} + (a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 + 5a^3b^4 + a^2b^5)e^{-8dx - 8c}) * d) - 1/8(5a^3 + 13a^2b + 11ab^2 + 3b^3 + (15a^3 + 13a^2b - 11ab^2 - 9b^3))e^{-2dx - 2c} + (15a^3 - a^2b + 9ab^2 + 9b^3)e^{-4dx - 4c} + (5a^3 - a^2b - 9ab^2 - 3b^3)e^{-6dx - 6c} / ((a^6 + 4a^5b + 6a^4b^2 + 4a^3b^3 + a^2b^4 + 4(a^6 + 2a^5b - 2a^3b^3 - a^2b^4))e^{-2dx - 2c} + 2(3a^6 + 4a^5b + 2a^4b^2 + 4a^3b^3 + 3a^2b^4)e^{-4dx - 4c} + 4(a^6 + 2a^5b - 2a^3b^3 - a^2b^4)e^{-6dx - 6c} + (a^6 + 4a^5b + 6a^4b^2 + 4a^3b^3 + a^2b^4)e^{-8dx - 8c}) * d) + 1/4 * log((a + b)e^{4dx + 4c} + 2(a - b)e^{2dx + 2c} + a + b) / ((a^3 + 3a^2b + 3ab^2 + b^3) * d) - 1/4 * log(2(a - b)e^{-2dx - 2c} + (a + b)e^{-4dx - 4c} + a + b) / ((a^3 + 3a^2b + 3ab^2 + b^3) * d) + 3/16 * arctan(1/2 * ((a + b)e^{-2dx - 2c} + a - b) / sqrt(a * b)) / (sqrt(a * b) * a^2 * d)$$

mupad [B] time = 3.47, size = 255, normalized size = 1.86

$$\frac{\frac{a^2 x}{(a+b)(a^2+2ab+b^2)} - \frac{\tanh(c+dx)(5a+b)}{8d(a^2+2ab+b^2)} + \frac{b^2 x \tanh(c+dx)^4}{a^3+3a^2b+3ab^2+b^3} + \frac{2abx \tanh(c+dx)^2}{a^3+3a^2b+3ab^2+b^3} - \frac{\tanh(c+dx)^3(3ab-b^2)}{8ad(a^2+2ab+b^2)}}{a^2 + 2ab \tanh(c+dx)^2 + b^2 \tanh(c+dx)^4} + \frac{\operatorname{atan}\left(\frac{b \tanh(c+dx)}{\sqrt{ab}}\right)}{\sqrt{ab} (8a^4d + a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3, x)

[Out] ((a^2*x)/((a + b)*(2*a*b + a^2 + b^2)) - (tanh(c + d*x)*(5*a + b))/(8*d*(2*a*b + a^2 + b^2))) + (b^2*x*tanh(c + d*x)^4)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) + (2*a*b*x*tanh(c + d*x)^2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c + d*x)^3*(3*a*b - b^2))/(8*a*d*(2*a*b + a^2 + b^2))/((a^2 + b^2*tanh(c + d*x)^4 + 2*a*b*tanh(c + d*x)^2) + (atan((b*tanh(c + d*x))/(a*b)^(1/2)))*(6*a*b - 3*a^2 + b^2))/((a*b)^(1/2)*(8*a^4*d + a*b*(24*a^2*d + 8*b^2*d + 24*a*b*d)))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3, x)

[Out] Timed out

$$3.195 \quad \int \frac{\tanh(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=94

$$\frac{1}{2d(a+b)^2(a+b \tanh^2(c+dx))} - \frac{1}{4d(a+b)(a+b \tanh^2(c+dx))^2} + \frac{\log(a+b \tanh^2(c+dx))}{2d(a+b)^3} + \frac{\log(\cosh(c+dx))}{d(a+b)^3}$$

[Out] $\ln(\cosh(d*x+c))/(a+b)^{3/d+1/2} \ln(a+b*\tanh(d*x+c)^2)/(a+b)^{3/d-1/4}/(a+b)/d/(a+b*\tanh(d*x+c)^2)^{2-1/2}/(a+b)^{2/d}/(a+b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.10, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3670, 444, 44}

$$\frac{1}{2d(a+b)^2(a+b \tanh^2(c+dx))} - \frac{1}{4d(a+b)(a+b \tanh^2(c+dx))^2} + \frac{\log(a+b \tanh^2(c+dx))}{2d(a+b)^3} + \frac{\log(\cosh(c+dx))}{d(a+b)^3}$$

Antiderivative was successfully verified.

[In] `Int[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3, x]`

[Out] `Log[Cosh[c + d*x]]/((a + b)^3*d) + Log[a + b*Tanh[c + d*x]^2]/(2*(a + b)^3*d) - 1/(4*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) - 1/(2*(a + b)^2*d*(a + b*Tanh[c + d*x]^2))`

Rule 44

`Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m + n + 2, 0])`

Rule 444

`Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]`

Rule 3670

`Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*((c_) * tan[(e_) + (f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))`

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(c+dx)}{(a+b \tanh^2(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{x}{(1-x^2)(a+bx^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^3} dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} + \frac{b}{(a+b)(a+bx)^3} + \frac{b}{(a+b)^2(a+bx)^2} + \frac{b}{(a+b)^3(a+bx)}\right) dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\log(\cosh(c+dx))}{(a+b)^3 d} + \frac{\log(a+b \tanh^2(c+dx))}{2(a+b)^3 d} - \frac{1}{4(a+b)d(a+b \tanh^2(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.59, size = 77, normalized size = 0.82

$$\frac{-\frac{(a+b)^2}{(a+b \tanh^2(c+dx))^2} - \frac{2(a+b)}{a+b \tanh^2(c+dx)} + 2 \log(a+b \tanh^2(c+dx)) + 4 \log(\cosh(c+dx))}{4d(a+b)^3}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3, x]

[Out] (4*Log[Cosh[c + d*x]] + 2*Log[a + b*Tanh[c + d*x]^2] - (a + b)^2/(a + b*Tanh[c + d*x]^2)^2 - (2*(a + b))/(a + b*Tanh[c + d*x]^2))/(4*(a + b)^3*d)

fricas [B] time = 0.47, size = 2554, normalized size = 27.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -1/2*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 16*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^8 + 8*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + (a^2 - b^2)*d*x + a*b + b^2)*sinh(d*x + c)^6 + 16*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + 3*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 - 2*a*b + 3*b^2)*d*x + 4*a*b - 2*b^2)*cosh(d*x + c)^4 + 4*(35*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + (3*a^2 - 2*a*b + 3*b^2)*d*x + 30*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)^2 + 4*a*b - 2*b^2)*sinh(d*x + c)^4 + 16*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^5 + 10*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)^3 + ((3*a^2 - 2*a*b + 3*b^2)*d*x + 4*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*d*x + 8*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)^2 + 8*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 15*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)^4 + (a^2 - b^2)*d*x + 3*((3*a^2 - 2*a*b + 3*b^2)*d*x + 4*a*b - 2*b^2)*cosh(d*x + c)^2 + a*b + b^2)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 4*(a^2 - b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 30*(a^2 - b^2)*cosh(d*x + c)^2 + 3*a^2 - 2*a*b + 3*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 10*(a^2 - b^2)*cosh(d*x + c)^3 + (3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 +

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4*(a^2 - b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 +
15*(a^2 - b^2)*cosh(d*x + c)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^2
+ a^2 - b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*c
osh(d*x + c)^7 + 3*(a^2 - b^2)*cosh(d*x + c)^5 + (3*a^2 - 2*a*b + 3*b^2)*co
sh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*((a + b)*co
sh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(
d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 16*((a^2 + 2*a*b + b^2)*d*x*co
sh(d*x + c)^7 + 3*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)^5 + ((3*a^2 -
2*a*b + 3*b^2)*d*x + 4*a*b - 2*b^2)*cosh(d*x + c)^3 + ((a^2 - b^2)*d*x + a
*b + b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a
^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 +
10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5 + 5*a^4
*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*sinh(d*x + c)^8 + 4*(a^5 +
3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^6 + 4*(7*(
a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^2
+ (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d)*sinh(d*x + c)^
6 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*
x + c)^4 + 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d
*cosh(d*x + c)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5
)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^4 + 30*(a^5 + 3*a^4*b + 2*a^3*b^2
- 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^2 + (3*a^5 + 7*a^4*b + 6*a^3*
b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d)*sinh(d*x + c)^4 + 4*(a^5 + 3*a^4*b +
2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^5 + 5*a^
4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*cosh(d*x + c)^5 + 10*(a^5
+ 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*d*cosh(d*x + c)^3 + (3*a
^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*d*cosh(d*x + c))*si
nh(d*x + c)^3 + 4*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b
^5)*d*cosh(d*x + c)^6 + 15*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4
- b^5)*d*cosh(d*x + c)^4 + 3*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*
a*b^4 + 3*b^5)*d*cosh(d*x + c)^2 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 -
3*a*b^4 - b^5)*d)*sinh(d*x + c)^2 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b
^3 + 5*a*b^4 + b^5)*d + 8*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b
^4 + b^5)*d*cosh(d*x + c)^7 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*
a*b^4 - b^5)*d*cosh(d*x + c)^5 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 +
7*a*b^4 + 3*b^5)*d*cosh(d*x + c)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^
3 - 3*a*b^4 - b^5)*d*cosh(d*x + c))*sinh(d*x + c))

```

giac [B] time = 0.39, size = 245, normalized size = 2.61

$$\frac{2 \log\left(\frac{a(e^{2dx+2c})+e^{(-2dx-2c)}}{a^3+3a^2b+3ab^2+b^3}\right)+b\left(\frac{e^{(2dx+2c)}+e^{(-2dx-2c)}}{a^2+2ab+b^2}\right)+2a-2b}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(d*x+c)/(a+b*tanh(d*x+c))^3,x, algorithm="giac")

```

[Out] 1/4*(2*log(abs(a*(e^(2*d*x + 2*c)) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c)
+ e^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a*(e
^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 3*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2
*c))^2 + 12*a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 4*b*(e^(2*d*x + 2*c) +
e^(-2*d*x - 2*c)) + 12*a - 4*b)/((a^2 + 2*a*b + b^2)*(a*(e^(2*d*x + 2*c) +
e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2*b)^2)
)/d

```

maple [B] time = 0.12, size = 193, normalized size = 2.05

$$\frac{\ln(\tanh(dx+c)-1)}{2d(a+b)^3} - \frac{\ln(1+\tanh(dx+c))}{2d(a+b)^3} - \frac{a^2}{4d(a+b)^3(a+b(\tanh^2(dx+c)))^2} - \frac{ab}{2d(a+b)^3(a+b(\tanh^2(dx+c)))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x)`

[Out]
$$-1/2/d/(a+b)^3 \ln(\tanh(d*x+c)-1) - 1/2/d/(a+b)^3 \ln(1+\tanh(d*x+c)) - 1/4/d/(a+b)^3 a^2/(a+b*tanh(d*x+c)^2)^2 - 1/2/d/(a+b)^3 a*b/(a+b*tanh(d*x+c)^2)^2 - 1/4/d*b^2/(a+b)^3/(a+b*tanh(d*x+c)^2)^2 + 1/2*\ln(a+b*tanh(d*x+c)^2)/(a+b)^3/d - 1/2/d/(a+b)^3 a/(a+b*tanh(d*x+c)^2) - 1/2/d/(a+b)^3/(a+b*tanh(d*x+c)^2)*b$$

maxima [B] time = 0.37, size = 378, normalized size = 4.02

$$\frac{dx + c}{(a^3 + 3a^2b + 3ab^2 + b^3)d} - \frac{(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5 + 4(a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3a^2b^2 + 3ab^3 + b^4))e^{-(2dx+c)}}{(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5 + 4(a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3a^2b^2 + 3ab^3 + b^4))e^{-(2dx+c)}} + \frac{(2ab - b^2)e^{-(4dx+4c)} + (ab + b^2)e^{-(6dx+6c)}}{(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5 + 4(a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3a^2b^2 + 3ab^3 + b^4))e^{-(2dx+c)}} + \frac{2(3a^5 + 7a^4b + 6a^3b^2 + 6a^2b^3 + 7ab^4 + 3b^5)e^{-(4dx+4c)} + 4(a^5 + 3a^4b + 2a^3b^2 + 2a^2b^3 - 2a^2b^2 + 3ab^3 + b^4)e^{-(6dx+6c)} + (a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5)e^{-(8dx+8c)}}{(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5)e^{-(8dx+8c)}} + \frac{1}{2} \log(2(a-b)e^{-(2dx+2c)} + (a+b)e^{-(4dx+4c)} + a+b) / ((a^3 + 3a^2b + 3ab^2 + b^3)d)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")`

[Out]
$$(dx + c) / ((a^3 + 3a^2b + 3ab^2 + b^3)d) - 4*((ab + b^2)*e^{-(2dx + 2c)} + (2ab - b^2)*e^{-(4dx + 4c)} + (ab + b^2)*e^{-(6dx + 6c)}) / ((a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5 + 4*(a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3a^2b^2 + 3ab^3 + b^4))*e^{-(2dx + 2c)} + 2*(3a^5 + 7a^4b + 6a^3b^2 + 6a^2b^3 + 7ab^4 + 3b^5)*e^{-(4dx + 4c)} + 4*(a^5 + 3a^4b + 2a^3b^2 + 2a^2b^3 - 2a^2b^2 + 3ab^3 + b^4)*e^{-(6dx + 6c)} + (a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5)*e^{-(8dx + 8c)}) * d + 1/2 * \log(2*(a - b)*e^{-(2dx + 2c)} + (a + b)*e^{-(4dx + 4c)} + a + b) / ((a^3 + 3a^2b + 3ab^2 + b^3)d)$$

mupad [B] time = 2.25, size = 235, normalized size = 2.50

$$\frac{\ln(b \tanh(c + dx)^2 + a)}{2da^3 + 6da^2b + 6dab^2 + 2db^3} - \frac{\ln(1 - \tanh(c + dx))}{2da^3 + 6da^2b + 6dab^2 + 2db^3} - \frac{\ln(\tanh(c + dx) + 1)}{2da^3 + 6da^2b + 6dab^2 + 2db^3} + \frac{1}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(c + d*x)/(a + b*tanh(c + d*x)^2)^3,x)`

[Out]
$$\log(a + b*\tanh(c + d*x)^2)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) - \log(1 - \tanh(c + d*x))/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) - \log(\tanh(c + d*x) + 1)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) + ((\tanh(c + d*x))^4*((3*a*b^2)/4 + b^3/4))/(a^2*d*(2*a*b + a^2 + b^2)) + (\tanh(c + d*x)^2*(a*b + b^2/2))/(a*d*(2*a*b + a^2 + b^2))/(a^2 + b^2*\tanh(c + d*x)^4 + 2*a*b*\tanh(c + d*x)^2)$$

sympy [A] time = 157.27, size = 3442, normalized size = 36.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)`

[Out]
$$\text{Piecewise}((\text{zoo}*x/\tanh(c)**5, \text{Eq}(a, 0) \ \& \ \text{Eq}(b, 0) \ \& \ \text{Eq}(d, 0)), ((x - \log(\tanh(c + d*x) + 1)/d)/a**3, \text{Eq}(b, 0)), (1/(6*b**3*d*\tanh(c + d*x)**6 - 18*b**3*d*\tanh(c + d*x)**4 + 18*b**3*d*\tanh(c + d*x)**2 - 6*b**3*d), \text{Eq}(a, -b)), (x*\tanh(c)/(a + b*\tanh(c)**2)**3, \text{Eq}(d, 0)), (4*a**2*d*x/(4*a**5*d + 8*a**4*b*d*\tanh(c + d*x)**2 + 12*a**4*b*d + 4*a**3*b**2*d*\tanh(c + d*x)**4 + 24*a**3*b**2*d*\tanh(c + d*x)**2 + 12*a**3*b**2*d + 12*a**2*b**3*d*\tanh(c + d*x)**4 + 24*a**2*b**3*d*\tanh(c + d*x)**2 + 4*a**2*b**3*d + 12*a*b**4*d*\tanh(c + d*x)**4 + 8*a*b**4*d*\tanh(c + d*x)**2 + 4*b**5*d*\tanh(c + d*x)**4) + 2*a**$$

$$\begin{aligned}
& 2*\log(-I*\sqrt{a}*\sqrt{1/b} + \tanh(c + d*x))/(4*a**5*d + 8*a**4*b*d*\tanh(c + \\
& d*x)**2 + 12*a**4*b*d + 4*a**3*b**2*d*\tanh(c + d*x)**4 + 24*a**3*b**2*d*\tanh(c + d*x)**2 + 12*a**3*b**2*d + 12*a**2*b**3*d*\tanh(c + d*x)**4 + 24*a**2 \\
& *b**3*d*\tanh(c + d*x)**2 + 4*a**2*b**3*d + 12*a*b**4*d*\tanh(c + d*x)**4 + 8 \\
& *a*b**4*d*\tanh(c + d*x)**2 + 4*b**5*d*\tanh(c + d*x)**4) + 2*a**2*\log(I*\sqrt{ \\
& a}*\sqrt{1/b} + \tanh(c + d*x))/(4*a**5*d + 8*a**4*b*d*\tanh(c + d*x)**2 + 12 \\
& *a**4*b*d + 4*a**3*b**2*d*\tanh(c + d*x)**4 + 24*a**3*b**2*d*\tanh(c + d*x)** \\
& 2 + 12*a**3*b**2*d + 12*a**2*b**3*d*\tanh(c + d*x)**4 + 24*a**2*b**3*d*\tanh(c \\
& + d*x)**2 + 4*a**2*b**3*d + 12*a*b**4*d*\tanh(c + d*x)**4 + 8*a*b**4*d*\tanh \\
& (c + d*x)**2 + 4*b**5*d*\tanh(c + d*x)**4) - 4*a**2*\log(\tanh(c + d*x) + 1)/ \\
& (4*a**5*d + 8*a**4*b*d*\tanh(c + d*x)**2 + 12*a**4*b*d + 4*a**3*b**2*d*\tanh(c \\
& + d*x)**4 + 24*a**3*b**2*d*\tanh(c + d*x)**2 + 12*a**3*b**2*d + 12*a**2*b* \\
& *3*d*\tanh(c + d*x)**4 + 24*a**2*b**3*d*\tanh(c + d*x)**2 + 4*a**2*b**3*d + 1 \\
& 2*a*b**4*d*\tanh(c + d*x)**4 + 8*a*b**4*d*\tanh(c + d*x)**2 + 4*b**5*d*\tanh(c \\
& + d*x)**4) - 3*a**2/(4*a**5*d + 8*a**4*b*d*\tanh(c + d*x)**2 + 12*a**4*b*d \\
& + 4*a**3*b**2*d*\tanh(c + d*x)**4 + 24*a**3*b**2*d*\tanh(c + d*x)**2 + 12*a** \\
& 3*b**2*d + 12*a**2*b**3*d*\tanh(c + d*x)**4 + 24*a**2*b**3*d*\tanh(c + d*x)** \\
& 2 + 4*a**2*b**3*d + 12*a*b**4*d*\tanh(c + d*x)**4 + 8*a*b**4*d*\tanh(c + d*x) \\
& **2 + 4*b**5*d*\tanh(c + d*x)**4) + 8*a*b*d*x*\tanh(c + d*x)**2/(4*a**5*d + 8 \\
& *a**4*b*d*\tanh(c + d*x)**2 + 12*a**4*b*d + 4*a**3*b**2*d*\tanh(c + d*x)**4 + \\
& 24*a**3*b**2*d*\tanh(c + d*x)**2 + 12*a**3*b**2*d + 12*a**2*b**3*d*\tanh(c + \\
& d*x)**4 + 24*a**2*b**3*d*\tanh(c + d*x)**2 + 4*a**2*b**3*d + 12*a*b**4*d*\tanh \\
& (c + d*x)**4 + 8*a*b**4*d*\tanh(c + d*x)**2 + 4*b**5*d*\tanh(c + d*x)**4) + \\
& 4*a*b*\log(-I*\sqrt{a}*\sqrt{1/b} + \tanh(c + d*x))*\tanh(c + d*x)**2/(4*a**5*d \\
& + 8*a**4*b*d*\tanh(c + d*x)**2 + 12*a**4*b*d + 4*a**3*b**2*d*\tanh(c + d*x)* \\
& *4 + 24*a**3*b**2*d*\tanh(c + d*x)**2 + 12*a**3*b**2*d + 12*a**2*b**3*d*\tanh \\
& (c + d*x)**4 + 24*a**2*b**3*d*\tanh(c + d*x)**2 + 4*a**2*b**3*d + 12*a*b**4* \\
& d*\tanh(c + d*x)**4 + 8*a*b**4*d*\tanh(c + d*x)**2 + 4*b**5*d*\tanh(c + d*x)** \\
& 4) + 4*a*b*\log(I*\sqrt{a}*\sqrt{1/b} + \tanh(c + d*x))*\tanh(c + d*x)**2/(4*a** \\
& 5*d + 8*a**4*b*d*\tanh(c + d*x)**2 + 12*a**4*b*d + 4*a**3*b**2*d*\tanh(c + d* \\
& x)**4 + 24*a**3*b**2*d*\tanh(c + d*x)**2 + 12*a**3*b**2*d + 12*a**2*b**3*d*\t \\
& anh(c + d*x)**4 + 24*a**2*b**3*d*\tanh(c + d*x)**2 + 4*a**2*b**3*d + 12*a*b* \\
& *4*d*\tanh(c + d*x)**4 + 8*a*b**4*d*\tanh(c + d*x)**2 + 4*b**5*d*\tanh(c + d*x \\
&)**4) - 8*a*b*\log(\tanh(c + d*x) + 1)*\tanh(c + d*x)**2/(4*a**5*d + 8*a**4*b* \\
& d*\tanh(c + d*x)**2 + 12*a**4*b*d + 4*a**3*b**2*d*\tanh(c + d*x)**4 + 24*a**3 \\
& *b**2*d*\tanh(c + d*x)**2 + 12*a**3*b**2*d + 12*a**2*b**3*d*\tanh(c + d*x)**4 \\
& + 24*a**2*b**3*d*\tanh(c + d*x)**2 + 4*a**2*b**3*d + 12*a*b**4*d*\tanh(c + d \\
& *x)**4 + 8*a*b**4*d*\tanh(c + d*x)**2 + 4*b**5*d*\tanh(c + d*x)**4) - 2*a*b*t \\
& anh(c + d*x)**2/(4*a**5*d + 8*a**4*b*d*\tanh(c + d*x)**2 + 12*a**4*b*d + 4*a \\
& **3*b**2*d*\tanh(c + d*x)**4 + 24*a**3*b**2*d*\tanh(c + d*x)**2 + 12*a**3*b** \\
& 2*d + 12*a**2*b**3*d*\tanh(c + d*x)**4 + 24*a**2*b**3*d*\tanh(c + d*x)**2 + 4 \\
& *a**2*b**3*d + 12*a*b**4*d*\tanh(c + d*x)**4 + 8*a*b**4*d*\tanh(c + d*x)**2 + \\
& 4*b**5*d*\tanh(c + d*x)**4) - 4*a*b/(4*a**5*d + 8*a**4*b*d*\tanh(c + d*x)**2 \\
& + 12*a**4*b*d + 4*a**3*b**2*d*\tanh(c + d*x)**4 + 24*a**3*b**2*d*\tanh(c + d \\
& *x)**2 + 12*a**3*b**2*d + 12*a**2*b**3*d*\tanh(c + d*x)**4 + 24*a**2*b**3*d* \\
& tanh(c + d*x)**2 + 4*a**2*b**3*d + 12*a*b**4*d*\tanh(c + d*x)**4 + 8*a*b**4* \\
& d*\tanh(c + d*x)**2 + 4*b**5*d*\tanh(c + d*x)**4) + 4*b**2*d*x*\tanh(c + d*x)* \\
& *4/(4*a**5*d + 8*a**4*b*d*\tanh(c + d*x)**2 + 12*a**4*b*d + 4*a**3*b**2*d*\tanh \\
& (c + d*x)**4 + 24*a**3*b**2*d*\tanh(c + d*x)**2 + 12*a**3*b**2*d + 12*a**2 \\
& *b**3*d*\tanh(c + d*x)**4 + 24*a**2*b**3*d*\tanh(c + d*x)**2 + 4*a**2*b**3*d \\
& + 12*a*b**4*d*\tanh(c + d*x)**4 + 8*a*b**4*d*\tanh(c + d*x)**2 + 4*b**5*d*\tanh \\
& (c + d*x)**4) + 2*b**2*\log(-I*\sqrt{a}*\sqrt{1/b} + \tanh(c + d*x))*\tanh(c + \\
& d*x)**4/(4*a**5*d + 8*a**4*b*d*\tanh(c + d*x)**2 + 12*a**4*b*d + 4*a**3*b**2 \\
& *d*\tanh(c + d*x)**4 + 24*a**3*b**2*d*\tanh(c + d*x)**2 + 12*a**3*b**2*d + 12 \\
& *a**2*b**3*d*\tanh(c + d*x)**4 + 24*a**2*b**3*d*\tanh(c + d*x)**2 + 4*a**2*b* \\
& *3*d + 12*a*b**4*d*\tanh(c + d*x)**4 + 8*a*b**4*d*\tanh(c + d*x)**2 + 4*b**5* \\
& d*\tanh(c + d*x)**4) + 2*b**2*\log(I*\sqrt{a}*\sqrt{1/b} + \tanh(c + d*x))*\tanh(c \\
& + d*x)**4/(4*a**5*d + 8*a**4*b*d*\tanh(c + d*x)**2 + 12*a**4*b*d + 4*a**3*b \\
& **2*d*\tanh(c + d*x)**4 + 24*a**3*b**2*d*\tanh(c + d*x)**2 + 12*a**3*b**2*d
\end{aligned}$$


```

+ 12*a**2*b**3*d*tanh(c + d*x)**4 + 24*a**2*b**3*d*tanh(c + d*x)**2 + 4*a**
2*b**3*d + 12*a*b**4*d*tanh(c + d*x)**4 + 8*a*b**4*d*tanh(c + d*x)**2 + 4*b
**5*d*tanh(c + d*x)**4) - 4*b**2*log(tanh(c + d*x) + 1)*tanh(c + d*x)**4/(4
*a**5*d + 8*a**4*b*d*tanh(c + d*x)**2 + 12*a**4*b*d + 4*a**3*b**2*d*tanh(c
+ d*x)**4 + 24*a**3*b**2*d*tanh(c + d*x)**2 + 12*a**3*b**2*d + 12*a**2*b**3
*d*tanh(c + d*x)**4 + 24*a**2*b**3*d*tanh(c + d*x)**2 + 4*a**2*b**3*d + 12*
a*b**4*d*tanh(c + d*x)**4 + 8*a*b**4*d*tanh(c + d*x)**2 + 4*b**5*d*tanh(c +
d*x)**4) - 2*b**2*tanh(c + d*x)**2/(4*a**5*d + 8*a**4*b*d*tanh(c + d*x)**2
+ 12*a**4*b*d + 4*a**3*b**2*d*tanh(c + d*x)**4 + 24*a**3*b**2*d*tanh(c + d
*x)**2 + 12*a**3*b**2*d + 12*a**2*b**3*d*tanh(c + d*x)**4 + 24*a**2*b**3*d*
tanh(c + d*x)**2 + 4*a**2*b**3*d + 12*a*b**4*d*tanh(c + d*x)**4 + 8*a*b**4*
d*tanh(c + d*x)**2 + 4*b**5*d*tanh(c + d*x)**4) - b**2/(4*a**5*d + 8*a**4*b
*d*tanh(c + d*x)**2 + 12*a**4*b*d + 4*a**3*b**2*d*tanh(c + d*x)**4 + 24*a**
3*b**2*d*tanh(c + d*x)**2 + 12*a**3*b**2*d + 12*a**2*b**3*d*tanh(c + d*x)**
4 + 24*a**2*b**3*d*tanh(c + d*x)**2 + 4*a**2*b**3*d + 12*a*b**4*d*tanh(c +
d*x)**4 + 8*a*b**4*d*tanh(c + d*x)**2 + 4*b**5*d*tanh(c + d*x)**4), True))

```

$$3.196 \quad \int \frac{1}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=142

$$\frac{b(7a+3b) \tanh(c+dx)}{8a^2d(a+b)^2(a+b \tanh^2(c+dx))} + \frac{\sqrt{b}(15a^2+10ab+3b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^3} + \frac{b \tanh(c+dx)}{4ad(a+b)(a+b \tanh^2(c+dx))}$$

[Out] x/(a+b)^3+1/8*(15*a^2+10*a*b+3*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/a^(5/2)/(a+b)^3/d+1/4*b*tanh(d*x+c)/a/(a+b)/d/(a+b*tanh(d*x+c)^2)^2+1/8*b*(7*a+3*b)*tanh(d*x+c)/a^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)

Rubi [A] time = 0.16, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3661, 414, 527, 522, 206, 205}

$$\frac{\sqrt{b}(15a^2+10ab+3b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^3} + \frac{b(7a+3b) \tanh(c+dx)}{8a^2d(a+b)^2(a+b \tanh^2(c+dx))} + \frac{b \tanh(c+dx)}{4ad(a+b)(a+b \tanh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tanh[c + d*x]^2)^(-3), x]

[Out] x/(a + b)^3 + (Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a + b)^3*d) + (b*Tanh[c + d*x])/(4*a*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) + (b*(7*a + 3*b)*Tanh[c + d*x])/(8*a^2*(a + b)^2*d*(a + b*Tanh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + b \tanh^2(c + dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{b \tanh(c + dx)}{4a(a + b)d (a + b \tanh^2(c + dx))^2} - \frac{\text{Subst}\left(\int \frac{b-4(a+b)+3bx^2}{(1-x^2)(a+bx^2)^2} dx, x, \tanh(c + dx)\right)}{4a(a + b)d} \\ &= \frac{b \tanh(c + dx)}{4a(a + b)d (a + b \tanh^2(c + dx))^2} + \frac{b(7a + 3b) \tanh(c + dx)}{8a^2(a + b)^2d (a + b \tanh^2(c + dx))} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{4a(a + b)d} \\ &= \frac{b \tanh(c + dx)}{4a(a + b)d (a + b \tanh^2(c + dx))^2} + \frac{b(7a + 3b) \tanh(c + dx)}{8a^2(a + b)^2d (a + b \tanh^2(c + dx))} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{4a(a + b)d} \\ &= \frac{x}{(a + b)^3} + \frac{\sqrt{b} (15a^2 + 10ab + 3b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a + b)^3d} + \frac{b \tanh(c + dx)}{4a(a + b)d (a + b \tanh^2(c + dx))} \end{aligned}$$

Mathematica [A] time = 0.32, size = 147, normalized size = 1.04

$$\frac{\frac{b(7a+3b)(a+b) \tanh(c+dx)}{a^2(a+b \tanh^2(c+dx))} + \frac{\sqrt{b}(15a^2+10ab+3b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}} + \frac{2b(a+b)^2 \tanh(c+dx)}{a(a+b \tanh^2(c+dx))^2} - 4 \log(1 - \tanh(c + dx)) + 4 \log(1 + \tanh(c + dx))}{8d(a + b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Tanh[c + d*x]^2)^(-3), x]
```

```
[Out] ((Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(5/2) - 4*Log[1 - Tanh[c + d*x]] + 4*Log[1 + Tanh[c + d*x]] + (2*b*(a + b)^2*Tanh[c + d*x])/(a*(a + b*Tanh[c + d*x]^2)^2) + (b*(a + b)*(7*a + 3*b)*Tanh[c + d*x])/(a^2*(a + b*Tanh[c + d*x]^2)))/(8*(a + b)^3*d)
```

fricas [B] time = 0.56, size = 7496, normalized size = 52.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(16*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^8 + 128*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 16*(a^4 + 2*a^3*b + a^2*b^2)*d*x*sinh(d*x + c)^8 - 4*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a^2*b^2)*d*x)*cosh(d*x + c)^6 + 4*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^2 - 9*a^3*b + a^2*b^2 + 13*a*b^3 + 3*b^4 + 16*(a^4 - a^2*b^2)*d*x)*sinh(d*x + c)^6 + 8*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^3 - 3*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 4*(27*a^3*b - 9*a^2*b^2 + 21*a*b^3 + 9*b^4 - 8*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c)^4 + 4*(280*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^4 - 27*a^3*b + 9*a^2*b^2 - 21*a*b^3 - 9*b^4 + 8*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*d*x - 15*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a^2*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 36*a^3*b - 84*a^2*b^2 - 60*a*b^3 - 12*b^4 + 16*(56*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^5 - 5*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a^2*b^2)*d*x)*cosh(d*x + c)^3 - (27*a^3*b - 9*a^2*b^2 + 21*a*b^3 + 9*b^4 - 8*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 16*(a^4 + 2*a^3*b + a^2*b^2)*d*x - 4*(27*a^3*b + 13*a^2*b^2 - 23*a*b^3 - 9*b^4 - 16*(a^4 - a^2*b^2)*d*x)*cosh(d*x + c)^2 + 4*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x + c)^6 - 15*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a^2*b^2)*d*x)*cosh(d*x + c)^4 - 27*a^3*b - 13*a^2*b^2 + 23*a*b^3 + 9*b^4 + 16*(a^4 - a^2*b^2)*d*x - 6*(27*a^3*b - 9*a^2*b^2 + 21*a*b^3 + 9*b^4 - 8*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 4*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4 + 7*(15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(45*a^4 + 34*a^2*b^2 + 24*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 45*a^4 + 34*a^2*b^2 + 24*a*b^3 + 9*b^4 + 30*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4 + 8*(7*(15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (45*a^4 + 34*a^2*b^2 + 24*a*b^3 + 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c)^2 + 4*(7*(15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4 + 3*(45*a^4 + 34*a^2*b^2 + 24*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (45*a^4 + 34*a^2*b^2 + 24*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 8*(16*(a^4 + 2*a^3*b + a^2*b^2)*d*x*cosh(d*x +

$$\begin{aligned}
& c)^7 - 3*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a^2*b^2)*d*x)*c \\
& \text{osh}(d*x + c)^5 - 2*(27*a^3*b - 9*a^2*b^2 + 21*a*b^3 + 9*b^4 - 8*(3*a^4 - 2* \\
& a^3*b + 3*a^2*b^2)*d*x)*\text{cosh}(d*x + c)^3 - (27*a^3*b + 13*a^2*b^2 - 23*a*b^3 \\
& - 9*b^4 - 16*(a^4 - a^2*b^2)*d*x)*\text{cosh}(d*x + c))*\text{sinh}(d*x + c))/((a^7 + 5* \\
& a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*\text{cosh}(d*x + c)^8 + \\
& 8*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*\text{cosh}(d* \\
& x + c)*\text{sinh}(d*x + c)^7 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b \\
& ^4 + a^2*b^5)*d*\text{sinh}(d*x + c)^8 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 \\
& - 3*a^3*b^4 - a^2*b^5)*d*\text{cosh}(d*x + c)^6 + 4*(7*(a^7 + 5*a^6*b + 10*a^5*b^2 \\
& + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*\text{cosh}(d*x + c)^2 + (a^7 + 3*a^6*b + 2 \\
& *a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d)*\text{sinh}(d*x + c)^6 + 2*(3*a^7 + \\
& 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*d*\text{cosh}(d*x + c)^4 \\
& + 8*(7*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*c \\
& \text{osh}(d*x + c)^3 + 3*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2 \\
& *b^5)*d*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)^5 + 2*(35*(a^7 + 5*a^6*b + 10*a^5*b^2 \\
& + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*\text{cosh}(d*x + c)^4 + 30*(a^7 + 3*a^6*b + \\
& 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*\text{cosh}(d*x + c)^2 + (3*a^7 + \\
& 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*d)*\text{sinh}(d*x + c)^4 \\
& + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*\text{cosh}(d \\
& *x + c)^2 + 8*(7*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2 \\
& *b^5)*d*\text{cosh}(d*x + c)^5 + 10*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3 \\
& *b^4 - a^2*b^5)*d*\text{cosh}(d*x + c)^3 + (3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^ \\
& 3 + 7*a^3*b^4 + 3*a^2*b^5)*d*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)^3 + 4*(7*(a^7 + 5 \\
& *a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*\text{cosh}(d*x + c)^6 + \\
& 15*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*\text{cosh}(d* \\
& x + c)^4 + 3*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b \\
& ^5)*d*\text{cosh}(d*x + c)^2 + (a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 \\
& - a^2*b^5)*d)*\text{sinh}(d*x + c)^2 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + \\
& 5*a^3*b^4 + a^2*b^5)*d + 8*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^ \\
& 3*b^4 + a^2*b^5)*d*\text{cosh}(d*x + c)^7 + 3*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b \\
& ^3 - 3*a^3*b^4 - a^2*b^5)*d*\text{cosh}(d*x + c)^5 + (3*a^7 + 7*a^6*b + 6*a^5*b^2 \\
& + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*d*\text{cosh}(d*x + c)^3 + (a^7 + 3*a^6*b + 2 \\
& *a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)) \\
& , 1/8*(8*(a^4 + 2*a^3*b + a^2*b^2)*d*x*\text{cosh}(d*x + c)^8 + 64*(a^4 + 2*a^3*b \\
& + a^2*b^2)*d*x*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^7 + 8*(a^4 + 2*a^3*b + a^2*b^2)* \\
& d*x*\text{sinh}(d*x + c)^8 - 2*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a \\
& ^2*b^2)*d*x)*\text{cosh}(d*x + c)^6 + 2*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*\text{cosh}(d* \\
& x + c)^2 - 9*a^3*b + a^2*b^2 + 13*a*b^3 + 3*b^4 + 16*(a^4 - a^2*b^2)*d*x)*s \\
& \text{inh}(d*x + c)^6 + 4*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*\text{cosh}(d*x + c)^3 - 3*(\\
& 9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a^2*b^2)*d*x)*\text{cosh}(d*x + c \\
&))*\text{sinh}(d*x + c)^5 - 2*(27*a^3*b - 9*a^2*b^2 + 21*a*b^3 + 9*b^4 - 8*(3*a^4 \\
& - 2*a^3*b + 3*a^2*b^2)*d*x)*\text{cosh}(d*x + c)^4 + 2*(280*(a^4 + 2*a^3*b + a^2*b \\
& ^2)*d*x*\text{cosh}(d*x + c)^4 - 27*a^3*b + 9*a^2*b^2 - 21*a*b^3 - 9*b^4 + 8*(3*a^ \\
& 4 - 2*a^3*b + 3*a^2*b^2)*d*x - 15*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 1 \\
& 6*(a^4 - a^2*b^2)*d*x)*\text{cosh}(d*x + c)^2)*\text{sinh}(d*x + c)^4 - 18*a^3*b - 42*a^2 \\
& *b^2 - 30*a*b^3 - 6*b^4 + 8*(56*(a^4 + 2*a^3*b + a^2*b^2)*d*x*\text{cosh}(d*x + c) \\
& ^5 - 5*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a^2*b^2)*d*x)*\text{cosh} \\
& (d*x + c)^3 - (27*a^3*b - 9*a^2*b^2 + 21*a*b^3 + 9*b^4 - 8*(3*a^4 - 2*a^3*b \\
& + 3*a^2*b^2)*d*x)*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)^3 + 8*(a^4 + 2*a^3*b + a^2* \\
& b^2)*d*x - 2*(27*a^3*b + 13*a^2*b^2 - 23*a*b^3 - 9*b^4 - 16*(a^4 - a^2*b^2) \\
& *d*x)*\text{cosh}(d*x + c)^2 + 2*(112*(a^4 + 2*a^3*b + a^2*b^2)*d*x*\text{cosh}(d*x + c)^ \\
& 6 - 15*(9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a^2*b^2)*d*x)*\text{cosh} \\
& (d*x + c)^4 - 27*a^3*b - 13*a^2*b^2 + 23*a*b^3 + 9*b^4 + 16*(a^4 - a^2*b^2) \\
& *d*x - 6*(27*a^3*b - 9*a^2*b^2 + 21*a*b^3 + 9*b^4 - 8*(3*a^4 - 2*a^3*b + 3* \\
& a^2*b^2)*d*x)*\text{cosh}(d*x + c)^2)*\text{sinh}(d*x + c)^2 + ((15*a^4 + 40*a^3*b + 38*a \\
& ^2*b^2 + 16*a*b^3 + 3*b^4)*\text{cosh}(d*x + c)^8 + 8*(15*a^4 + 40*a^3*b + 38*a^2* \\
& b^2 + 16*a*b^3 + 3*b^4)*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^7 + (15*a^4 + 40*a^3*b \\
& + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*\text{sinh}(d*x + c)^8 + 4*(15*a^4 + 10*a^3*b - 1 \\
& 2*a^2*b^2 - 10*a*b^3 - 3*b^4)*\text{cosh}(d*x + c)^6 + 4*(15*a^4 + 10*a^3*b - 12*a
\end{aligned}$$

$$\begin{aligned}
& ^2*b^2 - 10*a*b^3 - 3*b^4 + 7*(15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + \\
& 3*b^4)*\cosh(d*x + c)^2*\sinh(d*x + c)^6 + 8*(7*(15*a^4 + 40*a^3*b + 38*a^2* \\
& b^2 + 16*a*b^3 + 3*b^4)*\cosh(d*x + c)^3 + 3*(15*a^4 + 10*a^3*b - 12*a^2*b^2 \\
& - 10*a*b^3 - 3*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(45*a^4 + 34*a^2*b^ \\
& 2 + 24*a*b^3 + 9*b^4)*\cosh(d*x + c)^4 + 2*(35*(15*a^4 + 40*a^3*b + 38*a^2*b \\
& ^2 + 16*a*b^3 + 3*b^4)*\cosh(d*x + c)^4 + 45*a^4 + 34*a^2*b^2 + 24*a*b^3 + 9 \\
& *b^4 + 30*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*\cosh(d*x + c) \\
& ^2)*\sinh(d*x + c)^4 + 15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4 + 8 \\
& *(7*(15*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*\cosh(d*x + c)^5 + 1 \\
& 0*(15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*\cosh(d*x + c)^3 + (45 \\
& *a^4 + 34*a^2*b^2 + 24*a*b^3 + 9*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(1 \\
& 5*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*\cosh(d*x + c)^2 + 4*(7*(1 \\
& 5*a^4 + 40*a^3*b + 38*a^2*b^2 + 16*a*b^3 + 3*b^4)*\cosh(d*x + c)^6 + 15*(15 \\
& a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4)*\cosh(d*x + c)^4 + 15*a^4 + \\
& 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4 + 3*(45*a^4 + 34*a^2*b^2 + 24*a*b^ \\
& 3 + 9*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((15*a^4 + 40*a^3*b + 38*a^ \\
& 2*b^2 + 16*a*b^3 + 3*b^4)*\cosh(d*x + c)^7 + 3*(15*a^4 + 10*a^3*b - 12*a^2*b \\
& ^2 - 10*a*b^3 - 3*b^4)*\cosh(d*x + c)^5 + (45*a^4 + 34*a^2*b^2 + 24*a*b^3 + \\
& 9*b^4)*\cosh(d*x + c)^3 + (15*a^4 + 10*a^3*b - 12*a^2*b^2 - 10*a*b^3 - 3*b^4 \\
&)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{b/a}*\arctan(1/2*((a + b)*\cosh(d*x + c) \\
& ^2 + 2*(a + b)*\cosh(d*x + c)*\sinh(d*x + c) + (a + b)*\sinh(d*x + c)^2 + a - \\
& b)*\sqrt{b/a}/b) + 4*(16*(a^4 + 2*a^3*b + a^2*b^2)*d*x*\cosh(d*x + c)^7 - 3*(\\
& 9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4 - 16*(a^4 - a^2*b^2)*d*x)*\cosh(d*x + c \\
&)^5 - 2*(27*a^3*b - 9*a^2*b^2 + 21*a*b^3 + 9*b^4 - 8*(3*a^4 - 2*a^3*b + 3*a \\
& ^2*b^2)*d*x)*\cosh(d*x + c)^3 - (27*a^3*b + 13*a^2*b^2 - 23*a*b^3 - 9*b^4 - \\
& 16*(a^4 - a^2*b^2)*d*x)*\cosh(d*x + c))*\sinh(d*x + c))/((a^7 + 5*a^6*b + 10* \\
& a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^8 + 8*(a^7 + 5* \\
& a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)*\sinh \\
& (d*x + c)^7 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^ \\
& 5)*d*\sinh(d*x + c)^8 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 \\
& - a^2*b^5)*d*\cosh(d*x + c)^6 + 4*(7*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b \\
& ^3 + 5*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^2 + (a^7 + 3*a^6*b + 2*a^5*b^2 - \\
& 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d)*\sinh(d*x + c)^6 + 2*(3*a^7 + 7*a^6*b + \\
& 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*d*\cosh(d*x + c)^4 + 8*(7*(a^ \\
& 7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c \\
&)^3 + 3*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*\cos \\
& h(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^ \\
& 3 + 5*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^4 + 30*(a^7 + 3*a^6*b + 2*a^5*b^2 \\
& - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*\cosh(d*x + c)^2 + (3*a^7 + 7*a^6*b + 6 \\
& *a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*d)*\sinh(d*x + c)^4 + 4*(a^7 + \\
& 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*\cosh(d*x + c)^2 + \\
& 8*(7*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*\cos \\
& h(d*x + c)^5 + 10*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2* \\
& b^5)*d*\cosh(d*x + c)^3 + (3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b \\
& ^4 + 3*a^2*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^7 + 5*a^6*b + 10 \\
& *a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^6 + 15*(a^7 + \\
& 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*\cosh(d*x + c)^4 + \\
& 3*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*d*\cosh(\\
& d*x + c)^2 + (a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)* \\
& d)*\sinh(d*x + c)^2 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + \\
& a^2*b^5)*d + 8*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2 \\
& *b^5)*d*\cosh(d*x + c)^7 + 3*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3* \\
& b^4 - a^2*b^5)*d*\cosh(d*x + c)^5 + (3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 \\
& + 7*a^3*b^4 + 3*a^2*b^5)*d*\cosh(d*x + c)^3 + (a^7 + 3*a^6*b + 2*a^5*b^2 - \\
& 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c))]
\end{aligned}$$

giac [B] time = 0.21, size = 409, normalized size = 2.88

$$\frac{(15a^2b + 10ab^2 + 3b^3) \arctan\left(\frac{ae^{(2dx+2c)+be(2dx+2c)+a-b}}{2\sqrt{ab}}\right)}{(a^5 + 3a^4b + 3a^3b^2 + a^2b^3)\sqrt{ab}} + \frac{8(dx+c)}{a^3 + 3a^2b + 3ab^2 + b^3} - \frac{2(9a^3be^{(6dx+6c)} - a^2b^2e^{(6dx+6c)} - 13ab^3e^{(6dx+6c)} - 3b^4e^{(6dx+6c)})}{(a^5 + 3a^4b + 3a^3b^2 + a^2b^3)\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/8*((15*a^2*b + 10*a*b^2 + 3*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b)))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)) + 8*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(9*a^3*b*e^(6*d*x + 6*c) - a^2*b^2*e^(6*d*x + 6*c) - 13*a*b^3*e^(6*d*x + 6*c) - 3*b^4*e^(6*d*x + 6*c) + 27*a^3*b*e^(4*d*x + 4*c) - 9*a^2*b^2*e^(4*d*x + 4*c) + 21*a*b^3*e^(4*d*x + 4*c) + 9*b^4*e^(4*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + 13*a^2*b^2*e^(2*d*x + 2*c) - 23*a*b^3*e^(2*d*x + 2*c) - 9*b^4*e^(2*d*x + 2*c) + 9*a^3*b + 21*a^2*b^2 + 15*a*b^3 + 3*b^4)/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2))/d

maple [B] time = 0.15, size = 352, normalized size = 2.48

$$\frac{\ln(\tanh(dx+c)-1)}{2d(a+b)^3} + \frac{\ln(1+\tanh(dx+c))}{2d(a+b)^3} + \frac{7(\tanh^3(dx+c))b^2}{8d(a+b)^3(a+b(\tanh^2(dx+c)))^2} + \frac{5b^3(\tanh^3(dx+c))}{4d(a+b)^3(a+b(\tanh^2(dx+c)))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*tanh(d*x+c)^2)^3,x)

[Out] -1/2/d/(a+b)^3*ln(tanh(d*x+c)-1)+1/2/d/(a+b)^3*ln(1+tanh(d*x+c))+7/8/d/(a+b)^3/(a+b*tanh(d*x+c)^2)^2*tanh(d*x+c)^3*b^2+5/4/d/(a+b)^3/(a+b*tanh(d*x+c)^2)^2*b^3/a*tanh(d*x+c)^3+3/8/d/(a+b)^3*b^4/(a+b*tanh(d*x+c)^2)^2/a^2*tanh(d*x+c)^3+9/8/d/(a+b)^3/(a+b*tanh(d*x+c)^2)^2*a*b*tanh(d*x+c)+7/4/d/(a+b)^3/(a+b*tanh(d*x+c)^2)^2*tanh(d*x+c)*b^2+5/8/d/(a+b)^3*b^3/(a+b*tanh(d*x+c)^2)^2/a*tanh(d*x+c)+15/8/d/(a+b)^3*b/(a*b)^(1/2)*arctan(tanh(d*x+c)*b/(a*b)^(1/2))+5/4/d/(a+b)^3/a/(a*b)^(1/2)*arctan(tanh(d*x+c)*b/(a*b)^(1/2))*b^2+3/8/d/(a+b)^3*b^3/a^2/(a*b)^(1/2)*arctan(tanh(d*x+c)*b/(a*b)^(1/2))

maxima [B] time = 0.57, size = 507, normalized size = 3.57

$$\frac{(15a^2b + 10ab^2 + 3b^3) \arctan\left(\frac{(a+b)e^{(-2dx-2c)+a-b}}{2\sqrt{ab}}\right)}{8(a^5 + 3a^4b + 3a^3b^2 + a^2b^3)\sqrt{ab}d} + \frac{4(a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 + 5a^3b^4 + a^2b^5 + 4(a^7 + 5a^6b + 2a^5b^2 - 2a^4b^3 - 3a^3b^4 - a^2b^5)e^{(-2dx-2c)} + 2(3a^7 + 7a^6b + 6a^5b^2 + 6a^4b^3 + 7a^3b^4 + 3a^2b^5)e^{(-4dx-4c)} - 4(a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 + 5a^3b^4 + a^2b^5)e^{(-6dx-6c)} + (a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 + 5a^3b^4 + a^2b^5)e^{(-8dx-8c)})}{4(a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 + 5a^3b^4 + a^2b^5 + 4(a^7 + 5a^6b + 2a^5b^2 - 2a^4b^3 - 3a^3b^4 - a^2b^5)e^{(-2dx-2c)} + 2(3a^7 + 7a^6b + 6a^5b^2 + 6a^4b^3 + 7a^3b^4 + 3a^2b^5)e^{(-4dx-4c)} - 4(a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 + 5a^3b^4 + a^2b^5)e^{(-6dx-6c)} + (a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 + 5a^3b^4 + a^2b^5)e^{(-8dx-8c)})d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] -1/8*(15*a^2*b + 10*a*b^2 + 3*b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b)))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)*d) + 1/4*(9*a^3*b + 21*a^2*b^2 + 15*a*b^3 + 3*b^4 + (27*a^3*b + 13*a^2*b^2 - 23*a*b^3 - 9*b^4)*e^(-2*d*x - 2*c) + 3*(9*a^3*b - 3*a^2*b^2 + 7*a*b^3 + 3*b^4)*e^(-4*d*x - 4*c) + (9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4)*e^(-6*d*x - 6*c))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*e^(-4*d*x - 4*c) + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(-6*d*x - 6*c) + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*e^(-8*d*x - 8*c))*d) + (d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)

mupad [B] time = 0.90, size = 260, normalized size = 1.83

$$\frac{\ln(\tanh(c+dx)+1)}{2da^3+6da^2b+6dab^2+2db^3} - \frac{\ln(1-\tanh(c+dx))}{2da^3+6da^2b+6dab^2+2db^3} + \frac{\tanh(c+dx)^3\left(\frac{3b^3}{8}+\frac{7ab^2}{8}\right)}{a^2d(a^2+2ab+b^2)} + \frac{\tanh(c+dx)(5b^2+9a)}{8ad(a^2+2ab+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*tanh(c + d*x)^2)^3,x)

[Out] log(tanh(c + d*x) + 1)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) - log(1 - tanh(c + d*x))/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) + ((tanh(c + d*x)^3*((7*a*b^2)/8 + (3*b^3)/8))/(a^2*d*(2*a*b + a^2 + b^2)) + (tanh(c + d*x)*(9*a*b + 5*b^2))/(8*a*d*(2*a*b + a^2 + b^2)))/(a^2 + b^2*tanh(c + d*x)^4 + 2*a*b*tanh(c + d*x)^2) + (atan((b*tanh(c + d*x))/(a*b)^(1/2))*(10*a*b^2 + 15*a^2*b + 3*b^3))/((a*b)^(1/2)*(8*a^5*d + a*b*(24*a^3*d + a*b*(24*a*d + 8*b*d))))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tanh(d*x+c)**2)**3,x)

[Out] Timed out

$$3.197 \quad \int \frac{\coth(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=138

$$\frac{\log(\tanh(c+dx))}{a^3 d} + \frac{b(2a+b)}{2a^2 d(a+b)^2 (a+b \tanh^2(c+dx))} - \frac{b(3a^2+3ab+b^2) \log(a+b \tanh^2(c+dx))}{2a^3 d(a+b)^3} + \frac{1}{4ad(a+b)}$$

[Out] $\ln(\cosh(d*x+c))/(a+b)^3/d + \ln(\tanh(d*x+c))/a^3/d - 1/2*b*(3*a^2+3*a*b+b^2)*\ln(a+b*\tanh(d*x+c)^2)/a^3/(a+b)^3/d + 1/4*b/a/(a+b)/d/(a+b*\tanh(d*x+c)^2)^2 + 1/2*b*(2*a+b)/a^2/(a+b)^2/d/(a+b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.20, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3670, 446, 72}

$$-\frac{b(3a^2+3ab+b^2) \log(a+b \tanh^2(c+dx))}{2a^3 d(a+b)^3} + \frac{b(2a+b)}{2a^2 d(a+b)^2 (a+b \tanh^2(c+dx))} + \frac{\log(\tanh(c+dx))}{a^3 d} + \frac{1}{4ad(a+b)}$$

Antiderivative was successfully verified.

[In] `Int[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2)^3, x]`

[Out] $\text{Log}[\text{Cosh}[c + d*x]]/((a + b)^3*d) + \text{Log}[\text{Tanh}[c + d*x]]/(a^3*d) - (b*(3*a^2 + 3*a*b + b^2)*\text{Log}[a + b*\text{Tanh}[c + d*x]^2])/(2*a^3*(a + b)^3*d) + b/(4*a*(a + b)*d*(a + b*\text{Tanh}[c + d*x]^2)^2) + (b*(2*a + b))/(2*a^2*(a + b)^2*d*(a + b*\text{Tanh}[c + d*x]^2))$

Rule 72

`Int[((e_.) + (f_.)*(x_.))^(p_.)/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]`

Rule 446

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]`

Rule 3670

`Int[((d_.)*tan[(e_.) + (f_.)*(x_.)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f*f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))`

Rubi steps

$$\begin{aligned}
\int \frac{\coth(c+dx)}{(a+b \tanh^2(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(1-x^2)(a+bx^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{(1-x)x(a+bx)^3} dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} + \frac{1}{a^3x} - \frac{b^2}{a(a+b)(a+bx)^3} - \frac{b^2(2a+b)}{a^2(a+b)^2(a+bx)^2} - \frac{b^2(3a^2+3ab+b^2)}{a^3(a+b)^3(a+bx)}\right) dx, x}{2d} \right)}{d} \\
&= \frac{\log(\cosh(c+dx))}{(a+b)^3d} + \frac{\log(\tanh(c+dx))}{a^3d} - \frac{b(3a^2+3ab+b^2)\log(a+b \tanh^2(c+dx))}{2a^3(a+b)^3d}
\end{aligned}$$

Mathematica [A] time = 1.76, size = 117, normalized size = 0.85

$$\frac{\frac{b \left(\frac{a(a+b)(2b(2a+b) \tanh^2(c+dx) + a(5a+3b))}{(a+b \tanh^2(c+dx))^2} - 2(3a^2+3ab+b^2) \log(a+b \tanh^2(c+dx)) \right)}{(a+b)^3} + 4 \log(\tanh(c+dx))}{a^3} + \frac{4 \log(\cosh(c+dx))}{(a+b)^3}$$

$4d$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2)^3, x]

[Out] ((4*Log[Cosh[c + d*x]])/(a + b)^3 + (4*Log[Tanh[c + d*x]] + (b*(-2*(3*a^2 + 3*a*b + b^2)*Log[a + b*Tanh[c + d*x]^2] + (a*(a + b)*(a*(5*a + 3*b) + 2*b*(2*a + b)*Tanh[c + d*x]^2)))/(a + b*Tanh[c + d*x]^2)^2))/(a + b)^3/a^3/(4*d)

fricas [B] time = 0.87, size = 4800, normalized size = 34.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -1/2*(2*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^8 + 16*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^5 + 2*a^4*b + a^3*b^2)*d*x*sinh(d*x + c)^8 - 4*(3*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c)^6 - 4*(3*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 14*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^2 - 2*(a^5 - a^3*b^2)*d*x)*sinh(d*x + c)^6 + 8*(14*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^3 - 3*(3*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 4*(6*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 - (3*a^5 - 2*a^4*b + 3*a^3*b^2)*d*x)*cosh(d*x + c)^4 + 4*(35*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^4 - 6*a^3*b^2 + 2*a^2*b^3 + 2*a*b^4 + (3*a^5 - 2*a^4*b + 3*a^3*b^2)*d*x - 15*(3*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(7*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^5 - 5*(3*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c)^3 - (6*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 - (3*a^5 - 2*a^4*b + 3*a^3*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(a^5 + 2*a^4*b + a^3*b^2)*d*x - 4*(3*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c)^2 + 4*(14*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^6 - 3*a^3*b^2 - 4*a^2*b^3 - a*b^4 - 15*(3*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)*d*x)*cosh(d*x + c)^4 + 2*(a^5 - a^3*b^2)

$$\begin{aligned}
& *d*x - 6*(6*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 - (3*a^5 - 2*a^4*b + 3*a^3*b^2)*d \\
& *x)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + ((3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + \\
& 5*a*b^4 + b^5)*\cosh(d*x + c)^8 + 8*(3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + 5*a \\
& *b^4 + b^5)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (3*a^4*b + 9*a^3*b^2 + 10*a^2*b \\
& ^3 + 5*a*b^4 + b^5)*\sinh(d*x + c)^8 + 4*(3*a^4*b + 3*a^3*b^2 - 2*a^2*b^3 - \\
& 3*a*b^4 - b^5)*\cosh(d*x + c)^6 + 4*(3*a^4*b + 3*a^3*b^2 - 2*a^2*b^3 - 3*a*b \\
& ^4 - b^5 + 7*(3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*\cosh(d*x + \\
& c)^2)*\sinh(d*x + c)^6 + 8*(7*(3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + \\
& b^5)*\cosh(d*x + c)^3 + 3*(3*a^4*b + 3*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)* \\
& \cosh(d*x + c))*\sinh(d*x + c)^5 + 3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 \\
& + b^5 + 2*(9*a^4*b + 3*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*\cosh(d*x + c \\
&)^4 + 2*(9*a^4*b + 3*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5 + 35*(3*a^4*b + \\
& 9*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*\cosh(d*x + c)^4 + 30*(3*a^4*b + 3*a \\
& ^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(7 \\
& *(3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*\cosh(d*x + c)^5 + 10*(3 \\
& *a^4*b + 3*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*\cosh(d*x + c)^3 + (9*a^4*b \\
& + 3*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*\cosh(d*x + c))*\sinh(d*x + c)^3 + \\
& 4*(3*a^4*b + 3*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*\cosh(d*x + c)^2 + 4*(7 \\
& *(3*a^4*b + 9*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*\cosh(d*x + c)^6 + 3*a^4 \\
& *b + 3*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 15*(3*a^4*b + 3*a^3*b^2 - 2*a^ \\
& 2*b^3 - 3*a*b^4 - b^5)*\cosh(d*x + c)^4 + 3*(9*a^4*b + 3*a^3*b^2 + 6*a^2*b^3 \\
& + 7*a*b^4 + 3*b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((3*a^4*b + 9*a^3* \\
& b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*\cosh(d*x + c)^7 + 3*(3*a^4*b + 3*a^3*b^2 \\
& - 2*a^2*b^3 - 3*a*b^4 - b^5)*\cosh(d*x + c)^5 + (9*a^4*b + 3*a^3*b^2 + 6*a^2 \\
& *b^3 + 7*a*b^4 + 3*b^5)*\cosh(d*x + c)^3 + (3*a^4*b + 3*a^3*b^2 - 2*a^2*b^3 \\
& - 3*a*b^4 - b^5)*\cosh(d*x + c))*\sinh(d*x + c))*\log(2*((a + b)*\cosh(d*x + c) \\
& ^2 + (a + b)*\sinh(d*x + c)^2 + a - b)/(\cosh(d*x + c)^2 - 2*\cosh(d*x + c)*\si \\
& nh(d*x + c) + \sinh(d*x + c)^2)) - 2*((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b \\
& ^3 + 5*a*b^4 + b^5)*\cosh(d*x + c)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^ \\
& 2*b^3 + 5*a*b^4 + b^5)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^5 + 5*a^4*b + 10* \\
& a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*\sinh(d*x + c)^8 + 4*(a^5 + 3*a^4*b + \\
& 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*\cosh(d*x + c)^6 + 4*(a^5 + 3*a^4*b + \\
& 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10 \\
& *a^2*b^3 + 5*a*b^4 + b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(a^5 + 5* \\
& a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*\cosh(d*x + c)^3 + 3*(a^5 + \\
& 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*\cosh(d*x + c))*\sinh(d*x + \\
& c)^5 + a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5 + 2*(3*a^5 \\
& + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*\cosh(d*x + c)^4 + 2*(3 \\
& *a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5 + 35*(a^5 + 5*a^4*b \\
& b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*\cosh(d*x + c)^4 + 30*(a^5 + 3* \\
& a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*\cosh(d*x + c)^2)*\sinh(d*x + \\
& c)^4 + 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*\cosh(\\
& d*x + c)^5 + 10*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*\cos \\
& h(d*x + c)^3 + (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)* \\
& \cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - \\
& 3*a*b^4 - b^5)*\cosh(d*x + c)^2 + 4*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2 \\
& *b^3 + 5*a*b^4 + b^5)*\cosh(d*x + c)^6 + a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b \\
& ^3 - 3*a*b^4 - b^5 + 15*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - \\
& b^5)*\cosh(d*x + c)^4 + 3*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 \\
& + 3*b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((a^5 + 5*a^4*b + 10*a^3*b^2 \\
& + 10*a^2*b^3 + 5*a*b^4 + b^5)*\cosh(d*x + c)^7 + 3*(a^5 + 3*a^4*b + 2*a^3*b \\
& ^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*\cosh(d*x + c)^5 + (3*a^5 + 7*a^4*b + 6*a^3* \\
& b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*\cosh(d*x + c)^3 + (a^5 + 3*a^4*b + 2*a^3 \\
& *b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*\cosh(d*x + c))*\sinh(d*x + c))*\log(2*\sinh(\\
& d*x + c)/(\cosh(d*x + c) - \sinh(d*x + c))) + 8*(2*(a^5 + 2*a^4*b + a^3*b^2)* \\
& d*x*\cosh(d*x + c)^7 - 3*(3*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 2*(a^5 - a^3*b^2)* \\
& d*x)*\cosh(d*x + c)^5 - 2*(6*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 - (3*a^5 - 2*a^4* \\
& b + 3*a^3*b^2)*d*x)*\cosh(d*x + c)^3 - (3*a^3*b^2 + 4*a^2*b^3 + a*b^4 - 2*(a \\
& ^5 - a^3*b^2)*d*x)*\cosh(d*x + c))*\sinh(d*x + c))/((a^8 + 5*a^7*b + 10*a^6*b
\end{aligned}$$

$$\begin{aligned} &^2 + 10a^5b^3 + 5a^4b^4 + a^3b^5) * d * \cosh(dx + c)^8 + 8(a^8 + 5a^7b \\ &+ 10a^6b^2 + 10a^5b^3 + 5a^4b^4 + a^3b^5) * d * \cosh(dx + c) * \sinh(dx \\ &+ c)^7 + (a^8 + 5a^7b + 10a^6b^2 + 10a^5b^3 + 5a^4b^4 + a^3b^5) * d * \\ &\sinh(dx + c)^8 + 4(a^8 + 3a^7b + 2a^6b^2 - 2a^5b^3 - 3a^4b^4 - a^3b^5) * d * \cosh(dx + c)^6 + 4(7(a^8 + 5a^7b + 10a^6b^2 + 10a^5b^3 + \\ &5a^4b^4 + a^3b^5) * d * \cosh(dx + c)^2 + (a^8 + 3a^7b + 2a^6b^2 - 2a^5b^3 - 3a^4b^4 - a^3b^5) * d) * \sinh(dx + c)^6 + 2(3a^8 + 7a^7b + 6a^6 \\ &b^2 + 6a^5b^3 + 7a^4b^4 + 3a^3b^5) * d * \cosh(dx + c)^4 + 8(7(a^8 + 5 \\ &a^7b + 10a^6b^2 + 10a^5b^3 + 5a^4b^4 + a^3b^5) * d * \cosh(dx + c)^3 + \\ &3(a^8 + 3a^7b + 2a^6b^2 - 2a^5b^3 - 3a^4b^4 - a^3b^5) * d * \cosh(dx \\ &+ c)) * \sinh(dx + c)^5 + 2(35(a^8 + 5a^7b + 10a^6b^2 + 10a^5b^3 + 5 \\ &a^4b^4 + a^3b^5) * d * \cosh(dx + c)^4 + 30(a^8 + 3a^7b + 2a^6b^2 - 2a^5b^3 - 3a^4b^4 - a^3b^5) * d * \cosh(dx + c)^2 + (3a^8 + 7a^7b + 6a^6b^2 + 6a^5b^3 + 7a^4b^4 + 3a^3b^5) * d) * \sinh(dx + c)^4 + 4(a^8 + 3a^7b + 2a^6b^2 - 2a^5b^3 - 3a^4b^4 - a^3b^5) * d * \cosh(dx + c)^2 + 8(7 \\ &(a^8 + 5a^7b + 10a^6b^2 + 10a^5b^3 + 5a^4b^4 + a^3b^5) * d * \cosh(dx \\ &+ c)^5 + 10(a^8 + 3a^7b + 2a^6b^2 - 2a^5b^3 - 3a^4b^4 - a^3b^5) * \\ &d * \cosh(dx + c)^3 + (3a^8 + 7a^7b + 6a^6b^2 + 6a^5b^3 + 7a^4b^4 + \\ &3a^3b^5) * d * \cosh(dx + c)) * \sinh(dx + c)^3 + 4(7(a^8 + 5a^7b + 10a^6b^2 + 10a^5b^3 + 5a^4b^4 + a^3b^5) * d * \cosh(dx + c)^6 + 15(a^8 + 3a^7b + 2a^6b^2 - 2a^5b^3 - 3a^4b^4 - a^3b^5) * d * \cosh(dx + c)^4 + 3(3a^8 + 7a^7b + 6a^6b^2 + 6a^5b^3 + 7a^4b^4 + 3a^3b^5) * d * \cosh(dx + c)^2 + (a^8 + 3a^7b + 2a^6b^2 - 2a^5b^3 - 3a^4b^4 - a^3b^5) * d) * \sinh(dx + c)^2 + (a^8 + 5a^7b + 10a^6b^2 + 10a^5b^3 + 5a^4b^4 + a^3b^5) * d + 8((a^8 + 5a^7b + 10a^6b^2 + 10a^5b^3 + 5a^4b^4 + a^3b^5) * d * \cosh(dx + c)^7 + 3(a^8 + 3a^7b + 2a^6b^2 - 2a^5b^3 - 3a^4b^4 - a^3b^5) * d * \cosh(dx + c)^5 + (3a^8 + 7a^7b + 6a^6b^2 + 6a^5b^3 + 7a^4b^4 + 3a^3b^5) * d * \cosh(dx + c)^3 + (a^8 + 3a^7b + 2a^6b^2 - 2a^5b^3 - 3a^4b^4 - a^3b^5) * d * \cosh(dx + c)) * \sinh(dx + c)) \end{aligned}$$

giac [B] time = 0.49, size = 295, normalized size = 2.14

$$\frac{(3a^2b+3ab^2+b^3) \log(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b)}{a^6+3a^5b+3a^4b^2+a^3b^3} + \frac{2(dx+c)}{a^3+3a^2b+3ab^2+b^3} - \frac{2 \log(|e^{(2dx+2c)}-1|)}{a^3} - \frac{4 \left((3a^2b^2+ab^3)e^{(4dx+4c)} \right)}{(ae^{(4dx+4c)})}$$

2 d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(dx+c)/(a+b*tanh(dx+c)^2)^3,x, algorithm="giac")
 [Out] -1/2*((3a^2*b + 3a*b^2 + b^3)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^6 + 3a^5*b + 3a^4*b^2 + a^3*b^3) + 2*(d*x + c)/(a^3 + 3a^2*b + 3a*b^2 + b^3) - 2*log(abs(e^(2*d*x + 2*c) - 1))/a^3 - 4*((3a^2*b^2 + a*b^3)*e^(6*d*x + 6*c) + (3a^2*b^2 + a*b^3)*e^(2*d*x + 2*c) + 2*(3a^3*b^2 - a^2*b^3 - a*b^4)*e^(4*d*x + 4*c))/(a + b))/((a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2*(a + b)^2*a^3))/d

maple [B] time = 0.50, size = 952, normalized size = 6.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(dx+c)/(a+b*tanh(dx+c)^2)^3,x)
 [Out] -1/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)-1)-1/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)+1)-6/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^6-10/d*b^3/(a+b)^3/a/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*ta

$$\frac{\operatorname{nh}\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-4/d*b^4/(a+b)^3/a^2/\left(\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4*a+2*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2*a+4*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2*b+a\right)^2*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-12/d*b^2/(a+b)^3/\left(\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4*a+2*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2*a+4*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2*b+a\right)^2*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-40/d*b^3/(a+b)^3/a/\left(\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4*a+2*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2*a+4*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2*b+a\right)^2*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-40/d*b^4/(a+b)^3/a^2/\left(\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4*a+2*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2*a+4*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2*b+a\right)^2*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-12/d*b^5/(a+b)^3/a^3/\left(\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4*a+2*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2*a+4*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2*b+a\right)^2*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-6/d*b^2/(a+b)^3/\left(\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4*a+2*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2*a+4*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2*b+a\right)^2*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-10/d*b^3/(a+b)^3/a/\left(\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4*a+2*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2*a+4*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2*b+a\right)^2*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-4/d*b^4/(a+b)^3/a^2/\left(\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4*a+2*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2*a+4*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2*b+a\right)^2*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3/2/d*b/(a+b)^3/a*\ln\left(\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4*a+2*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2*a+4*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2*b+a\right)-3/2/d*b^2/(a+b)^3/a^2*2*\ln\left(\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4*a+2*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2*a+4*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2*b+a\right)-1/2/d*b^3/(a+b)^3/a^3*\ln\left(\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4*a+2*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2*a+4*\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2*b+a\right)+1/d/a^3*\ln\left(\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}$$

maxima [B] time = 0.40, size = 498, normalized size = 3.61

$$\frac{(3a^2b + 3ab^2 + b^3) \log(2(a-b)e^{(-2dx-2c)} + (a+b)e^{(-4dx-4c)} + a + b)}{2(a^6 + 3a^5b + 3a^4b^2 + a^3b^3)d} + \frac{dx + c}{(a^3 + 3a^2b + 3ab^2 + b^3)d} + \frac{1}{(a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 + 5a^3b^4 + a^2b^5 + 4(a^7 + 3a^6b + 2a^5b^2 - 2a^4b^3 - 3a^3b^4 - a^2b^5)e^{(-2dx-2c)} + 2(3a^7 + 7a^6b + 6a^5b^2 + 6a^4b^3 + 7a^3b^4 + 3a^2b^5)e^{(-4dx-4c)} + 4(a^7 + 3a^6b + 2a^5b^2 - 2a^4b^3 - 3a^3b^4 - a^2b^5)e^{(-6dx-6c)} + (a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 + 5a^3b^4 + a^2b^5)e^{(-8dx-8c)}) * d} + \log(e^{(-dx-c)} + 1)/(a^3d) + \log(e^{(-dx-c)} - 1)/(a^3d)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(dx+c)/(a+b*tanh(dx+c)^2)^3,x, algorithm="maxima")

[Out]
$$\frac{-1/2*(3*a^2*b + 3*a*b^2 + b^3)*\log(2*(a - b)*e^{(-2*d*x - 2*c)} + (a + b)*e^{(-4*d*x - 4*c)} + a + b)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) + (d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 2*((3*a^2*b^2 + 4*a*b^3 + b^4)*e^{(-2*d*x - 2*c)} + 2*(3*a^2*b^2 - a*b^3 - b^4)*e^{(-4*d*x - 4*c)} + (3*a^2*b^2 + 4*a*b^3 + b^4)*e^{(-6*d*x - 6*c)})/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^{(-2*d*x - 2*c)} + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*e^{(-4*d*x - 4*c)} + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^{(-6*d*x - 6*c)} + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*e^{(-8*d*x - 8*c)}) * d} + \log(e^{(-d*x - c)} + 1)/(a^3*d) + \log(e^{(-d*x - c)} - 1)/(a^3*d)$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{coth}(c + dx)}{(b \tanh(c + dx)^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + dx)/(a + b*tanh(c + dx)^2)^3,x)

[Out] int(coth(c + dx)/(a + b*tanh(c + dx)^2)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{coth}(c + dx)}{(a + b \tanh^2(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(dx+c)/(a+b*tanh(dx+c)**2)**3,x)

[Out] Integral(coth(c + dx)/(a + b*tanh(c + dx)**2)**3, x)

$$3.198 \quad \int \frac{\coth^2(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=178

$$\frac{b(9a+5b) \coth(c+dx)}{8a^2d(a+b)^2(a+b \tanh^2(c+dx))} - \frac{b^{3/2}(35a^2+42ab+15b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d(a+b)^3} - \frac{(8a^2+27ab+15b^2) \coth(c+dx)}{8a^3d(a+b)^2}$$

[Out] x/(a+b)^3-1/8*b^(3/2)*(35*a^2+42*a*b+15*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(7/2)/(a+b)^3/d-1/8*(8*a^2+27*a*b+15*b^2)*coth(d*x+c)/a^3/(a+b)^2/d+1/4*b*coth(d*x+c)/a/(a+b)/d/(a+b*tanh(d*x+c)^2)^2+1/8*b*(9*a+5*b)*coth(d*x+c)/a^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)

Rubi [A] time = 0.29, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3670, 472, 579, 583, 522, 206, 205}

$$-\frac{(8a^2+27ab+15b^2) \coth(c+dx)}{8a^3d(a+b)^2} - \frac{b^{3/2}(35a^2+42ab+15b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d(a+b)^3} + \frac{b(9a+5b) \coth(c+dx)}{8a^2d(a+b)^2(a+b \tanh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] x/(a + b)^3 - (b^(3/2)*(35*a^2 + 42*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(7/2)*(a + b)^3*d) - ((8*a^2 + 27*a*b + 15*b^2)*Coth[c + d*x])/(8*a^3*(a + b)^2*d) + (b*Coth[c + d*x])/(4*a*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) + (b*(9*a + 5*b)*Coth[c + d*x])/(8*a^2*(a + b)^2*d*(a + b*Tanh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 472

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*(e*x)^(m+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(a*e*n*(b*c-a*d)*(p+1)), x] + Dist[1/(a*n*(b*c-a*d)*(p+1)), Int[(e*x)^m*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[c*b*(m+1)+n*(b*c-a*d)*(p+1)+d*b*(m+n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.)*((e_.) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.)*((e_.) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_.) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))
```

Rubi steps

$$\begin{aligned} \int \frac{\coth^2(c + dx)}{(a + b \tanh^2(c + dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)(a+bx^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{b \coth(c + dx)}{4a(a + b)d (a + b \tanh^2(c + dx))^2} - \frac{\text{Subst}\left(\int \frac{-4a-5b+5bx^2}{x^2(1-x^2)(a+bx^2)^2} dx, x, \tanh(c + dx)\right)}{4a(a + b)d} \\ &= \frac{b \coth(c + dx)}{4a(a + b)d (a + b \tanh^2(c + dx))^2} + \frac{b(9a + 5b) \coth(c + dx)}{8a^2(a + b)^2d (a + b \tanh^2(c + dx))} + \frac{\text{Subst}\left(\int \frac{b}{x^2(1-x^2)} dx, x, \tanh(c + dx)\right)}{8a^2} \\ &= -\frac{(8a^2 + 27ab + 15b^2) \coth(c + dx)}{8a^3(a + b)^2d} + \frac{b \coth(c + dx)}{4a(a + b)d (a + b \tanh^2(c + dx))^2} + \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)} dx, x, \tanh(c + dx)\right)}{8a^2} \\ &= -\frac{(8a^2 + 27ab + 15b^2) \coth(c + dx)}{8a^3(a + b)^2d} + \frac{b \coth(c + dx)}{4a(a + b)d (a + b \tanh^2(c + dx))^2} + \frac{\text{Subst}\left(\int \frac{1}{x^2(1-x^2)} dx, x, \tanh(c + dx)\right)}{8a^2} \\ &= \frac{x}{(a + b)^3} - \frac{b^{3/2} (35a^2 + 42ab + 15b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}(a + b)^3d} - \frac{(8a^2 + 27ab + 15b^2) \coth(c + dx)}{8a^3(a + b)^2d} \end{aligned}$$

Mathematica [A] time = 6.31, size = 166, normalized size = 0.93

$$\frac{\frac{b^2(13a+7b) \sinh(2(c+dx))}{a^3(a+b)^2((a+b) \cosh(2(c+dx))+a-b)} + \frac{8 \coth(c+dx)}{a^3} + \frac{4b^3 \sinh(2(c+dx))}{a^2(a+b)^2((a+b) \cosh(2(c+dx))+a-b)^2} + \frac{b^{3/2}(35a^2+42ab+15b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}(a+b)^3}}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] -1/8*((-8*(c + d*x))/(a + b)^3 + (b^(3/2)*(35*a^2 + 42*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(7/2)*(a + b)^3) + (8*Coth[c + d*x])/a^3 + (4*b^3*Sinh[2*(c + d*x)]/(a^2*(a + b)^2*(a - b + (a + b)*Cosh[2*(c + d*x)]))^2 + (b^2*(13*a + 7*b)*Sinh[2*(c + d*x)]/(a^3*(a + b)^2*(a - b + (a + b)*Cosh[2*(c + d*x)])))/d

fricas [B] time = 0.68, size = 11865, normalized size = 66.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(16*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^10 + 160*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^9 + 16*(a^5 + 2*a^4*b + a^3*b^2)*d*x*sinh(d*x + c)^10 - 4*(8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c)^8 - 4*(8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 180*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^2 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*sinh(d*x + c)^8 + 32*(60*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^3 - (8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 - 8*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a*b^4 - 30*b^5 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^6 + 8*(420*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^4 - 16*a^5 - 48*a^4*b - 19*a^3*b^2 + 28*a^2*b^3 + 69*a*b^4 + 30*b^5 + 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x - 14*(8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 16*(252*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^5 - 14*(8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c)^3 - 3*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a*b^4 - 30*b^5 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 32*a^5 - 160*a^4*b - 372*a^3*b^2 - 452*a^2*b^3 - 268*a*b^4 - 60*b^5 - 8*(24*a^5 + 56*a^4*b + 48*a^3*b^2 + 33*a^2*b^3 + 86*a*b^4 + 45*b^5 + 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^4 + 8*(420*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^6 - 24*a^5 - 56*a^4*b - 48*a^3*b^2 - 33*a^2*b^3 - 86*a*b^4 - 45*b^5 - 35*(8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c)^4 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x - 15*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a*b^4 - 30*b^5 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 32*(60*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^7 - 7*(8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c)^5 - 5*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a*b^4 - 30*b^5 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c)^3 - (24*a^5 + 56*a^4*b + 48*a^3*b^2 + 33*a^2*b^3 + 86*a*b^4 + 45*b^5 + 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 - 16*(a^5 + 2*a^4*b + a^3*b^2)*d*x - 8*(16*a^5 + 48*a^4*b + 45*a^3*b^2 - 36*a^2*b^3 - 79*a*b^4 - 30*b^5 + 2*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(d*x + c)^2 + 8*(90*(a^5 + 2*a^4*b + a^3*b^2)*d*x*cosh(d*x + c)^8 - 14*(8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*cosh(

$$\begin{aligned}
& d*x + c)^6 - 16*a^5 - 48*a^4*b - 45*a^3*b^2 + 36*a^2*b^3 + 79*a*b^4 + 30*b^5 \\
& - 15*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a*b^4 - 30*b^5 - 4 \\
& *(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*\cosh(d*x + c)^4 - 2*(3*a^5 - 2*a^4*b - 5* \\
& a^3*b^2)*d*x - 6*(24*a^5 + 56*a^4*b + 48*a^3*b^2 + 33*a^2*b^3 + 86*a*b^4 + \\
& 45*b^5 + 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 \\
& + ((35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*\cosh(d*x + \\
& c)^10 + 10*(35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*\cosh(\\
& d*x + c)*\sinh(d*x + c)^9 + (35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 \\
& + 15*b^5)*\sinh(d*x + c)^10 + (105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a \\
& *b^4 - 75*b^5)*\cosh(d*x + c)^8 + (105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 24 \\
& 0*a*b^4 - 75*b^5 + 45*(35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15 \\
& *b^5)*\cosh(d*x + c)^2*\sinh(d*x + c)^8 + 8*(15*(35*a^4*b + 112*a^3*b^2 + 13 \\
& 4*a^2*b^3 + 72*a*b^4 + 15*b^5)*\cosh(d*x + c)^3 + (105*a^4*b + 56*a^3*b^2 - \\
& 214*a^2*b^3 - 240*a*b^4 - 75*b^5)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(35*a^ \\
& 4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5)*\cosh(d*x + c)^6 + 2*(3 \\
& 5*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5 + 105*(35*a^4*b + 1 \\
& 12*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*\cosh(d*x + c)^4 + 14*(105*a^4 \\
& *b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 75*b^5)*\cosh(d*x + c)^2*\sinh(d \\
& *x + c)^6 + 4*(63*(35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5 \\
&)*\cosh(d*x + c)^5 + 14*(105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - \\
& 75*b^5)*\cosh(d*x + c)^3 + 3*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^ \\
& 4 + 75*b^5)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 35*a^4*b - 112*a^3*b^2 - 134*a \\
& ^2*b^3 - 72*a*b^4 - 15*b^5 - 2*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a \\
& *b^4 + 75*b^5)*\cosh(d*x + c)^4 + 2*(105*(35*a^4*b + 112*a^3*b^2 + 134*a^2*b \\
& ^3 + 72*a*b^4 + 15*b^5)*\cosh(d*x + c)^6 - 35*a^4*b + 28*a^3*b^2 - 106*a^2*b \\
& ^3 - 180*a*b^4 - 75*b^5 + 35*(105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a* \\
& b^4 - 75*b^5)*\cosh(d*x + c)^4 + 15*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 1 \\
& 80*a*b^4 + 75*b^5)*\cosh(d*x + c)^2*\sinh(d*x + c)^4 + 8*(15*(35*a^4*b + 112 \\
& *a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*\cosh(d*x + c)^7 + 7*(105*a^4*b \\
& + 56*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 75*b^5)*\cosh(d*x + c)^5 + 5*(35*a^ \\
& 4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5)*\cosh(d*x + c)^3 - (35* \\
& a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5)*\cosh(d*x + c))*\sinh(\\
& d*x + c)^3 - (105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 75*b^5)*\co \\
& sh(d*x + c)^2 + (45*(35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b \\
& ^5)*\cosh(d*x + c)^8 + 28*(105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 \\
& - 75*b^5)*\cosh(d*x + c)^6 - 105*a^4*b - 56*a^3*b^2 + 214*a^2*b^3 + 240*a*b^ \\
& 4 + 75*b^5 + 30*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5)* \\
& \cosh(d*x + c)^4 - 12*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75* \\
& b^5)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 2*(5*(35*a^4*b + 112*a^3*b^2 + 134* \\
& a^2*b^3 + 72*a*b^4 + 15*b^5)*\cosh(d*x + c)^9 + 4*(105*a^4*b + 56*a^3*b^2 - \\
& 214*a^2*b^3 - 240*a*b^4 - 75*b^5)*\cosh(d*x + c)^7 + 6*(35*a^4*b - 28*a^3*b^ \\
& 2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5)*\cosh(d*x + c)^5 - 4*(35*a^4*b - 28*a^ \\
& 3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5)*\cosh(d*x + c)^3 - (105*a^4*b + 56 \\
& *a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 75*b^5)*\cosh(d*x + c))*\sinh(d*x + c))* \\
& \sqrt{-b/a}*\log(((a^2 + 2*a*b + b^2)*\cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2) \\
& *\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*\sinh(d*x + c)^4 + 2*(a \\
& ^2 - b^2)*\cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*\cosh(d*x + c)^2 + a^2 \\
& - b^2)*\sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*\cosh(d* \\
& x + c)^3 + (a^2 - b^2)*\cosh(d*x + c))*\sinh(d*x + c) - 4*((a^2 + a*b)*\cosh(d \\
& *x + c)^2 + 2*(a^2 + a*b)*\cosh(d*x + c)*\sinh(d*x + c) + (a^2 + a*b)*\sinh(d* \\
& x + c)^2 + a^2 - a*b)*\sqrt{-b/a}))/((a + b)*\cosh(d*x + c)^4 + 4*(a + b)*\cosh \\
& (d*x + c)*\sinh(d*x + c)^3 + (a + b)*\sinh(d*x + c)^4 + 2*(a - b)*\cosh(d*x + \\
& c)^2 + 2*(3*(a + b)*\cosh(d*x + c)^2 + a - b)*\sinh(d*x + c)^2 + 4*((a + b)*\c \\
& osh(d*x + c)^3 + (a - b)*\cosh(d*x + c))*\sinh(d*x + c) + a + b)) + 16*(10*(a \\
& ^5 + 2*a^4*b + a^3*b^2)*d*x*\cosh(d*x + c)^9 - 2*(8*a^5 + 40*a^4*b + 67*a^3* \\
& b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x) \\
& *\cosh(d*x + c)^7 - 3*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a*b^ \\
& 4 - 30*b^5 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*\cosh(d*x + c)^5 - 2*(24*a^5 \\
& + 56*a^4*b + 48*a^3*b^2 + 33*a^2*b^3 + 86*a*b^4 + 45*b^5 + 4*(a^5 - 2*a^4*
\end{aligned}$$

$$\begin{aligned}
& ^3*b^2)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(252*(a^5 + 2*a^4*b + a^3 \\
& *b^2)*d*x*\cosh(d*x + c)^5 - 14*(8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 \\
& + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*\cosh(d*x + c)^3 \\
& - 3*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a*b^4 - 30*b^5 - 4*(a \\
& ^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 16*a^5 - 80 \\
& *a^4*b - 186*a^3*b^2 - 226*a^2*b^3 - 134*a*b^4 - 30*b^5 - 4*(24*a^5 + 56*a^4 \\
& *b + 48*a^3*b^2 + 33*a^2*b^3 + 86*a*b^4 + 45*b^5 + 4*(a^5 - 2*a^4*b + 5*a^3 \\
& *b^2)*d*x)*\cosh(d*x + c)^4 + 4*(420*(a^5 + 2*a^4*b + a^3*b^2)*d*x*\cosh(d*x \\
& + c)^6 - 24*a^5 - 56*a^4*b - 48*a^3*b^2 - 33*a^2*b^3 - 86*a*b^4 - 45*b^5 - \\
& 35*(8*a^5 + 40*a^4*b + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3* \\
& a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*\cosh(d*x + c)^4 - 4*(a^5 - 2*a^4*b + 5*a^3* \\
& b^2)*d*x - 15*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a*b^4 - 30* \\
& b^5 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + \\
& 16*(60*(a^5 + 2*a^4*b + a^3*b^2)*d*x*\cosh(d*x + c)^7 - 7*(8*a^5 + 40*a^4*b \\
& + 67*a^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3 \\
& *b^2)*d*x)*\cosh(d*x + c)^5 - 5*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 \\
& - 69*a*b^4 - 30*b^5 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*\cosh(d*x + c)^3 - \\
& (24*a^5 + 56*a^4*b + 48*a^3*b^2 + 33*a^2*b^3 + 86*a*b^4 + 45*b^5 + 4*(a^5 \\
& - 2*a^4*b + 5*a^3*b^2)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 8*(a^5 + 2*a^4 \\
& *b + a^3*b^2)*d*x - 4*(16*a^5 + 48*a^4*b + 45*a^3*b^2 - 36*a^2*b^3 - 79*a*b \\
& ^4 - 30*b^5 + 2*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*\cosh(d*x + c)^2 + 4*(90* \\
& (a^5 + 2*a^4*b + a^3*b^2)*d*x*\cosh(d*x + c)^8 - 14*(8*a^5 + 40*a^4*b + 67*a \\
& ^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d \\
& *x)*\cosh(d*x + c)^6 - 16*a^5 - 48*a^4*b - 45*a^3*b^2 + 36*a^2*b^3 + 79*a*b^4 \\
& + 30*b^5 - 15*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a*b^4 - 3 \\
& 0*b^5 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*\cosh(d*x + c)^4 - 2*(3*a^5 - 2*a \\
& ^4*b - 5*a^3*b^2)*d*x - 6*(24*a^5 + 56*a^4*b + 48*a^3*b^2 + 33*a^2*b^3 + 86 \\
& *a*b^4 + 45*b^5 + 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*\cosh(d*x + c)^2)*\sinh(\\
& d*x + c)^2 - ((35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*\co \\
& sh(d*x + c)^10 + 10*(35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b \\
& ^5)*\cosh(d*x + c)*\sinh(d*x + c)^9 + (35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + \\
& 72*a*b^4 + 15*b^5)*\sinh(d*x + c)^10 + (105*a^4*b + 56*a^3*b^2 - 214*a^2*b^ \\
& 3 - 240*a*b^4 - 75*b^5)*\cosh(d*x + c)^8 + (105*a^4*b + 56*a^3*b^2 - 214*a^2 \\
& *b^3 - 240*a*b^4 - 75*b^5 + 45*(35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a \\
& *b^4 + 15*b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 8*(15*(35*a^4*b + 112*a^3 \\
& *b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*\cosh(d*x + c)^3 + (105*a^4*b + 56*a \\
& ^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 75*b^5)*\cosh(d*x + c))*\sinh(d*x + c)^7 + \\
& 2*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5)*\cosh(d*x + c) \\
& ^6 + 2*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5 + 105*(35* \\
& a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*\cosh(d*x + c)^4 + 14 \\
& *(105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 75*b^5)*\cosh(d*x + c)^ \\
& 2)*\sinh(d*x + c)^6 + 4*(63*(35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 \\
& + 15*b^5)*\cosh(d*x + c)^5 + 14*(105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240 \\
& *a*b^4 - 75*b^5)*\cosh(d*x + c)^3 + 3*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + \\
& 180*a*b^4 + 75*b^5)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 35*a^4*b - 112*a^3*b^ \\
& 2 - 134*a^2*b^3 - 72*a*b^4 - 15*b^5 - 2*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^ \\
& 3 + 180*a*b^4 + 75*b^5)*\cosh(d*x + c)^4 + 2*(105*(35*a^4*b + 112*a^3*b^2 + \\
& 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*\cosh(d*x + c)^6 - 35*a^4*b + 28*a^3*b^2 - \\
& 106*a^2*b^3 - 180*a*b^4 - 75*b^5 + 35*(105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 \\
& - 240*a*b^4 - 75*b^5)*\cosh(d*x + c)^4 + 15*(35*a^4*b - 28*a^3*b^2 + 106*a^ \\
& 2*b^3 + 180*a*b^4 + 75*b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(15*(35*a^ \\
& 4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*\cosh(d*x + c)^7 + 7*(1 \\
& 05*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 75*b^5)*\cosh(d*x + c)^5 + \\
& 5*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5)*\cosh(d*x + c) \\
& ^3 - (35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5)*\cosh(d*x + \\
& c))*\sinh(d*x + c)^3 - (105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 7 \\
& 5*b^5)*\cosh(d*x + c)^2 + (45*(35*a^4*b + 112*a^3*b^2 + 134*a^2*b^3 + 72*a*b \\
& ^4 + 15*b^5)*\cosh(d*x + c)^8 + 28*(105*a^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 2 \\
& 40*a*b^4 - 75*b^5)*\cosh(d*x + c)^6 - 105*a^4*b - 56*a^3*b^2 + 214*a^2*b^3 +
\end{aligned}$$

$$\begin{aligned}
& 240*a*b^4 + 75*b^5 + 30*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + \\
& 75*b^5)*\cosh(d*x + c)^4 - 12*(35*a^4*b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a* \\
& b^4 + 75*b^5)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 2*(5*(35*a^4*b + 112*a^3*b \\
& ^2 + 134*a^2*b^3 + 72*a*b^4 + 15*b^5)*\cosh(d*x + c)^9 + 4*(105*a^4*b + 56*a \\
& ^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 75*b^5)*\cosh(d*x + c)^7 + 6*(35*a^4*b - \\
& 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5)*\cosh(d*x + c)^5 - 4*(35*a^4*b \\
& b - 28*a^3*b^2 + 106*a^2*b^3 + 180*a*b^4 + 75*b^5)*\cosh(d*x + c)^3 - (105*a \\
& ^4*b + 56*a^3*b^2 - 214*a^2*b^3 - 240*a*b^4 - 75*b^5)*\cosh(d*x + c))*\sinh(d \\
& *x + c))*\sqrt{b/a}*\arctan(1/2*((a + b)*\cosh(d*x + c)^2 + 2*(a + b)*\cosh(d*x \\
& + c)*\sinh(d*x + c) + (a + b)*\sinh(d*x + c)^2 + a - b)*\sqrt{b/a}/b) + 8*(10 \\
& *(a^5 + 2*a^4*b + a^3*b^2)*d*x*\cosh(d*x + c)^9 - 2*(8*a^5 + 40*a^4*b + 67*a \\
& ^3*b^2 + 77*a^2*b^3 + 57*a*b^4 + 15*b^5 - 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d \\
& *x)*\cosh(d*x + c)^7 - 3*(16*a^5 + 48*a^4*b + 19*a^3*b^2 - 28*a^2*b^3 - 69*a \\
& *b^4 - 30*b^5 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*d*x)*\cosh(d*x + c)^5 - 2*(24* \\
& a^5 + 56*a^4*b + 48*a^3*b^2 + 33*a^2*b^3 + 86*a*b^4 + 45*b^5 + 4*(a^5 - 2*a \\
& ^4*b + 5*a^3*b^2)*d*x)*\cosh(d*x + c)^3 - (16*a^5 + 48*a^4*b + 45*a^3*b^2 - \\
& 36*a^2*b^3 - 79*a*b^4 - 30*b^5 + 2*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*d*x)*\cosh(\\
& d*x + c))*\sinh(d*x + c))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4* \\
& b^4 + a^3*b^5)*d*\cosh(d*x + c)^10 + 10*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5 \\
& *b^3 + 5*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c)*\sinh(d*x + c)^9 + (a^8 + 5*a^7* \\
& b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*\sinh(d*x + c)^10 + (3* \\
& a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*d*\cosh(d*x \\
& + c)^8 + (45*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^ \\
& 5)*d*\cosh(d*x + c)^2 + (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b \\
& ^4 - 5*a^3*b^5)*d)*\sinh(d*x + c)^8 + 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^ \\
& 3 + 13*a^4*b^4 + 5*a^3*b^5)*d*\cosh(d*x + c)^6 + 8*(15*(a^8 + 5*a^7*b + 10*a \\
& ^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c)^3 + (3*a^8 + 7*a \\
& ^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*d*\cosh(d*x + c))*\si \\
& nh(d*x + c)^7 + 2*(105*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 \\
& + a^3*b^5)*d*\cosh(d*x + c)^4 + 14*(3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^ \\
& 3 - 17*a^4*b^4 - 5*a^3*b^5)*d*\cosh(d*x + c)^2 + (a^8 + a^7*b + 2*a^6*b^2 + \\
& 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d)*\sinh(d*x + c)^6 - 2*(a^8 + a^7*b + \\
& 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d*\cosh(d*x + c)^4 + 4*(63* \\
& (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*\cosh(d*x \\
& + c)^5 + 14*(3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3* \\
& b^5)*d*\cosh(d*x + c)^3 + 3*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b \\
& ^4 + 5*a^3*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(105*(a^8 + 5*a^7*b + \\
& 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c)^6 + 35*(3*a^ \\
& 8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*d*\cosh(d*x + \\
& c)^4 + 15*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)* \\
& d*\cosh(d*x + c)^2 - (a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5* \\
& a^3*b^5)*d)*\sinh(d*x + c)^4 - (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 1 \\
& 7*a^4*b^4 - 5*a^3*b^5)*d*\cosh(d*x + c)^2 + 8*(15*(a^8 + 5*a^7*b + 10*a^6*b^ \\
& 2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c)^7 + 7*(3*a^8 + 7*a^7* \\
& b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*d*\cosh(d*x + c)^5 + 5* \\
& (a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d*\cosh(d*x \\
& + c)^3 - (a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d* \\
& \cosh(d*x + c))*\sinh(d*x + c)^3 + (45*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b \\
& ^3 + 5*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c)^8 + 28*(3*a^8 + 7*a^7*b - 2*a^6*b \\
& ^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*d*\cosh(d*x + c)^6 + 30*(a^8 + a^7 \\
& *b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d*\cosh(d*x + c)^4 - 1 \\
& 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d*\cosh(d* \\
& x + c)^2 - (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b \\
& ^5)*d)*\sinh(d*x + c)^2 - (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b \\
& ^4 + a^3*b^5)*d + 2*(5*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 \\
& + a^3*b^5)*d*\cosh(d*x + c)^9 + 4*(3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 \\
& - 17*a^4*b^4 - 5*a^3*b^5)*d*\cosh(d*x + c)^7 + 6*(a^8 + a^7*b + 2*a^6*b^2 + \\
& 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d*\cosh(d*x + c)^5 - 4*(a^8 + a^7*b + \\
& 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*d*\cosh(d*x + c)^3 - (3*a^8
\end{aligned}$$

+ 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)]

giac [B] time = 0.55, size = 437, normalized size = 2.46

$$\frac{(35a^2b^2+42ab^3+15b^4)\arctan\left(\frac{ae^{2dx+2c}+be^{2dx+2c}+a-b}{2\sqrt{ab}}\right)}{(a^6+3a^5b+3a^4b^2+a^3b^3)\sqrt{ab}} - \frac{8(dx+c)}{a^3+3a^2b+3ab^2+b^3} - \frac{2(13a^3b^2e^{6dx+6c}+3a^2b^3e^{6dx+6c}-17ab^4e^{6dx+6c}-7b^5e^{6dx+6c})}{a^3+3a^2b+3ab^2+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c))^2)^3,x, algorithm="giac")

[Out] -1/8*((35*a^2*b^2 + 42*a*b^3 + 15*b^4)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b)))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt(a*b)) - 8*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(13*a^3*b^2*e^(6*d*x + 6*c) + 3*a^2*b^3*e^(6*d*x + 6*c) - 17*a*b^4*e^(6*d*x + 6*c) - 7*b^5*e^(6*d*x + 6*c) + 39*a^3*b^2*e^(4*d*x + 4*c) - 5*a^2*b^3*e^(4*d*x + 4*c) + 25*a*b^4*e^(4*d*x + 4*c) + 21*b^5*e^(4*d*x + 4*c) + 39*a^3*b^2*e^(2*d*x + 2*c) + 25*a^2*b^3*e^(2*d*x + 2*c) - 35*a*b^4*e^(2*d*x + 2*c) - 21*b^5*e^(2*d*x + 2*c) + 13*a^3*b^2 + 33*a^2*b^3 + 27*a*b^4 + 7*b^5)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2) + 16/(a^3*(e^(2*d*x + 2*c) - 1))/d

maple [B] time = 0.48, size = 2045, normalized size = 11.49

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^2/(a+b*tanh(d*x+c))^2)^3,x)

[Out] -1/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)+1)+15/8/d*b^5/(a+b)^3/a^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/2/d/a^3*tanh(1/2*d*x+1/2*c)-1/2/d/a^3/tanh(1/2*d*x+1/2*c)+15/8/d*b^5/(a+b)^3/a^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+77/8/d*b^3/(a+b)^3/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+57/8/d*b^4/(a+b)^3/a^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+77/8/d*b^3/(a+b)^3/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+57/8/d*b^4/(a+b)^3/a^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-13/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7-55/2/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^5+35/8/d*b^2/(a+b)^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-99/4/d*b^4/(a+b)^3/a^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5-55/2/d*b^3/(a+b)^3/a/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3-99/4/d*b^4/(a+b)^3/a^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)-35/8/d*b^2/(a+b)^3/a/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+35/8/d*b^2/(a+b)^3/a/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-7/d*b^5/(a+b)^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x

$$\begin{aligned}
& +1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5+15/8/d*b^4 \\
& / (a+b)^3/a^3/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2* \\
& c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-11/2/d*b^3/(a+b)^3/a/(tanh(1/2*d*x+ \\
& 1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/ \\
& 2*d*x+1/2*c)^7-9/4/d*b^4/(a+b)^3/a^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d* \\
& x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7-7/d*b^5/(\\
& a+b)^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d* \\
& x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3-21/4/d*b^3/(a+b)^3/a^2/((2*(b*(a+b) \\
&)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a- \\
& 2*b)*a)^(1/2))-15/8/d*b^4/(a+b)^3/a^3/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)* \\
& arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))+21/4/d*b \\
& ^3/(a+b)^3/a^2/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/ \\
& 2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-9/4/d*b^4/(a+b)^3/a^2/(tanh(1/2*d \\
& *x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh \\
& (1/2*d*x+1/2*c)+35/8/d*b^2/(a+b)^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2* \\
& b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/ \\
& 2))-39/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4 \\
& *tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5-39/4/d*b^2/(a+b)^3/(tan \\
& h(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a) \\
& ^2*tanh(1/2*d*x+1/2*c)^3-13/4/d*b^2/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh \\
& (1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)
\end{aligned}$$

maxima [B] time = 0.94, size = 1944, normalized size = 10.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned}
& -1/4*(3*a^2*b + 3*a*b^2 + b^3)*\log((a + b)*e^{(4*d*x + 4*c)} + 2*(a - b)*e^{(2* \\
& *d*x + 2*c)} + a + b)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) + 1/4*(3*a^2 \\
& *b + 3*a*b^2 + b^3)*\log(2*(a - b)*e^{(-2*d*x - 2*c)} + (a + b)*e^{(-4*d*x - 4* \\
& c)} + a + b)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) + 1/32*(15*a^3*b - 25 \\
& *a^2*b^2 - 39*a*b^3 - 15*b^4)*\arctan(1/2*((a + b)*e^{(2*d*x + 2*c)} + a - b)/ \\
& \sqrt{a*b})/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*\sqrt{a*b}*d) - 1/32*(15*a \\
& ^3*b - 25*a^2*b^2 - 39*a*b^3 - 15*b^4)*\arctan(1/2*((a + b)*e^{(-2*d*x - 2*c)} \\
& + a - b)/\sqrt{a*b})/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*\sqrt{a*b}*d) + \\
& 1/16*(8*a^5 + 31*a^4*b + 72*a^3*b^2 + 98*a^2*b^3 + 64*a*b^4 + 15*b^5 + (8*a \\
& ^5 + 49*a^4*b + 18*a^3*b^2 + 38*a*b^4 + 15*b^5)*e^{(8*d*x + 8*c)} + 2*(16*a^5 \\
& + 57*a^4*b - 9*a^3*b^2 + 37*a^2*b^3 - 39*a*b^4 - 30*b^5)*e^{(6*d*x + 6*c)} + \\
& 2*(24*a^5 + 56*a^4*b + 83*a^3*b^2 - 37*a^2*b^3 + 53*a*b^4 + 45*b^5)*e^{(4*d \\
& *x + 4*c)} + 2*(16*a^5 + 39*a^4*b + 73*a^3*b^2 + 15*a^2*b^3 - 65*a*b^4 - 30* \\
& b^5)*e^{(2*d*x + 2*c)})/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 \\
& + a^3*b^5 - (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5 \\
&)*e^{(10*d*x + 10*c)} - (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 \\
& - 5*a^3*b^5)*e^{(8*d*x + 8*c)} - 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + \\
& 13*a^4*b^4 + 5*a^3*b^5)*e^{(6*d*x + 6*c)} + 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a \\
& ^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*e^{(4*d*x + 4*c)} + (3*a^8 + 7*a^7*b - 2*a^6 \\
& *b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*e^{(2*d*x + 2*c)})*d) - 1/16*(8*a \\
& ^5 + 31*a^4*b + 72*a^3*b^2 + 98*a^2*b^3 + 64*a*b^4 + 15*b^5 + 2*(16*a^5 + 3 \\
& 9*a^4*b + 73*a^3*b^2 + 15*a^2*b^3 - 65*a*b^4 - 30*b^5)*e^{(-2*d*x - 2*c)} + 2 \\
& *(24*a^5 + 56*a^4*b + 83*a^3*b^2 - 37*a^2*b^3 + 53*a*b^4 + 45*b^5)*e^{(-4*d* \\
& x - 4*c)} + 2*(16*a^5 + 57*a^4*b - 9*a^3*b^2 + 37*a^2*b^3 - 39*a*b^4 - 30*b^ \\
& 5)*e^{(-6*d*x - 6*c)} + (8*a^5 + 49*a^4*b + 18*a^3*b^2 + 38*a*b^4 + 15*b^5)*e \\
& ^{(-8*d*x - 8*c)})/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^ \\
& 3*b^5 + (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5) \\
& *e^{(-2*d*x - 2*c)} + 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + \\
& 5*a^3*b^5)*e^{(-4*d*x - 4*c)} - 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13* \\
& a^4*b^4 + 5*a^3*b^5)*e^{(-6*d*x - 6*c)} - (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a \\
& ^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*e^{(-8*d*x - 8*c)} - (a^8 + 5*a^7*b + 10*a^6
\end{aligned}$$

```

*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*e^(-10*d*x - 10*c))*d) - 1/8*(8*a^
4 + 41*a^3*b + 73*a^2*b^2 + 55*a*b^3 + 15*b^4 + 2*(16*a^4 + 41*a^3*b - 55*a
*b^3 - 30*b^4)*e^(-2*d*x - 2*c) + 2*(24*a^4 + 32*a^3*b + 5*a^2*b^2 + 50*a*b
^3 + 45*b^4)*e^(-4*d*x - 4*c) + 2*(16*a^4 + 23*a^3*b - 45*a*b^3 - 30*b^4)*e
^(-6*d*x - 6*c) + (8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 + 15*b^4)*e^(-8
*d*x - 8*c))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4 + (3*a^7 + 4
*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*e^(-2*d*x - 2*c) + 2*(a^7 + 2*
a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*e^(-4*d*x - 4*c) - 2*(a^7 + 2*a^5*b^2 + 8*
a^4*b^3 + 5*a^3*b^4)*e^(-6*d*x - 6*c) - (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a
^4*b^3 - 5*a^3*b^4)*e^(-8*d*x - 8*c) - (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b
^3 + a^3*b^4)*e^(-10*d*x - 10*c))*d) + 15/16*b*arctan(1/2*((a + b)*e^(-2*d*
x - 2*c) + a - b)/sqrt(a*b))/sqrt(a*b)*a^3*d) + 1/2*log(e^(2*d*x + 2*c) -
1)/(a^3*d) - 1/2*log(e^(-2*d*x - 2*c) - 1)/(a^3*d)

```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(c + dx)^2}{(b \tanh(c + dx)^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3, x)
```

```
[Out] int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(c + dx)}{(a + b \tanh^2(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3, x)
```

```
[Out] Integral(coth(c + d*x)**2/(a + b*tanh(c + d*x)**2)**3, x)
```

3.199 $\int \frac{\coth^3(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$

Optimal. Leaf size=171

$$\frac{(a-3b) \log(\tanh(c+dx))}{a^4 d} - \frac{b^2(3a+2b)}{2a^3 d(a+b)^2 (a+b \tanh^2(c+dx))} - \frac{\coth^2(c+dx)}{2a^3 d} - \frac{b^2}{4a^2 d(a+b) (a+b \tanh^2(c+dx))}$$

[Out] $-1/2*\coth(d*x+c)^2/a^3/d+\ln(\cosh(d*x+c))/(a+b)^3/d+(a-3*b)*\ln(\tanh(d*x+c))/a^4/d+1/2*b^2*(6*a^2+8*a*b+3*b^2)*\ln(a+b*\tanh(d*x+c)^2)/a^4/(a+b)^3/d-1/4*b^2/a^2/(a+b)/d/(a+b*\tanh(d*x+c)^2)^2-1/2*b^2*(3*a+2*b)/a^3/(a+b)^2/d/(a+b*\tanh(d*x+c)^2)$

Rubi [A] time = 0.26, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3670, 446, 88}

$$-\frac{b^2(3a+2b)}{2a^3 d(a+b)^2 (a+b \tanh^2(c+dx))} - \frac{b^2}{4a^2 d(a+b) (a+b \tanh^2(c+dx))^2} + \frac{b^2 (6a^2 + 8ab + 3b^2) \log(a+b \tanh^2(c+dx))}{2a^4 d(a+b)^3}$$

Antiderivative was successfully verified.

[In] `Int[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]`

[Out] $-\text{Coth}[c + d*x]^2/(2*a^3*d) + \text{Log}[\text{Cosh}[c + d*x]]/((a + b)^3*d) + ((a - 3*b)*\text{Log}[\text{Tanh}[c + d*x]])/(a^4*d) + (b^2*(6*a^2 + 8*a*b + 3*b^2)*\text{Log}[a + b*\text{Tanh}[c + d*x]^2])/(2*a^4*(a + b)^3*d) - b^2/(4*a^2*(a + b)*d*(a + b*\text{Tanh}[c + d*x]^2)^2) - (b^2*(3*a + 2*b))/(2*a^3*(a + b)^2*d*(a + b*\text{Tanh}[c + d*x]^2))$

Rule 88

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))`

Rule 446

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]`

Rule 3670

`Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))`

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(c+dx)}{(a+b\tanh^2(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^3(1-x^2)(a+bx^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{(1-x)x^2(a+bx)^3} dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)^3(-1+x)} + \frac{1}{a^3x^2} + \frac{a-3b}{a^4x} + \frac{b^3}{a^2(a+b)(a+bx)^3} + \frac{b^3(3a+2b)}{a^3(a+b)^2(a+bx)^2} + \frac{b^3(6a^2+8ab+3b^2)}{a^4(a+b)^3}\right) dx, x, \tanh^2(c+dx)\right)}{2d} \\
&= -\frac{\coth^2(c+dx)}{2a^3d} + \frac{\log(\cosh(c+dx))}{(a+b)^3d} + \frac{(a-3b)\log(\tanh(c+dx))}{a^4d} + \frac{b^2(6a^2+8ab+3b^2)\log(a\coth^2(c+dx)+b)}{a^4(a+b)^3} - \frac{2\log(\sinh(c+dx))}{(a+b)^3}
\end{aligned}$$

Mathematica [A] time = 1.91, size = 138, normalized size = 0.81

$$\frac{\frac{b^4}{2a^4(a+b)(a\coth^2(c+dx)+b)^2} - \frac{b^3(4a+3b)}{a^4(a+b)^2(a\coth^2(c+dx)+b)} + \frac{\coth^2(c+dx)}{a^3} - \frac{b^2(6a^2+8ab+3b^2)\log(a\coth^2(c+dx)+b)}{a^4(a+b)^3} - \frac{2\log(\sinh(c+dx))}{(a+b)^3}}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] -1/2*(Coth[c + d*x]^2/a^3 + b^4/(2*a^4*(a + b)*(b + a*Coth[c + d*x]^2)^2) - (b^3*(4*a + 3*b))/(a^4*(a + b)^2*(b + a*Coth[c + d*x]^2)) - (b^2*(6*a^2 + 8*a*b + 3*b^2)*Log[b + a*Coth[c + d*x]^2])/(a^4*(a + b)^3) - (2*Log[Sinh[c + d*x]])/(a + b)^3)/d

fricas [B] time = 1.29, size = 10720, normalized size = 62.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] -1/2*(2*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^12 + 24*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^11 + 2*(a^6 + 2*a^5*b + a^4*b^2)*d*x*sinh(d*x + c)^12 + 4*(a^6 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x)*cosh(d*x + c)^10 + 4*(a^6 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + 33*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^2 + (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x)*sinh(d*x + c)^10 + 40*(11*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^3 + (a^6 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^9 + 2*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 15*a^4*b^2)*d*x)*cosh(d*x + c)^8 + 2*(495*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^4 + 8*a^6 + 24*a^5*b + 16*a^4*b^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 15*a^4*b^2)*d*x + 90*(a^6 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(99*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^5 + 30*(a^6 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x)*cosh(d*x + c)^3 + (8*a^6 + 24*a^5*b + 16*a^4*b^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 15*a^4*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^7 + 8*(3*a^6 + 7*a^5*b + 6*a^4*b^2 + 2*a^3*b^3 + 15*a^2*b^4 + 9*a*b^5 - (a^6 - 2*a^5*b + 5*a^4*b^2)*d*x)*cosh(d*x + c)^6

$$\begin{aligned}
& + 8*(231*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^6 + 3*a^6 + 7*a^5*b + \\
& 6*a^4*b^2 + 2*a^3*b^3 + 15*a^2*b^4 + 9*a*b^5 + 105*(a^6 + 5*a^5*b + 10*a^4 \\
& *b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x) \\
& *cosh(d*x + c)^4 - (a^6 - 2*a^5*b + 5*a^4*b^2)*d*x + 7*(8*a^6 + 24*a^5*b + \\
& 16*a^4*b^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 15*a^4*b \\
& ^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 16*(99*(a^6 + 2*a^5*b + a^4*b^2 \\
&)*d*x*cosh(d*x + c)^7 + 63*(a^6 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^ \\
& 2*b^4 + 3*a*b^5 + (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x)*cosh(d*x + c)^5 + 7*(8*a \\
& ^6 + 24*a^5*b + 16*a^4*b^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2* \\
& a^5*b - 15*a^4*b^2)*d*x)*cosh(d*x + c)^3 + 3*(3*a^6 + 7*a^5*b + 6*a^4*b^2 + \\
& 2*a^3*b^3 + 15*a^2*b^4 + 9*a*b^5 - (a^6 - 2*a^5*b + 5*a^4*b^2)*d*x)*cosh(d \\
& *x + c))*sinh(d*x + c)^5 + 2*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 16*a^3*b^3 - \\
& 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 15*a^4*b^2)*d*x)*cosh(d*x + c)^4 + \\
& 2*(495*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^8 + 420*(a^6 + 5*a^5*b \\
& + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + (a^6 - 2*a^5*b - 3*a^4*b \\
& ^2)*d*x)*cosh(d*x + c)^6 + 8*a^6 + 24*a^5*b + 16*a^4*b^2 - 16*a^3*b^3 - 52* \\
& a^2*b^4 - 24*a*b^5 + 70*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 16*a^3*b^3 - 52*a^ \\
& 2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 15*a^4*b^2)*d*x)*cosh(d*x + c)^4 - (a^6 \\
& + 2*a^5*b - 15*a^4*b^2)*d*x + 60*(3*a^6 + 7*a^5*b + 6*a^4*b^2 + 2*a^3*b^3 \\
& + 15*a^2*b^4 + 9*a*b^5 - (a^6 - 2*a^5*b + 5*a^4*b^2)*d*x)*cosh(d*x + c)^2)* \\
& sinh(d*x + c)^4 + 8*(55*(a^6 + 2*a^5*b + a^4*b^2)*d*x*cosh(d*x + c)^9 + 60* \\
& (a^6 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + (a^6 - 2* \\
& a^5*b - 3*a^4*b^2)*d*x)*cosh(d*x + c)^7 + 14*(8*a^6 + 24*a^5*b + 16*a^4*b^2 \\
& - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 15*a^4*b^2)*d*x)*c \\
& osh(d*x + c)^5 + 20*(3*a^6 + 7*a^5*b + 6*a^4*b^2 + 2*a^3*b^3 + 15*a^2*b^4 + \\
& 9*a*b^5 - (a^6 - 2*a^5*b + 5*a^4*b^2)*d*x)*cosh(d*x + c)^3 + (8*a^6 + 24*a \\
& ^5*b + 16*a^4*b^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 1 \\
& 5*a^4*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(a^6 + 2*a^5*b + a^4*b^2 \\
&)*d*x + 4*(a^6 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + \\
& (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x)*cosh(d*x + c)^2 + 4*(33*(a^6 + 2*a^5*b + \\
& a^4*b^2)*d*x*cosh(d*x + c)^10 + 45*(a^6 + 5*a^5*b + 10*a^4*b^2 + 14*a^3*b^3 \\
& + 11*a^2*b^4 + 3*a*b^5 + (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x)*cosh(d*x + c)^8 \\
& + 14*(8*a^6 + 24*a^5*b + 16*a^4*b^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 - \\
& (a^6 + 2*a^5*b - 15*a^4*b^2)*d*x)*cosh(d*x + c)^6 + a^6 + 5*a^5*b + 10*a^4* \\
& b^2 + 14*a^3*b^3 + 11*a^2*b^4 + 3*a*b^5 + 30*(3*a^6 + 7*a^5*b + 6*a^4*b^2 + \\
& 2*a^3*b^3 + 15*a^2*b^4 + 9*a*b^5 - (a^6 - 2*a^5*b + 5*a^4*b^2)*d*x)*cosh(d \\
& *x + c)^4 + (a^6 - 2*a^5*b - 3*a^4*b^2)*d*x + 3*(8*a^6 + 24*a^5*b + 16*a^4* \\
& b^2 - 16*a^3*b^3 - 52*a^2*b^4 - 24*a*b^5 - (a^6 + 2*a^5*b - 15*a^4*b^2)*d*x) \\
&)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((6*a^4*b^2 + 20*a^3*b^3 + 25*a^2*b^4 \\
& + 14*a*b^5 + 3*b^6)*cosh(d*x + c)^12 + 12*(6*a^4*b^2 + 20*a^3*b^3 + 25*a^2* \\
& b^4 + 14*a*b^5 + 3*b^6)*cosh(d*x + c)*sinh(d*x + c)^11 + (6*a^4*b^2 + 20*a^ \\
& 3*b^3 + 25*a^2*b^4 + 14*a*b^5 + 3*b^6)*sinh(d*x + c)^12 + 2*(6*a^4*b^2 - 4* \\
& a^3*b^3 - 31*a^2*b^4 - 30*a*b^5 - 9*b^6)*cosh(d*x + c)^10 + 2*(6*a^4*b^2 - \\
& 4*a^3*b^3 - 31*a^2*b^4 - 30*a*b^5 - 9*b^6 + 33*(6*a^4*b^2 + 20*a^3*b^3 + 25 \\
& *a^2*b^4 + 14*a*b^5 + 3*b^6)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*(11*(6* \\
& a^4*b^2 + 20*a^3*b^3 + 25*a^2*b^4 + 14*a*b^5 + 3*b^6)*cosh(d*x + c)^3 + (6* \\
& a^4*b^2 - 4*a^3*b^3 - 31*a^2*b^4 - 30*a*b^5 - 9*b^6)*cosh(d*x + c))*sinh(d \\
& x + c)^9 - (6*a^4*b^2 + 20*a^3*b^3 - 71*a^2*b^4 - 114*a*b^5 - 45*b^6)*cosh(\\
& d*x + c)^8 - (6*a^4*b^2 + 20*a^3*b^3 - 71*a^2*b^4 - 114*a*b^5 - 45*b^6 - 49 \\
& 5*(6*a^4*b^2 + 20*a^3*b^3 + 25*a^2*b^4 + 14*a*b^5 + 3*b^6)*cosh(d*x + c)^4 \\
& - 90*(6*a^4*b^2 - 4*a^3*b^3 - 31*a^2*b^4 - 30*a*b^5 - 9*b^6)*cosh(d*x + c)^ \\
& 2)*sinh(d*x + c)^8 + 8*(99*(6*a^4*b^2 + 20*a^3*b^3 + 25*a^2*b^4 + 14*a*b^5 \\
& + 3*b^6)*cosh(d*x + c)^5 + 30*(6*a^4*b^2 - 4*a^3*b^3 - 31*a^2*b^4 - 30*a*b^ \\
& 5 - 9*b^6)*cosh(d*x + c)^3 - (6*a^4*b^2 + 20*a^3*b^3 - 71*a^2*b^4 - 114*a*b \\
& ^5 - 45*b^6)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(6*a^4*b^2 - 4*a^3*b^3 + 17 \\
& *a^2*b^4 + 34*a*b^5 + 15*b^6)*cosh(d*x + c)^6 + 4*(231*(6*a^4*b^2 + 20*a^3* \\
& b^3 + 25*a^2*b^4 + 14*a*b^5 + 3*b^6)*cosh(d*x + c)^6 - 6*a^4*b^2 + 4*a^3*b^ \\
& 3 - 17*a^2*b^4 - 34*a*b^5 - 15*b^6 + 105*(6*a^4*b^2 - 4*a^3*b^3 - 31*a^2*b^ \\
& 4 - 30*a*b^5 - 9*b^6)*cosh(d*x + c)^4 - 7*(6*a^4*b^2 + 20*a^3*b^3 - 71*a^2*
\end{aligned}$$

$$\begin{aligned}
& b^4 - 114ab^5 - 45b^6) \cosh(dx + c)^2 \sinh(dx + c)^6 + 6a^4b^2 + 20 \\
& a^3b^3 + 25a^2b^4 + 14ab^5 + 3b^6 + 8(99(6a^4b^2 + 20a^3b^3 + \\
& 25a^2b^4 + 14ab^5 + 3b^6) \cosh(dx + c)^7 + 63(6a^4b^2 - 4a^3b^3 \\
& - 31a^2b^4 - 30ab^5 - 9b^6) \cosh(dx + c)^5 - 7(6a^4b^2 + 20a^3b^3 \\
& - 71a^2b^4 - 114ab^5 - 45b^6) \cosh(dx + c)^3 - 3(6a^4b^2 - 4a^3 \\
& b^3 + 17a^2b^4 + 34ab^5 + 15b^6) \cosh(dx + c) \sinh(dx + c)^5 - (6 \\
& a^4b^2 + 20a^3b^3 - 71a^2b^4 - 114ab^5 - 45b^6) \cosh(dx + c)^4 + (\\
& 495(6a^4b^2 + 20a^3b^3 + 25a^2b^4 + 14ab^5 + 3b^6) \cosh(dx + c)^8 \\
& + 420(6a^4b^2 - 4a^3b^3 - 31a^2b^4 - 30ab^5 - 9b^6) \cosh(dx + \\
& c)^6 - 6a^4b^2 - 20a^3b^3 + 71a^2b^4 + 114ab^5 + 45b^6 - 70(6a^4 \\
& b^2 + 20a^3b^3 - 71a^2b^4 - 114ab^5 - 45b^6) \cosh(dx + c)^4 - 60(\\
& 6a^4b^2 - 4a^3b^3 + 17a^2b^4 + 34ab^5 + 15b^6) \cosh(dx + c)^2 \sinh \\
& (dx + c)^4 + 4(55(6a^4b^2 + 20a^3b^3 + 25a^2b^4 + 14ab^5 + 3b \\
& ^6) \cosh(dx + c)^9 + 60(6a^4b^2 - 4a^3b^3 - 31a^2b^4 - 30ab^5 - 9 \\
& b^6) \cosh(dx + c)^7 - 14(6a^4b^2 + 20a^3b^3 - 71a^2b^4 - 114ab^5 \\
& - 45b^6) \cosh(dx + c)^5 - 20(6a^4b^2 - 4a^3b^3 + 17a^2b^4 + 34ab \\
& b^5 + 15b^6) \cosh(dx + c)^3 - (6a^4b^2 + 20a^3b^3 - 71a^2b^4 - 114 \\
& ab^5 - 45b^6) \cosh(dx + c) \sinh(dx + c)^3 + 2(6a^4b^2 - 4a^3b^3 - \\
& 31a^2b^4 - 30ab^5 - 9b^6) \cosh(dx + c)^2 + 2(33(6a^4b^2 + 20a^3 \\
& b^3 + 25a^2b^4 + 14ab^5 + 3b^6) \cosh(dx + c)^10 + 45(6a^4b^2 - 4 \\
& a^3b^3 - 31a^2b^4 - 30ab^5 - 9b^6) \cosh(dx + c)^8 - 14(6a^4b^2 + \\
& 20a^3b^3 - 71a^2b^4 - 114ab^5 - 45b^6) \cosh(dx + c)^6 + 6a^4b^2 - \\
& 4a^3b^3 - 31a^2b^4 - 30ab^5 - 9b^6 - 30(6a^4b^2 - 4a^3b^3 + 17 \\
& a^2b^4 + 34ab^5 + 15b^6) \cosh(dx + c)^4 - 3(6a^4b^2 + 20a^3b^3 - \\
& 71a^2b^4 - 114ab^5 - 45b^6) \cosh(dx + c)^2 \sinh(dx + c)^2 + 4(3(\\
& 6a^4b^2 + 20a^3b^3 + 25a^2b^4 + 14ab^5 + 3b^6) \cosh(dx + c)^11 + \\
& 5(6a^4b^2 - 4a^3b^3 - 31a^2b^4 - 30ab^5 - 9b^6) \cosh(dx + c)^9 - \\
& 2(6a^4b^2 + 20a^3b^3 - 71a^2b^4 - 114ab^5 - 45b^6) \cosh(dx + c) \\
& ^7 - 6(6a^4b^2 - 4a^3b^3 + 17a^2b^4 + 34ab^5 + 15b^6) \cosh(dx + \\
& c)^5 - (6a^4b^2 + 20a^3b^3 - 71a^2b^4 - 114ab^5 - 45b^6) \cosh(dx \\
& + c)^3 + (6a^4b^2 - 4a^3b^3 - 31a^2b^4 - 30ab^5 - 9b^6) \cosh(dx + \\
& c) \sinh(dx + c) \log(2((a + b) \cosh(dx + c)^2 + (a + b) \sinh(dx + c) \\
& ^2 + a - b) / (\cosh(dx + c)^2 - 2 \cosh(dx + c) \sinh(dx + c) + \sinh(dx + c) \\
& ^2)) - 2((a^6 + 2a^5b - 5a^4b^2 - 20a^3b^3 - 25a^2b^4 - 14ab^5 - \\
& 3b^6) \cosh(dx + c)^12 + 12(a^6 + 2a^5b - 5a^4b^2 - 20a^3b^3 - 25 \\
& a^2b^4 - 14ab^5 - 3b^6) \cosh(dx + c) \sinh(dx + c)^11 + (a^6 + 2a^5b \\
& - 5a^4b^2 - 20a^3b^3 - 25a^2b^4 - 14ab^5 - 3b^6) \sinh(dx + c)^12 \\
& + 2(a^6 - 2a^5b - 9a^4b^2 + 4a^3b^3 + 31a^2b^4 + 30ab^5 + 9b^6 \\
&) \cosh(dx + c)^10 + 2(a^6 - 2a^5b - 9a^4b^2 + 4a^3b^3 + 31a^2b^4 \\
& + 30ab^5 + 9b^6 + 33(a^6 + 2a^5b - 5a^4b^2 - 20a^3b^3 - 25a^2b^4 \\
& - 14ab^5 - 3b^6) \cosh(dx + c)^2 \sinh(dx + c)^10 + 20(11(a^6 + 2a \\
& ^5b - 5a^4b^2 - 20a^3b^3 - 25a^2b^4 - 14ab^5 - 3b^6) \cosh(dx + c \\
&)^3 + (a^6 - 2a^5b - 9a^4b^2 + 4a^3b^3 + 31a^2b^4 + 30ab^5 + 9b^6 \\
&) \cosh(dx + c) \sinh(dx + c)^9 - (a^6 + 2a^5b - 21a^4b^2 - 20a^3b^3 \\
& + 71a^2b^4 + 114ab^5 + 45b^6) \cosh(dx + c)^8 - (a^6 + 2a^5b - 21 \\
& a^4b^2 - 20a^3b^3 + 71a^2b^4 + 114ab^5 + 45b^6 - 495(a^6 + 2a^5b \\
& - 5a^4b^2 - 20a^3b^3 - 25a^2b^4 - 14ab^5 - 3b^6) \cosh(dx + c)^4 \\
& - 90(a^6 - 2a^5b - 9a^4b^2 + 4a^3b^3 + 31a^2b^4 + 30ab^5 + 9b^6 \\
&) \cosh(dx + c)^2 \sinh(dx + c)^8 + 8(99(a^6 + 2a^5b - 5a^4b^2 - 20 \\
& a^3b^3 - 25a^2b^4 - 14ab^5 - 3b^6) \cosh(dx + c)^5 + 30(a^6 - 2a^5 \\
& b - 9a^4b^2 + 4a^3b^3 + 31a^2b^4 + 30ab^5 + 9b^6) \cosh(dx + c)^3 \\
& - (a^6 + 2a^5b - 21a^4b^2 - 20a^3b^3 + 71a^2b^4 + 114ab^5 + 45b^6 \\
&) \cosh(dx + c) \sinh(dx + c)^7 - 4(a^6 - 2a^5b - a^4b^2 + 4a^3b^3 \\
& - 17a^2b^4 - 34ab^5 - 15b^6) \cosh(dx + c)^6 + 4(231(a^6 + 2a^5b - \\
& 5a^4b^2 - 20a^3b^3 - 25a^2b^4 - 14ab^5 - 3b^6) \cosh(dx + c)^6 - \\
& a^6 + 2a^5b + a^4b^2 - 4a^3b^3 + 17a^2b^4 + 34ab^5 + 15b^6 + 105 \\
& (a^6 - 2a^5b - 9a^4b^2 + 4a^3b^3 + 31a^2b^4 + 30ab^5 + 9b^6) \cos \\
& h(dx + c)^4 - 7(a^6 + 2a^5b - 21a^4b^2 - 20a^3b^3 + 71a^2b^4 + 11 \\
& 4ab^5 + 45b^6) \cosh(dx + c)^2 \sinh(dx + c)^6 + a^6 + 2a^5b - 5a^4b^2
\end{aligned}$$

$$\begin{aligned}
& b^2 - 20a^3b^3 - 25a^2b^4 - 14ab^5 - 3b^6 + 8(99(a^6 + 2a^5b - 5a^4b^2 - 20a^3b^3 - 25a^2b^4 - 14ab^5 - 3b^6) \cosh(dx + c))^7 + 63 \\
& (a^6 - 2a^5b - 9a^4b^2 + 4a^3b^3 + 31a^2b^4 + 30ab^5 + 9b^6) \cosh(dx + c)^5 - 7(a^6 + 2a^5b - 21a^4b^2 - 20a^3b^3 + 71a^2b^4 + 1 \\
& 14ab^5 + 45b^6) \cosh(dx + c)^3 - 3(a^6 - 2a^5b - a^4b^2 + 4a^3b^3 - 17a^2b^4 - 34ab^5 - 15b^6) \cosh(dx + c) \sinh(dx + c)^5 - (a^6 + \\
& 2a^5b - 21a^4b^2 - 20a^3b^3 + 71a^2b^4 + 114ab^5 + 45b^6) \cosh(dx + c)^4 + (495(a^6 + 2a^5b - 5a^4b^2 - 20a^3b^3 - 25a^2b^4 - 14 \\
& ab^5 - 3b^6) \cosh(dx + c))^8 + 420(a^6 - 2a^5b - 9a^4b^2 + 4a^3b^3 + 31a^2b^4 + 30ab^5 + 9b^6) \cosh(dx + c)^6 - a^6 - 2a^5b + 21a^4b^2 \\
& + 20a^3b^3 - 71a^2b^4 - 114ab^5 - 45b^6 - 70(a^6 + 2a^5b - 21a^4b^2 - 20a^3b^3 + 71a^2b^4 + 114ab^5 + 45b^6) \cosh(dx + c)^4 - \\
& 60(a^6 - 2a^5b - a^4b^2 + 4a^3b^3 - 17a^2b^4 - 34ab^5 - 15b^6) \cosh(dx + c)^2 \sinh(dx + c)^4 + 4(55(a^6 + 2a^5b - 5a^4b^2 - 20a^3 \\
& b^3 - 25a^2b^4 - 14ab^5 - 3b^6) \cosh(dx + c))^9 + 60(a^6 - 2a^5b - 9a^4b^2 + 4a^3b^3 + 31a^2b^4 + 30ab^5 + 9b^6) \cosh(dx + c)^7 - 1 \\
& 4(a^6 + 2a^5b - 21a^4b^2 - 20a^3b^3 + 71a^2b^4 + 114ab^5 + 45b^6) \cosh(dx + c)^5 - 20(a^6 - 2a^5b - a^4b^2 + 4a^3b^3 - 17a^2b^4 - \\
& 34ab^5 - 15b^6) \cosh(dx + c)^3 - (a^6 + 2a^5b - 21a^4b^2 - 20a^3b^3 + 71a^2b^4 + 114ab^5 + 45b^6) \cosh(dx + c) \sinh(dx + c)^3 + 2(\\
& a^6 - 2a^5b - 9a^4b^2 + 4a^3b^3 + 31a^2b^4 + 30ab^5 + 9b^6) \cosh(dx + c)^2 + 2(33(a^6 + 2a^5b - 5a^4b^2 - 20a^3b^3 - 25a^2b^4 - \\
& 14ab^5 - 3b^6) \cosh(dx + c))^10 + 45(a^6 - 2a^5b - 9a^4b^2 + 4a^3b^3 + 31a^2b^4 + 30ab^5 + 9b^6) \cosh(dx + c)^8 - 14(a^6 + 2a^5b - \\
& 21a^4b^2 - 20a^3b^3 + 71a^2b^4 + 114ab^5 + 45b^6) \cosh(dx + c)^6 + a^6 - 2a^5b - 9a^4b^2 + 4a^3b^3 + 31a^2b^4 + 30ab^5 + 9b^6 - 3 \\
& 0(a^6 - 2a^5b - a^4b^2 + 4a^3b^3 - 17a^2b^4 - 34ab^5 - 15b^6) \cosh(dx + c)^4 - 3(a^6 + 2a^5b - 21a^4b^2 - 20a^3b^3 + 71a^2b^4 + 1 \\
& 14ab^5 + 45b^6) \cosh(dx + c)^2 \sinh(dx + c)^2 + 4(3(a^6 + 2a^5b - 5a^4b^2 - 20a^3b^3 - 25a^2b^4 - 14ab^5 - 3b^6) \cosh(dx + c))^11 + \\
& 5(a^6 - 2a^5b - 9a^4b^2 + 4a^3b^3 + 31a^2b^4 + 30ab^5 + 9b^6) \cosh(dx + c)^9 - 2(a^6 + 2a^5b - 21a^4b^2 - 20a^3b^3 + 71a^2b^4 + \\
& 114ab^5 + 45b^6) \cosh(dx + c)^7 - 6(a^6 - 2a^5b - a^4b^2 + 4a^3b^3 - 17a^2b^4 - 34ab^5 - 15b^6) \cosh(dx + c)^5 - (a^6 + 2a^5b - 21 \\
& a^4b^2 - 20a^3b^3 + 71a^2b^4 + 114ab^5 + 45b^6) \cosh(dx + c)^3 + (a^6 - 2a^5b - 9a^4b^2 + 4a^3b^3 + 31a^2b^4 + 30ab^5 + 9b^6) \cosh \\
& (dx + c) \sinh(dx + c) \log(2 \sinh(dx + c) / (\cosh(dx + c) - \sinh(dx + c))) + 8(3(a^6 + 2a^5b + a^4b^2) d * \cosh(dx + c))^11 + 5(a^6 + 5a^5b \\
& b + 10a^4b^2 + 14a^3b^3 + 11a^2b^4 + 3ab^5 + (a^6 - 2a^5b - 3a^4b^2) d * x) \cosh(dx + c)^9 + 2(8a^6 + 24a^5b + 16a^4b^2 - 16a^3b^3 \\
& - 52a^2b^4 - 24ab^5 - (a^6 + 2a^5b - 15a^4b^2) d * x) \cosh(dx + c)^7 + 6(3a^6 + 7a^5b + 6a^4b^2 + 2a^3b^3 + 15a^2b^4 + 9ab^5 - (a^6 \\
& - 2a^5b + 5a^4b^2) d * x) \cosh(dx + c)^5 + (8a^6 + 24a^5b + 16a^4b^2 - 16a^3b^3 - 52a^2b^4 - 24ab^5 - (a^6 + 2a^5b - 15a^4b^2) d * x) \\
& * \cosh(dx + c)^3 + (a^6 + 5a^5b + 10a^4b^2 + 14a^3b^3 + 11a^2b^4 + 3ab^5 + (a^6 - 2a^5b - 3a^4b^2) d * x) \cosh(dx + c) \sinh(dx + c) / ((\\
& a^9 + 5a^8b + 10a^7b^2 + 10a^6b^3 + 5a^5b^4 + a^4b^5) d * \cosh(dx + c))^12 + 12(a^9 + 5a^8b + 10a^7b^2 + 10a^6b^3 + 5a^5b^4 + a^4b^5) \\
& * d * \cosh(dx + c) \sinh(dx + c)^11 + (a^9 + 5a^8b + 10a^7b^2 + 10a^6b^3 + 5a^5b^4 + a^4b^5) d * \sinh(dx + c)^12 + 2(a^9 + a^8b - 6a^7b^2 - \\
& 14a^6b^3 - 11a^5b^4 - 3a^4b^5) d * \cosh(dx + c)^10 + 2(33(a^9 + 5a^8b + 10a^7b^2 + 10a^6b^3 + 5a^5b^4 + a^4b^5) d * \cosh(dx + c))^2 + (a \\
& ^9 + a^8b - 6a^7b^2 - 14a^6b^3 - 11a^5b^4 - 3a^4b^5) d * \sinh(dx + c)^10 - (a^9 + 5a^8b - 6a^7b^2 - 38a^6b^3 - 43a^5b^4 - 15a^4b^5) \\
& * d * \cosh(dx + c)^8 + 20(11(a^9 + 5a^8b + 10a^7b^2 + 10a^6b^3 + 5a^5b^4 + a^4b^5) d * \cosh(dx + c))^3 + (a^9 + a^8b - 6a^7b^2 - 14a^6b^3 \\
& - 11a^5b^4 - 3a^4b^5) d * \cosh(dx + c) \sinh(dx + c)^9 + (495(a^9 + 5a^8b + 10a^7b^2 + 10a^6b^3 + 5a^5b^4 + a^4b^5) d * \cosh(dx + c))^4 + \\
& 90(a^9 + a^8b - 6a^7b^2 - 14a^6b^3 - 11a^5b^4 - 3a^4b^5) d * \cosh(dx + c)
\end{aligned}$$

$$\begin{aligned} & *x + c)^2 - (a^9 + 5a^8b - 6a^7b^2 - 38a^6b^3 - 43a^5b^4 - 15a^4b^5) * d) * \sinh(dx + c)^8 - 4*(a^9 + a^8b + 2a^7b^2 + 10a^6b^3 + 13a^5b^4 + 5a^4b^5) * d * \cosh(dx + c)^6 + 8*(99*(a^9 + 5a^8b + 10a^7b^2 + 10a^6b^3 + 5a^5b^4 + a^4b^5) * d * \cosh(dx + c)^5 + 30*(a^9 + a^8b - 6a^7b^2 - 14a^6b^3 - 11a^5b^4 - 3a^4b^5) * d * \cosh(dx + c)^3 - (a^9 + 5a^8b * b - 6a^7b^2 - 38a^6b^3 - 43a^5b^4 - 15a^4b^5) * d * \cosh(dx + c)) * \sinh(dx + c)^7 + 4*(231*(a^9 + 5a^8b + 10a^7b^2 + 10a^6b^3 + 5a^5b^4 + a^4b^5) * d * \cosh(dx + c)^6 + 105*(a^9 + a^8b - 6a^7b^2 - 14a^6b^3 - 11a^5b^4 - 3a^4b^5) * d * \cosh(dx + c)^4 - 7*(a^9 + 5a^8b - 6a^7b^2 - 38a^6b^3 - 43a^5b^4 - 15a^4b^5) * d * \cosh(dx + c)^2 - (a^9 + a^8b + 2a^7b^2 + 10a^6b^3 + 13a^5b^4 + 5a^4b^5) * d) * \sinh(dx + c)^6 - (a^9 + 5a^8b - 6a^7b^2 - 38a^6b^3 - 43a^5b^4 - 15a^4b^5) * d * \cosh(dx + c)^4 + 8*(99*(a^9 + 5a^8b + 10a^7b^2 + 10a^6b^3 + 5a^5b^4 + a^4b^5) * d * \cosh(dx + c)^7 + 63*(a^9 + a^8b - 6a^7b^2 - 14a^6b^3 - 11a^5b^4 - 3a^4b^5) * d * \cosh(dx + c)^5 - 7*(a^9 + 5a^8b - 6a^7b^2 - 38a^6b^3 - 43a^5b^4 - 15a^4b^5) * d * \cosh(dx + c)^3 - 3*(a^9 + a^8b + 2a^7b^2 + 10a^6b^3 + 13a^5b^4 + 5a^4b^5) * d * \cosh(dx + c)) * \sinh(dx + c)^5 + (495*(a^9 + 5a^8b + 10a^7b^2 + 10a^6b^3 + 5a^5b^4 + a^4b^5) * d * \cosh(dx + c)^8 + 420*(a^9 + a^8b - 6a^7b^2 - 14a^6b^3 - 11a^5b^4 - 3a^4b^5) * d * \cosh(dx + c)^6 - 70*(a^9 + 5a^8b - 6a^7b^2 - 38a^6b^3 - 43a^5b^4 - 15a^4b^5) * d * \cosh(dx + c)^4 - 60*(a^9 + a^8b + 2a^7b^2 + 10a^6b^3 + 13a^5b^4 + 5a^4b^5) * d * \cosh(dx + c)^2 - (a^9 + 5a^8b - 6a^7b^2 - 38a^6b^3 - 43a^5b^4 - 15a^4b^5) * d) * \sinh(dx + c)^4 + 2*(a^9 + a^8b - 6a^7b^2 - 14a^6b^3 - 11a^5b^4 - 3a^4b^5) * d * \cosh(dx + c)^2 + 4*(55*(a^9 + 5a^8b + 10a^7b^2 + 10a^6b^3 + 5a^5b^4 + a^4b^5) * d * \cosh(dx + c)^9 + 60*(a^9 + a^8b - 6a^7b^2 - 14a^6b^3 - 11a^5b^4 - 3a^4b^5) * d * \cosh(dx + c)^7 - 14*(a^9 + 5a^8b - 6a^7b^2 - 38a^6b^3 - 43a^5b^4 - 15a^4b^5) * d * \cosh(dx + c)^5 - 20*(a^9 + a^8b + 2a^7b^2 + 10a^6b^3 + 13a^5b^4 + 5a^4b^5) * d * \cosh(dx + c)^3 - (a^9 + 5a^8b - 6a^7b^2 - 38a^6b^3 - 43a^5b^4 - 15a^4b^5) * d * \cosh(dx + c)) * \sinh(dx + c)^3 + 2*(33*(a^9 + 5a^8b + 10a^7b^2 + 10a^6b^3 + 5a^5b^4 + a^4b^5) * d * \cosh(dx + c)^10 + 45*(a^9 + a^8b - 6a^7b^2 - 14a^6b^3 - 11a^5b^4 - 3a^4b^5) * d * \cosh(dx + c)^8 - 14*(a^9 + 5a^8b - 6a^7b^2 - 38a^6b^3 - 43a^5b^4 - 15a^4b^5) * d * \cosh(dx + c)^6 - 30*(a^9 + a^8b + 2a^7b^2 + 10a^6b^3 + 13a^5b^4 + 5a^4b^5) * d * \cosh(dx + c)^4 - 3*(a^9 + 5a^8b * b - 6a^7b^2 - 38a^6b^3 - 43a^5b^4 - 15a^4b^5) * d * \cosh(dx + c)^2 + (a^9 + a^8b - 6a^7b^2 - 14a^6b^3 - 11a^5b^4 - 3a^4b^5) * d) * \sinh(dx + c)^2 + (a^9 + 5a^8b + 10a^7b^2 + 10a^6b^3 + 5a^5b^4 + a^4b^5) * d + 4*(3*(a^9 + 5a^8b + 10a^7b^2 + 10a^6b^3 + 5a^5b^4 + a^4b^5) * d * \cosh(dx + c)^11 + 5*(a^9 + a^8b - 6a^7b^2 - 14a^6b^3 - 11a^5b^4 - 3a^4b^5) * d * \cosh(dx + c)^9 - 2*(a^9 + 5a^8b - 6a^7b^2 - 38a^6b^3 - 43a^5b^4 - 15a^4b^5) * d * \cosh(dx + c)^7 - 6*(a^9 + a^8b + 2a^7b^2 + 10a^6b^3 + 13a^5b^4 + 5a^4b^5) * d * \cosh(dx + c)^5 - (a^9 + 5a^8b - 6a^7b^2 - 38a^6b^3 - 43a^5b^4 - 15a^4b^5) * d * \cosh(dx + c)^3 + (a^9 + a^8b - 6a^7b^2 - 14a^6b^3 - 11a^5b^4 - 3a^4b^5) * d * \cosh(dx + c)) * \sinh(dx + c)) \end{aligned}$$

giac [B] time = 1.06, size = 474, normalized size = 2.77

$$\frac{(6a^2b^2 + 8ab^3 + 3b^4) \log(ae^{(4dx+4c)} + be^{(4dx+4c)} + 2ae^{(2dx+2c)} - 2be^{(2dx+2c)} + a + b)}{a^7 + 3a^6b + 3a^5b^2 + a^4b^3} - \frac{2(dx+c)}{a^3 + 3a^2b + 3ab^2 + b^3} + \frac{2(a-3b) \log(|e^{(2dx+2c)} - 1|)}{a^4} - \frac{4((a^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(dx+c)^3/(a+b*tanh(dx+c)^2)^3,x, algorithm="giac")

[Out] 1/2*((6*a^2*b^2 + 8*a*b^3 + 3*b^4)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3) - 2*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(a - 3*

b)*log(abs(e^(2*d*x + 2*c) - 1))/a^4 - 4*((a^5 + 5*a^4*b + 10*a^3*b^2 + 14*a^2*b^3 + 11*a*b^4 + 3*b^5)*e^(10*d*x + 10*c) + 2*(2*a^5 + 6*a^4*b + 4*a^3*b^2 - 4*a^2*b^3 - 13*a*b^4 - 6*b^5)*e^(8*d*x + 8*c) + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 2*a^2*b^3 + 15*a*b^4 + 9*b^5)*e^(6*d*x + 6*c) + 2*(2*a^5 + 6*a^4*b + 4*a^3*b^2 - 4*a^2*b^3 - 13*a*b^4 - 6*b^5)*e^(4*d*x + 4*c) + (a^5 + 5*a^4*b + 10*a^3*b^2 + 14*a^2*b^3 + 11*a*b^4 + 3*b^5)*e^(2*d*x + 2*c))/((a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2*(a + b)^3*a^3*(e^(2*d*x + 2*c) - 1)^2))/d

maple [B] time = 0.50, size = 1020, normalized size = 5.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x)

[Out] -1/8/d*tanh(1/2*d*x+1/2*c)^2/a^3-1/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)-1)-1/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)+1)+8/d*b^3/(a+b)^3/a/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^6+14/d*b^4/(a+b)^3/a^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^6+6/d*b^5/a^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^6+16/d*b^3/(a+b)^3/a/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^4+56/d*b^4/(a+b)^3/a^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^4+60/d*b^5/(a+b)^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^4+20/d*b^6/a^4/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^4+8/d*b^3/(a+b)^3/a/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^2+14/d*b^4/(a+b)^3/a^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^2+6/d*b^5/a^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^2+3/d*b^2/(a+b)^3/a^2*ln(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)+4/d*b^3/(a+b)^3/a^3*ln(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)+3/2/d*b^4/a^4/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)-1/8/d/a^3/tanh(1/2*d*x+1/2*c)^2+1/d/a^3*ln(tanh(1/2*d*x+1/2*c))-3/d/a^4*ln(tanh(1/2*d*x+1/2*c))*b

maxima [B] time = 0.42, size = 770, normalized size = 4.50

$$\frac{(6a^2b^2 + 8ab^3 + 3b^4) \log(2(a - b)e^{(-2dx-2c)} + (a + b)e^{(-4dx-4c)} + a + b)}{2(a^7 + 3a^6b + 3a^5b^2 + a^4b^3)d} + \frac{dx + c}{(a^3 + 3a^2b + 3ab^2 + b^3)d} - \frac{1}{(a^8 + 5a^7b + 10a^6b^2 + 10a^5b^3 + 5a^4b^4 + a^3b^5 + 2(a^8 + a^7b - 6a^6b^2 - 14a^5b^3 - 11a^4b^4 - 3a^3b^5)e^{(-2dx - 2c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 1/2*(6*a^2*b^2 + 8*a*b^3 + 3*b^4)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d) + (d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 2*((a^5 + 5*a^4*b + 10*a^3*b^2 + 14*a^2*b^3 + 11*a*b^4 + 3*b^5)*e^(-2*d*x - 2*c) + 2*(2*a^5 + 6*a^4*b + 4*a^3*b^2 - 4*a^2*b^3 - 13*a*b^4 - 6*b^5)*e^(-4*d*x - 4*c) + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 2*a^2*b^3 + 15*a*b^4 + 9*b^5)*e^(-6*d*x - 6*c) + 2*(2*a^5 + 6*a^4*b + 4*a^3*b^2 - 4*a^2*b^3 - 13*a*b^4 - 6*b^5)*e^(-8*d*x - 8*c) + (a^5 + 5*a^4*b + 10*a^3*b^2 + 14*a^2*b^3 + 11*a*b^4 + 3*b^5)*e^(-10*d*x - 10*c))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5 + 2*(a^8 + a^7*b - 6*a^6*b^2 - 14*a^5*b^3 - 11*a^4*b^4 - 3*a^3*b^5)*e^(-2*d*x - 2*c)

) - (a⁸ + 5*a⁷*b - 6*a⁶*b² - 38*a⁵*b³ - 43*a⁴*b⁴ - 15*a³*b⁵)*e^(-4*d*x - 4*c) - 4*(a⁸ + a⁷*b + 2*a⁶*b² + 10*a⁵*b³ + 13*a⁴*b⁴ + 5*a³*b⁵)*e^(-6*d*x - 6*c) - (a⁸ + 5*a⁷*b - 6*a⁶*b² - 38*a⁵*b³ - 43*a⁴*b⁴ - 15*a³*b⁵)*e^(-8*d*x - 8*c) + 2*(a⁸ + a⁷*b - 6*a⁶*b² - 14*a⁵*b³ - 11*a⁴*b⁴ - 3*a³*b⁵)*e^(-10*d*x - 10*c) + (a⁸ + 5*a⁷*b + 10*a⁶*b² + 10*a⁵*b³ + 5*a⁴*b⁴ + a³*b⁵)*e^(-12*d*x - 12*c))*d) + (a - 3*b)*log(e^(-d*x - c) + 1)/(a⁴*d) + (a - 3*b)*log(e^(-d*x - c) - 1)/(a⁴*d)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(c + dx)^3}{(b \tanh(c + dx)^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^3/(a + b*tanh(c + d*x)^2)^3, x)

[Out] int(coth(c + d*x)^3/(a + b*tanh(c + d*x)^2)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(c + dx)}{(a + b \tanh^2(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3, x)

[Out] Integral(coth(c + d*x)**3/(a + b*tanh(c + d*x)**2)**3, x)

$$3.200 \quad \int \frac{\coth^4(c+dx)}{(a+b \tanh^2(c+dx))^3} dx$$

Optimal. Leaf size=228

$$\frac{b(11a+7b) \coth^3(c+dx)}{8a^2d(a+b)^2(a+b \tanh^2(c+dx))} + \frac{b^{5/2}(63a^2+90ab+35b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d(a+b)^3} - \frac{(8a^2+55ab+35b^2) \coth^3(c+dx)}{24a^3d(a+b)^2}$$

[Out] x/(a+b)^3+1/8*b^(5/2)*(63*a^2+90*a*b+35*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(9/2)/(a+b)^3/d-1/8*(8*a^3-8*a^2*b-55*a*b^2-35*b^3)*coth(d*x+c)/a^4/(a+b)^2/d-1/24*(8*a^2+55*a*b+35*b^2)*coth(d*x+c)^3/a^3/(a+b)^2/d+1/4*b*cot h(d*x+c)^3/a/(a+b)/d/(a+b*tanh(d*x+c)^2)^2+1/8*b*(11*a+7*b)*coth(d*x+c)^3/a^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)

Rubi [A] time = 0.37, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3670, 472, 579, 583, 522, 206, 205}

$$\frac{(8a^2+55ab+35b^2) \coth^3(c+dx)}{24a^3d(a+b)^2} - \frac{(-8a^2b+8a^3-55ab^2-35b^3) \coth(c+dx)}{8a^4d(a+b)^2} + \frac{b^{5/2}(63a^2+90ab+35b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d(a+b)^3}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] x/(a + b)^3 + (b^(5/2)*(63*a^2 + 90*a*b + 35*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(9/2)*(a + b)^3*d) - ((8*a^3 - 8*a^2*b - 55*a*b^2 - 35*b^3)*Coth[c + d*x])/(8*a^4*(a + b)^2*d) - ((8*a^2 + 55*a*b + 35*b^2)*Coth[c + d*x]^3)/(24*a^3*(a + b)^2*d) + (b*Coth[c + d*x]^3)/(4*a*(a + b)*d*(a + b*Tanh[c + d*x]^2)^2) + (b*(11*a + 7*b)*Coth[c + d*x]^3)/(8*a^2*(a + b)^2*d*(a + b*Tanh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 472

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(a*e*n*(b*c-a*d)*(p+1)), x] + Dist[1/(a*n*(b*c-a*d)*(p+1)), Int[(e*x)^m*(a+b*x^n)^(p+1)*(c+d*x^n)^q*Simp[c*b*(m+1)+n*(b*c-a*d)*(p+1)+d*b*(m+n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c-a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]

- Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 579

Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]

Rule 583

Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f*f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
\int \frac{\coth^4(c+dx)}{(a+b \tanh^2(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)(a+bx^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{b \coth^3(c+dx)}{4a(a+b)d(a+b \tanh^2(c+dx))^2} - \frac{\text{Subst}\left(\int \frac{-4a-7b+7bx^2}{x^4(1-x^2)(a+bx^2)^2} dx, x, \tanh(c+dx)\right)}{4a(a+b)d} \\
&= \frac{b \coth^3(c+dx)}{4a(a+b)d(a+b \tanh^2(c+dx))^2} + \frac{b(11a+7b) \coth^3(c+dx)}{8a^2(a+b)^2d(a+b \tanh^2(c+dx))} + \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)} dx, x, \tanh(c+dx)\right)}{8a^2(a+b)^2d} \\
&= -\frac{(8a^2+55ab+35b^2) \coth^3(c+dx)}{24a^3(a+b)^2d} + \frac{b \coth^3(c+dx)}{4a(a+b)d(a+b \tanh^2(c+dx))^2} + \frac{\text{Subst}\left(\int \frac{1}{x^4(1-x^2)} dx, x, \tanh(c+dx)\right)}{8a^2(a+b)^2d} \\
&= -\frac{(8a^3-8a^2b-55ab^2-35b^3) \coth(c+dx)}{8a^4(a+b)^2d} - \frac{(8a^2+55ab+35b^2) \coth^3(c+dx)}{24a^3(a+b)^2d} \\
&= -\frac{(8a^3-8a^2b-55ab^2-35b^3) \coth(c+dx)}{8a^4(a+b)^2d} - \frac{(8a^2+55ab+35b^2) \coth^3(c+dx)}{24a^3(a+b)^2d} \\
&= \frac{x}{(a+b)^3} + \frac{b^{5/2}(63a^2+90ab+35b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}(a+b)^3d} - \frac{(8a^3-8a^2b-55ab^2-35b^3) \coth(c+dx)}{8a^4(a+b)^2d}
\end{aligned}$$

Mathematica [A] time = 3.87, size = 194, normalized size = 0.85

$$\frac{\frac{3b^3(17a+11b) \sinh(2(c+dx))}{a^4(a+b)^2((a+b) \cosh(2(c+dx))+a-b)} + \frac{8(9b-4a) \coth(c+dx)}{a^4} + \frac{12b^4 \sinh(2(c+dx))}{a^3(a+b)^2((a+b) \cosh(2(c+dx))+a-b)^2} - \frac{8 \coth(c+dx) \text{csch}^2(c+dx)}{a^3} + \frac{3b^{5/2}(63a^2+90ab+35b^2) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}(a+b)^3d} - \frac{(8a^3-8a^2b-55ab^2-35b^3) \coth(c+dx)}{8a^4(a+b)^2d}}{24d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]

[Out] ((24*(c + d*x))/(a + b)^3 + (3*b^(5/2)*(63*a^2 + 90*a*b + 35*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/(a^(9/2)*(a + b)^3) + (8*(-4*a + 9*b)*Coth[c + d*x])/a^4 - (8*Coth[c + d*x]*Csch[c + d*x]^2)/a^3 + (12*b^4*Sinh[2*(c + d*x)]/(a^3*(a + b)^2*(a - b + (a + b)*Cosh[2*(c + d*x)]^2) + (3*b^3*(17*a + 11*b)*Sinh[2*(c + d*x)]/(a^4*(a + b)^2*(a - b + (a + b)*Cosh[2*(c + d*x)]))))/(24*d)

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] Timed out

giac [B] time = 1.02, size = 493, normalized size = 2.16

$$\frac{3(63a^2b^3 + 90ab^4 + 35b^5) \arctan\left(\frac{ae^{2dx+2c} + be^{2dx+2c} + a - b}{2\sqrt{ab}}\right)}{(a^7 + 3a^6b + 3a^5b^2 + a^4b^3)\sqrt{ab}} + \frac{24(dx+c)}{a^3 + 3a^2b + 3ab^2 + b^3} - \frac{6(17a^3b^3e^{6dx+6c} + 7a^2b^4e^{6dx+6c} - 21ab^5e^{6dx+6c} - 11b^6e^{6dx+6c} + 51a^3b^3e^{4dx+4c} - a^2b^4e^{4dx+4c} + 29a^2b^5e^{4dx+4c} + 33b^6e^{4dx+4c} + 51a^3b^3e^{2dx+2c} + 37a^2b^4e^{2dx+2c} - 47a^2b^5e^{2dx+2c} - 33b^6e^{2dx+2c} + 17a^3b^3 + 45a^2b^4 + 39a^2b^5 + 11b^6)}{(a^7 + 3a^6b + 3a^5b^2 + a^4b^3)(ae^{4dx+4c} + b^2e^{4dx+4c} + 2ae^{2dx+2c} - 2be^{2dx+2c} + a + b)^2} - \frac{16(6ae^{4dx+4c} - 9be^{4dx+4c} - 6ae^{2dx+2c} + 18be^{2dx+2c} + 4a - 9b)}{a^4(e^{2dx+2c} - 1)^3} / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/24*(3*(63*a^2*b^3 + 90*a*b^4 + 35*b^5)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b)))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*sqrt(a*b)) + 24*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 6*(17*a^3*b^3*e^(6*d*x + 6*c) + 7*a^2*b^4*e^(6*d*x + 6*c) - 21*a*b^5*e^(6*d*x + 6*c) - 11*b^6*e^(6*d*x + 6*c) + 51*a^3*b^3*e^(4*d*x + 4*c) - a^2*b^4*e^(4*d*x + 4*c) + 29*a^2*b^5*e^(4*d*x + 4*c) + 33*b^6*e^(4*d*x + 4*c) + 51*a^3*b^3*e^(2*d*x + 2*c) + 37*a^2*b^4*e^(2*d*x + 2*c) - 47*a^2*b^5*e^(2*d*x + 2*c) - 33*b^6*e^(2*d*x + 2*c) + 17*a^3*b^3 + 45*a^2*b^4 + 39*a^2*b^5 + 11*b^6)/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*(a*e^(4*d*x + 4*c) + b^2*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2) - 16*(6*a*e^(4*d*x + 4*c) - 9*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) + 18*b*e^(2*d*x + 2*c) + 4*a - 9*b)/(a^4*(e^(2*d*x + 2*c) - 1)^3)/d

maple [B] time = 0.50, size = 2139, normalized size = 9.38

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x)

[Out] -35/8/d*b^6/(a+b)^3/a^4/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-35/8/d*b^6/(a+b)^3/a^4/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-1/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/(a+b)^3*ln(tanh(1/2*d*x+1/2*c)+1)-125/8/d*b^5/(a+b)^3/a^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-5/8/d/a^3*tanh(1/2*d*x+1/2*c)-5/8/d/a^3/tanh(1/2*d*x+1/2*c)-125/8/d*b^5/(a+b)^3/a^3/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-63/8/d*b^3/(a+b)^3/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-153/8/d*b^4/(a+b)^3/a^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)-a-2*b)*a)^(1/2))-63/8/d*b^3/(a+b)^3/a/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))-153/8/d*b^4/(a+b)^3/a^2/(b*(a+b))^(1/2)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2))+51/4/d*b^3/(a+b)^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^5+75/2/d*b^4/(a+b)^3/a^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5+51/4/d*b^3/(a+b)^3/a/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3+75/2/d*b^4/(a+b)^3/a^2/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3+17/4/d*b^3/(a+b)^3/a/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)+3/2/d/a^4*tanh(1/2*d*x+1/2*c)*b+3/2/d/a^4/tanh(1/2*d*x+1/2*c)*b+143/4/d*b^5/(a+b)^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5-45/4/d*b^4/(a+b)^3/a^3/((2*(b*(a+b))^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c))

$$\begin{aligned} & /((2*(b*(a+b))^{(1/2)+a+2*b}*a)^{(1/2)}+17/4/d*b^3/(a+b)^3/a/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)^7+15/2/d*b^4/(a+b)^3/a^2/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)^7+143/4/d*b^5/(a+b)^3/a^3/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)^3+63/8/d*b^3/(a+b)^3/a^2/((2*(b*(a+b))^{(1/2)-a-2*b}*a)^{(1/2)}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)-a-2*b}*a)^{(1/2)}))-a-2*b)*a)^{(1/2)}+45/4/d*b^4/(a+b)^3/a^3/((2*(b*(a+b))^{(1/2)-a-2*b}*a)^{(1/2)}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)-a-2*b}*a)^{(1/2)}))-63/8/d*b^3/(a+b)^3/a^2/((2*(b*(a+b))^{(1/2)+a+2*b}*a)^{(1/2)}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)+a+2*b}*a)^{(1/2)}))+15/2/d*b^4/(a+b)^3/a^2/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)+35/8/d*b^5/(a+b)^3/a^4/((2*(b*(a+b))^{(1/2)-a-2*b}*a)^{(1/2)}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)-a-2*b}*a)^{(1/2)}))+13/4/d*b^5/(a+b)^3/a^3/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)^7-35/8/d*b^5/(a+b)^3/a^4/((2*(b*(a+b))^{(1/2)+a+2*b}*a)^{(1/2)}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(b*(a+b))^{(1/2)+a+2*b}*a)^{(1/2)}))+11/d*b^6/(a+b)^3/a^4/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)^3+11/d*b^6/(a+b)^3/a^4/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)^5+13/4/d*b^5/(a+b)^3/a^3/(\tanh(1/2*d*x+1/2*c)^4*a+2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)-1/24/d/a^3*\tanh(1/2*d*x+1/2*c)^3-1/24/d/a^3/\tanh(1/2*d*x+1/2*c)^3 \end{aligned}$$

maxima [B] time = 2.67, size = 4285, normalized size = 18.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c))^2)^3,x, algorithm="maxima"

[Out]
$$\begin{aligned} & -1/8*(3*a^3*b - 3*a^2*b^2 - 7*a*b^3 - 3*b^4)*\log((a + b)*e^{(4*d*x + 4*c)} + 2*(a - b)*e^{(2*d*x + 2*c)} + a + b)/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d) \\ & + 1/8*(3*a^3*b - 3*a^2*b^2 - 7*a*b^3 - 3*b^4)*\log(2*(a - b)*e^{(-2*d*x - 2*c)} + (a + b)*e^{(-4*d*x - 4*c)} + a + b)/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d) \\ & + 1/128*(15*a^4*b - 200*a^3*b^2 - 186*a^2*b^3 + 35*b^5)*\operatorname{arctan}(1/2*((a + b)*e^{(2*d*x + 2*c)} + a - b)/\sqrt{a*b}))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*\sqrt{a*b}*d) \\ & - 1/128*(15*a^4*b - 200*a^3*b^2 - 186*a^2*b^3 + 35*b^5)*\operatorname{arctan}(1/2*((a + b)*e^{(-2*d*x - 2*c)} + a - b)/\sqrt{a*b}))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*\sqrt{a*b}*d) \\ & + 1/192*(176*a^6 + 781*a^5*b + 1571*a^4*b^2 + 1538*a^3*b^3 + 502*a^2*b^4 - 175*a*b^5 - 105*b^6 + 3*(96*a^6 + 465*a^5*b + 665*a^4*b^2 + 706*a^3*b^3 + 506*a^2*b^4 + 61*a*b^5 - 35*b^6))*e^{(12*d*x + 12*c)} \\ & + 6*(120*a^6 + 192*a^5*b - 315*a^4*b^2 - 728*a^3*b^3 - 1070*a^2*b^4 - 240*a*b^5 + 105*b^6)*e^{(10*d*x + 10*c)} \\ & + (176*a^6 - 281*a^5*b + 3509*a^4*b^2 + 3950*a^3*b^3 + 12226*a^2*b^4 + 3755*a*b^5 - 1575*b^6)*e^{(8*d*x + 8*c)} \\ & - 4*(184*a^6 + 48*a^5*b + 473*a^4*b^2 + 970*a^3*b^3 + 3684*a^2*b^4 + 1070*a*b^5 - 525*b^6)*e^{(6*d*x + 6*c)} \\ & - (384*a^6 + 1127*a^5*b - 861*a^4*b^2 - 7146*a^3*b^3 - 11386*a^2*b^4 - 1965*a*b^5 + 1575*b^6)*e^{(4*d*x + 4*c)} \\ & + 2*(136*a^6 - 96*a^5*b - 1309*a^4*b^2 - 2996*a^3*b^3 - 2238*a^2*b^4 - 4*a*b^5 + 315*b^6)*e^{(2*d*x + 2*c)}/((a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4 + a^4*b^5 - (a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4 + a^4*b^5))*e^{(14*d*x + 14*c)} \\ & - (a^9 - 3*a^8*b - 22*a^7*b^2 - 38*a^6*b^3 - 27*a^5*b^4 - 7*a^4*b^5)*e^{(12*d*x + 12*c)} + (3*a^9 + 7*a^8*b - 18*a^7*b^2 - 66*a^6*b^3 - 65*a^5*b^4 - 21*a^4*b^5)*e^{(10*d*x + 10*c)} \\ & + (3*a^9 - a^8*b + 14*a^7*b^2 + 78*a^6*b^3 + 95*a^5*b^4 + 35*a^4*b^5)*e^{(8*d*x + 8*c)} - (3*a^9 - a^8*b + 14*a^7*b^2 + 78*a^6*b^3 + 95*a^5*b^4 + 35*a^4*b^5)*e^{(6*d*x + 6*c)} \\ & - (3*a^9 + 7*a^8*b - 18*a^7*b^2 - 66*a^6*b^3 - 65*a^5*b^4 - 21*a^4*b^5)*e^{(4*d*x + 4*c)} \\ & + (a^9 - 3*a^8*b - 22*a^7*b^2 - 38*a^6*b^3 - 27*a^5*b^4 - 7*a^4*b^5)*e^{(2*d*x + 2*c)})*d) - 1/192*(176*a^6 + 781*a^5*b + 1571*a^4*b^2 + 1 \end{aligned}$$

$$\begin{aligned}
& 538a^3b^3 + 502a^2b^4 - 175a^2b^5 - 105b^6 + 2(136a^6 - 96a^5b - 1 \\
& 309a^4b^2 - 2996a^3b^3 - 2238a^2b^4 - 4a^2b^5 + 315b^6)e^{(-2dx - 2c)} - (384a^6 + 1127a^5b - 861a^4b^2 - 7146a^3b^3 - 11386a^2b^4 - \\
& 1965a^2b^5 + 1575b^6)e^{(-4dx - 4c)} - 4(184a^6 + 48a^5b + 473a^4b^2 + 970a^3b^3 + 3684a^2b^4 + 1070a^2b^5 - 525b^6)e^{(-6dx - 6c)} + \\
& (176a^6 - 281a^5b + 3509a^4b^2 + 3950a^3b^3 + 12226a^2b^4 + 3755a^2b^5 - 1575b^6)e^{(-8dx - 8c)} + 6(120a^6 + 192a^5b - 315a^4b^2 - \\
& 728a^3b^3 - 1070a^2b^4 - 240a^2b^5 + 105b^6)e^{(-10dx - 10c)} + 3(\\
& 96a^6 + 465a^5b + 665a^4b^2 + 706a^3b^3 + 506a^2b^4 + 61a^2b^5 - 3 \\
& 5b^6)e^{(-12dx - 12c)}/((a^9 + 5a^8b + 10a^7b^2 + 10a^6b^3 + 5a^5b^4 + a^4b^5 + (a^9 - 3a^8b - 22a^7b^2 - 38a^6b^3 - 27a^5b^4 - 7 \\
& a^4b^5)e^{(-2dx - 2c)} - (3a^9 + 7a^8b - 18a^7b^2 - 66a^6b^3 - 6 \\
& 5a^5b^4 - 21a^4b^5)e^{(-4dx - 4c)} - (3a^9 - a^8b + 14a^7b^2 + 78 \\
& a^6b^3 + 95a^5b^4 + 35a^4b^5)e^{(-6dx - 6c)} + (3a^9 - a^8b + 14a^7b^2 + 78a^6b^3 + 95a^5b^4 + 35a^4b^5)e^{(-8dx - 8c)} + (3a^9 + \\
& 7a^8b - 18a^7b^2 - 66a^6b^3 - 65a^5b^4 - 21a^4b^5)e^{(-10dx - 10c)} - (a^9 - 3a^8b - 22a^7b^2 - 38a^6b^3 - 27a^5b^4 - 7a^4b^5) \\
& e^{(-12dx - 12c)} - (a^9 + 5a^8b + 10a^7b^2 + 10a^6b^3 + 5a^5b^4 + a^4b^5)e^{(-14dx - 14c)})d) + 1/48(32a^5 + 83a^4b - 60a^3b^2 - 3 \\
& 46a^2b^3 - 340a^2b^4 - 105b^5 + 3(32a^5 + 95a^4b + 154a^3b^2 + 84a^2b^3 - 42a^2b^4 - 35b^5)e^{(12dx + 12c)} + 6(48a^5 + 40a^4b - 117 \\
& a^3b^2 - 201a^2b^3 + 45a^2b^4 + 105b^5)e^{(10dx + 10c)} + (224a^5 + 281a^4b + 384a^3b^2 + 2318a^2b^3 - 160a^2b^4 - 1575b^5)e^{(8dx + \\
& 8c)} - 4(16a^5 - 136a^4b - 9a^3b^2 + 697a^2b^3 - 115a^2b^4 - 525b^5)e^{(6dx + 6c)} - (96a^5 + 137a^4b - 1262a^3b^2 - 1840a^2b^3 + 12 \\
& 30a^2b^4 + 1575b^5)e^{(4dx + 4c)} + 2(16a^5 - 136a^4b - 435a^3b^2 - 35a^2b^3 + 563a^2b^4 + 315b^5)e^{(2dx + 2c)}/((a^8 + 4a^7b + 6a^6b^2 + 4a^5b^3 + a^4b^4) \\
& e^{(14dx + 14c)} - (a^8 - 4a^7b - 18a^6b^2 - 20a^5b^3 - 7a^4b^4)e^{(12dx + 12c)} + (3a^8 + 4a^7b - 22a^6b^2 - 44a^5b^3 - 21a^4b^4) \\
& e^{(10dx + 10c)} + (3a^8 - 4a^7b + 18a^6b^2 + 60a^5b^3 + 35a^4b^4)e^{(8dx + 8c)} - (3a^8 - 4a^7b + 18a^6b^2 + 60a^5b^3 + 35a^4b^4) \\
& e^{(6dx + 6c)} - (3a^8 + 4a^7b - 22a^6b^2 - 44a^5b^3 - 21a^4b^4)e^{(4dx + 4c)} + (a^8 - 4a^7b - 18a^6b^2 - 20a^5b^3 - 7a^4b^4) \\
& e^{(2dx + 2c)})d) - 1/48(32a^5 + 83a^4b - 60a^3b^2 - 346a^2b^3 - 340a^2b^4 - 105b^5 + 2(16a^5 - 136a^4b - 435a^3b^2 - 35a^2b^3 \\
& + 563a^2b^4 + 315b^5)e^{(-2dx - 2c)} - (96a^5 + 137a^4b - 1262a^3b^2 - 1840a^2b^3 + 1230a^2b^4 + 1575b^5)e^{(-4dx - 4c)} - 4(16a^5 - 1 \\
& 36a^4b - 9a^3b^2 + 697a^2b^3 - 115a^2b^4 - 525b^5)e^{(-6dx - 6c)} + (224a^5 + 281a^4b + 384a^3b^2 + 2318a^2b^3 - 160a^2b^4 - 1575b^5) \\
& e^{(-8dx - 8c)} + 6(48a^5 + 40a^4b - 117a^3b^2 - 201a^2b^3 + 45a^2b^4 + 105b^5)e^{(-10dx - 10c)} + 3(32a^5 + 95a^4b + 154a^3b^2 + 8 \\
& 4a^2b^3 - 42a^2b^4 - 35b^5)e^{(-12dx - 12c)}/((a^8 + 4a^7b + 6a^6b^2 + 4a^5b^3 + a^4b^4) \\
& e^{(-2dx - 2c)} - (3a^8 + 4a^7b - 22a^6b^2 - 44a^5b^3 - 21a^4b^4) \\
& e^{(-4dx - 4c)} - (3a^8 - 4a^7b + 18a^6b^2 + 60a^5b^3 + 35a^4b^4) \\
& e^{(-6dx - 6c)} + (3a^8 - 4a^7b + 18a^6b^2 + 60a^5b^3 + 35a^4b^4) \\
& e^{(-8dx - 8c)} + (3a^8 + 4a^7b - 22a^6b^2 - 44a^5b^3 - 21a^4b^4) \\
& e^{(-10dx - 10c)} - (a^8 - 4a^7b - 18a^6b^2 - 20a^5b^3 - 7a^4b^4) \\
& e^{(-12dx - 12c)} - (a^8 + 4a^7b + 6a^6b^2 + 4a^5b^3 + a^4b^4) \\
& e^{(-14dx - 14c)})d) + 1/32(16a^4 + 147a^3b + 351a^2b^2 + 32 \\
& 5a^2b^3 + 105b^4 + 2(8a^4 + 32a^3b - 251a^2b^2 - 590a^2b^3 - 315b^4) \\
& e^{(-2dx - 2c)} - (96a^4 + 313a^3b + 19a^2b^2 - 1725a^2b^3 - 1575b^4) \\
& e^{(-4dx - 4c)} - 4(56a^4 + 80a^3b - 65a^2b^2 + 400a^2b^3 + 525b^4) \\
& e^{(-6dx - 6c)} - (176a^4 + 135a^3b + 15a^2b^2 - 1375a^2b^3 - 15 \\
& 75b^4)e^{(-8dx - 8c)} - 6(8a^4 + 45a^2b^2 + 150a^2b^3 + 105b^4)e^{(-10dx - 10c)} + 15(3a^3b + 13a^2b^2 + 17a^2b^3 + 7b^4)e^{(-12dx - 12c)}/((a^7 + 3a^6b + 3a^5b^2 + a^4b^3 + (a^7 - 5a^6b - 13a^5b^2 - 7a^4b^3) \\
& e^{(-2dx - 2c)} - (3a^7 + a^6b - 23a^5b^2 - 21a^4b^3)*)
\end{aligned}$$

$$e^{(-4dx - 4c)} - (3a^7 - 7a^6b + 25a^5b^2 + 35a^4b^3)e^{(-6dx - 6c)} + (3a^7 - 7a^6b + 25a^5b^2 + 35a^4b^3)e^{(-8dx - 8c)} + (3a^7 + a^6b - 23a^5b^2 - 21a^4b^3)e^{(-10dx - 10c)} - (a^7 - 5a^6b - 13a^5b^2 - 7a^4b^3)e^{(-12dx - 12c)} - (a^7 + 3a^6b + 3a^5b^2 + a^4b^3)e^{(-14dx - 14c)} * d + 3/4 * b * \log((a + b) * e^{(4dx + 4c)} + 2 * (a - b) * e^{(2dx + 2c)} + a + b) / (a^4 * d) - 3/4 * b * \log(2 * (a - b) * e^{(-2dx - 2c)} + (a + b) * e^{(-4dx - 4c)} + a + b) / (a^4 * d) + 1/4 * (2a - 3b) * \log(e^{(2dx + 2c)} - 1) / (a^4 * d) - 3/2 * b * \log(e^{(2dx + 2c)} - 1) / (a^4 * d) - 1/4 * (2a - 3b) * \log(e^{(-2dx - 2c)} - 1) / (a^4 * d) + 3/2 * b * \log(e^{(-2dx - 2c)} - 1) / (a^4 * d) - 5/32 * (3ab - 7b^2) * \arctan(1/2 * ((a + b) * e^{(2dx + 2c)} + a - b) / \sqrt{ab}) / (\sqrt{ab} * a^4 * d) - 15/64 * (3ab + 7b^2) * \arctan(1/2 * ((a + b) * e^{(-2dx - 2c)} + a - b) / \sqrt{ab}) / (\sqrt{ab} * a^4 * d) + 5/32 * (3ab - 7b^2) * \arctan(1/2 * ((a + b) * e^{(-2dx - 2c)} + a - b) / \sqrt{ab}) / (\sqrt{ab} * a^4 * d)$$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\coth(c + dx)^4}{(b \tanh(c + dx)^2 + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2)^3, x)

[Out] int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^4(c + dx)}{(a + b \tanh^2(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3, x)

[Out] Integral(coth(c + d*x)**4/(a + b*tanh(c + d*x)**2)**3, x)

$$3.201 \quad \int \frac{1}{(a+b \tanh^2(c+dx))^4} dx$$

Optimal. Leaf size=201

$$\frac{b(11a+5b) \tanh(c+dx)}{24a^2d(a+b)^2(a+b \tanh^2(c+dx))^2} + \frac{b(19a^2+16ab+5b^2) \tanh(c+dx)}{16a^3d(a+b)^3(a+b \tanh^2(c+dx))} + \frac{\sqrt{b}(35a^3+35a^2b+21ab^2+5b^3)}{16a^{7/2}d(a+b)}$$

[Out] x/(a+b)^4+1/16*(35*a^3+35*a^2*b+21*a*b^2+5*b^3)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))*b^(1/2)/a^(7/2)/(a+b)^4/d+1/6*b*tanh(d*x+c)/a/(a+b)/d/(a+b*tanh(d*x+c)^2)^3+1/24*b*(11*a+5*b)*tanh(d*x+c)/a^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)^2+1/16*b*(19*a^2+16*a*b+5*b^2)*tanh(d*x+c)/a^3/(a+b)^3/d/(a+b*tanh(d*x+c)^2)

Rubi [A] time = 0.28, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3661, 414, 527, 522, 206, 205}

$$\frac{b(19a^2+16ab+5b^2) \tanh(c+dx)}{16a^3d(a+b)^3(a+b \tanh^2(c+dx))} + \frac{\sqrt{b}(35a^2b+35a^3+21ab^2+5b^3) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{16a^{7/2}d(a+b)^4} + \frac{b(11a+5b)}{24a^2d(a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tanh[c + d*x]^2)^(-4), x]

[Out] x/(a + b)^4 + (Sqrt[b]*(35*a^3 + 35*a^2*b + 21*a*b^2 + 5*b^3)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(16*a^(7/2)*(a + b)^4*d) + (b*Tanh[c + d*x])/(6*a*(a + b)*d*(a + b*Tanh[c + d*x]^2)^3) + (b*(11*a + 5*b)*Tanh[c + d*x])/(24*a^2*(a + b)^2*d*(a + b*Tanh[c + d*x]^2)^2) + (b*(19*a^2 + 16*a*b + 5*b^2)*Tanh[c + d*x])/(16*a^3*(a + b)^3*d*(a + b*Tanh[c + d*x]^2))

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 527

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3661

```
Int[((a_) + (b_)*((c_)*tan[(e_) + (f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + b \tanh^2(c + dx))^4} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^2)^4} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{b \tanh(c + dx)}{6a(a + b)d (a + b \tanh^2(c + dx))^3} - \frac{\text{Subst}\left(\int \frac{b-6(a+b)+5bx^2}{(1-x^2)(a+bx^2)^3} dx, x, \tanh(c + dx)\right)}{6a(a + b)d} \\ &= \frac{b \tanh(c + dx)}{6a(a + b)d (a + b \tanh^2(c + dx))^3} + \frac{b(11a + 5b) \tanh(c + dx)}{24a^2(a + b)^2d (a + b \tanh^2(c + dx))^2} + \frac{b(11a + 5b) \tanh(c + dx)}{24a^2(a + b)^2d (a + b \tanh^2(c + dx))^2} + \frac{b(11a + 5b) \tanh(c + dx)}{24a^2(a + b)^2d (a + b \tanh^2(c + dx))^2} + \frac{b(11a + 5b) \tanh(c + dx)}{24a^2(a + b)^2d (a + b \tanh^2(c + dx))^2} \\ &= \frac{b \tanh(c + dx)}{6a(a + b)d (a + b \tanh^2(c + dx))^3} + \frac{b(11a + 5b) \tanh(c + dx)}{24a^2(a + b)^2d (a + b \tanh^2(c + dx))^2} + \frac{b(11a + 5b) \tanh(c + dx)}{24a^2(a + b)^2d (a + b \tanh^2(c + dx))^2} + \frac{b(11a + 5b) \tanh(c + dx)}{24a^2(a + b)^2d (a + b \tanh^2(c + dx))^2} + \frac{b(11a + 5b) \tanh(c + dx)}{24a^2(a + b)^2d (a + b \tanh^2(c + dx))^2} \\ &= \frac{x}{(a + b)^4} + \frac{\sqrt{b} (35a^3 + 35a^2b + 21ab^2 + 5b^3) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{16a^{7/2}(a + b)^4d} + \frac{b \tanh(c + dx)}{6a(a + b)d (a + b \tanh^2(c + dx))^3} \end{aligned}$$

Mathematica [A] time = 0.68, size = 203, normalized size = 1.01

$$\frac{2b(11a+5b)(a+b)^2 \tanh(c+dx)}{a^2(a+b \tanh^2(c+dx))^2} + \frac{3b(19a^2+16ab+5b^2)(a+b) \tanh(c+dx)}{a^3(a+b \tanh^2(c+dx))} + \frac{3\sqrt{b}(35a^3+35a^2b+21ab^2+5b^3) \tan^{-1}\left(\frac{\sqrt{b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}} + \frac{8b(a+b)^3 \tanh(c+dx)}{a(a+b \tanh^2(c+dx))} \Bigg/ 48d(a+b)^4$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Tanh[c + d*x]^2)^(-4), x]

[Out] ((3*sqrt[b]*(35*a^3 + 35*a^2*b + 21*a*b^2 + 5*b^3)*ArcTan[(sqrt[b]*Tanh[c + d*x])/sqrt[a]])/a^(7/2) - 24*Log[1 - Tanh[c + d*x]] + 24*Log[1 + Tanh[c + d*x]])/48*d*(a + b)^4

$$d*x]] + (8*b*(a + b)^3*\text{Tanh}[c + d*x])/(a*(a + b*\text{Tanh}[c + d*x]^2)^3) + (2*b*(a + b)^2*(11*a + 5*b)*\text{Tanh}[c + d*x])/(a^2*(a + b*\text{Tanh}[c + d*x]^2)^2) + (3*b*(a + b)*(19*a^2 + 16*a*b + 5*b^2)*\text{Tanh}[c + d*x])/(a^3*(a + b*\text{Tanh}[c + d*x]^2)))/(48*(a + b)^4*d)$$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tanh(d*x+c)^2)^4,x, algorithm="fricas")

[Out] Timed out

giac [B] time = 0.53, size = 1356, normalized size = 6.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tanh(d*x+c)^2)^4,x, algorithm="giac")

[Out]
$$\frac{1}{48} * (3 * (35 * a^3 * b + 35 * a^2 * b^2 + 21 * a * b^3 + 5 * b^4) * \arctan(1/2 * (a * e^{(2 * d * x + 2 * c)} + b * e^{(2 * d * x + 2 * c)} + a - b) / \sqrt{a * b})) / ((a^7 + 4 * a^6 * b + 6 * a^5 * b^2 + 4 * a^4 * b^3 + a^3 * b^4) * \sqrt{a * b}) + 48 * (d * x + c) / (a^4 + 4 * a^3 * b + 6 * a^2 * b^2 + 4 * a * b^3 + b^4) - 2 * (87 * a^{11} * b * e^{(10 * d * x + 10 * c)} + 591 * a^{10} * b^2 * e^{(10 * d * x + 10 * c)} + 1533 * a^9 * b^3 * e^{(10 * d * x + 10 * c)} + 1413 * a^8 * b^4 * e^{(10 * d * x + 10 * c)} - 1674 * a^7 * b^5 * e^{(10 * d * x + 10 * c)} - 6426 * a^6 * b^6 * e^{(10 * d * x + 10 * c)} - 8694 * a^5 * b^7 * e^{(10 * d * x + 10 * c)} - 6822 * a^4 * b^8 * e^{(10 * d * x + 10 * c)} - 3357 * a^3 * b^9 * e^{(10 * d * x + 10 * c)} - 1029 * a^2 * b^{10} * e^{(10 * d * x + 10 * c)} - 183 * a * b^{11} * e^{(10 * d * x + 10 * c)} - 15 * b^{12} * e^{(10 * d * x + 10 * c)} + 435 * a^{11} * b * e^{(8 * d * x + 8 * c)} + 2661 * a^{10} * b^2 * e^{(8 * d * x + 8 * c)} + 6657 * a^9 * b^3 * e^{(8 * d * x + 8 * c)} + 8871 * a^8 * b^4 * e^{(8 * d * x + 8 * c)} + 7950 * a^7 * b^5 * e^{(8 * d * x + 8 * c)} + 8610 * a^6 * b^6 * e^{(8 * d * x + 8 * c)} + 12306 * a^5 * b^7 * e^{(8 * d * x + 8 * c)} + 13182 * a^4 * b^8 * e^{(8 * d * x + 8 * c)} + 8751 * a^3 * b^9 * e^{(8 * d * x + 8 * c)} + 3465 * a^2 * b^{10} * e^{(8 * d * x + 8 * c)} + 765 * a * b^{11} * e^{(8 * d * x + 8 * c)} + 75 * b^{12} * e^{(8 * d * x + 8 * c)} + 870 * a^{11} * b * e^{(6 * d * x + 6 * c)} + 5278 * a^{10} * b^2 * e^{(6 * d * x + 6 * c)} + 13722 * a^9 * b^3 * e^{(6 * d * x + 6 * c)} + 19602 * a^8 * b^4 * e^{(6 * d * x + 6 * c)} + 14908 * a^7 * b^5 * e^{(6 * d * x + 6 * c)} + 300 * a^6 * b^6 * e^{(6 * d * x + 6 * c)} - 14412 * a^5 * b^7 * e^{(6 * d * x + 6 * c)} - 19228 * a^4 * b^8 * e^{(6 * d * x + 6 * c)} - 13698 * a^3 * b^9 * e^{(6 * d * x + 6 * c)} - 5802 * a^2 * b^{10} * e^{(6 * d * x + 6 * c)} - 1390 * a * b^{11} * e^{(6 * d * x + 6 * c)} - 150 * b^{12} * e^{(6 * d * x + 6 * c)} + 870 * a^{11} * b * e^{(4 * d * x + 4 * c)} + 5778 * a^{10} * b^2 * e^{(4 * d * x + 4 * c)} + 16362 * a^9 * b^3 * e^{(4 * d * x + 4 * c)} + 26190 * a^8 * b^4 * e^{(4 * d * x + 4 * c)} + 27996 * a^7 * b^5 * e^{(4 * d * x + 4 * c)} + 25620 * a^6 * b^6 * e^{(4 * d * x + 4 * c)} + 24948 * a^5 * b^7 * e^{(4 * d * x + 4 * c)} + 22332 * a^4 * b^8 * e^{(4 * d * x + 4 * c)} + 14430 * a^3 * b^9 * e^{(4 * d * x + 4 * c)} + 5946 * a^2 * b^{10} * e^{(4 * d * x + 4 * c)} + 1410 * a * b^{11} * e^{(4 * d * x + 4 * c)} + 150 * b^{12} * e^{(4 * d * x + 4 * c)} + 435 * a^{11} * b * e^{(2 * d * x + 2 * c)} + 3411 * a^{10} * b^2 * e^{(2 * d * x + 2 * c)} + 11433 * a^9 * b^3 * e^{(2 * d * x + 2 * c)} + 20793 * a^8 * b^4 * e^{(2 * d * x + 2 * c)} + 20526 * a^7 * b^5 * e^{(2 * d * x + 2 * c)} + 6510 * a^6 * b^6 * e^{(2 * d * x + 2 * c)} - 9534 * a^5 * b^7 * e^{(2 * d * x + 2 * c)} - 14622 * a^4 * b^8 * e^{(2 * d * x + 2 * c)} - 9777 * a^3 * b^9 * e^{(2 * d * x + 2 * c)} - 3729 * a^2 * b^{10} * e^{(2 * d * x + 2 * c)} - 795 * a * b^{11} * e^{(2 * d * x + 2 * c)} - 75 * b^{12} * e^{(2 * d * x + 2 * c)} - 549755813673 * a^{11} * b + 841 * a^{10} * b^2 + 3669 * a^9 * b^3 + 9531 * a^8 * b^4 + 16374 * a^7 * b^5 + 19530 * a^6 * b^6 + 16506 * a^5 * b^7 + 9894 * a^4 * b^8 + 4131 * a^3 * b^9 + 1149 * a^2 * b^{10} + 193 * a * b^{11} + 15 * b^{12}) / ((a^{13} + 10 * a^{12} * b + 45 * a^{11} * b^2 + 120 * a^{10} * b^3 + 210 * a^9 * b^4 + 252 * a^8 * b^5 + 210 * a^7 * b^6 + 120 * a^6 * b^7 + 45 * a^5 * b^8 + 10 * a^4 * b^9 + a^3 * b^{10}) * (a * e^{(4 * d * x + 4 * c)} + b * e^{(4 * d * x + 4 * c)} + 2 * a * e^{(2 * d * x + 2 * c)} - 2 * b * e^{(2 * d * x + 2 * c)} + a + b)^3) / d$$

maple [B] time = 0.16, size = 608, normalized size = 3.02

$$\frac{\ln(\tanh(dx+c)-1)}{2d(a+b)^4} + \frac{\ln(1+\tanh(dx+c))}{2d(a+b)^4} + \frac{19b^3(\tanh^5(dx+c))}{16d(a+b)^4(a+b(\tanh^2(dx+c)))^3} + \frac{35b^4(\tanh^5(dx+c))}{16d(a+b)^4(a+b(\tanh^2(dx+c)))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*tanh(d*x+c)^2)^4,x)`

[Out]
$$-1/2/d/(a+b)^4 \ln(\tanh(d*x+c)-1) + 1/2/d/(a+b)^4 \ln(1+\tanh(d*x+c)) + 19/16/d/(a+b)^4 b^3 / (a+b \tanh(d*x+c)^2)^3 \tanh(d*x+c)^5 + 35/16/d/(a+b)^4 b^4 / (a+b \tanh(d*x+c)^2)^3 / a \tanh(d*x+c)^5 + 21/16/d/(a+b)^4 b^5 / (a+b \tanh(d*x+c)^2)^3 / a^2 \tanh(d*x+c)^5 + 5/16/d/(a+b)^4 b^6 / (a+b \tanh(d*x+c)^2)^3 / a^3 \tanh(d*x+c)^5 + 17/6/d/(a+b)^4 b^2 / (a+b \tanh(d*x+c)^2)^3 a \tanh(d*x+c)^3 + 11/2/d/(a+b)^4 b^3 / (a+b \tanh(d*x+c)^2)^3 \tanh(d*x+c)^3 + 7/2/d/(a+b)^4 b^4 / (a+b \tanh(d*x+c)^2)^3 / a \tanh(d*x+c)^3 + 5/6/d/(a+b)^4 b^5 / (a+b \tanh(d*x+c)^2)^3 / a^2 \tanh(d*x+c)^3 + 29/16/d/(a+b)^4 b / (a+b \tanh(d*x+c)^2)^3 a^2 \tanh(d*x+c) + 61/16/d/(a+b)^4 b^2 / (a+b \tanh(d*x+c)^2)^3 a \tanh(d*x+c) + 43/16/d/(a+b)^4 b^3 / (a+b \tanh(d*x+c)^2)^3 \tanh(d*x+c) + 11/16/d/(a+b)^4 b^4 / (a+b \tanh(d*x+c)^2)^3 / a \tanh(d*x+c) + 35/16/d/(a+b)^4 b / (a*b)^{(1/2)} \arctan(\tanh(d*x+c)*b/(a*b)^{(1/2)}) + 35/16/d/(a+b)^4 b^2/a/(a*b)^{(1/2)} \arctan(\tanh(d*x+c)*b/(a*b)^{(1/2)}) + 21/16/d/(a+b)^4 b^3/a^2/(a*b)^{(1/2)} \arctan(\tanh(d*x+c)*b/(a*b)^{(1/2)}) + 5/16/d/(a+b)^4 b^4/a^3/(a*b)^{(1/2)} \arctan(\tanh(d*x+c)*b/(a*b)^{(1/2)})$$

maxima [B] time = 0.74, size = 925, normalized size = 4.60

$$\frac{(35 a^3 b + 35 a^2 b^2 + 21 a b^3 + 5 b^4) \arctan\left(\frac{(a+b)e^{(-2dx-2c)+a-b}}{2\sqrt{ab}}\right)}{16(a^7 + 4a^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4)\sqrt{abd}} + \frac{24(a^{10} + 7a^9b + 21a^8b^2 + 35a^7b^3 + 35a^6b^4 + 21a^5b^5 + 7a^4b^6 + a^3b^7 + 6a^2b^8 + 3a^2b^9 + 3a^2b^{10})}{24(a^{10} + 7a^9b + 21a^8b^2 + 35a^7b^3 + 35a^6b^4 + 21a^5b^5 + 7a^4b^6 + a^3b^7 + 6a^2b^8 + 3a^2b^9 + 3a^2b^{10})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*tanh(d*x+c)^2)^4,x, algorithm="maxima")`

[Out]
$$-1/16*(35*a^3*b + 35*a^2*b^2 + 21*a*b^3 + 5*b^4)*\arctan(1/2*((a + b)*e^{(-2*d*x - 2*c) + a - b}/\sqrt{a*b}))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*\sqrt{a*b}*d) + 1/24*(87*a^5*b + 319*a^4*b^2 + 450*a^3*b^3 + 306*a^2*b^4 + 103*a*b^5 + 15*b^6 + 3*(145*a^5*b + 267*a^4*b^2 + 34*a^3*b^3 - 178*a^2*b^4 - 115*a*b^5 - 25*b^6)*e^{(-2*d*x - 2*c)} + 6*(145*a^5*b + 93*a^4*b^2 - 6*a^3*b^3 + 106*a^2*b^4 + 85*a*b^5 + 25*b^6)*e^{(-4*d*x - 4*c)} + 2*(435*a^5*b + 29*a^4*b^2 + 162*a^3*b^3 - 306*a^2*b^4 - 245*a*b^5 - 75*b^6)*e^{(-6*d*x - 6*c)} + 3*(145*a^5*b + 17*a^4*b^2 - 58*a^3*b^3 + 150*a^2*b^4 + 105*a*b^5 + 25*b^6)*e^{(-8*d*x - 8*c)} + 3*(29*a^5*b + 23*a^4*b^2 - 62*a^3*b^3 - 82*a^2*b^4 - 31*a*b^5 - 5*b^6)*e^{(-10*d*x - 10*c)})/((a^10 + 7*a^9*b + 21*a^8*b^2 + 35*a^7*b^3 + 35*a^6*b^4 + 21*a^5*b^5 + 7*a^4*b^6 + a^3*b^7 + 6*(a^10 + 5*a^9*b + 9*a^8*b^2 + 5*a^7*b^3 - 5*a^6*b^4 - 9*a^5*b^5 - 5*a^4*b^6 - a^3*b^7)*e^{(-2*d*x - 2*c)} + 3*(5*a^10 + 19*a^9*b + 25*a^8*b^2 + 15*a^7*b^3 + 15*a^6*b^4 + 25*a^5*b^5 + 19*a^4*b^6 + 5*a^3*b^7)*e^{(-4*d*x - 4*c)} + 4*(5*a^10 + 17*a^9*b + 21*a^8*b^2 + 9*a^7*b^3 - 9*a^6*b^4 - 21*a^5*b^5 - 17*a^4*b^6 - 5*a^3*b^7)*e^{(-6*d*x - 6*c)} + 3*(5*a^10 + 19*a^9*b + 25*a^8*b^2 + 15*a^7*b^3 + 15*a^6*b^4 + 25*a^5*b^5 + 19*a^4*b^6 + 5*a^3*b^7)*e^{(-8*d*x - 8*c)} + 6*(a^10 + 5*a^9*b + 9*a^8*b^2 + 5*a^7*b^3 - 5*a^6*b^4 - 9*a^5*b^5 - 5*a^4*b^6 - a^3*b^7)*e^{(-10*d*x - 10*c)} + (a^10 + 7*a^9*b + 21*a^8*b^2 + 35*a^7*b^3 + 35*a^6*b^4 + 21*a^5*b^5 + 7*a^4*b^6 + a^3*b^7)*e^{(-12*d*x - 12*c)})*d) + (d*x + c)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d)$$

mupad [B] time = 1.38, size = 3685, normalized size = 18.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + b*tanh(c + d*x)^2)^4,x)`

[Out]
$$\log(\tanh(c + d*x) + 1)/(2*a^4*d + 2*b^4*d + 12*a^2*b^2*d + 8*a*b^3*d + 8*a^3*b*d) + ((\tanh(c + d*x)^3*(16*a*b^3 + 5*b^4 + 17*a^2*b^2))/(6*a^2*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + (\tanh(c + d*x)*(32*a*b^2 + 29*a^2*b + 11*b^3)))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d)$$

$$\begin{aligned}
& 16*a*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + (b^2*\tanh(c + d*x)^5*(16*a*b^2 + 19 \\
& *a^2*b + 5*b^3))/(16*a^2*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)))/(a^3*d + b^3 \\
& *d*\tanh(c + d*x)^6 + 3*a^2*b*d*\tanh(c + d*x)^2 + 3*a*b^2*d*\tanh(c + d*x)^4) \\
& - \log(\tanh(c + d*x) - 1)/(2*d*(a + b)^4) - (\operatorname{atan}(\frac{(-a^7*b)^{1/2}*(\tanh(c + d*x)*(210*a*b^8 + 25*b^9 + 791*a^2*b^7 + 1820*a^3*b^6 + 2695*a^4*b^5 + 2 \\
& 450*a^5*b^4 + 1481*a^6*b^3))}{(128*(a^{12}*d^2 + 6*a^{11}*b*d^2 + a^6*b^6*d^2 + 6*a^7*b^5*d^2 + 15*a^8*b^4*d^2 + 20*a^9*b^3*d^2 + 15*a^{10}*b^2*d^2))} + (((5 \\
& *a^3*b^{13}*d^2)/4 + 14*a^4*b^{12}*d^2 + (287*a^5*b^{11}*d^2)/4 + 224*a^6*b^{10}*d^2 + (953*a^7*b^9*d^2)/2 + 728*a^8*b^8*d^2 + (1631*a^9*b^7*d^2)/2 + 668*a^{10} \\
& *b^6*d^2 + (1561*a^{11}*b^5*d^2)/4 + 154*a^{12}*b^4*d^2 + (147*a^{13}*b^3*d^2)/4 + 4*a^{14}*b^2*d^2)/(a^{15}*d^3 + 9*a^{14}*b*d^3 + a^6*b^9*d^3 + 9*a^7*b^8*d^3 + \\
& 36*a^8*b^7*d^3 + 84*a^9*b^6*d^3 + 126*a^{10}*b^5*d^3 + 126*a^{11}*b^4*d^3 + 84*a^{12}*b^3*d^3 + 36*a^{13}*b^2*d^3) - (\tanh(c + d*x)*(-a^7*b)^{1/2}*(21*a*b^2 + \\
& 35*a^2*b + 35*a^3 + 5*b^3)*(1024*a^6*b^{11}*d^2 + 7168*a^7*b^{10}*d^2 + 20480*a^8*b^9*d^2 + 28672*a^9*b^8*d^2 + 14336*a^{10}*b^7*d^2 - 14336*a^{11}*b^6*d^2 - \\
& 28672*a^{12}*b^5*d^2 - 20480*a^{13}*b^4*d^2 - 7168*a^{14}*b^3*d^2 - 1024*a^{15}*b^2*d^2))/(4096*(a^{11}*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^{10}*b*d) \\
& *(a^{12}*d^2 + 6*a^{11}*b*d^2 + a^6*b^6*d^2 + 6*a^7*b^5*d^2 + 15*a^8*b^4*d^2 + 20*a^9*b^3*d^2 + 15*a^{10}*b^2*d^2)))*(-a^7*b)^{1/2}*(21*a*b^2 + 35*a^2*b + 3 \\
& 5*a^3 + 5*b^3))/(32*(a^{11}*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^{10}*b*d)))*(21*a*b^2 + 35*a^2*b + 35*a^3 + 5*b^3)*1i)/(32*(a^{11}*d + a^7*b^4*d \\
& + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^{10}*b*d)) + ((-a^7*b)^{1/2}*(\tanh(c + d*x)*(210*a*b^8 + 25*b^9 + 791*a^2*b^7 + 1820*a^3*b^6 + 2695*a^4*b^5 + 2450*a \\
& ^5*b^4 + 1481*a^6*b^3)))/(128*(a^{12}*d^2 + 6*a^{11}*b*d^2 + a^6*b^6*d^2 + 6*a^7*b^5*d^2 + 15*a^8*b^4*d^2 + 20*a^9*b^3*d^2 + 15*a^{10}*b^2*d^2)) - (((5*a^3*b \\
& ^{13}*d^2)/4 + 14*a^4*b^{12}*d^2 + (287*a^5*b^{11}*d^2)/4 + 224*a^6*b^{10}*d^2 + (953*a^7*b^9*d^2)/2 + 728*a^8*b^8*d^2 + (1631*a^9*b^7*d^2)/2 + 668*a^{10}*b^6* \\
& d^2 + (1561*a^{11}*b^5*d^2)/4 + 154*a^{12}*b^4*d^2 + (147*a^{13}*b^3*d^2)/4 + 4*a^{14}*b^2*d^2)/(a^{15}*d^3 + 9*a^{14}*b*d^3 + a^6*b^9*d^3 + 9*a^7*b^8*d^3 + 36*a^8*b^7*d^3 \\
& + 84*a^9*b^6*d^3 + 126*a^{10}*b^5*d^3 + 126*a^{11}*b^4*d^3 + 84*a^{12}*b^3*d^3 + 36*a^{13}*b^2*d^3) + (\tanh(c + d*x)*(-a^7*b)^{1/2}*(21*a*b^2 + 35*a^2*b + 35*a^3 \\
& ^2*b + 35*a^3 + 5*b^3)*(1024*a^6*b^{11}*d^2 + 7168*a^7*b^{10}*d^2 + 20480*a^8*b^9*d^2 + 28672*a^9*b^8*d^2 + 14336*a^{10}*b^7*d^2 - 14336*a^{11}*b^6*d^2 - 2867 \\
& 2*a^{12}*b^5*d^2 - 20480*a^{13}*b^4*d^2 - 7168*a^{14}*b^3*d^2 - 1024*a^{15}*b^2*d^2))/(4096*(a^{11}*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^{10}*b*d)*(a^{12}*d^2 + 6*a^{11}*b*d^2 + a^6*b^6*d^2 + 6*a^7*b^5*d^2 + 15*a^8*b^4*d^2 + 20*a^9*b^3*d^2 + 15*a^{10}*b^2*d^2)))*(-a^7*b)^{1/2}*(21*a*b^2 + 35*a^2*b + 35*a^3 \\
& + 5*b^3))/(32*(a^{11}*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^{10}*b*d)))*(21*a*b^2 + 35*a^2*b + 35*a^3 + 5*b^3)*1i)/(32*(a^{11}*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^{10}*b*d)))/(((185*a*b^7)/128 + (25*b^8)/128 + \\
& (303*a^2*b^6)/64 + (567*a^3*b^5)/64 + (1225*a^4*b^4)/128 + (665*a^5*b^3)/128)/(a^{15}*d^3 + 9*a^{14}*b*d^3 + a^6*b^9*d^3 + 9*a^7*b^8*d^3 + 36*a^8*b^7*d^3 + 84*a^9*b^6*d^3 + 126*a^{10}*b^5*d^3 + 126*a^{11}*b^4*d^3 + 84*a^{12}*b^3*d^3 + \\
& 36*a^{13}*b^2*d^3) + ((-a^7*b)^{1/2}*(\tanh(c + d*x)*(210*a*b^8 + 25*b^9 + 791*a^2*b^7 + 1820*a^3*b^6 + 2695*a^4*b^5 + 2450*a^5*b^4 + 1481*a^6*b^3)))/(12 \\
& 8*(a^{12}*d^2 + 6*a^{11}*b*d^2 + a^6*b^6*d^2 + 6*a^7*b^5*d^2 + 15*a^8*b^4*d^2 + 20*a^9*b^3*d^2 + 15*a^{10}*b^2*d^2)) + (((5*a^3*b^{13}*d^2)/4 + 14*a^4*b^{12}*d^2 + (287*a^5*b^{11}*d^2)/4 + 224*a^6*b^{10}*d^2 + (953*a^7*b^9*d^2)/2 + 728*a^8*b^8*d^2 + (1631*a^9*b^7*d^2)/2 + 668*a^{10}*b^6*d^2 + (1561*a^{11}*b^5*d^2)/4 + 154*a^{12}*b^4*d^2 + (147*a^{13}*b^3*d^2)/4 + 4*a^{14}*b^2*d^2)/(a^{15}*d^3 + 9*a^{14}*b*d^3 + a^6*b^9*d^3 + 9*a^7*b^8*d^3 + 36*a^8*b^7*d^3 + 84*a^9*b^6*d^3 + 126*a^{10}*b^5*d^3 + 126*a^{11}*b^4*d^3 + 84*a^{12}*b^3*d^3 + 36*a^{13}*b^2*d^3) - (\tanh(c + d*x)*(-a^7*b)^{1/2}*(21*a*b^2 + 35*a^2*b + 35*a^3 + 5*b^3)*(1024*a^6*b^{11}*d^2 + 7168*a^7*b^{10}*d^2 + 20480*a^8*b^9*d^2 + 28672*a^9*b^8*d^2 + 14336*a^{10}*b^7*d^2 - 14336*a^{11}*b^6*d^2 - 28672*a^{12}*b^5*d^2 - 20480*a^{13}*b^4*d^2 - 7168*a^{14}*b^3*d^2 - 1024*a^{15}*b^2*d^2))/(4096*(a^{11}*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^{10}*b*d)*(a^{12}*d^2 + 6*a^{11}*b*d^2 + a^6*b^6*d^2 + 6*a^7*b^5*d^2 + 15*a^8*b^4*d^2 + 20*a^9*b^3*d^2 + 15*a^{10}*b^2*d^2)))*(-a^7*b)^{1/2}*(21*a*b^2 + 35*a^2*b + 35*a^3 + 5*b^3))/(32*(a^{11}*d + a^
\end{aligned}$$

```

7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d)))*(21*a*b^2 + 35*a^2*b +
35*a^3 + 5*b^3))/(32*(a^11*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^
10*b*d)) - ((-a^7*b)^(1/2))*((tanh(c + d*x))*(210*a*b^8 + 25*b^9 + 791*a^2*b^
7 + 1820*a^3*b^6 + 2695*a^4*b^5 + 2450*a^5*b^4 + 1481*a^6*b^3)))/(128*(a^12*
d^2 + 6*a^11*b*d^2 + a^6*b^6*d^2 + 6*a^7*b^5*d^2 + 15*a^8*b^4*d^2 + 20*a^9*
b^3*d^2 + 15*a^10*b^2*d^2)) - (((5*a^3*b^13*d^2)/4 + 14*a^4*b^12*d^2 + (28
7*a^5*b^11*d^2)/4 + 224*a^6*b^10*d^2 + (953*a^7*b^9*d^2)/2 + 728*a^8*b^8*d^
2 + (1631*a^9*b^7*d^2)/2 + 668*a^10*b^6*d^2 + (1561*a^11*b^5*d^2)/4 + 154*a^
12*b^4*d^2 + (147*a^13*b^3*d^2)/4 + 4*a^14*b^2*d^2)/(a^15*d^3 + 9*a^14*b*d^
3 + a^6*b^9*d^3 + 9*a^7*b^8*d^3 + 36*a^8*b^7*d^3 + 84*a^9*b^6*d^3 + 126*a^
10*b^5*d^3 + 126*a^11*b^4*d^3 + 84*a^12*b^3*d^3 + 36*a^13*b^2*d^3) + (tanh(
c + d*x))*(-a^7*b)^(1/2)*(21*a*b^2 + 35*a^2*b + 35*a^3 + 5*b^3)*(1024*a^6*b^
11*d^2 + 7168*a^7*b^10*d^2 + 20480*a^8*b^9*d^2 + 28672*a^9*b^8*d^2 + 14336*
a^10*b^7*d^2 - 14336*a^11*b^6*d^2 - 28672*a^12*b^5*d^2 - 20480*a^13*b^4*d^2
- 7168*a^14*b^3*d^2 - 1024*a^15*b^2*d^2))/(4096*(a^11*d + a^7*b^4*d + 4*a^
8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d))*(a^12*d^2 + 6*a^11*b*d^2 + a^6*b^6*d^2
+ 6*a^7*b^5*d^2 + 15*a^8*b^4*d^2 + 20*a^9*b^3*d^2 + 15*a^10*b^2*d^2))*(-a^
7*b)^(1/2)*(21*a*b^2 + 35*a^2*b + 35*a^3 + 5*b^3))/(32*(a^11*d + a^7*b^4*d
+ 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d)))*(21*a*b^2 + 35*a^2*b + 35*a^3 +
5*b^3))/(32*(a^11*d + a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d))
))*(-a^7*b)^(1/2)*(21*a*b^2 + 35*a^2*b + 35*a^3 + 5*b^3)*1i)/(16*(a^11*d +
a^7*b^4*d + 4*a^8*b^3*d + 6*a^9*b^2*d + 4*a^10*b*d))

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tanh(d*x+c)**2)**4,x)

[Out] Timed out

$$3.202 \quad \int \sqrt{1 - \tanh^2(x)} dx$$

Optimal. Leaf size=3

$$\sin^{-1}(\tanh(x))$$

[Out] arcsin(tanh(x))

Rubi [A] time = 0.02, antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3657, 4122, 216}

$$\sin^{-1}(\tanh(x))$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - Tanh[x]^2], x]

[Out] ArcSin[Tanh[x]]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 3657

Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2)^(p_), x_Symbol] := Int[ActivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a, b]

Rule 4122

Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^2)^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \sqrt{1 - \tanh^2(x)} dx &= \int \sqrt{\operatorname{sech}^2(x)} dx \\ &= \operatorname{Subst}\left(\int \frac{1}{\sqrt{1 - x^2}} dx, x, \tanh(x)\right) \\ &= \sin^{-1}(\tanh(x)) \end{aligned}$$

Mathematica [B] time = 0.01, size = 19, normalized size = 6.33

$$2 \cosh(x) \sqrt{\operatorname{sech}^2(x)} \tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - Tanh[x]^2], x]

[Out] 2*ArcTan[Tanh[x/2]]*Cosh[x]*Sqrt[Sech[x]^2]

fricas [B] time = 0.74, size = 8, normalized size = 2.67

$$2 \arctan(\cosh(x) + \sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-tanh(x)^2)^(1/2),x, algorithm="fricas")

[Out] 2*arctan(cosh(x) + sinh(x))

giac [A] time = 0.13, size = 5, normalized size = 1.67

$$2 \arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-tanh(x)^2)^(1/2),x, algorithm="giac")

[Out] 2*arctan(e^x)

maple [A] time = 0.08, size = 4, normalized size = 1.33

$$\arcsin(\tanh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-tanh(x)^2)^(1/2),x)

[Out] arcsin(tanh(x))

maxima [A] time = 0.47, size = 5, normalized size = 1.67

$$2 \arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] 2*arctan(e^x)

mupad [B] time = 0.04, size = 3, normalized size = 1.00

$$\operatorname{asin}(\tanh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - tanh(x)^2)^(1/2),x)

[Out] asin(tanh(x))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{1 - \tanh^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-tanh(x)**2)**(1/2),x)

[Out] Integral(sqrt(1 - tanh(x)**2), x)

$$3.203 \quad \int \sqrt{-1 + \tanh^2(x)} dx$$

Optimal. Leaf size=16

$$-\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}\right)$$

[Out] -arctanh(tanh(x)/(-sech(x)^2)^(1/2))

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3657, 4122, 217, 206}

$$-\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-1 + Tanh[x]^2], x]

[Out] -ArcTanh[Tanh[x]/Sqrt[-Sech[x]^2]]

Rule 206

Int[(a_) + (b_.)*(x_)^2]^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 3657

Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2)^(p_), x_Symbol] := Int[ActivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a, b]

Rule 4122

Int[(b_.)*sec[(e_.) + (f_.)*(x_)])^2)^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \sqrt{-1 + \tanh^2(x)} \, dx &= \int \sqrt{-\operatorname{sech}^2(x)} \, dx \\
&= -\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1 + x^2}} \, dx, x, \tanh(x)\right) \\
&= -\operatorname{Subst}\left(\int \frac{1}{1 - x^2} \, dx, x, \frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}\right) \\
&= -\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.31

$$2 \cosh(x) \sqrt{-\operatorname{sech}^2(x)} \tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-1 + Tanh[x]^2], x]

[Out] 2*ArcTan[Tanh[x/2]]*Cosh[x]*Sqrt[-Sech[x]^2]

fricas [A] time = 2.08, size = 1, normalized size = 0.06

0

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+tanh(x)^2)^(1/2), x, algorithm="fricas")

[Out] 0

giac [A] time = 0.13, size = 1, normalized size = 0.06

0

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+tanh(x)^2)^(1/2), x, algorithm="giac")

[Out] 0

maple [A] time = 0.11, size = 15, normalized size = 0.94

$$-\ln\left(\tanh(x) + \sqrt{-1 + \tanh^2(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+tanh(x)^2)^(1/2), x)

[Out] -ln(tanh(x)+(-1+tanh(x)^2)^(1/2))

maxima [C] time = 0.49, size = 5, normalized size = 0.31

$$2i \operatorname{arctan}(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] 2*I*arctan(e^x)

mupad [B] time = 0.25, size = 14, normalized size = 0.88

$$-\ln\left(\tanh(x) + \sqrt{\tanh(x)^2 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tanh(x)^2 - 1)^(1/2),x)

[Out] -log(tanh(x) + (tanh(x)^2 - 1)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tanh^2(x) - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+tanh(x)**2)**(1/2),x)

[Out] Integral(sqrt(tanh(x)**2 - 1), x)

3.204 $\int (1 - \tanh^2(x))^{3/2} dx$

Optimal. Leaf size=22

$$\frac{1}{2} \sin^{-1}(\tanh(x)) + \frac{1}{2} \tanh(x) \sqrt{\operatorname{sech}^2(x)}$$

[Out] 1/2*arcsin(tanh(x))+1/2*(sech(x)^2)^(1/2)*tanh(x)

Rubi [A] time = 0.02, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3657, 4122, 195, 216}

$$\frac{1}{2} \sin^{-1}(\tanh(x)) + \frac{1}{2} \tanh(x) \sqrt{\operatorname{sech}^2(x)}$$

Antiderivative was successfully verified.

[In] Int[(1 - Tanh[x]^2)^(3/2), x]

[Out] ArcSin[Tanh[x]]/2 + (Sqrt[Sech[x]^2]*Tanh[x])/2

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 3657

Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)^2])^(p_), x_Symbol] := Int[ActivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a, b]

Rule 4122

Int[((b_.)*sec[(e_.) + (f_.)*(x_)^2])^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int (1 - \tanh^2(x))^{3/2} dx &= \int \operatorname{sech}^2(x)^{3/2} dx \\ &= \operatorname{Subst}\left(\int \sqrt{1 - x^2} dx, x, \tanh(x)\right) \\ &= \frac{1}{2} \sqrt{\operatorname{sech}^2(x)} \tanh(x) + \frac{1}{2} \operatorname{Subst}\left(\int \frac{1}{\sqrt{1 - x^2}} dx, x, \tanh(x)\right) \\ &= \frac{1}{2} \sin^{-1}(\tanh(x)) + \frac{1}{2} \sqrt{\operatorname{sech}^2(x)} \tanh(x) \end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.32

$$\frac{\operatorname{sech}(x) \left(2 \tan^{-1} \left(\tanh \left(\frac{x}{2} \right) \right) + \tanh(x) \operatorname{sech}(x) \right)}{2 \sqrt{\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - Tanh[x]^2)^(3/2), x]

[Out] (Sech[x]*(2*ArcTan[Tanh[x/2]] + Sech[x]*Tanh[x]))/(2*Sqrt[Sech[x]^2])

fricas [B] time = 0.45, size = 140, normalized size = 6.36

$$\frac{\cosh(x)^3 + 3 \cosh(x) \sinh(x)^2 + \sinh(x)^3 + (\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2(3 \cosh(x)^2 + 1))}{\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2(3 \cosh(x)^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-tanh(x)^2)^(3/2), x, algorithm="fricas")

[Out] (cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)

giac [B] time = 0.12, size = 45, normalized size = 2.05

$$\frac{1}{4} \pi - \frac{e^{(-x)} - e^x}{(e^{(-x)} - e^x)^2 + 4} + \frac{1}{2} \arctan \left(\frac{1}{2} (e^{(2x)} - 1) e^{(-x)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-tanh(x)^2)^(3/2), x, algorithm="giac")

[Out] 1/4*pi - (e^(-x) - e^x)/((e^(-x) - e^x)^2 + 4) + 1/2*arctan(1/2*(e^(2*x) - 1)*e^(-x))

maple [A] time = 0.07, size = 21, normalized size = 0.95

$$\frac{\tanh(x) \sqrt{1 - (\tanh^2(x))}}{2} + \frac{\arcsin(\tanh(x))}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-tanh(x)^2)^(3/2), x)

[Out] 1/2*tanh(x)*(1-tanh(x)^2)^(1/2)+1/2*arcsin(tanh(x))

maxima [A] time = 0.48, size = 28, normalized size = 1.27

$$\frac{e^{(3x)} - e^x}{e^{(4x)} + 2e^{(2x)} + 1} + \arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-tanh(x)^2)^(3/2), x, algorithm="maxima")

[Out] (e^(3*x) - e^x)/(e^(4*x) + 2*e^(2*x) + 1) + arctan(e^x)

mupad [B] time = 0.10, size = 20, normalized size = 0.91

$$\frac{\operatorname{asin}(\tanh(x))}{2} + \frac{\tanh(x)\sqrt{1-\tanh(x)^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1 - tanh(x)^2)^(3/2), x)`

[Out] `asin(tanh(x))/2 + (tanh(x)*(1 - tanh(x)^2)^(1/2))/2`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (1 - \tanh^2(x))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-tanh(x)**2)**(3/2), x)`

[Out] `Integral((1 - tanh(x)**2)**(3/2), x)`

3.205 $\int (-1 + \tanh^2(x))^{3/2} dx$

Optimal. Leaf size=35

$$\frac{1}{2} \tanh^{-1} \left(\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}} \right) - \frac{1}{2} \tanh(x) \sqrt{-\operatorname{sech}^2(x)}$$

[Out] 1/2*arctanh(tanh(x)/(-sech(x)^2)^(1/2))-1/2*(-sech(x)^2)^(1/2)*tanh(x)

Rubi [A] time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3657, 4122, 195, 217, 206}

$$\frac{1}{2} \tanh^{-1} \left(\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}} \right) - \frac{1}{2} \tanh(x) \sqrt{-\operatorname{sech}^2(x)}$$

Antiderivative was successfully verified.

[In] Int[(-1 + Tanh[x]^2)^(3/2), x]

[Out] ArcTanh[Tanh[x]/Sqrt[-Sech[x]^2]]/2 - (Sqrt[-Sech[x]^2]*Tanh[x])/2

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 3657

Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] := Int[ActivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a, b]

Rule 4122

Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int (-1 + \tanh^2(x))^{3/2} dx &= \int (-\operatorname{sech}^2(x))^{3/2} dx \\
&= -\operatorname{Subst}\left(\int \sqrt{-1+x^2} dx, x, \tanh(x)\right) \\
&= -\frac{1}{2}\sqrt{-\operatorname{sech}^2(x)} \tanh(x) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+x^2}} dx, x, \tanh(x)\right) \\
&= -\frac{1}{2}\sqrt{-\operatorname{sech}^2(x)} \tanh(x) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}\right) \\
&= \frac{1}{2}\tanh^{-1}\left(\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}\right) - \frac{1}{2}\sqrt{-\operatorname{sech}^2(x)} \tanh(x)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 0.80

$$-\frac{1}{2}\sqrt{-\operatorname{sech}^2(x)}\left(\tanh(x) + 2\cosh(x)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + Tanh[x]^2)^(3/2), x]

[Out] -1/2*(Sqrt[-Sech[x]^2]*(2*ArcTan[Tanh[x/2]]*Cosh[x] + Tanh[x]))

fricas [A] time = 0.61, size = 1, normalized size = 0.03

0

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+tanh(x)^2)^(3/2),x, algorithm="fricas")

[Out] 0

giac [A] time = 0.12, size = 41, normalized size = 1.17

$$\frac{\sqrt{-e^{(2x)}} + \frac{1}{\sqrt{-e^{(2x)}}}}{\left(\sqrt{-e^{(2x)}} + \frac{1}{\sqrt{-e^{(2x)}}}\right)^2 - 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+tanh(x)^2)^(3/2),x, algorithm="giac")

[Out] (sqrt(-e^(2*x)) + 1/sqrt(-e^(2*x)))/((sqrt(-e^(2*x)) + 1/sqrt(-e^(2*x)))^2 - 4)

maple [A] time = 0.12, size = 28, normalized size = 0.80

$$-\frac{\tanh(x)\sqrt{-1+\tanh^2(x)}}{2} + \frac{\ln\left(\tanh(x) + \sqrt{-1+\tanh^2(x)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+tanh(x)^2)^(3/2), x)

[Out] -1/2*tanh(x)*(-1+tanh(x)^2)^(1/2)+1/2*ln(tanh(x)+(-1+tanh(x)^2)^(1/2))

maxima [C] time = 0.46, size = 32, normalized size = 0.91

$$\frac{-ie^{(3x)} + ie^x}{e^{(4x)} + 2e^{(2x)} + 1} - i \arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+tanh(x)^2)^(3/2), x, algorithm="maxima")

[Out] (-I*e^(3*x) + I*e^x)/(e^(4*x) + 2*e^(2*x) + 1) - I*arctan(e^x)

mupad [B] time = 1.17, size = 27, normalized size = 0.77

$$\frac{\ln\left(\tanh(x) + \sqrt{\tanh(x)^2 - 1}\right)}{2} - \frac{\tanh(x) \sqrt{\tanh(x)^2 - 1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tanh(x)^2 - 1)^(3/2), x)

[Out] log(tanh(x) + (tanh(x)^2 - 1)^(1/2))/2 - (tanh(x)*(tanh(x)^2 - 1)^(1/2))/2

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\tanh^2(x) - 1)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+tanh(x)**2)**(3/2), x)

[Out] Integral((tanh(x)**2 - 1)**(3/2), x)

$$3.206 \quad \int \frac{1}{\sqrt{1-\tanh^2(x)}} dx$$

Optimal. Leaf size=11

$$\frac{\tanh(x)}{\sqrt{\operatorname{sech}^2(x)}}$$

[Out] $\tanh(x)/(\operatorname{sech}(x)^2)^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3657, 4122, 191}

$$\frac{\tanh(x)}{\sqrt{\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

[In] `Int[1/Sqrt[1 - Tanh[x]^2], x]`

[Out] `Tanh[x]/Sqrt[Sech[x]^2]`

Rule 191

`Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]`

Rule 3657

`Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2)^(p_), x_Symbol] := Int[ActivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a, b]`

Rule 4122

`Int[((b_.)*sec[(e_.) + (f_.)*(x_)])^2)^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[Int[(b + b*ff^2*x^2)^(p - 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] && !IntegerQ[p]`

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{1-\tanh^2(x)}} dx &= \int \frac{1}{\sqrt{\operatorname{sech}^2(x)}} dx \\ &= \operatorname{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}} dx, x, \tanh(x)\right) \\ &= \frac{\tanh(x)}{\sqrt{\operatorname{sech}^2(x)}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\frac{\tanh(x)}{\sqrt{\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[1 - Tanh[x]^2], x]

[Out] Tanh[x]/Sqrt[Sech[x]^2]

fricas [A] time = 0.55, size = 2, normalized size = 0.18

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-tanh(x)^2)^(1/2), x, algorithm="fricas")

[Out] sinh(x)

giac [A] time = 0.14, size = 11, normalized size = 1.00

$$-\frac{1}{2}e^{(-x)} + \frac{1}{2}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-tanh(x)^2)^(1/2), x, algorithm="giac")

[Out] -1/2*e^(-x) + 1/2*e^x

maple [A] time = 0.05, size = 14, normalized size = 1.27

$$\frac{\tanh(x)}{\sqrt{1 - (\tanh^2(x))}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-tanh(x)^2)^(1/2), x)

[Out] 1/(1-tanh(x)^2)^(1/2)*tanh(x)

maxima [A] time = 0.42, size = 11, normalized size = 1.00

$$-\frac{1}{2}e^{(-x)} + \frac{1}{2}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-tanh(x)^2)^(1/2), x, algorithm="maxima")

[Out] -1/2*e^(-x) + 1/2*e^x

mupad [B] time = 0.14, size = 2, normalized size = 0.18

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1 - tanh(x)^2)^(1/2), x)

[Out] sinh(x)

sympy [A] time = 0.41, size = 12, normalized size = 1.09

$$\frac{\tanh(x)}{\sqrt{1 - \tanh^2(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-tanh(x)**2)**(1/2), x)

[Out] tanh(x)/sqrt(1 - tanh(x)**2)

$$3.207 \quad \int \frac{1}{\sqrt{-1+\tanh^2(x)}} dx$$

Optimal. Leaf size=13

$$\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}$$

[Out] $\tanh(x)/(-\operatorname{sech}(x)^2)^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3657, 4122, 191}

$$\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[1/\text{Sqrt}[-1 + \text{Tanh}[x]^2], x]$

[Out] $\text{Tanh}[x]/\text{Sqrt}[-\text{Sech}[x]^2]$

Rule 191

$\text{Int}[(a_ + (b_ \cdot (x_)^{n_})^{p_}), x_Symbol] \rightarrow \text{Simp}[(x \cdot (a + b \cdot x^n)^{p+1})/a, x] /;$ $\text{FreeQ}\{a, b, n, p\}, x \ \&\& \ \text{EqQ}[1/n + p + 1, 0]$

Rule 3657

$\text{Int}[(u_ \cdot ((a_ + (b_ \cdot \tan[(e_ + (f_ \cdot (x_)]^2)^{p_}), x_Symbol] \rightarrow \text{Int}[\text{ActivateTrig}[u \cdot (a \cdot \sec[e + f \cdot x]^2)^p], x] /;$ $\text{FreeQ}\{a, b, e, f, p\}, x \ \&\& \ \text{EqQ}[a, b]$

Rule 4122

$\text{Int}[(b_ \cdot \sec[(e_ + (f_ \cdot (x_)]^2)^{p_}), x_Symbol] \rightarrow \text{With}\{\{ff = \text{FreeFactors}[\text{Tan}[e + f \cdot x], x]\}, \text{Dist}[(b \cdot ff)/f, \text{Subst}[\text{Int}[(b + b \cdot ff^2 \cdot x^2)^{p-1}], x], x, \text{Tan}[e + f \cdot x]/ff], x]\} /;$ $\text{FreeQ}\{b, e, f, p\}, x \ \&\& \ !\text{IntegerQ}[p]$

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-1+\tanh^2(x)}} dx &= \int \frac{1}{\sqrt{-\operatorname{sech}^2(x)}} dx \\ &= -\text{Subst} \left(\int \frac{1}{(-1+x^2)^{3/2}} dx, x, \tanh(x) \right) \\ &= \frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-1 + Tanh[x]^2], x]

[Out] Tanh[x]/Sqrt[-Sech[x]^2]

fricas [A] time = 0.46, size = 1, normalized size = 0.08

0

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+tanh(x)^2)^(1/2), x, algorithm="fricas")

[Out] 0

giac [A] time = 0.13, size = 21, normalized size = 1.62

$$-\frac{1}{2} \sqrt{-e^{(2x)}} - \frac{1}{2 \sqrt{-e^{(2x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+tanh(x)^2)^(1/2), x, algorithm="giac")

[Out] -1/2*sqrt(-e^(2*x)) - 1/2/sqrt(-e^(2*x))

maple [A] time = 0.09, size = 12, normalized size = 0.92

$$\frac{\tanh(x)}{\sqrt{-1 + \tanh^2(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-1+tanh(x)^2)^(1/2), x)

[Out] tanh(x)/(-1+tanh(x)^2)^(1/2)

maxima [B] time = 0.42, size = 25, normalized size = 1.92

$$-\frac{e^{(-2x)}}{2 \sqrt{-e^{(-2x)}}} + \frac{1}{2 \sqrt{-e^{(-2x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+tanh(x)^2)^(1/2), x, algorithm="maxima")

[Out] -1/2*e^(-2*x)/sqrt(-e^(-2*x)) + 1/2/sqrt(-e^(-2*x))

mupad [B] time = 0.09, size = 14, normalized size = 1.08

$$\frac{\sinh(2x) \sqrt{-\frac{1}{\cosh(x)^2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(tanh(x)^2 - 1)^(1/2), x)

[Out] -(sinh(2*x)*(-1/cosh(x)^2)^(1/2))/2

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\tanh^2(x) - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+tanh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(tanh(x)**2 - 1), x)
```

3.208 $\int \tanh^5(x) \sqrt{a + b \tanh^2(x)} dx$

Optimal. Leaf size=87

$$-\frac{(a + b \tanh^2(x))^{5/2}}{5b^2} + \frac{(a - b)(a + b \tanh^2(x))^{3/2}}{3b^2} - \sqrt{a + b \tanh^2(x)} + \sqrt{a + b} \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}} \right)$$

[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)-(a+b*tanh(x)^2)^(1/2)+1/3*(a-b)*(a+b*tanh(x)^2)^(3/2)/b^2-1/5*(a+b*tanh(x)^2)^(5/2)/b^2

Rubi [A] time = 0.16, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3670, 446, 88, 50, 63, 208}

$$-\frac{(a + b \tanh^2(x))^{5/2}}{5b^2} + \frac{(a - b)(a + b \tanh^2(x))^{3/2}}{3b^2} - \sqrt{a + b \tanh^2(x)} + \sqrt{a + b} \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}} \right)$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^5*Sqrt[a + b*Tanh[x]^2], x]

[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Sqrt[a + b*Tanh[x]^2] + ((a - b)*(a + b*Tanh[x]^2)^(3/2))/(3*b^2) - (a + b*Tanh[x]^2)^(5/2)/(5*b^2)

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x]]

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_.)])^(m_.)*((a_.) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_.)])^(n_.))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \tanh^5(x) \sqrt{a + b \tanh^2(x)} dx &= \text{Subst} \left(\int \frac{x^5 \sqrt{a + bx^2}}{1 - x^2} dx, x, \tanh(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{x^2 \sqrt{a + bx}}{1 - x} dx, x, \tanh^2(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{(a - b) \sqrt{a + bx}}{b} + \frac{\sqrt{a + bx}}{1 - x} - \frac{(a + bx)^{3/2}}{b} \right) dx, x, \tanh^2(x) \right) \\
&= \frac{(a - b) (a + b \tanh^2(x))^{3/2}}{3b^2} - \frac{(a + b \tanh^2(x))^{5/2}}{5b^2} + \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{a + bx}}{1 - x} dx, x \right) \\
&= -\sqrt{a + b \tanh^2(x)} + \frac{(a - b) (a + b \tanh^2(x))^{3/2}}{3b^2} - \frac{(a + b \tanh^2(x))^{5/2}}{5b^2} + \frac{1}{2} (a + b \tanh^2(x)) \\
&= -\sqrt{a + b \tanh^2(x)} + \frac{(a - b) (a + b \tanh^2(x))^{3/2}}{3b^2} - \frac{(a + b \tanh^2(x))^{5/2}}{5b^2} + \frac{(a + b \tanh^2(x))^{3/2}}{\sqrt{a + b}} \\
&= \sqrt{a + b} \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}} \right) - \sqrt{a + b \tanh^2(x)} + \frac{(a - b) (a + b \tanh^2(x))^{3/2}}{3b^2}
\end{aligned}$$

Mathematica [A] time = 0.52, size = 85, normalized size = 0.98

$$\frac{\sqrt{a + b \tanh^2(x)} (2a^2 - b(a + 5b) \tanh^2(x) - 5ab - 3b^2 \tanh^4(x) - 15b^2)}{15b^2} + \sqrt{a + b} \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^5*Sqrt[a + b*Tanh[x]^2], x]
```

```
[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] + (Sqrt[a + b*Tanh[x]^2]*(2*a^2 - 5*a*b - 15*b^2 - b*(a + 5*b)*Tanh[x]^2 - 3*b^2*Tanh[x]^4))/(15*b^2)
```

fricas [B] time = 0.87, size = 4529, normalized size = 52.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^5,x, algorithm="fricas")
```

```
[Out] [1/60*(15*(b^2*cosh(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b^2*cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*(3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2*cosh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh(x)^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cosh(x)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*b^2*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5 + 4*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 15*(b^2*cosh(x)^10 + 10*b^2*cosh(x)*sinh(x)^9 + b^2*sinh(x)^10 + 5*b^2*cosh(x)^8 + 5*(9*b^2*cosh(x)^2 + b^2)*sinh(x)^8 + 10*b^2*cosh(x)^6 + 40*(3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^7 + 10*(21*b^2*cosh(x)^4 + 14*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 10*b^2*cosh(x)^4 + 4*(63*b^2*cosh(x)^5 + 70*b^2*cosh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh(x)^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cosh(x)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*b^2*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5 + 4*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a + b)*log(-(a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*sqrt(2)*((2*a^2 - 6*a*b - 23*b^2)*cosh(x)^8 + 8*(2*a^2 - 6*a*b - 23*b^2)*cosh(x)*sinh(x)^7 + (2*a^2 - 6*a*b - 23*b^2)*sinh(x)^8 + 4*(2*a^2 - 5*a*b - 12*b^2)*cosh(x)^6 + 4*(7*(2*a^2 - 6*a*b - 23*b^2)*cosh(x)^2 + 2*a^2 - 5*a*b - 12*b^2)*sinh(x)^6 + 8*(7*(2*a^2 - 6*a*b - 23*b^2)*cosh(x)^3 + 3*(2*a^2 - 5*a*b - 12*b^2)*cosh(x))*sinh(x)^5 + 2*(6*a^2 - 14*a*b - 49*b^2)*cosh(x)^4 + 2*(35*(2*a^2 - 6*a*b - 23*b^2)*cosh(x)^4 + 30*(2*a^2 - 5*a*b - 12*b^2)*cosh(x)^2 + 6*a^2 - 14*a*b - 49*b^2)*sinh(x)^4 + 8*(7*(2*a^2 - 6*a*b - 23*b^2)*cosh(x)^5 + 10*(2*a^2 - 5*a*b - 12*b^2)*cosh(x)^3 + (6*a^2 - 14*a*b - 49*b^2)*cosh(x))*sinh(x)^3 + 4*(2*a^2 - 5*a*b - 12*b^2)*cosh(x)^2 + 4*(7*(2*a^2 - 6*a*b - 23*b^2)*cosh(x)^6 + 15*(2*a^2 - 5*a*b - 12*b^2)*cosh(x)^4 + 3*(6*a^2 - 14*a*b - 49*b^2)*cosh(x)^2 + 2*a^2 - 5*a*b - 12*b^2)*sinh(x)^2 + 2*a^2 - 6*a*b - 23*b^2 + 8*((2*a^2 - 6*a*b - 23*b^2)*cosh(x)^7 + 3*(2*a^2 - 5*a*b - 12*b^2)*cosh(x)^5 + (6*a^2 - 14*a*b - 49*b^2)*cosh(x)^3 + (2*a^2 - 5*a*b - 12*b^2)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(
```

$$\begin{aligned}
& \cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) / (b^2*\cosh(x)^{10} + 10*b^2*\cosh(x)*\sinh(x)^9 + b^2*\sinh(x)^{10} + 5*b^2*\cosh(x)^8 + 5*(9*b^2*\cosh(x)^2 + b^2)*\sinh(x)^8 + 10*b^2*\cosh(x)^6 + 40*(3*b^2*\cosh(x)^3 + b^2*\cosh(x))*\sinh(x)^7 + 10*(21*b^2*\cosh(x)^4 + 14*b^2*\cosh(x)^2 + b^2)*\sinh(x)^6 + 10*b^2*\cosh(x)^4 + 4*(63*b^2*\cosh(x)^5 + 70*b^2*\cosh(x)^3 + 15*b^2*\cosh(x))*\sinh(x)^5 + 10*(21*b^2*\cosh(x)^6 + 35*b^2*\cosh(x)^4 + 15*b^2*\cosh(x)^2 + b^2)*\sinh(x)^4 + 5*b^2*\cosh(x)^2 + 40*(3*b^2*\cosh(x)^7 + 7*b^2*\cosh(x)^5 + 5*b^2*\cosh(x)^3 + b^2*\cosh(x))*\sinh(x)^3 + 5*(9*b^2*\cosh(x)^8 + 28*b^2*\cosh(x)^6 + 30*b^2*\cosh(x)^4 + 12*b^2*\cosh(x)^2 + b^2)*\sinh(x)^2 + b^2 + 10*(b^2*\cosh(x)^9 + 4*b^2*\cosh(x)^7 + 6*b^2*\cosh(x)^5 + 4*b^2*\cosh(x)^3 + b^2*\cosh(x))*\sinh(x)), \\
& -1/30*(15*(b^2*\cosh(x)^{10} + 10*b^2*\cosh(x)*\sinh(x)^9 + b^2*\sinh(x)^{10} + 5*b^2*\cosh(x)^8 + 5*(9*b^2*\cosh(x)^2 + b^2)*\sinh(x)^8 + 10*b^2*\cosh(x)^6 + 40*(3*b^2*\cosh(x)^3 + b^2*\cosh(x))*\sinh(x)^7 + 10*(21*b^2*\cosh(x)^4 + 14*b^2*\cosh(x)^2 + b^2)*\sinh(x)^6 + 10*b^2*\cosh(x)^4 + 4*(63*b^2*\cosh(x)^5 + 70*b^2*\cosh(x)^3 + 15*b^2*\cosh(x))*\sinh(x)^5 + 10*(21*b^2*\cosh(x)^6 + 35*b^2*\cosh(x)^4 + 15*b^2*\cosh(x)^2 + b^2)*\sinh(x)^4 + 5*b^2*\cosh(x)^2 + 40*(3*b^2*\cosh(x)^7 + 7*b^2*\cosh(x)^5 + 5*b^2*\cosh(x)^3 + b^2*\cosh(x))*\sinh(x)^3 + 5*(9*b^2*\cosh(x)^8 + 28*b^2*\cosh(x)^6 + 30*b^2*\cosh(x)^4 + 12*b^2*\cosh(x)^2 + b^2)*\sinh(x)^2 + b^2 + 10*(b^2*\cosh(x)^9 + 4*b^2*\cosh(x)^7 + 6*b^2*\cosh(x)^5 + 4*b^2*\cosh(x)^3 + b^2*\cosh(x))*\sinh(x))*\sqrt{-a - b}*\arctan(\sqrt{2}*(a*\cosh(x)^2 + 2*a*\cosh(x)*\sinh(x) + a*\sinh(x)^2 + a + b))*\sqrt{-a - b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))}/((a^2 + a*b)*\cosh(x)^4 + 4*(a^2 + a*b)*\cosh(x)*\sinh(x)^3 + (a^2 + a*b)*\sinh(x)^4 + (2*a^2 + a*b - b^2)*\cosh(x)^2 + (6*(a^2 + a*b)*\cosh(x)^2 + 2*a^2 + a*b - b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*\cosh(x)^3 + (2*a^2 + a*b - b^2)*\cosh(x))*\sinh(x))) + 15*(b^2*\cosh(x)^{10} + 10*b^2*\cosh(x)*\sinh(x)^9 + b^2*\sinh(x)^{10} + 5*b^2*\cosh(x)^8 + 5*(9*b^2*\cosh(x)^2 + b^2)*\sinh(x)^8 + 10*b^2*\cosh(x)^6 + 40*(3*b^2*\cosh(x)^3 + b^2*\cosh(x))*\sinh(x)^7 + 10*(21*b^2*\cosh(x)^4 + 14*b^2*\cosh(x)^2 + b^2)*\sinh(x)^6 + 10*b^2*\cosh(x)^4 + 4*(63*b^2*\cosh(x)^5 + 70*b^2*\cosh(x)^3 + 15*b^2*\cosh(x))*\sinh(x)^5 + 10*(21*b^2*\cosh(x)^6 + 35*b^2*\cosh(x)^4 + 15*b^2*\cosh(x)^2 + b^2)*\sinh(x)^4 + 5*b^2*\cosh(x)^2 + 40*(3*b^2*\cosh(x)^7 + 7*b^2*\cosh(x)^5 + 5*b^2*\cosh(x)^3 + b^2*\cosh(x))*\sinh(x)^3 + 5*(9*b^2*\cosh(x)^8 + 28*b^2*\cosh(x)^6 + 30*b^2*\cosh(x)^4 + 12*b^2*\cosh(x)^2 + b^2)*\sinh(x)^2 + b^2 + 10*(b^2*\cosh(x)^9 + 4*b^2*\cosh(x)^7 + 6*b^2*\cosh(x)^5 + 4*b^2*\cosh(x)^3 + b^2*\cosh(x))*\sinh(x))*\sqrt{-a - b}*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1))*\sqrt{-a - b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))}/((a + b)*\cosh(x)^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a + b)*\sinh(x)^4 + 2*(a - b)*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 + a - b)*\sinh(x)^2 + 4*((a + b)*\cosh(x)^3 + (a - b)*\cosh(x))*\sinh(x) + a + b) - 2*\sqrt{2}*((2*a^2 - 6*a*b - 23*b^2)*\cosh(x)^8 + 8*(2*a^2 - 6*a*b - 23*b^2)*\cosh(x)*\sinh(x)^7 + (2*a^2 - 6*a*b - 23*b^2)*\sinh(x)^8 + 4*(2*a^2 - 5*a*b - 12*b^2)*\cosh(x)^6 + 4*(7*(2*a^2 - 6*a*b - 23*b^2)*\cosh(x)^2 + 2*a^2 - 5*a*b - 12*b^2)*\sinh(x)^6 + 8*(7*(2*a^2 - 6*a*b - 23*b^2)*\cosh(x)^3 + 3*(2*a^2 - 5*a*b - 12*b^2)*\cosh(x))*\sinh(x)^5 + 2*(6*a^2 - 14*a*b - 49*b^2)*\cosh(x)^4 + 2*(35*(2*a^2 - 6*a*b - 23*b^2)*\cosh(x)^4 + 30*(2*a^2 - 5*a*b - 12*b^2)*\cosh(x)^2 + 6*a^2 - 14*a*b - 49*b^2)*\sinh(x)^4 + 8*(7*(2*a^2 - 6*a*b - 23*b^2)*\cosh(x)^5 + 10*(2*a^2 - 5*a*b - 12*b^2)*\cosh(x)^3 + (6*a^2 - 14*a*b - 49*b^2)*\cosh(x))*\sinh(x)^3 + 4*(2*a^2 - 5*a*b - 12*b^2)*\cosh(x)^2 + 4*(7*(2*a^2 - 6*a*b - 23*b^2)*\cosh(x)^6 + 15*(2*a^2 - 5*a*b - 12*b^2)*\cosh(x)^4 + 3*(6*a^2 - 14*a*b - 49*b^2)*\cosh(x)^2 + 2*a^2 - 5*a*b - 12*b^2)*\sinh(x)^2 + 2*a^2 - 6*a*b - 23*b^2 + 8*((2*a^2 - 6*a*b - 23*b^2)*\cosh(x)^7 + 3*(2*a^2 - 5*a*b - 12*b^2)*\cosh(x)^5 + (6*a^2 - 14*a*b - 49*b^2)*\cosh(x)^3 + (2*a^2 - 5*a*b - 12*b^2)*\cosh(x))*\sinh(x))*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))} / (b^2*\cosh(x)^{10} + 10*b^2*\cosh(x)*\sinh(x)^9 + b^2*\sinh(x)^{10} + 5*b^2*\cosh(x)^8 + 5*(9*b^2*\cosh(x)^2 + b^2)*\sinh(x)^8 + 10*b^2*\cosh(x)^6 + 40*(3*b^2*\cosh(x)^3 + b^2*\cosh(x))*\sinh(x)^7 + 10*(21*b^2*\cosh(x)^4 + 14*b^2*\cosh(x)^2 + b^2)*\sinh(x)^6 + 10*b^2*\cosh(x)^4 + 4*(63*b^2*\cosh(x)^5 + 70*b^2*\cosh(x)^3 + 15*b^2*\cosh(x))*\sinh(x)^5 + 10*(21*b^2*\cosh(x)^6 + 35*b^2*\cosh(x)^4 + 15*b^2*\cosh(x)^2 + b^2)*\sinh(x)^4 + 5*b^2*\cosh(x)^2 + 40*(3*b^2*\cosh(x)^7 + 7*b^2*\cosh(x)^5 + 5*b^2*\cosh(x)^3 + b^2*\cosh(x))*\sinh(x)^3 + 5*(9*b^2*\cosh(x)^8 + 28*b^2*\cosh(x)^6 + 30*b^2*\cosh(x)^4 + 12*b^2*\cosh(x)^2 + b^2)*\sinh(x)^2 + b^2 + 10*(b^2*\cosh(x)^9 + 4*b^2*\cosh(x)^7 + 6*b^2*\cosh(x)^5 + 4*b^2*\cosh(x)^3 + b^2*\cosh(x))*\sinh(x))
\end{aligned}$$


```
osh(x)^3 + 15*b^2*cosh(x))*sinh(x)^5 + 10*(21*b^2*cosh(x)^6 + 35*b^2*cosh(x)
)^4 + 15*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 5*b^2*cosh(x)^2 + 40*(3*b^2*cosh(
x)^7 + 7*b^2*cosh(x)^5 + 5*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)^3 + 5*(9*b^
2*cosh(x)^8 + 28*b^2*cosh(x)^6 + 30*b^2*cosh(x)^4 + 12*b^2*cosh(x)^2 + b^2)
*sinh(x)^2 + b^2 + 10*(b^2*cosh(x)^9 + 4*b^2*cosh(x)^7 + 6*b^2*cosh(x)^5 +
4*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))]
```

giac [B] time = 2.95, size = 980, normalized size = 11.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^5,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) + 1/
2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log(abs
(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b) - sqrt(a + b))) - 4/15*(15*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*
x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^9*(2*a + 3*b) + 15*(sq
rt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
+ a + b))^8*(10*a + 9*b)*sqrt(a + b) + 20*(18*a^2 + 23*a*b + b^2)*(sqrt(a +
b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
b))^7 + 20*(30*a^2 - 7*a*b - 65*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^6*sqrt(a + b) + 2*(330*a^
3 - 705*a^2*b - 1480*a*b^2 + 19*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5 + 10*(18*a^3 - 279*a^2*
b + 68*a*b^2 + 349*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqrt(a + b) - 20*(30*a^4 + 81*a^3*b
- 149*a^2*b^2 - 245*a*b^3 + 19*b^4)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) +
b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3 - 20*(42*a^4 - 33*a^3*b
- 139*a^2*b^2 + 69*a*b^3 + 325*b^4)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) +
b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b) - 5*(90*a^5
- 121*a^4*b - 184*a^3*b^2 + 658*a^2*b^3 + 166*a*b^4 - 1233*b^5)*(sqrt(a + b
)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b
)) - (90*a^5 - 215*a^4*b + 240*a^3*b^2 + 638*a^2*b^3 - 2034*a*b^4 + 1713*b^5
)*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(
2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) +
b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)^5
```

maple [B] time = 0.13, size = 288, normalized size = 3.31

$$\frac{(\tanh^2(x)) \left(a + b (\tanh^2(x)) \right)^{\frac{3}{2}}}{5b} + \frac{2a \left(a + b (\tanh^2(x)) \right)^{\frac{3}{2}}}{15b^2} - \frac{\left(a + b (\tanh^2(x)) \right)^{\frac{3}{2}}}{3b} - \frac{\sqrt{(\tanh(x) - 1)^2 b + 2 (\tanh(x) - 1) b + b^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*tanh(x)^2)^(1/2)*tanh(x)^5,x)
```

```
[Out] -1/5*tanh(x)^2*(a+b*tanh(x)^2)^(3/2)/b+2/15*a/b^2*(a+b*tanh(x)^2)^(3/2)-1/3
*(a+b*tanh(x)^2)^(3/2)/b-1/2*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)-1/
2*b^(1/2)*ln(((tanh(x)-1)*b+b)/b^(1/2)+((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b
)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2))*((tanh(x
)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))-1/2*((1+tanh(x))^2*b-2*(1
+tanh(x))*b+a+b)^(1/2)+1/2*b^(1/2)*ln(((1+tanh(x))*b-b)/b^(1/2)+((1+tanh(x)
)^2*b-2*(1+tanh(x))*b+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b-2*(1+tanh(x)
)*b+2*(a+b)^(1/2))*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tanh(x)^2 + a} \tanh(x)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^5,x, algorithm="maxima")

[Out] integrate(sqrt(b*tanh(x)^2 + a)*tanh(x)^5, x)

mupad [B] time = 9.60, size = 119, normalized size = 1.37

$$-\frac{(b \tanh(x)^2 + a)^{5/2}}{5b^2} - 2 \operatorname{atan} \left(\frac{2 \sqrt{b \tanh(x)^2 + a} \sqrt{-\frac{a}{4} - \frac{b}{4}}}{a + b} \right) \sqrt{-\frac{a}{4} - \frac{b}{4}} - \sqrt{b \tanh(x)^2 + a} \left((a + b) \left(\frac{a + b}{b^2} - \frac{2a}{b^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^5*(a + b*tanh(x)^2)^(1/2),x)

[Out] - (a + b*tanh(x)^2)^(5/2)/(5*b^2) - 2*atan((2*(a + b*tanh(x)^2)^(1/2)*(- a/4 - b/4)^(1/2))/(a + b))*(- a/4 - b/4)^(1/2) - (a + b*tanh(x)^2)^(1/2)*((a + b)*((a + b)/b^2 - (2*a)/b^2) + a^2/b^2) - ((a + b)/(3*b^2) - (2*a)/(3*b^2))*(a + b*tanh(x)^2)^(3/2)

sympy [A] time = 6.39, size = 97, normalized size = 1.11

$$-\frac{2 \left(\frac{b^3 \sqrt{a+b \tanh^2(x)}}{2} + \frac{b^3(a+b) \operatorname{atan} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{-a-b}} \right)}{2\sqrt{-a-b}} + \frac{b(a+b \tanh^2(x))^{5/2}}{10} + \frac{(a+b \tanh^2(x))^{3/2} \left(-\frac{ab}{2} + \frac{b^2}{2} \right)}{3} \right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(x)**2)**(1/2)*tanh(x)**5,x)

[Out] -2*(b**3*sqrt(a + b*tanh(x)**2)/2 + b**3*(a + b)*atan(sqrt(a + b*tanh(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a + b*tanh(x)**2)**(5/2)/10 + (a + b*tanh(x)**2)**(3/2)*(-a*b/2 + b**2/2)/3)/b**3

3.209 $\int \tanh^4(x) \sqrt{a + b \tanh^2(x)} dx$

Optimal. Leaf size=121

$$\frac{(a^2 - 4ab - 8b^2) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{8b^{3/2}} - \frac{(a+4b) \tanh(x) \sqrt{a+b \tanh^2(x)}}{8b} + \sqrt{a+b} \tanh^{-1}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)$$

[Out] 1/8*(a^2-4*a*b-8*b^2)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(3/2)+arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))*(a+b)^(1/2)-1/8*(a+4*b)*(a+b*tanh(x)^2)^(1/2)*tanh(x)/b-1/4*(a+b*tanh(x)^2)^(1/2)*tanh(x)^3

Rubi [A] time = 0.20, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3670, 478, 582, 523, 217, 206, 377}

$$\frac{(a^2 - 4ab - 8b^2) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{8b^{3/2}} - \frac{1}{4} \tanh^3(x) \sqrt{a+b \tanh^2(x)} - \frac{(a+4b) \tanh(x) \sqrt{a+b \tanh^2(x)}}{8b} + \sqrt{a+b} \tanh^{-1}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^4*Sqrt[a + b*Tanh[x]^2], x]

[Out] ((a^2 - 4*a*b - 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/(8*b^(3/2)) + Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - ((a + 4*b)*Tanh[x]*Sqrt[a + b*Tanh[x]^2])/(8*b) - (Tanh[x]^3*Sqrt[a + b*Tanh[x]^2])/4

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 478

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n-1)*(e*x)^(m-n+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^q)/(b*(m+n*(p+q)+1)), x] - Dist[e^n/(b*(m+n*(p+q)+1)), Int[(e*x)^(m-n)*(a+b*x^n)^p*(c+d*x^n)^(q-1)*Simp[a*c*(m-n+1)+(a*d*(m-n+1)-n*q*(b*c-a*d)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m-n+1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 523

```
Int[((e_) + (f_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_))*Sqrt[(c_) + (d_)*(x_)^(n_)], x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]
```

Rule 582

```
Int[((g_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(f*g^(n - 1)*(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1) + 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3670

```
Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_))*((c_)*tan[(e_) + (f_)*(x_)])^(n_)]^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))
```

Rubi steps

$$\int \tanh^4(x)\sqrt{a + b \tanh^2(x)} dx = \text{Subst} \left(\int \frac{x^4 \sqrt{a + bx^2}}{1 - x^2} dx, x, \tanh(x) \right)$$

$$= -\frac{1}{4} \tanh^3(x)\sqrt{a + b \tanh^2(x)} + \frac{1}{4} \text{Subst} \left(\int \frac{x^2 (3a + (a + 4b)x^2)}{(1 - x^2) \sqrt{a + bx^2}} dx, x, \tanh(x) \right)$$

$$= -\frac{(a + 4b) \tanh(x)\sqrt{a + b \tanh^2(x)}}{8b} - \frac{1}{4} \tanh^3(x)\sqrt{a + b \tanh^2(x)} + \frac{\text{Subst} \left(\int \frac{x^2 (3a + (a + 4b)x^2)}{(1 - x^2) \sqrt{a + bx^2}} dx, x, \tanh(x) \right)}{4}$$

$$= -\frac{(a + 4b) \tanh(x)\sqrt{a + b \tanh^2(x)}}{8b} - \frac{1}{4} \tanh^3(x)\sqrt{a + b \tanh^2(x)} + (a + b) \text{Subst} \left(\int \frac{x^2 (3a + (a + 4b)x^2)}{(1 - x^2) \sqrt{a + bx^2}} dx, x, \tanh(x) \right)$$

$$= -\frac{(a + 4b) \tanh(x)\sqrt{a + b \tanh^2(x)}}{8b} - \frac{1}{4} \tanh^3(x)\sqrt{a + b \tanh^2(x)} + (a + b) \text{Subst} \left(\int \frac{x^2 (3a + (a + 4b)x^2)}{(1 - x^2) \sqrt{a + bx^2}} dx, x, \tanh(x) \right)$$

$$= \frac{(a^2 - 4ab - 8b^2) \tanh^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right)}{8b^{3/2}} + \sqrt{a + b} \tanh^{-1} \left(\frac{\sqrt{a + b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right)$$

Mathematica [C] time = 6.22, size = 580, normalized size = 4.79

$$\sqrt{\frac{a \cosh(2x) + a + b \cosh(2x) - b}{\cosh(2x) + 1}} \left(\frac{\operatorname{sech}(x)(-a \sinh(x) - 6b \sinh(x))}{8b} + \frac{1}{4} \tanh(x) \operatorname{sech}^2(x) \right) + \frac{b(a^2 - 4b^2) \sinh^4(x)}{\dots}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^4*Sqrt[a + b*Tanh[x]^2], x]

[Out]
$$\begin{aligned} & -((b*(a^2 - 4*b^2)*\operatorname{Sqrt}[(a - b + (a + b)*\operatorname{Cosh}[2*x])/(1 + \operatorname{Cosh}[2*x])])*\operatorname{Sqrt}[-((a*\operatorname{Coth}[x]^2)/b)])*\operatorname{Sqrt}[-((a*(1 + \operatorname{Cosh}[2*x])*\operatorname{Csch}[x]^2)/b)])*\operatorname{Sqrt}[(a - b + (a + b)*\operatorname{Cosh}[2*x])*\operatorname{Csch}[x]^2/b]*\operatorname{Csch}[2*x]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[(a - b + (a + b)*\operatorname{Cosh}[2*x])*\operatorname{Csch}[x]^2/b]/\operatorname{Sqrt}[2]], 1]*\operatorname{Sinh}[x]^4)/(a*(a - b + (a + b)*\operatorname{Cosh}[2*x])) - ((4*I)*b*(4*a*b + 4*b^2)*\operatorname{Sqrt}[1 + \operatorname{Cosh}[2*x]])* \operatorname{Sqrt}[(a - b + (a + b)*\operatorname{Cosh}[2*x])/(1 + \operatorname{Cosh}[2*x])]*(((-1/4*I)*\operatorname{Sqrt}[-((a*\operatorname{Coth}[x]^2)/b)])*\operatorname{Sqrt}[-((a*(1 + \operatorname{Cosh}[2*x])*\operatorname{Csch}[x]^2)/b)])*\operatorname{Sqrt}[(a - b + (a + b)*\operatorname{Cosh}[2*x])* \operatorname{Csch}[x]^2/b]*\operatorname{Csch}[2*x]*\operatorname{EllipticF}[\operatorname{ArcSin}[\operatorname{Sqrt}[(a - b + (a + b)*\operatorname{Cosh}[2*x])* \operatorname{Csch}[x]^2/b]/\operatorname{Sqrt}[2]], 1]*\operatorname{Sinh}[x]^4)/(a*\operatorname{Sqrt}[1 + \operatorname{Cosh}[2*x]])* \operatorname{Sqrt}[a - b + (a + b)*\operatorname{Cosh}[2*x]]) + ((I/2)*\operatorname{Sqrt}[-((a*\operatorname{Coth}[x]^2)/b)])*\operatorname{Sqrt}[-((a*(1 + \operatorname{Cosh}[2*x])* \operatorname{Csch}[x]^2)/b)])*\operatorname{Sqrt}[(a - b + (a + b)*\operatorname{Cosh}[2*x])* \operatorname{Csch}[x]^2/b]*\operatorname{Csch}[2*x]*\operatorname{EllipticPi}[b/(a + b), \operatorname{ArcSin}[\operatorname{Sqrt}[(a - b + (a + b)*\operatorname{Cosh}[2*x])* \operatorname{Csch}[x]^2/b]/\operatorname{Sqrt}[2]], 1]*\operatorname{Sinh}[x]^4)/((a + b)*\operatorname{Sqrt}[1 + \operatorname{Cosh}[2*x]])* \operatorname{Sqrt}[a - b + (a + b)*\operatorname{Cosh}[2*x]])/(\operatorname{Sqrt}[a - b + (a + b)*\operatorname{Cosh}[2*x]])/(4*b) + \operatorname{Sqrt}[(a - b + a*\operatorname{Cosh}[2*x] + b*\operatorname{Cosh}[2*x])/(1 + \operatorname{Cosh}[2*x])]*((\operatorname{Sech}[x]*(-a*\operatorname{Sinh}[x]) - 6*b*\operatorname{Sinh}[x]))/(8*b) + (\operatorname{Sech}[x]^2*\operatorname{Tanh}[x])/4) \end{aligned}$$

fricas [B] time = 0.95, size = 9360, normalized size = 77.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^4,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/16*(4*(b^2*\cosh(x)^8 + 8*b^2*\cosh(x)*\sinh(x)^7 + b^2*\sinh(x)^8 + 4*b^2*\cosh(x)^6 + 4*(7*b^2*\cosh(x)^2 + b^2)*\sinh(x)^6 + 6*b^2*\cosh(x)^4 + 8*(7*b^2*\cosh(x)^3 + 3*b^2*\cosh(x))*\sinh(x)^5 + 2*(35*b^2*\cosh(x)^4 + 30*b^2*\cosh(x)^2 + 3*b^2)*\sinh(x)^4 + 4*b^2*\cosh(x)^2 + 8*(7*b^2*\cosh(x)^5 + 10*b^2*\cosh(x)^3 + 3*b^2*\cosh(x))*\sinh(x)^3 + 4*(7*b^2*\cosh(x)^6 + 15*b^2*\cosh(x)^4 + 9*b^2*\cosh(x)^2 + b^2)*\sinh(x)^2 + b^2 + 8*(b^2*\cosh(x)^7 + 3*b^2*\cosh(x)^5 + 3*b^2*\cosh(x)^3 + b^2*\cosh(x))*\sinh(x))*\operatorname{sqrt}(a + b)*\log(-((a*b^2 + b^3)*\cosh(x)^8 + 8*(a*b^2 + b^3)*\cosh(x)*\sinh(x)^7 + (a*b^2 + b^3)*\sinh(x)^8 - 2*(a*b^2 + 2*b^3)*\cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*\cosh(x)^2)*\sinh(x)^6 + 4*(14*(a*b^2 + b^3)*\cosh(x)^3 - 3*(a*b^2 + 2*b^3)*\cosh(x))*\sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*\cosh(x)^4 + (70*(a*b^2 + b^3)*\cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*\cosh(x)^2)*\sinh(x)^4 + 4*(14*(a*b^2 + b^3)*\cosh(x)^5 - 10*(a*b^2 + 2*b^3)*\cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*\cosh(x))*\sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*\cosh(x)^2 + 2*(14*(a*b^2 + b^3)*\cosh(x)^6 - 15*(a*b^2 + 2*b^3)*\cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*\cosh(x)^2)*\sinh(x)^2 + \operatorname{sqrt}(2)*(b^2*\cosh(x)^6 + 6*b^2*\cosh(x))*\sinh(x)^5 + b^2*\sinh(x)^6 - 3*b^2*\cosh(x)^4 + 3*(5*b^2*\cosh(x)^2 - b^2)*\sinh(x)^4 + 4*(5*b^2*\cosh(x)^3 - 3*b^2*\cosh(x))*\sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*\cosh(x)^2 + (15*b^2*\cosh(x)^4 - 18*b^2*\cosh(x)^2 - a^2 + 2*a*b + 3*b^2) \end{aligned}$$

$$\begin{aligned}
&) * \sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a \\
& ^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + \\
& (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + \\
& 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + \\
& 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(co \\
& sh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(\\
& x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) - ((a^2 - \\
& 4*a*b - 8*b^2)*cosh(x)^8 + 8*(a^2 - 4*a*b - 8*b^2)*cosh(x)*sinh(x)^7 + (a^ \\
& 2 - 4*a*b - 8*b^2)*sinh(x)^8 + 4*(a^2 - 4*a*b - 8*b^2)*cosh(x)^6 + 4*(7*(a^ \\
& 2 - 4*a*b - 8*b^2)*cosh(x)^2 + a^2 - 4*a*b - 8*b^2)*sinh(x)^6 + 8*(7*(a^2 - \\
& 4*a*b - 8*b^2)*cosh(x)^3 + 3*(a^2 - 4*a*b - 8*b^2)*cosh(x))*sinh(x)^5 + 6* \\
& (a^2 - 4*a*b - 8*b^2)*cosh(x)^4 + 2*(35*(a^2 - 4*a*b - 8*b^2)*cosh(x)^4 + 3 \\
& 0*(a^2 - 4*a*b - 8*b^2)*cosh(x)^2 + 3*a^2 - 12*a*b - 24*b^2)*sinh(x)^4 + 8* \\
& (7*(a^2 - 4*a*b - 8*b^2)*cosh(x)^5 + 10*(a^2 - 4*a*b - 8*b^2)*cosh(x)^3 + 3 \\
& *(a^2 - 4*a*b - 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 - 4*a*b - 8*b^2)*cosh(x) \\
& ^2 + 4*(7*(a^2 - 4*a*b - 8*b^2)*cosh(x)^6 + 15*(a^2 - 4*a*b - 8*b^2)*cosh(x) \\
&)^4 + 9*(a^2 - 4*a*b - 8*b^2)*cosh(x)^2 + a^2 - 4*a*b - 8*b^2)*sinh(x)^2 + \\
& a^2 - 4*a*b - 8*b^2 + 8*((a^2 - 4*a*b - 8*b^2)*cosh(x)^7 + 3*(a^2 - 4*a*b - \\
& 8*b^2)*cosh(x)^5 + 3*(a^2 - 4*a*b - 8*b^2)*cosh(x)^3 + (a^2 - 4*a*b - 8*b^ \\
& 2)*cosh(x))*sinh(x))*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x) \\
&)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)* \\
& cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + \\
& sinh(x)^2 - 1))*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b \\
&)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh(x)^3 + (\\
& a - 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sin \\
& h(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(\\
& x))*sinh(x) + 1)) + 4*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x) \\
&)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x) \\
& ^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + \\
& 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 \\
& + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2 \\
& *cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3* \\
& b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a + b)*log(((a \\
& + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cos \\
& h(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cos \\
& h(x)*sinh(x) + sinh(x)^2 + 1))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b) \\
& *sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + \\
& b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + \\
& sinh(x)^2)) - 2*sqrt(2)*((a*b + 6*b^2)*cosh(x)^6 + 6*(a*b + 6*b^2)*cosh(x) \\
& *sinh(x)^5 + (a*b + 6*b^2)*sinh(x)^6 + (a*b - 2*b^2)*cosh(x)^4 + (15*(a*b + \\
& 6*b^2)*cosh(x)^2 + a*b - 2*b^2)*sinh(x)^4 + 4*(5*(a*b + 6*b^2)*cosh(x)^3 + \\
& (a*b - 2*b^2)*cosh(x))*sinh(x)^3 - (a*b - 2*b^2)*cosh(x)^2 + (15*(a*b + 6* \\
& b^2)*cosh(x)^4 + 6*(a*b - 2*b^2)*cosh(x)^2 - a*b + 2*b^2)*sinh(x)^2 - a*b - \\
& 6*b^2 + 2*(3*(a*b + 6*b^2)*cosh(x)^5 + 2*(a*b - 2*b^2)*cosh(x)^3 - (a*b - \\
& 2*b^2)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - \\
& b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b^2*cosh(x)^8 + 8*b^2*cos \\
& h(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2) \\
&)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x) \\
& ^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh \\
& (x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + \\
& 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + \\
& b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*s \\
& inh(x)), -1/8*(((a^2 - 4*a*b - 8*b^2)*cosh(x)^8 + 8*(a^2 - 4*a*b - 8*b^2)*c \\
& osh(x)*sinh(x)^7 + (a^2 - 4*a*b - 8*b^2)*sinh(x)^8 + 4*(a^2 - 4*a*b - 8*b^2) \\
&)*cosh(x)^6 + 4*(7*(a^2 - 4*a*b - 8*b^2)*cosh(x)^2 + a^2 - 4*a*b - 8*b^2)*s \\
& inh(x)^6 + 8*(7*(a^2 - 4*a*b - 8*b^2)*cosh(x)^3 + 3*(a^2 - 4*a*b - 8*b^2)*c \\
& osh(x))*sinh(x)^5 + 6*(a^2 - 4*a*b - 8*b^2)*cosh(x)^4 + 2*(35*(a^2 - 4*a*b \\
& - 8*b^2)*cosh(x)^4 + 30*(a^2 - 4*a*b - 8*b^2)*cosh(x)^2 + 3*a^2 - 12*a*b - \\
& 24*b^2)*sinh(x)^4 + 8*(7*(a^2 - 4*a*b - 8*b^2)*cosh(x)^5 + 10*(a^2 - 4*a*b
\end{aligned}$$

$$\begin{aligned}
& - 8*b^2*cosh(x)^3 + 3*(a^2 - 4*a*b - 8*b^2)*cosh(x)*sinh(x)^3 + 4*(a^2 - \\
& 4*a*b - 8*b^2)*cosh(x)^2 + 4*(7*(a^2 - 4*a*b - 8*b^2)*cosh(x)^6 + 15*(a^2 - \\
& 4*a*b - 8*b^2)*cosh(x)^4 + 9*(a^2 - 4*a*b - 8*b^2)*cosh(x)^2 + a^2 - 4*a*b \\
& - 8*b^2)*sinh(x)^2 + a^2 - 4*a*b - 8*b^2 + 8*((a^2 - 4*a*b - 8*b^2)*cosh(x) \\
&)^7 + 3*(a^2 - 4*a*b - 8*b^2)*cosh(x)^5 + 3*(a^2 - 4*a*b - 8*b^2)*cosh(x)^3 \\
& + (a^2 - 4*a*b - 8*b^2)*cosh(x))*sinh(x))*sqrt(-b)*arctan(sqrt(2)*(cosh(x) \\
& ^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + \\
& (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a \\
& + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - \\
& b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x) \\
&)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - 2*(b^2*cosh(x)^8 + 8*b^2*cosh(x) \\
&)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*s \\
& inh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 \\
& + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x) \\
& ^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(\\
& 7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 \\
& + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh \\
& (x))*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*s \\
& inh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 \\
& + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x) \\
&)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^ \\
& 3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 \\
& - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - \\
& 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*s \\
& inh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x) \\
& ^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3 \\
& *a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + s \\
& qrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cos \\
& h(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*c \\
& osh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 1 \\
& 8*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3 \\
& *b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))* \\
& sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 \\
& - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^ \\
& 2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3 \\
& *a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cos \\
& h(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh \\
& (x)*sinh(x)^5 + sinh(x)^6)) - 2*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + \\
& b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b \\
& ^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*co \\
& sh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2* \\
& cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^ \\
& 6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh \\
& (x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a + \\
& b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 \\
& + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x) \\
& ^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 \\
& + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) \\
& + 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x) \\
&)*sinh(x) + sinh(x)^2)) + sqrt(2)*((a*b + 6*b^2)*cosh(x)^6 + 6*(a*b + 6*b^2) \\
&)*cosh(x)*sinh(x)^5 + (a*b + 6*b^2)*sinh(x)^6 + (a*b - 2*b^2)*cosh(x)^4 + (1 \\
& 5*(a*b + 6*b^2)*cosh(x)^2 + a*b - 2*b^2)*sinh(x)^4 + 4*(5*(a*b + 6*b^2)*cos \\
& h(x)^3 + (a*b - 2*b^2)*cosh(x))*sinh(x)^3 - (a*b - 2*b^2)*cosh(x)^2 + (15*(\\
& a*b + 6*b^2)*cosh(x)^4 + 6*(a*b - 2*b^2)*cosh(x)^2 - a*b + 2*b^2)*sinh(x)^2 \\
& - a*b - 6*b^2 + 2*(3*(a*b + 6*b^2)*cosh(x)^5 + 2*(a*b - 2*b^2)*cosh(x)^3 - \\
& (a*b - 2*b^2)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^ \\
& 2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b^2*cosh(x)^8 + 8 \\
& *b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x) \\
& ^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))
\end{aligned}$$

$$\begin{aligned}
& * \sinh(x)^5 + 2*(35*b^2*\cosh(x)^4 + 30*b^2*\cosh(x)^2 + 3*b^2)*\sinh(x)^4 + 4* \\
& b^2*\cosh(x)^2 + 8*(7*b^2*\cosh(x)^5 + 10*b^2*\cosh(x)^3 + 3*b^2*\cosh(x))*\sinh \\
& (x)^3 + 4*(7*b^2*\cosh(x)^6 + 15*b^2*\cosh(x)^4 + 9*b^2*\cosh(x)^2 + b^2)*\sinh \\
& (x)^2 + b^2 + 8*(b^2*\cosh(x)^7 + 3*b^2*\cosh(x)^5 + 3*b^2*\cosh(x)^3 + b^2*\cosh(x)) \\
& *\sinh(x)), -1/16*(8*(b^2*\cosh(x)^8 + 8*b^2*\cosh(x)*\sinh(x)^7 + b^2*\sinh(x)^8 + 4*b^2*\cosh(x)^6 + 4*(7*b^2*\cosh(x)^2 + b^2)*\sinh(x)^6 + 6*b^2*\cosh(x)^4 + 8*(7*b^2*\cosh(x)^3 + 3*b^2*\cosh(x))*\sinh(x)^5 + 2*(35*b^2*\cosh(x)^4 + 30*b^2*\cosh(x)^2 + 3*b^2)*\sinh(x)^4 + 4*b^2*\cosh(x)^2 + 8*(7*b^2*\cosh(x))^5 + 10*b^2*\cosh(x)^3 + 3*b^2*\cosh(x))*\sinh(x)^3 + 4*(7*b^2*\cosh(x)^6 + 15*b^2*\cosh(x)^4 + 9*b^2*\cosh(x)^2 + b^2)*\sinh(x)^2 + b^2 + 8*(b^2*\cosh(x)^7 + 3*b^2*\cosh(x)^5 + 3*b^2*\cosh(x)^3 + b^2*\cosh(x))*\sinh(x))*\sqrt{-a-b}*\arctan(\sqrt{2}*(b*\cosh(x)^2 + 2*b*\cosh(x)*\sinh(x) + b*\sinh(x)^2 - a - b)*\sqrt{-a-b}*\sqrt{((a+b)*\cosh(x)^2 + (a+b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)})/((a*b + b^2)*\cosh(x)^4 + 4*(a*b + b^2)*\cosh(x)*\sinh(x)^3 + (a*b + b^2)*\sinh(x)^4 + (a^2 - a*b - 2*b^2)*\cosh(x)^2 + (6*(a*b + b^2)*\cosh(x)^2 + a^2 - a*b - 2*b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*\cosh(x)^3 + (a^2 - a*b - 2*b^2)*\cosh(x))*\sinh(x))) + 8*(b^2*\cosh(x)^8 + 8*b^2*\cosh(x)*\sinh(x)^7 + b^2*\sinh(x)^8 + 4*b^2*\cosh(x)^6 + 4*(7*b^2*\cosh(x)^2 + b^2)*\sinh(x)^6 + 6*b^2*\cosh(x)^4 + 8*(7*b^2*\cosh(x)^3 + 3*b^2*\cosh(x))*\sinh(x)^5 + 2*(35*b^2*\cosh(x)^4 + 30*b^2*\cosh(x)^2 + 3*b^2)*\sinh(x)^4 + 4*b^2*\cosh(x)^2 + 8*(7*b^2*\cosh(x)^5 + 10*b^2*\cosh(x)^3 + 3*b^2*\cosh(x))*\sinh(x)^3 + 4*(7*b^2*\cosh(x)^6 + 15*b^2*\cosh(x)^4 + 9*b^2*\cosh(x)^2 + b^2)*\sinh(x)^2 + b^2 + 8*(b^2*\cosh(x)^7 + 3*b^2*\cosh(x)^5 + 3*b^2*\cosh(x)^3 + b^2*\cosh(x))*\sinh(x))*\sqrt{-a-b}*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1)*\sqrt{-a-b}*\sqrt{((a+b)*\cosh(x)^2 + (a+b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)})/((a+b)*\cosh(x)^4 + 4*(a+b)*\cosh(x)*\sinh(x)^3 + (a+b)*\sinh(x)^4 + 2*(a-b)*\cosh(x)^2 + 2*(3*(a+b)*\cosh(x)^2 + a - b)*\sinh(x)^2 + 4*((a+b)*\cosh(x)^3 + (a-b)*\cosh(x))*\sinh(x) + a + b)) + ((a^2 - 4*a*b - 8*b^2)*\cosh(x)^8 + 8*(a^2 - 4*a*b - 8*b^2)*\cosh(x)*\sinh(x)^7 + (a^2 - 4*a*b - 8*b^2)*\sinh(x)^8 + 4*(a^2 - 4*a*b - 8*b^2)*\cosh(x)^6 + 4*(7*(a^2 - 4*a*b - 8*b^2)*\cosh(x)^2 + a^2 - 4*a*b - 8*b^2)*\sinh(x)^6 + 8*(7*(a^2 - 4*a*b - 8*b^2)*\cosh(x)^3 + 3*(a^2 - 4*a*b - 8*b^2)*\cosh(x))*\sinh(x)^5 + 6*(a^2 - 4*a*b - 8*b^2)*\cosh(x)^4 + 2*(35*(a^2 - 4*a*b - 8*b^2)*\cosh(x)^4 + 30*(a^2 - 4*a*b - 8*b^2)*\cosh(x)^2 + 3*a^2 - 12*a*b - 24*b^2)*\sinh(x)^4 + 8*(7*(a^2 - 4*a*b - 8*b^2)*\cosh(x)^5 + 10*(a^2 - 4*a*b - 8*b^2)*\cosh(x)^3 + 3*(a^2 - 4*a*b - 8*b^2)*\cosh(x))*\sinh(x)^3 + 4*(a^2 - 4*a*b - 8*b^2)*\cosh(x)^2 + 4*(7*(a^2 - 4*a*b - 8*b^2)*\cosh(x)^6 + 15*(a^2 - 4*a*b - 8*b^2)*\cosh(x)^4 + 9*(a^2 - 4*a*b - 8*b^2)*\cosh(x)^2 + a^2 - 4*a*b - 8*b^2)*\sinh(x)^2 + a^2 - 4*a*b - 8*b^2 + 8*((a^2 - 4*a*b - 8*b^2)*\cosh(x)^7 + 3*(a^2 - 4*a*b - 8*b^2)*\cosh(x)^5 + 3*(a^2 - 4*a*b - 8*b^2)*\cosh(x)^3 + (a^2 - 4*a*b - 8*b^2)*\cosh(x))*\sinh(x))*\sqrt{b}*\log(-((a+2*b)*\cosh(x)^4 + 4*(a+2*b)*\cosh(x)*\sinh(x)^3 + (a+2*b)*\sinh(x)^4 + 2*(a-2*b)*\cosh(x)^2 + 2*(3*(a+2*b)*\cosh(x)^2 + a - 2*b)*\sinh(x)^2 - 2*\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1)*\sqrt{b}*\sqrt{((a+b)*\cosh(x)^2 + (a+b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)}) + 4*((a+2*b)*\cosh(x)^3 + (a-2*b)*\cosh(x))*\sinh(x) + a + 2*b)/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 + 1)*\sinh(x)^2 + 2*\cosh(x)^2 + 4*(\cosh(x)^3 + \cosh(x))*\sinh(x) + 1)) + 2*\sqrt{2})*((a*b + 6*b^2)*\cosh(x)^6 + 6*(a*b + 6*b^2)*\cosh(x)*\sinh(x)^5 + (a*b + 6*b^2)*\sinh(x)^6 + (a*b - 2*b^2)*\cosh(x)^4 + (15*(a*b + 6*b^2)*\cosh(x)^2 + a*b - 2*b^2)*\sinh(x)^4 + 4*(5*(a*b + 6*b^2)*\cosh(x)^3 + (a*b - 2*b^2)*\cosh(x))*\sinh(x)^3 - (a*b - 2*b^2)*\cosh(x)^2 + (15*(a*b + 6*b^2)*\cosh(x)^4 + 6*(a*b - 2*b^2)*\cosh(x)^2 - a*b + 2*b^2)*\sinh(x)^2 - a*b - 6*b^2 + 2*(3*(a*b + 6*b^2)*\cosh(x)^5 + 2*(a*b - 2*b^2)*\cosh(x)^3 - (a*b - 2*b^2)*\cosh(x))*\sinh(x))*\sqrt{((a+b)*\cosh(x)^2 + (a+b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(b^2*\cosh(x)^8 + 8*b^2*\cosh(x)*\sinh(x)^7 + b^2*\sinh(x)^8 + 4*b^2*\cosh(x)^6 + 4*(7*b^2*\cosh(x)^2 + b^2)*\sinh(x)^6 + 6*b^2*\cosh(x)^4 + 8*(7*b^2*\cosh(x)^3 + 3*b^2*\cosh(x))*\sinh(x)^5 + 2*(35*b^2*\cosh(x)^4 + 30*b^2*\cosh(x)^2 + 3*b^2)*\sinh(x)^4 + 4*b^2*\cosh(x)^2 + 8*(7*b^2*\cosh(x)
\end{aligned}$$

$$\begin{aligned}
&)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15 \\
& *b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 \\
& + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x)), -1/8*(4*(b^2*c \\
& osh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7 \\
& *b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3* \\
& b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*si \\
& nh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*c \\
& osh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 \\
& + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x) \\
&)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b* \\
& cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 \\
& + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/ \\
& ((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh \\
& (x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a* \\
& b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 \\
& - a*b - 2*b^2)*cosh(x))*sinh(x))) + 4*(b^2*cosh(x)^8 + 8*b^2*cosh(x)*sinh \\
& (x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7*b^2*cosh(x)^2 + b^2)*sinh(x) \\
& ^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^5 + 2*(3 \\
& 5*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*sinh(x)^4 + 4*b^2*cosh(x)^2 + 8 \\
& *(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 4*(7*b^2* \\
& cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 8*(\\
& b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*s \\
& qrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)* \\
& sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^ \\
& 2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)* \\
& sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^ \\
& 2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a \\
& + b)) + ((a^2 - 4*a*b - 8*b^2)*cosh(x)^8 + 8*(a^2 - 4*a*b - 8*b^2)*cosh(x)* \\
& sinh(x)^7 + (a^2 - 4*a*b - 8*b^2)*sinh(x)^8 + 4*(a^2 - 4*a*b - 8*b^2)*cosh(\\
& x)^6 + 4*(7*(a^2 - 4*a*b - 8*b^2)*cosh(x)^2 + a^2 - 4*a*b - 8*b^2)*sinh(x)^ \\
& 6 + 8*(7*(a^2 - 4*a*b - 8*b^2)*cosh(x)^3 + 3*(a^2 - 4*a*b - 8*b^2)*cosh(x)) \\
& *sinh(x)^5 + 6*(a^2 - 4*a*b - 8*b^2)*cosh(x)^4 + 2*(35*(a^2 - 4*a*b - 8*b^2) \\
&)*cosh(x)^4 + 30*(a^2 - 4*a*b - 8*b^2)*cosh(x)^2 + 3*a^2 - 12*a*b - 24*b^2) \\
& *sinh(x)^4 + 8*(7*(a^2 - 4*a*b - 8*b^2)*cosh(x)^5 + 10*(a^2 - 4*a*b - 8*b^2) \\
&)*cosh(x)^3 + 3*(a^2 - 4*a*b - 8*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 - 4*a*b - \\
& 8*b^2)*cosh(x)^2 + 4*(7*(a^2 - 4*a*b - 8*b^2)*cosh(x)^6 + 15*(a^2 - 4*a*b \\
& - 8*b^2)*cosh(x)^4 + 9*(a^2 - 4*a*b - 8*b^2)*cosh(x)^2 + a^2 - 4*a*b - 8*b^ \\
& 2)*sinh(x)^2 + a^2 - 4*a*b - 8*b^2 + 8*((a^2 - 4*a*b - 8*b^2)*cosh(x)^7 + 3 \\
& *(a^2 - 4*a*b - 8*b^2)*cosh(x)^5 + 3*(a^2 - 4*a*b - 8*b^2)*cosh(x)^3 + (a^2 \\
& - 4*a*b - 8*b^2)*cosh(x))*sinh(x))*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2* \\
& cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b) \\
& *sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/((a + b)*c \\
& osh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh \\
& (x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + \\
& (a - b)*cosh(x))*sinh(x) + a + b)) + sqrt(2)*((a*b + 6*b^2)*cosh(x)^6 + 6*(\\
& a*b + 6*b^2)*cosh(x)*sinh(x)^5 + (a*b + 6*b^2)*sinh(x)^6 + (a*b - 2*b^2)*co \\
& sh(x)^4 + (15*(a*b + 6*b^2)*cosh(x)^2 + a*b - 2*b^2)*sinh(x)^4 + 4*(5*(a*b \\
& + 6*b^2)*cosh(x)^3 + (a*b - 2*b^2)*cosh(x))*sinh(x)^3 - (a*b - 2*b^2)*cosh(\\
& x)^2 + (15*(a*b + 6*b^2)*cosh(x)^4 + 6*(a*b - 2*b^2)*cosh(x)^2 - a*b + 2*b^ \\
& 2)*sinh(x)^2 - a*b - 6*b^2 + 2*(3*(a*b + 6*b^2)*cosh(x)^5 + 2*(a*b - 2*b^2) \\
&)*cosh(x)^3 - (a*b - 2*b^2)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + \\
& b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b^2*c \\
& osh(x)^8 + 8*b^2*cosh(x)*sinh(x)^7 + b^2*sinh(x)^8 + 4*b^2*cosh(x)^6 + 4*(7 \\
& *b^2*cosh(x)^2 + b^2)*sinh(x)^6 + 6*b^2*cosh(x)^4 + 8*(7*b^2*cosh(x)^3 + 3* \\
& b^2*cosh(x))*sinh(x)^5 + 2*(35*b^2*cosh(x)^4 + 30*b^2*cosh(x)^2 + 3*b^2)*si \\
& nh(x)^4 + 4*b^2*cosh(x)^2 + 8*(7*b^2*cosh(x)^5 + 10*b^2*cosh(x)^3 + 3*b^2*c \\
& osh(x))*sinh(x)^3 + 4*(7*b^2*cosh(x)^6 + 15*b^2*cosh(x)^4 + 9*b^2*cosh(x)^2 \\
& + b^2)*sinh(x)^2 + b^2 + 8*(b^2*cosh(x)^7 + 3*b^2*cosh(x)^5 + 3*b^2*cosh(x) \\
&)^3 + b^2*cosh(x))*sinh(x))]
\end{aligned}$$

giac [B] time = 3.04, size = 938, normalized size = 7.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^4,x, algorithm="giac")

[Out]
$$-1/2*\sqrt{a+b}*\log(\text{abs}(-(\sqrt{a+b})e^{2x} - \sqrt{ae^{4x} + be^{4x}} + 2ae^{2x} - 2be^{2x} + a + b))*(a+b) - \sqrt{a+b}*(a-b)) - 1/2*\sqrt{a+b}*\log(\text{abs}(-\sqrt{a+b})e^{2x} + \sqrt{ae^{4x} + be^{4x}} + 2ae^{2x} - 2be^{2x} + a + b) + \sqrt{a+b})) + 1/2*\sqrt{a+b}*\log(\text{abs}(-\sqrt{a+b})e^{2x} + \sqrt{ae^{4x} + be^{4x}} + 2ae^{2x} - 2be^{2x} + a + b) - \sqrt{a+b})) + 1/4*(a^2 - 4ab - 8b^2)*\arctan(-1/2*(\sqrt{a+b})e^{2x} - \sqrt{ae^{4x} + be^{4x}} + 2ae^{2x} - 2be^{2x} + a + b) + \sqrt{a+b})/\sqrt{-b})/(\sqrt{-b}*b) - 1/2*((a^2 + 12ab + 16b^2)*(\sqrt{a+b})e^{2x} - \sqrt{ae^{4x} + be^{4x}} + 2ae^{2x} - 2be^{2x} + a + b))^7 + (7a^2 + 52ab + 16b^2)*(\sqrt{a+b})e^{2x} - \sqrt{ae^{4x} + be^{4x}} + 2ae^{2x} - 2be^{2x} + a + b))^6*\sqrt{a+b} + (21a^3 + 109a^2b + 28ab^2 - 48b^3)*(\sqrt{a+b})e^{2x} - \sqrt{ae^{4x} + be^{4x}} + 2ae^{2x} - 2be^{2x} + a + b))^5 + (35a^3 + 115a^2b - 156ab^2 - 176b^3)*(\sqrt{a+b})e^{2x} - \sqrt{ae^{4x} + be^{4x}} + 2ae^{2x} - 2be^{2x} + a + b))^4*\sqrt{a+b} + (35a^4 + 130a^3b - 317a^2b^2 - 156ab^3 + 304b^4)*(\sqrt{a+b})e^{2x} - \sqrt{ae^{4x} + be^{4x}} + 2ae^{2x} - 2be^{2x} + a + b))^3 + (21a^4 + 94a^3b - 379a^2b^2 + 476ab^3 + 48b^4)*(\sqrt{a+b})e^{2x} - \sqrt{ae^{4x} + be^{4x}} + 2ae^{2x} - 2be^{2x} + a + b))^2*\sqrt{a+b} + (7a^5 + 53a^4b - 135a^3b^2 + 271a^2b^3 - 140ab^4 - 272b^5)*(\sqrt{a+b})e^{2x} - \sqrt{ae^{4x} + be^{4x}} + 2ae^{2x} - 2be^{2x} + a + b)) + (a^5 + 11a^4b - 17a^3b^2 + 65a^2b^3 - 116ab^4 + 112b^5)*\sqrt{a+b})/(((\sqrt{a+b})e^{2x} - \sqrt{ae^{4x} + be^{4x}} + 2ae^{2x} - 2be^{2x} + a + b))^2 + 2*(\sqrt{a+b})e^{2x} - \sqrt{ae^{4x} + be^{4x}} + 2ae^{2x} - 2be^{2x} + a + b))*\sqrt{a+b} + a - 3b)^4*b)$$

maple [B] time = 0.10, size = 337, normalized size = 2.79

$$-\frac{\tanh(x) \left(a + b \left(\tanh^2(x) \right)^{\frac{3}{2}} \right)}{4b} + \frac{a \tanh(x) \sqrt{a + b \left(\tanh^2(x) \right)}}{8b} + \frac{a^2 \ln \left(\sqrt{b} \tanh(x) + \sqrt{a + b \left(\tanh^2(x) \right)} \right)}{8b^{\frac{3}{2}}} - \frac{\sqrt{a + b}}{8b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*tanh(x)^2)^(1/2)*tanh(x)^4,x)

[Out]
$$-1/4*\tanh(x)*(a+b*\tanh(x)^2)^(3/2)/b+1/8*a/b*\tanh(x)*(a+b*\tanh(x)^2)^(1/2)+1/8*a^2/b^(3/2)*\ln(b^(1/2)*\tanh(x)+(a+b*\tanh(x)^2)^(1/2))-1/2*(a+b*\tanh(x)^2)^(1/2)*\tanh(x)-1/2*a/b^(1/2)*\ln(b^(1/2)*\tanh(x)+(a+b*\tanh(x)^2)^(1/2))-1/2*((\tanh(x)-1)^2*b+2*(\tanh(x)-1)*b+a+b)^(1/2)-1/2*b^(1/2)*\ln(((\tanh(x)-1)*b+b)/b^(1/2)+((\tanh(x)-1)^2*b+2*(\tanh(x)-1)*b+a+b)^(1/2))+1/2*(a+b)^(1/2)*\ln(((2*a+2*b+2*(\tanh(x)-1)*b+2*(a+b)^(1/2))*((\tanh(x)-1)^2*b+2*(\tanh(x)-1)*b+a+b)^(1/2)))/(\tanh(x)-1))+1/2*((1+\tanh(x))^2*b-2*(1+\tanh(x))*b+a+b)^(1/2)-1/2*b^(1/2)*\ln(((1+\tanh(x))*b-b)/b^(1/2)+((1+\tanh(x))^2*b-2*(1+\tanh(x))*b+a+b)^(1/2))-1/2*(a+b)^(1/2)*\ln((2*a+2*b-2*(1+\tanh(x))*b+2*(a+b)^(1/2))*((1+\tanh(x))^2*b-2*(1+\tanh(x))*b+a+b)^(1/2))/(1+\tanh(x)))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tanh(x)^2 + a} \tanh(x)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^4,x, algorithm="maxima")

[Out] integrate(sqrt(b*tanh(x)^2 + a)*tanh(x)^4, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x)^4 \sqrt{b \tanh(x)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^4*(a + b*tanh(x)^2)^(1/2),x)

[Out] int(tanh(x)^4*(a + b*tanh(x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \tanh^2(x)} \tanh^4(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(x)**2)**(1/2)*tanh(x)**4,x)

[Out] Integral(sqrt(a + b*tanh(x)**2)*tanh(x)**4, x)

3.210 $\int \tanh^3(x) \sqrt{a + b \tanh^2(x)} dx$

Optimal. Leaf size=63

$$-\frac{(a + b \tanh^2(x))^{3/2}}{3b} - \sqrt{a + b \tanh^2(x)} + \sqrt{a + b} \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}} \right)$$

[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)-(a+b*tanh(x)^2)^(1/2)-1/3*(a+b*tanh(x)^2)^(3/2)/b

Rubi [A] time = 0.12, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3670, 446, 80, 50, 63, 208}

$$-\frac{(a + b \tanh^2(x))^{3/2}}{3b} - \sqrt{a + b \tanh^2(x)} + \sqrt{a + b} \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}} \right)$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^3*Sqrt[a + b*Tanh[x]^2],x]

[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Sqrt[a + b*Tanh[x]^2] - (a + b*Tanh[x]^2)^(3/2)/(3*b)

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p

$*(c + d*x)^q, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, m, n, p, q\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 3670

$\text{Int}[\{(d_)*\tan[(e_)] + (f_)*(x_)]^{(m_)}*((a_)+ (b_)*((c_)*\tan[(e_)] + (f_)*(x_)]^{(n_)}^{(p_)}, x_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Tan}[e + f*x], x]\}, \text{Dist}[(c*ff)/f, \text{Subst}[\text{Int}[\{(d*ff*x)/c\}^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*\text{Tan}[e + f*x])/ff], x]] /; \text{FreeQ}[\{a, b, c, d, e, f, m, n, p\}, x] \ \&\& \ (\text{IGtQ}[p, 0] \ || \ \text{EqQ}[n, 2] \ || \ \text{EqQ}[n, 4] \ || \ (\text{IntegerQ}[p] \ \&\& \ \text{RationalQ}[n]))$

Rubi steps

$$\begin{aligned} \int \tanh^3(x) \sqrt{a + b \tanh^2(x)} dx &= \text{Subst} \left(\int \frac{x^3 \sqrt{a + bx^2}}{1 - x^2} dx, x, \tanh(x) \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{x \sqrt{a + bx}}{1 - x} dx, x, \tanh^2(x) \right) \\ &= -\frac{(a + b \tanh^2(x))^{3/2}}{3b} + \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{a + bx}}{1 - x} dx, x, \tanh^2(x) \right) \\ &= -\sqrt{a + b \tanh^2(x)} - \frac{(a + b \tanh^2(x))^{3/2}}{3b} + \frac{1}{2}(a + b) \text{Subst} \left(\int \frac{1}{(1 - x)\sqrt{a + bx}} dx, x, \tanh^2(x) \right) \\ &= -\sqrt{a + b \tanh^2(x)} - \frac{(a + b \tanh^2(x))^{3/2}}{3b} + \frac{(a + b) \text{Subst} \left(\int \frac{1}{1 + \frac{a}{b} - \frac{x^2}{b}} dx, x, \sqrt{a + bx} \right)}{b} \\ &= \sqrt{a + b} \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}} \right) - \sqrt{a + b \tanh^2(x)} - \frac{(a + b \tanh^2(x))^{3/2}}{3b} \end{aligned}$$

Mathematica [A] time = 0.18, size = 60, normalized size = 0.95

$$\sqrt{a + b} \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}} \right) - \frac{\sqrt{a + b \tanh^2(x)} (a + b \tanh^2(x) + 3b)}{3b}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^3*Sqrt[a + b*Tanh[x]^2], x]

[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Sqrt[a + b*Tanh[x]^2]*(a + 3*b + b*Tanh[x]^2))/(3*b)

fricas [B] time = 0.58, size = 2329, normalized size = 36.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^3,x, algorithm="fricas")

[Out] [1/12*(3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^4 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh(x)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 + 6*(

$$\begin{aligned} &^3 + (a + 2*b)*\cosh(x))*\sinh(x) + a + 4*b)*\sqrt{((a + b)*\cosh(x)^2 + (a + b) \\ &)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))/ (b*\cosh(x) \\ &)^6 + 6*b*\cosh(x)*\sinh(x)^5 + b*\sinh(x)^6 + 3*b*\cosh(x)^4 + 3*(5*b*\cosh(x) \\ &)^2 + b)*\sinh(x)^4 + 4*(5*b*\cosh(x)^3 + 3*b*\cosh(x))*\sinh(x)^3 + 3*b*\cosh(x) \\ &)^2 + 3*(5*b*\cosh(x)^4 + 6*b*\cosh(x)^2 + b)*\sinh(x)^2 + 6*(b*\cosh(x)^5 + 2*b \\ &)*\cosh(x)^3 + b*\cosh(x))*\sinh(x) + b)] \end{aligned}$$

giac [B] time = 2.18, size = 630, normalized size = 10.00

$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{2x}-\sqrt{ae^{4x}+be^{4x}+2ae^{2x}-2be^{2x}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)+\frac{1}{2}\sqrt{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} &-1/2*\sqrt{a+b}*\log(\text{abs}(-(\sqrt{a+b}*e^{2*x}-\sqrt{a*e^{4*x}+b*e^{4*x}+2*a*e^{2*x}-2*b*e^{2*x}+a+b})*(a+b)-\sqrt{a+b}*(a-b))))+1/ \\ &2*\sqrt{a+b}*\log(\text{abs}(-\sqrt{a+b}*e^{2*x}+\sqrt{a*e^{4*x}+b*e^{4*x}+2*a*e^{2*x}-2*b*e^{2*x}+a+b}+\sqrt{a+b}))) \\ &-1/2*\sqrt{a+b}*\log(\text{abs}(-\sqrt{a+b}*e^{2*x}+\sqrt{a*e^{4*x}+b*e^{4*x}+2*a*e^{2*x}-2*b*e^{2*x}+a+b}-\sqrt{a+b}))) \\ &-4/3*(3*(\sqrt{a+b}*e^{2*x}-\sqrt{a*e^{4*x}+b*e^{4*x}+2*a*e^{2*x}-2*b*e^{2*x}+a+b})^5*(a+2*b)+3*(\sqrt{a+b} \\ &)*e^{2*x}-\sqrt{a*e^{4*x}+b*e^{4*x}+2*a*e^{2*x}-2*b*e^{2*x}+a+b})^4*(3*a+2*b)*\sqrt{a+b}+2*(3*a^2-3*a*b-10*b^2)*(\sqrt{a+b})*e^{2*x} \\ &-\sqrt{a*e^{4*x}+b*e^{4*x}+2*a*e^{2*x}-2*b*e^{2*x}+a+b})^3-6*(a^2+3*a*b+6*b^2)*(\sqrt{a+b})*e^{2*x}-\sqrt{a*e^{4*x}+b*e^{4*x}+2*a*e^{2*x}-2*b*e^{2*x}+a+b})^2*\sqrt{a+b} \\ &-3*(3*a^3+4*a^2*b-9*a*b^2-26*b^3)*(\sqrt{a+b})*e^{2*x}-\sqrt{a*e^{4*x}+b*e^{4*x}+2*a*e^{2*x}-2*b*e^{2*x}+a+b}) \\ &-(3*a^3-17*a*b^2+34*b^3)*\sqrt{a+b})/((\sqrt{a+b}*e^{2*x}-\sqrt{a*e^{4*x}+b*e^{4*x}+2*a*e^{2*x}-2*b*e^{2*x}+a+b})^2+2*(\sqrt{a+b})*e^{2*x} \\ &-\sqrt{a*e^{4*x}+b*e^{4*x}+2*a*e^{2*x}-2*b*e^{2*x}+a+b})*\sqrt{a+b}+a-3*b)^3 \end{aligned}$$

maple [B] time = 0.09, size = 253, normalized size = 4.02

$$\frac{(a+b(\tanh^2(x)))^{\frac{3}{2}}}{3b} - \frac{\sqrt{(\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a + b}}{2} - \frac{\sqrt{b} \ln\left(\frac{(\tanh(x)-1)b+b}{\sqrt{b}} + \sqrt{(\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a + b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*tanh(x)^2)^(1/2)*tanh(x)^3,x)

[Out]
$$\begin{aligned} &-1/3*(a+b*\tanh(x)^2)^(3/2)/b-1/2*((\tanh(x)-1)^2*b+2*(\tanh(x)-1)*b+a+b)^(1/2) \\ &-1/2*b^(1/2)*\ln(((\tanh(x)-1)*b+b)/b^(1/2)+((\tanh(x)-1)^2*b+2*(\tanh(x)-1)*b \\ &+a+b)^(1/2))+1/2*(a+b)^(1/2)*\ln((2*a+2*b+2*(\tanh(x)-1)*b+2*(a+b)^(1/2))*((\tanh(x)-1)^2*b+2*(\tanh(x)-1)*b+a+b)^(1/2))/(\tanh(x)-1))-1/2*((1+\tanh(x))^2*b-2*(1+\tanh(x))*b+a+b)^(1/2)+1/2*b^(1/2)*\ln(((1+\tanh(x))*b-b)/b^(1/2)+((1+\tanh(x))^2*b-2*(1+\tanh(x))*b+a+b)^(1/2))+1/2*(a+b)^(1/2)*\ln((2*a+2*b-2*(1+\tanh(x))*b+2*(a+b)^(1/2))*((1+\tanh(x))^2*b-2*(1+\tanh(x))*b+a+b)^(1/2))/(1+\tanh(x)))) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tanh(x)^2 + a} \tanh(x)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^3,x, algorithm="maxima")

[Out] integrate(sqrt(b*tanh(x)^2 + a)*tanh(x)^3, x)

mupad [B] time = 3.47, size = 66, normalized size = 1.05

$$-\sqrt{b \tanh(x)^2 + a} - \frac{(b \tanh(x)^2 + a)^{3/2}}{3b} - 2 \operatorname{atan} \left(\frac{2 \sqrt{b \tanh(x)^2 + a} \sqrt{-\frac{a}{4} - \frac{b}{4}}}{a + b} \right) \sqrt{-\frac{a}{4} - \frac{b}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^3*(a + b*tanh(x)^2)^(1/2),x)

[Out] $-(a + b \tanh(x)^2)^{1/2} - (a + b \tanh(x)^2)^{3/2}/(3b) - 2 \operatorname{atan}((2*(a + b \tanh(x)^2)^{1/2}*(-a/4 - b/4)^{1/2})/(a + b))*(-a/4 - b/4)^{1/2}$

sympy [A] time = 4.28, size = 71, normalized size = 1.13

$$\frac{2 \left(\frac{b^2 \sqrt{a+b \tanh^2(x)}}{2} + \frac{b^2(a+b) \operatorname{atan} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{-a-b}} \right)}{2\sqrt{-a-b}} + \frac{b(a+b \tanh^2(x))^{3/2}}{6} \right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(x)**2)**(1/2)*tanh(x)**3,x)

[Out] $-2*(b**2*\sqrt{a + b*tanh(x)**2})/2 + b**2*(a + b)*\operatorname{atan}(\sqrt{a + b*tanh(x)**2})/\sqrt{-a - b})/(2*\sqrt{-a - b}) + b*(a + b*tanh(x)**2)**(3/2)/6)/b**2$

3.211 $\int \tanh^2(x) \sqrt{a + b \tanh^2(x)} dx$

Optimal. Leaf size=85

$$-\frac{1}{2} \tanh(x) \sqrt{a + b \tanh^2(x)} - \frac{(a + 2b) \tanh^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right)}{2\sqrt{b}} + \sqrt{a + b} \tanh^{-1} \left(\frac{\sqrt{a + b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right)$$

[Out] $-1/2*(a+2*b)*\operatorname{arctanh}(b^{(1/2)}*\tanh(x)/(a+b*\tanh(x)^2)^{(1/2)})/b^{(1/2)}+\operatorname{arctanh}((a+b)^{(1/2)}*\tanh(x)/(a+b*\tanh(x)^2)^{(1/2)})*(a+b)^{(1/2)}-1/2*(a+b*\tanh(x)^2)^{(1/2)}*\tanh(x)$

Rubi [A] time = 0.12, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3670, 478, 523, 217, 206, 377}

$$-\frac{(a + 2b) \tanh^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right)}{2\sqrt{b}} + \sqrt{a + b} \tanh^{-1} \left(\frac{\sqrt{a + b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right) - \frac{1}{2} \tanh(x) \sqrt{a + b \tanh^2(x)}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^2*Sqrt[a + b*Tanh[x]^2], x]

[Out] $-((a + 2*b)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Tanh}[x])/\operatorname{Sqrt}[a + b*\operatorname{Tanh}[x]^2]])/(2*\operatorname{Sqrt}[b]) + \operatorname{Sqrt}[a + b]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a + b]*\operatorname{Tanh}[x])/\operatorname{Sqrt}[a + b*\operatorname{Tanh}[x]^2]] - (\operatorname{Tanh}[x]*\operatorname{Sqrt}[a + b*\operatorname{Tanh}[x]^2])/2$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 478

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n + 1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 523

```
Int[((e_) + (f_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_))*Sqrt[(c_) + (d_)*(x_)^(n_)], x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]
```

Rule 3670

```
Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*((c_)*tan[(e_) + (f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff, x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))
```

Rubi steps

$$\begin{aligned} \int \tanh^2(x) \sqrt{a + b \tanh^2(x)} dx &= \text{Subst} \left(\int \frac{x^2 \sqrt{a + bx^2}}{1 - x^2} dx, x, \tanh(x) \right) \\ &= -\frac{1}{2} \tanh(x) \sqrt{a + b \tanh^2(x)} + \frac{1}{2} \text{Subst} \left(\int \frac{a + (a + 2b)x^2}{(1 - x^2) \sqrt{a + bx^2}} dx, x, \tanh(x) \right) \\ &= -\frac{1}{2} \tanh(x) \sqrt{a + b \tanh^2(x)} + \frac{1}{2} (-a - 2b) \text{Subst} \left(\int \frac{1}{\sqrt{a + bx^2}} dx, x, \tanh(x) \right) \\ &= -\frac{1}{2} \tanh(x) \sqrt{a + b \tanh^2(x)} + \frac{1}{2} (-a - 2b) \text{Subst} \left(\int \frac{1}{1 - bx^2} dx, x, \frac{\tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right) \\ &= -\frac{(a + 2b) \tanh^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right)}{2\sqrt{b}} + \sqrt{a + b} \tanh^{-1} \left(\frac{\sqrt{a + b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right) - \frac{1}{2} \tanh(x) \sqrt{a + b \tanh^2(x)} \end{aligned}$$

Mathematica [C] time = 3.44, size = 193, normalized size = 2.27

$$\frac{\tanh(x) \left(-\left(\text{sech}^2(x) \left((a + b) \cosh(2x) + a - b \right) + \sqrt{2} a \sqrt{\frac{\text{csch}^2(x) \left((a + b) \cosh(2x) + a - b \right)}{b}} \right) F \left(\sin^{-1} \left(\frac{\sqrt{\frac{(a - b + (a + b) \cosh(2x)) \text{csch}^2(x)}{b}}}{\sqrt{2}} \right) \right)}{2\sqrt{2} \sqrt{\text{sech}^2(x) \left((a + b) \cosh(2x) + a - b \right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2*Sqrt[a + b*Tanh[x]^2], x]
```

```
[Out] ((Sqrt[2]*a*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b])*EllipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] - 2*Sqrt[2]*a*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b])*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] - (a - b + (a + b)*Cosh[2*x])*Sech[x]^2*Tanh[x])/(2*Sqrt[2]*Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2])
```

fricas [B] time = 0.68, size = 4825, normalized size = 56.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/4*((b*\cosh(x)^4 + 4*b*\cosh(x)*\sinh(x)^3 + b*\sinh(x)^4 + 2*b*\cosh(x)^2 + \\ & 2*(3*b*\cosh(x)^2 + b)*\sinh(x)^2 + 4*(b*\cosh(x)^3 + b*\cosh(x))*\sinh(x) + b)* \\ & \sqrt{a + b}*\log(-((a*b^2 + b^3)*\cosh(x)^8 + 8*(a*b^2 + b^3)*\cosh(x)*\sinh(x) \\ & ^7 + (a*b^2 + b^3)*\sinh(x)^8 - 2*(a*b^2 + 2*b^3)*\cosh(x)^6 - 2*(a*b^2 + 2*b \\ & ^3 - 14*(a*b^2 + b^3)*\cosh(x)^2)*\sinh(x)^6 + 4*(14*(a*b^2 + b^3)*\cosh(x)^3 \\ & - 3*(a*b^2 + 2*b^3)*\cosh(x))*\sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*\co \\ & sh(x)^4 + (70*(a*b^2 + b^3)*\cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30* \\ & (a*b^2 + 2*b^3)*\cosh(x)^2)*\sinh(x)^4 + 4*(14*(a*b^2 + b^3)*\cosh(x)^5 - 10*(\\ & a*b^2 + 2*b^3)*\cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*\cosh(x))*\sinh(x) \\ & ^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*\cosh(x)^2 + \\ & 2*(14*(a*b^2 + b^3)*\cosh(x)^6 - 15*(a*b^2 + 2*b^3)*\cosh(x)^4 + a^3 - 3*a*b^ \\ & 2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*\cosh(x)^2)*\sinh(x)^2 + \sqrt{2} \\ &)*(b^2*\cosh(x)^6 + 6*b^2*\cosh(x)*\sinh(x)^5 + b^2*\sinh(x)^6 - 3*b^2*\cosh(x)^ \\ & 4 + 3*(5*b^2*\cosh(x)^2 - b^2)*\sinh(x)^4 + 4*(5*b^2*\cosh(x)^3 - 3*b^2*\cosh(x) \\ &))*\sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*\cosh(x)^2 + (15*b^2*\cosh(x)^4 - 18*b^2 \\ & *\cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*\sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2* \\ & \cosh(x)^5 - 6*b^2*\cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*\cosh(x))*\sinh(x))*\sqrt{ \\ & a + b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2* \\ & \cosh(x)*\sinh(x) + \sinh(x)^2))} + 4*(2*(a*b^2 + b^3)*\cosh(x)^7 - 3*(a*b^2 + 2 \\ & *b^3)*\cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*\cosh(x)^3 + (a^3 - 3*a*b^ \\ & 2 - 2*b^3)*\cosh(x))*\sinh(x))/(\cosh(x)^6 + 6*\cosh(x)^5*\sinh(x) + 15*\cosh(x)^ \\ & 4*\sinh(x)^2 + 20*\cosh(x)^3*\sinh(x)^3 + 15*\cosh(x)^2*\sinh(x)^4 + 6*\cosh(x)*\s \\ & inh(x)^5 + \sinh(x)^6)) + ((a + 2*b)*\cosh(x)^4 + 4*(a + 2*b)*\cosh(x)*\sinh(x) \\ & ^3 + (a + 2*b)*\sinh(x)^4 + 2*(a + 2*b)*\cosh(x)^2 + 2*(3*(a + 2*b)*\cosh(x)^2 \\ & + a + 2*b)*\sinh(x)^2 + 4*((a + 2*b)*\cosh(x)^3 + (a + 2*b)*\cosh(x))*\sinh(x) \\ & + a + 2*b)*\sqrt{b}*\log(-((a + 2*b)*\cosh(x)^4 + 4*(a + 2*b)*\cosh(x)*\sinh(x) \\ & ^3 + (a + 2*b)*\sinh(x)^4 + 2*(a - 2*b)*\cosh(x)^2 + 2*(3*(a + 2*b)*\cosh(x)^2 \\ & + a - 2*b)*\sinh(x)^2 - 2*\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^ \\ & 2 - 1)*\sqrt{b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x) \\ &)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))} + 4*((a + 2*b)*\cosh(x)^3 + (a - 2*b)* \\ & \cosh(x))*\sinh(x) + a + 2*b)/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + \\ & 2*(3*\cosh(x)^2 + 1)*\sinh(x)^2 + 2*\cosh(x)^2 + 4*(\cosh(x)^3 + \cosh(x))*\sinh(\\ & x) + 1)) + (b*\cosh(x)^4 + 4*b*\cosh(x)*\sinh(x)^3 + b*\sinh(x)^4 + 2*b*\cosh(x) \\ & ^2 + 2*(3*b*\cosh(x)^2 + b)*\sinh(x)^2 + 4*(b*\cosh(x)^3 + b*\cosh(x))*\sinh(x) \\ & + b)*\sqrt{a + b}*\log(((a + b)*\cosh(x)^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a \\ & + b)*\sinh(x)^4 + 2*a*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 + a)*\sinh(x)^2 + sq \\ & rt(2)*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1))*\sqrt{a + b}*\sqrt{((a \\ & + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) \\ & + \sinh(x)^2))} + 4*((a + b)*\cosh(x)^3 + a*\cosh(x))*\sinh(x) + a + b)/(\cosh(x) \\ & ^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) - 2*\sqrt{2}*(b*\cosh(x)^2 + 2*b*\cosh(x) \\ & *\sinh(x) + b*\sinh(x)^2 - b)*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a \\ & - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))}/(b*\cosh(x)^4 + 4*b*\cosh \\ & (x)*\sinh(x)^3 + b*\sinh(x)^4 + 2*b*\cosh(x)^2 + 2*(3*b*\cosh(x)^2 + b)*\sinh(x) \\ & ^2 + 4*(b*\cosh(x)^3 + b*\cosh(x))*\sinh(x) + b), 1/4*(2*((a + 2*b)*\cosh(x)^4 \\ & + 4*(a + 2*b)*\cosh(x)*\sinh(x)^3 + (a + 2*b)*\sinh(x)^4 + 2*(a + 2*b)*\cosh(x) \\ & ^2 + 2*(3*(a + 2*b)*\cosh(x)^2 + a + 2*b)*\sinh(x)^2 + 4*((a + 2*b)*\cosh(x)^3 \\ & + (a + 2*b)*\cosh(x))*\sinh(x) + a + 2*b)*\sqrt{-b}*\arctan(\sqrt{2}*(\cosh(x)^2 \\ & + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1)*\sqrt{-b}*\sqrt{((a + b)*\cosh(x)^2 + (a \\ & + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))}/((a + \\ & b)*\cosh(x)^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a + b)*\sinh(x)^4 + 2*(a - b) \\ & *\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 + a - b)*\sinh(x)^2 + 4*((a + b)*\cosh(x) \\ & ^3 + (a - b)*\cosh(x))*\sinh(x) + a + b)) + (b*\cosh(x)^4 + 4*b*\cosh(x)*\sinh(x) \\ &)^3 + b*\sinh(x)^4 + 2*b*\cosh(x)^2 + 2*(3*b*\cosh(x)^2 + b)*\sinh(x)^2 + 4*(b* \\ & \cosh(x)^3 + b*\cosh(x))*\sinh(x) + b)*\sqrt{a + b}*\log(-((a*b^2 + b^3)*\cosh(x) \\ & ^8 + 8*(a*b^2 + b^3)*\cosh(x)*\sinh(x)^7 + (a*b^2 + b^3)*\sinh(x)^8 - 2*(a*b^2 \\ & + 2*b^3)*\cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*\cosh(x)^2)*\sinh(x) \\ &)^6 + 4*(14*(a*b^2 + b^3)*\cosh(x)^3 - 3*(a*b^2 + 2*b^3)*\cosh(x))*\sinh(x)^5 \\ & + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*\cosh(x)^4 + (70*(a*b^2 + b^3)*\cosh(x)^4 + \end{aligned}$$

$$\begin{aligned}
& a^3 - a^2b + 4ab^2 + 6b^3 - 30(a^2b^2 + 2b^3)\cosh(x)^2 \sinh(x)^4 + \\
& 4(14(a^2b^2 + b^3)\cosh(x)^5 - 10(a^2b^2 + 2b^3)\cosh(x)^3 + (a^3 - a^2b + \\
& 4ab^2 + 6b^3)\cosh(x))\sinh(x)^3 + a^3 + 3a^2b + 3ab^2 + b^3 + 2 \\
& (a^3 - 3a^2b - 2b^3)\cosh(x)^2 + 2(14(a^2b^2 + b^3)\cosh(x)^6 - 15(a^2b \\
& ^2 + 2b^3)\cosh(x)^4 + a^3 - 3a^2b - 2b^3 + 3(a^3 - a^2b + 4ab^2 + \\
& 6b^3)\cosh(x)^2)\sinh(x)^2 + \sqrt{2}(b^2\cosh(x)^6 + 6b^2\cosh(x)\sinh(x) \\
&)^5 + b^2\sinh(x)^6 - 3b^2\cosh(x)^4 + 3(5b^2\cosh(x)^2 - b^2)\sinh(x)^4 \\
& + 4(5b^2\cosh(x)^3 - 3b^2\cosh(x))\sinh(x)^3 - (a^2 - 2ab - 3b^2)\co \\
& sh(x)^2 + (15b^2\cosh(x)^4 - 18b^2\cosh(x)^2 - a^2 + 2ab + 3b^2)\sinh(\\
& x)^2 - a^2 - 2ab - b^2 + 2(3b^2\cosh(x)^5 - 6b^2\cosh(x)^3 - (a^2 - 2 \\
& ab - 3b^2)\cosh(x))\sinh(x))\sqrt{a+b}\sqrt{((a+b)\cosh(x)^2 + (a+b) \\
&)\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)} + 4(2(a \\
& *b^2 + b^3)\cosh(x)^7 - 3(a^2b^2 + 2b^3)\cosh(x)^5 + (a^3 - a^2b + 4ab^2 \\
& ^2 + 6b^3)\cosh(x)^3 + (a^3 - 3a^2b - 2b^3)\cosh(x))\sinh(x))/(\cosh(x)^6 \\
& + 6\cosh(x)^5\sinh(x) + 15\cosh(x)^4\sinh(x)^2 + 20\cosh(x)^3\sinh(x)^3 + \\
& 15\cosh(x)^2\sinh(x)^4 + 6\cosh(x)\sinh(x)^5 + \sinh(x)^6) + (b\cosh(x)^4 + \\
& 4b\cosh(x)\sinh(x)^3 + b\sinh(x)^4 + 2b\cosh(x)^2 + 2(3b\cosh(x)^2 + b \\
&)\sinh(x)^2 + 4(b\cosh(x)^3 + b\cosh(x))\sinh(x) + b)\sqrt{a+b}\log(((a \\
& + b)\cosh(x)^4 + 4(a+b)\cosh(x)\sinh(x)^3 + (a+b)\sinh(x)^4 + 2a\cosh \\
& (x)^2 + 2(3(a+b)\cosh(x)^2 + a)\sinh(x)^2 + \sqrt{2}(\cosh(x)^2 + 2\cosh \\
& (x)\sinh(x) + \sinh(x)^2 + 1)\sqrt{a+b}\sqrt{((a+b)\cosh(x)^2 + (a+b) \\
&)\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)} + 4((a+b) \\
&)\cosh(x)^3 + a\cosh(x))\sinh(x) + a + b)/(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \\
& \sinh(x)^2) - 2\sqrt{2}(b\cosh(x)^2 + 2b\cosh(x)\sinh(x) + b\sinh(x)^2 - \\
& b)\sqrt{((a+b)\cosh(x)^2 + (a+b)\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2\cosh \\
& (x)\sinh(x) + \sinh(x)^2))}/(b\cosh(x)^4 + 4b\cosh(x)\sinh(x)^3 + b\sinh(x) \\
& ^4 + 2b\cosh(x)^2 + 2(3b\cosh(x)^2 + b)\sinh(x)^2 + 4(b\cosh(x)^3 + b\c \\
& osh(x))\sinh(x) + b), -1/4(2(b\cosh(x)^4 + 4b\cosh(x)\sinh(x)^3 + b\sinh \\
& (x)^4 + 2b\cosh(x)^2 + 2(3b\cosh(x)^2 + b)\sinh(x)^2 + 4(b\cosh(x)^3 + \\
& b\cosh(x))\sinh(x) + b)\sqrt{-a-b}\arctan(\sqrt{2}(b\cosh(x)^2 + 2b\cosh \\
& (x)\sinh(x) + b\sinh(x)^2 - a - b)\sqrt{-a-b}\sqrt{((a+b)\cosh(x)^2 + (\\
& a+b)\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)})/((a \\
& b + b^2)\cosh(x)^4 + 4(ab + b^2)\cosh(x)\sinh(x)^3 + (ab + b^2)\sinh(x)^ \\
& 4 + (a^2 - ab - 2b^2)\cosh(x)^2 + (6(ab + b^2)\cosh(x)^2 + a^2 - ab - \\
& 2b^2)\sinh(x)^2 + a^2 + 2ab + b^2 + 2(2(ab + b^2)\cosh(x)^3 + (a^2 - \\
& ab - 2b^2)\cosh(x))\sinh(x))) + 2(b\cosh(x)^4 + 4b\cosh(x)\sinh(x)^3 + \\
& b\sinh(x)^4 + 2b\cosh(x)^2 + 2(3b\cosh(x)^2 + b)\sinh(x)^2 + 4(b\cosh(x) \\
&)^3 + b\cosh(x))\sinh(x) + b)\sqrt{-a-b}\arctan(\sqrt{2}(\cosh(x)^2 + 2\co \\
& sh(x)\sinh(x) + \sinh(x)^2 + 1)\sqrt{-a-b}\sqrt{((a+b)\cosh(x)^2 + (a+b) \\
&)\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)})/((a+b) \\
&)\cosh(x)^4 + 4(a+b)\cosh(x)\sinh(x)^3 + (a+b)\sinh(x)^4 + 2(a-b)\co \\
& sh(x)^2 + 2(3(a+b)\cosh(x)^2 + a - b)\sinh(x)^2 + 4((a+b)\cosh(x)^3 \\
& + (a-b)\cosh(x))\sinh(x) + a + b) - ((a + 2b)\cosh(x)^4 + 4(a + 2b)\c \\
& osh(x)\sinh(x)^3 + (a + 2b)\sinh(x)^4 + 2(a + 2b)\cosh(x)^2 + 2(3(a + \\
& 2b)\cosh(x)^2 + a + 2b)\sinh(x)^2 + 4((a + 2b)\cosh(x)^3 + (a + 2b)\co \\
& sh(x))\sinh(x) + a + 2b)\sqrt{b}\log(-((a + 2b)\cosh(x)^4 + 4(a + 2b)\c \\
& osh(x)\sinh(x)^3 + (a + 2b)\sinh(x)^4 + 2(a - 2b)\cosh(x)^2 + 2(3(a + \\
& 2b)\cosh(x)^2 + a - 2b)\sinh(x)^2 - 2\sqrt{2}(\cosh(x)^2 + 2\cosh(x)\sinh \\
& (x) + \sinh(x)^2 - 1)\sqrt{b}\sqrt{((a+b)\cosh(x)^2 + (a+b)\sinh(x)^2 + \\
& a - b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)} + 4((a + 2b)\cosh(x) \\
& ^3 + (a - 2b)\cosh(x))\sinh(x) + a + 2b)/(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 \\
& + \sinh(x)^4 + 2(3\cosh(x)^2 + 1)\sinh(x)^2 + 2\cosh(x)^2 + 4(\cosh(x)^3 + \\
& \cosh(x))\sinh(x) + 1) + 2\sqrt{2}(b\cosh(x)^2 + 2b\cosh(x)\sinh(x) + b\si \\
& nh(x)^2 - b)\sqrt{((a+b)\cosh(x)^2 + (a+b)\sinh(x)^2 + a - b)/(\cosh(x) \\
& ^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2))}/(b\cosh(x)^4 + 4b\cosh(x)\sinh(x)^3 \\
& + b\sinh(x)^4 + 2b\cosh(x)^2 + 2(3b\cosh(x)^2 + b)\sinh(x)^2 + 4(b\cosh \\
& (x)^3 + b\cosh(x))\sinh(x) + b), -1/2((b\cosh(x)^4 + 4b\cosh(x)\sinh(x)^3 \\
& + b\sinh(x)^4 + 2b\cosh(x)^2 + 2(3b\cosh(x)^2 + b)\sinh(x)^2 + 4(b\cos \\
& h(x)^3 + b\cosh(x))\sinh(x) + b)\sqrt{-a-b}\arctan(\sqrt{2}(b\cosh(x)^2 +
\end{aligned}$$

$$2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + (b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - ((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a + 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^2 + a + 2*b)*sinh(x)^2 + 4*((a + 2*b)*cosh(x)^3 + (a + 2*b)*cosh(x))*sinh(x) + a + 2*b)*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 + 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 + b)*sinh(x)^2 + 4*(b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)]$$

giac [B] time = 1.74, size = 554, normalized size = 6.52

$$\frac{(a + 2b) \arctan\left(\frac{\sqrt{a+b}e^{2x} - \sqrt{ae^{4x} + be^{4x} + 2ae^{2x} - 2be^{2x} + a + b} + \sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{1}{2} \sqrt{a+b} \log\left(\left| -\left(\sqrt{a+b}e^{2x} - \sqrt{ae^{4x} + be^{4x} + 2ae^{2x} - 2be^{2x} + a + b}\right) \right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^2,x, algorithm="giac")

[Out] $-(a + 2*b)*arctan(-1/2*(sqrt(a + b)*e^{2*x} - sqrt(a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^{2*x} - sqrt(a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b))*(a + b) - sqrt(a + b)*(a - b))) - 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^{2*x} + sqrt(a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b) + sqrt(a + b))) + 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^{2*x} + sqrt(a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b) - sqrt(a + b))) - 2*((sqrt(a + b)*e^{2*x} - sqrt(a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b))^3*(a + 2*b) + (sqrt(a + b)*e^{2*x} - sqrt(a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b))^2*(3*a - 2*b)*sqrt(a + b) + (3*a^2 - 3*a*b - 2*b^2)*(sqrt(a + b)*e^{2*x} - sqrt(a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b)) + (a^2 - a*b + 2*b^2)*sqrt(a + b))/((sqrt(a + b)*e^{2*x} - sqrt(a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b))^2 + 2*(sqrt(a + b)*e^{2*x} - sqrt(a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b))*sqrt(a + b) + a - 3*b)^2$

maple [B] time = 0.09, size = 276, normalized size = 3.25

$$\frac{\sqrt{a + b (\tanh^2(x))} \tanh(x)}{2} - \frac{a \ln\left(\sqrt{b} \tanh(x) + \sqrt{a + b (\tanh^2(x))}\right)}{2\sqrt{b}} - \frac{\sqrt{(\tanh(x) - 1)^2 b + 2 (\tanh(x) - 1)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*tanh(x)^2)^(1/2)*tanh(x)^2,x)`

[Out]
$$-1/2*(a+b*\tanh(x)^2)^{(1/2)}*\tanh(x)-1/2*a/b^{(1/2)}*\ln(b^{(1/2)}*\tanh(x)+(a+b*\tanh(x)^2)^{(1/2)})-1/2*((\tanh(x)-1)^2*b+2*(\tanh(x)-1)*b+a+b)^{(1/2)}-1/2*b^{(1/2)}*\ln(((\tanh(x)-1)*b+b)/b^{(1/2)}+((\tanh(x)-1)^2*b+2*(\tanh(x)-1)*b+a+b)^{(1/2)}))+1/2*(a+b)^{(1/2)}*\ln((2*a+2*b+2*(\tanh(x)-1)*b+2*(a+b)^{(1/2)}*((\tanh(x)-1)^2*b+2*(\tanh(x)-1)*b+a+b)^{(1/2)}))/(\tanh(x)-1))+1/2*((1+\tanh(x))^2*b-2*(1+\tanh(x))*b+a+b)^{(1/2)}-1/2*b^{(1/2)}*\ln(((1+\tanh(x))*b-b)/b^{(1/2)}+((1+\tanh(x))^2*b-2*(1+\tanh(x))*b+a+b)^{(1/2)})-1/2*(a+b)^{(1/2)}*\ln((2*a+2*b-2*(1+\tanh(x))*b+2*(a+b)^{(1/2)}*((1+\tanh(x))^2*b-2*(1+\tanh(x))*b+a+b)^{(1/2)}))/(1+\tanh(x)))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tanh(x)^2 + a} \tanh(x)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tanh(x)^2)^(1/2)*tanh(x)^2,x, algorithm="maxima")`

[Out] `integrate(sqrt(b*tanh(x)^2 + a)*tanh(x)^2, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x)^2 \sqrt{b \tanh(x)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)^2*(a + b*tanh(x)^2)^(1/2),x)`

[Out] `int(tanh(x)^2*(a + b*tanh(x)^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \tanh^2(x)} \tanh^2(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*tanh(x)**2)**(1/2)*tanh(x)**2,x)`

[Out] `Integral(sqrt(a + b*tanh(x)**2)*tanh(x)**2, x)`

3.212 $\int \tanh(x) \sqrt{a + b \tanh^2(x)} dx$

Optimal. Leaf size=44

$$\sqrt{a+b} \tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}} \right) - \sqrt{a+b \tanh^2(x)}$$

[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)-(a+b*tanh(x)^2)^(1/2)

Rubi [A] time = 0.08, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3670, 444, 50, 63, 208}

$$\sqrt{a+b} \tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}} \right) - \sqrt{a+b \tanh^2(x)}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]*Sqrt[a + b*Tanh[x]^2], x]

[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Sqrt[a + b*Tanh[x]^2]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 444

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x],

$x\}}, \text{Dist}[(c*ff)/f, \text{Subst}[\text{Int}[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f^2*x^2), x], x, (c*\text{Tan}[e + f*x])/ff], x]] /; \text{FreeQ}\{a, b, c, d, e, f, m, n, p\}, x\} \&\& (\text{IGtQ}[p, 0] \|\ \text{EqQ}[n, 2] \|\ \text{EqQ}[n, 4] \|\ (\text{IntegerQ}[p] \&\& \text{RationalQ}[n]))$

Rubi steps

$$\begin{aligned} \int \tanh(x)\sqrt{a + b \tanh^2(x)} dx &= \text{Subst}\left(\int \frac{x\sqrt{a + bx^2}}{1 - x^2} dx, x, \tanh(x)\right) \\ &= \frac{1}{2} \text{Subst}\left(\int \frac{\sqrt{a + bx}}{1 - x} dx, x, \tanh^2(x)\right) \\ &= -\sqrt{a + b \tanh^2(x)} + \frac{1}{2}(a + b) \text{Subst}\left(\int \frac{1}{(1 - x)\sqrt{a + bx}} dx, x, \tanh^2(x)\right) \\ &= -\sqrt{a + b \tanh^2(x)} + \frac{(a + b) \text{Subst}\left(\int \frac{1}{1 + \frac{a}{b} - \frac{x^2}{b}} dx, x, \sqrt{a + b \tanh^2(x)}\right)}{b} \\ &= \sqrt{a + b} \tanh^{-1}\left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}}\right) - \sqrt{a + b \tanh^2(x)} \end{aligned}$$

Mathematica [A] time = 0.03, size = 44, normalized size = 1.00

$$\sqrt{a + b} \tanh^{-1}\left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}}\right) - \sqrt{a + b \tanh^2(x)}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]*Sqrt[a + b*Tanh[x]^2], x]

[Out] Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Sqrt[a + b*Tanh[x]^2]

fricas [B] time = 0.51, size = 1543, normalized size = 35.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x), x, algorithm="fricas")

[Out] [1/4*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*log((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2

[In] int((a+b*tanh(x)^2)^(1/2)*tanh(x),x)

[Out] $-1/2*((\tanh(x)-1)^2*b+2*(\tanh(x)-1)*b+a+b)^{(1/2)}-1/2*b^{(1/2)}*\ln(((\tanh(x)-1)*b+b)/b^{(1/2)}+((\tanh(x)-1)^2*b+2*(\tanh(x)-1)*b+a+b)^{(1/2)})+1/2*(a+b)^{(1/2)}*\ln((2*a+2*b+2*(\tanh(x)-1)*b+2*(a+b)^{(1/2)}*((\tanh(x)-1)^2*b+2*(\tanh(x)-1)*b+a+b)^{(1/2)))/(\tanh(x)-1))-1/2*((1+\tanh(x))^2*b-2*(1+\tanh(x))*b+a+b)^{(1/2)}+1/2*b^{(1/2)}*\ln(((1+\tanh(x))*b-b)/b^{(1/2)}+((1+\tanh(x))^2*b-2*(1+\tanh(x))*b+a+b)^{(1/2)})+1/2*(a+b)^{(1/2)}*\ln((2*a+2*b-2*(1+\tanh(x))*b+2*(a+b)^{(1/2)}*((1+\tanh(x))^2*b-2*(1+\tanh(x))*b+a+b)^{(1/2)))/(1+\tanh(x)))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tanh(x)^2 + a} \tanh(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(x)^2)^(1/2)*tanh(x),x, algorithm="maxima")

[Out] integrate(sqrt(b*tanh(x)^2 + a)*tanh(x), x)

mupad [B] time = 1.69, size = 51, normalized size = 1.16

$$-\sqrt{b \tanh(x)^2 + a} - 2 \operatorname{atan} \left(\frac{2 \sqrt{b \tanh(x)^2 + a} \sqrt{-\frac{a}{4} - \frac{b}{4}}}{a + b} \right) \sqrt{-\frac{a}{4} - \frac{b}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)*(a + b*tanh(x)^2)^(1/2),x)

[Out] $-(a + b*\tanh(x)^2)^{(1/2)} - 2*\operatorname{atan}((2*(a + b*\tanh(x)^2)^{(1/2)}*(-a/4 - b/4)^{(1/2)))/(a + b))*(-a/4 - b/4)^{(1/2)}$

sympy [A] time = 2.26, size = 51, normalized size = 1.16

$$\frac{2 \left(\frac{b \sqrt{a+b \tanh^2(x)}}{2} + \frac{b(a+b) \operatorname{atan} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{-a-b}} \right)}{2 \sqrt{-a-b}} \right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(x)**2)**(1/2)*tanh(x),x)

[Out] $-2*(b*\sqrt{a + b*\tanh(x)**2})/2 + b*(a + b)*\operatorname{atan}(\sqrt{a + b*\tanh(x)**2}/\sqrt{-a - b})/(2*\sqrt{-a - b}))/b$

3.213 $\int \sqrt{a + b \tanh^2(x)} dx$

Optimal. Leaf size=60

$$\sqrt{a+b} \tanh^{-1} \left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right) - \sqrt{b} \tanh^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right)$$

[Out] $-\operatorname{arctanh}(b^{(1/2)} \tanh(x) / (a+b \tanh(x)^2)^{(1/2)}) * b^{(1/2)} + \operatorname{arctanh}((a+b)^{(1/2)} * \tanh(x) / (a+b \tanh(x)^2)^{(1/2)}) * (a+b)^{(1/2)}$

Rubi [A] time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3661, 402, 217, 206, 377}

$$\sqrt{a+b} \tanh^{-1} \left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right) - \sqrt{b} \tanh^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Tanh[x]^2], x]

[Out] $-(\operatorname{Sqrt}[b] * \operatorname{ArcTanh}[(\operatorname{Sqrt}[b] * \operatorname{Tanh}[x]) / \operatorname{Sqrt}[a + b * \operatorname{Tanh}[x]^2]]) + \operatorname{Sqrt}[a + b] * \operatorname{ArcTanh}[(\operatorname{Sqrt}[a + b] * \operatorname{Tanh}[x]) / \operatorname{Sqrt}[a + b * \operatorname{Tanh}[x]^2]]$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 402

Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/d, Int[(a + b*x^2)^(p-1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p-1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])

Rule 3661

Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(ff*x)^n]^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned}
\int \sqrt{a + b \tanh^2(x)} dx &= \text{Subst} \left(\int \frac{\sqrt{a + bx^2}}{1 - x^2} dx, x, \tanh(x) \right) \\
&= - \left((-a - b) \text{Subst} \left(\int \frac{1}{(1 - x^2) \sqrt{a + bx^2}} dx, x, \tanh(x) \right) \right) - b \text{Subst} \left(\int \frac{1}{\sqrt{a + bx^2}} dx, x, \tanh(x) \right) \\
&= - \left((-a - b) \text{Subst} \left(\int \frac{1}{1 - (a + b)x^2} dx, x, \frac{\tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right) \right) - b \text{Subst} \left(\int \frac{1}{1 - bx^2} dx, x, \tanh(x) \right) \\
&= -\sqrt{b} \tanh^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right) + \sqrt{a + b} \tanh^{-1} \left(\frac{\sqrt{a + b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right)
\end{aligned}$$

Mathematica [B] time = 0.24, size = 137, normalized size = 2.28

$$\frac{1}{2} \left(-2\sqrt{b} \log \left(\sqrt{b} \sqrt{a + b \tanh^2(x)} + b \tanh(x) \right) - \sqrt{a + b} \log \left(\sqrt{a + b} \sqrt{a + b \tanh^2(x)} + a - b \tanh(x) \right) + \sqrt{a + b} \log \left(\sqrt{a + b} \sqrt{a + b \tanh^2(x)} - a + b \tanh(x) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Tanh[x]^2], x]

[Out] $(-\text{Sqrt}[a + b] \text{Log}[1 - \text{Tanh}[x]]) + \text{Sqrt}[a + b] \text{Log}[1 + \text{Tanh}[x]] - 2 \text{Sqrt}[b] \text{Log}[b \text{Tanh}[x] + \text{Sqrt}[b] \text{Sqrt}[a + b \text{Tanh}[x]^2]] - \text{Sqrt}[a + b] \text{Log}[a - b \text{Tanh}[x] + \text{Sqrt}[a + b] \text{Sqrt}[a + b \text{Tanh}[x]^2]] + \text{Sqrt}[a + b] \text{Log}[a + b \text{Tanh}[x] + \text{Sqrt}[a + b] \text{Sqrt}[a + b \text{Tanh}[x]^2]])/2$

fricas [B] time = 0.57, size = 3443, normalized size = 57.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")

[Out] $[1/4 \text{sqrt}(a + b) \log(-((a*b^2 + b^3) \cosh(x)^8 + 8*(a*b^2 + b^3) \cosh(x) \sinh(x)^7 + (a*b^2 + b^3) \sinh(x)^8 - 2*(a*b^2 + 2*b^3) \cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3) \cosh(x)^2) \sinh(x)^6 + 4*(14*(a*b^2 + b^3) \cosh(x)^3 - 3*(a*b^2 + 2*b^3) \cosh(x)) \sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3) \cosh(x)^4 + (70*(a*b^2 + b^3) \cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3) \cosh(x)^2) \sinh(x)^4 + 4*(14*(a*b^2 + b^3) \cosh(x)^5 - 10*(a*b^2 + 2*b^3) \cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3) \cosh(x)) \sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3) \cosh(x)^2 + 2*(14*(a*b^2 + b^3) \cosh(x)^6 - 15*(a*b^2 + 2*b^3) \cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3) \cosh(x)^2) \sinh(x)^2 + \text{sqrt}(2) \text{sqrt}(b^2 \cosh(x)^6 + 6*b^2 \cosh(x) \sinh(x)^5 + b^2 \sinh(x)^6 - 3*b^2 \cosh(x)^4 + 3*(5*b^2 \cosh(x)^2 - b^2) \sinh(x)^4 + 4*(5*b^2 \cosh(x)^3 - 3*b^2 \cosh(x)) \sinh(x)^3 - (a^2 - 2*a*b - 3*b^2) \cosh(x)^2 + (15*b^2 \cosh(x)^4 - 18*b^2 \cosh(x)^2 - a^2 + 2*a*b + 3*b^2) \sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2 \cosh(x)^5 - 6*b^2 \cosh(x)^3 - (a^2 - 2*a*b - 3*b^2) \cosh(x)) \sinh(x)) \text{sqrt}(a + b) \text{sqrt}(((a + b) \cosh(x)^2 + (a + b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)) + 4*(2*(a*b^2 + b^3) \cosh(x)^7 - 3*(a*b^2 + 2*b^3) \cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3) \cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3) \cosh(x)) \sinh(x)) / (\cosh(x)^6 + 6 \cosh(x)^5 \sinh(x) + 15 \cosh(x)^4 \sinh(x)^2 + 20 \cosh(x)^3 \sinh(x)^3 + 15 \cosh(x)^2 \sinh(x)^4 + 6 \cosh(x) \sinh(x)^5 + \sinh(x)^6) + 1/2 \text{sqrt}(b) \log(-((a + 2*b) \cosh(x)^4 + 4*(a + 2*b) \cosh(x) \sinh(x)^3 + (a + 2*b) \sinh(x)^4 + 2*(a - 2*b) \cosh(x)^2 + 2*$

$x^4 + 2*(3*\cosh(x)^2 + 1)*\sinh(x)^2 + 2*\cosh(x)^2 + 4*(\cosh(x)^3 + \cosh(x))*\sinh(x) + 1)), -1/2*\sqrt{-a - b}*\arctan(\sqrt{2}*(b*\cosh(x)^2 + 2*b*\cosh(x))*\sinh(x) + b*\sinh(x)^2 - a - b)*\sqrt{-a - b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)})/((a*b + b^2)*\cosh(x)^4 + 4*(a*b + b^2)*\cosh(x)*\sinh(x)^3 + (a*b + b^2)*\sinh(x)^4 + (a^2 - a*b - 2*b^2)*\cosh(x)^2 + (6*(a*b + b^2)*\cosh(x)^2 + a^2 - a*b - 2*b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*\cosh(x)^3 + (a^2 - a*b - 2*b^2)*\cosh(x))*\sinh(x))) - 1/2*\sqrt{-a - b}*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1)*\sqrt{-a - b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)})/((a + b)*\cosh(x)^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a + b)*\sinh(x)^4 + 2*(a - b)*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 + a - b)*\sinh(x)^2 + 4*((a + b)*\cosh(x)^3 + (a - b)*\cosh(x))*\sinh(x) + a + b)) + \sqrt{-b}*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1)*\sqrt{-b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)})/((a + b)*\cosh(x)^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a + b)*\sinh(x)^4 + 2*(a - b)*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 + a - b)*\sinh(x)^2 + 4*((a + b)*\cosh(x)^3 + (a - b)*\cosh(x))*\sinh(x) + a + b))]$

giac [B] time = 0.69, size = 253, normalized size = 4.22

$$-\frac{2b \arctan\left(-\frac{\sqrt{a+b}e^{2x}-\sqrt{ae^{4x}+be^{4x}+2ae^{2x}-2be^{2x}+a+b}+\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{1}{2}\sqrt{a+b} \log\left(\left|-\left(\sqrt{a+b}e^{2x}-\sqrt{ae^{4x}+be^{4x}+2ae^{2x}-2be^{2x}+a+b}\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(x)^2)^(1/2),x, algorithm="giac")

[Out] $-2*b*\arctan(-1/2*(\sqrt{a + b}*e^{(2*x)} - \sqrt{a*e^{(4*x)} + b*e^{(4*x)} + 2*a*e^{(2*x)} - 2*b*e^{(2*x)} + a + b) + \sqrt{a + b})/\sqrt{-b})/\sqrt{-b} - 1/2*\sqrt{a + b}*\log(\text{abs}(-(\sqrt{a + b}*e^{(2*x)} - \sqrt{a*e^{(4*x)} + b*e^{(4*x)} + 2*a*e^{(2*x)} - 2*b*e^{(2*x)} + a + b))*(a + b) - \sqrt{a + b}*(a - b))) - 1/2*\sqrt{a + b}*\log(\text{abs}(-\sqrt{a + b}*e^{(2*x)} + \sqrt{a*e^{(4*x)} + b*e^{(4*x)} + 2*a*e^{(2*x)} - 2*b*e^{(2*x)} + a + b) + \sqrt{a + b})) + 1/2*\sqrt{a + b}*\log(\text{abs}(-\sqrt{a + b}*e^{(2*x)} + \sqrt{a*e^{(4*x)} + b*e^{(4*x)} + 2*a*e^{(2*x)} - 2*b*e^{(2*x)} + a + b) - \sqrt{a + b}))$

maple [B] time = 0.11, size = 238, normalized size = 3.97

$$\frac{\sqrt{(\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a + b}}{2} - \frac{\sqrt{b} \ln\left(\frac{(\tanh(x)-1)b+b}{\sqrt{b}} + \sqrt{(\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a + b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*tanh(x)^2)^(1/2),x)

[Out] $-1/2*((\tanh(x)-1)^2*b+2*(\tanh(x)-1)*b+a+b)^(1/2)-1/2*b^(1/2)*\ln(((\tanh(x)-1)*b+b)/b^(1/2)+((\tanh(x)-1)^2*b+2*(\tanh(x)-1)*b+a+b)^(1/2))+1/2*(a+b)^(1/2))*\ln((2*a+2*b+2*(\tanh(x)-1)*b+2*(a+b)^(1/2))*((\tanh(x)-1)^2*b+2*(\tanh(x)-1)*b+a+b)^(1/2))/(\tanh(x)-1))+1/2*((1+\tanh(x))^2*b-2*(1+\tanh(x))*b+a+b)^(1/2)-1/2*b^(1/2)*\ln(((1+\tanh(x))*b-b)/b^(1/2)+((1+\tanh(x))^2*b-2*(1+\tanh(x))*b+a+b)^(1/2))-1/2*(a+b)^(1/2)*\ln((2*a+2*b-2*(1+\tanh(x))*b+2*(a+b)^(1/2))*((1+\tanh(x))^2*b-2*(1+\tanh(x))*b+a+b)^(1/2))/(1+\tanh(x)))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tanh(x)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*tanh(x)^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{b \tanh(x)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tanh(x)^2)^(1/2),x)

[Out] int((a + b*tanh(x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \tanh^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(x)**2)**(1/2),x)

[Out] Integral(sqrt(a + b*tanh(x)**2), x)

3.214 $\int \coth(x) \sqrt{a + b \tanh^2(x)} dx$

Optimal. Leaf size=56

$$\sqrt{a+b} \tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}} \right) - \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a}} \right)$$

[Out] $-\operatorname{arctanh}((a+b*\tanh(x)^2)^{(1/2)/a^{(1/2))}*a^{(1/2)}+\operatorname{arctanh}((a+b*\tanh(x)^2)^{(1/2)/(a+b)^{(1/2))}*(a+b)^{(1/2)}$

Rubi [A] time = 0.11, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3670, 446, 83, 63, 208}

$$\sqrt{a+b} \tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}} \right) - \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a}} \right)$$

Antiderivative was successfully verified.

[In] `Int[Coth[x]*Sqrt[a + b*Tanh[x]^2],x]`

[Out] $-(\operatorname{Sqrt}[a]*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tanh}[x]^2]/\operatorname{Sqrt}[a]]) + \operatorname{Sqrt}[a + b]*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tanh}[x]^2]/\operatorname{Sqrt}[a + b]]$

Rule 63

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 83

`Int[((e_.) + (f_.)*(x_))^(p_.)/((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]`

Rule 208

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 446

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]`

Rule 3670

`Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n`

, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
 \int \coth(x)\sqrt{a+b \tanh^2(x)} dx &= \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x(1-x^2)} dx, x, \tanh(x)\right) \\
 &= \frac{1}{2} \text{Subst}\left(\int \frac{\sqrt{a+bx}}{(1-x)x} dx, x, \tanh^2(x)\right) \\
 &= \frac{1}{2} a \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \tanh^2(x)\right) + \frac{1}{2}(a+b) \text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}} dx, x, \tanh^2(x)\right) \\
 &= \frac{a \text{Subst}\left(\int \frac{1}{\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a+b \tanh^2(x)}\right)}{b} + \frac{(a+b) \text{Subst}\left(\int \frac{1}{1 + \frac{a}{b} - \frac{x^2}{b}} dx, x, \sqrt{a+b \tanh^2(x)}\right)}{b} \\
 &= -\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a}}\right) + \sqrt{a+b} \tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)
 \end{aligned}$$

Mathematica [A] time = 0.03, size = 56, normalized size = 1.00

$$\sqrt{a+b} \tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right) - \sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]*Sqrt[a + b*Tanh[x]^2], x]

[Out] -(Sqrt[a]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]) + Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]

fricas [B] time = 0.60, size = 3467, normalized size = 61.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(a+b*tanh(x)^2)^(1/2), x, algorithm="fricas")

[Out] [1/4*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*s

osh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 1/2*sqrt(a)*log(-((2*a + b)*cosh(x)^4 + 4*(2*a + b)*cosh(x)*sinh(x)^3 + (2*a + b)*sinh(x)^4 + 2*(2*a - b)*cosh(x)^2 + 2*(3*(2*a + b)*cosh(x)^2 + 2*a - b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((2*a + b)*cosh(x)^3 + (2*a - b)*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh(x) + 1)), sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - 1/2*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x))*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) - 1/2*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b))]

giac [B] time = 0.76, size = 255, normalized size = 4.55

$$\frac{2 a \arctan\left(\frac{-\sqrt{a+b} e^{(2 x)}-\sqrt{a e^{(4 x)}+b e^{(4 x)}+2 a e^{(2 x)}-2 b e^{(2 x)}+a+b}-\sqrt{a+b}}{2 \sqrt{-a}}\right)}{\sqrt{-a}}-\frac{1}{2} \sqrt{a+b} \log\left(\left|-\left(\sqrt{a+b} e^{(2 x)}-\sqrt{a e^{(4 x)}+b e^{(4 x)}}\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
[Out] 2*a*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/sqrt(-a))/sqrt(-a) - 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) + 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))
```

maple [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \coth(x)\sqrt{a + b(\tanh^2(x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a+b*tanh(x)^2)^(1/2),x)
[Out] int(coth(x)*(a+b*tanh(x)^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tanh(x)^2 + a} \coth(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*tanh(x)^2 + a)*coth(x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \coth(x) \sqrt{b \tanh(x)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)*(a + b*tanh(x)^2)^(1/2),x)

[Out] int(coth(x)*(a + b*tanh(x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \tanh^2(x)} \coth(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(a+b*tanh(x)**2)**(1/2),x)

[Out] Integral(sqrt(a + b*tanh(x)**2)*coth(x), x)

3.215 $\int \coth^2(x) \sqrt{a + b \tanh^2(x)} dx$

Optimal. Leaf size=48

$$\sqrt{a+b} \tanh^{-1} \left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right) - \coth(x) \sqrt{a+b \tanh^2(x)}$$

[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))*(a+b)^(1/2)-coth(x)*(a+b*tanh(x)^2)^(1/2)

Rubi [A] time = 0.09, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3670, 475, 12, 377, 206}

$$\sqrt{a+b} \tanh^{-1} \left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right) - \coth(x) \sqrt{a+b \tanh^2(x)}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^2*Sqrt[a + b*Tanh[x]^2], x]

[Out] Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - Coth[x]*Sqrt[a + b*Tanh[x]^2]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 475

Int[((e_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m+1)*(a + b*x^n)^(p+1)*(c + d*x^n)^q)/(a*e*(m+1)), x] - Dist[1/(a*e^n*(m+1)), Int[(e*x)^(m+n)*(a + b*x^n)^p*(c + d*x^n)^(q-1)*Simp[c*b*(m+1) + n*(b*c*(p+1) + a*d*q) + d*(b*(m+1) + b*n*(p+q+1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n}

, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
 \int \coth^2(x) \sqrt{a + b \tanh^2(x)} dx &= \text{Subst} \left(\int \frac{\sqrt{a + bx^2}}{x^2(1-x^2)} dx, x, \tanh(x) \right) \\
 &= -\coth(x) \sqrt{a + b \tanh^2(x)} + \text{Subst} \left(\int \frac{a+b}{(1-x^2)\sqrt{a+bx^2}} dx, x, \tanh(x) \right) \\
 &= -\coth(x) \sqrt{a + b \tanh^2(x)} + (a+b) \text{Subst} \left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}} dx, x, \tanh(x) \right) \\
 &= -\coth(x) \sqrt{a + b \tanh^2(x)} + (a+b) \text{Subst} \left(\int \frac{1}{1-(a+b)x^2} dx, x, \frac{\tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right) \\
 &= \sqrt{a+b} \tanh^{-1} \left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right) - \coth(x) \sqrt{a + b \tanh^2(x)}
 \end{aligned}$$

Mathematica [C] time = 0.13, size = 42, normalized size = 0.88

$$-\coth(x) \sqrt{a + b \tanh^2(x)} {}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{(a+b) \tanh^2(x)}{b \tanh^2(x) + a} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]^2*Sqrt[a + b*Tanh[x]^2], x]

[Out] -(Coth[x]*Hypergeometric2F1[-1/2, 1, 1/2, ((a + b)*Tanh[x]^2)/(a + b*Tanh[x]^2)]*Sqrt[a + b*Tanh[x]^2])

fricas [B] time = 0.51, size = 1539, normalized size = 32.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2*(a+b*tanh(x)^2)^(1/2), x, algorithm="fricas")

[Out] [1/4*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt((a + b)*

$$\begin{aligned} & \cosh(x)^2 + (a + b)\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2) + 4*(2*(a*b^2 + b^3)*\cosh(x)^7 - 3*(a*b^2 + 2*b^3)*\cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*\cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*\cosh(x))*\sinh(x))/(\cosh(x)^6 + 6*\cosh(x)^5*\sinh(x) + 15*\cosh(x)^4*\sinh(x)^2 + 20*\cosh(x)^3*\sinh(x)^3 + 15*\cosh(x)^2*\sinh(x)^4 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6) \\ &) + (\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1)*\sqrt{a + b}*\log(((a + b)*\cosh(x)^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a + b)*\sinh(x)^4 + 2*a*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 + a)*\sinh(x)^2 + \sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1)*\sqrt{a + b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)}) + 4*((a + b)*\cosh(x)^3 + a*\cosh(x))*\sinh(x) + a + b)/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) - 4*\sqrt{2}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1), -1/2*((\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1)*\sqrt{-a - b}*\arctan(\sqrt{2}*(b*\cosh(x)^2 + 2*b*\cosh(x)*\sinh(x) + b*\sinh(x)^2 - a - b)*\sqrt{-a - b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)})/((a*b + b^2)*\cosh(x)^4 + 4*(a*b + b^2)*\cosh(x)*\sinh(x)^3 + (a*b + b^2)*\sinh(x)^4 + (a^2 - a*b - 2*b^2)*\cosh(x)^2 + (6*(a*b + b^2)*\cosh(x)^2 + a^2 - a*b - 2*b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*\cosh(x)^3 + (a^2 - a*b - 2*b^2)*\cosh(x))*\sinh(x))) + (\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1)*\sqrt{-a - b}*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1)*\sqrt{-a - b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)})/((a + b)*\cosh(x)^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a + b)*\sinh(x)^4 + 2*(a - b)*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 + a - b)*\sinh(x)^2 + 4*((a + b)*\cosh(x)^3 + (a - b)*\cosh(x))*\sinh(x) + a + b)) + 2*\sqrt{2}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1)] \end{aligned}$$

giac [B] time = 1.07, size = 348, normalized size = 7.25

$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{2x}-\sqrt{ae^{4x}+be^{4x}+2ae^{2x}-2be^{2x}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{2x}+\sqrt{ae^{4x}+be^{4x}+2ae^{2x}-2be^{2x}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")

[Out] -1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) - 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))) + 4*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*a + sqrt(a + b)*a)/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a + b)

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int (\coth^2(x)) \sqrt{a + b(\tanh^2(x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x)

[Out] int(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tanh(x)^2 + a} \coth(x)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*tanh(x)^2 + a)*coth(x)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \coth(x)^2 \sqrt{b \tanh(x)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^2*(a + b*tanh(x)^2)^(1/2),x)

[Out] int(coth(x)^2*(a + b*tanh(x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \tanh^2(x)} \coth^2(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**2*(a+b*tanh(x)**2)**(1/2),x)

[Out] Integral(sqrt(a + b*tanh(x)**2)*coth(x)**2, x)

3.216 $\int \coth^3(x) \sqrt{a + b \tanh^2(x)} dx$

Optimal. Leaf size=83

$$-\frac{(2a+b) \tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a}}\right)}{2\sqrt{a}} + \sqrt{a+b} \tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{1}{2} \coth^2(x) \sqrt{a+b \tanh^2(x)}$$

[Out] $-1/2*(2*a+b)*\operatorname{arctanh}((a+b*\tanh(x)^2)^{(1/2)}/a^{(1/2)})/a^{(1/2)}+\operatorname{arctanh}((a+b*\tanh(x)^2)^{(1/2)}/(a+b)^{(1/2))}*(a+b)^{(1/2)}-1/2*\coth(x)^2*(a+b*\tanh(x)^2)^{(1/2)}$

Rubi [A] time = 0.16, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3670, 446, 99, 156, 63, 208}

$$-\frac{(2a+b) \tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a}}\right)}{2\sqrt{a}} + \sqrt{a+b} \tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{1}{2} \coth^2(x) \sqrt{a+b \tanh^2(x)}$$

Antiderivative was successfully verified.

[In] `Int[Coth[x]^3*Sqrt[a + b*Tanh[x]^2], x]`

[Out] $-\left(\frac{(2a+b)\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a}}\right]}{2\sqrt{a}} + \sqrt{a+b}\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right] - \frac{\coth^2(x)\sqrt{a+b \tanh^2(x)}}{2}\right)$

Rule 63

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 99

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n + p + 2)*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])`

Rule 156

`Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]`

Rule 208

`Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*((c_)*tan[(e_) +
(f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x],
x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \coth^3(x) \sqrt{a + b \tanh^2(x)} dx &= \text{Subst} \left(\int \frac{\sqrt{a + bx^2}}{x^3(1-x^2)} dx, x, \tanh(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{a + bx}}{(1-x)x^2} dx, x, \tanh^2(x) \right) \\
&= -\frac{1}{2} \coth^2(x) \sqrt{a + b \tanh^2(x)} + \frac{1}{2} \text{Subst} \left(\int \frac{\frac{1}{2}(2a + b) + \frac{bx}{2}}{(1-x)x\sqrt{a + bx}} dx, x, \tanh^2(x) \right) \\
&= -\frac{1}{2} \coth^2(x) \sqrt{a + b \tanh^2(x)} + \frac{1}{2}(a + b) \text{Subst} \left(\int \frac{1}{(1-x)\sqrt{a + bx}} dx, x, \tanh^2(x) \right) \\
&= -\frac{1}{2} \coth^2(x) \sqrt{a + b \tanh^2(x)} + \frac{(a + b) \text{Subst} \left(\int \frac{1}{1 + \frac{a}{b} - \frac{x^2}{b}} dx, x, \sqrt{a + b \tanh^2(x)} \right)}{b} \\
&= -\frac{(2a + b) \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a}} \right)}{2\sqrt{a}} + \sqrt{a + b} \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}} \right) - \frac{1}{2} \coth^2(x) \sqrt{a + b \tanh^2(x)}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 83, normalized size = 1.00

$$-\frac{(2a + b) \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a}} \right)}{2\sqrt{a}} + \sqrt{a + b} \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}} \right) - \frac{1}{2} \coth^2(x) \sqrt{a + b \tanh^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3*Sqrt[a + b*Tanh[x]^2], x]
```

```
[Out] -1/2*((2*a + b)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]/Sqrt[a] + Sqrt[a +
b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Coth[x]^2*Sqrt[a + b*Tanh[
x]^2])/2
```

fricas [B] time = 0.79, size = 4891, normalized size = 58.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/4*((a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 - 2*a*\cosh(x)^2 + \\ & 2*(3*a*\cosh(x)^2 - a)*\sinh(x)^2 + 4*(a*\cosh(x)^3 - a*\cosh(x))*\sinh(x) + a)* \\ & \sqrt{a + b}*\log(((a^3 + a^2*b)*\cosh(x)^8 + 8*(a^3 + a^2*b)*\cosh(x)*\sinh(x)^7 + \\ & (a^3 + a^2*b)*\sinh(x)^8 + 2*(2*a^3 + a^2*b)*\cosh(x)^6 + 2*(2*a^3 + a^2*b \\ & b + 14*(a^3 + a^2*b)*\cosh(x)^2)*\sinh(x)^6 + 4*(14*(a^3 + a^2*b)*\cosh(x)^3 + \\ & 3*(2*a^3 + a^2*b)*\cosh(x))*\sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh \\ & h(x)^4 + (70*(a^3 + a^2*b)*\cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(\\ & 2*a^3 + a^2*b)*\cosh(x)^2)*\sinh(x)^4 + 4*(14*(a^3 + a^2*b)*\cosh(x)^5 + 10*(2 \\ & *a^3 + a^2*b)*\cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x))*\sinh(x)^3 \\ & + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*\cosh(x)^2 + 2 \\ & *(14*(a^3 + a^2*b)*\cosh(x)^6 + 15*(2*a^3 + a^2*b)*\cosh(x)^4 + 2*a^3 + 3*a^2 \\ & *b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x)^2)*\sinh(x)^2 + \sqrt{2} \\ & *(a^2*\cosh(x)^6 + 6*a^2*\cosh(x)*\sinh(x)^5 + a^2*\sinh(x)^6 + 3*a^2*\cosh(x)^4 \\ & + 3*(5*a^2*\cosh(x)^2 + a^2)*\sinh(x)^4 + 4*(5*a^2*\cosh(x)^3 + 3*a^2*\cosh(x) \\ &)*\sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*\cosh(x)^2 + (15*a^2*\cosh(x)^4 + 18*a^2* \\ & \cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*c \\ & \cosh(x)^5 + 6*a^2*\cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*\cosh(x))*\sinh(x))*\sqrt{a \\ & + b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*c \\ & \cosh(x)*\sinh(x) + \sinh(x)^2)} + 4*(2*(a^3 + a^2*b)*\cosh(x)^7 + 3*(2*a^3 + a^ \\ & 2*b)*\cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x)^3 + (2*a^3 + 3*a^2 \\ & *b - b^3)*\cosh(x))*\sinh(x))/(\cosh(x)^6 + 6*\cosh(x)^5*\sinh(x) + 15*\cosh(x)^4 \\ & *\sinh(x)^2 + 20*\cosh(x)^3*\sinh(x)^3 + 15*\cosh(x)^2*\sinh(x)^4 + 6*\cosh(x)*\sinh \\ & (x)^5 + \sinh(x)^6)) + ((2*a + b)*\cosh(x)^4 + 4*(2*a + b)*\cosh(x)*\sinh(x)^3 \\ & + (2*a + b)*\sinh(x)^4 - 2*(2*a + b)*\cosh(x)^2 + 2*(3*(2*a + b)*\cosh(x)^2 \\ & - 2*a - b)*\sinh(x)^2 + 4*((2*a + b)*\cosh(x)^3 - (2*a + b)*\cosh(x))*\sinh(x) \\ & + 2*a + b)*\sqrt{a}*\log(-((2*a + b)*\cosh(x)^4 + 4*(2*a + b)*\cosh(x)*\sinh(x)^3 \\ & + (2*a + b)*\sinh(x)^4 + 2*(2*a - b)*\cosh(x)^2 + 2*(3*(2*a + b)*\cosh(x)^2 \\ & + 2*a - b)*\sinh(x)^2 - 2*\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 \\ & + 1)*\sqrt{a}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x) \\ & ^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)} + 4*((2*a + b)*\cosh(x)^3 + (2*a - b)*c \\ & \cosh(x))*\sinh(x) + 2*a + b)/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2 \\ & *(3*\cosh(x)^2 - 1)*\sinh(x)^2 - 2*\cosh(x)^2 + 4*(\cosh(x)^3 - \cosh(x))*\sinh(x) \\ &) + 1)) + (a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x)^3 + a*\sinh(x)^4 - 2*a*\cosh(x)^2 \\ & + 2*(3*a*\cosh(x)^2 - a)*\sinh(x)^2 + 4*(a*\cosh(x)^3 - a*\cosh(x))*\sinh(x) + \\ & a)*\sqrt{a + b}*\log(-((a + b)*\cosh(x)^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a \\ & + b)*\sinh(x)^4 - 2*b*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 - b)*\sinh(x)^2 + sq \\ & rt(2)*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1)*\sqrt{a + b}*\sqrt{((a \\ & + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) \\ & + \sinh(x)^2)} + 4*((a + b)*\cosh(x)^3 - b*\cosh(x))*\sinh(x) + a + b)/(\cosh(x) \\ & ^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) - 2*\sqrt{2}*(a*\cosh(x)^2 + 2*a*\cosh(x) \\ & *\sinh(x) + a*\sinh(x)^2 + a)*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a \\ & - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(a*\cosh(x)^4 + 4*a*\cosh \\ & (x)*\sinh(x)^3 + a*\sinh(x)^4 - 2*a*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 - a)*\sinh(x) \\ & ^2 + 4*(a*\cosh(x)^3 - a*\cosh(x))*\sinh(x) + a), 1/4*(2*((2*a + b)*\cosh(x)^4 \\ & + 4*(2*a + b)*\cosh(x)*\sinh(x)^3 + (2*a + b)*\sinh(x)^4 - 2*(2*a + b)*\cosh(x) \\ & ^2 + 2*(3*(2*a + b)*\cosh(x)^2 - 2*a - b)*\sinh(x)^2 + 4*((2*a + b)*\cosh(x)^3 \\ & - (2*a + b)*\cosh(x))*\sinh(x) + 2*a + b)*\sqrt{-a}*\arctan(\sqrt{2}*(\cosh(x)^2 \\ & + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1)*\sqrt{-a}*\sqrt{((a + b)*\cosh(x)^2 + (a \\ & + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/((a + \\ & b)*\cosh(x)^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a + b)*\sinh(x)^4 + 2*(a - b) \\ & *\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 + a - b)*\sinh(x)^2 + 4*((a + b)*\cosh(x) \\ & ^3 + (a - b)*\cosh(x))*\sinh(x) + a + b)) + (a*\cosh(x)^4 + 4*a*\cosh(x)*\sinh(x) \\ &)^3 + a*\sinh(x)^4 - 2*a*\cosh(x)^2 + 2*(3*a*\cosh(x)^2 - a)*\sinh(x)^2 + 4*(a* \\ & \cosh(x)^3 - a*\cosh(x))*\sinh(x) + a)*\sqrt{a + b}*\log(((a^3 + a^2*b)*\cosh(x)^8 \\ & + 8*(a^3 + a^2*b)*\cosh(x)*\sinh(x)^7 + (a^3 + a^2*b)*\sinh(x)^8 + 2*(2*a^3 \\ & + a^2*b)*\cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*\cosh(x)^2)*\sinh(x) \\ & ^6 + 4*(14*(a^3 + a^2*b)*\cosh(x)^3 + 3*(2*a^3 + a^2*b)*\cosh(x))*\sinh(x)^5 + \\ & (6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x)^4 + (70*(a^3 + a^2*b)*\cosh(x)^4 + \end{aligned}$$

h(x))*sinh(x) + 2*a + b)*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - (a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh(x)^3 - a*cosh(x))*sinh(x) + a)*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) - (a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh(x)^3 - a*cosh(x))*sinh(x) + a)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 - 2*a*cosh(x)^2 + 2*(3*a*cosh(x)^2 - a)*sinh(x)^2 + 4*(a*cosh(x)^3 - a*cosh(x))*sinh(x) + a)]

giac [B] time = 1.76, size = 557, normalized size = 6.71

$$\frac{(2a + b) \arctan\left(-\frac{\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{1}{2} \sqrt{a+b} \log\left(\left|-\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")

[Out] (2*a + b)*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/sqrt(-a))/sqrt(-a) - 1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) + 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))) + 2*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3*(2*a + b) + (sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*(2*a - 3*b)*sqrt(a + b) - (2*a^2 + 3*a*b - 3*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) - (2*a^2 - a*b + b^2)*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a + b)^2

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int (\coth^3(x)) \sqrt{a + b (\tanh^2(x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x)

[Out] `int(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tanh(x)^2 + a} \coth(x)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*tanh(x)^2 + a)*coth(x)^3, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(x)^3 \sqrt{b \tanh(x)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(x)^3*(a + b*tanh(x)^2)^(1/2),x)`

[Out] `int(coth(x)^3*(a + b*tanh(x)^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \tanh^2(x)} \coth^3(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)**3*(a+b*tanh(x)**2)**(1/2),x)`

[Out] `Integral(sqrt(a + b*tanh(x)**2)*coth(x)**3, x)`

3.217 $\int \coth^4(x) \sqrt{a + b \tanh^2(x)} dx$

Optimal. Leaf size=78

$$\sqrt{a+b} \tanh^{-1} \left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right) - \frac{1}{3} \coth^3(x) \sqrt{a+b \tanh^2(x)} - \frac{(3a+b) \coth(x) \sqrt{a+b \tanh^2(x)}}{3a}$$

[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))*(a+b)^(1/2)-1/3*(3*a+b)*coth(x)*(a+b*tanh(x)^2)^(1/2)/a-1/3*coth(x)^3*(a+b*tanh(x)^2)^(1/2)

Rubi [A] time = 0.15, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3670, 475, 583, 12, 377, 206}

$$\sqrt{a+b} \tanh^{-1} \left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right) - \frac{1}{3} \coth^3(x) \sqrt{a+b \tanh^2(x)} - \frac{(3a+b) \coth(x) \sqrt{a+b \tanh^2(x)}}{3a}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^4*Sqrt[a + b*Tanh[x]^2], x]

[Out] Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - ((3*a + b)*Coth[x]*Sqrt[a + b*Tanh[x]^2])/(3*a) - (Coth[x]^3*Sqrt[a + b*Tanh[x]^2])/3

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 475

Int[((e_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m+1)*(a + b*x^n)^(p+1)*(c + d*x^n)^q)/(a*e*(m+1)), x] - Dist[1/(a*e^n*(m+1)), Int[(e*x)^(m+n)*(a + b*x^n)^p*(c + d*x^n)^(q-1)*Simp[c*b*(m+1) + n*(b*c*(p+1) + a*d*q) + d*(b*(m+1) + b*n*(p+q+1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 583

Int[((g_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m+1)*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*c*g*(m+1)), x] + Dist[1/(a*c*g^n*(

```
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
+ 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*((c_)*tan[(e_) +
(f_)*(x_)])^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\int \coth^4(x)\sqrt{a + b \tanh^2(x)} dx = \text{Subst}\left(\int \frac{\sqrt{a + bx^2}}{x^4(1 - x^2)} dx, x, \tanh(x)\right)$$

$$= -\frac{1}{3} \coth^3(x)\sqrt{a + b \tanh^2(x)} + \frac{1}{3} \text{Subst}\left(\int \frac{3a + b + 2bx^2}{x^2(1 - x^2)\sqrt{a + bx^2}} dx, x, \tanh(x)\right)$$

$$= -\frac{(3a + b) \coth(x)\sqrt{a + b \tanh^2(x)}}{3a} - \frac{1}{3} \coth^3(x)\sqrt{a + b \tanh^2(x)} - \frac{\text{Subst}\left(\int \frac{1}{x^2(1 - x^2)\sqrt{a + bx^2}} dx, x, \tanh(x)\right)}{3}$$

$$= -\frac{(3a + b) \coth(x)\sqrt{a + b \tanh^2(x)}}{3a} - \frac{1}{3} \coth^3(x)\sqrt{a + b \tanh^2(x)} - (-a - b) \text{Subst}\left(\int \frac{1}{x^2(1 - x^2)\sqrt{a + bx^2}} dx, x, \tanh(x)\right)$$

$$= -\frac{(3a + b) \coth(x)\sqrt{a + b \tanh^2(x)}}{3a} - \frac{1}{3} \coth^3(x)\sqrt{a + b \tanh^2(x)} - (-a - b) \text{Subst}\left(\int \frac{1}{x^2(1 - x^2)\sqrt{a + bx^2}} dx, x, \tanh(x)\right)$$

$$= \sqrt{a + b} \tanh^{-1}\left(\frac{\sqrt{a + b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}}\right) - \frac{(3a + b) \coth(x)\sqrt{a + b \tanh^2(x)}}{3a} - \frac{1}{3} \coth^3(x)\sqrt{a + b \tanh^2(x)}$$

Mathematica [C] time = 6.22, size = 161, normalized size = 2.06

$$\frac{\cosh^4(x) \coth^3(x) \left(\frac{b \tanh^2(x)}{a} + 1\right) \left(-\frac{4(a+b)(a \tanh(x) + b \tanh^3(x))^2 {}_2F_1\left(2, 2; \frac{3}{2}; -\frac{(a+b)\sinh^2(x)}{a}\right)}{a^2} - \frac{\text{sech}^4(x)(a - 2b \tanh^2(x)) \left(\sqrt{-\frac{(a+b)\sinh^2(x)}{a}}\right)}{\sqrt{b \sinh^2(x)}} \right)}{3\sqrt{a + b \tanh^2(x)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Coth[x]^4*Sqrt[a + b*Tanh[x]^2], x]
[Out] (Cosh[x]^4*Coth[x]^3*(1 + (b*Tanh[x]^2)/a)*(-((Sech[x]^4*(ArcSin[Sqrt[-((a
+ b)*Sinh[x]^2)/a]])*Sqrt[-((a + b)*Sinh[x]^2)/a]] + Sqrt[Cosh[x]^2 + (b*
Sinh[x]^2)/a])*(a - 2*b*Tanh[x]^2))/Sqrt[Cosh[x]^2 + (b*Sinh[x]^2)/a]) - (4
*(a + b)*Hypergeometric2F1[2, 2, 3/2, -((a + b)*Sinh[x]^2)/a])*(a*Tanh[x]
+ b*Tanh[x]^3)^2/a^2)/(3*Sqrt[a + b*Tanh[x]^2])
```

fricas [B] time = 0.60, size = 2355, normalized size = 30.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & \frac{1}{12} (3(a \cosh(x)^6 + 6a \cosh(x) \sinh(x)^5 + a \sinh(x)^6 - 3a \cosh(x)^4 \\ & + 3(5a \cosh(x)^2 - a) \sinh(x)^4 + 4(5a \cosh(x)^3 - 3a \cosh(x)) \sinh(x)^3 + 3a \cosh(x)^2 + 3(5a \cosh(x)^4 - 6a \cosh(x)^2 + a) \sinh(x)^2 + 6(a \cosh(x)^5 - 2a \cosh(x)^3 + a \cosh(x)) \sinh(x) - a) \sqrt{a+b} \log(-((a b^2 + b^3) \cosh(x)^8 + 8(a b^2 + b^3) \cosh(x) \sinh(x)^7 + (a b^2 + b^3) \sinh(x)^8 - 2(a b^2 + 2b^3) \cosh(x)^6 - 2(a b^2 + 2b^3 - 14(a b^2 + b^3) \cosh(x)^2) \sinh(x)^6 + 4(14(a b^2 + b^3) \cosh(x)^3 - 3(a b^2 + 2b^3) \cosh(x)) \sinh(x)^5 + (a^3 - a^2 b + 4a b^2 + 6b^3) \cosh(x)^4 + (70(a b^2 + b^3) \cosh(x)^4 + a^3 - a^2 b + 4a b^2 + 6b^3 - 30(a b^2 + 2b^3) \cosh(x)^2) \sinh(x)^4 + 4(14(a b^2 + b^3) \cosh(x)^5 - 10(a b^2 + 2b^3) \cosh(x)^3 + (a^3 - a^2 b + 4a b^2 + 6b^3) \cosh(x)) \sinh(x)^3 + a^3 + 3a^2 b + 3a b^2 + b^3 + 2(a^3 - 3a b^2 - 2b^3) \cosh(x)^2 + 2(14(a b^2 + b^3) \cosh(x)^6 - 15(a b^2 + 2b^3) \cosh(x)^4 + a^3 - 3a b^2 - 2b^3 + 3(a^3 - a^2 b + 4a b^2 + 6b^3) \cosh(x)^2) \sinh(x)^2 + \sqrt{2} (b^2 \cosh(x)^6 + 6b^2 \cosh(x) \sinh(x)^5 + b^2 \sinh(x)^6 - 3b^2 \cosh(x)^4 + 3(5b^2 \cosh(x)^2 - b^2) \sinh(x)^4 + 4(5b^2 \cosh(x)^3 - 3b^2 \cosh(x)) \sinh(x)^3 - (a^2 - 2a b - 3b^2) \cosh(x)^2 + (15b^2 \cosh(x)^4 - 18b^2 \cosh(x)^2 - a^2 + 2a b + 3b^2) \sinh(x)^2 - a^2 - 2a b - b^2 + 2(3b^2 \cosh(x)^5 - 6b^2 \cosh(x)^3 - (a^2 - 2a b - 3b^2) \cosh(x)) \sinh(x)) \sqrt{a+b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a-b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} + 4(2(a b^2 + b^3) \cosh(x)^7 - 3(a b^2 + 2b^3) \cosh(x)^5 + (a^3 - a^2 b + 4a b^2 + 6b^3) \cosh(x)^3 + (a^3 - 3a b^2 - 2b^3) \cosh(x)) \sinh(x) / (\cosh(x)^6 + 6 \cosh(x)^5 \sinh(x) + 15 \cosh(x)^4 \sinh(x)^2 + 20 \cosh(x)^3 \sinh(x)^3 + 15 \cosh(x)^2 \sinh(x)^4 + 6 \cosh(x) \sinh(x)^5 + \sinh(x)^6) + 3(a \cosh(x)^6 + 6a \cosh(x) \sinh(x)^5 + a \sinh(x)^6 - 3a \cosh(x)^4 + 3(5a \cosh(x)^2 - a) \sinh(x)^4 + 4(5a \cosh(x)^3 - 3a \cosh(x)) \sinh(x)^3 + 3a \cosh(x)^2 + 3(5a \cosh(x)^4 - 6a \cosh(x)^2 + a) \sinh(x)^2 + 6(a \cosh(x)^5 - 2a \cosh(x)^3 + a \cosh(x)) \sinh(x) - a) \sqrt{a+b} \log(((a+b) \cosh(x)^4 + 4(a+b) \cosh(x) \sinh(x)^3 + (a+b) \sinh(x)^4 + 2a \cosh(x)^2 + 2(3(a+b) \cosh(x)^2 + a) \sinh(x)^2 + \sqrt{2} (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1) \sqrt{a+b}) \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a-b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} + 4((a+b) \cosh(x)^3 + a \cosh(x)) \sinh(x) + a+b) / (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2)) - 4 \sqrt{2} ((4a+b) \cosh(x)^4 + 4(4a+b) \cosh(x) \sinh(x)^3 + (4a+b) \sinh(x)^4 - 2(2a+b) \cosh(x)^2 + 2(3(4a+b) \cosh(x)^2 - 2a-b) \sinh(x)^2 + 4((4a+b) \cosh(x)^3 - (2a+b) \cosh(x)) \sinh(x) + 4(a+b) \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a-b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))}) / (a \cosh(x)^6 + 6a \cosh(x) \sinh(x)^5 + a \sinh(x)^6 - 3a \cosh(x)^4 + 3(5a \cosh(x)^2 - a) \sinh(x)^4 + 4(5a \cosh(x)^3 - 3a \cosh(x)) \sinh(x)^3 + 3a \cosh(x)^2 + 3(5a \cosh(x)^4 - 6a \cosh(x)^2 + a) \sinh(x)^2 + a) \sinh(x)^2 + 6(a \cosh(x)^5 - 2a \cosh(x)^3 + a \cosh(x)) \sinh(x) - a), -1/6(3(a \cosh(x)^6 + 6a \cosh(x) \sinh(x)^5 + a \sinh(x)^6 - 3a \cosh(x)^4 + 3(5a \cosh(x)^2 - a) \sinh(x)^4 + 4(5a \cosh(x)^3 - 3a \cosh(x)) \sinh(x)^3 + 3a \cosh(x)^2 + 3(5a \cosh(x)^4 - 6a \cosh(x)^2 + a) \sinh(x)^2 + 6(a \cosh(x)^5 - 2a \cosh(x)^3 + a \cosh(x)) \sinh(x) - a) \sqrt{-a-b} \arctan(\sqrt{2} (b \cosh(x)^2 + 2b \cosh(x) \sinh(x) + b \sinh(x)^2 - a - b) \sqrt{-a-b}) \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a-b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} / ((a b + b^2) \cosh(x)^4 + 4(a b + b^2) \cosh(x) \sinh(x)^3 + (a b + b^2) \sinh(x)^4 + (a^2 - a b - 2b^2) \cosh(x)^2 + (6(a b + b^2) \cosh(x)^2 + a^2 - a b - 2b^2) \sinh(x)^2 + a^2 + 2a b + b^2 + 2(2(a b + b^2) \cosh(x)^3 + (a^2 - a b - 2b^2) \cosh(x)) \sinh(x))) + 3(a \cosh(x)^6 + 6a \cosh(x) \sinh(x)^5 + a \sinh(x)^6 - 3a \cosh(x)^4 + 3(5a \cosh(x)^2 - a) \sinh(x)^4 + 4(5a \cosh(x)^3 - 3a \cosh(x)) \sinh(x)^3 + 3a \cosh(x)^2 + 3(5a \cosh(x)^4 - 6a \cosh(x)^2 + a) \sinh(x)^2 + 6(a \cosh(x)^5 - 2a \cosh(x)^3 + a \cosh(x)) \sinh(x) - a) \sqrt{-a-b} \arctan(\sqrt{2} (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1) \sqrt{-a-b}) \sqrt{((a+b) \cosh(x)^2$$

+ (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2))/(
(a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a
- b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cos
h(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 2*sqrt(2)*((4*a + b)*cosh(x)^
4 + 4*(4*a + b)*cosh(x)*sinh(x)^3 + (4*a + b)*sinh(x)^4 - 2*(2*a + b)*cosh(
x)^2 + 2*(3*(4*a + b)*cosh(x)^2 - 2*a - b)*sinh(x)^2 + 4*((4*a + b)*cosh(x)
^3 - (2*a + b)*cosh(x))*sinh(x) + 4*a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b
)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a*cosh(
x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 - 3*a*cosh(x)^4 + 3*(5*a*cosh(x)
^2 - a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)
^2 + 3*(5*a*cosh(x)^4 - 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 - 2*a
*cosh(x)^3 + a*cosh(x))*sinh(x) - a]

giac [B] time = 2.22, size = 629, normalized size = 8.06

$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{2x}-\sqrt{ae^{4x}+be^{4x}+2ae^{2x}-2be^{2x}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)-\frac{1}{2}\sqrt{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")

[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) - 1/
2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*sqrt(a + b)*log(abs
(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
x) + a + b) - sqrt(a + b))) + 4/3(3*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5*(2*a + b) - 3*(sqrt(a
+ b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
b))^4*(2*a + 3*b)*sqrt(a + b) - 2*(10*a^2 + 3*a*b - 3*b^2)*(sqrt(a + b)*e^(
2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3
+ 6*(6*a^2 + 3*a*b + b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b) + 3*(26*a^3 + 9*a^2*b
- 4*a*b^2 - 3*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*
e^(2*x) - 2*b*e^(2*x) + a + b)) + (34*a^3 - 17*a^2*b + 3*b^3)*sqrt(a + b))/
((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
x) + a + b))^2 - 2(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a
*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a + b)^3

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int (\coth^4(x)) \sqrt{a + b (\tanh^2(x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x)

[Out] int(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tanh(x)^2 + a} \coth(x)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*tanh(x)^2 + a)*coth(x)^4, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(x)^4 \sqrt{b \tanh(x)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^4*(a + b*tanh(x)^2)^(1/2), x)

[Out] int(coth(x)^4*(a + b*tanh(x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \tanh^2(x)} \coth^4(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**4*(a+b*tanh(x)**2)**(1/2), x)

[Out] Integral(sqrt(a + b*tanh(x)**2)*coth(x)**4, x)

Rule 156

Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
 ((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
 f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
 Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
 *(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
 b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
 (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
 f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n,
 p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
 alQ[n]))

Rubi steps

$$\begin{aligned}
 \int \coth^5(x) \sqrt{a + b \tanh^2(x)} dx &= \text{Subst} \left(\int \frac{\sqrt{a + bx^2}}{x^5(1-x^2)} dx, x, \tanh(x) \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{a + bx}}{(1-x)x^3} dx, x, \tanh^2(x) \right) \\
 &= -\frac{1}{4} \coth^4(x) \sqrt{a + b \tanh^2(x)} + \frac{1}{4} \text{Subst} \left(\int \frac{\frac{1}{2}(4a + b) + \frac{3bx}{2}}{(1-x)x^2 \sqrt{a + bx}} dx, x, \tanh^2(x) \right) \\
 &= -\frac{(4a + b) \coth^2(x) \sqrt{a + b \tanh^2(x)}}{8a} - \frac{1}{4} \coth^4(x) \sqrt{a + b \tanh^2(x)} - \frac{\text{Subst} \left(\int \frac{1}{(1-x)x^2 \sqrt{a + bx}} dx, x, \tanh^2(x) \right)}{4} \\
 &= -\frac{(4a + b) \coth^2(x) \sqrt{a + b \tanh^2(x)}}{8a} - \frac{1}{4} \coth^4(x) \sqrt{a + b \tanh^2(x)} - \frac{1}{2} (-a + b \tanh^2(x)) \sqrt{a + b \tanh^2(x)} \\
 &= -\frac{(4a + b) \coth^2(x) \sqrt{a + b \tanh^2(x)}}{8a} - \frac{1}{4} \coth^4(x) \sqrt{a + b \tanh^2(x)} + \frac{(a + b) \tanh^2(x) \sqrt{a + b \tanh^2(x)}}{2} \\
 &= -\frac{(8a^2 + 4ab - b^2) \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a}} \right)}{8a^{3/2}} + \sqrt{a + b} \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}} \right)
 \end{aligned}$$

Mathematica [A] time = 0.63, size = 111, normalized size = 0.92

$$\frac{(-8a^2 - 4ab + b^2) \tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a}}\right) + \sqrt{a} \left(8a\sqrt{a+b} \tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right) - \coth^2(x)\sqrt{a+b \tanh^2(x)}\right)}{8a^{3/2}} (2a$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]^5*Sqrt[a + b*Tanh[x]^2], x]

[Out] $((-8a^2 - 4ab + b^2) \operatorname{ArcTanh}[\operatorname{Sqrt}[a + b \operatorname{Tanh}[x]^2] / \operatorname{Sqrt}[a]] + \operatorname{Sqrt}[a] * (8a * \operatorname{Sqrt}[a + b] * \operatorname{ArcTanh}[\operatorname{Sqrt}[a + b \operatorname{Tanh}[x]^2] / \operatorname{Sqrt}[a + b]] - \operatorname{Coth}[x]^2 * (4a + b + 2a * \operatorname{Coth}[x]^2) * \operatorname{Sqrt}[a + b \operatorname{Tanh}[x]^2])) / (8a^{3/2})$

fricas [B] time = 0.96, size = 9642, normalized size = 79.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^5*(a+b*tanh(x)^2)^(1/2), x, algorithm="fricas")

[Out] $[1/16*(4*(a^2*\cosh(x)^8 + 8*a^2*\cosh(x)*\sinh(x)^7 + a^2*\sinh(x)^8 - 4*a^2*\cosh(x)^6 + 4*(7*a^2*\cosh(x)^2 - a^2)*\sinh(x)^6 + 6*a^2*\cosh(x)^4 + 8*(7*a^2*\cosh(x)^3 - 3*a^2*\cosh(x))*\sinh(x)^5 + 2*(35*a^2*\cosh(x)^4 - 30*a^2*\cosh(x)^2 + 3*a^2)*\sinh(x)^4 - 4*a^2*\cosh(x)^2 + 8*(7*a^2*\cosh(x)^5 - 10*a^2*\cosh(x)^3 + 3*a^2*\cosh(x))*\sinh(x)^3 + 4*(7*a^2*\cosh(x)^6 - 15*a^2*\cosh(x)^4 + 9*a^2*\cosh(x)^2 - a^2)*\sinh(x)^2 + a^2 + 8*(a^2*\cosh(x)^7 - 3*a^2*\cosh(x)^5 + 3*a^2*\cosh(x)^3 - a^2*\cosh(x))*\sinh(x))*\operatorname{sqrt}(a + b)*\log(((a^3 + a^2*b)*\cosh(x)^8 + 8*(a^3 + a^2*b)*\cosh(x)*\sinh(x)^7 + (a^3 + a^2*b)*\sinh(x)^8 + 2*(2*a^3 + a^2*b)*\cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*\cosh(x)^2)*\sinh(x)^6 + 4*(14*(a^3 + a^2*b)*\cosh(x)^3 + 3*(2*a^3 + a^2*b)*\cosh(x))*\sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x)^4 + (70*(a^3 + a^2*b)*\cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*\cosh(x)^2)*\sinh(x)^4 + 4*(14*(a^3 + a^2*b)*\cosh(x)^5 + 10*(2*a^3 + a^2*b)*\cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x))*\sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*\cosh(x)^2 + 2*(14*(a^3 + a^2*b)*\cosh(x)^6 + 15*(2*a^3 + a^2*b)*\cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x)^2)*\sinh(x)^2 + \operatorname{sqrt}(2)*(a^2*\cosh(x)^6 + 6*a^2*\cosh(x)*\sinh(x)^5 + a^2*\sinh(x)^6 + 3*a^2*\cosh(x)^4 + 3*(5*a^2*\cosh(x)^2 + a^2)*\sinh(x)^4 + 4*(5*a^2*\cosh(x)^3 + 3*a^2*\cosh(x))*\sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*\cosh(x)^2 + (15*a^2*\cosh(x)^4 + 18*a^2*\cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*\cosh(x)^5 + 6*a^2*\cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*\cosh(x))*\sinh(x))*\operatorname{sqrt}(a + b)*\operatorname{sqrt}(((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b) / (\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*\cosh(x)^7 + 3*(2*a^3 + a^2*b)*\cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*\cosh(x))*\sinh(x)) / (\cosh(x)^6 + 6*\cosh(x)^5*\sinh(x) + 15*\cosh(x)^4*\sinh(x)^2 + 20*\cosh(x)^3*\sinh(x)^3 + 15*\cosh(x)^2*\sinh(x)^4 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6)) - ((8*a^2 + 4*a*b - b^2)*\cosh(x)^8 + 8*(8*a^2 + 4*a*b - b^2)*\cosh(x)*\sinh(x)^7 + (8*a^2 + 4*a*b - b^2)*\sinh(x)^8 - 4*(8*a^2 + 4*a*b - b^2)*\cosh(x)^6 + 4*(7*(8*a^2 + 4*a*b - b^2)*\cosh(x)^2 - 8*a^2 - 4*a*b + b^2)*\sinh(x)^6 + 8*(7*(8*a^2 + 4*a*b - b^2)*\cosh(x)^3 - 3*(8*a^2 + 4*a*b - b^2)*\cosh(x))*\sinh(x)^5 + 6*(8*a^2 + 4*a*b - b^2)*\cosh(x)^4 + 2*(35*(8*a^2 + 4*a*b - b^2)*\cosh(x)^4 - 30*(8*a^2 + 4*a*b - b^2)*\cosh(x)^2 + 24*a^2 + 12*a*b - 3*b^2)*\sinh(x)^4 + 8*(7*(8*a^2 + 4*a*b - b^2)*\cosh(x)^5 - 10*(8*a^2 + 4*a*b - b^2)*\cosh(x)^3 + 3*(8*a^2 + 4*a*b - b^2)*\cosh(x))*\sinh(x)^3 - 4*(8*a^2 + 4*a*b - b^2)*\cosh(x)^2 + 4*(7*(8*a^2 + 4*a*b - b^2)*\cosh(x)^6 - 15*(8*a^2 + 4*a*b - b^2)*\cosh(x)^4 + 9*(8*a^2 + 4*a*b - b^2)*\cosh(x)^2 - 8*a^2 - 4*a*b + b^2)*\sinh(x)^2 + 8*a^2 + 4*a*b - b^2 + 8*((8*a^2 + 4*a*b - b^2)*\cosh(x)^7 - 3*(8*a^2 + 4*a*b - b^2)*\cosh(x)^5 + 3*(8*a^2 + 4*a*b - b^2)*\cosh(x)^3 - (8*a^2 + 4*a*b - b^2)$

$$\begin{aligned}
&) * \cosh(x) * \sinh(x) * \sqrt{a} * \log(-((2*a + b) * \cosh(x)^4 + 4*(2*a + b) * \cosh(x) \\
& * \sinh(x)^3 + (2*a + b) * \sinh(x)^4 + 2*(2*a - b) * \cosh(x)^2 + 2*(3*(2*a + b) * \cosh(x)^2 + 2*a - b) * \sinh(x)^2 + 2*\sqrt{2} * (\cosh(x)^2 + 2*\cosh(x) * \sinh(x) + \sinh(x)^2 + 1) * \sqrt{a} * \sqrt{((a + b) * \cosh(x)^2 + (a + b) * \sinh(x)^2 + a - b) / ((\cosh(x)^2 - 2*\cosh(x) * \sinh(x) + \sinh(x)^2))} + 4*((2*a + b) * \cosh(x)^3 + (2*a - b) * \cosh(x) * \sinh(x) + 2*a + b) / (\cosh(x)^4 + 4*\cosh(x) * \sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 - 1) * \sinh(x)^2 - 2*\cosh(x)^2 + 4*(\cosh(x)^3 - \cosh(x)) * \sinh(x) + 1)) + 4*(a^2 * \cosh(x)^8 + 8*a^2 * \cosh(x) * \sinh(x)^7 + a^2 * \sinh(x)^8 - 4*a^2 * \cosh(x)^6 + 4*(7*a^2 * \cosh(x)^2 - a^2) * \sinh(x)^6 + 6*a^2 * \cosh(x)^4 + 8*(7*a^2 * \cosh(x)^3 - 3*a^2 * \cosh(x)) * \sinh(x)^5 + 2*(35*a^2 * \cosh(x)^4 - 30*a^2 * \cosh(x)^2 + 3*a^2) * \sinh(x)^4 - 4*a^2 * \cosh(x)^2 + 8*(7*a^2 * \cosh(x)^5 - 10*a^2 * \cosh(x)^3 + 3*a^2 * \cosh(x)) * \sinh(x)^3 + 4*(7*a^2 * \cosh(x)^6 - 15*a^2 * \cosh(x)^4 + 9*a^2 * \cosh(x)^2 - a^2) * \sinh(x)^2 + a^2 + 8*(a^2 * \cosh(x)^7 - 3*a^2 * \cosh(x)^5 + 3*a^2 * \cosh(x)^3 - a^2 * \cosh(x)) * \sinh(x)) * \sqrt{a + b} * \log(-((a + b) * \cosh(x)^4 + 4*(a + b) * \cosh(x) * \sinh(x)^3 + (a + b) * \sinh(x)^4 - 2*b * \cosh(x)^2 + 2*(3*(a + b) * \cosh(x)^2 - b) * \sinh(x)^2 + \sqrt{2} * (\cosh(x)^2 + 2*\cosh(x) * \sinh(x) + \sinh(x)^2 - 1) * \sqrt{a + b} * \sqrt{((a + b) * \cosh(x)^2 + (a + b) * \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2*\cosh(x) * \sinh(x) + \sinh(x)^2))} + 4*((a + b) * \cosh(x)^3 - b * \cosh(x)) * \sinh(x) + a + b) / (\cosh(x)^2 + 2*\cosh(x) * \sinh(x) + \sinh(x)^2)) - 2*\sqrt{2} * ((6*a^2 + a*b) * \cosh(x)^6 + 6*(6*a^2 + a*b) * \cosh(x) * \sinh(x)^5 + (6*a^2 + a*b) * \sinh(x)^6 + (2*a^2 - a*b) * \cosh(x)^4 + (15*(6*a^2 + a*b) * \cosh(x)^2 + 2*a^2 - a*b) * \sinh(x)^4 + 4*(5*(6*a^2 + a*b) * \cosh(x)^3 + (2*a^2 - a*b) * \cosh(x)) * \sinh(x)^3 + (2*a^2 - a*b) * \cosh(x)^2 + (15*(6*a^2 + a*b) * \cosh(x)^4 + 6*(2*a^2 - a*b) * \cosh(x)^2 + 2*a^2 - a*b) * \sinh(x)^2 + 6*a^2 + a*b + 2*(3*(6*a^2 + a*b) * \cosh(x)^5 + 2*(2*a^2 - a*b) * \cosh(x)^3 + (2*a^2 - a*b) * \cosh(x)) * \sinh(x)) * \sqrt{((a + b) * \cosh(x)^2 + (a + b) * \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2*\cosh(x) * \sinh(x) + \sinh(x)^2))} / (a^2 * \cosh(x)^8 + 8*a^2 * \cosh(x) * \sinh(x)^7 + a^2 * \sinh(x)^8 - 4*a^2 * \cosh(x)^6 + 4*(7*a^2 * \cosh(x)^2 - a^2) * \sinh(x)^6 + 6*a^2 * \cosh(x)^4 + 8*(7*a^2 * \cosh(x)^3 - 3*a^2 * \cosh(x)) * \sinh(x)^5 + 2*(35*a^2 * \cosh(x)^4 - 30*a^2 * \cosh(x)^2 + 3*a^2) * \sinh(x)^4 - 4*a^2 * \cosh(x)^2 + 8*(7*a^2 * \cosh(x)^5 - 10*a^2 * \cosh(x)^3 + 3*a^2 * \cosh(x)) * \sinh(x)^3 + 4*(7*a^2 * \cosh(x)^6 - 15*a^2 * \cosh(x)^4 + 9*a^2 * \cosh(x)^2 - a^2) * \sinh(x)^2 + a^2 + 8*(a^2 * \cosh(x)^7 - 3*a^2 * \cosh(x)^5 + 3*a^2 * \cosh(x)^3 - a^2 * \cosh(x)) * \sinh(x)), 1/8 * (((8*a^2 + 4*a*b - b^2) * \cosh(x)^8 + 8*(8*a^2 + 4*a*b - b^2) * \cosh(x) * \sinh(x)^7 + (8*a^2 + 4*a*b - b^2) * \sinh(x)^8 - 4*(8*a^2 + 4*a*b - b^2) * \cosh(x)^6 + 4*(7*(8*a^2 + 4*a*b - b^2) * \cosh(x)^2 - 8*a^2 - 4*a*b + b^2) * \sinh(x)^6 + 8*(7*(8*a^2 + 4*a*b - b^2) * \cosh(x)^3 - 3*(8*a^2 + 4*a*b - b^2) * \cosh(x)) * \sinh(x)^5 + 6*(8*a^2 + 4*a*b - b^2) * \cosh(x)^4 + 2*(35*(8*a^2 + 4*a*b - b^2) * \cosh(x)^4 - 30*(8*a^2 + 4*a*b - b^2) * \cosh(x)^2 + 24*a^2 + 12*a*b - 3*b^2) * \sinh(x)^4 + 8*(7*(8*a^2 + 4*a*b - b^2) * \cosh(x)^5 - 10*(8*a^2 + 4*a*b - b^2) * \cosh(x)^3 + 3*(8*a^2 + 4*a*b - b^2) * \cosh(x)) * \sinh(x)^3 - 4*(8*a^2 + 4*a*b - b^2) * \cosh(x)^2 + 4*(7*(8*a^2 + 4*a*b - b^2) * \cosh(x)^6 - 15*(8*a^2 + 4*a*b - b^2) * \cosh(x)^4 + 9*(8*a^2 + 4*a*b - b^2) * \cosh(x)^2 - 8*a^2 - 4*a*b + b^2) * \sinh(x)^2 + 8*a^2 + 4*a*b - b^2 + 8*((8*a^2 + 4*a*b - b^2) * \cosh(x)^7 - 3*(8*a^2 + 4*a*b - b^2) * \cosh(x)^5 + 3*(8*a^2 + 4*a*b - b^2) * \cosh(x)^3 - (8*a^2 + 4*a*b - b^2) * \cosh(x)) * \sinh(x)) * \sqrt{-a} * \arctan(\sqrt{2} * (\cosh(x)^2 + 2*\cosh(x) * \sinh(x) + \sinh(x)^2 + 1) * \sqrt{-a} * \sqrt{((a + b) * \cosh(x)^2 + (a + b) * \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2*\cosh(x) * \sinh(x) + \sinh(x)^2))} / ((a + b) * \cosh(x)^4 + 4*(a + b) * \cosh(x) * \sinh(x)^3 + (a + b) * \sinh(x)^4 + 2*(a - b) * \cosh(x)^2 + 2*(3*(a + b) * \cosh(x)^2 + a - b) * \sinh(x)^2 + 4*((a + b) * \cosh(x)^3 + (a - b) * \cosh(x)) * \sinh(x) + a + b)) + 2*(a^2 * \cosh(x)^8 + 8*a^2 * \cosh(x) * \sinh(x)^7 + a^2 * \sinh(x)^8 - 4*a^2 * \cosh(x)^6 + 4*(7*a^2 * \cosh(x)^2 - a^2) * \sinh(x)^6 + 6*a^2 * \cosh(x)^4 + 8*(7*a^2 * \cosh(x)^3 - 3*a^2 * \cosh(x)) * \sinh(x)^5 + 2*(35*a^2 * \cosh(x)^4 - 30*a^2 * \cosh(x)^2 + 3*a^2) * \sinh(x)^4 - 4*a^2 * \cosh(x)^2 + 8*(7*a^2 * \cosh(x)^5 - 10*a^2 * \cosh(x)^3 + 3*a^2 * \cosh(x)) * \sinh(x)^3 + 4*(7*a^2 * \cosh(x)^6 - 15*a^2 * \cosh(x)^4 + 9*a^2 * \cosh(x)^2 - a^2) * \sinh(x)^2 + a^2 + 8*(a^2 * \cosh(x)^7 - 3*a^2 * \cosh(x)^5 + 3*a^2 * \cosh(x)^3 - a^2 * \cosh(x)) * \sinh(x)) * \sqrt{a + b} * \log(((a^3 + a^2*b) * \cosh(x)^8 + 8*(a^3 + a^2*b) * \cosh(x) * \sinh(x)^7 + (a^3 + a^2*b) * \sinh(x)^8 + 2*(2*a^3 + a^2*b) * \cosh(x)^6 + 2*(2*a^3 +
\end{aligned}$$

$$\begin{aligned}
& a^2b + 14*(a^3 + a^2b)*\cosh(x)^2*\sinh(x)^6 + 4*(14*(a^3 + a^2b)*\cosh(x) \\
& ^3 + 3*(2*a^3 + a^2b)*\cosh(x))*\sinh(x)^5 + (6*a^3 + 4*a^2b - a*b^2 + b^3) \\
& *\cosh(x)^4 + (70*(a^3 + a^2b)*\cosh(x)^4 + 6*a^3 + 4*a^2b - a*b^2 + b^3 + \\
& 30*(2*a^3 + a^2b)*\cosh(x)^2)*\sinh(x)^4 + 4*(14*(a^3 + a^2b)*\cosh(x)^5 + 1 \\
& 0*(2*a^3 + a^2b)*\cosh(x)^3 + (6*a^3 + 4*a^2b - a*b^2 + b^3)*\cosh(x))*\sinh \\
& (x)^3 + a^3 + 3*a^2b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2b - b^3)*\cosh(x)^2 \\
& + 2*(14*(a^3 + a^2b)*\cosh(x)^6 + 15*(2*a^3 + a^2b)*\cosh(x)^4 + 2*a^3 + 3 \\
& *a^2b - b^3 + 3*(6*a^3 + 4*a^2b - a*b^2 + b^3)*\cosh(x)^2)*\sinh(x)^2 + \text{sqrt} \\
& (2)*(a^2*\cosh(x)^6 + 6*a^2*\cosh(x)*\sinh(x)^5 + a^2*\sinh(x)^6 + 3*a^2*\cosh(x) \\
& ^4 + 3*(5*a^2*\cosh(x)^2 + a^2)*\sinh(x)^4 + 4*(5*a^2*\cosh(x)^3 + 3*a^2*\cosh \\
& (x))*\sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*\cosh(x)^2 + (15*a^2*\cosh(x)^4 + 18* \\
& a^2*\cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a \\
& ^2*\cosh(x)^5 + 6*a^2*\cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*\cosh(x))*\sinh(x))*\text{sq} \\
& \text{rt}(a + b)*\text{sqrt}(((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - \\
& 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) + 4*(2*(a^3 + a^2b)*\cosh(x)^7 + 3*(2*a^3 \\
& + a^2b)*\cosh(x)^5 + (6*a^3 + 4*a^2b - a*b^2 + b^3)*\cosh(x)^3 + (2*a^3 + 3 \\
& *a^2b - b^3)*\cosh(x))*\sinh(x))/(\cosh(x)^6 + 6*\cosh(x)^5*\sinh(x) + 15*\cosh(x) \\
& ^4*\sinh(x)^2 + 20*\cosh(x)^3*\sinh(x)^3 + 15*\cosh(x)^2*\sinh(x)^4 + 6*\cosh(x) \\
&)*\sinh(x)^5 + \sinh(x)^6)) + 2*(a^2*\cosh(x)^8 + 8*a^2*\cosh(x)*\sinh(x)^7 + a^ \\
& 2*\sinh(x)^8 - 4*a^2*\cosh(x)^6 + 4*(7*a^2*\cosh(x)^2 - a^2)*\sinh(x)^6 + 6*a^2 \\
& *\cosh(x)^4 + 8*(7*a^2*\cosh(x)^3 - 3*a^2*\cosh(x))*\sinh(x)^5 + 2*(35*a^2*\cosh \\
& (x)^4 - 30*a^2*\cosh(x)^2 + 3*a^2)*\sinh(x)^4 - 4*a^2*\cosh(x)^2 + 8*(7*a^2*\cosh \\
& (x)^5 - 10*a^2*\cosh(x)^3 + 3*a^2*\cosh(x))*\sinh(x)^3 + 4*(7*a^2*\cosh(x)^6 \\
& - 15*a^2*\cosh(x)^4 + 9*a^2*\cosh(x)^2 - a^2)*\sinh(x)^2 + a^2 + 8*(a^2*\cosh(x) \\
&)^7 - 3*a^2*\cosh(x)^5 + 3*a^2*\cosh(x)^3 - a^2*\cosh(x))*\sinh(x))*\text{sqrt}(a + b) \\
& *\log(-((a + b)*\cosh(x)^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a + b)*\sinh(x)^4 \\
& - 2*b*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 - b)*\sinh(x)^2 + \text{sqrt}(2)*(\cosh(x)^ \\
& 2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1))*\text{sqrt}(a + b)*\text{sqrt}(((a + b)*\cosh(x)^2 \\
& + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) + \\
& 4*((a + b)*\cosh(x)^3 - b*\cosh(x))*\sinh(x) + a + b)/(\cosh(x)^2 + 2*\cosh(x)* \\
& \sinh(x) + \sinh(x)^2)) - \text{sqrt}(2)*((6*a^2 + a*b)*\cosh(x)^6 + 6*(6*a^2 + a*b)* \\
& \cosh(x)*\sinh(x)^5 + (6*a^2 + a*b)*\sinh(x)^6 + (2*a^2 - a*b)*\cosh(x)^4 + (15 \\
& *(6*a^2 + a*b)*\cosh(x)^2 + 2*a^2 - a*b)*\sinh(x)^4 + 4*(5*(6*a^2 + a*b)*\cosh \\
& (x)^3 + (2*a^2 - a*b)*\cosh(x))*\sinh(x)^3 + (2*a^2 - a*b)*\cosh(x)^2 + (15*(6 \\
& *a^2 + a*b)*\cosh(x)^4 + 6*(2*a^2 - a*b)*\cosh(x)^2 + 2*a^2 - a*b)*\sinh(x)^2 \\
& + 6*a^2 + a*b + 2*(3*(6*a^2 + a*b)*\cosh(x)^5 + 2*(2*a^2 - a*b)*\cosh(x)^3 + \\
& (2*a^2 - a*b)*\cosh(x))*\sinh(x))*\text{sqrt}(((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 \\
& + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(a^2*\cosh(x)^8 + 8* \\
& a^2*\cosh(x)*\sinh(x)^7 + a^2*\sinh(x)^8 - 4*a^2*\cosh(x)^6 + 4*(7*a^2*\cosh(x)^ \\
& 2 - a^2)*\sinh(x)^6 + 6*a^2*\cosh(x)^4 + 8*(7*a^2*\cosh(x)^3 - 3*a^2*\cosh(x))* \\
& \sinh(x)^5 + 2*(35*a^2*\cosh(x)^4 - 30*a^2*\cosh(x)^2 + 3*a^2)*\sinh(x)^4 - 4*a \\
& ^2*\cosh(x)^2 + 8*(7*a^2*\cosh(x)^5 - 10*a^2*\cosh(x)^3 + 3*a^2*\cosh(x))*\sinh \\
& (x)^3 + 4*(7*a^2*\cosh(x)^6 - 15*a^2*\cosh(x)^4 + 9*a^2*\cosh(x)^2 - a^2)*\sinh \\
& (x)^2 + a^2 + 8*(a^2*\cosh(x)^7 - 3*a^2*\cosh(x)^5 + 3*a^2*\cosh(x)^3 - a^2*\cosh \\
& (x))*\sinh(x)), -1/16*(8*(a^2*\cosh(x)^8 + 8*a^2*\cosh(x)*\sinh(x)^7 + a^2*\sinh \\
& (x)^8 - 4*a^2*\cosh(x)^6 + 4*(7*a^2*\cosh(x)^2 - a^2)*\sinh(x)^6 + 6*a^2*\cosh \\
& (x)^4 + 8*(7*a^2*\cosh(x)^3 - 3*a^2*\cosh(x))*\sinh(x)^5 + 2*(35*a^2*\cosh(x)^4 \\
& - 30*a^2*\cosh(x)^2 + 3*a^2)*\sinh(x)^4 - 4*a^2*\cosh(x)^2 + 8*(7*a^2*\cosh(x) \\
& ^5 - 10*a^2*\cosh(x)^3 + 3*a^2*\cosh(x))*\sinh(x)^3 + 4*(7*a^2*\cosh(x)^6 - 15* \\
& a^2*\cosh(x)^4 + 9*a^2*\cosh(x)^2 - a^2)*\sinh(x)^2 + a^2 + 8*(a^2*\cosh(x)^7 - \\
& 3*a^2*\cosh(x)^5 + 3*a^2*\cosh(x)^3 - a^2*\cosh(x))*\sinh(x))*\text{sqrt}(-a - b)*\text{arc} \\
& \text{tan}(\text{sqrt}(2)*(a*\cosh(x)^2 + 2*a*\cosh(x)*\sinh(x) + a*\sinh(x)^2 + a + b)*\text{sqrt} \\
& (-a - b)*\text{sqrt}(((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2 \\
& *\cosh(x)*\sinh(x) + \sinh(x)^2)))/((a^2 + a*b)*\cosh(x)^4 + 4*(a^2 + a*b)*\cosh \\
& (x)*\sinh(x)^3 + (a^2 + a*b)*\sinh(x)^4 + (2*a^2 + a*b - b^2)*\cosh(x)^2 + (6*(\\
& a^2 + a*b)*\cosh(x)^2 + 2*a^2 + a*b - b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 2 \\
& *(2*(a^2 + a*b)*\cosh(x)^3 + (2*a^2 + a*b - b^2)*\cosh(x))*\sinh(x))) + 8*(a^2 \\
& *\cosh(x)^8 + 8*a^2*\cosh(x)*\sinh(x)^7 + a^2*\sinh(x)^8 - 4*a^2*\cosh(x)^6 + 4* \\
& (7*a^2*\cosh(x)^2 - a^2)*\sinh(x)^6 + 6*a^2*\cosh(x)^4 + 8*(7*a^2*\cosh(x)^3 -
\end{aligned}$$


```

b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*s
inh(x) + a + b)) - 4*(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)
^8 - 4*a^2*cosh(x)^6 + 4*(7*a^2*cosh(x)^2 - a^2)*sinh(x)^6 + 6*a^2*cosh(x)^
4 + 8*(7*a^2*cosh(x)^3 - 3*a^2*cosh(x))*sinh(x)^5 + 2*(35*a^2*cosh(x)^4 - 3
0*a^2*cosh(x)^2 + 3*a^2)*sinh(x)^4 - 4*a^2*cosh(x)^2 + 8*(7*a^2*cosh(x)^5 -
10*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 - 15*a^2*
cosh(x)^4 + 9*a^2*cosh(x)^2 - a^2)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 - 3*a
^2*cosh(x)^5 + 3*a^2*cosh(x)^3 - a^2*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(
sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a -
b))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cos
h(x)*sinh(x) + sinh(x)^2)))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*s
inh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2
+ a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*
(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) - 4*(a^2*cos
h(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 - 4*a^2*cosh(x)^6 + 4*(7*a
^2*cosh(x)^2 - a^2)*sinh(x)^6 + 6*a^2*cosh(x)^4 + 8*(7*a^2*cosh(x)^3 - 3*a^
2*cosh(x))*sinh(x)^5 + 2*(35*a^2*cosh(x)^4 - 30*a^2*cosh(x)^2 + 3*a^2)*sinh
(x)^4 - 4*a^2*cosh(x)^2 + 8*(7*a^2*cosh(x)^5 - 10*a^2*cosh(x)^3 + 3*a^2*cos
h(x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 - 15*a^2*cosh(x)^4 + 9*a^2*cosh(x)^2 -
a^2)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 - 3*a^2*cosh(x)^5 + 3*a^2*cosh(x)^
3 - a^2*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)
)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*s
inh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cos
h(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)
^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a
- b)*cosh(x))*sinh(x) + a + b)) - sqrt(2)*((6*a^2 + a*b)*cosh(x)^6 + 6*(6*
a^2 + a*b)*cosh(x)*sinh(x)^5 + (6*a^2 + a*b)*sinh(x)^6 + (2*a^2 - a*b)*cosh
(x)^4 + (15*(6*a^2 + a*b)*cosh(x)^2 + 2*a^2 - a*b)*sinh(x)^4 + 4*(5*(6*a^2
+ a*b)*cosh(x)^3 + (2*a^2 - a*b)*cosh(x))*sinh(x)^3 + (2*a^2 - a*b)*cosh(x)
^2 + (15*(6*a^2 + a*b)*cosh(x)^4 + 6*(2*a^2 - a*b)*cosh(x)^2 + 2*a^2 - a*b)
*sinh(x)^2 + 6*a^2 + a*b + 2*(3*(6*a^2 + a*b)*cosh(x)^5 + 2*(2*a^2 - a*b)*c
osh(x)^3 + (2*a^2 - a*b)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)
)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^2*cos
h(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 - 4*a^2*cosh(x)^6 + 4*(7*a
^2*cosh(x)^2 - a^2)*sinh(x)^6 + 6*a^2*cosh(x)^4 + 8*(7*a^2*cosh(x)^3 - 3*a^
2*cosh(x))*sinh(x)^5 + 2*(35*a^2*cosh(x)^4 - 30*a^2*cosh(x)^2 + 3*a^2)*sinh
(x)^4 - 4*a^2*cosh(x)^2 + 8*(7*a^2*cosh(x)^5 - 10*a^2*cosh(x)^3 + 3*a^2*cos
h(x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 - 15*a^2*cosh(x)^4 + 9*a^2*cosh(x)^2 -
a^2)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 - 3*a^2*cosh(x)^5 + 3*a^2*cosh(x)^
3 - a^2*cosh(x))*sinh(x))]

```

giac [B] time = 3.12, size = 947, normalized size = 7.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")

```

[Out] -1/2*sqrt(a + b)*log(abs(-(sqrt(a + b))*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) + 1/
2*sqrt(a + b)*log(abs(-sqrt(a + b))*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log(abs
(-sqrt(a + b))*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b) - sqrt(a + b))) + 1/4*(8*a^2 + 4*a*b - b^2)*arctan(-1/2*(sqrt(
a + b))*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
+ b) - sqrt(a + b))/sqrt(-a))/sqrt(-a)*a + 1/2*((16*a^2 + 12*a*b + b^2)*
(sqrt(a + b))*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b))^7 - (16*a^2 + 52*a*b + 7*b^2)*(sqrt(a + b))*e^(2*x) - sqrt(a*e^
(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^6*sqrt(a + b) - (48
*a^3 - 28*a^2*b - 109*a*b^2 - 21*b^3)*(sqrt(a + b))*e^(2*x) - sqrt(a*e^(4*x)

```

$$\begin{aligned}
& + b e^{4x} + 2 a e^{2x} - 2 b e^{2x} + a + b)^5 + (176 a^3 + 156 a^2 b \\
& - 115 a b^2 - 35 b^3) (\sqrt{a + b} e^{2x} - \sqrt{a e^{4x} + b e^{4x} + \\
& 2 a e^{2x} - 2 b e^{2x} + a + b})^4 \sqrt{a + b} + (304 a^4 - 156 a^3 b - \\
& 317 a^2 b^2 + 130 a b^3 + 35 b^4) (\sqrt{a + b} e^{2x} - \sqrt{a e^{4x} + b \\
& e^{4x} + 2 a e^{2x} - 2 b e^{2x} + a + b})^3 - (48 a^4 + 476 a^3 b - 37 \\
& 9 a^2 b^2 + 94 a b^3 + 21 b^4) (\sqrt{a + b} e^{2x} - \sqrt{a e^{4x} + b e^{4x} \\
& (4x) + 2 a e^{2x} - 2 b e^{2x} + a + b})^2 \sqrt{a + b} - (272 a^5 + 140 a \\
& a^4 b - 271 a^3 b^2 + 135 a^2 b^3 - 53 a b^4 - 7 b^5) (\sqrt{a + b} e^{2x} \\
& - \sqrt{a e^{4x} + b e^{4x} + 2 a e^{2x} - 2 b e^{2x} + a + b}) - (112 a \\
& ^5 - 116 a^4 b + 65 a^3 b^2 - 17 a^2 b^3 + 11 a b^4 + b^5) \sqrt{a + b} / (((\\
& \sqrt{a + b} e^{2x} - \sqrt{a e^{4x} + b e^{4x} + 2 a e^{2x} - 2 b e^{2x} \\
&) + a + b))^2 - 2 (\sqrt{a + b} e^{2x} - \sqrt{a e^{4x} + b e^{4x} + 2 a e \\
& ^{2x} - 2 b e^{2x} + a + b}) \sqrt{a + b} - 3 (a + b)^4 a)
\end{aligned}$$

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int (\coth^5(x) \sqrt{a + b (\tanh^2(x))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x)

[Out] int(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tanh(x)^2 + a} \coth(x)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*tanh(x)^2 + a)*coth(x)^5, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(x)^5 \sqrt{b \tanh(x)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^5*(a + b*tanh(x)^2)^(1/2),x)

[Out] int(coth(x)^5*(a + b*tanh(x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \tanh^2(x)} \coth^5(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**5*(a+b*tanh(x)**2)**(1/2),x)

[Out] Integral(sqrt(a + b*tanh(x)**2)*coth(x)**5, x)

3.219 $\int \tanh^3(x) (a + b \tanh^2(x))^{3/2} dx$

Optimal. Leaf size=82

$$-\frac{(a + b \tanh^2(x))^{5/2}}{5b} - \frac{1}{3} (a + b \tanh^2(x))^{3/2} - (a+b)\sqrt{a + b \tanh^2(x)} + (a+b)^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}} \right)$$

[Out] (a+b)^(3/2)*arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))-(a+b)*(a+b*tanh(x)^2)^(1/2)-1/3*(a+b*tanh(x)^2)^(3/2)-1/5*(a+b*tanh(x)^2)^(5/2)/b

Rubi [A] time = 0.15, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3670, 446, 80, 50, 63, 208}

$$-\frac{(a + b \tanh^2(x))^{5/2}}{5b} - \frac{1}{3} (a + b \tanh^2(x))^{3/2} - (a+b)\sqrt{a + b \tanh^2(x)} + (a+b)^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}} \right)$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^3*(a + b*Tanh[x]^2)^(3/2),x]

[Out] (a + b)^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (a + b)*Sqrt[a + b*Tanh[x]^2] - (a + b*Tanh[x]^2)^(3/2)/3 - (a + b*Tanh[x]^2)^(5/2)/(5*b)

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p

$*(c + d*x)^q, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, m, n, p, q\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 3670

$\text{Int}[\{(d_)*\tan[(e_)] + (f_)*(x_)]^{(m_)}*((a_)] + (b_)*((c_)*\tan[(e_)] + (f_)*(x_)]^{(n_)}\}^{(p_)}, x_Symbol] \ :> \ \text{With}[\{ff = \text{FreeFactors}[\text{Tan}[e + f*x], x]\}, \text{Dist}[(c*ff)/f, \text{Subst}[\text{Int}[\{(d*ff*x)/c\}^{m*}(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*\text{Tan}[e + f*x])/ff], x]] /; \text{FreeQ}[\{a, b, c, d, e, f, m, n, p\}, x] \ \&\& \ (\text{IGtQ}[p, 0] \ || \ \text{EqQ}[n, 2] \ || \ \text{EqQ}[n, 4] \ || \ (\text{IntegerQ}[p] \ \&\& \ \text{RationalQ}[n]))$

Rubi steps

$$\begin{aligned} \int \tanh^3(x) (a + b \tanh^2(x))^{3/2} dx &= \text{Subst} \left(\int \frac{x^3 (a + bx^2)^{3/2}}{1 - x^2} dx, x, \tanh(x) \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{x(a + bx)^{3/2}}{1 - x} dx, x, \tanh^2(x) \right) \\ &= -\frac{(a + b \tanh^2(x))^{5/2}}{5b} + \frac{1}{2} \text{Subst} \left(\int \frac{(a + bx)^{3/2}}{1 - x} dx, x, \tanh^2(x) \right) \\ &= -\frac{1}{3} (a + b \tanh^2(x))^{3/2} - \frac{(a + b \tanh^2(x))^{5/2}}{5b} + \frac{1}{2} (a + b) \text{Subst} \left(\int \frac{\sqrt{a + bx}}{1 - x} dx, x, \tanh^2(x) \right) \\ &= -(a + b) \sqrt{a + b \tanh^2(x)} - \frac{1}{3} (a + b \tanh^2(x))^{3/2} - \frac{(a + b \tanh^2(x))^{5/2}}{5b} + \frac{1}{2} (a + b) \text{Subst} \left(\int \frac{\sqrt{a + bx}}{1 - x} dx, x, \tanh^2(x) \right) \\ &= -(a + b) \sqrt{a + b \tanh^2(x)} - \frac{1}{3} (a + b \tanh^2(x))^{3/2} - \frac{(a + b \tanh^2(x))^{5/2}}{5b} + \frac{1}{2} (a + b) \text{Subst} \left(\int \frac{\sqrt{a + bx}}{1 - x} dx, x, \tanh^2(x) \right) \\ &= (a + b)^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}} \right) - (a + b) \sqrt{a + b \tanh^2(x)} - \frac{1}{3} (a + b \tanh^2(x))^{3/2} - \frac{(a + b \tanh^2(x))^{5/2}}{5b} \end{aligned}$$

Mathematica [A] time = 0.44, size = 86, normalized size = 1.05

$$(a+b)^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}} \right) - \frac{\sqrt{a + b \tanh^2(x)} (3a^2 + b(6a + 5b) \tanh^2(x) + 20ab + 3b^2 \tanh^4(x) + 15b^2 \tanh^6(x))}{15b}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^3*(a + b*Tanh[x]^2)^(3/2), x]

[Out] (a + b)^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Sqrt[a + b*Tanh[x]^2]*(3*a^2 + 20*a*b + 15*b^2 + b*(6*a + 5*b)*Tanh[x]^2 + 3*b^2*Tanh[x]^4))/(15*b)

fricas [B] time = 0.83, size = 4941, normalized size = 60.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3*(a+b*tanh(x)^2)^(3/2), x, algorithm="fricas")

```
[Out] [1/60*(15*((a*b + b^2)*cosh(x)^10 + 10*(a*b + b^2)*cosh(x)*sinh(x)^9 + (a*b
+ b^2)*sinh(x)^10 + 5*(a*b + b^2)*cosh(x)^8 + 5*(9*(a*b + b^2)*cosh(x)^2 +
a*b + b^2)*sinh(x)^8 + 40*(3*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*
sinh(x)^7 + 10*(a*b + b^2)*cosh(x)^6 + 10*(21*(a*b + b^2)*cosh(x)^4 + 14*(a
*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^6 + 4*(63*(a*b + b^2)*cosh(x)^5 +
70*(a*b + b^2)*cosh(x)^3 + 15*(a*b + b^2)*cosh(x))*sinh(x)^5 + 10*(a*b + b^
2)*cosh(x)^4 + 10*(21*(a*b + b^2)*cosh(x)^6 + 35*(a*b + b^2)*cosh(x)^4 + 15
*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^4 + 40*(3*(a*b + b^2)*cosh(x)^7
+ 7*(a*b + b^2)*cosh(x)^5 + 5*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))
*sinh(x)^3 + 5*(a*b + b^2)*cosh(x)^2 + 5*(9*(a*b + b^2)*cosh(x)^8 + 28*(a*b
+ b^2)*cosh(x)^6 + 30*(a*b + b^2)*cosh(x)^4 + 12*(a*b + b^2)*cosh(x)^2 + a
*b + b^2)*sinh(x)^2 + a*b + b^2 + 10*((a*b + b^2)*cosh(x)^9 + 4*(a*b + b^2)
*cosh(x)^7 + 6*(a*b + b^2)*cosh(x)^5 + 4*(a*b + b^2)*cosh(x)^3 + (a*b + b^2
)*cosh(x))*sinh(x)*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2
*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)
^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3
+ a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*
b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b
- a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*
b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^
3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b
- b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(
x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)
*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)
^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(
x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2
*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*
b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh
(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(
x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)
^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*si
nh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh
(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 15*((a*b + b^2)*cosh(x)^10 + 10
*(a*b + b^2)*cosh(x)*sinh(x)^9 + (a*b + b^2)*sinh(x)^10 + 5*(a*b + b^2)*cos
h(x)^8 + 5*(9*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^8 + 40*(3*(a*b + b
^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*sinh(x)^7 + 10*(a*b + b^2)*cosh(x)^6 +
10*(21*(a*b + b^2)*cosh(x)^4 + 14*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(
x)^6 + 4*(63*(a*b + b^2)*cosh(x)^5 + 70*(a*b + b^2)*cosh(x)^3 + 15*(a*b + b
^2)*cosh(x))*sinh(x)^5 + 10*(a*b + b^2)*cosh(x)^4 + 10*(21*(a*b + b^2)*cosh
(x)^6 + 35*(a*b + b^2)*cosh(x)^4 + 15*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*si
nh(x)^4 + 40*(3*(a*b + b^2)*cosh(x)^7 + 7*(a*b + b^2)*cosh(x)^5 + 5*(a*b +
b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*sinh(x)^3 + 5*(a*b + b^2)*cosh(x)^2 +
5*(9*(a*b + b^2)*cosh(x)^8 + 28*(a*b + b^2)*cosh(x)^6 + 30*(a*b + b^2)*cos
h(x)^4 + 12*(a*b + b^2)*cosh(x)^2 + a*b + b^2)*sinh(x)^2 + a*b + b^2 + 10*(
a*b + b^2)*cosh(x)^9 + 4*(a*b + b^2)*cosh(x)^7 + 6*(a*b + b^2)*cosh(x)^5 +
4*(a*b + b^2)*cosh(x)^3 + (a*b + b^2)*cosh(x))*sinh(x))*sqrt(a + b)*log(-
(a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*c
osh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*c
osh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a +
b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a
+ b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x)
+ sinh(x)^2)) - 4*sqrt(2)*((3*a^2 + 26*a*b + 23*b^2)*cosh(x)^8 + 8*(3*a^2
+ 26*a*b + 23*b^2)*cosh(x)*sinh(x)^7 + (3*a^2 + 26*a*b + 23*b^2)*sinh(x)^8
+ 4*(3*a^2 + 20*a*b + 12*b^2)*cosh(x)^6 + 4*(7*(3*a^2 + 26*a*b + 23*b^2)*co
sh(x)^2 + 3*a^2 + 20*a*b + 12*b^2)*sinh(x)^6 + 8*(7*(3*a^2 + 26*a*b + 23*b^
2)*cosh(x)^3 + 3*(3*a^2 + 20*a*b + 12*b^2)*cosh(x))*sinh(x)^5 + 2*(9*a^2 +
54*a*b + 49*b^2)*cosh(x)^4 + 2*(35*(3*a^2 + 26*a*b + 23*b^2)*cosh(x)^4 + 30
*(3*a^2 + 20*a*b + 12*b^2)*cosh(x)^2 + 9*a^2 + 54*a*b + 49*b^2)*sinh(x)^4 +
```

$$\begin{aligned}
& 8*(7*(3*a^2 + 26*a*b + 23*b^2)*\cosh(x)^5 + 10*(3*a^2 + 20*a*b + 12*b^2)*\cosh(x)^3 + (9*a^2 + 54*a*b + 49*b^2)*\cosh(x)*\sinh(x)^3 + 4*(3*a^2 + 20*a*b + 12*b^2)*\cosh(x)^2 + 4*(7*(3*a^2 + 26*a*b + 23*b^2)*\cosh(x)^6 + 15*(3*a^2 + 20*a*b + 12*b^2)*\cosh(x)^4 + 3*(9*a^2 + 54*a*b + 49*b^2)*\cosh(x)^2 + 3*a^2 + 20*a*b + 12*b^2)*\sinh(x)^2 + 3*a^2 + 26*a*b + 23*b^2 + 8*((3*a^2 + 26*a*b + 23*b^2)*\cosh(x)^7 + 3*(3*a^2 + 20*a*b + 12*b^2)*\cosh(x)^5 + (9*a^2 + 54*a*b + 49*b^2)*\cosh(x)^3 + (3*a^2 + 20*a*b + 12*b^2)*\cosh(x))*\sinh(x))*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))}/(b*\cosh(x)^10 + 10*b*\cosh(x)*\sinh(x)^9 + b*\sinh(x)^10 + 5*b*\cosh(x)^8 + 5*(9*b*\cosh(x)^2 + b)*\sinh(x)^8 + 40*(3*b*\cosh(x)^3 + b*\cosh(x))*\sinh(x)^7 + 10*b*\cosh(x)^6 + 10*(21*b*\cosh(x)^4 + 14*b*\cosh(x)^2 + b)*\sinh(x)^6 + 4*(63*b*\cosh(x)^5 + 70*b*\cosh(x)^3 + 15*b*\cosh(x))*\sinh(x)^5 + 10*b*\cosh(x)^4 + 10*(21*b*\cosh(x)^6 + 35*b*\cosh(x)^4 + 15*b*\cosh(x)^2 + b)*\sinh(x)^4 + 40*(3*b*\cosh(x)^7 + 7*b*\cosh(x)^5 + 5*b*\cosh(x)^3 + b*\cosh(x))*\sinh(x)^3 + 5*b*\cosh(x)^2 + 5*(9*b*\cosh(x)^8 + 28*b*\cosh(x)^6 + 30*b*\cosh(x)^4 + 12*b*\cosh(x)^2 + b)*\sinh(x)^2 + 10*(b*\cosh(x)^9 + 4*b*\cosh(x)^7 + 6*b*\cosh(x)^5 + 4*b*\cosh(x)^3 + b*\cosh(x))*\sinh(x) + b), -1/30*(15*((a*b + b^2)*\cosh(x)^10 + 10*(a*b + b^2)*\cosh(x)*\sinh(x)^9 + (a*b + b^2)*\sinh(x)^10 + 5*(a*b + b^2)*\cosh(x)^8 + 5*(9*(a*b + b^2)*\cosh(x)^2 + a*b + b^2)*\sinh(x)^8 + 40*(3*(a*b + b^2)*\cosh(x)^3 + (a*b + b^2)*\cosh(x))*\sinh(x)^7 + 10*(a*b + b^2)*\cosh(x)^6 + 10*(21*(a*b + b^2)*\cosh(x)^4 + 14*(a*b + b^2)*\cosh(x)^2 + a*b + b^2)*\sinh(x)^6 + 4*(63*(a*b + b^2)*\cosh(x)^5 + 70*(a*b + b^2)*\cosh(x)^3 + 15*(a*b + b^2)*\cosh(x))*\sinh(x)^5 + 10*(a*b + b^2)*\cosh(x)^4 + 10*(21*(a*b + b^2)*\cosh(x)^6 + 35*(a*b + b^2)*\cosh(x)^4 + 15*(a*b + b^2)*\cosh(x)^2 + a*b + b^2)*\sinh(x)^4 + 40*(3*(a*b + b^2)*\cosh(x)^7 + 7*(a*b + b^2)*\cosh(x)^5 + 5*(a*b + b^2)*\cosh(x)^3 + (a*b + b^2)*\cosh(x))*\sinh(x)^3 + 5*(a*b + b^2)*\cosh(x)^2 + 5*(9*(a*b + b^2)*\cosh(x)^8 + 28*(a*b + b^2)*\cosh(x)^6 + 30*(a*b + b^2)*\cosh(x)^4 + 12*(a*b + b^2)*\cosh(x)^2 + a*b + b^2)*\sinh(x)^2 + a*b + b^2 + 10*((a*b + b^2)*\cosh(x)^9 + 4*(a*b + b^2)*\cosh(x)^7 + 6*(a*b + b^2)*\cosh(x)^5 + 4*(a*b + b^2)*\cosh(x)^3 + (a*b + b^2)*\cosh(x))*\sinh(x))*\sqrt{-a - b}*\arctan(\sqrt{2}*(a*\cosh(x)^2 + 2*a*\cosh(x)*\sinh(x) + a*\sinh(x)^2 + a + b))*\sqrt{-a - b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))}/((a^2 + a*b)*\cosh(x)^4 + 4*(a^2 + a*b)*\cosh(x)*\sinh(x)^3 + (a^2 + a*b)*\sinh(x)^4 + (2*a^2 + a*b - b^2)*\cosh(x)^2 + (6*(a^2 + a*b)*\cosh(x)^2 + 2*a^2 + a*b - b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*\cosh(x)^3 + (2*a^2 + a*b - b^2)*\cosh(x))*\sinh(x))) + 15*((a*b + b^2)*\cosh(x)^10 + 10*(a*b + b^2)*\cosh(x)*\sinh(x)^9 + (a*b + b^2)*\sinh(x)^10 + 5*(a*b + b^2)*\cosh(x)^8 + 5*(9*(a*b + b^2)*\cosh(x)^2 + a*b + b^2)*\sinh(x)^8 + 40*(3*(a*b + b^2)*\cosh(x)^3 + (a*b + b^2)*\cosh(x))*\sinh(x)^7 + 10*(a*b + b^2)*\cosh(x)^6 + 10*(21*(a*b + b^2)*\cosh(x)^4 + 14*(a*b + b^2)*\cosh(x)^2 + a*b + b^2)*\sinh(x)^6 + 4*(63*(a*b + b^2)*\cosh(x)^5 + 70*(a*b + b^2)*\cosh(x)^3 + 15*(a*b + b^2)*\cosh(x))*\sinh(x)^5 + 10*(a*b + b^2)*\cosh(x)^4 + 10*(21*(a*b + b^2)*\cosh(x)^6 + 35*(a*b + b^2)*\cosh(x)^4 + 15*(a*b + b^2)*\cosh(x)^2 + a*b + b^2)*\sinh(x)^4 + 40*(3*(a*b + b^2)*\cosh(x)^7 + 7*(a*b + b^2)*\cosh(x)^5 + 5*(a*b + b^2)*\cosh(x)^3 + (a*b + b^2)*\cosh(x))*\sinh(x)^3 + 5*(a*b + b^2)*\cosh(x)^2 + 5*(9*(a*b + b^2)*\cosh(x)^8 + 28*(a*b + b^2)*\cosh(x)^6 + 30*(a*b + b^2)*\cosh(x)^4 + 12*(a*b + b^2)*\cosh(x)^2 + a*b + b^2 + 10*((a*b + b^2)*\cosh(x)^9 + 4*(a*b + b^2)*\cosh(x)^7 + 6*(a*b + b^2)*\cosh(x)^5 + 4*(a*b + b^2)*\cosh(x)^3 + (a*b + b^2)*\cosh(x))*\sinh(x))*\sqrt{-a - b}*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1))*\sqrt{-a - b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))}/((a + b)*\cosh(x)^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a + b)*\sinh(x)^4 + 2*(a - b)*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 + a - b)*\sinh(x)^2 + 4*((a + b)*\cosh(x)^3 + (a - b)*\cosh(x))*\sinh(x) + a + b)) + 2*\sqrt{2}*((3*a^2 + 26*a*b + 23*b^2)*\cosh(x)^8 + 8*(3*a^2 + 26*a*b + 23*b^2)*\cosh(x)*\sinh(x)^7 + (3*a^2 + 26*a*b + 23*b^2)*\sinh(x)^8 + 4*(3*a^2 + 20*a*b + 12*b^2)*\cosh(x)^6 + 4*(7*(3*a^2 + 26*a*b + 23*b^2)*\cosh(x)^2 + 3*a^2 + 20*a*b + 12*b^2)*\sinh(x)^6 + 8*(7*(3*a^2 + 26*a*b + 23*b^2)*\cosh(x)^3 + 3*(3*a^2 + 20*a*b + 12*b^2)*\cosh(x))*\sinh(x)
\end{aligned}$$

```

)^5 + 2*(9*a^2 + 54*a*b + 49*b^2)*cosh(x)^4 + 2*(35*(3*a^2 + 26*a*b + 23*b^
2)*cosh(x)^4 + 30*(3*a^2 + 20*a*b + 12*b^2)*cosh(x)^2 + 9*a^2 + 54*a*b + 49
*b^2)*sinh(x)^4 + 8*(7*(3*a^2 + 26*a*b + 23*b^2)*cosh(x)^5 + 10*(3*a^2 + 20
*a*b + 12*b^2)*cosh(x)^3 + (9*a^2 + 54*a*b + 49*b^2)*cosh(x))*sinh(x)^3 + 4
*(3*a^2 + 20*a*b + 12*b^2)*cosh(x)^2 + 4*(7*(3*a^2 + 26*a*b + 23*b^2)*cosh(
x)^6 + 15*(3*a^2 + 20*a*b + 12*b^2)*cosh(x)^4 + 3*(9*a^2 + 54*a*b + 49*b^2)
*cosh(x)^2 + 3*a^2 + 20*a*b + 12*b^2)*sinh(x)^2 + 3*a^2 + 26*a*b + 23*b^2 +
8*((3*a^2 + 26*a*b + 23*b^2)*cosh(x)^7 + 3*(3*a^2 + 20*a*b + 12*b^2)*cosh(
x)^5 + (9*a^2 + 54*a*b + 49*b^2)*cosh(x)^3 + (3*a^2 + 20*a*b + 12*b^2)*cosh
(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(b*cosh(x)^10 + 10*b*cosh(x)*sinh(x)^
9 + b*sinh(x)^10 + 5*b*cosh(x)^8 + 5*(9*b*cosh(x)^2 + b)*sinh(x)^8 + 40*(3*
b*cosh(x)^3 + b*cosh(x))*sinh(x)^7 + 10*b*cosh(x)^6 + 10*(21*b*cosh(x)^4 +
14*b*cosh(x)^2 + b)*sinh(x)^6 + 4*(63*b*cosh(x)^5 + 70*b*cosh(x)^3 + 15*b*c
osh(x))*sinh(x)^5 + 10*b*cosh(x)^4 + 10*(21*b*cosh(x)^6 + 35*b*cosh(x)^4 +
15*b*cosh(x)^2 + b)*sinh(x)^4 + 40*(3*b*cosh(x)^7 + 7*b*cosh(x)^5 + 5*b*cos
h(x)^3 + b*cosh(x))*sinh(x)^3 + 5*b*cosh(x)^2 + 5*(9*b*cosh(x)^8 + 28*b*cos
h(x)^6 + 30*b*cosh(x)^4 + 12*b*cosh(x)^2 + b)*sinh(x)^2 + 10*(b*cosh(x)^9 +
4*b*cosh(x)^7 + 6*b*cosh(x)^5 + 4*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)]

```

giac [B] time = 4.08, size = 1063, normalized size = 12.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")

```

[Out] 1/2*(a + b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*(a + b)^(3/2)*l
og(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*
b*e^(2*x) + a + b) - sqrt(a + b))) - 1/2*(a^2 + 2*a*b + b^2)*log(abs(-(sqrt
(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 4/15*(15*(a^2 + 4*a*b
+ 3*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
2*b*e^(2*x) + a + b))^9 + 15*(7*a^2 + 20*a*b + 9*b^2)*(sqrt(a + b)*e^(2*x)
- sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^8*sqrt(
a + b) + 20*(15*a^3 + 39*a^2*b + 21*a*b^2 + b^3)*(sqrt(a + b)*e^(2*x) - sqr
t(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^7 + 20*(21*a^
3 + 21*a^2*b - 57*a*b^2 - 65*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b
*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^6*sqrt(a + b) + 2*(105*a^4 -
210*a^3*b - 1860*a^2*b^2 - 1590*a*b^3 + 19*b^4)*(sqrt(a + b)*e^(2*x) - sqr
t(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5 - 10*(21*a^
4 + 126*a^3*b + 288*a^2*b^2 - 390*a*b^3 - 349*b^4)*(sqrt(a + b)*e^(2*x) - s
qrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqrt(a +
b) - 20*(21*a^5 + 63*a^4*b - 18*a^3*b^2 - 378*a^2*b^3 - 235*a*b^4 + 19*b^5)
*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b))^3 - 20*(15*a^5 + 21*a^4*b - 126*a^3*b^2 - 90*a^2*b^3 + 367*a*
b^4 + 325*b^5)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b) - 5*(21*a^6 + 24*a^5*b - 243*a^4*
b^2 + 280*a^3*b^3 + 815*a^2*b^4 - 944*a*b^5 - 1233*b^6)*(sqrt(a + b)*e^(2*x)
) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) - (15*
a^6 - 165*a^4*b^2 + 920*a^3*b^3 - 1147*a^2*b^4 - 504*a*b^5 + 1713*b^6)*sqrt
(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4
*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)^5

```

maple [B] time = 0.08, size = 593, normalized size = 7.23

$$\frac{(a + b(\tanh^2(x)))^{\frac{5}{2}}}{5b} - \frac{((\tanh(x) - 1)^2 b + 2(\tanh(x) - 1)b + a + b)^{\frac{3}{2}}}{6} - \frac{b\sqrt{(\tanh(x) - 1)^2 b + 2(\tanh(x) - 1)b + a + b}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^3*(a+b*tanh(x)^2)^(3/2),x)

[Out]
$$-1/5*(a+b*tanh(x)^2)^(5/2)/b-1/6*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(3/2)-1/4*b*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*tanh(x)-3/4*b^(1/2)*ln(((tanh(x)-1)*b+b)/b^(1/2)+((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))*a+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2))*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))*a-1/2*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*a-1/2*b^(3/2)*ln(((tanh(x)-1)*b+b)/b^(1/2)+((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2))*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))*b-1/2*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*b-1/6*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(3/2)+1/4*b*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)*tanh(x)+3/4*b^(1/2)*ln(((1+tanh(x))*b-b)/b^(1/2)+((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))*a+1/2*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2))*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))*(a+b)^(1/2)*a-1/2*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)*a+1/2*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2))*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))*(a+b)^(1/2)*b+1/2*b^(3/2)*ln(((1+tanh(x))*b-b)/b^(1/2)+((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))-1/2*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)*b$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \tanh(x)^2 + a)^{\frac{3}{2}} \tanh(x)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*tanh(x)^2 + a)^(3/2)*tanh(x)^3, x)

mupad [B] time = 10.99, size = 112, normalized size = 1.37

$$-\frac{(b \tanh(x)^2 + a)^{5/2}}{5b} - \left(\frac{a+b}{3b} - \frac{a}{3b}\right) (b \tanh(x)^2 + a)^{3/2} - (a+b) \left(\frac{a+b}{b} - \frac{a}{b}\right) \sqrt{b \tanh(x)^2 + a} - \operatorname{atan}\left(\frac{(a+b)}{\dots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^3*(a + b*tanh(x)^2)^(3/2),x)

[Out]
$$-(a + b*tanh(x)^2)^(5/2)/(5*b) - ((a + b)/(3*b) - a/(3*b))*(a + b*tanh(x)^2)^(3/2) - \operatorname{atan}(((a + b)^(3/2)*(a + b*tanh(x)^2)^(1/2)*1i)/(2*a*b + a^2 + b^2))*(a + b)^(3/2)*1i - (a + b)*((a + b)/b - a/b)*(a + b*tanh(x)^2)^(1/2)$$

sympy [B] time = 29.54, size = 175, normalized size = 2.13

$$\frac{2a \left(\frac{b^2 \sqrt{a+b \tanh^2(x)}}{2} + \frac{b^2(a+b) \operatorname{atan}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}} + \frac{b(a+b \tanh^2(x))^{\frac{3}{2}}}{6} \right)}{b^2} - \frac{2 \left(\frac{b^3 \sqrt{a+b \tanh^2(x)}}{2} + \frac{b^3(a+b) \operatorname{atan}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}} \right)}{b^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)**3*(a+b*tanh(x)**2)**(3/2),x)

[Out]
$$-2*a*(b**2*\sqrt{a + b*tanh(x)**2})/2 + b**2*(a + b)*\operatorname{atan}(\sqrt{a + b*tanh(x)**2})/\sqrt{-a - b})/(2*\sqrt{-a - b}) + b*(a + b*tanh(x)**2)**(3/2)/6)/b**2 - 2*(b**3*\sqrt{a + b*tanh(x)**2})/2 + b**3*(a + b)*\operatorname{atan}(\sqrt{a + b*tanh(x)**2})/\sqrt{-a - b})/(2*\sqrt{-a - b}) + b*(a + b*tanh(x)**2)**(5/2)/10 + (a + b*tanh(x)**2)**(3/2)*(-a*b/2 + b**2/2)/3)/b**2$$

3.220 $\int \tanh^2(x) (a + b \tanh^2(x))^{3/2} dx$

Optimal. Leaf size=123

$$\frac{(3a^2 + 12ab + 8b^2) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{8\sqrt{b}} - \frac{1}{8}(5a+4b) \tanh(x) \sqrt{a + b \tanh^2(x)} + (a+b)^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}}\right)$$

[Out] (a+b)^(3/2)*arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))-1/8*(3*a^2+12*a*b+8*b^2)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(1/2)-1/8*(5*a+4*b)*(a+b*tanh(x)^2)^(1/2)*tanh(x)-1/4*b*(a+b*tanh(x)^2)^(1/2)*tanh(x)^3

Rubi [A] time = 0.24, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3670, 477, 582, 523, 217, 206, 377}

$$\frac{(3a^2 + 12ab + 8b^2) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{8\sqrt{b}} - \frac{1}{4}b \tanh^3(x) \sqrt{a + b \tanh^2(x)} - \frac{1}{8}(5a+4b) \tanh(x) \sqrt{a + b \tanh^2(x)} +$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^2*(a + b*Tanh[x]^2)^(3/2), x]

[Out] -((3*a^2 + 12*a*b + 8*b^2)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/(8*Sqrt[b]) + (a + b)^(3/2)*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - ((5*a + 4*b)*Tanh[x]*Sqrt[a + b*Tanh[x]^2])/8 - (b*Tanh[x]^3*Sqrt[a + b*Tanh[x]^2])/4

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 477

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), Int[(e*x)^(m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) + c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n*(p + q))*x^n, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 523

Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*Sqrt[(c_) + (d_)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 582

Int[((g_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(f*g^(n-1)*(g*x)^(m-n+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(b*d*(m+n*(p+q+1)+1)), x] - Dist[g^n/(b*d*(m+n*(p+q+1)+1)), Int[(g*x)^(m-n)*(a+b*x^n)^p*(c+d*x^n)^q*Simp[a*f*c*(m-n+1) + (a*f*d*(m+n*q+1) + b*(f*c*(m+n*p+1) - e*d*(m+n*(p+q+1)+1))]*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n-1]

Rule 3670

Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*((c_)*tan[(e_) + (f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
 \int \tanh^2(x) (a + b \tanh^2(x))^{3/2} dx &= \text{Subst} \left(\int \frac{x^2 (a + bx^2)^{3/2}}{1 - x^2} dx, x, \tanh(x) \right) \\
 &= -\frac{1}{4} b \tanh^3(x) \sqrt{a + b \tanh^2(x)} - \frac{1}{4} \text{Subst} \left(\int \frac{x^2 (-a(4a + 3b) - b(5a + 4b))}{(1 - x^2) \sqrt{a + bx^2}} dx, x, \tanh(x) \right) \\
 &= -\frac{1}{8} (5a + 4b) \tanh(x) \sqrt{a + b \tanh^2(x)} - \frac{1}{4} b \tanh^3(x) \sqrt{a + b \tanh^2(x)} - \frac{1}{4} \text{Subst} \left(\int \frac{x^2 (-a(4a + 3b) - b(5a + 4b))}{(1 - x^2) \sqrt{a + bx^2}} dx, x, \tanh(x) \right) \\
 &= -\frac{1}{8} (5a + 4b) \tanh(x) \sqrt{a + b \tanh^2(x)} - \frac{1}{4} b \tanh^3(x) \sqrt{a + b \tanh^2(x)} - \frac{1}{4} \text{Subst} \left(\int \frac{x^2 (-a(4a + 3b) - b(5a + 4b))}{(1 - x^2) \sqrt{a + bx^2}} dx, x, \tanh(x) \right) \\
 &= -\frac{1}{8} (5a + 4b) \tanh(x) \sqrt{a + b \tanh^2(x)} - \frac{1}{4} b \tanh^3(x) \sqrt{a + b \tanh^2(x)} - \frac{1}{4} \text{Subst} \left(\int \frac{x^2 (-a(4a + 3b) - b(5a + 4b))}{(1 - x^2) \sqrt{a + bx^2}} dx, x, \tanh(x) \right) \\
 &= -\frac{(3a^2 + 12ab + 8b^2) \tanh^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right)}{8\sqrt{b}} + (a + b)^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}} \right)
 \end{aligned}$$

Mathematica [C] time = 6.27, size = 584, normalized size = 4.75

$$\sqrt{\frac{a \cosh(2x) + a + b \cosh(2x) - b}{\cosh(2x) + 1}} \left(\frac{1}{8} \operatorname{sech}(x)(-5a \sinh(x) - 6b \sinh(x)) + \frac{1}{4} b \tanh(x) \operatorname{sech}^2(x) \right) + \frac{1}{4} \frac{b(a^2 - 4ab + 4b^2)}{\dots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^2*(a + b*Tanh[x]^2)^(3/2),x]
[Out] (-((b*(a^2 - 4*a*b - 4*b^2)*Sqrt[(a - b + (a + b)*Cosh[2*x])/(1 + Cosh[2*x])]*Sqrt[-((a*Coth[x]^2)/b)]*Sqrt[-((a*(1 + Cosh[2*x])*Csch[x]^2)/b)]*Sqrt[(a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*Csch[2*x]*EllipticF[ArcSin[Sqrt[(a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]*Sinh[x]^4)/(a*(a - b + (a + b)*Cosh[2*x]))) - ((4*I)*b*(4*a^2 + 8*a*b + 4*b^2)*Sqrt[1 + Cosh[2*x]]*Sqrt[(a - b + (a + b)*Cosh[2*x])/(1 + Cosh[2*x])]*((-1/4*I)*Sqrt[-((a*Coth[x]^2)/b)]*Sqrt[-((a*(1 + Cosh[2*x])*Csch[x]^2)/b)]*Sqrt[(a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*Csch[2*x]*EllipticF[ArcSin[Sqrt[(a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]*Sinh[x]^4)/(a*Sqrt[1 + Cosh[2*x]]*Sqrt[a - b + (a + b)*Cosh[2*x]]) + ((I/2)*Sqrt[-((a*Coth[x]^2)/b)]*Sqrt[-((a*(1 + Cosh[2*x])*Csch[x]^2)/b)]*Sqrt[(a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*Csch[2*x]*EllipticPi[b/(a + b), ArcSin[Sqrt[(a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]*Sinh[x]^4)/((a + b)*Sqrt[1 + Cosh[2*x]]*Sqrt[a - b + (a + b)*Cosh[2*x]]))/Sqrt[a - b + (a + b)*Cosh[2*x]]/4 + Sqrt[(a - b + a*Cosh[2*x] + b*Cosh[2*x])/(1 + Cosh[2*x])]*((Sech[x]*(-5*a*Sinh[x] - 6*b*Sinh[x]))/8 + (b*Sech[x]^2*Tanh[x])/4)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

giac [B] time = 3.67, size = 949, normalized size = 7.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
[Out] -1/2*(a + b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*(a + b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))) - 1/4*(3*a^2 + 12*a*b + 8*b^2)*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*(a^2 + 2*a*b + b^2)
```

```
*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 1/2*((
5*a^2 + 20*a*b + 16*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^7 + (35*a^2 + 76*a*b + 16*b^2)*(sqrt(
a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
+ b))^6*sqrt(a + b) + (105*a^3 + 153*a^2*b - 28*a*b^2 - 48*b^3)*(sqrt(a +
b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b
))^5 + (175*a^3 - 25*a^2*b - 260*a*b^2 - 176*b^3)*(sqrt(a + b)*e^(2*x) - sq
rt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqrt(a + b
) + (175*a^4 - 110*a^3*b - 417*a^2*b^2 + 60*a*b^3 + 304*b^4)*(sqrt(a + b)*e
^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3
+ (105*a^4 - 210*a^3*b - 55*a^2*b^2 + 484*a*b^3 + 48*b^4)*(sqrt(a + b)*e^(
2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*s
qrt(a + b) + (35*a^5 - 79*a^4*b + 53*a^3*b^2 + 195*a^2*b^3 - 308*a*b^4 - 27
2*b^5)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*
b*e^(2*x) + a + b)) + (5*a^5 - 17*a^4*b + 51*a^3*b^2 - 19*a^2*b^3 - 44*a*b^
4 + 112*b^5)*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a
*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a
- 3*b)^4
```

maple [B] time = 0.07, size = 633, normalized size = 5.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^2*(a+b*tanh(x)^2)^(3/2), x)
```

```
[Out] -1/4*tanh(x)*(a+b*tanh(x)^2)^(3/2)-3/8*a*tanh(x)*(a+b*tanh(x)^2)^(1/2)-3/8*
a^2/b^(1/2)*ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2))-1/6*((tanh(x)-1)^2*b+
2*(tanh(x)-1)*b+a+b)^(3/2)-1/4*b*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2
)*tanh(x)-3/4*b^(1/2)*ln(((tanh(x)-1)*b+b)/b^(1/2)+((tanh(x)-1)^2*b+2*(tanh
(x)-1)*b+a+b)^(1/2))*a+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(
1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))*a-1/2*((tan
h(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*a-1/2*b^(3/2)*ln(((tanh(x)-1)*b+b)/b
^(1/2)+((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a
+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1
/2))/(tanh(x)-1))*b-1/2*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*b+1/6*(
(1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(3/2)-1/4*b*((1+tanh(x))^2*b-2*(1+tanh
(x))*b+a+b)^(1/2)*tanh(x)-3/4*b^(1/2)*ln(((1+tanh(x))*b-b)/b^(1/2)+((1+tanh
(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))*a-1/2*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a
+b)^(1/2)*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))*(a+b)^(
1/2)*a+1/2*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)*a-1/2*b^(3/2)*ln(((1
+tanh(x))*b-b)/b^(1/2)+((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))-1/2*ln(
(2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b
)^(1/2))/(1+tanh(x)))*(a+b)^(1/2)*b+1/2*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+
b)^(1/2)*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \tanh(x)^2 + a)^{\frac{3}{2}} \tanh(x)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2*(a+b*tanh(x)^2)^(3/2), x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(x)^2 + a)^(3/2)*tanh(x)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x)^2 (b \tanh(x)^2 + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)^2*(a + b*tanh(x)^2)^(3/2), x)`

[Out] `int(tanh(x)^2*(a + b*tanh(x)^2)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(x))^{\frac{3}{2}} \tanh^2(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)**2*(a+b*tanh(x)**2)**(3/2), x)`

[Out] `Integral((a + b*tanh(x)**2)**(3/2)*tanh(x)**2, x)`

3.221 $\int \tanh(x) (a + b \tanh^2(x))^{3/2} dx$

Optimal. Leaf size=63

$$-(a+b)\sqrt{a+b \tanh^2(x)} - \frac{1}{3}(a+b \tanh^2(x))^{3/2} + (a+b)^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)$$

[Out] (a+b)^(3/2)*arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))-(a+b)*(a+b*tanh(x)^2)^(1/2)-1/3*(a+b*tanh(x)^2)^(3/2)

Rubi [A] time = 0.10, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3670, 444, 50, 63, 208}

$$(a+b)^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right) - (a+b)\sqrt{a+b \tanh^2(x)} - \frac{1}{3}(a+b \tanh^2(x))^{3/2}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]*(a + b*Tanh[x]^2)^(3/2), x]

[Out] (a + b)^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (a + b)*Sqrt[a + b*Tanh[x]^2] - (a + b*Tanh[x]^2)^(3/2)/3

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x],

x]], Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
 \int \tanh(x) (a + b \tanh^2(x))^{3/2} dx &= \text{Subst} \left(\int \frac{x (a + bx^2)^{3/2}}{1 - x^2} dx, x, \tanh(x) \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{(a + bx)^{3/2}}{1 - x} dx, x, \tanh^2(x) \right) \\
 &= -\frac{1}{3} (a + b \tanh^2(x))^{3/2} + \frac{1}{2} (a + b) \text{Subst} \left(\int \frac{\sqrt{a + bx}}{1 - x} dx, x, \tanh^2(x) \right) \\
 &= -(a + b) \sqrt{a + b \tanh^2(x)} - \frac{1}{3} (a + b \tanh^2(x))^{3/2} + \frac{1}{2} (a + b)^2 \text{Subst} \left(\int \frac{1}{1 - \frac{a + bx}{b}} dx, x, \tanh^2(x) \right) \\
 &= -(a + b) \sqrt{a + b \tanh^2(x)} - \frac{1}{3} (a + b \tanh^2(x))^{3/2} + \frac{(a + b)^2 \text{Subst} \left(\int \frac{1}{1 + \frac{a - x^2}{b}} dx, x, \tanh^2(x) \right)}{b} \\
 &= (a + b)^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}} \right) - (a + b) \sqrt{a + b \tanh^2(x)} - \frac{1}{3} (a + b \tanh^2(x))^{3/2}
 \end{aligned}$$

Mathematica [A] time = 0.18, size = 59, normalized size = 0.94

$$(a + b)^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}} \right) - \frac{1}{3} \sqrt{a + b \tanh^2(x)} (4a + b \tanh^2(x) + 3b)$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]*(a + b*Tanh[x]^2)^(3/2), x]

[Out] (a + b)^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Sqrt[a + b*Tanh[x]^2]*(4*a + 3*b + b*Tanh[x]^2))/3

fricas [B] time = 2.12, size = 2385, normalized size = 37.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)*(a+b*tanh(x)^2)^(3/2), x, algorithm="fricas")

[Out] [1/12*(3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x)^6 + 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 + a + b)*sinh(x)^4 + 4*(5*(a + b)*cosh(x)^3 + 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2 + 3*(5*(a + b)*cosh(x)^4 + 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 6*((a + b)*cosh(x)^5 + 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3

$$\begin{aligned}
& + a^2 b) \cosh(x)^2 \sinh(x)^4 + 4*(14*(a^3 + a^2 b) \cosh(x)^5 + 10*(2*a^3 + \\
& a^2 b) \cosh(x)^3 + (6*a^3 + 4*a^2 b - a*b^2 + b^3) \cosh(x) \sinh(x)^3 + a^3 \\
& + 3*a^2 b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2 b - b^3) \cosh(x)^2 + 2*(14*(\\
& a^3 + a^2 b) \cosh(x)^6 + 15*(2*a^3 + a^2 b) \cosh(x)^4 + 2*a^3 + 3*a^2 b - b \\
& ^3 + 3*(6*a^3 + 4*a^2 b - a*b^2 + b^3) \cosh(x)^2 \sinh(x)^2 + \sqrt{2}*(a^2 * \\
& \cosh(x)^6 + 6*a^2 \cosh(x) \sinh(x)^5 + a^2 \sinh(x)^6 + 3*a^2 \cosh(x)^4 + 3*(\\
& 5*a^2 \cosh(x)^2 + a^2) \sinh(x)^4 + 4*(5*a^2 \cosh(x)^3 + 3*a^2 \cosh(x)) \sinh \\
& (x)^3 + (3*a^2 + 2*a*b - b^2) \cosh(x)^2 + (15*a^2 \cosh(x)^4 + 18*a^2 \cosh(x) \\
&)^2 + 3*a^2 + 2*a*b - b^2) \sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2 \cosh(x) \\
& ^5 + 6*a^2 \cosh(x)^3 + (3*a^2 + 2*a*b - b^2) \cosh(x)) \sinh(x)) \sqrt{a + b} * \\
& \sqrt{((a + b) \cosh(x)^2 + (a + b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \\
& * \sinh(x) + \sinh(x)^2))} + 4*(2*(a^3 + a^2 b) \cosh(x)^7 + 3*(2*a^3 + a^2 b) * \\
& \cosh(x)^5 + (6*a^3 + 4*a^2 b - a*b^2 + b^3) \cosh(x)^3 + (2*a^3 + 3*a^2 b - b \\
& ^3) \cosh(x) \sinh(x)) / (\cosh(x)^6 + 6 \cosh(x)^5 \sinh(x) + 15 \cosh(x)^4 \sinh(x) \\
& ^2 + 20 \cosh(x)^3 \sinh(x)^3 + 15 \cosh(x)^2 \sinh(x)^4 + 6 \cosh(x) \sinh(x)^5 \\
& + \sinh(x)^6)) + 3*((a + b) \cosh(x)^6 + 6*(a + b) \cosh(x) \sinh(x)^5 + (a + \\
& b) \sinh(x)^6 + 3*(a + b) \cosh(x)^4 + 3*(5*(a + b) \cosh(x)^2 + a + b) \sinh(x) \\
& ^4 + 4*(5*(a + b) \cosh(x)^3 + 3*(a + b) \cosh(x)) \sinh(x)^3 + 3*(a + b) \cosh \\
& (x)^2 + 3*(5*(a + b) \cosh(x)^4 + 6*(a + b) \cosh(x)^2 + a + b) \sinh(x)^2 + \\
& 6*((a + b) \cosh(x)^5 + 2*(a + b) \cosh(x)^3 + (a + b) \cosh(x)) \sinh(x) + a \\
& + b) \sqrt{a + b} \log(-((a + b) \cosh(x)^4 + 4*(a + b) \cosh(x) \sinh(x)^3 + (a \\
& + b) \sinh(x)^4 - 2*b \cosh(x)^2 + 2*(3*(a + b) \cosh(x)^2 - b) \sinh(x)^2 + \sqrt{2} \\
& * (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) \sqrt{a + b} \sqrt{((a \\
& + b) \cosh(x)^2 + (a + b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) \\
& + \sinh(x)^2))} + 4*((a + b) \cosh(x)^3 - b \cosh(x)) \sinh(x) + a + b) / (\cosh(x) \\
& ^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2)) - 16 \sqrt{2} * ((a + b) \cosh(x)^4 + 4*(\\
& a + b) \cosh(x) \sinh(x)^3 + (a + b) \sinh(x)^4 + (2*a + b) \cosh(x)^2 + (6*(a \\
& + b) \cosh(x)^2 + 2*a + b) \sinh(x)^2 + 2*(2*(a + b) \cosh(x)^3 + (2*a + b) \cosh \\
& (x)) \sinh(x) + a + b) \sqrt{((a + b) \cosh(x)^2 + (a + b) \sinh(x)^2 + a - b) \\
& / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} / (\cosh(x)^6 + 6 \cosh(x) \sinh \\
& (x)^5 + \sinh(x)^6 + 3*(5 \cosh(x)^2 + 1) \sinh(x)^4 + 3 \cosh(x)^4 + 4*(5 \cosh \\
& (x)^3 + 3 \cosh(x)) \sinh(x)^3 + 3*(5 \cosh(x)^4 + 6 \cosh(x)^2 + 1) \sinh(x)^2 \\
& + 3 \cosh(x)^2 + 6*(\cosh(x)^5 + 2 \cosh(x)^3 + \cosh(x)) \sinh(x) + 1), -1/6*(3 \\
& * ((a + b) \cosh(x)^6 + 6*(a + b) \cosh(x) \sinh(x)^5 + (a + b) \sinh(x)^6 + 3*(\\
& a + b) \cosh(x)^4 + 3*(5*(a + b) \cosh(x)^2 + a + b) \sinh(x)^4 + 4*(5*(a + b) \\
& * \cosh(x)^3 + 3*(a + b) \cosh(x)) \sinh(x)^3 + 3*(a + b) \cosh(x)^2 + 3*(5*(a + \\
& b) \cosh(x)^4 + 6*(a + b) \cosh(x)^2 + a + b) \sinh(x)^2 + 6*((a + b) \cosh(x) \\
& ^5 + 2*(a + b) \cosh(x)^3 + (a + b) \cosh(x)) \sinh(x) + a + b) \sqrt{-a - b} * \\
& \arctan(\sqrt{2} * (a \cosh(x)^2 + 2*a \cosh(x) \sinh(x) + a \sinh(x)^2 + a + b) \sqrt{ \\
& -a - b} \sqrt{((a + b) \cosh(x)^2 + (a + b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - \\
& 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / ((a^2 + a*b) \cosh(x)^4 + 4*(a^2 + a*b) \cosh \\
& (x) \sinh(x)^3 + (a^2 + a*b) \sinh(x)^4 + (2*a^2 + a*b - b^2) \cosh(x)^2 + (6 \\
& *(a^2 + a*b) \cosh(x)^2 + 2*a^2 + a*b - b^2) \sinh(x)^2 + a^2 + 2*a*b + b^2 + \\
& 2*(2*(a^2 + a*b) \cosh(x)^3 + (2*a^2 + a*b - b^2) \cosh(x)) \sinh(x))) + 3*((\\
& a + b) \cosh(x)^6 + 6*(a + b) \cosh(x) \sinh(x)^5 + (a + b) \sinh(x)^6 + 3*(a + \\
& b) \cosh(x)^4 + 3*(5*(a + b) \cosh(x)^2 + a + b) \sinh(x)^4 + 4*(5*(a + b) \cosh \\
& (x)^3 + 3*(a + b) \cosh(x)) \sinh(x)^3 + 3*(a + b) \cosh(x)^2 + 3*(5*(a + b) \\
& * \cosh(x)^4 + 6*(a + b) \cosh(x)^2 + a + b) \sinh(x)^2 + 6*((a + b) \cosh(x)^5 \\
& + 2*(a + b) \cosh(x)^3 + (a + b) \cosh(x)) \sinh(x) + a + b) \sqrt{-a - b} * \arctan \\
& (\sqrt{2} * (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) \sqrt{-a - b} \sqrt{ \\
& ((a + b) \cosh(x)^2 + (a + b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh \\
& (x) + \sinh(x)^2)}) / ((a + b) \cosh(x)^4 + 4*(a + b) \cosh(x) \sinh(x)^3 + (a + \\
& b) \sinh(x)^4 + 2*(a - b) \cosh(x)^2 + 2*(3*(a + b) \cosh(x)^2 + a - b) \sinh(x) \\
& ^2 + 4*((a + b) \cosh(x)^3 + (a - b) \cosh(x)) \sinh(x) + a + b)) + 8 \sqrt{2} \\
& * ((a + b) \cosh(x)^4 + 4*(a + b) \cosh(x) \sinh(x)^3 + (a + b) \sinh(x)^4 + (2 \\
& * a + b) \cosh(x)^2 + (6*(a + b) \cosh(x)^2 + 2*a + b) \sinh(x)^2 + 2*(2*(a + b) \\
&) \cosh(x)^3 + (2*a + b) \cosh(x)) \sinh(x) + a + b) \sqrt{((a + b) \cosh(x)^2 + \\
& (a + b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} / (\\
& \cosh(x)^6 + 6 \cosh(x) \sinh(x)^5 + \sinh(x)^6 + 3*(5 \cosh(x)^2 + 1) \sinh(x)^4
\end{aligned}$$

+ 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 + 2*cosh(x)^3 + cosh(x))*sinh(x) + 1]

giac [B] time = 2.59, size = 662, normalized size = 10.51

$$\frac{1}{2}(a+b)^{\frac{3}{2}} \log\left(\left|-\sqrt{a+b}e^{2x} + \sqrt{ae^{4x} + be^{4x} + 2ae^{2x} - 2be^{2x} + a + b} + \sqrt{a+b}\right|\right) - \frac{1}{2}(a+b)^{\frac{3}{2}} \log\left(\left|-\sqrt{a+b}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)*(a+b*tanh(x)^2)^(3/2), x, algorithm="giac")

[Out] 1/2*(a+b)^(3/2)*log(abs(-sqrt(a+b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a+b))) - 1/2*(a+b)^(3/2)*log(abs(-sqrt(a+b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a+b))) - 1/2*(a^2 + 2*a*b + b^2)*log(abs(-(sqrt(a+b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a+b) - sqrt(a+b)*(a-b)))/sqrt(a+b) - 8/3*(3*(a*b + b^2)*(sqrt(a+b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5 + 3*(3*a*b + b^2)*(sqrt(a+b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqrt(a+b) + 2*(3*a^2*b - 6*a*b^2 - 5*b^3)*(sqrt(a+b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3 - 6*(a^2*b + 4*a*b^2 + 3*b^3)*(sqrt(a+b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a+b) - 3*(3*a^3*b + a^2*b^2 - 15*a*b^3 - 13*b^4)*(sqrt(a+b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) - (3*a^3*b - 9*a^2*b^2 + 5*a*b^3 + 17*b^4)*sqrt(a+b))/((sqrt(a+b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a+b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a+b) + a - 3*b)^3

maple [B] time = 0.06, size = 578, normalized size = 9.17

$$\frac{\left((\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a + b\right)^{\frac{3}{2}}}{6} \frac{b \sqrt{(\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a + b} \tanh(x)}{4} \frac{3\sqrt{b}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)*(a+b*tanh(x)^2)^(3/2), x)

[Out] -1/6*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(3/2)-1/4*b*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*tanh(x)-3/4*b^(1/2)*ln(((tanh(x)-1)*b+b)/b^(1/2))+((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))*a+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2))*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))*a-1/2*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))*a+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2))*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))*b-1/2*b^(3/2)*ln(((tanh(x)-1)*b+b)/b^(1/2))+((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))-1/2*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))*b-1/6*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(3/2)+1/4*b*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)*tanh(x)+3/4*b^(1/2)*ln(((1+tanh(x))*b-b)/b^(1/2))+((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))*a+1/2*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2))*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))*(a+b)^(1/2)*a-1/2*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))*a+1/2*b^(3/2)*ln(((1+tanh(x))*b-b)/b^(1/2))+((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))+1/2*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2))*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))*(a+b)^(1/2))*b-1/2*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))*b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \tanh(x)^2 + a)^{\frac{3}{2}} \tanh(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*tanh(x)^2 + a)^(3/2)*tanh(x), x)

mupad [B] time = 3.71, size = 64, normalized size = 1.02

$$\operatorname{atanh}\left(\frac{(a+b)^{3/2}\sqrt{b\tanh(x)^2+a}}{a^2+2ab+b^2}\right)(a+b)^{3/2} - (a+b)\sqrt{b\tanh(x)^2+a} - \frac{(b\tanh(x)^2+a)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)*(a + b*tanh(x)^2)^(3/2),x)

[Out] atanh(((a + b)^(3/2)*(a + b*tanh(x)^2)^(1/2))/(2*a*b + a^2 + b^2))*(a + b)^(3/2) - (a + b)*(a + b*tanh(x)^2)^(1/2) - (a + b*tanh(x)^2)^(3/2)/3

sympy [B] time = 17.58, size = 128, normalized size = 2.03

$$\frac{2a\left(\frac{b\sqrt{a+b\tanh^2(x)}}{2} + \frac{b(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}\right)}{b} - \frac{2\left(\frac{b^2\sqrt{a+b\tanh^2(x)}}{2} + \frac{b^2(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}} + \frac{b(a+b\tanh^2(x))^{\frac{3}{2}}}{6}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)*(a+b*tanh(x)**2)**(3/2),x)

[Out] -2*a*(b*sqrt(a + b*tanh(x)**2)/2 + b*(a + b)*atan(sqrt(a + b*tanh(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)))/b - 2*(b**2*sqrt(a + b*tanh(x)**2)/2 + b**2*(a + b)*atan(sqrt(a + b*tanh(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a + b*tanh(x)**2)**(3/2)/6)/b

3.222 $\int (a + b \tanh^2(x))^{3/2} dx$

Optimal. Leaf size=88

$$-\frac{1}{2}b \tanh(x) \sqrt{a + b \tanh^2(x)} + (a+b)^{3/2} \tanh^{-1} \left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right) - \frac{1}{2} \sqrt{b} (3a+2b) \tanh^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right)$$

[Out] (a+b)^(3/2)*arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))-1/2*(3*a+2*b)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))*b^(1/2)-1/2*b*(a+b*tanh(x)^2)^(1/2)*tanh(x)

Rubi [A] time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3661, 416, 523, 217, 206, 377}

$$(a+b)^{3/2} \tanh^{-1} \left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right) - \frac{1}{2} \sqrt{b} (3a+2b) \tanh^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right) - \frac{1}{2} b \tanh(x) \sqrt{a + b \tanh^2(x)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tanh[x]^2)^(3/2), x]

[Out] -(Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/2 + (a + b)^(3/2)*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - (b*Tanh[x]*Sqrt[a + b*Tanh[x]^2])/2

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 416

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q-1))/(b*(n*(p+q)+1)), x] + Dist[1/(b*(n*(p+q)+1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q-2)*Simp[c*(b*c*(n*(p+q)+1) - a*d] + d*(b*c*(n*(p+2*q-1)+1) - a*d*(n*(q-1)+1))*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && NeQ[n*(p+q)+1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 523

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x_)^(n_)], x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e_

$- a*f)/b$, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 3661

Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(ff*x)^n]^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int (a + b \tanh^2(x))^{3/2} dx &= \text{Subst} \left(\int \frac{(a + bx^2)^{3/2}}{1 - x^2} dx, x, \tanh(x) \right) \\ &= -\frac{1}{2} b \tanh(x) \sqrt{a + b \tanh^2(x)} - \frac{1}{2} \text{Subst} \left(\int \frac{-a(2a + b) - b(3a + 2b)x^2}{(1 - x^2) \sqrt{a + bx^2}} dx, x, \tanh(x) \right) \\ &= -\frac{1}{2} b \tanh(x) \sqrt{a + b \tanh^2(x)} + (a + b)^2 \text{Subst} \left(\int \frac{1}{(1 - x^2) \sqrt{a + bx^2}} dx, x, \tanh(x) \right) \\ &= -\frac{1}{2} b \tanh(x) \sqrt{a + b \tanh^2(x)} + (a + b)^2 \text{Subst} \left(\int \frac{1}{1 - (a + b)x^2} dx, x, \frac{\tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right) \\ &= -\frac{1}{2} \sqrt{b} (3a + 2b) \tanh^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right) + (a + b)^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right) \end{aligned}$$

Mathematica [A] time = 0.37, size = 161, normalized size = 1.83

$$\frac{1}{2} \left(-b \tanh(x) \sqrt{a + b \tanh^2(x)} - (a + b)^{3/2} \log \left(\sqrt{a + b} \sqrt{a + b \tanh^2(x)} + a - b \tanh(x) \right) + (a + b)^{3/2} \log \left(\sqrt{a + b} \sqrt{a + b \tanh^2(x)} + a + b \tanh(x) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Tanh[x]^2)^(3/2), x]

[Out] $(-((a + b)^{(3/2)} \text{Log}[1 - \text{Tanh}[x]]) + (a + b)^{(3/2)} \text{Log}[1 + \text{Tanh}[x]] - \text{Sqrt}[b] * (3*a + 2*b) * \text{Log}[b * \text{Tanh}[x] + \text{Sqrt}[b] * \text{Sqrt}[a + b * \text{Tanh}[x]^2]] - (a + b)^{(3/2)} * \text{Log}[a - b * \text{Tanh}[x] + \text{Sqrt}[a + b] * \text{Sqrt}[a + b * \text{Tanh}[x]^2]] + (a + b)^{(3/2)} * \text{Log}[a + b * \text{Tanh}[x] + \text{Sqrt}[a + b] * \text{Sqrt}[a + b * \text{Tanh}[x]^2]] - b * \text{Tanh}[x] * \text{Sqrt}[a + b * \text{Tanh}[x]^2]) / 2$

fricas [B] time = 3.05, size = 4841, normalized size = 55.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(x)^2)^(3/2), x, algorithm="fricas")

[Out] $[1/4 * (((a + b) * \cosh(x)^4 + 4 * (a + b) * \cosh(x) * \sinh(x)^3 + (a + b) * \sinh(x)^4 + 2 * (a + b) * \cosh(x)^2 + 2 * (3 * (a + b) * \cosh(x)^2 + a + b) * \sinh(x)^2 + 4 * ((a + b) * \cosh(x)^3 + (a + b) * \cosh(x)) * \sinh(x) + a + b) * \sqrt{a + b} * \log(-((a * b^2 + b^3) * \cosh(x)^8 + 8 * (a * b^2 + b^3) * \cosh(x) * \sinh(x)^7 + (a * b^2 + b^3) * \sinh(x)^6 + (a * b^2 + b^3) * \cosh(x)^7 + (a * b^2 + b^3) * \sinh(x)^6 + (a * b^2 + b^3) * \cosh(x)^7 + (a * b^2 + b^3) * \sinh(x)^6)) / 4$

$$\begin{aligned}
&)^8 - 2*(a*b^2 + 2*b^3)*\cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*\cos \\
& h(x)^2)*\sinh(x)^6 + 4*(14*(a*b^2 + b^3)*\cosh(x)^3 - 3*(a*b^2 + 2*b^3)*\cosh(\\
& x))*\sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*\cosh(x)^4 + (70*(a*b^2 + b^ \\
& 3)*\cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*\cosh(x)^2 \\
&)*\sinh(x)^4 + 4*(14*(a*b^2 + b^3)*\cosh(x)^5 - 10*(a*b^2 + 2*b^3)*\cosh(x)^3 \\
& + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*\cosh(x))*\sinh(x)^3 + a^3 + 3*a^2*b + 3*a* \\
& b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*\cosh(x)^2 + 2*(14*(a*b^2 + b^3)*\cosh(\\
& x)^6 - 15*(a*b^2 + 2*b^3)*\cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2* \\
& b + 4*a*b^2 + 6*b^3)*\cosh(x)^2)*\sinh(x)^2 + \sqrt{2}*(b^2*\cosh(x)^6 + 6*b^2* \\
& \cosh(x)*\sinh(x)^5 + b^2*\sinh(x)^6 - 3*b^2*\cosh(x)^4 + 3*(5*b^2*\cosh(x)^2 - \\
& b^2)*\sinh(x)^4 + 4*(5*b^2*\cosh(x)^3 - 3*b^2*\cosh(x))*\sinh(x)^3 - (a^2 - 2*a \\
& *b - 3*b^2)*\cosh(x)^2 + (15*b^2*\cosh(x)^4 - 18*b^2*\cosh(x)^2 - a^2 + 2*a*b \\
& + 3*b^2)*\sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*\cosh(x)^5 - 6*b^2*\cosh(x) \\
& ^3 - (a^2 - 2*a*b - 3*b^2)*\cosh(x))*\sinh(x))*\sqrt{a + b}*\sqrt{((a + b)*\cosh \\
& (x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x) \\
& ^2))} + 4*(2*(a*b^2 + b^3)*\cosh(x)^7 - 3*(a*b^2 + 2*b^3)*\cosh(x)^5 + (a^3 - \\
& a^2*b + 4*a*b^2 + 6*b^3)*\cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*\cosh(x))*\sinh(\\
& x))/(\cosh(x)^6 + 6*\cosh(x)^5*\sinh(x) + 15*\cosh(x)^4*\sinh(x)^2 + 20*\cosh(x)^ \\
& 3*\sinh(x)^3 + 15*\cosh(x)^2*\sinh(x)^4 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6) + \\
& ((3*a + 2*b)*\cosh(x)^4 + 4*(3*a + 2*b)*\cosh(x)*\sinh(x)^3 + (3*a + 2*b)*\sinh \\
& (x)^4 + 2*(3*a + 2*b)*\cosh(x)^2 + 2*(3*(3*a + 2*b)*\cosh(x)^2 + 3*a + 2*b)*s \\
& inh(x)^2 + 4*((3*a + 2*b)*\cosh(x)^3 + (3*a + 2*b)*\cosh(x))*\sinh(x) + 3*a + \\
& 2*b)*\sqrt{b}*\log(-((a + 2*b)*\cosh(x)^4 + 4*(a + 2*b)*\cosh(x)*\sinh(x)^3 + (a \\
& + 2*b)*\sinh(x)^4 + 2*(a - 2*b)*\cosh(x)^2 + 2*(3*(a + 2*b)*\cosh(x)^2 + a - \\
& 2*b)*\sinh(x)^2 - 2*\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1)* \\
& \sqrt{b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2 \\
& *\cosh(x)*\sinh(x) + \sinh(x)^2)} + 4*((a + 2*b)*\cosh(x)^3 + (a - 2*b)*\cosh(x) \\
&)*\sinh(x) + a + 2*b)/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cos \\
& h(x)^2 + 1)*\sinh(x)^2 + 2*\cosh(x)^2 + 4*(\cosh(x)^3 + \cosh(x))*\sinh(x) + 1) \\
&) + ((a + b)*\cosh(x)^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a + b)*\sinh(x)^4 + \\
& 2*(a + b)*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 + a + b)*\sinh(x)^2 + 4*((a + b) \\
&)*\cosh(x)^3 + (a + b)*\cosh(x))*\sinh(x) + a + b)*\sqrt{a + b}*\log(((a + b)*\cos \\
& h(x)^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a + b)*\sinh(x)^4 + 2*a*\cosh(x)^2 + \\
& 2*(3*(a + b)*\cosh(x)^2 + a)*\sinh(x)^2 + \sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sin \\
& h(x) + \sinh(x)^2 + 1)*\sqrt{a + b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x) \\
& ^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)} + 4*((a + b)*\cosh(\\
& x)^3 + a*\cosh(x))*\sinh(x) + a + b)/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x) \\
& ^2)) - 2*\sqrt{2}*(b*\cosh(x)^2 + 2*b*\cosh(x)*\sinh(x) + b*\sinh(x)^2 - b)*\sqrt{ \\
& ((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sin \\
& h(x) + \sinh(x)^2)))/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cos \\
& h(x)^2 + 1)*\sinh(x)^2 + 2*\cosh(x)^2 + 4*(\cosh(x)^3 + \cosh(x))*\sinh(x) + 1), \\
& 1/4*(2*((3*a + 2*b)*\cosh(x)^4 + 4*(3*a + 2*b)*\cosh(x)*\sinh(x)^3 + (3*a + 2 \\
& *b)*\sinh(x)^4 + 2*(3*a + 2*b)*\cosh(x)^2 + 2*(3*(3*a + 2*b)*\cosh(x)^2 + 3*a \\
& + 2*b)*\sinh(x)^2 + 4*((3*a + 2*b)*\cosh(x)^3 + (3*a + 2*b)*\cosh(x))*\sinh(x) \\
& + 3*a + 2*b)*\sqrt{-b}*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(\\
& x)^2 - 1)*\sqrt{-b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(co \\
& sh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/((a + b)*\cosh(x)^4 + 4*(a + b)*\cos \\
& h(x)*\sinh(x)^3 + (a + b)*\sinh(x)^4 + 2*(a - b)*\cosh(x)^2 + 2*(3*(a + b)*\cos \\
& h(x)^2 + a - b)*\sinh(x)^2 + 4*((a + b)*\cosh(x)^3 + (a - b)*\cosh(x))*\sinh(x) \\
&) + a + b) + ((a + b)*\cosh(x)^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a + b)*\si \\
& nh(x)^4 + 2*(a + b)*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 + a + b)*\sinh(x)^2 + \\
& 4*((a + b)*\cosh(x)^3 + (a + b)*\cosh(x))*\sinh(x) + a + b)*\sqrt{a + b}*\log(- \\
& ((a*b^2 + b^3)*\cosh(x)^8 + 8*(a*b^2 + b^3)*\cosh(x)*\sinh(x)^7 + (a*b^2 + b^3) \\
&)*\sinh(x)^8 - 2*(a*b^2 + 2*b^3)*\cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + \\
& b^3)*\cosh(x)^2)*\sinh(x)^6 + 4*(14*(a*b^2 + b^3)*\cosh(x)^3 - 3*(a*b^2 + 2*b^ \\
& 3)*\cosh(x))*\sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*\cosh(x)^4 + (70*(a* \\
& b^2 + b^3)*\cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*\c \\
& osh(x)^2)*\sinh(x)^4 + 4*(14*(a*b^2 + b^3)*\cosh(x)^5 - 10*(a*b^2 + 2*b^3)*\co \\
& sh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*\cosh(x))*\sinh(x)^3 + a^3 + 3*a^2*
\end{aligned}$$

$$\begin{aligned}
& b + 3ab^2 + b^3 + 2(a^3 - 3ab^2 - 2b^3)\cosh(x)^2 + 2(14(ab^2 + b^3)\cosh(x)^6 - 15(ab^2 + 2b^3)\cosh(x)^4 + a^3 - 3ab^2 - 2b^3 + 3(a^3 - a^2b + 4ab^2 + 6b^3)\cosh(x)^2)\sinh(x)^2 + \sqrt{2}(b^2\cosh(x)^6 + 6b^2\cosh(x)\sinh(x)^5 + b^2\sinh(x)^6 - 3b^2\cosh(x)^4 + 3(5b^2\cosh(x)^2 - b^2)\sinh(x)^4 + 4(5b^2\cosh(x)^3 - 3b^2\cosh(x))\sinh(x)^3 - (a^2 - 2ab - 3b^2)\cosh(x)^2 + (15b^2\cosh(x)^4 - 18b^2\cosh(x)^2 - a^2 + 2ab + 3b^2)\sinh(x)^2 - a^2 - 2ab - b^2 + 2(3b^2\cosh(x)^5 - 6b^2\cosh(x)^3 - (a^2 - 2ab - 3b^2)\cosh(x))\sinh(x))\sqrt{a+b}\sqrt{((a+b)\cosh(x)^2 + (a+b)\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)} + 4(2(ab^2 + b^3)\cosh(x)^7 - 3(ab^2 + 2b^3)\cosh(x)^5 + (a^3 - a^2b + 4ab^2 + 6b^3)\cosh(x)^3 + (a^3 - 3ab^2 - 2b^3)\cosh(x)\sinh(x))/(\cosh(x)^6 + 6\cosh(x)^5\sinh(x) + 15\cosh(x)^4\sinh(x)^2 + 20\cosh(x)^3\sinh(x)^3 + 15\cosh(x)^2\sinh(x)^4 + 6\cosh(x)\sinh(x)^5 + \sinh(x)^6) + ((a+b)\cosh(x)^4 + 4(a+b)\cosh(x)\sinh(x)^3 + (a+b)\sinh(x)^4 + 2(a+b)\cosh(x)^2 + 2(3(a+b)\cosh(x)^2 + a + b)\sinh(x)^2 + 4((a+b)\cosh(x)^3 + (a+b)\cosh(x))\sinh(x) + a + b)\sqrt{a+b}\log(((a+b)\cosh(x)^4 + 4(a+b)\cosh(x)\sinh(x)^3 + (a+b)\sinh(x)^4 + 2a\cosh(x)^2 + 2(3(a+b)\cosh(x)^2 + a)\sinh(x)^2 + \sqrt{2}(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1)\sqrt{a+b}\sqrt{((a+b)\cosh(x)^2 + (a+b)\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)} + 4((a+b)\cosh(x)^3 + a\cosh(x))\sinh(x) + a + b)/(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2)) - 2\sqrt{2}(b\cosh(x)^2 + 2b\cosh(x)\sinh(x) + b\sinh(x)^2 - b)\sqrt{((a+b)\cosh(x)^2 + (a+b)\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)}/(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2(3\cosh(x)^2 + 1)\sinh(x)^2 + 2\cosh(x)^2 + 4(\cosh(x)^3 + \cosh(x))\sinh(x) + 1), -1/4(2((a+b)\cosh(x)^4 + 4(a+b)\cosh(x)\sinh(x)^3 + (a+b)\sinh(x)^4 + 2(a+b)\cosh(x)^2 + 2(3(a+b)\cosh(x)^2 + a + b)\sinh(x)^2 + 4((a+b)\cosh(x)^3 + (a+b)\cosh(x))\sinh(x) + a + b)\sqrt{-a-b}\arctan(\sqrt{2}(b\cosh(x)^2 + 2b\cosh(x)\sinh(x) + b\sinh(x)^2 - a - b)\sqrt{-a-b}\sqrt{((a+b)\cosh(x)^2 + (a+b)\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)})/((a*b + b^2)\cosh(x)^4 + 4(a*b + b^2)\cosh(x)\sinh(x)^3 + (a*b + b^2)\sinh(x)^4 + (a^2 - a*b - 2*b^2)\cosh(x)^2 + (6(a*b + b^2)\cosh(x)^2 + a^2 - a*b - 2*b^2)\sinh(x)^2 + a^2 + 2*a*b + b^2 + 2(2(a*b + b^2)\cosh(x)^3 + (a^2 - a*b - 2*b^2)\cosh(x))\sinh(x))) + 2((a+b)\cosh(x)^4 + 4(a+b)\cosh(x)\sinh(x)^3 + (a+b)\sinh(x)^4 + 2(a+b)\cosh(x)^2 + 2(3(a+b)\cosh(x)^2 + a + b)\sinh(x)^2 + 4((a+b)\cosh(x)^3 + (a+b)\cosh(x))\sinh(x) + a + b)\sqrt{-a-b}\arctan(\sqrt{2}\sqrt{-a-b}\sqrt{((a+b)\cosh(x)^2 + (a+b)\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)})/((a+b)\cosh(x)^2 + 2(a+b)\cosh(x)\sinh(x) + (a+b)\sinh(x)^2 + a + b)) - ((3a + 2b)\cosh(x)^4 + 4(3a + 2b)\cosh(x)\sinh(x)^3 + (3a + 2b)\sinh(x)^4 + 2(3a + 2b)\cosh(x)^2 + 2(3(3a + 2b)\cosh(x)^2 + 3a + 2b)\sinh(x)^2 + 4((3a + 2b)\cosh(x)^3 + (3a + 2b)\cosh(x))\sinh(x) + 3a + 2b)\sqrt{b}\log(-(a + 2b)\cosh(x)^4 + 4(a + 2b)\cosh(x)\sinh(x)^3 + (a + 2b)\sinh(x)^4 + 2(a - 2b)\cosh(x)^2 + 2(3(a + 2b)\cosh(x)^2 + a - 2b)\sinh(x)^2 - 2\sqrt{2}(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1)\sqrt{b}\sqrt{((a+b)\cosh(x)^2 + (a+b)\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)} + 4((a + 2b)\cosh(x)^3 + (a - 2b)\cosh(x))\sinh(x) + a + 2b)/(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2(3\cosh(x)^2 + 1)\sinh(x)^2 + 2\cosh(x)^2 + 4(\cosh(x)^3 + \cosh(x))\sinh(x) + 1)) + 2\sqrt{2}(b\cosh(x)^2 + 2b\cosh(x)\sinh(x) + b\sinh(x)^2 - b)\sqrt{((a+b)\cosh(x)^2 + (a+b)\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)}/(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2(3\cosh(x)^2 + 1)\sinh(x)^2 + 2\cosh(x)^2 + 4(\cosh(x)^3 + \cosh(x))\sinh(x) + 1), -1/2(((a+b)\cosh(x)^4 + 4(a+b)\cosh(x)\sinh(x)^3 + (a+b)\sinh(x)^4 + 2(a+b)\cosh(x)^2 + 2(3(a+b)\cosh(x)^2 + a + b)\sinh(x)^2 + 4((a+b)\cosh(x)^3 + (a+b)\cosh(x))\sinh(x) + a + b)\sqrt{-a-b}\arctan(\sqrt{2}(b\cosh(x)^2 + 2b\cosh(x)\sinh(x) + b\sinh(x)^2 - a - b)\sqrt{-a-b}\sqrt{((a+b)\cosh(x)^2 + (a+b)\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)})/((a*b + b^2)\cosh(x)^4 +
\end{aligned}$$

```

4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b
^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a
^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x)
)*sinh(x))) - ((3*a + 2*b)*cosh(x)^4 + 4*(3*a + 2*b)*cosh(x)*sinh(x)^3 + (3
*a + 2*b)*sinh(x)^4 + 2*(3*a + 2*b)*cosh(x)^2 + 2*(3*(3*a + 2*b)*cosh(x)^2
+ 3*a + 2*b)*sinh(x)^2 + 4*((3*a + 2*b)*cosh(x)^3 + (3*a + 2*b)*cosh(x))*si
nh(x) + 3*a + 2*b)*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^4 + 4*(a +
b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a +
b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*
sinh(x) + a + b)) + ((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a +
b)*sinh(x)^4 + 2*(a + b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a + b)*sinh(
x)^2 + 4*((a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*sqrt(-a - b
)*arctan(sqrt(2)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^2 + 2
*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)) + sqrt(2)*(b*cosh(x)
^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt(((a + b)*cosh(x)^2 + (a +
b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)
)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*c
osh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)]

```

giac [B] time = 2.00, size = 584, normalized size = 6.64

$$-\frac{1}{2}(a+b)^{\frac{3}{2}} \log\left(\left|-\sqrt{a+b}e^{2x} + \sqrt{ae^{4x} + be^{4x} + 2ae^{2x} - 2be^{2x} + a + b} + \sqrt{a+b}\right|\right) + \frac{1}{2}(a+b)^{\frac{3}{2}} \log\left(\left|-\sqrt{a+b}e^{2x} + \sqrt{ae^{4x} + be^{4x} + 2ae^{2x} - 2be^{2x} + a + b} + \sqrt{a+b}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
[Out] -1/2*(a + b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x)
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*(a + b)^(3/2)*
log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2
*b*e^(2*x) + a + b) - sqrt(a + b))) - (3*a*b + 2*b^2)*arctan(-1/2*(sqrt(a +
b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*(a^2 + 2*a*b + b^2)*log(abs(-sq
rt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
+ a + b))*(a + b) - sqrt(a + b)*(a - b))/sqrt(a + b) - 2*((a*b + 2*b^2)*(s
qrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
+ a + b))^3 + (3*a*b - 2*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^
(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b) + (3*a^2*b - 3*a*
b^2 - 2*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
) - 2*b*e^(2*x) + a + b)) + (a^2*b - a*b^2 + 2*b^3)*sqrt(a + b))/((sqrt(a +
b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)^2

```

maple [B] time = 0.09, size = 578, normalized size = 6.57

$$\frac{\left(\frac{(\tanh(x) - 1)^2 b + 2(\tanh(x) - 1)b + a + b}{6}\right)^{\frac{3}{2}} b \sqrt{\frac{(\tanh(x) - 1)^2 b + 2(\tanh(x) - 1)b + a + b}{4}} \tanh(x) + 3\sqrt{b}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] int((a+b*tanh(x)^2)^(3/2),x)

```



```
[Out] -1/6*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(3/2)-1/4*b*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*tanh(x)-3/4*b^(1/2)*ln(((tanh(x)-1)*b+b)/b^(1/2))+((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))*a+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2))*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))*a-1/2*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))*a-1/2*b^(3/2)*ln(((tanh(x)-1)*b+b)/b^(1/2))+((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2))*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))*b-1/2*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))*b+1/6*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(3/2)-1/4*b*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)*tanh(x)-3/4*b^(1/2)*ln(((1+tanh(x))*b-b)/b^(1/2))+((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))*a-1/2*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2))*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))*(a+b)^(1/2))*a+1/2*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))*a-1/2*b^(3/2)*ln(((1+tanh(x))*b-b)/b^(1/2))+((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))-1/2*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2))*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))*(a+b)^(1/2))*b+1/2*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \tanh(x)^2 + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)^2)^(3/2), x, algorithm="maxima")
```

```
[Out] integrate((b*tanh(x)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (b \tanh(x)^2 + a)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*tanh(x)^2)^(3/2), x)
```

```
[Out] int((a + b*tanh(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(x))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*tanh(x)**2)**(3/2), x)
```

```
[Out] Integral((a + b*tanh(x)**2)**(3/2), x)
```

3.223 $\int \coth(x) (a + b \tanh^2(x))^{3/2} dx$

Optimal. Leaf size=71

$$a^{3/2} \left(-\tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a}} \right) \right) - b \sqrt{a + b \tanh^2(x)} + (a + b)^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}} \right)$$

[Out] $-a^{(3/2)}*\operatorname{arctanh}((a+b*\tanh(x)^2)^{(1/2)}/a^{(1/2)})+(a+b)^{(3/2)}*\operatorname{arctanh}((a+b*\tanh(x)^2)^{(1/2)}/(a+b)^{(1/2)})-b*(a+b*\tanh(x)^2)^{(1/2)}$

Rubi [A] time = 0.14, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3670, 446, 84, 156, 63, 208}

$$a^{3/2} \left(-\tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a}} \right) \right) + (a + b)^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}} \right) - b \sqrt{a + b \tanh^2(x)}$$

Antiderivative was successfully verified.

[In] `Int[Coth[x]*(a + b*Tanh[x]^2)^(3/2),x]`

[Out] $-(a^{(3/2)}*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tanh}[x]^2]/\operatorname{Sqrt}[a]]) + (a + b)^{(3/2)}*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tanh}[x]^2]/\operatorname{Sqrt}[a + b]] - b*\operatorname{Sqrt}[a + b*\operatorname{Tanh}[x]^2]$

Rule 63

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 84

`Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Simp[(f*(e + f*x)^(p - 1))/(b*d*(p - 1)), x] + Dist[1/(b*d), Int[(b*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x*(e + f*x)^(p - 2)]/((a + b*x)*(c + d*x)), x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]`

Rule 156

`Int[(((e_.) + (f_.)*(x_))^(p_))*((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]`

Rule 208

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 446

`Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]`

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_.)])^(m_.)*((a_.) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f*f^2*x^2), x], x, (c*Tan[e + f*x])/ff, x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
 \int \coth(x) (a + b \tanh^2(x))^{3/2} dx &= \text{Subst} \left(\int \frac{(a + bx^2)^{3/2}}{x(1-x^2)} dx, x, \tanh(x) \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{(a + bx)^{3/2}}{(1-x)x} dx, x, \tanh^2(x) \right) \\
 &= -b\sqrt{a + b \tanh^2(x)} - \frac{1}{2} \text{Subst} \left(\int \frac{-a^2 + (-2a - b)bx}{(1-x)x\sqrt{a + bx}} dx, x, \tanh^2(x) \right) \\
 &= -b\sqrt{a + b \tanh^2(x)} + \frac{1}{2} a^2 \text{Subst} \left(\int \frac{1}{x\sqrt{a + bx}} dx, x, \tanh^2(x) \right) + \frac{1}{2} (a + b) \text{Subst} \left(\int \frac{1}{x\sqrt{a + bx}} dx, x, \tanh^2(x) \right) \\
 &= -b\sqrt{a + b \tanh^2(x)} + \frac{a^2 \text{Subst} \left(\int \frac{1}{\frac{-a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + b \tanh^2(x)} \right)}{b} + \frac{(a + b) \text{Subst} \left(\int \frac{1}{x\sqrt{a + bx}} dx, x, \tanh^2(x) \right)}{2} \\
 &= -a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a}} \right) + (a + b)^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}} \right)
 \end{aligned}$$

Mathematica [A] time = 0.07, size = 71, normalized size = 1.00

$$a^{3/2} \left(-\tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a}} \right) \right) - b\sqrt{a + b \tanh^2(x)} + (a + b)^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]*(a + b*Tanh[x]^2)^(3/2), x]

[Out] -(a^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]) + (a + b)^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - b*Sqrt[a + b*Tanh[x]^2]

fricas [B] time = 0.65, size = 4039, normalized size = 56.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(a+b*tanh(x)^2)^(3/2), x, algorithm="fricas")

[Out] [1/4*(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5

$$\begin{aligned}
 & a + b) \cdot \log(-((a + b) \cdot \cosh(x)^4 + 4 \cdot (a + b) \cdot \cosh(x) \cdot \sinh(x)^3 + (a + b) \cdot \sinh(x)^4 - 2 \cdot b \cdot \cosh(x)^2 + 2 \cdot (3 \cdot (a + b) \cdot \cosh(x)^2 - b) \cdot \sinh(x)^2 + \sqrt{2} \cdot (\cosh(x)^2 + 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2 - 1) \cdot \sqrt{a + b} \cdot \sqrt{((a + b) \cdot \cosh(x)^2 + (a + b) \cdot \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2)}) + 4 \cdot ((a + b) \cdot \cosh(x)^3 - b \cdot \cosh(x)) \cdot \sinh(x) + a + b) / (\cosh(x)^2 + 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2)) - 4 \cdot \sqrt{2} \cdot b \cdot \sqrt{((a + b) \cdot \cosh(x)^2 + (a + b) \cdot \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2)}) / (\cosh(x)^2 + 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2 + 1), -1/2 \cdot (((a + b) \cdot \cosh(x)^2 + 2 \cdot (a + b) \cdot \cosh(x) \cdot \sinh(x) + (a + b) \cdot \sinh(x)^2 + a + b) \cdot \sqrt{-a - b} \cdot \arctan(\sqrt{2} \cdot (a \cdot \cosh(x)^2 + 2 \cdot a \cdot \cosh(x) \cdot \sinh(x) + a \cdot \sinh(x)^2 + a + b) \cdot \sqrt{-a - b} \cdot \sqrt{((a + b) \cdot \cosh(x)^2 + (a + b) \cdot \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2)}) / ((a^2 + a \cdot b) \cdot \cosh(x)^4 + 4 \cdot (a^2 + a \cdot b) \cdot \cosh(x) \cdot \sinh(x)^3 + (a^2 + a \cdot b) \cdot \sinh(x)^4 + (2 \cdot a^2 + a \cdot b - b^2) \cdot \cosh(x)^2 + (6 \cdot (a^2 + a \cdot b) \cdot \cosh(x)^2 + 2 \cdot a^2 + a \cdot b - b^2) \cdot \sinh(x)^2 + a^2 + 2 \cdot a \cdot b + b^2 + 2 \cdot (2 \cdot (a^2 + a \cdot b) \cdot \cosh(x)^3 + (2 \cdot a^2 + a \cdot b - b^2) \cdot \cosh(x)) \cdot \sinh(x))) + ((a + b) \cdot \cosh(x)^2 + 2 \cdot (a + b) \cdot \cosh(x) \cdot \sinh(x) + (a + b) \cdot \sinh(x)^2 + a + b) \cdot \sqrt{-a - b} \cdot \arctan(\sqrt{2} \cdot (\cosh(x)^2 + 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2 - 1) \cdot \sqrt{-a - b} \cdot \sqrt{((a + b) \cdot \cosh(x)^2 + (a + b) \cdot \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2)}) / ((a + b) \cdot \cosh(x)^4 + 4 \cdot (a + b) \cdot \cosh(x) \cdot \sinh(x)^3 + (a + b) \cdot \sinh(x)^4 + 2 \cdot (a - b) \cdot \cosh(x)^2 + 2 \cdot (3 \cdot (a + b) \cdot \cosh(x)^2 + a - b) \cdot \sinh(x)^2 + 4 \cdot ((a + b) \cdot \cosh(x)^3 + (a - b) \cdot \cosh(x)) \cdot \sinh(x) + a + b)) - (a \cdot \cosh(x)^2 + 2 \cdot a \cdot \cosh(x) \cdot \sinh(x) + a \cdot \sinh(x)^2 + a) \cdot \sqrt{a} \cdot \log(-((2 \cdot a + b) \cdot \cosh(x)^4 + 4 \cdot (2 \cdot a + b) \cdot \cosh(x) \cdot \sinh(x)^3 + (2 \cdot a + b) \cdot \sinh(x)^4 + 2 \cdot (2 \cdot a - b) \cdot \cosh(x)^2 + 2 \cdot (3 \cdot (2 \cdot a + b) \cdot \cosh(x)^2 + 2 \cdot a - b) \cdot \sinh(x)^2 - 2 \cdot \sqrt{2} \cdot (\cosh(x)^2 + 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2 + 1) \cdot \sqrt{a} \cdot \sqrt{((a + b) \cdot \cosh(x)^2 + (a + b) \cdot \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2)}) + 4 \cdot ((2 \cdot a + b) \cdot \cosh(x)^3 + (2 \cdot a - b) \cdot \cosh(x)) \cdot \sinh(x) + 2 \cdot a + b) / (\cosh(x)^4 + 4 \cdot \cosh(x) \cdot \sinh(x)^3 + \sinh(x)^4 + 2 \cdot (3 \cdot \cosh(x)^2 - 1) \cdot \sinh(x)^2 - 2 \cdot \cosh(x)^2 + 4 \cdot (\cosh(x)^3 - \cosh(x)) \cdot \sinh(x) + 1)) + 2 \cdot \sqrt{2} \cdot b \cdot \sqrt{((a + b) \cdot \cosh(x)^2 + (a + b) \cdot \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2)}) / (\cosh(x)^2 + 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2 + 1), 1/2 \cdot (2 \cdot (a \cdot \cosh(x)^2 + 2 \cdot \cosh(x) \cdot \sinh(x) + a \cdot \sinh(x)^2 + a) \cdot \sqrt{-a} \cdot \arctan(\sqrt{2} \cdot (\cosh(x)^2 + 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2 + 1) \cdot \sqrt{-a} \cdot \sqrt{((a + b) \cdot \cosh(x)^2 + (a + b) \cdot \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2)}) / ((a + b) \cdot \cosh(x)^4 + 4 \cdot (a + b) \cdot \cosh(x) \cdot \sinh(x)^3 + (a + b) \cdot \sinh(x)^4 + 2 \cdot (a - b) \cdot \cosh(x)^2 + 2 \cdot (3 \cdot (a + b) \cdot \cosh(x)^2 + a - b) \cdot \sinh(x)^2 + 4 \cdot ((a + b) \cdot \cosh(x)^3 + (a - b) \cdot \cosh(x)) \cdot \sinh(x) + a + b)) - ((a + b) \cdot \cosh(x)^2 + 2 \cdot (a + b) \cdot \cosh(x) \cdot \sinh(x) + (a + b) \cdot \sinh(x)^2 + a + b) \cdot \sqrt{-a - b} \cdot \arctan(\sqrt{2} \cdot (a \cdot \cosh(x)^2 + 2 \cdot a \cdot \cosh(x) \cdot \sinh(x) + a \cdot \sinh(x)^2 + a + b) \cdot \sqrt{-a - b} \cdot \sqrt{((a + b) \cdot \cosh(x)^2 + (a + b) \cdot \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2)}) / ((a^2 + a \cdot b) \cdot \cosh(x)^4 + 4 \cdot (a^2 + a \cdot b) \cdot \cosh(x) \cdot \sinh(x)^3 + (a^2 + a \cdot b) \cdot \sinh(x)^4 + (2 \cdot a^2 + a \cdot b - b^2) \cdot \cosh(x)^2 + (6 \cdot (a^2 + a \cdot b) \cdot \cosh(x)^2 + 2 \cdot a^2 + a \cdot b - b^2) \cdot \sinh(x)^2 + a^2 + 2 \cdot a \cdot b + b^2 + 2 \cdot (2 \cdot (a^2 + a \cdot b) \cdot \cosh(x)^3 + (2 \cdot a^2 + a \cdot b - b^2) \cdot \cosh(x)) \cdot \sinh(x))) - ((a + b) \cdot \cosh(x)^2 + 2 \cdot (a + b) \cdot \cosh(x) \cdot \sinh(x) + (a + b) \cdot \sinh(x)^2 + a + b) \cdot \sqrt{-a - b} \cdot \arctan(\sqrt{2} \cdot (\cosh(x)^2 + 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2 - 1) \cdot \sqrt{-a - b} \cdot \sqrt{((a + b) \cdot \cosh(x)^2 + (a + b) \cdot \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2)}) / ((a + b) \cdot \cosh(x)^4 + 4 \cdot (a + b) \cdot \cosh(x) \cdot \sinh(x)^3 + (a + b) \cdot \sinh(x)^4 + 2 \cdot (a - b) \cdot \cosh(x)^2 + 2 \cdot (3 \cdot (a + b) \cdot \cosh(x)^2 + a - b) \cdot \sinh(x)^2 + 4 \cdot ((a + b) \cdot \cosh(x)^3 + (a - b) \cdot \cosh(x)) \cdot \sinh(x) + a + b)) - 2 \cdot \sqrt{2} \cdot b \cdot \sqrt{((a + b) \cdot \cosh(x)^2 + (a + b) \cdot \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2)}) / (\cosh(x)^2 + 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2 + 1)]
 \end{aligned}$$

giac [B] time = 2.78, size = 433, normalized size = 6.10

$$\frac{2a^2 \arctan\left(-\frac{\sqrt{a+b}e^{2x} - \sqrt{ae^{4x} + be^{4x} + 2ae^{2x} - 2be^{2x} + a + b} - \sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{1}{2}(a+b)^{\frac{3}{2}} \log\left(\left|-\sqrt{a+b}e^{2x} + \sqrt{ae^{4x} + be^{4x} + 2ae^{2x} - 2be^{2x} + a + b}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")

[Out] $2*a^2*\arctan(-1/2*(\sqrt{a+b}*e^{2*x} - \sqrt{a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b})/\sqrt{-a})/\sqrt{-a} + 1/2*(a+b)^{3/2}*\log(\text{abs}(-\sqrt{a+b}*e^{2*x} + \sqrt{a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b})) - 1/2*(a+b)^{3/2}*\log(\text{abs}(-\sqrt{a+b}*e^{2*x} + \sqrt{a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b}) - \sqrt{a+b})) - 1/2*(a^2 + 2*a*b + b^2)*\log(\text{abs}(-(\sqrt{a+b}*e^{2*x} - \sqrt{a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b))*(a+b) - \sqrt{a+b}*(a-b)))/\sqrt{a+b} - 4*((\sqrt{a+b}*e^{2*x} - \sqrt{a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b})*b^2 - \sqrt{a+b}*b^2)/((\sqrt{a+b}*e^{2*x} - \sqrt{a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b})^2 + 2*(\sqrt{a+b}*e^{2*x} - \sqrt{a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b})*\sqrt{a+b} + a - 3*b)$

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \coth(x) \left(a + b \left(\tanh^2(x) \right) \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)*(a+b*tanh(x)^2)^(3/2),x)

[Out] int(coth(x)*(a+b*tanh(x)^2)^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \tanh(x)^2 + a \right)^{\frac{3}{2}} \coth(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*tanh(x)^2 + a)^(3/2)*coth(x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(x) \left(b \tanh(x)^2 + a \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)*(a + b*tanh(x)^2)^(3/2),x)

[Out] int(coth(x)*(a + b*tanh(x)^2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \tanh^2(x) \right)^{\frac{3}{2}} \coth(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(a+b*tanh(x)**2)**(3/2),x)

[Out] Integral((a + b*tanh(x)**2)**(3/2)*coth(x), x)

3.224 $\int \coth^2(x) (a + b \tanh^2(x))^{3/2} dx$

Optimal. Leaf size=77

$$b^{3/2} \left(-\tanh^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right) \right) + (a+b)^{3/2} \tanh^{-1} \left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right) - a \coth(x) \sqrt{a + b \tanh^2(x)}$$

[Out] $-b^{(3/2)} * \operatorname{arctanh}(b^{(1/2)} * \tanh(x) / (a + b * \tanh(x)^2)^{(1/2)}) + (a+b)^{(3/2)} * \operatorname{arctanh}((a+b)^{(1/2)} * \tanh(x) / (a + b * \tanh(x)^2)^{(1/2)}) - a * \coth(x) * (a + b * \tanh(x)^2)^{(1/2)}$

Rubi [A] time = 0.13, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3670, 474, 523, 217, 206, 377}

$$b^{3/2} \left(-\tanh^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right) \right) + (a+b)^{3/2} \tanh^{-1} \left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right) - a \coth(x) \sqrt{a + b \tanh^2(x)}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^2*(a + b*Tanh[x]^2)^(3/2), x]

[Out] $-(b^{(3/2)} * \operatorname{ArcTanh}[(\operatorname{Sqrt}[b] * \operatorname{Tanh}[x]) / \operatorname{Sqrt}[a + b * \operatorname{Tanh}[x]^2]]) + (a + b)^{(3/2)} * \operatorname{ArcTanh}[(\operatorname{Sqrt}[a + b] * \operatorname{Tanh}[x]) / \operatorname{Sqrt}[a + b * \operatorname{Tanh}[x]^2]] - a * \operatorname{Coth}[x] * \operatorname{Sqrt}[a + b * \operatorname{Tanh}[x]^2]$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 474

Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(c*(e*x)^(m+1)*(a + b*x^n)^(p+1)*(c + d*x^n)^(q-1))/(a*e*(m+1)), x] - Dist[1/(a*e^n*(m+1)), Int[(e*x)^(m+n)*(a + b*x^n)^p*(c + d*x^n)^(q-2)*Simp[c*(c*b - a*d)*(m+1) + c*n*(b*c*(p+1) + a*d*(q-1)) + d*((c*b - a*d)*(m+1) + c*b*n*(p+q))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 523

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d}

, e, f, n}, x]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_.)])^(m_.)*((a_.) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_.)])^(n_.))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
 \int \coth^2(x) (a + b \tanh^2(x))^{3/2} dx &= \text{Subst} \left(\int \frac{(a + bx^2)^{3/2}}{x^2 (1 - x^2)} dx, x, \tanh(x) \right) \\
 &= -a \coth(x) \sqrt{a + b \tanh^2(x)} + \text{Subst} \left(\int \frac{a(a + 2b) + b^2 x^2}{(1 - x^2) \sqrt{a + bx^2}} dx, x, \tanh(x) \right) \\
 &= -a \coth(x) \sqrt{a + b \tanh^2(x)} - b^2 \text{Subst} \left(\int \frac{1}{\sqrt{a + bx^2}} dx, x, \tanh(x) \right) + (a + b) \text{Subst} \left(\int \frac{1}{1 - bx^2} dx, x, \frac{\tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right) \\
 &= -a \coth(x) \sqrt{a + b \tanh^2(x)} - b^2 \text{Subst} \left(\int \frac{1}{1 - bx^2} dx, x, \frac{\tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right) \\
 &= -b^{3/2} \tanh^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right) + (a + b)^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b} \tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right) - a \coth(x) \sqrt{a + b \tanh^2(x)}
 \end{aligned}$$

Mathematica [C] time = 3.00, size = 197, normalized size = 2.56

$$\frac{a \tanh(x) \left(\text{csch}^2(x) ((a + b) \cosh(2x) + a - b) - \sqrt{2} (a + 2b) \sqrt{\frac{\text{csch}^2(x) ((a + b) \cosh(2x) + a - b)}{b}} F \left(\sin^{-1} \left(\frac{\sqrt{\frac{(a - b + (a + b) \cosh(2x))}{b}}}{\sqrt{2}} \right) \right) \right)}{\sqrt{2} \sqrt{\text{sech}^2(x) ((a + b) \cosh(2x) + a - b)}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]^2*(a + b*Tanh[x]^2)^(3/2), x]

[Out] -((a*((a - b + (a + b)*Cosh[2*x])*Csch[x]^2 - Sqrt[2]*(a + 2*b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] + Sqrt[2]*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1])*Tanh[x])/(Sqrt[2]*Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2]))

fricas [B] time = 0.64, size = 3913, normalized size = 50.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2*(a+b*tanh(x)^2)^(3/2), x, algorithm="fricas")


```
[Out] [1/4*(((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 -
a - b)*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*
sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^
2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cos
h(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*
b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^
3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5
- 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*
sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(
x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 -
3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 +
sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*c
osh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2
*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 -
18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*
(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x)
)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*
b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 -
3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*c
osh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*co
sh(x)*sinh(x)^5 + sinh(x)^6)) + 2*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*si
nh(x)^2 - b)*sqrt(b)*log(-((a + 2*b)*cosh(x)^4 + 4*(a + 2*b)*cosh(x)*sinh(x)
)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(x)^2 + 2*(3*(a + 2*b)*cosh(x)^
2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)
^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(
x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + 2*b)*cosh(x)^3 + (a - 2*b)
*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 +
2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh
(x) + 1)) + ((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)
)^2 - a - b)*sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)
^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)
)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*s
qrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*
sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/
(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*sqrt(2)*a*sqrt(((a + b)*co
sh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(
x)^2)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1), 1/4*(4*(b*cosh(x)^
2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - b)*sqrt(-b)*arctan(sqrt(2)*(cosh(x)
^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 +
(a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a
+ b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a -
b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(
x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + ((a + b)*cosh(x)^2 + 2*(a + b)*
cosh(x)*sinh(x) + (a + b)*sinh(x)^2 - a - b)*sqrt(a + b)*log(-((a*b^2 + b^3)
)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 -
2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^
2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*s
inh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*co
sh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sin
h(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^
3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 +
b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6
- 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4
*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(
x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*
sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b -
3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b
^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 -
```

$$\begin{aligned}
& (a^2 - 2ab - 3b^2) \cosh(x) \sinh(x) \sqrt{a+b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)} \\
& + 4(2(a^2b^2 + b^3) \cosh(x)^7 - 3(a^2b^2 + 2b^3) \cosh(x)^5 + (a^3 - a^2b + 4a^2b^2 + 6b^3) \cosh(x)^3 + (a^3 - 3a^2b^2 - 2b^3) \cosh(x) \sinh(x)) / \\
& (\cosh(x)^6 + 6 \cosh(x)^5 \sinh(x) + 15 \cosh(x)^4 \sinh(x)^2 + 20 \cosh(x)^3 \sinh(x)^3 + 15 \cosh(x)^2 \sinh(x)^4 + 6 \cosh(x) \sinh(x)^5 + \sinh(x)^6) + ((a+b) \cosh(x)^2 + 2(a+b) \cosh(x) \sinh(x) + (a+b) \sinh(x)^2 - a - b) \sqrt{a+b} \log(((a+b) \cosh(x)^4 + 4(a+b) \cosh(x) \sinh(x)^3 + (a+b) \sinh(x)^4 + 2a \cosh(x)^2 + 2(3(a+b) \cosh(x)^2 + a) \sinh(x)^2 + \sqrt{2}(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1) \sqrt{a+b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)})) + 4((a+b) \cosh(x)^3 + a \cosh(x) \sinh(x) + a + b) / (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2) - 4 \sqrt{2} a \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1), -1/2(((a+b) \cosh(x)^2 + 2(a+b) \cosh(x) \sinh(x) + (a+b) \sinh(x)^2 - a - b) \sqrt{-a-b} \arctan(\sqrt{2} (b \cosh(x)^2 + 2b \cosh(x) \sinh(x) + b \sinh(x)^2 - a - b) \sqrt{-a-b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)})) / ((a^2b + b^2) \cosh(x)^4 + 4(a^2b + b^2) \cosh(x) \sinh(x)^3 + (a^2b + b^2) \sinh(x)^4 + (a^2 - a^2b - 2b^2) \cosh(x)^2 + (6(a^2b + b^2) \cosh(x)^2 + a^2 - a^2b - 2b^2) \sinh(x)^2 + a^2 + 2a^2b + b^2 + 2(2(a^2b + b^2) \cosh(x)^3 + (a^2 - a^2b - 2b^2) \cosh(x) \sinh(x))) + ((a+b) \cosh(x)^2 + 2(a+b) \cosh(x) \sinh(x) + (a+b) \sinh(x)^2 - a - b) \sqrt{-a-b} \arctan(\sqrt{2} \sqrt{-a-b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / ((a+b) \cosh(x)^2 + 2(a+b) \cosh(x) \sinh(x) + (a+b) \sinh(x)^2 + a + b)) - (b \cosh(x)^2 + 2b \cosh(x) \sinh(x) + b \sinh(x)^2 - b) \sqrt{b} \log(-((a+2b) \cosh(x)^4 + 4(a+2b) \cosh(x) \sinh(x)^3 + (a+2b) \sinh(x)^4 + 2(a-2b) \cosh(x)^2 + 2(3(a+2b) \cosh(x)^2 + a - 2b) \sinh(x)^2 - 2 \sqrt{2} (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) \sqrt{b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)})) + 4((a+2b) \cosh(x)^3 + (a-2b) \cosh(x) \sinh(x) + a + 2b) / (\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2(3 \cosh(x)^2 + 1) \sinh(x)^2 + 2 \cosh(x)^2 + 4(\cosh(x)^3 + \cosh(x)) \sinh(x) + 1)) + 2 \sqrt{2} a \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1), -1/2(((a+b) \cosh(x)^2 + 2(a+b) \cosh(x) \sinh(x) + (a+b) \sinh(x)^2 - a - b) \sqrt{-a-b} \arctan(\sqrt{2} (b \cosh(x)^2 + 2b \cosh(x) \sinh(x) + b \sinh(x)^2 - a - b) \sqrt{-a-b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)})) / ((a^2b + b^2) \cosh(x)^4 + 4(a^2b + b^2) \cosh(x) \sinh(x)^3 + (a^2b + b^2) \sinh(x)^4 + (a^2 - a^2b - 2b^2) \cosh(x)^2 + (6(a^2b + b^2) \cosh(x)^2 + a^2 - a^2b - 2b^2) \sinh(x)^2 + a^2 + 2a^2b + b^2 + 2(2(a^2b + b^2) \cosh(x)^3 + (a^2 - a^2b - 2b^2) \cosh(x) \sinh(x))) - 2(b \cosh(x)^2 + 2b \cosh(x) \sinh(x) + b \sinh(x)^2 - b) \sqrt{-b} \arctan(\sqrt{2} (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) \sqrt{-b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / ((a+b) \cosh(x)^4 + 4(a+b) \cosh(x) \sinh(x)^3 + (a+b) \sinh(x)^4 + 2(a-b) \cosh(x)^2 + 2(3(a+b) \cosh(x)^2 + a - b) \sinh(x)^2 + 4((a+b) \cosh(x)^3 + (a-b) \cosh(x) \sinh(x) + a + b)) + ((a+b) \cosh(x)^2 + 2(a+b) \cosh(x) \sinh(x) + (a+b) \sinh(x)^2 - a - b) \sqrt{-a-b} \arctan(\sqrt{2} \sqrt{-a-b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / ((a+b) \cosh(x)^2 + 2(a+b) \cosh(x) \sinh(x) + (a+b) \sinh(x)^2 + a + b)) + 2 \sqrt{2} a \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1)}]
\end{aligned}$$

giac [B] time = 2.79, size = 430, normalized size = 5.58

$$\frac{2b^2 \arctan\left(-\frac{\sqrt{a+b}e^{2x} - \sqrt{ae^{4x} + be^{4x} + 2ae^{2x} - 2be^{2x} + a + b} + \sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{1}{2}(a+b)^{\frac{3}{2}} \log\left(\left|-\sqrt{a+b}e^{2x} + \sqrt{ae^{4x} + be^{4x} + 2ae^{2x} - 2be^{2x} + a + b}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")

[Out] $-2*b^2*\arctan(-1/2*(\sqrt{a+b}*e^{2*x} - \sqrt{a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b})/\sqrt{-b})/\sqrt{-b} - 1/2*(a + b)^{(3/2)}*\log(\text{abs}(-\sqrt{a+b}*e^{2*x} + \sqrt{a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b})) + 1/2*(a + b)^{(3/2)}*\log(\text{abs}(-\sqrt{a+b}*e^{2*x} + \sqrt{a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b}) - \sqrt{a+b})) - 1/2*(a^2 + 2*a*b + b^2)*\log(\text{abs}(-(\sqrt{a+b}*e^{2*x} - \sqrt{a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b))* (a + b) - \sqrt{a+b}*(a - b)))/\sqrt{a+b} + 4*((\sqrt{a+b}*e^{2*x} - \sqrt{a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b})*a^2 + \sqrt{a+b}*a^2)/((\sqrt{a+b}*e^{2*x} - \sqrt{a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b})^2 - 2*(\sqrt{a+b}*e^{2*x} - \sqrt{a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b})*\sqrt{a+b} - 3*a + b)$

maple [F] time = 0.30, size = 0, normalized size = 0.00

$$\int (\coth^2(x) (a + b (\tanh^2(x)))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x)

[Out] int(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \tanh(x)^2 + a)^{\frac{3}{2}} \coth(x)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*tanh(x)^2 + a)^(3/2)*coth(x)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \coth(x)^2 (b \tanh(x)^2 + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^2*(a + b*tanh(x)^2)^(3/2),x)

[Out] int(coth(x)^2*(a + b*tanh(x)^2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^2(x))^{\frac{3}{2}} \coth^2(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**2*(a+b*tanh(x)**2)**(3/2),x)

[Out] Integral((a + b*tanh(x)**2)**(3/2)*coth(x)**2, x)

$$3.225 \quad \int \sqrt{1 + \tanh^2(x)} dx$$

Optimal. Leaf size=31

$$\sqrt{2} \tanh^{-1} \left(\frac{\sqrt{2} \tanh(x)}{\sqrt{\tanh^2(x) + 1}} \right) - \sinh^{-1}(\tanh(x))$$

[Out] -arcsinh(tanh(x))+arctanh(2^(1/2)*tanh(x)/(1+tanh(x)^2)^(1/2))*2^(1/2)

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3661, 402, 215, 377, 206}

$$\sqrt{2} \tanh^{-1} \left(\frac{\sqrt{2} \tanh(x)}{\sqrt{\tanh^2(x) + 1}} \right) - \sinh^{-1}(\tanh(x))$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + Tanh[x]^2], x]

[Out] -ArcSinh[Tanh[x]] + Sqrt[2]*ArcTanh[(Sqrt[2]*Tanh[x])/Sqrt[1 + Tanh[x]^2]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 402

Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])

Rule 3661

Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(ff*x)^n]^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned}
\int \sqrt{1 + \tanh^2(x)} \, dx &= \text{Subst} \left(\int \frac{\sqrt{1+x^2}}{1-x^2} \, dx, x, \tanh(x) \right) \\
&= 2 \text{Subst} \left(\int \frac{1}{(1-x^2)\sqrt{1+x^2}} \, dx, x, \tanh(x) \right) - \text{Subst} \left(\int \frac{1}{\sqrt{1+x^2}} \, dx, x, \tanh(x) \right) \\
&= -\sinh^{-1}(\tanh(x)) + 2 \text{Subst} \left(\int \frac{1}{1-2x^2} \, dx, x, \frac{\tanh(x)}{\sqrt{1+\tanh^2(x)}} \right) \\
&= -\sinh^{-1}(\tanh(x)) + \sqrt{2} \tanh^{-1} \left(\frac{\sqrt{2} \tanh(x)}{\sqrt{1+\tanh^2(x)}} \right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 51, normalized size = 1.65

$$\frac{\cosh(x)\sqrt{\tanh^2(x)+1} \left(\sqrt{2} \sinh^{-1}(\sqrt{2} \sinh(x)) - \tanh^{-1}\left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}}\right) \right)}{\sqrt{\cosh(2x)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + Tanh[x]^2], x]

[Out] ((Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[x]] - ArcTanh[Sinh[x]/Sqrt[Cosh[2*x]])*Cosh[x]*Sqrt[1 + Tanh[x]^2])/Sqrt[Cosh[2*x]]

fricas [B] time = 0.46, size = 679, normalized size = 21.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+tanh(x)^2)^(1/2), x, algorithm="fricas")

[Out] 1/4*sqrt(2)*log(-2*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(x)^2 - 3)*sinh(x)^6 - 3*cosh(x)^6 + 2*(28*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 + 5*(14*cosh(x)^4 - 9*cosh(x)^2 + 1)*sinh(x)^4 + 5*cosh(x)^4 + 4*(14*cosh(x)^5 - 15*cosh(x)^3 + 5*cosh(x))*sinh(x)^3 + (28*cosh(x)^6 - 45*cosh(x)^4 + 30*cosh(x)^2 - 4)*sinh(x)^2 - 4*cosh(x)^2 + 2*(4*cosh(x)^7 - 9*cosh(x)^5 + 10*cosh(x)^3 - 4*cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^4 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x)^3 + (15*sqrt(2)*cosh(x)^4 - 18*sqrt(2)*cosh(x)^2 + 4*sqrt(2))*sinh(x)^2 + 4*sqrt(2)*cosh(x)^2 + 2*(3*sqrt(2)*cosh(x)^5 - 6*sqrt(2)*cosh(x)^3 + 4*sqrt(2)*cosh(x))*sinh(x) - 4*sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4)/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 1/4*sqrt(2)*log(2*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*cosh(x)^3 + cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1/2*log((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 2*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 1/2*log((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 2*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))

giac [B] time = 0.15, size = 104, normalized size = 3.35

$$-\frac{1}{2}\sqrt{2}\left(\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}+1}{\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}-1}\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)-\log\left(-\sqrt{e^{(4x)}+1}+e^{(2x)}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+tanh(x)^2)^(1/2),x, algorithm="giac")

[Out] -1/2*sqrt(2)*(sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)/(sqrt(2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) + log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) + log(sqrt(e^(4*x) + 1) - e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))

maple [B] time = 0.14, size = 97, normalized size = 3.13

$$-\frac{\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}{2}-\operatorname{arcsinh}(\tanh(x))+\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{(2\tanh(x)+2)\sqrt{2}}{4\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}\right)}{2}+\frac{\sqrt{(1+\tanh(x))^2-2\tanh(x)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+tanh(x)^2)^(1/2),x)

[Out] -1/2*((tanh(x)-1)^2+2*tanh(x))^(1/2)-arcsinh(tanh(x))+1/2*2^(1/2)*arctanh(1/4*(2*tanh(x)+2)*2^(1/2)/((tanh(x)-1)^2+2*tanh(x))^(1/2))+1/2*((1+tanh(x))^2-2*tanh(x))^(1/2)-1/2*2^(1/2)*arctanh(1/4*(2-2*tanh(x))*2^(1/2)/((1+tanh(x))^2-2*tanh(x))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tanh(x)^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(tanh(x)^2 + 1), x)

mupad [B] time = 0.23, size = 68, normalized size = 2.19

$$\frac{\sqrt{2}\left(\ln(\tanh(x)+1)-\ln\left(\sqrt{2}\sqrt{\tanh(x)^2+1}-\tanh(x)+1\right)\right)}{2}-\operatorname{asinh}(\tanh(x))+\frac{\sqrt{2}\left(\ln\left(\tanh(x)+\sqrt{2}\sqrt{\tanh(x)^2+1}\right)-\ln\left(\sqrt{2}\sqrt{\tanh(x)^2+1}-\tanh(x)+1\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tanh(x)^2 + 1)^(1/2),x)

[Out] (2^(1/2)*(log(tanh(x) + 1) - log(2^(1/2)*(tanh(x)^2 + 1)^(1/2) - tanh(x) + 1)))/2 - asinh(tanh(x)) + (2^(1/2)*(log(tanh(x) + 2^(1/2)*(tanh(x)^2 + 1)^(1/2) + 1) - log(tanh(x) - 1)))/2

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\tanh^2(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+tanh(x)**2)**(1/2),x)

[Out] Integral(sqrt(tanh(x)**2 + 1), x)

$$3.226 \quad \int \sqrt{-1 - \tanh^2(x)} dx$$

Optimal. Leaf size=45

$$\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right) - \sqrt{2} \tan^{-1}\left(\frac{\sqrt{2} \tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)$$

[Out] arctan(tanh(x)/(-1-tanh(x)^2)^(1/2))-arctan(2^(1/2)*tanh(x)/(-1-tanh(x)^2)^(1/2))*2^(1/2)

Rubi [A] time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3661, 402, 217, 203, 377}

$$\tan^{-1}\left(\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right) - \sqrt{2} \tan^{-1}\left(\frac{\sqrt{2} \tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-1 - Tanh[x]^2], x]

[Out] ArcTan[Tanh[x]/Sqrt[-1 - Tanh[x]^2]] - Sqrt[2]*ArcTan[(Sqrt[2]*Tanh[x])/Sqrt[-1 - Tanh[x]^2]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 402

Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])

Rule 3661

Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(ff*x)^n]^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned}
\int \sqrt{-1 - \tanh^2(x)} \, dx &= \text{Subst} \left(\int \frac{\sqrt{-1 - x^2}}{1 - x^2} \, dx, x, \tanh(x) \right) \\
&= - \left(2 \text{Subst} \left(\int \frac{1}{\sqrt{-1 - x^2} (1 - x^2)} \, dx, x, \tanh(x) \right) \right) + \text{Subst} \left(\int \frac{1}{\sqrt{-1 - x^2}} \, dx, x, \tanh(x) \right) \\
&= - \left(2 \text{Subst} \left(\int \frac{1}{1 + 2x^2} \, dx, x, \frac{\tanh(x)}{\sqrt{-1 - \tanh^2(x)}} \right) \right) + \text{Subst} \left(\int \frac{1}{1 + x^2} \, dx, x, \frac{\tanh(x)}{\sqrt{-1 - \tanh^2(x)}} \right) \\
&= \tan^{-1} \left(\frac{\tanh(x)}{\sqrt{-1 - \tanh^2(x)}} \right) - \sqrt{2} \tan^{-1} \left(\frac{\sqrt{2} \tanh(x)}{\sqrt{-1 - \tanh^2(x)}} \right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 1.18

$$\frac{\cosh(x) \sqrt{-\tanh^2(x) - 1} \left(\sqrt{2} \sinh^{-1} \left(\sqrt{2} \sinh(x) \right) - \tanh^{-1} \left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}} \right) \right)}{\sqrt{\cosh(2x)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-1 - Tanh[x]^2], x]

[Out] ((Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[x]] - ArcTanh[Sinh[x]/Sqrt[Cosh[2*x]]])*Cosh[x]*Sqrt[-1 - Tanh[x]^2])/Sqrt[Cosh[2*x]]

fricas [C] time = 0.43, size = 228, normalized size = 5.07

$$-\frac{1}{4} \sqrt{-2} \log \left(- \left(\sqrt{-2} \sqrt{-2 e^{4x} - 2} + 2 e^{2x} + 2 \right) e^{(-2x)} \right) + \frac{1}{4} \sqrt{-2} \log \left(\left(\sqrt{-2} \sqrt{-2 e^{4x} - 2} - 2 e^{2x} - 2 \right) e^{(-2x)} \right) + \frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-tanh(x)^2)^(1/2), x, algorithm="fricas")

[Out] -1/4*sqrt(-2)*log(-(sqrt(-2)*sqrt(-2*e^(4*x) - 2) + 2*e^(2*x) + 2)*e^(-2*x)) + 1/4*sqrt(-2)*log((sqrt(-2)*sqrt(-2*e^(4*x) - 2) - 2*e^(2*x) - 2)*e^(-2*x)) + 1/4*sqrt(-2)*log(-2*(sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 2) + sqrt(-2)*e^(4*x) - sqrt(-2)*e^(2*x) + 2*sqrt(-2))*e^(-4*x)) - 1/4*sqrt(-2)*log(-2*(sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 2) - sqrt(-2)*e^(4*x) + sqrt(-2)*e^(2*x) - 2*sqrt(-2))*e^(-4*x)) - 1/2*I*log((4*I*sqrt(-2*e^(4*x) - 2) - 4*e^(2*x) + 4)*e^(-2*x)) + 1/2*I*log((-4*I*sqrt(-2*e^(4*x) - 2) - 4*e^(2*x) + 4)*e^(-2*x))

giac [C] time = 0.17, size = 104, normalized size = 2.31

$$-\frac{1}{2} i \sqrt{2} \left(\sqrt{2} \log \left(\frac{\sqrt{2} - \sqrt{e^{4x} + 1} + e^{2x} + 1}{\sqrt{2} + \sqrt{e^{4x} + 1} - e^{2x} - 1} \right) + \log \left(\sqrt{e^{4x} + 1} - e^{2x} + 1 \right) + \log \left(\sqrt{e^{4x} + 1} - e^{2x} \right) - \log \left(- \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-tanh(x)^2)^(1/2), x, algorithm="giac")

[Out] -1/2*I*sqrt(2)*(sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)/(sqrt(2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) + log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) + log(sqrt(e^(4*x) + 1) - e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))

maple [B] time = 0.12, size = 142, normalized size = 3.16

$$\frac{\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}{2} + \frac{\arctan\left(\frac{\tanh(x)}{\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}\right)}{2} + \frac{\sqrt{2}\arctan\left(\frac{(-2-2\tanh(x))\sqrt{2}}{4\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}\right)}{2} + \frac{\sqrt{-(\tanh(x)+1)^2-2\tanh(x)}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1-tanh(x)^2)^(1/2),x)

[Out] -1/2*(-(tanh(x)-1)^2-2*tanh(x))^(1/2)+1/2*arctan(tanh(x)/(-(tanh(x)-1)^2-2*tanh(x))^(1/2))+1/2*2^(1/2)*arctan(1/4*(-2-2*tanh(x))*2^(1/2)/(-(tanh(x)-1)^2-2*tanh(x))^(1/2))+1/2*(-(1+tanh(x))^2+2*tanh(x))^(1/2)+1/2*arctan(tanh(x)/(-(1+tanh(x))^2+2*tanh(x))^(1/2))-1/2*2^(1/2)*arctan(1/4*(-2+2*tanh(x))*2^(1/2)/(-(1+tanh(x))^2+2*tanh(x))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\tanh(x)^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-tanh(x)^2 - 1), x)

mupad [B] time = 1.34, size = 43, normalized size = 0.96

$$-\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh(x)^2-1}}\right) - \ln\left(\tanh(x) - \sqrt{-\tanh(x)^2-1}\right) \operatorname{li}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-tanh(x)^2-1)^(1/2),x)

[Out] -log(tanh(x) - (-tanh(x)^2-1)^(1/2)*1i)*1i - 2^(1/2)*atan((2^(1/2)*tanh(x))/(-tanh(x)^2-1)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\tanh^2(x) - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-tanh(x)**2)**(1/2),x)

[Out] Integral(sqrt(-tanh(x)**2 - 1), x)

3.227 $\int (1 + \tanh^2(x))^{3/2} dx$

Optimal. Leaf size=50

$$-\frac{1}{2} \tanh(x) \sqrt{\tanh^2(x) + 1} + 2\sqrt{2} \tanh^{-1} \left(\frac{\sqrt{2} \tanh(x)}{\sqrt{\tanh^2(x) + 1}} \right) - \frac{5}{2} \sinh^{-1}(\tanh(x))$$

[Out] $-5/2*\operatorname{arcsinh}(\tanh(x))+2*\operatorname{arctanh}(2^{(1/2)}*\tanh(x)/(1+\tanh(x)^2)^{(1/2}))*2^{(1/2)}-1/2*(1+\tanh(x)^2)^{(1/2)}*\tanh(x)$

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3661, 416, 523, 215, 377, 206}

$$2\sqrt{2} \tanh^{-1} \left(\frac{\sqrt{2} \tanh(x)}{\sqrt{\tanh^2(x) + 1}} \right) - \frac{1}{2} \tanh(x) \sqrt{\tanh^2(x) + 1} - \frac{5}{2} \sinh^{-1}(\tanh(x))$$

Antiderivative was successfully verified.

[In] Int[(1 + Tanh[x]^2)^(3/2), x]

[Out] $(-5*\operatorname{ArcSinh}[\operatorname{Tanh}[x]])/2 + 2*\operatorname{Sqrt}[2]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[2]*\operatorname{Tanh}[x])/\operatorname{Sqrt}[1 + \operatorname{Tanh}[x]^2]] - (\operatorname{Tanh}[x]*\operatorname{Sqrt}[1 + \operatorname{Tanh}[x]^2])/2$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 416

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q-1))/(b*(n*(p+q)+1)), x] + Dist[1/(b*(n*(p+q)+1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q-2)*Simp[c*(b*c*(n*(p+q)+1) - a*d] + d*(b*c*(n*(p+2*q-1)+1) - a*d*(n*(q-1)+1))*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && NeQ[n*(p+q)+1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 523

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x_)^(n_)], x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :=
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n]^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned} \int (1 + \tanh^2(x))^{3/2} dx &= \text{Subst} \left(\int \frac{(1+x^2)^{3/2}}{1-x^2} dx, x, \tanh(x) \right) \\ &= -\frac{1}{2} \tanh(x) \sqrt{1 + \tanh^2(x)} - \frac{1}{2} \text{Subst} \left(\int \frac{-3-5x^2}{(1-x^2)\sqrt{1+x^2}} dx, x, \tanh(x) \right) \\ &= -\frac{1}{2} \tanh(x) \sqrt{1 + \tanh^2(x)} - \frac{5}{2} \text{Subst} \left(\int \frac{1}{\sqrt{1+x^2}} dx, x, \tanh(x) \right) + 4 \text{Subst} \left(\int \frac{1}{1-2x^2} dx, x, \frac{\tanh(x)}{\sqrt{1+\tanh^2(x)}} \right) \\ &= -\frac{5}{2} \sinh^{-1}(\tanh(x)) - \frac{1}{2} \tanh(x) \sqrt{1 + \tanh^2(x)} + 4 \text{Subst} \left(\int \frac{1}{1-2x^2} dx, x, \frac{\tanh(x)}{\sqrt{1+\tanh^2(x)}} \right) \\ &= -\frac{5}{2} \sinh^{-1}(\tanh(x)) + 2\sqrt{2} \tanh^{-1} \left(\frac{\sqrt{2} \tanh(x)}{\sqrt{1 + \tanh^2(x)}} \right) - \frac{1}{2} \tanh(x) \sqrt{1 + \tanh^2(x)} \end{aligned}$$

Mathematica [A] time = 0.15, size = 74, normalized size = 1.48

$$\frac{(\tanh^2(x) + 1)^{3/2} \left(-4\sqrt{2} \sinh^{-1}(\sqrt{2} \sinh(x)) \cosh^3(x) + \sinh(x) \sqrt{\cosh(2x)} \cosh(x) + 5 \cosh^3(x) \tanh^{-1} \left(\frac{\sqrt{2} \tanh(x)}{\sqrt{1 + \tanh^2(x)}} \right) \right)}{2 \cosh^2(2x)}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Tanh[x]^2)^(3/2), x]

[Out] -1/2*((-4*Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[x]]*Cosh[x]^3 + 5*ArcTanh[Sinh[x]/Sqrt[Cosh[2*x]])*Cosh[x]^3 + Cosh[x]*Sqrt[Cosh[2*x]]*Sinh[x])*(1 + Tanh[x]^2)^(3/2))/Cosh[2*x]^(3/2)

fricas [B] time = 0.43, size = 1027, normalized size = 20.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+tanh(x)^2)^(3/2), x, algorithm="fricas")

[Out] 1/4*(2*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + 2*(3*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 2*sqrt(2)*cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(-2*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(x)^2 - 3)*sinh(x)^6 - 3*cosh(x)^6 + 2*(28*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 + 5*(14*cosh(x)^4 - 9*cosh(x)^2 + 1)*sinh(x)^4 + 5*cosh(x)^4 + 4*(14*cosh(x)^5 - 15*cosh(x)^3 + 5*cosh(x))*sinh(x)^3 + (28*cosh(x)^6 - 45*cosh(x)^4 + 30*cosh(x)^2 - 4)*sinh(x)^2 - 4*cosh(x)^2 + 2*(4*cosh(x)^7 - 9*cosh(x)^5 + 10*cosh(x)^3 - 4*cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6 +

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3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^4 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x)*sinh(x)^3 + (15*sqrt(2)*cosh(x)^4 - 18*sqrt(2)*cosh(x)^2 + 4*sqrt(2))*sinh(x)^2 + 4*sqrt(2)*cosh(x)^2 + 2*(3*sqrt(2)*cosh(x)^5 - 6*sqrt(2)*cosh(x)^3 + 4*sqrt(2)*cosh(x))*sinh(x) - 4*sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4)/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6) + 2*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + 2*(3*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 2*sqrt(2)*cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(2*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*cosh(x)^3 + cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 5*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*log((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 2*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) + 5*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*log((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 2*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) - 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) - 4*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)

```

giac [B] time = 0.14, size = 202, normalized size = 4.04

$$-\frac{1}{4}\sqrt{2}\left(5\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{4x}+1}+e^{2x}+1}{\sqrt{2}+\sqrt{e^{4x}+1}-e^{2x}-1}\right)-\frac{4\left(3\left(\sqrt{e^{4x}+1}-e^{2x}\right)^3-\left(\sqrt{e^{4x}+1}-e^{2x}\right)^2-\sqrt{e^{4x}+1}\right)}{\left(\left(\sqrt{e^{4x}+1}-e^{2x}\right)^2-2\sqrt{e^{4x}+1}+2e^{2x}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+tanh(x)^2)^(3/2),x, algorithm="giac")

```

[Out] -1/4*sqrt(2)*(5*sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)/(sqrt(2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) - 4*(3*(sqrt(e^(4*x) + 1) - e^(2*x))^3 - (sqrt(e^(4*x) + 1) - e^(2*x))^2 - sqrt(e^(4*x) + 1) + e^(2*x) - 1)/(sqrt(e^(4*x) + 1) - e^(2*x))^2 - 2*sqrt(e^(4*x) + 1) + 2*e^(2*x) - 1)^2 + 4*log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) + 4*log(sqrt(e^(4*x) + 1) - e^(2*x)) - 4*log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))

```

maple [B] time = 0.10, size = 158, normalized size = 3.16

$$-\frac{\left((\tanh(x)-1)^2+2\tanh(x)\right)^{\frac{3}{2}}}{6}-\frac{\tanh(x)\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}{4}-\frac{5\operatorname{arcsinh}(\tanh(x))}{2}-\sqrt{(\tanh(x)-1)^2+2\tanh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+tanh(x)^2)^(3/2),x)

```

[Out] -1/6*((tanh(x)-1)^2+2*tanh(x))^(3/2)-1/4*tanh(x)*((tanh(x)-1)^2+2*tanh(x))^(1/2)-5/2*arcsinh(tanh(x))-((tanh(x)-1)^2+2*tanh(x))^(1/2)+2^(1/2)*arctanh(1/4*(2*tanh(x)+2)*2^(1/2)/((tanh(x)-1)^2+2*tanh(x))^(1/2))+1/6*((1+tanh(x))^2-2*tanh(x))^(3/2)-1/4*tanh(x)*((1+tanh(x))^2-2*tanh(x))^(1/2)+((1+tanh(x)

```

)²-2*tanh(x))^(1/2)-2^(1/2)*arctanh(1/4*(2-2*tanh(x))*2^(1/2)/((1+tanh(x))²-2*tanh(x))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\tanh(x)^2 + 1)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+tanh(x)^2)^(3/2), x, algorithm="maxima")

[Out] integrate((tanh(x)² + 1)^(3/2), x)

mupad [B] time = 0.29, size = 78, normalized size = 1.56

$$\sqrt{2} \left(\ln(\tanh(x) + 1) - \ln\left(\sqrt{2} \sqrt{\tanh(x)^2 + 1} - \tanh(x) + 1\right) \right) - \frac{5 \operatorname{asinh}(\tanh(x))}{2} - \frac{\tanh(x) \sqrt{\tanh(x)^2 + 1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((tanh(x)² + 1)^(3/2), x)

[Out] 2^(1/2)*(log(tanh(x) + 1) - log(2^(1/2)*(tanh(x)² + 1)^(1/2) - tanh(x) + 1)) - (5*asinh(tanh(x)))/2 - (tanh(x)*(tanh(x)² + 1)^(1/2))/2 + 2^(1/2)*(log(tanh(x) + 2^(1/2)*(tanh(x)² + 1)^(1/2) + 1) - log(tanh(x) - 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (\tanh^2(x) + 1)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+tanh(x)**2)**(3/2), x)

[Out] Integral((tanh(x)**2 + 1)**(3/2), x)

$$3.228 \quad \int (-1 - \tanh^2(x))^{3/2} dx$$

Optimal. Leaf size=67

$$\frac{1}{2} \tanh(x) \sqrt{-\tanh^2(x) - 1} - \frac{5}{2} \tan^{-1} \left(\frac{\tanh(x)}{\sqrt{-\tanh^2(x) - 1}} \right) + 2\sqrt{2} \tan^{-1} \left(\frac{\sqrt{2} \tanh(x)}{\sqrt{-\tanh^2(x) - 1}} \right)$$

[Out] $-5/2*\arctan(\tanh(x)/(-1-\tanh(x)^2)^{(1/2)})+2*\arctan(2^{(1/2)}*\tanh(x)/(-1-\tanh(x)^2)^{(1/2)})*2^{(1/2)}+1/2*(-1-\tanh(x)^2)^{(1/2)}*\tanh(x)$

Rubi [A] time = 0.05, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3661, 416, 523, 217, 203, 377}

$$\frac{1}{2} \tanh(x) \sqrt{-\tanh^2(x) - 1} - \frac{5}{2} \tan^{-1} \left(\frac{\tanh(x)}{\sqrt{-\tanh^2(x) - 1}} \right) + 2\sqrt{2} \tan^{-1} \left(\frac{\sqrt{2} \tanh(x)}{\sqrt{-\tanh^2(x) - 1}} \right)$$

Antiderivative was successfully verified.

[In] Int[(-1 - Tanh[x]^2)^(3/2), x]

[Out] $(-5*\text{ArcTan}[\text{Tanh}[x]/\text{Sqrt}[-1 - \text{Tanh}[x]^2]])/2 + 2*\text{Sqrt}[2]*\text{ArcTan}[(\text{Sqrt}[2]*\text{Tanh}[x])/\text{Sqrt}[-1 - \text{Tanh}[x]^2]] + (\text{Tanh}[x]*\text{Sqrt}[-1 - \text{Tanh}[x]^2])/2$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 416

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q-1))/(b*(n*(p+q)+1)), x] + Dist[1/(b*(n*(p+q)+1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q-2)*Simp[c*(b*c*(n*(p+q)+1) - a*d] + d*(b*c*(n*(p+2*q-1)+1) - a*d*(n*(q-1)+1))*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && NeQ[n*(p+q)+1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 523

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x_)^(n_)], x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :=
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n]^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned} \int (-1 - \tanh^2(x))^{3/2} dx &= \text{Subst} \left(\int \frac{(-1 - x^2)^{3/2}}{1 - x^2} dx, x, \tanh(x) \right) \\ &= \frac{1}{2} \tanh(x) \sqrt{-1 - \tanh^2(x)} - \frac{1}{2} \text{Subst} \left(\int \frac{-3 - 5x^2}{\sqrt{-1 - x^2} (1 - x^2)} dx, x, \tanh(x) \right) \\ &= \frac{1}{2} \tanh(x) \sqrt{-1 - \tanh^2(x)} - \frac{5}{2} \text{Subst} \left(\int \frac{1}{\sqrt{-1 - x^2}} dx, x, \tanh(x) \right) + 4 \text{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \tanh(x) \right) \\ &= \frac{1}{2} \tanh(x) \sqrt{-1 - \tanh^2(x)} - \frac{5}{2} \text{Subst} \left(\int \frac{1}{1 + x^2} dx, x, \frac{\tanh(x)}{\sqrt{-1 - \tanh^2(x)}} \right) + 4 \text{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \frac{\tanh(x)}{\sqrt{-1 - \tanh^2(x)}} \right) \\ &= -\frac{5}{2} \tan^{-1} \left(\frac{\tanh(x)}{\sqrt{-1 - \tanh^2(x)}} \right) + 2\sqrt{2} \tan^{-1} \left(\frac{\sqrt{2} \tanh(x)}{\sqrt{-1 - \tanh^2(x)}} \right) + \frac{1}{2} \tanh(x) \sqrt{-1 - \tanh^2(x)} \end{aligned}$$

Mathematica [A] time = 0.07, size = 76, normalized size = 1.13

$$\frac{(-\tanh^2(x) - 1)^{3/2} \left(-4\sqrt{2} \sinh^{-1}(\sqrt{2} \sinh(x)) \cosh^3(x) + \sinh(x) \sqrt{\cosh(2x)} \cosh(x) + 5 \cosh^3(x) \tanh^{-1}(\sqrt{2} \sinh(x)) \right)}{2 \cosh^3(2x)}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 - Tanh[x]^2)^(3/2), x]

[Out] -1/2*((-4*Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[x]]*Cosh[x]^3 + 5*ArcTanh[Sinh[x]/Sqrt[Cosh[2*x]])*Cosh[x]^3 + Cosh[x]*Sqrt[Cosh[2*x]]*Sinh[x])*(-1 - Tanh[x]^2)^(3/2))/Cosh[2*x]^(3/2)

fricas [C] time = 0.41, size = 361, normalized size = 5.39

$$2 \left(\sqrt{-2} e^{4x} + 2 \sqrt{-2} e^{2x} + \sqrt{-2} \right) \log \left(2 \left(\sqrt{-2} \sqrt{-2} e^{4x} - 2 + 2 e^{2x} + 2 \right) e^{(-2x)} \right) - 2 \left(\sqrt{-2} e^{4x} + 2 \sqrt{-2} e^{2x} + \sqrt{-2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-tanh(x)^2)^(3/2),x, algorithm="fricas")

[Out] 1/4*(2*(sqrt(-2)*e^(4*x) + 2*sqrt(-2)*e^(2*x) + sqrt(-2))*log(2*(sqrt(-2)*sqrt(-2)*e^(4*x) - 2) + 2*e^(2*x) + 2)*e^(-2*x)) - 2*(sqrt(-2)*e^(4*x) + 2*sqrt(-2)*e^(2*x) + sqrt(-2))*log(-2*(sqrt(-2)*sqrt(-2)*e^(4*x) - 2) - 2*e^(2*x) - 2)*e^(-2*x)) + (5*I*e^(4*x) + 10*I*e^(2*x) + 5*I)*log((4*I*sqrt(-2)*e^(4*x) - 2) - 4*e^(2*x) + 4)*e^(-2*x)) + (-5*I*e^(4*x) - 10*I*e^(2*x) - 5*I)*log((-4*I*sqrt(-2)*e^(4*x) - 2) - 4*e^(2*x) + 4)*e^(-2*x)) - 2*(sqrt(-2)*e^(4*x) + 2*sqrt(-2)*e^(2*x) + sqrt(-2))*log(4*(sqrt(-2)*e^(4*x) - 2)*(e^(2*x) - 2))

2) + sqrt(-2)*e^(4*x) - sqrt(-2)*e^(2*x) + 2*sqrt(-2))*e^(-4*x)) + 2*(sqrt(-2)*e^(4*x) + 2*sqrt(-2)*e^(2*x) + sqrt(-2))*log(4*(sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 2) - sqrt(-2)*e^(4*x) + sqrt(-2)*e^(2*x) - 2*sqrt(-2))*e^(-4*x)) + 2*sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 1))/(e^(4*x) + 2*e^(2*x) + 1)

giac [C] time = 0.17, size = 204, normalized size = 3.04

$$-\frac{1}{4}\sqrt{2}\left[-5i\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{4x}+1}+e^{2x}+1}{\sqrt{2}+\sqrt{e^{4x}+1}-e^{2x}-1}\right)-\frac{4\left(-3i\left(\sqrt{e^{4x}+1}-e^{2x}\right)^3+i\left(\sqrt{e^{4x}+1}-e^{2x}\right)^2+i\sqrt{e^{4x}+1}-e^{2x}\right)}{\left(\left(\sqrt{e^{4x}+1}-e^{2x}\right)^2-2\sqrt{e^{4x}+1}+2e^{2x}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-tanh(x)^2)^(3/2),x, algorithm="giac")

[Out] -1/4*sqrt(2)*(-5*I*sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)/(sqrt(2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) - 4*(-3*I*(sqrt(e^(4*x) + 1) - e^(2*x))^3 + I*(sqrt(e^(4*x) + 1) - e^(2*x))^2 + I*sqrt(e^(4*x) + 1) - I*e^(2*x) + I)/((sqrt(e^(4*x) + 1) - e^(2*x))^2 - 2*sqrt(e^(4*x) + 1) + 2*e^(2*x) - 1)^2 - 4*I*log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) - 4*I*log(sqrt(e^(4*x) + 1) - e^(2*x)) + 4*I*log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))

maple [B] time = 0.11, size = 211, normalized size = 3.15

$$-\frac{(-\tanh(x)-1)^2-2\tanh(x)}{6}+\frac{\tanh(x)\sqrt{-\tanh(x)-1)^2-2\tanh(x)}}{4}-\frac{5\arctan\left(\frac{\tanh(x)}{\sqrt{-\tanh(x)-1)^2-2\tanh(x)}}\right)}{4}+\sqrt{-\tanh(x)-1)^2-2\tanh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1-tanh(x)^2)^(3/2),x)

[Out] -1/6*(-(tanh(x)-1)^2-2*tanh(x))^(3/2)+1/4*tanh(x)*(-(tanh(x)-1)^2-2*tanh(x))^(1/2))-5/4*arctan(tanh(x)/(-(tanh(x)-1)^2-2*tanh(x))^(1/2)))+(-(tanh(x)-1)^2-2*tanh(x))^(1/2)-2^(1/2)*arctan(1/4*(-2-2*tanh(x))*2^(1/2)/(-(tanh(x)-1)^2-2*tanh(x))^(1/2))+1/6*(-(1+tanh(x))^2+2*tanh(x))^(3/2)+1/4*tanh(x)*(-(1+tanh(x))^2+2*tanh(x))^(1/2))-5/4*arctan(tanh(x)/(-(1+tanh(x))^2+2*tanh(x))^(1/2)))-(-(1+tanh(x))^2+2*tanh(x))^(1/2)+2^(1/2)*arctan(1/4*(-2+2*tanh(x))*2^(1/2)/(-(1+tanh(x))^2+2*tanh(x))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-\tanh(x)^2 - 1)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-tanh(x)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((-tanh(x)^2 - 1)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (-\tanh(x)^2 - 1)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((- tanh(x)^2 - 1)^(3/2),x)


```
[Out] int((- tanh(x)^2 - 1)^(3/2), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \left(-\tanh^2(x) - 1\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-tanh(x)**2)**(3/2), x)
```

```
[Out] Integral((-tanh(x)**2 - 1)**(3/2), x)
```

$$3.229 \quad \int \frac{\tanh^5(x)}{\sqrt{a+b \tanh^2(x)}} dx$$

Optimal. Leaf size=70

$$-\frac{(a+b \tanh^2(x))^{3/2}}{3b^2} + \frac{(a-b)\sqrt{a+b \tanh^2(x)}}{b^2} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)+(a-b)*(a+b*tanh(x)^2)^(1/2)/b^2-1/3*(a+b*tanh(x)^2)^(3/2)/b^2

Rubi [A] time = 0.14, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3670, 446, 88, 63, 208}

$$-\frac{(a+b \tanh^2(x))^{3/2}}{3b^2} + \frac{(a-b)\sqrt{a+b \tanh^2(x)}}{b^2} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^5/Sqrt[a + b*Tanh[x]^2], x]

[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b] + ((a - b)*Sqrt[a + b*Tanh[x]^2])/b^2 - (a + b*Tanh[x]^2)^(3/2)/(3*b^2)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 88

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],

$x\}}, \text{Dist}[(c*ff)/f, \text{Subst}[\text{Int}[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f^2*x^2), x], x, (c*\text{Tan}[e + f*x])/ff], x]] /; \text{FreeQ}\{a, b, c, d, e, f, m, n, p\}, x] \&\& (\text{IGtQ}[p, 0] \|\ \text{EqQ}[n, 2] \|\ \text{EqQ}[n, 4] \|\ (\text{IntegerQ}[p] \&\& \text{RationalQ}[n]))$

Rubi steps

$$\begin{aligned}
 \int \frac{\tanh^5(x)}{\sqrt{a + b \tanh^2(x)}} dx &= \text{Subst} \left(\int \frac{x^5}{(1-x^2)\sqrt{a+bx^2}} dx, x, \tanh(x) \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{(1-x)\sqrt{a+bx}} dx, x, \tanh^2(x) \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{a-b}{b\sqrt{a+bx}} + \frac{1}{(1-x)\sqrt{a+bx}} - \frac{\sqrt{a+bx}}{b} \right) dx, x, \tanh^2(x) \right) \\
 &= \frac{(a-b)\sqrt{a+b \tanh^2(x)}}{b^2} - \frac{(a+b \tanh^2(x))^{3/2}}{3b^2} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1-x)\sqrt{a+bx}} dx, x, \right. \\
 &= \frac{(a-b)\sqrt{a+b \tanh^2(x)}}{b^2} - \frac{(a+b \tanh^2(x))^{3/2}}{3b^2} + \frac{\text{Subst} \left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}} dx, x, \sqrt{a+b \tanh^2(x)} \right)}{b} \\
 &= \frac{\tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}} \right)}{\sqrt{a+b}} + \frac{(a-b)\sqrt{a+b \tanh^2(x)}}{b^2} - \frac{(a+b \tanh^2(x))^{3/2}}{3b^2}
 \end{aligned}$$

Mathematica [A] time = 0.54, size = 68, normalized size = 0.97

$$\frac{\text{sech}^2(x)((a-2b) \cosh(2x) + a-b)\sqrt{a+b \tanh^2(x)}}{3b^2} + \frac{\tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}} \right)}{\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^5/Sqrt[a + b*Tanh[x]^2], x]

[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b] + ((a - b + (a - 2*b)*Cosh[2*x])*Sech[x]^2*Sqrt[a + b*Tanh[x]^2])/(3*b^2)

fricas [B] time = 0.67, size = 2827, normalized size = 40.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(1/2), x, algorithm="fricas")

[Out] [1/12*(3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 + 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3*b^2*cosh(x)^2 + 4*(5*b^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*cosh(x)^4 + 6*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 +

$$\begin{aligned}
& b^3 + 30*(2*a^3 + a^2*b)*\cosh(x)^2*\sinh(x)^4 + 4*(14*(a^3 + a^2*b)*\cosh(x) \\
&)^5 + 10*(2*a^3 + a^2*b)*\cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x) \\
&)*\sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*\cosh(x) \\
&)^2 + 2*(14*(a^3 + a^2*b)*\cosh(x)^6 + 15*(2*a^3 + a^2*b)*\cosh(x)^4 + 2* \\
& a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x)^2)*\sinh(x)^2 \\
& + \sqrt{2}*(a^2*\cosh(x)^6 + 6*a^2*\cosh(x)*\sinh(x)^5 + a^2*\sinh(x)^6 + 3*a^2 \\
& *\cosh(x)^4 + 3*(5*a^2*\cosh(x)^2 + a^2)*\sinh(x)^4 + 4*(5*a^2*\cosh(x)^3 + 3* \\
& a^2*\cosh(x))*\sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*\cosh(x)^2 + (15*a^2*\cosh(x)^4 \\
& + 18*a^2*\cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + \\
& 2*(3*a^2*\cosh(x)^5 + 6*a^2*\cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*\cosh(x))*\sinh \\
& (x))*\sqrt{a + b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh \\
& (x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)} + 4*(2*(a^3 + a^2*b)*\cosh(x)^7 + 3* \\
& (2*a^3 + a^2*b)*\cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x)^3 + (2* \\
& a^3 + 3*a^2*b - b^3)*\cosh(x))*\sinh(x))/(\cosh(x)^6 + 6*\cosh(x)^5*\sinh(x) + 1 \\
& 5*\cosh(x)^4*\sinh(x)^2 + 20*\cosh(x)^3*\sinh(x)^3 + 15*\cosh(x)^2*\sinh(x)^4 + 6 \\
& *\cosh(x)*\sinh(x)^5 + \sinh(x)^6) + 3*(b^2*\cosh(x)^6 + 6*b^2*\cosh(x)*\sinh(x) \\
& ^5 + b^2*\sinh(x)^6 + 3*b^2*\cosh(x)^4 + 3*(5*b^2*\cosh(x)^2 + b^2)*\sinh(x)^4 \\
& + 3*b^2*\cosh(x)^2 + 4*(5*b^2*\cosh(x)^3 + 3*b^2*\cosh(x))*\sinh(x)^3 + 3*(5*b^2 \\
& *\cosh(x)^4 + 6*b^2*\cosh(x)^2 + b^2)*\sinh(x)^2 + b^2 + 6*(b^2*\cosh(x)^5 + 2 \\
& *b^2*\cosh(x)^3 + b^2*\cosh(x))*\sinh(x))*\sqrt{a + b}*\log(-((a + b)*\cosh(x)^4 \\
& + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a + b)*\sinh(x)^4 - 2*b*\cosh(x)^2 + 2*(3*(a \\
& + b)*\cosh(x)^2 - b)*\sinh(x)^2 + \sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + s \\
& \sinh(x)^2 - 1))*\sqrt{a + b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - \\
& b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)} + 4*((a + b)*\cosh(x)^3 - b \\
& *\cosh(x))*\sinh(x) + a + b)/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2) + 8 \\
& *\sqrt{2}*((a^2 - a*b - 2*b^2)*\cosh(x)^4 + 4*(a^2 - a*b - 2*b^2)*\cosh(x)*\sinh \\
& (x)^3 + (a^2 - a*b - 2*b^2)*\sinh(x)^4 + 2*(a^2 - b^2)*\cosh(x)^2 + 2*(3*(a^2 \\
& - a*b - 2*b^2)*\cosh(x)^2 + a^2 - b^2)*\sinh(x)^2 + a^2 - a*b - 2*b^2 + 4*(\\
& (a^2 - a*b - 2*b^2)*\cosh(x)^3 + (a^2 - b^2)*\cosh(x))*\sinh(x))*\sqrt{((a + b) \\
& *\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh \\
& (x)^2)}))/((a*b^2 + b^3)*\cosh(x)^6 + 6*(a*b^2 + b^3)*\cosh(x)*\sinh(x)^5 + (\\
& a*b^2 + b^3)*\sinh(x)^6 + 3*(a*b^2 + b^3)*\cosh(x)^4 + 3*(a*b^2 + b^3 + 5*(a* \\
& b^2 + b^3)*\cosh(x)^2)*\sinh(x)^4 + 4*(5*(a*b^2 + b^3)*\cosh(x)^3 + 3*(a*b^2 + \\
& b^3)*\cosh(x))*\sinh(x)^3 + a*b^2 + b^3 + 3*(a*b^2 + b^3)*\cosh(x)^2 + 3*(5*(\\
& a*b^2 + b^3)*\cosh(x)^4 + a*b^2 + b^3 + 6*(a*b^2 + b^3)*\cosh(x)^2)*\sinh(x)^2 \\
& + 6*((a*b^2 + b^3)*\cosh(x)^5 + 2*(a*b^2 + b^3)*\cosh(x)^3 + (a*b^2 + b^3)*\c \\
& \cosh(x))*\sinh(x)), -1/6*(3*(b^2*\cosh(x)^6 + 6*b^2*\cosh(x)*\sinh(x)^5 + b^2*\sinh \\
& (x)^6 + 3*b^2*\cosh(x)^4 + 3*(5*b^2*\cosh(x)^2 + b^2)*\sinh(x)^4 + 3*b^2*\cosh \\
& (x)^2 + 4*(5*b^2*\cosh(x)^3 + 3*b^2*\cosh(x))*\sinh(x)^3 + 3*(5*b^2*\cosh(x)^4 \\
& + 6*b^2*\cosh(x)^2 + b^2)*\sinh(x)^2 + b^2 + 6*(b^2*\cosh(x)^5 + 2*b^2*\cosh(x) \\
&)^3 + b^2*\cosh(x))*\sinh(x))*\sqrt{-a - b}*\arctan(\sqrt{2}*(a*\cosh(x)^2 + 2*a* \\
& \cosh(x)*\sinh(x) + a*\sinh(x)^2 + a + b))*\sqrt{-a - b}*\sqrt{((a + b)*\cosh(x)^2 \\
& + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)}/ \\
& ((a^2 + a*b)*\cosh(x)^4 + 4*(a^2 + a*b)*\cosh(x)*\sinh(x)^3 + (a^2 + a*b)*\sinh \\
& (x)^4 + (2*a^2 + a*b - b^2)*\cosh(x)^2 + (6*(a^2 + a*b)*\cosh(x)^2 + 2*a^2 + \\
& a*b - b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*\cosh(x)^3 + (2* \\
& a^2 + a*b - b^2)*\cosh(x))*\sinh(x)) + 3*(b^2*\cosh(x)^6 + 6*b^2*\cosh(x)*\sinh \\
& (x)^5 + b^2*\sinh(x)^6 + 3*b^2*\cosh(x)^4 + 3*(5*b^2*\cosh(x)^2 + b^2)*\sinh(x) \\
& ^4 + 3*b^2*\cosh(x)^2 + 4*(5*b^2*\cosh(x)^3 + 3*b^2*\cosh(x))*\sinh(x)^3 + 3*(5 \\
& *b^2*\cosh(x)^4 + 6*b^2*\cosh(x)^2 + b^2)*\sinh(x)^2 + b^2 + 6*(b^2*\cosh(x)^5 \\
& + 2*b^2*\cosh(x)^3 + b^2*\cosh(x))*\sinh(x))*\sqrt{-a - b}*\arctan(\sqrt{2}*(\cosh \\
& (x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1))*\sqrt{-a - b}*\sqrt{((a + b)*\cosh \\
& (x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2) \\
&)}/((a + b)*\cosh(x)^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a + b)*\sinh(x)^4 + \\
& 2*(a - b)*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 + a - b)*\sinh(x)^2 + 4*((a + b) \\
&)*\cosh(x)^3 + (a - b)*\cosh(x))*\sinh(x) + a + b) - 4*\sqrt{2}*((a^2 - a*b - \\
& 2*b^2)*\cosh(x)^4 + 4*(a^2 - a*b - 2*b^2)*\cosh(x)*\sinh(x)^3 + (a^2 - a*b - 2 \\
& *b^2)*\sinh(x)^4 + 2*(a^2 - b^2)*\cosh(x)^2 + 2*(3*(a^2 - a*b - 2*b^2)*\cosh(x) \\
&)^2 + a^2 - b^2)*\sinh(x)^2 + a^2 - a*b - 2*b^2 + 4*((a^2 - a*b - 2*b^2)*\cos
\end{aligned}$$

$$h(x)^3 + (a^2 - b^2) \cosh(x) \sinh(x) \sqrt{((a + b) \cosh(x)^2 + (a + b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} / ((a b^2 + b^3) \cosh(x)^6 + 6(a b^2 + b^3) \cosh(x) \sinh(x)^5 + (a b^2 + b^3) \sinh(x)^6 + 3(a b^2 + b^3) \cosh(x)^4 + 3(a b^2 + b^3 + 5(a b^2 + b^3) \cosh(x)^2) \sinh(x)^4 + 4(5(a b^2 + b^3) \cosh(x)^3 + 3(a b^2 + b^3) \cosh(x) \sinh(x)^3 + a b^2 + b^3 + 3(a b^2 + b^3) \cosh(x)^2 + 3(5(a b^2 + b^3) \cosh(x)^4 + a b^2 + b^3 + 6(a b^2 + b^3) \cosh(x)^2) \sinh(x)^2 + 6((a b^2 + b^3) \cosh(x)^2 \sinh(x)^2 + 2(a b^2 + b^3) \cosh(x)^3 + (a b^2 + b^3) \cosh(x) \sinh(x)))$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep+1)]Evaluation time: 0.99Error: Bad Argument Type

maple [B] time = 0.11, size = 164, normalized size = 2.34

$$-\frac{(\tanh^2(x))\sqrt{a+b(\tanh^2(x))}}{3b} + \frac{2a\sqrt{a+b(\tanh^2(x))}}{3b^2} - \frac{\sqrt{a+b(\tanh^2(x))}}{b} + \frac{\ln\left(\frac{2a+2b+2(\tanh(x)-1)b+2\sqrt{a+b}\sqrt{(\tanh(x)-1)^2+b+2\sqrt{a+b}}}{\tanh(x)-1}\right)}{2\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x)

[Out] $-\frac{1}{3} \frac{\tanh(x)^2}{b} (a+b \tanh(x)^2)^{(1/2)} + \frac{2}{3} \frac{a}{b^2} (a+b \tanh(x)^2)^{(1/2)} - (a+b \tanh(x)^2)^{(1/2)} / b + \frac{1}{2} (a+b)^{(1/2)} \ln\left(\frac{(2a+2b+2(\tanh(x)-1)b+2\sqrt{a+b})\sqrt{(\tanh(x)-1)^2+b+2\sqrt{a+b}}}{(\tanh(x)-1)}\right) + \frac{1}{2} (a+b)^{(1/2)} \ln\left(\frac{(2a+2b-2(1+\tanh(x))b+2(a+b)^{(1/2)}((1+\tanh(x))^2b-2(1+\tanh(x))b+a+b)^{(1/2)})}{(1+\tanh(x))}\right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^5}{\sqrt{b \tanh(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(tanh(x)^5/sqrt(b*tanh(x)^2 + a), x)

mupad [B] time = 2.17, size = 65, normalized size = 0.93

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b \tanh(x)^2 + a}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{(b \tanh(x)^2 + a)^{3/2}}{3b^2} - \left(\frac{a+b}{b^2} - \frac{2a}{b^2}\right) \sqrt{b \tanh(x)^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^5/(a + b*tanh(x)^2)^(1/2),x)

[Out] $\operatorname{atanh}\left(\frac{(a + b \tanh(x)^2)^{(1/2)}}{(a + b)^{(1/2)}}\right) / (a + b)^{(1/2)} - (a + b \tanh(x)^2)^{(3/2)} / (3 * b^2) - ((a + b) / b^2 - (2 * a) / b^2) * (a + b \tanh(x)^2)^{(1/2)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{\sqrt{a + b \tanh^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)**5/(a+b*tanh(x)**2)**(1/2), x)

[Out] Integral(tanh(x)**5/sqrt(a + b*tanh(x)**2), x)

$$3.230 \quad \int \frac{\tanh^4(x)}{\sqrt{a+b \tanh^2(x)}} dx$$

Optimal. Leaf size=88

$$\frac{(a-2b) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{2b^{3/2}} - \frac{\tanh(x) \sqrt{a+b \tanh^2(x)}}{2b} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{\sqrt{a+b}}$$

[Out] 1/2*(a-2*b)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(3/2)+arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(1/2)-1/2*(a+b*tanh(x)^2)^(1/2)*tanh(x)/b

Rubi [A] time = 0.12, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3670, 479, 523, 217, 206, 377}

$$\frac{(a-2b) \tanh^{-1}\left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{2b^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{\sqrt{a+b}} - \frac{\tanh(x) \sqrt{a+b \tanh^2(x)}}{2b}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^4/Sqrt[a + b*Tanh[x]^2], x]

[Out] ((a - 2*b)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(2*b^(3/2)) + ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b] - (Tanh[x]*Sqrt[a + b*Tanh[x]^2])/(2*b)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 479

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 523

```
Int[((e_) + (f_)*(x_)^(n_))/((a_) + (b_)*(x_)^n)*Sqrt[(c_) + (d_)*(x_)^n], x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]
```

Rule 3670

```
Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*((c_)*tan[(e_) + (f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff, x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))
```

Rubi steps

$$\int \frac{\tanh^4(x)}{\sqrt{a + b \tanh^2(x)}} dx = \text{Subst} \left(\int \frac{x^4}{(1-x^2)\sqrt{a+bx^2}} dx, x, \tanh(x) \right)$$

$$= -\frac{\tanh(x)\sqrt{a + b \tanh^2(x)}}{2b} + \frac{\text{Subst} \left(\int \frac{a+(-a+2b)x^2}{(1-x^2)\sqrt{a+bx^2}} dx, x, \tanh(x) \right)}{2b}$$

$$= -\frac{\tanh(x)\sqrt{a + b \tanh^2(x)}}{2b} + \frac{(a-2b) \text{Subst} \left(\int \frac{1}{\sqrt{a+bx^2}} dx, x, \tanh(x) \right)}{2b} + \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \tanh(x) \right)$$

$$= -\frac{\tanh(x)\sqrt{a + b \tanh^2(x)}}{2b} + \frac{(a-2b) \text{Subst} \left(\int \frac{1}{1-bx^2} dx, x, \frac{\tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right)}{2b} + \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \tanh(x) \right)$$

$$= \frac{(a-2b) \tanh^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right)}{2b^{3/2}} + \frac{\tanh^{-1} \left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right)}{\sqrt{a+b}} - \frac{\tanh(x)\sqrt{a + b \tanh^2(x)}}{2b}$$

Mathematica [C] time = 4.90, size = 208, normalized size = 2.36

$$\frac{\tanh(x) \left(-((a+b)\text{sech}^2(x)((a+b)\cosh(2x)+a-b)) + \sqrt{2}a(a+b)\sqrt{\frac{\text{csch}^2(x)((a+b)\cosh(2x)+a-b)}{b}} F \left(\sin^{-1} \left(\frac{\sqrt{a-b+(a+b)\cosh(2x)}}{\sqrt{a+b}} \right) \right) \right)}{2\sqrt{2}b(a+b)\sqrt{\text{sech}^2(x)((a+b)\cosh(2x)+a-b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/Sqrt[a + b*Tanh[x]^2], x]
[Out] ((Sqrt[2]*a*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] - 2*Sqrt[2]*a*b*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] - (a + b)*(a - b + (a + b)*Cosh[2*x])*Sech[x]^2*Tanh[x])/(2*Sqrt[2]*b*(a + b)*Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2])
```

fricas [B] time = 0.77, size = 5494, normalized size = 62.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")

[Out]
$$\frac{1}{4} \left((b^2 \cosh(x)^4 + 4b^2 \cosh(x) \sinh(x)^3 + b^2 \sinh(x)^4 + 2b^2 \cosh(x)^2 + 2(3b^2 \cosh(x)^2 + b^2) \sinh(x)^2 + b^2 + 4(b^2 \cosh(x)^3 + b^2 \cosh(x) \sinh(x)) \sqrt{a+b} \log(-((a b^2 + b^3) \cosh(x)^8 + 8(a b^2 + b^3) \cosh(x) \sinh(x)^7 + (a b^2 + b^3) \sinh(x)^8 - 2(a b^2 + 2b^3) \cosh(x)^6 - 2(a b^2 + 2b^3 - 14(a b^2 + b^3) \cosh(x)^2) \sinh(x)^6 + 4(14(a b^2 + b^3) \cosh(x)^3 - 3(a b^2 + 2b^3) \cosh(x) \sinh(x)^5 + (a^3 - a^2 b + 4a b^2 + 6b^3) \cosh(x)^4 + (70(a b^2 + b^3) \cosh(x)^4 + a^3 - a^2 b + 4a b^2 + 6b^3 - 30(a b^2 + 2b^3) \cosh(x)^2) \sinh(x)^4 + 4(14(a b^2 + b^3) \cosh(x)^5 - 10(a b^2 + 2b^3) \cosh(x)^3 + (a^3 - a^2 b + 4a b^2 + 6b^3) \cosh(x) \sinh(x)^3 + a^3 + 3a^2 b + 3a b^2 + b^3 + 2(a^3 - 3a b^2 - 2b^3) \cosh(x)^2 + 2(14(a b^2 + b^3) \cosh(x)^6 - 15(a b^2 + 2b^3) \cosh(x)^4 + a^3 - 3a b^2 - 2b^3 + 3(a^3 - a^2 b + 4a b^2 + 6b^3) \cosh(x)^2) \sinh(x)^2 + \sqrt{2}(b^2 \cosh(x)^6 + 6b^2 \cosh(x) \sinh(x)^5 + b^2 \sinh(x)^6 - 3b^2 \cosh(x)^4 + 3(5b^2 \cosh(x)^2 - b^2) \sinh(x)^4 + 4(5b^2 \cosh(x)^3 - 3b^2 \cosh(x) \sinh(x)^3 - (a^2 - 2a b - 3b^2) \cosh(x)^2 + (15b^2 \cosh(x)^4 - 18b^2 \cosh(x)^2 - a^2 + 2a b + 3b^2) \sinh(x)^2 - a^2 - 2a b - b^2 + 2(3b^2 \cosh(x)^5 - 6b^2 \cosh(x)^3 - (a^2 - 2a b - 3b^2) \cosh(x) \sinh(x)) \sqrt{a+b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a-b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)} + 4(2(a b^2 + b^3) \cosh(x)^7 - 3(a b^2 + 2b^3) \cosh(x)^5 + (a^3 - a^2 b + 4a b^2 + 6b^3) \cosh(x)^3 + (a^3 - 3a b^2 - 2b^3) \cosh(x) \sinh(x)) / (\cosh(x)^6 + 6 \cosh(x)^5 \sinh(x) + 15 \cosh(x)^4 \sinh(x)^2 + 20 \cosh(x)^3 \sinh(x)^3 + 15 \cosh(x)^2 \sinh(x)^4 + 6 \cosh(x) \sinh(x)^5 + \sinh(x)^6) - ((a^2 - a b - 2b^2) \cosh(x)^4 + 4(a^2 - a b - 2b^2) \cosh(x) \sinh(x)^3 + (a^2 - a b - 2b^2) \sinh(x)^4 + 2(a^2 - a b - 2b^2) \cosh(x)^2 + 2(3(a^2 - a b - 2b^2) \cosh(x)^2 + a^2 - a b - 2b^2) \sinh(x)^2 + a^2 - a b - 2b^2 + 4((a^2 - a b - 2b^2) \cosh(x)^3 + (a^2 - a b - 2b^2) \cosh(x) \sinh(x)) \sqrt{b} \log(-((a+2b) \cosh(x)^4 + 4(a+2b) \cosh(x) \sinh(x)^3 + (a+2b) \sinh(x)^4 + 2(a-2b) \cosh(x)^2 + 2(3(a+2b) \cosh(x)^2 + a-2b) \sinh(x)^2 - 2 \sqrt{2} (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) \sqrt{b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a-b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)} + 4((a+2b) \cosh(x)^3 + (a-2b) \cosh(x) \sinh(x) + a+2b) / (\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2(3 \cosh(x)^2 + 1) \sinh(x)^2 + 2 \cosh(x)^2 + 4(\cosh(x)^3 + \cosh(x) \sinh(x) + 1)) + (b^2 \cosh(x)^4 + 4b^2 \cosh(x) \sinh(x)^3 + b^2 \sinh(x)^4 + 2b^2 \cosh(x)^2 + 2(3b^2 \cosh(x)^2 + b^2) \sinh(x)^2 + b^2 + 4(b^2 \cosh(x)^3 + b^2 \cosh(x) \sinh(x)) \sqrt{a+b} \log(((a+b) \cosh(x)^4 + 4(a+b) \cosh(x) \sinh(x)^3 + (a+b) \sinh(x)^4 + 2a \cosh(x)^2 + 2(3(a+b) \cosh(x)^2 + a) \sinh(x)^2 + \sqrt{2} (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1) \sqrt{a+b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a-b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)} + 4((a+b) \cosh(x)^3 + a \cosh(x) \sinh(x) + a+b) / (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2) - 2 \sqrt{2} ((a b + b^2) \cosh(x)^2 + 2(a b + b^2) \cosh(x) \sinh(x) + (a b + b^2) \sinh(x)^2 - a b - b^2) \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a-b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)})) / ((a b^2 + b^3) \cosh(x)^4 + 4(a b^2 + b^3) \cosh(x) \sinh(x)^3 + (a b^2 + b^3) \sinh(x)^4 + a b^2 + b^3 + 2(a b^2 + b^3) \cosh(x)^2 + 2(a b^2 + b^3 + 3(a b^2 + b^3) \cosh(x)^2) \sinh(x)^2 + 4((a b^2 + b^3) \cosh(x)^3 + (a b^2 + b^3) \cosh(x) \sinh(x)), -1/4(2((a^2 - a b - 2b^2) \cosh(x)^4 + 4(a^2 - a b - 2b^2) \cosh(x) \sinh(x)^3 + (a^2 - a b - 2b^2) \sinh(x)^4 + 2(a^2 - a b - 2b^2) \cosh(x)^2 + 2(3(a^2 - a b - 2b^2) \cosh(x)^2 + a^2 - a b - 2b^2) \sinh(x)^2 + a^2 - a b - 2b^2 + 4((a^2 - a b - 2b^2) \cosh(x)^3 + (a^2 - a b - 2b^2) \cosh(x) \sinh(x)) \sqrt{-b} \arctan(\sqrt{2} (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) \sqrt{-b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a-b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / ((a+b) \cosh(x)^4 + 4(a+b) \cosh(x) \sinh(x)^3 + (a+b) \sinh(x)^4 + 2(a-b) \cosh(x)^2 + 2(3(a+b) \cosh(x)^2 + a-b) \sinh(x)^2 + 4((a+b) \cosh(x)^3 + (a-b) \cos$$

$$\begin{aligned}
& h(x)) * \sinh(x) + a + b)) - (b^2 * \cosh(x)^4 + 4 * b^2 * \cosh(x) * \sinh(x)^3 + b^2 * \sinh(x)^4 + 2 * b^2 * \cosh(x)^2 + 2 * (3 * b^2 * \cosh(x)^2 + b^2) * \sinh(x)^2 + b^2 + 4 * (\\
& b^2 * \cosh(x)^3 + b^2 * \cosh(x)) * \sinh(x)) * \sqrt{a + b} * \log(-((a * b^2 + b^3) * \cosh(x)^8 + 8 * (a * b^2 + b^3) * \cosh(x) * \sinh(x)^7 + (a * b^2 + b^3) * \sinh(x)^8 - 2 * (a * b \\
& ^2 + 2 * b^3) * \cosh(x)^6 - 2 * (a * b^2 + 2 * b^3 - 14 * (a * b^2 + b^3) * \cosh(x)^2) * \sinh(x)^6 + 4 * (14 * (a * b^2 + b^3) * \cosh(x)^3 - 3 * (a * b^2 + 2 * b^3) * \cosh(x)) * \sinh(x)^5 + (a^3 - a^2 * b + 4 * a * b^2 + 6 * b^3) * \cosh(x)^4 + (70 * (a * b^2 + b^3) * \cosh(x)^4 + a^3 - a^2 * b + 4 * a * b^2 + 6 * b^3 - 30 * (a * b^2 + 2 * b^3) * \cosh(x)^2) * \sinh(x)^4 + 4 * (14 * (a * b^2 + b^3) * \cosh(x)^5 - 10 * (a * b^2 + 2 * b^3) * \cosh(x)^3 + (a^3 - a^2 * b + 4 * a * b^2 + 6 * b^3) * \cosh(x)) * \sinh(x)^3 + a^3 + 3 * a^2 * b + 3 * a * b^2 + b^3 + 2 * (a^3 - 3 * a * b^2 - 2 * b^3) * \cosh(x)^2 + 2 * (14 * (a * b^2 + b^3) * \cosh(x)^6 - 15 * (a * b^2 + 2 * b^3) * \cosh(x)^4 + a^3 - 3 * a * b^2 - 2 * b^3 + 3 * (a^3 - a^2 * b + 4 * a * b^2 + 6 * b^3) * \cosh(x)^2) * \sinh(x)^2 + \sqrt{2} * (b^2 * \cosh(x)^6 + 6 * b^2 * \cosh(x) * \sinh(x)^5 + b^2 * \sinh(x)^6 - 3 * b^2 * \cosh(x)^4 + 3 * (5 * b^2 * \cosh(x)^2 - b^2) * \sinh(x)^4 + 4 * (5 * b^2 * \cosh(x)^3 - 3 * b^2 * \cosh(x)) * \sinh(x)^3 - (a^2 - 2 * a * b - 3 * b^2) * \cosh(x)^2 + (15 * b^2 * \cosh(x)^4 - 18 * b^2 * \cosh(x)^2 - a^2 + 2 * a * b + 3 * b^2) * \sinh(x)^2 - a^2 - 2 * a * b - b^2 + 2 * (3 * b^2 * \cosh(x)^5 - 6 * b^2 * \cosh(x)^3 - (a^2 - 2 * a * b - 3 * b^2) * \cosh(x)) * \sinh(x)) * \sqrt{a + b} * \sqrt{((a + b) * \cosh(x)^2 + (a + b) * \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2))} + 4 * (2 * (a * b^2 + b^3) * \cosh(x)^7 - 3 * (a * b^2 + 2 * b^3) * \cosh(x)^5 + (a^3 - a^2 * b + 4 * a * b^2 + 6 * b^3) * \cosh(x)^3 + (a^3 - 3 * a * b^2 - 2 * b^3) * \cosh(x)) * \sinh(x)) / (\cosh(x)^6 + 6 * \cosh(x)^5 * \sinh(x) + 15 * \cosh(x)^4 * \sinh(x)^2 + 20 * \cosh(x)^3 * \sinh(x)^3 + 15 * \cosh(x)^2 * \sinh(x)^4 + 6 * \cosh(x) * \sinh(x)^5 + \sinh(x)^6)) - (b^2 * \cosh(x)^4 + 4 * b^2 * \cosh(x) * \sinh(x)^3 + b^2 * \sinh(x)^4 + 2 * b^2 * \cosh(x)^2 + 2 * (3 * b^2 * \cosh(x)^2 + b^2) * \sinh(x)^2 + b^2 + 4 * (b^2 * \cosh(x)^3 + b^2 * \cosh(x)) * \sinh(x)) * \sqrt{a + b} * \log(((a + b) * \cosh(x)^4 + 4 * (a + b) * \cosh(x) * \sinh(x)^3 + (a + b) * \sinh(x)^4 + 2 * a * \cosh(x)^2 + 2 * (3 * (a + b) * \cosh(x)^2 + a) * \sinh(x)^2 + \sqrt{2} * (\cosh(x)^2 + 2 * \cosh(x) * \sinh(x) + \sinh(x)^2 + 1) * \sqrt{a + b} * \sqrt{((a + b) * \cosh(x)^2 + (a + b) * \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2))} + 4 * ((a + b) * \cosh(x)^3 + a * \cosh(x)) * \sinh(x) + a + b) / (\cosh(x)^2 + 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)) + 2 * \sqrt{2} * ((a * b + b^2) * \cosh(x)^2 + 2 * (a * b + b^2) * \cosh(x) * \sinh(x) + (a * b + b^2) * \sinh(x)^2 - a * b - b^2) * \sqrt{((a + b) * \cosh(x)^2 + (a + b) * \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2))} / ((a * b^2 + b^3) * \cosh(x)^4 + 4 * (a * b^2 + b^3) * \cosh(x) * \sinh(x)^3 + (a * b^2 + b^3) * \sinh(x)^4 + a * b^2 + b^3 + 2 * (a * b^2 + b^3) * \cosh(x)^2 + 2 * (a * b^2 + b^3 + 3 * (a * b^2 + b^3) * \cosh(x)^2) * \sinh(x)^2 + 4 * ((a * b^2 + b^3) * \cosh(x)^3 + (a * b^2 + b^3) * \cosh(x)) * \sinh(x)), -1/4 * (2 * (b^2 * \cosh(x)^4 + 4 * b^2 * \cosh(x) * \sinh(x)^3 + b^2 * \sinh(x)^4 + 2 * b^2 * \cosh(x)^2 + 2 * (3 * b^2 * \cosh(x)^2 + b^2) * \sinh(x)^2 + b^2 + 4 * (b^2 * \cosh(x)^3 + b^2 * \cosh(x)) * \sinh(x)) * \sqrt{-a - b} * \arctan(\sqrt{2} * (b * \cosh(x)^2 + 2 * b * \cosh(x) * \sinh(x) + b * \sinh(x)^2 - a - b) * \sqrt{-a - b} * \sqrt{((a + b) * \cosh(x)^2 + (a + b) * \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)}) / ((a * b + b^2) * \cosh(x)^4 + 4 * (a * b + b^2) * \cosh(x) * \sinh(x)^3 + (a * b + b^2) * \sinh(x)^4 + (a^2 - a * b - 2 * b^2) * \cosh(x)^2 + (6 * (a * b + b^2) * \cosh(x)^2 + a^2 - a * b - 2 * b^2) * \sinh(x)^2 + a^2 + 2 * a * b + b^2 + 2 * (2 * (a * b + b^2) * \cosh(x)^3 + (a^2 - a * b - 2 * b^2) * \cosh(x)) * \sinh(x))) + 2 * (b^2 * \cosh(x)^4 + 4 * b^2 * \cosh(x) * \sinh(x)^3 + b^2 * \sinh(x)^4 + 2 * b^2 * \cosh(x)^2 + 2 * (3 * b^2 * \cosh(x)^2 + b^2) * \sinh(x)^2 + b^2 + 4 * (b^2 * \cosh(x)^3 + b^2 * \cosh(x)) * \sinh(x)) * \sqrt{-a - b} * \arctan(\sqrt{2} * \sqrt{-a - b} * \sqrt{((a + b) * \cosh(x)^2 + (a + b) * \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)}) / ((a + b) * \cosh(x)^2 + 2 * (a + b) * \cosh(x) * \sinh(x) + (a + b) * \sinh(x)^2 + a + b)) + ((a^2 - a * b - 2 * b^2) * \cosh(x)^4 + 4 * (a^2 - a * b - 2 * b^2) * \cosh(x) * \sinh(x)^3 + (a^2 - a * b - 2 * b^2) * \sinh(x)^4 + 2 * (a^2 - a * b - 2 * b^2) * \cosh(x)^2 + 2 * (3 * (a^2 - a * b - 2 * b^2) * \cosh(x)^2 + a^2 - a * b - 2 * b^2) * \sinh(x)^2 + a^2 - a * b - 2 * b^2 + 4 * ((a^2 - a * b - 2 * b^2) * \cosh(x)^3 + (a^2 - a * b - 2 * b^2) * \cosh(x)) * \sinh(x)) * \sqrt{b} * \log(-((a + 2 * b) * \cosh(x)^4 + 4 * (a + 2 * b) * \cosh(x) * \sinh(x)^3 + (a + 2 * b) * \sinh(x)^4 + 2 * (a - 2 * b) * \cosh(x)^2 + 2 * (3 * (a + 2 * b) * \cosh(x)^2 + a - 2 * b) * \sinh(x)^2 - 2 * \sqrt{2} * (\cosh(x)^2 + 2 * \cosh(x) * \sinh(x) + \sinh(x)^2 - 1) * \sqrt{b} * \sqrt{((a + b) * \cosh(x)^2 + (a + b) * \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)}) + 4 * ((a + 2 * b) * \cosh(x)^3 + (a - 2 * b) * \cosh(x)) * \sinh(x)
\end{aligned}$$

```

x) + a + 2*b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2
+ 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + 2*sq
rt(2)*((a*b + b^2)*cosh(x)^2 + 2*(a*b + b^2)*cosh(x)*sinh(x) + (a*b + b^2)
*sinh(x)^2 - a*b - b^2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a*b^2 + b^3)*cosh(x)^4 +
4*(a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a*b^2 + b^3)*sinh(x)^4 + a*b^2 + b^3 +
2*(a*b^2 + b^3)*cosh(x)^2 + 2*(a*b^2 + b^3 + 3*(a*b^2 + b^3)*cosh(x)^2)*si
nh(x)^2 + 4*((a*b^2 + b^3)*cosh(x)^3 + (a*b^2 + b^3)*cosh(x))*sinh(x)), -1/
2*((b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*sinh(x)^4 + 2*b^2*cosh(x)
^2 + 2*(3*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 4*(b^2*cosh(x)^3 + b^2*cos
h(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(
x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*si
nh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a*b + b^2)*
cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2
- a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*si
nh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b
^2)*cosh(x))*sinh(x)) + ((a^2 - a*b - 2*b^2)*cosh(x)^4 + 4*(a^2 - a*b - 2*
b^2)*cosh(x)*sinh(x)^3 + (a^2 - a*b - 2*b^2)*sinh(x)^4 + 2*(a^2 - a*b - 2*b
^2)*cosh(x)^2 + 2*(3*(a^2 - a*b - 2*b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sin
h(x)^2 + a^2 - a*b - 2*b^2 + 4*((a^2 - a*b - 2*b^2)*cosh(x)^3 + (a^2 - a*b
- 2*b^2)*cosh(x))*sinh(x))*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*s
inh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^
2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^4
+ 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2
*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*c
osh(x))*sinh(x) + a + b)) + (b^2*cosh(x)^4 + 4*b^2*cosh(x)*sinh(x)^3 + b^2*
sinh(x)^4 + 2*b^2*cosh(x)^2 + 2*(3*b^2*cosh(x)^2 + b^2)*sinh(x)^2 + b^2 + 4
*(b^2*cosh(x)^3 + b^2*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*sqrt(-a
- b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x)
+ (a + b)*sinh(x)^2 + a + b)) + sqrt(2)*((a*b + b^2)*cosh(x)^2 + 2*(a*b +
b^2)*cosh(x)*sinh(x) + (a*b + b^2)*sinh(x)^2 - a*b - b^2)*sqrt(((a + b)*cos
h(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)
^2)))/((a*b^2 + b^3)*cosh(x)^4 + 4*(a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a*b^
2 + b^3)*sinh(x)^4 + a*b^2 + b^3 + 2*(a*b^2 + b^3)*cosh(x)^2 + 2*(a*b^2 + b
^3 + 3*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a*b^2 + b^3)*cosh(x)^3 + (a
*b^2 + b^3)*cosh(x))*sinh(x))]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep+1)]Evaluation time: 0.6Error: Bad Argument Type

maple [B] time = 0.13, size = 178, normalized size = 2.02

$$\frac{\sqrt{a + b(\tanh^2(x))} \tanh(x)}{2b} + \frac{a \ln\left(\sqrt{b} \tanh(x) + \sqrt{a + b(\tanh^2(x))}\right)}{2b^{\frac{3}{2}}} - \frac{\ln\left(\sqrt{b} \tanh(x) + \sqrt{a + b(\tanh^2(x))}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x)

[Out] $-1/2*(a+b*\tanh(x)^2)^{(1/2)}*\tanh(x)/b+1/2*a/b^{(3/2)}*\ln(b^{(1/2)}*\tanh(x)+(a+b*\tanh(x)^2)^{(1/2)})-\ln(b^{(1/2)}*\tanh(x)+(a+b*\tanh(x)^2)^{(1/2)})/b^{(1/2)}+1/2/(a+b)^{(1/2)}*\ln((2*a+2*b+2*(\tanh(x)-1)*b+2*(a+b)^{(1/2)}*((\tanh(x)-1)^2*b+2*(\tanh(x)-1)*b+a+b)^{(1/2)})/(\tanh(x)-1))-1/2/(a+b)^{(1/2)}*\ln((2*a+2*b-2*(1+\tanh(x))*b+2*(a+b)^{(1/2)}*((1+\tanh(x))^2*b-2*(1+\tanh(x))*b+a+b)^{(1/2)})/(1+\tanh(x)))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^4}{\sqrt{b \tanh(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(tanh(x)^4/sqrt(b*tanh(x)^2 + a), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^4}{\sqrt{b \tanh(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)^4/(a + b*tanh(x)^2)^(1/2),x)`

[Out] `int(tanh(x)^4/(a + b*tanh(x)^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{\sqrt{a + b \tanh^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)**4/(a+b*tanh(x)**2)**(1/2),x)`

[Out] `Integral(tanh(x)**4/sqrt(a + b*tanh(x)**2), x)`

$$3.231 \quad \int \frac{\tanh^3(x)}{\sqrt{a+b \tanh^2(x)}} dx$$

Optimal. Leaf size=47

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\sqrt{a+b \tanh^2(x)}}{b}$$

[Out] $\operatorname{arctanh}\left(\frac{(a+b \tanh(x)^2)^{1/2}}{(a+b)^{1/2}}\right) / (a+b)^{1/2} - (a+b \tanh(x)^2)^{1/2} / b$

Rubi [A] time = 0.11, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3670, 446, 80, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\sqrt{a+b \tanh^2(x)}}{b}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^3/Sqrt[a + b*Tanh[x]^2], x]

[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b] - Sqrt[a + b*Tanh[x]^2]/b

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x],

x]], Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f*f^2*x^2), x], x, (c*Tan[e + f*x])/ff, x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
 \int \frac{\tanh^3(x)}{\sqrt{a + b \tanh^2(x)}} dx &= \text{Subst} \left(\int \frac{x^3}{(1-x^2)\sqrt{a+bx^2}} dx, x, \tanh(x) \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{x}{(1-x)\sqrt{a+bx}} dx, x, \tanh^2(x) \right) \\
 &= -\frac{\sqrt{a + b \tanh^2(x)}}{b} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1-x)\sqrt{a+bx}} dx, x, \tanh^2(x) \right) \\
 &= -\frac{\sqrt{a + b \tanh^2(x)}}{b} + \frac{\text{Subst} \left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}} dx, x, \sqrt{a + b \tanh^2(x)} \right)}{b} \\
 &= \frac{\tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}} \right)}{\sqrt{a+b}} - \frac{\sqrt{a + b \tanh^2(x)}}{b}
 \end{aligned}$$

Mathematica [A] time = 0.11, size = 47, normalized size = 1.00

$$\frac{\tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}} \right)}{\sqrt{a+b}} - \frac{\sqrt{a + b \tanh^2(x)}}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^3/Sqrt[a + b*Tanh[x]^2], x]

[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b] - Sqrt[a + b*Tanh[x]^2]/b

fricas [B] time = 0.51, size = 1625, normalized size = 34.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(1/2), x, algorithm="fricas")

[Out] [1/4*((b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cos

$$\begin{aligned}
 & h(x)^2 + a^2) \sinh(x)^4 + 4*(5*a^2*\cosh(x)^3 + 3*a^2*\cosh(x))*\sinh(x)^3 + (\\
 & 3*a^2 + 2*a*b - b^2)*\cosh(x)^2 + (15*a^2*\cosh(x)^4 + 18*a^2*\cosh(x)^2 + 3*a \\
 & ^2 + 2*a*b - b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*\cosh(x)^5 + 6*a^ \\
 & 2*\cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*\cosh(x))*\sinh(x))*\sqrt{a + b}*\sqrt{((a \\
 & + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) \\
 & + \sinh(x)^2))} + 4*(2*(a^3 + a^2*b)*\cosh(x)^7 + 3*(2*a^3 + a^2*b)*\cosh(x)^5 \\
 & + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*\cosh(\\
 & x))*\sinh(x))/(\cosh(x)^6 + 6*\cosh(x)^5*\sinh(x) + 15*\cosh(x)^4*\sinh(x)^2 + 20 \\
 & *\cosh(x)^3*\sinh(x)^3 + 15*\cosh(x)^2*\sinh(x)^4 + 6*\cosh(x)*\sinh(x)^5 + \sinh(\\
 & x)^6) + (b*\cosh(x)^2 + 2*b*\cosh(x)*\sinh(x) + b*\sinh(x)^2 + b)*\sqrt{a + b}* \\
 & \log(-((a + b)*\cosh(x)^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a + b)*\sinh(x)^4 - \\
 & 2*b*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 - b)*\sinh(x)^2 + \sqrt{2}*(\cosh(x)^2 \\
 & + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1))*\sqrt{a + b}*\sqrt{((a + b)*\cosh(x)^2 + \\
 & (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))} + \\
 & 4*((a + b)*\cosh(x)^3 - b*\cosh(x))*\sinh(x) + a + b)/(\cosh(x)^2 + 2*\cosh(x)*\s \\
 & \sinh(x) + \sinh(x)^2)) - 4*\sqrt{2}*(a + b)*\sqrt{((a + b)*\cosh(x)^2 + (a + b)* \\
 & \sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))}/((a*b + b^ \\
 & 2)*\cosh(x)^2 + 2*(a*b + b^2)*\cosh(x)*\sinh(x) + (a*b + b^2)*\sinh(x)^2 + a*b \\
 & + b^2), -1/2*((b*\cosh(x)^2 + 2*b*\cosh(x)*\sinh(x) + b*\sinh(x)^2 + b)*\sqrt{-a \\
 & - b}*\arctan(\sqrt{2}*(a*\cosh(x)^2 + 2*a*\cosh(x)*\sinh(x) + a*\sinh(x)^2 + a + \\
 & b)*\sqrt{-a - b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh \\
 & (x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))}/((a^2 + a*b)*\cosh(x)^4 + 4*(a^2 + a \\
 & *b)*\cosh(x)*\sinh(x)^3 + (a^2 + a*b)*\sinh(x)^4 + (2*a^2 + a*b - b^2)*\cosh(x) \\
 & ^2 + (6*(a^2 + a*b)*\cosh(x)^2 + 2*a^2 + a*b - b^2)*\sinh(x)^2 + a^2 + 2*a*b \\
 & + b^2 + 2*(2*(a^2 + a*b)*\cosh(x)^3 + (2*a^2 + a*b - b^2)*\cosh(x))*\sinh(x))) \\
 & + (b*\cosh(x)^2 + 2*b*\cosh(x)*\sinh(x) + b*\sinh(x)^2 + b)*\sqrt{-a - b}*\arcta \\
 & n(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1))*\sqrt{-a - b}*\sqrt{ \\
 & (((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sin \\
 & h(x) + \sinh(x)^2))}/((a + b)*\cosh(x)^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a + \\
 & b)*\sinh(x)^4 + 2*(a - b)*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 + a - b)*\sinh(x) \\
 &)^2 + 4*((a + b)*\cosh(x)^3 + (a - b)*\cosh(x))*\sinh(x) + a + b) + 2*\sqrt{2} \\
 & *(a + b)*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - \\
 & 2*\cosh(x)*\sinh(x) + \sinh(x)^2))}/((a*b + b^2)*\cosh(x)^2 + 2*(a*b + b^2)*\cos \\
 & h(x)*\sinh(x) + (a*b + b^2)*\sinh(x)^2 + a*b + b^2)]
 \end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
 constant sign by intervals (correct if the argument is real):Check [abs(t_n
 ostep+1)]Error: Bad Argument Type

maple [B] time = 0.10, size = 129, normalized size = 2.74

$$\frac{\sqrt{a + b \left(\tanh^2(x)\right)}}{b} + \frac{\ln\left(\frac{2a+2b+2(\tanh(x)-1)b+2\sqrt{a+b} \sqrt{(\tanh(x)-1)^2b+2(\tanh(x)-1)b+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a + b}} + \frac{\ln\left(\frac{2a+2b-2(1+\tanh(x))b+2\sqrt{a+b}}{1}\right)}{2\sqrt{a + b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x)

[Out] -(a+b*tanh(x)^2)^(1/2)/b+1/2/(a+b)^(1/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))+1/2/(a+b)

$$\frac{\sqrt{1/2} \ln\left(\frac{(2a+2b-2(1+\tanh(x))b+2(a+b)^{1/2})((1+\tanh(x))^{2b-2(1+\tanh(x))b+a+b)^{1/2}}{(1+\tanh(x))}\right)}{1}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^3}{\sqrt{b \tanh(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(tanh(x)^3/sqrt(b*tanh(x)^2 + a), x)

mupad [B] time = 1.69, size = 39, normalized size = 0.83

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b \tanh(x)^2 + a}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\sqrt{b \tanh(x)^2 + a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^3/(a + b*tanh(x)^2)^(1/2),x)

[Out] atanh((a + b*tanh(x)^2)^(1/2)/(a + b)^(1/2))/(a + b)^(1/2) - (a + b*tanh(x)^2)^(1/2)/b

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{\sqrt{a + b \tanh^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)**3/(a+b*tanh(x)**2)**(1/2),x)

[Out] Integral(tanh(x)**3/sqrt(a + b*tanh(x)**2), x)

$$3.232 \quad \int \frac{\tanh^2(x)}{\sqrt{a+b \tanh^2(x)}} dx$$

Optimal. Leaf size=60

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{\sqrt{a+b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{\sqrt{b}}$$

[Out] $-\operatorname{arctanh}(b^{1/2} \tanh(x) / (a+b \tanh(x)^2)^{1/2}) / b^{1/2} + \operatorname{arctanh}((a+b)^{1/2} \tanh(x) / (a+b \tanh(x)^2)^{1/2}) / (a+b)^{1/2}$

Rubi [A] time = 0.09, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3670, 483, 217, 206, 377}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{\sqrt{a+b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^2/Sqrt[a + b*Tanh[x]^2], x]

[Out] $-(\operatorname{ArcTanh}[(\operatorname{Sqrt}[b] \operatorname{Tanh}[x]) / \operatorname{Sqrt}[a + b \operatorname{Tanh}[x]^2]] / \operatorname{Sqrt}[b]) + \operatorname{ArcTanh}[(\operatorname{Sqrt}[a + b] \operatorname{Tanh}[x]) / \operatorname{Sqrt}[a + b \operatorname{Tanh}[x]^2]] / \operatorname{Sqrt}[a + b]$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 483

Int[(((e_.)*(x_)^(m_))*((c_) + (d_.)*(x_)^(n_))^(q_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Dist[e^n/b, Int[(e*x)^(m-n)*(c + d*x^n)^q, x], x] - Dist[(a*e^n)/b, Int[((e*x)^(m-n)*(c + d*x^n)^q)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m, 2*n - 1] && IntBinomialQ[a, b, c, d, e, m, n, -1, q, x]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n}

, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned} \int \frac{\tanh^2(x)}{\sqrt{a+b \tanh^2(x)}} dx &= \text{Subst} \left(\int \frac{x^2}{(1-x^2)\sqrt{a+bx^2}} dx, x, \tanh(x) \right) \\ &= -\text{Subst} \left(\int \frac{1}{\sqrt{a+bx^2}} dx, x, \tanh(x) \right) + \text{Subst} \left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}} dx, x, \tanh(x) \right) \\ &= -\text{Subst} \left(\int \frac{1}{1-bx^2} dx, x, \frac{\tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right) + \text{Subst} \left(\int \frac{1}{1-(a+b)x^2} dx, x, \frac{\tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right) \\ &= -\frac{\tanh^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right)}{\sqrt{b}} + \frac{\tanh^{-1} \left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right)}{\sqrt{a+b}} \end{aligned}$$

Mathematica [C] time = 0.51, size = 101, normalized size = 1.68

$$\frac{a \coth(x) \sqrt{\text{sech}^2(x)((a+b) \cosh(2x) + a - b)} \Pi \left(\frac{b}{a+b}; \sin^{-1} \left(\frac{\sqrt{\frac{(a-b+(a+b) \cosh(2x)) \text{csch}^2(x)}{b}}}{\sqrt{2}} \right) \right) \Big|_1}{b(a+b) \sqrt{\frac{\text{csch}^2(x)((a+b) \cosh(2x) + a - b)}{b}}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^2/Sqrt[a + b*Tanh[x]^2], x]

[Out] -((a*Coth[x]*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]*Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2])/(b*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]))

fricas [B] time = 0.67, size = 3361, normalized size = 56.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(1/2), x, algorithm="fricas")

[Out] [1/4*(sqrt(a + b)*b*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4

$$\begin{aligned}
& - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2 \\
& *(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x) \\
&)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x) \\
&)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a \\
& *b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 \\
& - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15* \\
& cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*c \\
& osh(x)*sinh(x)^5 + sinh(x)^6)) + 2*(a + b)*sqrt(b)*log(-((a + 2*b)*cosh(x)^ \\
& 4 + 4*(a + 2*b)*cosh(x)*sinh(x)^3 + (a + 2*b)*sinh(x)^4 + 2*(a - 2*b)*cosh(\\
& x)^2 + 2*(3*(a + 2*b)*cosh(x)^2 + a - 2*b)*sinh(x)^2 - 2*sqrt(2)*(cosh(x)^2 \\
& + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(b)*sqrt(((a + b)*cosh(x)^2 + (a \\
& + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((\\
& a + 2*b)*cosh(x)^3 + (a - 2*b)*cosh(x))*sinh(x) + a + 2*b)/(cosh(x)^4 + 4*c \\
& osh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 \\
& + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)) + sqrt(a + b)*b*log(((a + b)*cosh(x) \\
&)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(\\
& 3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) \\
& + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + \\
& a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 \\
& + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)) \\
&)/(a*b + b^2), 1/4*(4*(a + b)*sqrt(-b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x) \\
&)*sinh(x) + sinh(x)^2 - 1)*sqrt(-b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(\\
& x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x) \\
& ^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 \\
& + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b) \\
&)*cosh(x))*sinh(x) + a + b)) + sqrt(a + b)*b*log(-((a*b^2 + b^3)*cosh(x)^8 \\
& + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + \\
& 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 \\
& + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (\\
& a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^ \\
& 3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(\\
& 14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + \\
& 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^ \\
& 3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 \\
& + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b \\
& ^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 \\
& + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + \\
& 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(\\
& x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^ \\
& 2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b \\
& - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*s \\
& inh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^ \\
& 2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + \\
& 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + \\
& 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15* \\
& cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + sqrt(a + b)*b*log \\
& (((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a \\
& *cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2 \\
& *cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a \\
& + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((\\
& a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(\\
& x) + sinh(x)^2)))/(a*b + b^2), -1/2*(sqrt(-a - b)*b*arctan(sqrt(2)*(b*cosh(\\
& x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b) \\
&)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + s \\
& inh(x)^2)))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b \\
& + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 \\
& + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh \\
& (x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x))) + sqrt(-a - b)*b*arctan(sqrt \\
& (2)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh
\end{aligned}$$

$$\frac{(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)}{((a + b)*\cosh(x)^2 + 2*(a + b)*\cosh(x)*\sinh(x) + (a + b)*\sinh(x)^2 + a + b)} - (a + b)*\sqrt{b}*\log(-((a + 2*b)*\cosh(x)^4 + 4*(a + 2*b)*\cosh(x)*\sinh(x)^3 + (a + 2*b)*\sinh(x)^4 + 2*(a - 2*b)*\cosh(x)^2 + 2*(3*(a + 2*b)*\cosh(x)^2 + a - 2*b)*\sinh(x)^2 - 2*\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1)*\sqrt{b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)}) + 4*((a + 2*b)*\cosh(x)^3 + (a - 2*b)*\cosh(x))*\sinh(x) + a + 2*b)/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 + 1)*\sinh(x)^2 + 2*\cosh(x)^2 + 4*(\cosh(x)^3 + \cosh(x))*\sinh(x) + 1)))/(a*b + b^2), -1/2*(\sqrt{-a - b}*b*\arctan(\sqrt{2}*(b*\cosh(x)^2 + 2*b*\cosh(x)*\sinh(x) + b*\sinh(x)^2 - a - b)*\sqrt{-a - b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)})/((a*b + b^2)*\cosh(x)^4 + 4*(a*b + b^2)*\cosh(x)*\sinh(x)^3 + (a*b + b^2)*\sinh(x)^4 + (a^2 - a*b - 2*b^2)*\cosh(x)^2 + (6*(a*b + b^2)*\cosh(x)^2 + a^2 - a*b - 2*b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*\cosh(x)^3 + (a^2 - a*b - 2*b^2)*\cosh(x))*\sinh(x))) - 2*(a + b)*\sqrt{-b}*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1)*\sqrt{-b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)})/((a + b)*\cosh(x)^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a + b)*\sinh(x)^4 + 2*(a - b)*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 + a - b)*\sinh(x)^2 + 4*((a + b)*\cosh(x)^3 + (a - b)*\cosh(x))*\sinh(x) + a + b)) + \sqrt{-a - b}*b*\arctan(\sqrt{2}*\sqrt{-a - b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)})/((a + b)*\cosh(x)^2 + 2*(a + b)*\cosh(x)*\sinh(x) + (a + b)*\sinh(x)^2 + a + b)))/(a*b + b^2]}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep+1)]Error: Bad Argument Type

maple [B] time = 0.10, size = 137, normalized size = 2.28

$$\frac{\ln\left(\sqrt{b} \tanh(x) + \sqrt{a + b \left(\tanh^2(x)\right)}\right)}{\sqrt{b}} + \frac{\ln\left(\frac{2a+2b+2(\tanh(x)-1)b+2\sqrt{a+b} \sqrt{(\tanh(x)-1)^2b+2(\tanh(x)-1)b+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}} - \frac{\ln\left(\frac{2a+2b-2(\tanh(x)-1)b+2\sqrt{a+b} \sqrt{(\tanh(x)-1)^2b+2(\tanh(x)-1)b+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x)

[Out] $-\ln(b^{(1/2)}*\tanh(x)+(a+b*\tanh(x)^2)^{(1/2)})/b^{(1/2)}+1/2/(a+b)^{(1/2)}*\ln((2*a+2*b+2*(\tanh(x)-1)*b+2*(a+b)^{(1/2))*((\tanh(x)-1)^2*b+2*(\tanh(x)-1)*b+a+b)^{(1/2)})/(\tanh(x)-1))-1/2/(a+b)^{(1/2)}*\ln((2*a+2*b-2*(1+\tanh(x))*b+2*(a+b)^{(1/2))*((1+\tanh(x))^2*b-2*(1+\tanh(x))*b+a+b)^{(1/2)})/(1+\tanh(x)))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^2}{\sqrt{b \tanh(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(tanh(x)^2/sqrt(b*tanh(x)^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tanh(x)^2}{\sqrt{b \tanh(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^2/(a + b*tanh(x)^2)^(1/2), x)

[Out] int(tanh(x)^2/(a + b*tanh(x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{\sqrt{a + b \tanh^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)**2/(a+b*tanh(x)**2)**(1/2), x)

[Out] Integral(tanh(x)**2/sqrt(a + b*tanh(x)**2), x)

$$3.233 \quad \int \frac{\tanh(x)}{\sqrt{a+b \tanh^2(x)}} dx$$

Optimal. Leaf size=29

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)

Rubi [A] time = 0.06, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3670, 444, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]/Sqrt[a + b*Tanh[x]^2], x]

[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(x)}{\sqrt{a+b \tanh^2(x)}} dx &= \text{Subst} \left(\int \frac{x}{(1-x^2)\sqrt{a+bx^2}} dx, x, \tanh(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1-x)\sqrt{a+bx}} dx, x, \tanh^2(x) \right) \\
&= \frac{\text{Subst} \left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}} dx, x, \sqrt{a+b \tanh^2(x)} \right)}{b} \\
&= \frac{\tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}} \right)}{\sqrt{a+b}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 1.00

$$\frac{\tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}} \right)}{\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]/Sqrt[a + b*Tanh[x]^2],x]

[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b]

fricas [B] time = 0.50, size = 1361, normalized size = 46.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")

[Out] [1/4*(sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + sqrt(a + b)*log(-(a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2

```
- 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 - b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a + b), -1/2*(sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x))) + sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)))/(a + b)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep+1)]Error: Bad Argument Type

maple [B] time = 0.10, size = 114, normalized size = 3.93

$$\frac{\ln\left(\frac{2a+2b+2(\tanh(x)-1)b+2\sqrt{a+b}\sqrt{(\tanh(x)-1)^2b+2(\tanh(x)-1)b+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}} + \frac{\ln\left(\frac{2a+2b-2(1+\tanh(x))b+2\sqrt{a+b}\sqrt{(1+\tanh(x))^2b-2(1+\tanh(x))b}}{1+\tanh(x)}\right)}{2\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)/(a+b*tanh(x)^2)^(1/2),x)

[Out] 1/2/(a+b)^(1/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)))/(tanh(x)-1))+1/2/(a+b)^(1/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)))/(1+tanh(x)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{b \tanh(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(tanh(x)/sqrt(b*tanh(x)^2 + a), x)

mupad [B] time = 1.62, size = 23, normalized size = 0.79

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b \tanh(x)^2 + a}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)/(a + b*tanh(x)^2)^(1/2), x)`

[Out] `atanh((a + b*tanh(x)^2)^(1/2)/(a + b)^(1/2))/(a + b)^(1/2)`

sympy [A] time = 1.30, size = 31, normalized size = 1.07

$$-\frac{\operatorname{atan}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)/(a+b*tanh(x)**2)**(1/2), x)`

[Out] `-atan(sqrt(a + b*tanh(x)**2)/sqrt(-a - b))/sqrt(-a - b)`

$$3.234 \quad \int \frac{1}{\sqrt{a+b \tanh^2(x)}} dx$$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{\sqrt{a+b}}$$

[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(1/2)

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3661, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b*Tanh[x]^2], x]

[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 3661

Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(ff*x)^n]^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a+b \tanh^2(x)}} dx &= \text{Subst} \left(\int \frac{1}{(1-x^2) \sqrt{a+bx^2}} dx, x, \tanh(x) \right) \\ &= \text{Subst} \left(\int \frac{1}{1-(a+b)x^2} dx, x, \frac{\tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right) \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{\sqrt{a+b}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 31, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b}\tanh^2(x)}\right)}{\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a + b*Tanh[x]^2], x]

[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b]

fricas [B] time = 0.48, size = 1287, normalized size = 41.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tanh(x)^2)^(1/2), x, algorithm="fricas")

[Out] [1/4*(sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a + b), -1/2*(sqrt(-a - b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a - b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x)) + sqrt(-a - b)*arctan(sqrt(2)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)))/(a + b)]

giac [B] time = 0.32, size = 188, normalized size = 6.06

$$\frac{\log\left(\left|-\left(\sqrt{a+b}e^{2x}-\sqrt{ae^{4x}+be^{4x}+2ae^{2x}-2be^{2x}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}} \log\left(\left|-\sqrt{a+b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")

[Out]
$$-1/2*\log(\text{abs}(-(\text{sqrt}(a+b)*e^{2*x} - \text{sqrt}(a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b))*(a+b) - \text{sqrt}(a+b)*(a-b)))/\text{sqrt}(a+b) - 1/2*\log(\text{abs}(-\text{sqrt}(a+b)*e^{2*x} + \text{sqrt}(a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b) + \text{sqrt}(a+b)))/\text{sqrt}(a+b) + 1/2*\log(\text{abs}(-\text{sqrt}(a+b)*e^{2*x} + \text{sqrt}(a*e^{4*x} + b*e^{4*x} + 2*a*e^{2*x} - 2*b*e^{2*x} + a + b) - \text{sqrt}(a+b)))/\text{sqrt}(a+b)$$

maple [B] time = 0.11, size = 114, normalized size = 3.68

$$\frac{\ln\left(\frac{2a+2b+2(\tanh(x)-1)b+2\sqrt{a+b}\sqrt{(\tanh(x)-1)^2b+2(\tanh(x)-1)b+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}} - \frac{\ln\left(\frac{2a+2b-2(1+\tanh(x))b+2\sqrt{a+b}\sqrt{(1+\tanh(x))^2b-2(1+\tanh(x))b+a+b}}{1+\tanh(x)}\right)}{2\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*tanh(x)^2)^(1/2),x)

[Out]
$$1/2/(a+b)^{(1/2)}*\ln((2*a+2*b+2*(\tanh(x)-1)*b+2*(a+b)^{(1/2)}*((\tanh(x)-1)^{2*b+2*(\tanh(x)-1)*b+a+b})^{(1/2)})/(\tanh(x)-1))-1/2/(a+b)^{(1/2)}*\ln((2*a+2*b-2*(1+\tanh(x))*b+2*(a+b)^{(1/2)}*((1+\tanh(x))^{2*b-2*(1+\tanh(x))*b+a+b})^{(1/2)})/(1+\tanh(x)))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \tanh(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(b*tanh(x)^2 + a), x)

mupad [B] time = 1.57, size = 25, normalized size = 0.81

$$\frac{\text{atanh}\left(\frac{\tanh(x)\sqrt{a+b}}{\sqrt{b \tanh(x)^2 + a}}\right)}{\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*tanh(x)^2)^(1/2),x)

[Out]
$$\text{atanh}((\tanh(x)*(a+b)^{(1/2)})/(a+b*\tanh(x)^2)^{(1/2)})/(a+b)^{(1/2)}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \tanh^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tanh(x)**2)**(1/2),x)

[Out] Integral(1/sqrt(a + b*tanh(x)**2), x)

$$3.235 \quad \int \frac{\coth(x)}{\sqrt{a+b \tanh^2(x)}} dx$$

Optimal. Leaf size=56

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

[Out] $-\operatorname{arctanh}((a+b*\tanh(x)^2)^{(1/2)/a^{(1/2))}/a^{(1/2)}+\operatorname{arctanh}((a+b*\tanh(x)^2)^{(1/2)/(a+b)^{(1/2))}/(a+b)^{(1/2)})$

Rubi [A] time = 0.11, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3670, 446, 86, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]/Sqrt[a + b*Tanh[x]^2], x]

[Out] $-(\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tanh}[x]^2]/\operatorname{Sqrt}[a]]/\operatorname{Sqrt}[a]) + \operatorname{ArcTanh}[\operatorname{Sqrt}[a + b*\operatorname{Tanh}[x]^2]/\operatorname{Sqrt}[a + b]]/\operatorname{Sqrt}[a + b]$

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 86

Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && !IntegerQ[p]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f

$f^2 x^2), x], x, (c \cdot \text{Tan}[e + f \cdot x]) / ff], x]] /; \text{FreeQ}[\{a, b, c, d, e, f, m, n, p\}, x] \&\& (\text{IGtQ}[p, 0] \mid \mid \text{EqQ}[n, 2] \mid \mid \text{EqQ}[n, 4] \mid \mid (\text{IntegerQ}[p] \&\& \text{RationalQ}[n]))$

Rubi steps

$$\begin{aligned} \int \frac{\coth(x)}{\sqrt{a + b \tanh^2(x)}} dx &= \text{Subst} \left(\int \frac{1}{x(1-x^2)\sqrt{a+bx^2}} dx, x, \tanh(x) \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1-x)x\sqrt{a+bx}} dx, x, \tanh^2(x) \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1-x)\sqrt{a+bx}} dx, x, \tanh^2(x) \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \tanh^2(x) \right) \\ &= \frac{\text{Subst} \left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}} dx, x, \sqrt{a+b \tanh^2(x)} \right)}{b} + \frac{\text{Subst} \left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+b \tanh^2(x)} \right)}{b} \\ &= -\frac{\tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a}} \right)}{\sqrt{a}} + \frac{\tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}} \right)}{\sqrt{a+b}} \end{aligned}$$

Mathematica [A] time = 0.05, size = 56, normalized size = 1.00

$$\frac{\tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}} \right)}{\sqrt{a+b}} - \frac{\tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a}} \right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]/Sqrt[a + b*Tanh[x]^2], x]

[Out] -(ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]/Sqrt[a]) + ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b]

fricas [B] time = 0.61, size = 3527, normalized size = 62.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*tanh(x)^2)^(1/2), x, algorithm="fricas")

[Out] [1/4*(sqrt(a + b)*a*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 +

$$b \cdot \cosh(x)^2 + (a + b) \cdot \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2) / ((a + b) \cdot \cosh(x)^4 + 4 \cdot (a + b) \cdot \cosh(x) \cdot \sinh(x)^3 + (a + b) \cdot \sinh(x)^4 + 2 \cdot (a - b) \cdot \cosh(x)^2 + 2 \cdot (3 \cdot (a + b) \cdot \cosh(x)^2 + a - b) \cdot \sinh(x)^2 + 4 \cdot ((a + b) \cdot \cosh(x)^3 + (a - b) \cdot \cosh(x)) \cdot \sinh(x) + a + b) - (a + b) \cdot \sqrt{a} \cdot \log(-((2 \cdot a + b) \cdot \cosh(x)^4 + 4 \cdot (2 \cdot a + b) \cdot \cosh(x) \cdot \sinh(x)^3 + (2 \cdot a + b) \cdot \sinh(x)^4 + 2 \cdot (2 \cdot a - b) \cdot \cosh(x)^2 + 2 \cdot (3 \cdot (2 \cdot a + b) \cdot \cosh(x)^2 + 2 \cdot a - b) \cdot \sinh(x)^2 - 2 \cdot \sqrt{2} \cdot (\cosh(x)^2 + 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2 + 1) \cdot \sqrt{a} \cdot \sqrt{((a + b) \cdot \cosh(x)^2 + (a + b) \cdot \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2)} + 4 \cdot ((2 \cdot a + b) \cdot \cosh(x)^3 + (2 \cdot a - b) \cdot \cosh(x)) \cdot \sinh(x) + 2 \cdot a + b) / (\cosh(x)^4 + 4 \cdot \cosh(x) \cdot \sinh(x)^3 + \sinh(x)^4 + 2 \cdot (3 \cdot \cosh(x)^2 - 1) \cdot \sinh(x)^2 - 2 \cdot \cosh(x)^2 + 4 \cdot (\cosh(x)^3 - \cosh(x)) \cdot \sinh(x) + 1))) / (a^2 + a \cdot b), 1/2 \cdot (2 \cdot \sqrt{-a}) \cdot (a + b) \cdot \arctan(\sqrt{2} \cdot (\cosh(x)^2 + 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2 + 1) \cdot \sqrt{-a} \cdot \sqrt{((a + b) \cdot \cosh(x)^2 + (a + b) \cdot \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2)}) / ((a + b) \cdot \cosh(x)^4 + 4 \cdot (a + b) \cdot \cosh(x) \cdot \sinh(x)^3 + (a + b) \cdot \sinh(x)^4 + 2 \cdot (a - b) \cdot \cosh(x)^2 + 2 \cdot (3 \cdot (a + b) \cdot \cosh(x)^2 + a - b) \cdot \sinh(x)^2 + 4 \cdot ((a + b) \cdot \cosh(x)^3 + (a - b) \cdot \cosh(x)) \cdot \sinh(x) + a + b)) - a \cdot \sqrt{-a - b} \cdot \arctan(\sqrt{2} \cdot (a \cdot \cosh(x)^2 + 2 \cdot a \cdot \cosh(x) \cdot \sinh(x) + a \cdot \sinh(x)^2 + a + b) \cdot \sqrt{-a - b} \cdot \sqrt{((a + b) \cdot \cosh(x)^2 + (a + b) \cdot \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2)}) / ((a^2 + a \cdot b) \cdot \cosh(x)^4 + 4 \cdot (a^2 + a \cdot b) \cdot \cosh(x) \cdot \sinh(x)^3 + (a^2 + a \cdot b) \cdot \sinh(x)^4 + (2 \cdot a^2 + a \cdot b - b^2) \cdot \cosh(x)^2 + (6 \cdot (a^2 + a \cdot b) \cdot \cosh(x)^2 + 2 \cdot a^2 + a \cdot b - b^2) \cdot \sinh(x)^2 + a^2 + 2 \cdot a \cdot b + b^2 + 2 \cdot (2 \cdot (a^2 + a \cdot b) \cdot \cosh(x)^3 + (2 \cdot a^2 + a \cdot b - b^2) \cdot \cosh(x)) \cdot \sinh(x))) - a \cdot \sqrt{-a - b} \cdot \arctan(\sqrt{2} \cdot (\cosh(x)^2 + 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2 - 1) \cdot \sqrt{-a - b} \cdot \sqrt{((a + b) \cdot \cosh(x)^2 + (a + b) \cdot \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2)}) / ((a + b) \cdot \cosh(x)^4 + 4 \cdot (a + b) \cdot \cosh(x) \cdot \sinh(x)^3 + (a + b) \cdot \sinh(x)^4 + 2 \cdot (a - b) \cdot \cosh(x)^2 + 2 \cdot (3 \cdot (a + b) \cdot \cosh(x)^2 + a - b) \cdot \sinh(x)^2 + 4 \cdot ((a + b) \cdot \cosh(x)^3 + (a - b) \cdot \cosh(x)) \cdot \sinh(x) + a + b))) / (a^2 + a \cdot b)]$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep+1)]Error: Bad Argument Type

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\sqrt{a + b(\tanh^2(x))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)/(a+b*tanh(x)^2)^(1/2),x)

[Out] int(coth(x)/(a+b*tanh(x)^2)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{\sqrt{b \tanh(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(coth(x)/sqrt(b*tanh(x)^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{coth}(x)}{\sqrt{b \tanh(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)/(a + b*tanh(x)^2)^(1/2), x)

[Out] int(coth(x)/(a + b*tanh(x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{coth}(x)}{\sqrt{a + b \tanh^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*tanh(x)**2)**(1/2), x)

[Out] Integral(coth(x)/sqrt(a + b*tanh(x)**2), x)

$$3.236 \quad \int \frac{\coth^2(x)}{\sqrt{a+b \tanh^2(x)}} dx$$

Optimal. Leaf size=51

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{\sqrt{a+b}} - \frac{\coth(x)\sqrt{a+b \tanh^2(x)}}{a}$$

[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(1/2)-coth(x)*(a+b*tanh(x)^2)^(1/2)/a

Rubi [A] time = 0.09, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3670, 480, 12, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{\sqrt{a+b}} - \frac{\coth(x)\sqrt{a+b \tanh^2(x)}}{a}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^2/Sqrt[a + b*Tanh[x]^2], x]

[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b] - (Coth[x]*Sqrt[a + b*Tanh[x]^2])/a

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 480

Int[((e_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m+1)*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*c*e*(m+1)), x] - Dist[1/(a*c*e^n*(m+1)), Int[(e*x)^(m+n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m+n+1) + n*(b*c*p + a*d*q) + b*d*(m+n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x],

x]], Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f*ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
 \int \frac{\coth^2(x)}{\sqrt{a + b \tanh^2(x)}} dx &= \text{Subst} \left(\int \frac{1}{x^2 (1-x^2) \sqrt{a + bx^2}} dx, x, \tanh(x) \right) \\
 &= -\frac{\coth(x) \sqrt{a + b \tanh^2(x)}}{a} + \frac{\text{Subst} \left(\int \frac{a}{(1-x^2) \sqrt{a+bx^2}} dx, x, \tanh(x) \right)}{a} \\
 &= -\frac{\coth(x) \sqrt{a + b \tanh^2(x)}}{a} + \text{Subst} \left(\int \frac{1}{(1-x^2) \sqrt{a + bx^2}} dx, x, \tanh(x) \right) \\
 &= -\frac{\coth(x) \sqrt{a + b \tanh^2(x)}}{a} + \text{Subst} \left(\int \frac{1}{1 - (a+b)x^2} dx, x, \frac{\tanh(x)}{\sqrt{a + b \tanh^2(x)}} \right) \\
 &= \frac{\tanh^{-1} \left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right)}{\sqrt{a+b}} - \frac{\coth(x) \sqrt{a + b \tanh^2(x)}}{a}
 \end{aligned}$$

Mathematica [C] time = 6.18, size = 123, normalized size = 2.41

$$\frac{\tanh(x) \left(\frac{(a+b)^2((a+b) \cosh(2x)+a-b)^2 {}_2F_1 \left(2, 2; \frac{5}{2}; -\frac{(a+b) \sinh^2(x)}{a} \right)}{a^3} + 3 \text{csch}^2(x) (a \coth^2(x) + 2b) \sin^{-1} \left(\sqrt{-\frac{(a+b) \sinh^2(x)}{a}} \right) \sqrt{-\frac{(a+b) \sinh^2(x)}{a}} \right)}{3(a+b) \sqrt{a + b \tanh^2(x)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Coth[x]^2/Sqrt[a + b*Tanh[x]^2], x]

[Out] (((a + b)^2*(a - b + (a + b)*Cosh[2*x])^2*Hypergeometric2F1[2, 2, 5/2, -((a + b)*Sinh[x]^2)/a])/a^3 + 3*ArcSin[Sqrt[-((a + b)*Sinh[x]^2)/a]]*(2*b + a*Coth[x]^2)*Csch[x]^2*Sqrt[-((a + b)*(b + a*Coth[x]^2)*Sinh[x]^4)/a^2])*Tanh[x]/(3*(a + b)*Sqrt[a + b*Tanh[x]^2])

fricas [B] time = 0.60, size = 1565, normalized size = 30.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(1/2), x, algorithm="fricas")

[Out] [1/4*((a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*

```

cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^
2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 +
b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(
a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^
6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*co
sh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 -
(a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^
2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b
^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a
+ b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
+ sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5
+ (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh
(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 2
0*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh
(x)^6)) + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(a + b)
*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 +
2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 +
(a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) +
4*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*s
inh(x) + sinh(x)^2)) - 4*sqrt(2)*(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*
sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^2 + a*
b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh(x)^2 - a^2
- a*b), -1/2*((a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(-a
- b)*arctan(sqrt(2)*(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 - a -
b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh
(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a*b + b^2)*cosh(x)^4 + 4*(a*b + b
^2)*cosh(x)*sinh(x)^3 + (a*b + b^2)*sinh(x)^4 + (a^2 - a*b - 2*b^2)*cosh(x)
^2 + (6*(a*b + b^2)*cosh(x)^2 + a^2 - a*b - 2*b^2)*sinh(x)^2 + a^2 + 2*a*b
+ b^2 + 2*(2*(a*b + b^2)*cosh(x)^3 + (a^2 - a*b - 2*b^2)*cosh(x))*sinh(x)))
+ (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(-a - b)*arcta
n(sqrt(2)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)
/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^2 + 2*(a + b
)*cosh(x)*sinh(x) + (a + b)*sinh(x)^2 + a + b)) + 2*sqrt(2)*(a + b)*sqrt(((
a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
+ sinh(x)^2)))/((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (
a^2 + a*b)*sinh(x)^2 - a^2 - a*b)]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep+1)]Error: Bad Argument Type

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{\sqrt{a + b(\tanh^2(x))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x)

[Out] int(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)^2}{\sqrt{b \tanh(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(coth(x)^2/sqrt(b*tanh(x)^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\coth(x)^2}{\sqrt{b \tanh(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^2/(a + b*tanh(x)^2)^(1/2),x)

[Out] int(coth(x)^2/(a + b*tanh(x)^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{\sqrt{a + b \tanh^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**2/(a+b*tanh(x)**2)**(1/2),x)

[Out] Integral(coth(x)**2/sqrt(a + b*tanh(x)**2), x)

$$3.237 \quad \int \frac{\coth^3(x)}{\sqrt{a+b \tanh^2(x)}} dx$$

Optimal. Leaf size=88

$$-\frac{(2a-b) \tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\coth^2(x) \sqrt{a+b \tanh^2(x)}}{2a}$$

[Out] $-1/2*(2*a-b)*\operatorname{arctanh}((a+b*\tanh(x)^2)^{(1/2)}/a^{(1/2)})/a^{(3/2)}+\operatorname{arctanh}((a+b*\tanh(x)^2)^{(1/2)}/(a+b)^{(1/2)})/(a+b)^{(1/2)}-1/2*\coth(x)^2*(a+b*\tanh(x)^2)^{(1/2)}/a$

Rubi [A] time = 0.17, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3670, 446, 103, 156, 63, 208}

$$-\frac{(2a-b) \tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\coth^2(x) \sqrt{a+b \tanh^2(x)}}{2a}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^3/Sqrt[a + b*Tanh[x]^2], x]

[Out] $-\frac{(2*a-b)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tanh}[x]^2]/\operatorname{Sqrt}[a]]}{(2*a^{(3/2)})} + \operatorname{ArcTanh}[\operatorname{Sqrt}[a+b*\operatorname{Tanh}[x]^2]/\operatorname{Sqrt}[a+b]]/\operatorname{Sqrt}[a+b] - \frac{\operatorname{Coth}[x]^2*\operatorname{Sqrt}[a+b*\operatorname{Tanh}[x]^2]}{(2*a)}$

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m+1)-1)*(c-(a*d)/b+(d*x^p)/b)^n, x], x, (a+b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c-a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 103

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[(b*(a+b*x)^(m+1)*(c+d*x)^(n+1)*(e+f*x)^(p+1))/((m+1)*(b*c-a*d)*(b*e-a*f)), x] + Dist[1/((m+1)*(b*c-a*d)*(b*e-a*f)), Int[(a+b*x)^(m+1)*(c+d*x)^n*(e+f*x)^p*Simp[a*d*f*(m+1)-b*(d*e*(m+n+2)+c*f*(m+p+2))-b*d*f*(m+n+p+3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])

Rule 156

Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g-a*h)/(b*c-a*d), Int[(e+f*x)^p/(a+b*x), x], x] - Dist[(d*g-c*h)/(b*c-a*d), Int[(e+f*x)^p/(c+d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*((c_)*tan[(e_) +
(f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x],
x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(x)}{\sqrt{a + b \tanh^2(x)}} dx &= \text{Subst} \left(\int \frac{1}{x^3 (1-x^2) \sqrt{a + bx^2}} dx, x, \tanh(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1-x)x^2 \sqrt{a + bx}} dx, x, \tanh^2(x) \right) \\
&= -\frac{\coth^2(x) \sqrt{a + b \tanh^2(x)}}{2a} - \frac{\text{Subst} \left(\int \frac{\frac{1}{2}(-2a+b) - \frac{bx}{2}}{(1-x)x \sqrt{a+bx}} dx, x, \tanh^2(x) \right)}{2a} \\
&= -\frac{\coth^2(x) \sqrt{a + b \tanh^2(x)}}{2a} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1-x) \sqrt{a + bx}} dx, x, \tanh^2(x) \right) + \frac{(2a - b)}{2a} \text{Subst} \left(\int \frac{1}{1 + \frac{a}{b} - \frac{x^2}{b}} dx, x, \sqrt{a + b \tanh^2(x)} \right) \\
&= -\frac{\coth^2(x) \sqrt{a + b \tanh^2(x)}}{2a} + \frac{\text{Subst} \left(\int \frac{1}{1 + \frac{a}{b} - \frac{x^2}{b}} dx, x, \sqrt{a + b \tanh^2(x)} \right)}{b} + \frac{(2a - b)}{2a} \text{Subst} \left(\int \frac{1}{1 + \frac{a}{b} - \frac{x^2}{b}} dx, x, \sqrt{a + b \tanh^2(x)} \right) \\
&= -\frac{(2a - b) \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a}} \right)}{2a^{3/2}} + \frac{\tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}} \right)}{\sqrt{a + b}} - \frac{\coth^2(x) \sqrt{a + b \tanh^2(x)}}{2a}
\end{aligned}$$

Mathematica [A] time = 0.47, size = 107, normalized size = 1.22

$$\frac{(-2a^2 - ab + b^2) \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a}} \right) + \sqrt{a} \left(2a \sqrt{a + b} \tanh^{-1} \left(\frac{\sqrt{a + b \tanh^2(x)}}{\sqrt{a + b}} \right) - (a + b) \coth^2(x) \sqrt{a + b \tanh^2(x)} \right)}{2a^{3/2}(a + b)}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]^3/Sqrt[a + b*Tanh[x]^2], x]

[Out] ((-2*a^2 - a*b + b^2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]] + Sqrt[a]*(2*a*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (a + b)*Coth[x]^2*Sqrt[a + b*Tanh[x]^2]))/(2*a^(3/2)*(a + b))

fricas [B] time = 0.99, size = 5711, normalized size = 64.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/4*((a^2*\cosh(x)^4 + 4*a^2*\cosh(x)*\sinh(x)^3 + a^2*\sinh(x)^4 - 2*a^2*\cosh(x)^2 + 2*(3*a^2*\cosh(x)^2 - a^2)*\sinh(x)^2 + a^2 + 4*(a^2*\cosh(x)^3 - a^2*\cosh(x))*\sinh(x))*\sqrt{a+b}*\log(((a^3 + a^2*b)*\cosh(x)^8 + 8*(a^3 + a^2*b)*\cosh(x)*\sinh(x)^7 + (a^3 + a^2*b)*\sinh(x)^8 + 2*(2*a^3 + a^2*b)*\cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*\cosh(x)^2)*\sinh(x)^6 + 4*(14*(a^3 + a^2*b)*\cosh(x)^3 + 3*(2*a^3 + a^2*b)*\cosh(x))*\sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x)^4 + (70*(a^3 + a^2*b)*\cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*\cosh(x)^2)*\sinh(x)^4 + 4*(14*(a^3 + a^2*b)*\cosh(x)^5 + 10*(2*a^3 + a^2*b)*\cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x))*\sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*\cosh(x)^2 + 2*(14*(a^3 + a^2*b)*\cosh(x)^6 + 15*(2*a^3 + a^2*b)*\cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x)^2)*\sinh(x)^2 + \sqrt{2}*(a^2*\cosh(x)^6 + 6*a^2*\cosh(x)*\sinh(x)^5 + a^2*\sinh(x)^6 + 3*a^2*\cosh(x)^4 + 3*(5*a^2*\cosh(x)^2 + a^2)*\sinh(x)^4 + 4*(5*a^2*\cosh(x)^3 + 3*a^2*\cosh(x))*\sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*\cosh(x)^2 + (15*a^2*\cosh(x)^4 + 18*a^2*\cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*\cosh(x)^5 + 6*a^2*\cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*\cosh(x))*\sinh(x))*\sqrt{a+b}*\sqrt{((a+b)*\cosh(x)^2 + (a+b)*\sinh(x)^2 + a-b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)} + 4*(2*(a^3 + a^2*b)*\cosh(x)^7 + 3*(2*a^3 + a^2*b)*\cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*\cosh(x))*\sinh(x))/(\cosh(x)^6 + 6*\cosh(x)^5*\sinh(x) + 15*\cosh(x)^4*\sinh(x)^2 + 20*\cosh(x)^3*\sinh(x)^3 + 15*\cosh(x)^2*\sinh(x)^4 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6)) - ((2*a^2 + a*b - b^2)*\cosh(x)^4 + 4*(2*a^2 + a*b - b^2)*\cosh(x)*\sinh(x)^3 + (2*a^2 + a*b - b^2)*\sinh(x)^4 - 2*(2*a^2 + a*b - b^2)*\cosh(x)^2 + 2*(3*(2*a^2 + a*b - b^2)*\cosh(x)^2 - 2*a^2 - a*b + b^2)*\sinh(x)^2 + 2*a^2 + a*b - b^2 + 4*((2*a^2 + a*b - b^2)*\cosh(x))^3 - (2*a^2 + a*b - b^2)*\cosh(x))*\sinh(x))*\sqrt{a}*\log(-((2*a + b)*\cosh(x)^4 + 4*(2*a + b)*\cosh(x)*\sinh(x)^3 + (2*a + b)*\sinh(x)^4 + 2*(2*a - b)*\cosh(x)^2 + 2*(3*(2*a + b)*\cosh(x)^2 + 2*a - b)*\sinh(x)^2 + 2*\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1)*\sqrt{a}*\sqrt{((a+b)*\cosh(x)^2 + (a+b)*\sinh(x)^2 + a-b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)} + 4*((2*a + b)*\cosh(x)^3 + (2*a - b)*\cosh(x))*\sinh(x) + 2*a + b)/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 - 1)*\sinh(x)^2 - 2*\cosh(x)^2 + 4*(\cosh(x)^3 - \cosh(x))*\sinh(x) + 1)) + (a^2*\cosh(x)^4 + 4*a^2*\cosh(x)*\sinh(x)^3 + a^2*\sinh(x)^4 - 2*a^2*\cosh(x)^2 + 2*(3*a^2*\cosh(x)^2 - a^2)*\sinh(x)^2 + a^2 + 4*(a^2*\cosh(x)^3 - a^2*\cosh(x))*\sinh(x))*\sqrt{a+b}*\log(-((a+b)*\cosh(x)^4 + 4*(a+b)*\cosh(x)*\sinh(x)^3 + (a+b)*\sinh(x)^4 - 2*b*\cosh(x)^2 + 2*(3*(a+b)*\cosh(x)^2 - b)*\sinh(x)^2 + \sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1)*\sqrt{a+b}*\sqrt{((a+b)*\cosh(x)^2 + (a+b)*\sinh(x)^2 + a-b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)} + 4*((a+b)*\cosh(x)^3 - b*\cosh(x))*\sinh(x) + a+b)/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) - 2*\sqrt{2}*((a^2 + a*b)*\cosh(x)^2 + 2*(a^2 + a*b)*\cosh(x)*\sinh(x) + (a^2 + a*b)*\sinh(x)^2 + a^2 + a*b)*\sqrt{((a+b)*\cosh(x)^2 + (a+b)*\sinh(x)^2 + a-b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/((a^3 + a^2*b)*\cosh(x)^4 + 4*(a^3 + a^2*b)*\cosh(x)*\sinh(x)^3 + (a^3 + a^2*b)*\sinh(x)^4 + a^3 + a^2*b - 2*(a^3 + a^2*b)*\cosh(x)^2 - 2*(a^3 + a^2*b - 3*(a^3 + a^2*b)*\cosh(x)^2)*\sinh(x)^2 + 4*((a^3 + a^2*b)*\cosh(x)^3 - (a^3 + a^2*b)*\cosh(x))*\sinh(x)), 1/4*(2*((2*a^2 + a*b - b^2)*\cosh(x)^4 + 4*(2*a^2 + a*b - b^2)*\cosh(x)*\sinh(x)^3 + (2*a^2 + a*b - b^2)*\sinh(x)^4 - 2*(2*a^2 + a*b - b^2)*\cosh(x)^2 + 2*(3*(2*a^2 + a*b - b^2)*\cosh(x)^2 - 2*a^2 - a*b + b^2)*\sinh(x)^2 + 2*a^2 + a*b - b^2 + 4*((2*a^2 + a*b - b^2)*\cosh(x))^3 - (2*a^2 + a*b - b^2)*\cosh(x))*\sinh(x))*\sqrt{-a}*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1)*\sqrt{-a}*\sqrt{((a+b)*\cosh(x)^2 + (a+b)*\sinh(x)^2 + a-b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)})/((a+b)*\cosh(x)^4 + 4*(a+b)*\cosh(x)*\sinh(x)^3 + (a+b)*\sinh(x)^4 + 2*(a-b)*\cosh(x)^2 + 2*(3*(a+b)*\cosh(x)^2 + a-b)*\sinh(x)^2 + 4*((a+b)*\cosh(x)^3 + (a-b)*\cosh(x))*\sinh(x) + a+b)) + (a^2*\cosh(x)^4 + 4*a^2*\cosh(x)*\sinh(x)^3 + a^2*\sinh(x)^4 - 2*a^2*\cosh(x)^2 + 2*(3*a^2*\cosh(x)^2 - a^2)*\sinh(x)^2 + a^2 + 4*(a$$

$$\begin{aligned}
& ^2\cosh(x)^3 - a^2\cosh(x))\sinh(x))\sqrt{a + b}\log(((a^3 + a^2b)\cosh(x) \\
& ^8 + 8*(a^3 + a^2b)\cosh(x)\sinh(x)^7 + (a^3 + a^2b)\sinh(x)^8 + 2*(2a^3 \\
& + a^2b)\cosh(x)^6 + 2*(2a^3 + a^2b + 14*(a^3 + a^2b)\cosh(x)^2)\sinh(x) \\
&)^6 + 4*(14*(a^3 + a^2b)\cosh(x)^3 + 3*(2a^3 + a^2b)\cosh(x)\sinh(x)^5 \\
& + (6a^3 + 4a^2b - ab^2 + b^3)\cosh(x)^4 + (70*(a^3 + a^2b)\cosh(x)^4 + \\
& 6a^3 + 4a^2b - ab^2 + b^3 + 30*(2a^3 + a^2b)\cosh(x)^2)\sinh(x)^4 + \\
& 4*(14*(a^3 + a^2b)\cosh(x)^5 + 10*(2a^3 + a^2b)\cosh(x)^3 + (6a^3 + 4a \\
& ^2b - ab^2 + b^3)\cosh(x)\sinh(x)^3 + a^3 + 3a^2b + 3ab^2 + b^3 + 2* \\
& (2a^3 + 3a^2b - b^3)\cosh(x)^2 + 2*(14*(a^3 + a^2b)\cosh(x)^6 + 15*(2a \\
& ^3 + a^2b)\cosh(x)^4 + 2a^3 + 3a^2b - b^3 + 3*(6a^3 + 4a^2b - ab^2 \\
& + b^3)\cosh(x)^2)\sinh(x)^2 + \sqrt{2}*(a^2\cosh(x)^6 + 6a^2\cosh(x)\sinh(x) \\
&)^5 + a^2\sinh(x)^6 + 3a^2\cosh(x)^4 + 3*(5a^2\cosh(x)^2 + a^2)\sinh(x)^4 \\
& + 4*(5a^2\cosh(x)^3 + 3a^2\cosh(x)\sinh(x)^3 + (3a^2 + 2ab - b^2)*\co \\
& sh(x)^2 + (15a^2\cosh(x)^4 + 18a^2\cosh(x)^2 + 3a^2 + 2ab - b^2)\sinh(\\
& x)^2 + a^2 + 2ab + b^2 + 2*(3a^2\cosh(x)^5 + 6a^2\cosh(x)^3 + (3a^2 + \\
& 2ab - b^2)\cosh(x)\sinh(x))\sqrt{a + b}\sqrt{((a + b)\cosh(x)^2 + (a + b) \\
&)\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)) + 4*(2*(a \\
& ^3 + a^2b)\cosh(x)^7 + 3*(2a^3 + a^2b)\cosh(x)^5 + (6a^3 + 4a^2b - a* \\
& b^2 + b^3)\cosh(x)^3 + (2a^3 + 3a^2b - b^3)\cosh(x)\sinh(x))/(\cosh(x)^6 \\
& + 6\cosh(x)^5\sinh(x) + 15\cosh(x)^4\sinh(x)^2 + 20\cosh(x)^3\sinh(x)^3 + \\
& 15\cosh(x)^2\sinh(x)^4 + 6\cosh(x)\sinh(x)^5 + \sinh(x)^6)) + (a^2\cosh(x)^4 \\
& + 4a^2\cosh(x)\sinh(x)^3 + a^2\sinh(x)^4 - 2a^2\cosh(x)^2 + 2*(3a^2\cos \\
& h(x)^2 - a^2)\sinh(x)^2 + a^2 + 4*(a^2\cosh(x)^3 - a^2\cosh(x))\sinh(x))\sqrt{ \\
& rt(a + b)\log(-((a + b)\cosh(x)^4 + 4*(a + b)\cosh(x)\sinh(x)^3 + (a + b)*\s \\
& inh(x)^4 - 2b\cosh(x)^2 + 2*(3*(a + b)\cosh(x)^2 - b)\sinh(x)^2 + \sqrt{2}* \\
& (\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1)\sqrt{a + b}\sqrt{((a + b)*\c \\
& osh(x)^2 + (a + b)\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh \\
& (x)^2)) + 4*((a + b)\cosh(x)^3 - b\cosh(x))\sinh(x) + a + b)/(\cosh(x)^2 + 2 \\
& *\cosh(x)\sinh(x) + \sinh(x)^2)) - 2\sqrt{2}*((a^2 + ab)\cosh(x)^2 + 2*(a^2 \\
& + ab)\cosh(x)\sinh(x) + (a^2 + ab)\sinh(x)^2 + a^2 + ab)\sqrt{((a + b)*\c \\
& osh(x)^2 + (a + b)\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh \\
& (x)^2)))/((a^3 + a^2b)\cosh(x)^4 + 4*(a^3 + a^2b)\cosh(x)\sinh(x)^3 + (a^ \\
& 3 + a^2b)\sinh(x)^4 + a^3 + a^2b - 2*(a^3 + a^2b)\cosh(x)^2 - 2*(a^3 + a \\
& ^2b - 3*(a^3 + a^2b)\cosh(x)^2)\sinh(x)^2 + 4*((a^3 + a^2b)\cosh(x)^3 - \\
& (a^3 + a^2b)\cosh(x)\sinh(x)), -1/4*(2*(a^2\cosh(x)^4 + 4a^2\cosh(x)\sin \\
& h(x)^3 + a^2\sinh(x)^4 - 2a^2\cosh(x)^2 + 2*(3a^2\cosh(x)^2 - a^2)\sinh(x) \\
&)^2 + a^2 + 4*(a^2\cosh(x)^3 - a^2\cosh(x))\sinh(x))\sqrt{-a - b}\arctan(\sqrt{ \\
& rt(2)*(a\cosh(x)^2 + 2a\cosh(x)\sinh(x) + a\sinh(x)^2 + a + b)\sqrt{-a - b} \\
&)\sqrt{((a + b)\cosh(x)^2 + (a + b)\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2\cosh(\\
& x)\sinh(x) + \sinh(x)^2)))/((a^2 + ab)\cosh(x)^4 + 4*(a^2 + ab)\cosh(x)\sin \\
& h(x)^3 + (a^2 + ab)\sinh(x)^4 + (2a^2 + ab - b^2)\cosh(x)^2 + (6*(a^2 + \\
& ab)\cosh(x)^2 + 2a^2 + ab - b^2)\sinh(x)^2 + a^2 + 2ab + b^2 + 2*(2*(a \\
& ^2 + ab)\cosh(x)^3 + (2a^2 + ab - b^2)\cosh(x)\sinh(x))) + 2*(a^2\cosh(\\
& x)^4 + 4a^2\cosh(x)\sinh(x)^3 + a^2\sinh(x)^4 - 2a^2\cosh(x)^2 + 2*(3a^2 \\
& *\cosh(x)^2 - a^2)\sinh(x)^2 + a^2 + 4*(a^2\cosh(x)^3 - a^2\cosh(x))\sinh(x) \\
&)\sqrt{-a - b}\arctan(\sqrt{2}*(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - \\
& 1)\sqrt{-a - b}\sqrt{((a + b)\cosh(x)^2 + (a + b)\sinh(x)^2 + a - b)/(\cosh(\\
& x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)))/((a + b)\cosh(x)^4 + 4*(a + b)\cosh(\\
& x)\sinh(x)^3 + (a + b)\sinh(x)^4 + 2*(a - b)\cosh(x)^2 + 2*(3*(a + b)\cosh(\\
& x)^2 + a - b)\sinh(x)^2 + 4*((a + b)\cosh(x)^3 + (a - b)\cosh(x))\sinh(x) + \\
& a + b) + ((2a^2 + ab - b^2)\cosh(x)^4 + 4*(2a^2 + ab - b^2)\cosh(x)*\s \\
& inh(x)^3 + (2a^2 + ab - b^2)\sinh(x)^4 - 2*(2a^2 + ab - b^2)\cosh(x)^2 \\
& + 2*(3*(2a^2 + ab - b^2)\cosh(x)^2 - 2a^2 - ab + b^2)\sinh(x)^2 + 2a^2 \\
& + ab - b^2 + 4*((2a^2 + ab - b^2)\cosh(x)^3 - (2a^2 + ab - b^2)\cosh(\\
& x))\sinh(x))\sqrt{a}\log(-((2a + b)\cosh(x)^4 + 4*(2a + b)\cosh(x)\sinh(x) \\
&)^3 + (2a + b)\sinh(x)^4 + 2*(2a - b)\cosh(x)^2 + 2*(3*(2a + b)\cosh(x)^ \\
& 2 + 2a - b)\sinh(x)^2 + 2\sqrt{2}*(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x) \\
& ^2 + 1)\sqrt{a}\sqrt{((a + b)\cosh(x)^2 + (a + b)\sinh(x)^2 + a - b)/(\cosh(\\
& x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)) + 4*((2a + b)\cosh(x)^3 + (2a - b)
\end{aligned}$$

```

*cosh(x))*sinh(x) + 2*a + b)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 +
  2*(3*cosh(x)^2 - 1)*sinh(x)^2 - 2*cosh(x)^2 + 4*(cosh(x)^3 - cosh(x))*sinh
(x) + 1)) + 2*sqrt(2)*((a^2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x)
) + (a^2 + a*b)*sinh(x)^2 + a^2 + a*b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*si
nh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3 + a^2*b
)*cosh(x)^4 + 4*(a^3 + a^2*b)*cosh(x)*sinh(x)^3 + (a^3 + a^2*b)*sinh(x)^4
+ a^3 + a^2*b - 2*(a^3 + a^2*b)*cosh(x)^2 - 2*(a^3 + a^2*b - 3*(a^3 + a^2*b
)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + a^2*b)*cosh(x)^3 - (a^3 + a^2*b)*cosh(x)
)*sinh(x)), 1/2*(((2*a^2 + a*b - b^2)*cosh(x)^4 + 4*(2*a^2 + a*b - b^2)*cos
h(x)*sinh(x)^3 + (2*a^2 + a*b - b^2)*sinh(x)^4 - 2*(2*a^2 + a*b - b^2)*cosh
(x)^2 + 2*(3*(2*a^2 + a*b - b^2)*cosh(x)^2 - 2*a^2 - a*b + b^2)*sinh(x)^2 +
  2*a^2 + a*b - b^2 + 4*((2*a^2 + a*b - b^2)*cosh(x)^3 - (2*a^2 + a*b - b^2)
*cosh(x))*sinh(x))*sqrt(-a)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) +
  sinh(x)^2 + 1))*sqrt(-a)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a -
  b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^4 + 4*(a +
  b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a +
  b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*
sinh(x) + a + b)) - (a^2*cosh(x)^4 + 4*a^2*cosh(x)*sinh(x)^3 + a^2*sinh(x)^
4 - 2*a^2*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 - a^2)*sinh(x)^2 + a^2 + 4*(a^2*co
sh(x)^3 - a^2*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)^2 +
  2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(
x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*
sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^
2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 +
  (2*a^2 + a*b - b^2)*cosh(x))*sinh(x)) - (a^2*cosh(x)^4 + 4*a^2*cosh(x)*si
nh(x)^3 + a^2*sinh(x)^4 - 2*a^2*cosh(x)^2 + 2*(3*a^2*cosh(x)^2 - a^2)*sinh(
x)^2 + a^2 + 4*(a^2*cosh(x)^3 - a^2*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(s
qrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1))*sqrt(-a - b)*sqrt(((
a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)
) + sinh(x)^2)))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*
sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2
+ 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) - sqrt(2)*((a^
2 + a*b)*cosh(x)^2 + 2*(a^2 + a*b)*cosh(x)*sinh(x) + (a^2 + a*b)*sinh(x)^2
+ a^2 + a*b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^
2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3 + a^2*b)*cosh(x)^4 + 4*(a^3 + a^
2*b)*cosh(x)*sinh(x)^3 + (a^3 + a^2*b)*sinh(x)^4 + a^3 + a^2*b - 2*(a^3 + a
^2*b)*cosh(x)^2 - 2*(a^3 + a^2*b - 3*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^2 + 4
*((a^3 + a^2*b)*cosh(x)^3 - (a^3 + a^2*b)*cosh(x))*sinh(x))]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep+1)]Evaluation time: 0.68Error: Bad Argument Type

maple [F] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(x)}{\sqrt{a + b(\tanh^2(x))}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x)`

[Out] `int(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)^3}{\sqrt{b \tanh(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(coth(x)^3/sqrt(b*tanh(x)^2 + a), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)^3}{\sqrt{b \tanh(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(x)^3/(a + b*tanh(x)^2)^(1/2),x)`

[Out] `int(coth(x)^3/(a + b*tanh(x)^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^3(x)}{\sqrt{a + b \tanh^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)**3/(a+b*tanh(x)**2)**(1/2),x)`

[Out] `Integral(coth(x)**3/sqrt(a + b*tanh(x)**2), x)`

$$3.238 \quad \int \frac{\tanh^5(x)}{(a+b \tanh^2(x))^{3/2}} dx$$

Optimal. Leaf size=72

$$-\frac{a^2}{b^2(a+b)\sqrt{a+b \tanh^2(x)}} - \frac{\sqrt{a+b \tanh^2(x)}}{b^2} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}$$

[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)-a^2/b^2/(a+b)/(a+b*tanh(x)^2)^(1/2)-(a+b*tanh(x)^2)^(1/2)/b^2

Rubi [A] time = 0.16, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3670, 446, 87, 63, 208}

$$-\frac{a^2}{b^2(a+b)\sqrt{a+b \tanh^2(x)}} - \frac{\sqrt{a+b \tanh^2(x)}}{b^2} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^5/(a + b*Tanh[x]^2)^(3/2),x]

[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) - a^2/(b^2*(a + b)*Sqrt[a + b*Tanh[x]^2]) - Sqrt[a + b*Tanh[x]^2]/b^2

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 87

Int[(((c_.) + (d_.)*(x_))^(n_.))*((e_.) + (f_.)*(x_))^(p_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], ((c + d*x)^n*(e + f*x)^IntegerPart[p])/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],

x]], Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f*ff*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
 \int \frac{\tanh^5(x)}{(a + b \tanh^2(x))^{3/2}} dx &= \text{Subst} \left(\int \frac{x^5}{(1-x^2)(a+bx^2)^{3/2}} dx, x, \tanh(x) \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{(1-x)(a+bx)^{3/2}} dx, x, \tanh^2(x) \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{a^2}{b(a+b)(a+bx)^{3/2}} - \frac{1}{b\sqrt{a+bx}} - \frac{1}{(a+b)(-1+x)\sqrt{a+bx}} \right) dx, x, \tanh^2(x) \right) \\
 &= -\frac{a^2}{b^2(a+b)\sqrt{a+b \tanh^2(x)}} - \frac{\sqrt{a+b \tanh^2(x)}}{b^2} - \frac{\text{Subst} \left(\int \frac{1}{(-1+x)\sqrt{a+bx}} dx, x, \tanh^2(x) \right)}{2(a+b)} \\
 &= -\frac{a^2}{b^2(a+b)\sqrt{a+b \tanh^2(x)}} - \frac{\sqrt{a+b \tanh^2(x)}}{b^2} - \frac{\text{Subst} \left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+b \tanh^2(x)} \right)}{b(a+b)} \\
 &= \frac{\tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}} \right)}{(a+b)^{3/2}} - \frac{a^2}{b^2(a+b)\sqrt{a+b \tanh^2(x)}} - \frac{\sqrt{a+b \tanh^2(x)}}{b^2}
 \end{aligned}$$

Mathematica [C] time = 0.11, size = 67, normalized size = 0.93

$$\frac{b^2 \left(-{}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{b \tanh^2(x)+a}{a+b} \right) \right) - (a+b)(2a + b \tanh^2(x) - b)}{b^2(a+b)\sqrt{a+b \tanh^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^5/(a + b*Tanh[x]^2)^(3/2), x]

[Out] (-(b^2*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tanh[x]^2)/(a + b)]) - (a + b)*(2*a - b + b*Tanh[x]^2))/(b^2*(a + b)*Sqrt[a + b*Tanh[x]^2])

fricas [B] time = 0.86, size = 3991, normalized size = 55.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(3/2), x, algorithm="fricas")

[Out] [1/4*(((a*b^2 + b^3)*cosh(x)^6 + 6*(a*b^2 + b^3)*cosh(x)*sinh(x)^5 + (a*b^2 + b^3)*sinh(x)^6 + (3*a*b^2 - b^3)*cosh(x)^4 + (3*a*b^2 - b^3 + 15*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a*b^2 + b^3)*cosh(x)^3 + (3*a*b^2 - b^3)*cosh(x))*sinh(x)^3 + a*b^2 + b^3 + (3*a*b^2 - b^3)*cosh(x)^2 + (15*(a*b^2 + b^3)*cosh(x)^4 + 3*a*b^2 - b^3 + 6*(3*a*b^2 - b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a*b^2 + b^3)*cosh(x)^5 + 2*(3*a*b^2 - b^3)*cosh(x)^3 + (3*a*b^2 - b^3)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a


```
(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 + 2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x)) + ((a*b^2 + b^3)*cosh(x)^6 + 6*(a*b^2 + b^3)*cosh(x)*sinh(x)^5 + (a*b^2 + b^3)*sinh(x)^6 + (3*a*b^2 - b^3)*cosh(x)^4 + (3*a*b^2 - b^3 + 15*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a*b^2 + b^3)*cosh(x)^3 + (3*a*b^2 - b^3)*cosh(x))*sinh(x)^3 + a*b^2 + b^3 + (3*a*b^2 - b^3)*cosh(x)^2 + (15*(a*b^2 + b^3)*cosh(x)^4 + 3*a*b^2 - b^3 + 6*(3*a*b^2 - b^3)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a*b^2 + b^3)*cosh(x)^5 + 2*(3*a*b^2 - b^3)*cosh(x)^3 + (3*a*b^2 - b^3)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 2*sqrt(2)*(((2*a^3 + 4*a^2*b + 3*a*b^2 + b^3)*cosh(x)^4 + 4*(2*a^3 + 4*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (2*a^3 + 4*a^2*b + 3*a*b^2 + b^3)*sinh(x)^4 + 2*a^3 + 4*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 2*a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(2*a^3 + 2*a^2*b - a*b^2 - b^3 + 3*(2*a^3 + 4*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 + 4*a^2*b + 3*a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 2*a^2*b - a*b^2 - b^3)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^6 + 6*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)*sinh(x)^5 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*sinh(x)^6 + a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5 + (3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5)*cosh(x)^4 + (3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5 + 15*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^3 + (3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5)*cosh(x))*sinh(x)^3 + (3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2 + (3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5 + 15*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^4 + 6*(3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^5 + 2*(3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5)*cosh(x)^3 + (3*a^3*b^2 + 5*a^2*b^3 + a*b^4 - b^5)*cosh(x))*sinh(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep+1)]Evaluation time: 1.35Error: Bad Argument Type
```

maple [B] time = 0.08, size = 322, normalized size = 4.47

$$-\frac{\tanh^2(x)}{b\sqrt{a+b(\tanh^2(x))}} - \frac{2a}{b^2\sqrt{a+b(\tanh^2(x))}} + \frac{1}{b\sqrt{a+b(\tanh^2(x))}} - \frac{1}{2(a+b)\sqrt{(\tanh(x)-1)^2b+2(\tanh(x))}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x)
```

```
[Out] -tanh(x)^2/b/(a+b*tanh(x)^2)^(1/2)-2*a/b^2/(a+b*tanh(x)^2)^(1/2)+1/b/(a+b*t
anh(x)^2)^(1/2)-1/2/(a+b)/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+b/(a+
b)*(2*(tanh(x)-1)*b+2*b)/(4*b*(a+b)-4*b^2)/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b
```

$$+a+b)^{1/2}+1/2/(a+b)^{3/2}*\ln((2*a+2*b+2*(\tanh(x)-1)*b+2*(a+b)^{1/2}*(\tanh(x)-1)^{2*b+2*(\tanh(x)-1)*b+a+b)^{1/2})/(\tanh(x)-1))-1/2/(a+b)/((1+\tanh(x))^{2*b-2*(1+\tanh(x))*b+a+b)^{1/2}-b/(a+b)*(2*(1+\tanh(x))*b-2*b)/(4*b*(a+b)-4*b^2)/((1+\tanh(x))^{2*b-2*(1+\tanh(x))*b+a+b})^{1/2}+1/2/(a+b)^{3/2}*\ln((2*a+2*b-2*(1+\tanh(x))*b+2*(a+b)^{1/2}*((1+\tanh(x))^{2*b-2*(1+\tanh(x))*b+a+b})^{1/2})/(1+\tanh(x)))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^5}{(b \tanh(x)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(tanh(x)^5/(b*tanh(x)^2 + a)^(3/2), x)

mupad [B] time = 2.52, size = 70, normalized size = 0.97

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b \tanh(x)^2 + a} (2a + 2b)}{2(a+b)^{3/2}}\right)}{(a+b)^{3/2}} - \frac{\sqrt{b \tanh(x)^2 + a}}{b^2} - \frac{a^2}{b^2 (a+b) \sqrt{b \tanh(x)^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^5/(a + b*tanh(x)^2)^(3/2),x)

[Out] atanh(((a + b*tanh(x)^2)^(1/2)*(2*a + 2*b))/(2*(a + b)^(3/2)))/(a + b)^(3/2) - (a + b*tanh(x)^2)^(1/2)/b^2 - a^2/(b^2*(a + b)*(a + b*tanh(x)^2)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{(a + b \tanh^2(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)**5/(a+b*tanh(x)**2)**(3/2),x)

[Out] Integral(tanh(x)**5/(a + b*tanh(x)**2)**(3/2), x)

$$3.239 \quad \int \frac{\tanh^4(x)}{(a+b \tanh^2(x))^{3/2}} dx$$

Optimal. Leaf size=84

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{b^{3/2}} + \frac{a \tanh(x)}{b(a+b)\sqrt{a+b \tanh^2(x)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{(a+b)^{3/2}}$$

[Out] $-\operatorname{arctanh}(b^{(1/2)} \tanh(x) / (a+b \tanh(x)^2)^{(1/2)}) / b^{(3/2)} + \operatorname{arctanh}((a+b)^{(1/2)} \tanh(x) / (a+b \tanh(x)^2)^{(1/2)}) / (a+b)^{(3/2)} + a \tanh(x) / b / (a+b) / (a+b \tanh(x)^2)^{(1/2)}$

Rubi [A] time = 0.13, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3670, 470, 523, 217, 206, 377}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{b^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{(a+b)^{3/2}} + \frac{a \tanh(x)}{b(a+b)\sqrt{a+b \tanh^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^4/(a + b*Tanh[x]^2)^(3/2), x]

[Out] $-(\operatorname{ArcTanh}[(\operatorname{Sqrt}[b] \operatorname{Tanh}[x]) / \operatorname{Sqrt}[a + b \operatorname{Tanh}[x]^2]] / b^{(3/2)}) + \operatorname{ArcTanh}[(\operatorname{Sqrt}[a + b] \operatorname{Tanh}[x]) / \operatorname{Sqrt}[a + b \operatorname{Tanh}[x]^2]] / (a + b)^{(3/2)} + (a \operatorname{Tanh}[x]) / (b(a + b) \operatorname{Sqrt}[a + b \operatorname{Tanh}[x]^2])$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 523

```
Int[((e_) + (f_)*(x_)^(n_))/((a_) + (b_)*(x_)^n)*Sqrt[(c_) + (d_)*(x_)^n], x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]
```

Rule 3670

```
Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*((c_)*tan[(e_) + (f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))
```

Rubi steps

$$\begin{aligned} \int \frac{\tanh^4(x)}{(a + b \tanh^2(x))^{3/2}} dx &= \text{Subst} \left(\int \frac{x^4}{(1-x^2)(a+bx^2)^{3/2}} dx, x, \tanh(x) \right) \\ &= \frac{a \tanh(x)}{b(a+b)\sqrt{a+b \tanh^2(x)}} - \frac{\text{Subst} \left(\int \frac{a+(-a-b)x^2}{(1-x^2)\sqrt{a+bx^2}} dx, x, \tanh(x) \right)}{b(a+b)} \\ &= \frac{a \tanh(x)}{b(a+b)\sqrt{a+b \tanh^2(x)}} - \frac{\text{Subst} \left(\int \frac{1}{\sqrt{a+bx^2}} dx, x, \tanh(x) \right)}{b} + \frac{\text{Subst} \left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}} dx, x, \tanh(x) \right)}{a+b} \\ &= \frac{a \tanh(x)}{b(a+b)\sqrt{a+b \tanh^2(x)}} - \frac{\text{Subst} \left(\int \frac{1}{1-bx^2} dx, x, \frac{\tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right)}{b} + \frac{\text{Subst} \left(\int \frac{1}{1-(a+b)x^2} dx, x, \frac{\tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right)}{a-b} \\ &= -\frac{\tanh^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right)}{b^{3/2}} + \frac{\tanh^{-1} \left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right)}{(a+b)^{3/2}} + \frac{a \tanh(x)}{b(a+b)\sqrt{a+b \tanh^2(x)}} \end{aligned}$$

Mathematica [C] time = 2.45, size = 188, normalized size = 2.24

$$\frac{a \tanh(x) \left(\sqrt{2} (a+b) \sqrt{\frac{\text{csch}^2(x)((a+b) \cosh(2x)+a-b)}{b}} F \left(\sin^{-1} \left(\frac{\sqrt{(a-b+(a+b) \cosh(2x)) \text{csch}^2(x)}}{\sqrt{2}} \right) \right) + \sqrt{2} b \sqrt{\frac{\text{csch}^2(x)((a+b) \cosh(2x)+a-b)}{b}} \right)}{\sqrt{2} b (a+b)^2 \sqrt{\text{sech}^2(x)((a+b) \cosh(2x)+a-b)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^4/(a + b*Tanh[x]^2)^(3/2), x]
```

```
[Out] -((a*(-2*a - 2*b + Sqrt[2]*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b])*EllipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] + Sqrt[2]*b*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b])*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1])*Tanh[x])/(Sqrt[2]*b*(a + b)^2*Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2]))
```

fricas [B] time = 0.83, size = 6973, normalized size = 83.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")

[Out]
$$\frac{1}{4} \left((a^2 b^2 + b^3) \cosh(x)^4 + 4(a^2 b^2 + b^3) \cosh(x) \sinh(x)^3 + (a^2 b^2 + b^3) \sinh(x)^4 + a^2 b^2 + b^3 + 2(a^2 b^2 - b^3) \cosh(x)^2 + 2(a^2 b^2 - b^3) \sinh(x)^2 + 3(a^2 b^2 + b^3) \cosh(x)^2 \sinh(x)^2 + 4((a^2 b^2 + b^3) \cosh(x)^3 + (a^2 b^2 - b^3) \cosh(x) \sinh(x)) \sqrt{a+b} \log(-((a^2 b^2 + b^3) \cosh(x)^8 + 8(a^2 b^2 + b^3) \cosh(x) \sinh(x)^7 + (a^2 b^2 + b^3) \sinh(x)^8 - 2(a^2 b^2 + 2b^3) \cosh(x)^6 - 2(a^2 b^2 + 2b^3 - 14(a^2 b^2 + b^3) \cosh(x)^2) \sinh(x)^6 + 4(14(a^2 b^2 + b^3) \cosh(x)^3 - 3(a^2 b^2 + 2b^3) \cosh(x)) \sinh(x)^5 + (a^3 - a^2 b + 4a^2 b^2 + 6b^3) \cosh(x)^4 + (70(a^2 b^2 + b^3) \cosh(x)^4 + a^3 - a^2 b + 4a^2 b^2 + 6b^3 - 30(a^2 b^2 + 2b^3) \cosh(x)^2) \sinh(x)^4 + 4(14(a^2 b^2 + b^3) \cosh(x)^5 - 10(a^2 b^2 + 2b^3) \cosh(x)^3 + (a^3 - a^2 b + 4a^2 b^2 + 6b^3) \cosh(x)) \sinh(x)^3 + a^3 + 3a^2 b + 3a^2 b^2 + b^3 + 2(a^3 - 3a^2 b - 2b^3) \cosh(x)^2 + 2(14(a^2 b^2 + b^3) \cosh(x)^6 - 15(a^2 b^2 + 2b^3) \cosh(x)^4 + a^3 - 3a^2 b - 2b^3 + 3(a^3 - a^2 b + 4a^2 b^2 + 6b^3) \cosh(x)^2) \sinh(x)^2 + \sqrt{2} (b^2 \cosh(x)^6 + 6b^2 \cosh(x) \sinh(x)^5 + b^2 \sinh(x)^6 - 3b^2 \cosh(x)^4 + 3(5b^2 \cosh(x)^2 - b^2) \sinh(x)^4 + 4(5b^2 \cosh(x)^3 - 3b^2 \cosh(x)) \sinh(x)^3 - (a^2 - 2ab - 3b^2) \cosh(x)^2 + (15b^2 \cosh(x)^4 - 18b^2 \cosh(x)^2 - a^2 + 2ab + 3b^2) \sinh(x)^2 - a^2 - 2ab - b^2 + 2(3b^2 \cosh(x)^5 - 6b^2 \cosh(x)^3 - (a^2 - 2ab - 3b^2) \cosh(x)) \sinh(x)) \sqrt{a+b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)} + 4(2(a^2 b^2 + b^3) \cosh(x)^7 - 3(a^2 b^2 + 2b^3) \cosh(x)^5 + (a^3 - a^2 b + 4a^2 b^2 + 6b^3) \cosh(x)^3 + (a^3 - 3a^2 b - 2b^3) \cosh(x)) \sinh(x) / (\cosh(x)^6 + 6 \cosh(x)^5 \sinh(x) + 15 \cosh(x)^4 \sinh(x)^2 + 20 \cosh(x)^3 \sinh(x)^3 + 15 \cosh(x)^2 \sinh(x)^4 + 6 \cosh(x) \sinh(x)^5 + \sinh(x)^6) + 2((a^3 + 3a^2 b + 3a^2 b^2 + b^3) \cosh(x)^4 + 4(a^3 + 3a^2 b + 3a^2 b^2 + b^3) \cosh(x) \sinh(x)^3 + (a^3 + 3a^2 b + 3a^2 b^2 + b^3) \sinh(x)^4 + a^3 + 3a^2 b + 3a^2 b^2 + b^3 + 2(a^3 + a^2 b - ab^2 - b^3) \cosh(x)^2 + 2(a^3 + a^2 b - ab^2 - b^3 + 3(a^3 + 3a^2 b + 3a^2 b^2 + b^3) \cosh(x)^2) \sinh(x)^2 + 4((a^3 + 3a^2 b + 3a^2 b^2 + b^3) \cosh(x)^3 + (a^3 + a^2 b - ab^2 - b^3) \cosh(x)) \sinh(x) \sqrt{b} \log(-((a+2b) \cosh(x)^4 + 4(a+2b) \cosh(x) \sinh(x)^3 + (a+2b) \sinh(x)^4 + 2(a-2b) \cosh(x)^2 + 2(3(a+2b) \cosh(x)^2 + a-2b) \sinh(x)^2 - 2\sqrt{2} (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) \sqrt{b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)} + 4((a+2b) \cosh(x)^3 + (a-2b) \cosh(x)) \sinh(x) + a + 2b) / (\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2(3 \cosh(x)^2 + 1) \sinh(x)^2 + 2 \cosh(x)^2 + 4(\cosh(x)^3 + \cosh(x)) \sinh(x) + 1) + ((a^2 b^2 + b^3) \cosh(x)^4 + 4(a^2 b^2 + b^3) \cosh(x) \sinh(x)^3 + (a^2 b^2 + b^3) \sinh(x)^4 + a^2 b^2 + b^3 + 2(a^2 b^2 - b^3) \cosh(x)^2 + 2(a^2 b^2 - b^3 + 3(a^2 b^2 + b^3) \cosh(x)^2) \sinh(x)^2 + 4((a^2 b^2 + b^3) \cosh(x)^3 + (a^2 b^2 - b^3) \cosh(x)) \sinh(x)) \sqrt{a+b} \log(((a+b) \cosh(x)^4 + 4(a+b) \cosh(x) \sinh(x)^3 + (a+b) \sinh(x)^4 + 2a \cosh(x)^2 + 2(3(a+b) \cosh(x)^2 + a) \sinh(x)^2 + \sqrt{2} (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1) \sqrt{a+b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)} + 4((a+b) \cosh(x)^3 + a \cosh(x)) \sinh(x) + a + b) / (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2)} - 4\sqrt{2} (a^2 b + a^2 b^2 - (a^2 b + a^2 b^2) \cosh(x)^2 - 2(a^2 b + a^2 b^2) \cosh(x) \sinh(x) - (a^2 b + a^2 b^2) \sinh(x)^2) \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)} / (a^3 b^2 + 3a^2 b^3 + 3a^2 b^4 + b^5 + (a^3 b^2 + 3a^2 b^3 + 3a^2 b^4 + b^5) \cosh(x)^4 + 4(a^3 b^2 + 3a^2 b^3 + 3a^2 b^4 + b^5) \cosh(x) \sinh(x)^3 + (a^3 b^2 + 3a^2 b^3 + 3a^2 b^4 + b^5) \sinh(x)^4 + 2(a^3 b^2 + a^2 b^3 - ab^4 - b^5) \cosh(x)^2 + 2(a^3 b^2 + a^2 b^3 - ab^4 - b^5 + 3(a^3 b^2 + 3a^2 b^3 + 3a^2 b^4 + b^5)$$

$$\begin{aligned}
& * \cosh(x)^2 * \sinh(x)^2 + 4 * ((a^3 * b^2 + 3 * a^2 * b^3 + 3 * a * b^4 + b^5) * \cosh(x)^3 \\
& + (a^3 * b^2 + a^2 * b^3 - a * b^4 - b^5) * \cosh(x) * \sinh(x)), 1/4 * (4 * ((a^3 + 3 * a^2 \\
& * b + 3 * a * b^2 + b^3) * \cosh(x)^4 + 4 * (a^3 + 3 * a^2 * b + 3 * a * b^2 + b^3) * \cosh(x) * \sinh(x)^3 \\
& + (a^3 + 3 * a^2 * b + 3 * a * b^2 + b^3) * \sinh(x)^4 + a^3 + 3 * a^2 * b + 3 * a * b^2 + b^3 + 2 * (a^3 + a^2 * b - a * b^2 - b^3) * \cosh(x)^2 \\
& + 2 * (a^3 + a^2 * b - a * b^2 - b^3 + 3 * (a^3 + 3 * a^2 * b + 3 * a * b^2 + b^3) * \cosh(x)^2) * \sinh(x)^2 + 4 * ((a^3 + 3 * a^2 * b + 3 * a * b^2 + b^3) * \cosh(x)^3 \\
& + (a^3 + a^2 * b - a * b^2 - b^3) * \cosh(x) * \sinh(x)) * \sqrt{-b} * \arctan(\sqrt{2} * (\cosh(x)^2 + 2 * \cosh(x) * \sinh(x) + \sinh(x)^2 - 1) * \sqrt{-b} * \sqrt{((a + b) * \cosh(x)^2 + (a + b) * \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)}) / ((a + b) * \cosh(x)^4 + 4 * (a + b) * \cosh(x) * \sinh(x)^3 + (a + b) * \sinh(x)^4 + 2 * (a - b) * \cosh(x)^2 + 2 * (3 * (a + b) * \cosh(x)^2 + a - b) * \sinh(x)^2 + 4 * ((a + b) * \cosh(x)^3 + (a - b) * \cosh(x) * \sinh(x) + a + b)) + ((a * b^2 + b^3) * \cosh(x)^4 + 4 * (a * b^2 + b^3) * \cosh(x) * \sinh(x)^3 + (a * b^2 + b^3) * \sinh(x)^4 + a * b^2 + b^3 + 2 * (a * b^2 - b^3) * \cosh(x)^2 + 2 * (a * b^2 - b^3 + 3 * (a * b^2 + b^3) * \cosh(x)^2) * \sinh(x)^2 + 4 * ((a * b^2 + b^3) * \cosh(x)^3 + (a * b^2 - b^3) * \cosh(x) * \sinh(x)) * \sqrt{a + b} * \log(-((a * b^2 + b^3) * \cosh(x)^8 + 8 * (a * b^2 + b^3) * \cosh(x) * \sinh(x)^7 + (a * b^2 + b^3) * \sinh(x)^8 - 2 * (a * b^2 + 2 * b^3) * \cosh(x)^6 - 2 * (a * b^2 + 2 * b^3 - 14 * (a * b^2 + b^3) * \cosh(x)^2) * \sinh(x)^6 + 4 * (14 * (a * b^2 + b^3) * \cosh(x)^3 - 3 * (a * b^2 + 2 * b^3) * \cosh(x) * \sinh(x)^5 + (a^3 - a^2 * b + 4 * a * b^2 + 6 * b^3) * \cosh(x)^4 + (70 * (a * b^2 + b^3) * \cosh(x)^4 + a^3 - a^2 * b + 4 * a * b^2 + 6 * b^3 - 30 * (a * b^2 + 2 * b^3) * \cosh(x)^2) * \sinh(x)^4 + 4 * (14 * (a * b^2 + b^3) * \cosh(x)^5 - 10 * (a * b^2 + 2 * b^3) * \cosh(x)^3 + (a^3 - a^2 * b + 4 * a * b^2 + 6 * b^3) * \cosh(x)) * \sinh(x)^3 + a^3 + 3 * a^2 * b + 3 * a * b^2 + b^3 + 2 * (a^3 - 3 * a * b^2 - 2 * b^3) * \cosh(x)^2 + 2 * (14 * (a * b^2 + b^3) * \cosh(x)^6 - 15 * (a * b^2 + 2 * b^3) * \cosh(x)^4 + a^3 - 3 * a * b^2 - 2 * b^3 + 3 * (a^3 - a^2 * b + 4 * a * b^2 + 6 * b^3) * \cosh(x)^2) * \sinh(x)^2 + \sqrt{2} * (b^2 * \cosh(x)^6 + 6 * b^2 * \cosh(x) * \sinh(x)^5 + b^2 * \sinh(x)^6 - 3 * b^2 * \cosh(x)^4 + 3 * (5 * b^2 * \cosh(x)^2 - b^2) * \sinh(x)^4 + 4 * (5 * b^2 * \cosh(x)^3 - 3 * b^2 * \cosh(x)) * \sinh(x)^3 - (a^2 - 2 * a * b - 3 * b^2) * \cosh(x)^2 + (15 * b^2 * \cosh(x)^4 - 18 * b^2 * \cosh(x)^2 - a^2 + 2 * a * b + 3 * b^2) * \sinh(x)^2 - a^2 - 2 * a * b - b^2 + 2 * (3 * b^2 * \cosh(x)^5 - 6 * b^2 * \cosh(x)^3 - (a^2 - 2 * a * b - 3 * b^2) * \cosh(x)) * \sinh(x)) * \sqrt{a + b} * \sqrt{((a + b) * \cosh(x)^2 + (a + b) * \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)} + 4 * (2 * (a * b^2 + b^3) * \cosh(x)^7 - 3 * (a * b^2 + 2 * b^3) * \cosh(x)^5 + (a^3 - a^2 * b + 4 * a * b^2 + 6 * b^3) * \cosh(x)^3 + (a^3 - 3 * a * b^2 - 2 * b^3) * \cosh(x) * \sinh(x)) / (\cosh(x)^6 + 6 * \cosh(x)^5 * \sinh(x) + 15 * \cosh(x)^4 * \sinh(x)^2 + 20 * \cosh(x)^3 * \sinh(x)^3 + 15 * \cosh(x)^2 * \sinh(x)^4 + 6 * \cosh(x) * \sinh(x)^5 + \sinh(x)^6)) + ((a * b^2 + b^3) * \cosh(x)^4 + 4 * (a * b^2 + b^3) * \cosh(x) * \sinh(x)^3 + (a * b^2 + b^3) * \sinh(x)^4 + a * b^2 + b^3 + 2 * (a * b^2 - b^3) * \cosh(x)^2 + 2 * (a * b^2 - b^3 + 3 * (a * b^2 + b^3) * \cosh(x)^2) * \sinh(x)^2 + 4 * ((a * b^2 + b^3) * \cosh(x)^3 + (a * b^2 - b^3) * \cosh(x) * \sinh(x)) * \sqrt{a + b} * \log(((a + b) * \cosh(x)^4 + 4 * (a + b) * \cosh(x) * \sinh(x)^3 + (a + b) * \sinh(x)^4 + 2 * a * \cosh(x)^2 + 2 * (3 * (a + b) * \cosh(x)^2 + a) * \sinh(x)^2 + \sqrt{2} * (\cosh(x)^2 + 2 * \cosh(x) * \sinh(x) + \sinh(x)^2 + 1) * \sqrt{a + b} * \sqrt{((a + b) * \cosh(x)^2 + (a + b) * \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)} + 4 * ((a + b) * \cosh(x)^3 + a * \cosh(x)) * \sinh(x) + a + b) / (\cosh(x)^2 + 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)) - 4 * \sqrt{2} * (a^2 * b + a * b^2 - (a^2 * b + a * b^2) * \cosh(x)^2 - 2 * (a^2 * b + a * b^2) * \cosh(x) * \sinh(x) - (a^2 * b + a * b^2) * \sinh(x)^2) * \sqrt{((a + b) * \cosh(x)^2 + (a + b) * \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2))} / (a^3 * b^2 + 3 * a^2 * b^3 + 3 * a * b^4 + b^5 + (a^3 * b^2 + 3 * a^2 * b^3 + 3 * a * b^4 + b^5) * \cosh(x)^4 + 4 * (a^3 * b^2 + 3 * a^2 * b^3 + 3 * a * b^4 + b^5) * \cosh(x) * \sinh(x)^3 + (a^3 * b^2 + 3 * a^2 * b^3 + 3 * a * b^4 + b^5) * \sinh(x)^4 + 2 * (a^3 * b^2 + a^2 * b^3 - a * b^4 - b^5) * \cosh(x)^2 + 2 * (a^3 * b^2 + a^2 * b^3 - a * b^4 - b^5 + 3 * (a^3 * b^2 + 3 * a^2 * b^3 + 3 * a * b^4 + b^5) * \cosh(x)^2) * \sinh(x)^2 + 4 * ((a^3 * b^2 + 3 * a^2 * b^3 + 3 * a * b^4 + b^5) * \cosh(x)^3 + (a^3 * b^2 + a^2 * b^3 - a * b^4 - b^5) * \cosh(x)) * \sinh(x)), -1/2 * (((a * b^2 + b^3) * \cosh(x)^4 + 4 * (a * b^2 + b^3) * \cosh(x) * \sinh(x)^3 + (a * b^2 + b^3) * \sinh(x)^4 + a * b^2 + b^3 + 2 * (a * b^2 - b^3) * \cosh(x)^2 + 2 * (a * b^2 - b^3 + 3 * (a * b^2 + b^3) * \cosh(x)^2) * \sinh(x)^2 + 4 * ((a * b^2 + b^3) * \cosh(x)^3 + (a * b^2 - b^3) * \cosh(x)) * \sinh(x)) * \sqrt{-a - b} * \arctan(\sqrt{2} * (b * \cosh(x)^2 + 2 * b * \cosh(x) * \sinh(x) + b * \sinh(x)^2 - a - b) * \sqrt{-a - b} * \sqrt{((a + b) * \cosh(x)^2 + (a + b) * \sinh(x)^2 + a - b) / (\cosh(x)
\end{aligned}$$


```
) *sinh(x) + sinh(x)^2))/((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 +
(a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*s
inh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)) + 2*sq
rt(2)*(a^2*b + a*b^2 - (a^2*b + a*b^2)*cosh(x)^2 - 2*(a^2*b + a*b^2)*cosh(x
)*sinh(x) - (a^2*b + a*b^2)*sinh(x)^2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*si
nh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^3*b^2 + 3
*a^2*b^3 + 3*a*b^4 + b^5 + (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)^4
+ 4*(a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*cosh(x)*sinh(x)^3 + (a^3*b^2 + 3*
a^2*b^3 + 3*a*b^4 + b^5)*sinh(x)^4 + 2*(a^3*b^2 + a^2*b^3 - a*b^4 - b^5)*co
sh(x)^2 + 2*(a^3*b^2 + a^2*b^3 - a*b^4 - b^5 + 3*(a^3*b^2 + 3*a^2*b^3 + 3*a
*b^4 + b^5)*cosh(x)^2)*sinh(x)^2 + 4*((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)
*cosh(x)^3 + (a^3*b^2 + a^2*b^3 - a*b^4 - b^5)*cosh(x))*sinh(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep+1)]Error: Bad Argument Type

maple [B] time = 0.10, size = 328, normalized size = 3.90

$$\frac{\tanh(x)}{b\sqrt{a+b(\tanh^2(x))}} - \frac{\ln\left(\sqrt{b}\tanh(x) + \sqrt{a+b(\tanh^2(x))}\right)}{b^{\frac{3}{2}}} - \frac{\tanh(x)}{a\sqrt{a+b(\tanh^2(x))}} - \frac{1}{2(a+b)\sqrt{(\tanh(x)-1)^2 b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x)

[Out] tanh(x)/b/(a+b*tanh(x)^2)^(1/2)-1/b^(3/2)*ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2))-tanh(x)/a/(a+b*tanh(x)^2)^(1/2)-1/2/(a+b)/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+b/(a+b)*(2*(tanh(x)-1)*b+2*b)/(4*b*(a+b)-4*b^2)/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))+1/2/(a+b)/(((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)+b/(a+b)*(2*(1+tanh(x))*b-2*b)/(4*b*(a+b)-4*b^2)/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)-1/2/(a+b)^(3/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x))))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^4}{(b \tanh(x)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(tanh(x)^4/(b*tanh(x)^2 + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^4}{(b \tanh(x)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)^4/(a + b*tanh(x)^2)^(3/2), x)`

[Out] `int(tanh(x)^4/(a + b*tanh(x)^2)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{(a + b \tanh^2(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)**4/(a+b*tanh(x)**2)**(3/2), x)`

[Out] `Integral(tanh(x)**4/(a + b*tanh(x)**2)**(3/2), x)`

$$3.240 \quad \int \frac{\tanh^3(x)}{(a+b \tanh^2(x))^{3/2}} dx$$

Optimal. Leaf size=52

$$\frac{a}{b(a+b)\sqrt{a+b \tanh^2(x)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}$$

[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)+a/b/(a+b)/(a+b*tanh(x)^2)^(1/2)

Rubi [A] time = 0.12, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3670, 446, 78, 63, 208}

$$\frac{a}{b(a+b)\sqrt{a+b \tanh^2(x)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^3/(a + b*Tanh[x]^2)^(3/2),x]

[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) + a/(b*(a + b)*Sqrt[a + b*Tanh[x]^2])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 3670


```
Int[((d_.)*tan[(e_.) + (f_.)*(x_.)])^(m_.)*((a_.) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned} \int \frac{\tanh^3(x)}{(a + b \tanh^2(x))^{3/2}} dx &= \text{Subst} \left(\int \frac{x^3}{(1-x^2)(a+bx^2)^{3/2}} dx, x, \tanh(x) \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{x}{(1-x)(a+bx)^{3/2}} dx, x, \tanh^2(x) \right) \\ &= \frac{a}{b(a+b)\sqrt{a+b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{1}{(1-x)\sqrt{a+bx}} dx, x, \tanh^2(x) \right)}{2(a+b)} \\ &= \frac{a}{b(a+b)\sqrt{a+b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}} dx, x, \sqrt{a+b \tanh^2(x)} \right)}{b(a+b)} \\ &= \frac{\tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}} \right)}{(a+b)^{3/2}} + \frac{a}{b(a+b)\sqrt{a+b \tanh^2(x)}} \end{aligned}$$

Mathematica [A] time = 0.13, size = 52, normalized size = 1.00

$$\frac{a}{b(a+b)\sqrt{a+b \tanh^2(x)}} + \frac{\tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}} \right)}{(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Tanh[x]^2)^(3/2), x]
```

```
[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) + a/(b*(a + b)*Sqr
t[a + b*Tanh[x]^2])
```

fricas [B] time = 0.54, size = 2525, normalized size = 48.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^2
)*sinh(x)^4 + 2*(a*b - b^2)*cosh(x)^2 + 2*(3*(a*b + b^2)*cosh(x)^2 + a*b -
b^2)*sinh(x)^2 + a*b + b^2 + 4*((a*b + b^2)*cosh(x)^3 + (a*b - b^2)*cosh(x)
)*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(
x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2
*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*
cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2
```

$$\begin{aligned}
& + b^3) \cosh(x)^4 + (70(a^3 + a^2b) \cosh(x)^4 + 6a^3 + 4a^2b - ab^2 + \\
& b^3 + 30(2a^3 + a^2b) \cosh(x)^2) \sinh(x)^4 + 4(14(a^3 + a^2b) \cosh(x) \\
&)^5 + 10(2a^3 + a^2b) \cosh(x)^3 + (6a^3 + 4a^2b - ab^2 + b^3) \cosh(x) \\
&) \sinh(x)^3 + a^3 + 3a^2b + 3ab^2 + b^3 + 2(2a^3 + 3a^2b - b^3) \cosh(x)^2 \\
& + 2(14(a^3 + a^2b) \cosh(x)^6 + 15(2a^3 + a^2b) \cosh(x)^4 + 2a^3 + 3a^2b - b^3 + 3(6a^3 + 4a^2b - ab^2 + b^3) \cosh(x)^2) \sinh(x)^2 \\
& + \sqrt{2}(a^2 \cosh(x)^6 + 6a^2 \cosh(x) \sinh(x)^5 + a^2 \sinh(x)^6 + 3a^2 \cosh(x)^4 + 3(5a^2 \cosh(x)^2 + a^2) \sinh(x)^4 + 4(5a^2 \cosh(x)^3 + 3a^2 \cosh(x)) \sinh(x)^3 + (3a^2 + 2ab - b^2) \cosh(x)^2 + (15a^2 \cosh(x)^4 + 18a^2 \cosh(x)^2 + 3a^2 + 2ab - b^2) \sinh(x)^2 + a^2 + 2ab + b^2 + 2(3a^2 \cosh(x)^5 + 6a^2 \cosh(x)^3 + (3a^2 + 2ab - b^2) \cosh(x)) \sinh(x)) \sqrt{a+b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} + 4(2(a^3 + a^2b) \cosh(x)^7 + 3(2a^3 + a^2b) \cosh(x)^5 + (6a^3 + 4a^2b - ab^2 + b^3) \cosh(x)^3 + (2a^3 + 3a^2b - b^3) \cosh(x)) \sinh(x) / (\cosh(x)^6 + 6 \cosh(x)^5 \sinh(x) + 15 \cosh(x)^4 \sinh(x)^2 + 20 \cosh(x)^3 \sinh(x)^3 + 15 \cosh(x)^2 \sinh(x)^4 + 6 \cosh(x) \sinh(x)^5 + \sinh(x)^6) + ((ab + b^2) \cosh(x)^4 + 4(ab + b^2) \cosh(x) \sinh(x)^3 + (ab + b^2) \sinh(x)^4 + 2(ab - b^2) \cosh(x)^2 + 2(3(ab + b^2) \cosh(x)^2 + ab - b^2) \sinh(x)^2 + ab + b^2 + 4((ab + b^2) \cosh(x)^3 + (ab - b^2) \cosh(x)) \sinh(x)) \sqrt{a+b} \log(-((a+b) \cosh(x)^4 + 4(a+b) \cosh(x) \sinh(x)^3 + (a+b) \sinh(x)^4 - 2b \cosh(x)^2 + 2(3(a+b) \cosh(x)^2 - b) \sinh(x)^2 + \sqrt{2}(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) \sqrt{a+b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} + 4((a+b) \cosh(x)^3 - b \cosh(x)) \sinh(x) + a + b) / (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2)) + 4\sqrt{2}((a^2 + ab) \cosh(x)^2 + 2(a^2 + ab) \cosh(x) \sinh(x) + (a^2 + ab) \sinh(x)^2 + a^2 + ab) \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} / ((a^3b + 3a^2b^2 + 3ab^3 + b^4) \cosh(x)^4 + 4(a^3b + 3a^2b^2 + 3ab^3 + b^4) \cosh(x) \sinh(x)^3 + (a^3b + 3a^2b^2 + 3ab^3 + b^4) \sinh(x)^4 + a^3b + 3a^2b^2 + 3ab^3 + b^4 + 2(a^3b + a^2b^2 - ab^3 - b^4) \cosh(x)^2 + 2(a^3b + a^2b^2 - ab^3 - b^4 + 3(a^3b + 3a^2b^2 + 3ab^3 + b^4) \cosh(x)^2) \sinh(x)^2 + 4(((a^3b + 3a^2b^2 + 3ab^3 + b^4) \cosh(x)^3 + (a^3b + a^2b^2 - ab^3 - b^4) \cosh(x)) \sinh(x)), -1/2(((ab + b^2) \cosh(x)^4 + 4(ab + b^2) \cosh(x) \sinh(x)^3 + (ab + b^2) \sinh(x)^4 + 2(ab - b^2) \cosh(x)^2 + 2(3(ab + b^2) \cosh(x)^2 + ab - b^2) \sinh(x)^2 + ab + b^2 + 4((ab + b^2) \cosh(x)^3 + (ab - b^2) \cosh(x)) \sinh(x)) \sqrt{-a-b} \arctan(\sqrt{2}(a \cosh(x)^2 + 2a \cosh(x) \sinh(x) + a \sinh(x)^2 + a + b) \sqrt{-a-b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / ((a^2 + ab) \cosh(x)^4 + 4(a^2 + ab) \cosh(x) \sinh(x)^3 + (a^2 + ab) \sinh(x)^4 + (2a^2 + ab - b^2) \cosh(x)^2 + (6(a^2 + ab) \cosh(x)^2 + 2a^2 + ab - b^2) \sinh(x)^2 + a^2 + 2ab + b^2 + 2(2(a^2 + ab) \cosh(x)^3 + (2a^2 + ab - b^2) \cosh(x)) \sinh(x))) + ((ab + b^2) \cosh(x)^4 + 4(ab + b^2) \cosh(x) \sinh(x)^3 + (ab + b^2) \sinh(x)^4 + 2(ab - b^2) \cosh(x)^2 + 2(3(ab + b^2) \cosh(x)^2 + ab - b^2) \sinh(x)^2 + ab + b^2 + 4((ab + b^2) \cosh(x)^3 + (ab - b^2) \cosh(x)) \sinh(x)) \sqrt{-a-b} \arctan(\sqrt{2}(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) \sqrt{-a-b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / ((a+b) \cosh(x)^4 + 4(a+b) \cosh(x) \sinh(x)^3 + (a+b) \sinh(x)^4 + 2(a-b) \cosh(x)^2 + 2(3(a+b) \cosh(x)^2 + a - b) \sinh(x)^2 + 4((a+b) \cosh(x)^3 + (a-b) \cosh(x)) \sinh(x) + a + b)) - 2\sqrt{2}((a^2 + ab) \cosh(x)^2 + 2(a^2 + ab) \cosh(x) \sinh(x) + (a^2 + ab) \sinh(x)^2 + a^2 + ab) \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} / ((a^3b + 3a^2b^2 + 3ab^3 + b^4) \cosh(x)^4 + 4(a^3b + 3a^2b^2 + 3ab^3 + b^4) \cosh(x) \sinh(x)^3 + (a^3b + 3a^2b^2 + 3ab^3 + b^4) \sinh(x)^4 + a^3b + 3a^2b^2 + 3ab^3 + b^4 + 2(a^3b + a^2b^2 - ab^3 - b^4) \cosh(x)^2 + 2(a^3b + a^2b^2 - ab^3 - b^4 + 3(a^3b + 3a^2b^2 + 3ab^3 + b^4) \cosh(x)^2) \sinh(x)^2 + 4(((a^3b + 3a^2b^2 + 3ab^3 + b^4) \cosh(x)^3 + (a^3b + a^2b^2 -
\end{aligned}$$

$a*b^3 - b^4*\cosh(x))*\sinh(x)]$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep+1)]Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Error: Bad Argument Type

maple [B] time = 0.08, size = 287, normalized size = 5.52

$$\frac{1}{b\sqrt{a+b(\tanh^2(x))}} - \frac{1}{2(a+b)\sqrt{(\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a + b}} + \frac{b(2(\tanh(x)-1)b + a + b)}{(a+b)(4b(a+b) - 4b^2)\sqrt{(\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a + b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x)

[Out] 1/b/(a+b*tanh(x)^2)^(1/2)-1/2/(a+b)/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+b/(a+b)*(2*(tanh(x)-1)*b+2*b)/(4*b*(a+b)-4*b^2)/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2))*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))-1/2/(a+b)/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)-b/(a+b)*(2*(1+tanh(x))*b-2*b)/(4*b*(a+b)-4*b^2)/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2))*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^3}{(b \tanh(x)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(tanh(x)^3/(b*tanh(x)^2 + a)^(3/2), x)

mupad [B] time = 2.06, size = 45, normalized size = 0.87

$$\frac{a \operatorname{atanh}\left(\frac{\sqrt{b \tanh(x)^2 + a}}{\sqrt{a + b}}\right)}{(a + b)^{3/2}} + \frac{a}{(b^2 + a b) \sqrt{b \tanh(x)^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^3/(a + b*tanh(x)^2)^(3/2),x)

[Out] atanh((a + b*tanh(x)^2)^(1/2)/(a + b)^(1/2))/(a + b)^(3/2) + a/((a*b + b^2)*(a + b*tanh(x)^2)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{(a + b \tanh^2(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)**3/(a+b*tanh(x)**2)**(3/2), x)

[Out] Integral(tanh(x)**3/(a + b*tanh(x)**2)**(3/2), x)

$$3.241 \quad \int \frac{\tanh^2(x)}{(a+b \tanh^2(x))^{3/2}} dx$$

Optimal. Leaf size=53

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{(a+b)^{3/2}} - \frac{\tanh(x)}{(a+b)\sqrt{a+b \tanh^2(x)}}$$

[Out] $\operatorname{arctanh}((a+b)^{1/2} \tanh(x) / (a+b \tanh(x)^2)^{1/2}) / (a+b)^{3/2} - \tanh(x) / (a+b \tanh(x)^2)^{1/2}$

Rubi [A] time = 0.10, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3670, 471, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{(a+b)^{3/2}} - \frac{\tanh(x)}{(a+b)\sqrt{a+b \tanh^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^2/(a + b*Tanh[x]^2)^(3/2), x]

[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(3/2) - Tanh[x]/((a + b)*Sqrt[a + b*Tanh[x]^2])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 471

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m - n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n + 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^2(x)}{(a + b \tanh^2(x))^{3/2}} dx &= \text{Subst} \left(\int \frac{x^2}{(1-x^2)(a+bx^2)^{3/2}} dx, x, \tanh(x) \right) \\
&= -\frac{\tanh(x)}{(a+b)\sqrt{a+b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}} dx, x, \tanh(x) \right)}{a+b} \\
&= -\frac{\tanh(x)}{(a+b)\sqrt{a+b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{1}{1-(a+b)x^2} dx, x, \frac{\tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right)}{a+b} \\
&= \frac{\tanh^{-1} \left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right)}{(a+b)^{3/2}} - \frac{\tanh(x)}{(a+b)\sqrt{a+b \tanh^2(x)}}
\end{aligned}$$

Mathematica [B] time = 1.83, size = 112, normalized size = 2.11

$$\frac{\tanh(x) \left(\tanh^{-1} \left(\frac{\sqrt{\frac{(a+b)\tanh^2(x)}{a}}}{\sqrt{\frac{b\tanh^2(x)}{a}+1}} \right) \sqrt{\frac{(a+b)\tanh^2(x)}{a}} (a \coth^2(x) + b) - (a+b) \sqrt{\frac{b\tanh^2(x)}{a}+1} \right)}{(a+b)^2 \sqrt{a+b \tanh^2(x)} \sqrt{\frac{b\tanh^2(x)}{a}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^2/(a + b*Tanh[x]^2)^(3/2), x]

[Out] (Tanh[x]*(ArcTanh[Sqrt[((a + b)*Tanh[x]^2)/a]/Sqrt[1 + (b*Tanh[x]^2)/a]]*(b + a*Coth[x]^2)*Sqrt[((a + b)*Tanh[x]^2)/a] - (a + b)*Sqrt[1 + (b*Tanh[x]^2)/a]))/((a + b)^2*Sqrt[a + b*Tanh[x]^2]*Sqrt[1 + (b*Tanh[x]^2)/a])

fricas [B] time = 0.55, size = 2281, normalized size = 43.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(3/2), x, algorithm="fricas")

[Out] [1/4*(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 -

$$\begin{aligned}
& b^2 \sinh(x)^4 + 4(5b^2 \cosh(x)^3 - 3b^2 \cosh(x)) \sinh(x)^3 - (a^2 - 2ab - 3b^2) \cosh(x)^2 + (15b^2 \cosh(x)^4 - 18b^2 \cosh(x)^2 - a^2 + 2ab + 3b^2) \sinh(x)^2 - a^2 - 2ab - b^2 + 2(3b^2 \cosh(x)^5 - 6b^2 \cosh(x)^3 - (a^2 - 2ab - 3b^2) \cosh(x)) \sinh(x) \sqrt{a+b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a-b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)} \\
& + 4(2(a^2 b^2 + b^3) \cosh(x)^7 - 3(a^2 b^2 + 2b^3) \cosh(x)^5 + (a^3 - a^2 b + 4ab^2 + 6b^3) \cosh(x)^3 + (a^3 - 3a^2 b - 2b^3) \cosh(x)) \sinh(x) / (\cosh(x)^6 + 6 \cosh(x)^5 \sinh(x) + 15 \cosh(x)^4 \sinh(x)^2 + 20 \cosh(x)^3 \sinh(x)^3 + 15 \cosh(x)^2 \sinh(x)^4 + 6 \cosh(x) \sinh(x)^5 + \sinh(x)^6) + ((a+b) \cosh(x)^4 + 4(a+b) \cosh(x) \sinh(x)^3 + (a+b) \sinh(x)^4 + 2(a-b) \cosh(x)^2 + 2(3(a+b) \cosh(x)^2 + a-b) \sinh(x)^2 + 4((a+b) \cosh(x)^3 + (a-b) \cosh(x)) \sinh(x) + a+b) \sqrt{a+b} \log(((a+b) \cosh(x))^4 + 4(a+b) \cosh(x) \sinh(x)^3 + (a+b) \sinh(x)^4 + 2a \cosh(x)^2 + 2(3(a+b) \cosh(x)^2 + a) \sinh(x)^2 + \sqrt{2}(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1) \sqrt{a+b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a-b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)} + 4((a+b) \cosh(x)^3 + a \cosh(x)) \sinh(x) + a+b) / (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2)) - 4 \sqrt{2}((a+b) \cosh(x)^2 + 2(a+b) \cosh(x) \sinh(x) + (a+b) \sinh(x)^2 - a-b) \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a-b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} / ((a^3 + 3a^2 b + 3a^2 b^2 + b^3) \cosh(x)^4 + 4(a^3 + 3a^2 b + 3a^2 b^2 + b^3) \cosh(x) \sinh(x)^3 + (a^3 + 3a^2 b + 3a^2 b^2 + b^3) \sinh(x)^4 + a^3 + 3a^2 b + 3a^2 b^2 + b^3 + 2(a^3 + a^2 b - ab^2 - b^3) \cosh(x)^2 + 2(a^3 + a^2 b - ab^2 - b^3 + 3(a^3 + 3a^2 b + 3a^2 b^2 + b^3) \cosh(x)^2) \sinh(x)^2 + 4((a^3 + 3a^2 b + 3a^2 b^2 + b^3) \cosh(x)^3 + (a^3 + a^2 b - ab^2 - b^3) \cosh(x)) \sinh(x)), -1/2(((a+b) \cosh(x)^4 + 4(a+b) \cosh(x) \sinh(x)^3 + (a+b) \sinh(x)^4 + 2(a-b) \cosh(x)^2 + 2(3(a+b) \cosh(x)^2 + a-b) \sinh(x)^2 + 4((a+b) \cosh(x)^3 + (a-b) \cosh(x)) \sinh(x) + a+b) \sqrt{-a-b} \arctan(\sqrt{2}(b \cosh(x)^2 + 2b \cosh(x) \sinh(x) + b \sinh(x)^2 - a-b) \sqrt{-a-b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a-b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)})) / ((a^2 b + b^2) \cosh(x)^4 + 4(a^2 b + b^2) \cosh(x) \sinh(x)^3 + (a^2 b + b^2) \sinh(x)^4 + (a^2 - ab - 2b^2) \cosh(x)^2 + (6(a^2 b + b^2) \cosh(x)^2 + a^2 - ab - 2b^2) \sinh(x)^2 + a^2 + 2ab + b^2 + 2(2(a^2 b + b^2) \cosh(x)^3 + (a^2 - ab - 2b^2) \cosh(x)) \sinh(x))) + ((a+b) \cosh(x)^4 + 4(a+b) \cosh(x) \sinh(x)^3 + (a+b) \sinh(x)^4 + 2(a-b) \cosh(x)^2 + 2(3(a+b) \cosh(x)^2 + a-b) \sinh(x)^2 + 4((a+b) \cosh(x)^3 + (a-b) \cosh(x)) \sinh(x) + a+b) \sqrt{-a-b} \arctan(\sqrt{2}(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1) \sqrt{-a-b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a-b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)})) / ((a+b) \cosh(x)^4 + 4(a+b) \cosh(x) \sinh(x)^3 + (a+b) \sinh(x)^4 + 2(a-b) \cosh(x)^2 + 2(3(a+b) \cosh(x)^2 + a-b) \sinh(x)^2 + 4((a+b) \cosh(x)^3 + (a-b) \cosh(x)) \sinh(x) + a+b) + 2 \sqrt{2}((a+b) \cosh(x)^2 + 2(a+b) \cosh(x) \sinh(x) + (a+b) \sinh(x)^2 - a-b) \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a-b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2))} / ((a^3 + 3a^2 b + 3a^2 b^2 + b^3) \cosh(x)^4 + 4(a^3 + 3a^2 b + 3a^2 b^2 + b^3) \cosh(x) \sinh(x)^3 + (a^3 + 3a^2 b + 3a^2 b^2 + b^3) \sinh(x)^4 + a^3 + 3a^2 b + 3a^2 b^2 + b^3 + 2(a^3 + a^2 b - ab^2 - b^3) \cosh(x)^2 + 2(a^3 + a^2 b - ab^2 - b^3 + 3(a^3 + 3a^2 b + 3a^2 b^2 + b^3) \cosh(x)^2) \sinh(x)^2 + 4((a^3 + 3a^2 b + 3a^2 b^2 + b^3) \cosh(x)^3 + (a^3 + a^2 b - ab^2 - b^3) \cosh(x)) \sinh(x))]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes

constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep+1)]Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Error: Bad Argument Type

maple [B] time = 0.08, size = 289, normalized size = 5.45

$$\frac{\tanh(x)}{a\sqrt{a+b(\tanh^2(x))}} - \frac{1}{2(a+b)\sqrt{(\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a+b}} + \frac{b(2(\tanh(x)-1)b + a+b)}{(a+b)(4b(a+b) - 4b^2)\sqrt{(\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^2/(a+b*tanh(x)^2)^(3/2), x)

[Out] -tanh(x)/a/(a+b*tanh(x)^2)^(1/2)-1/2/(a+b)/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+b/(a+b)*(2*(tanh(x)-1)*b+2*b)/(4*b*(a+b)-4*b^2)/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))+1/2/(a+b)/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)+b/(a+b)*(2*(1+tanh(x))*b-2*b)/(4*b*(a+b)-4*b^2)/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)-1/2/(a+b)^(3/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^2}{(b \tanh(x)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(3/2), x, algorithm="maxima")

[Out] integrate(tanh(x)^2/(b*tanh(x)^2 + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tanh(x)^2}{(b \tanh(x)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^2/(a + b*tanh(x)^2)^(3/2), x)

[Out] int(tanh(x)^2/(a + b*tanh(x)^2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{(a + b \tanh^2(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)**2/(a+b*tanh(x)**2)**(3/2), x)

[Out] Integral(tanh(x)**2/(a + b*tanh(x)**2)**(3/2), x)

$$3.242 \quad \int \frac{\tanh(x)}{(a+b \tanh^2(x))^{3/2}} dx$$

Optimal. Leaf size=49

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{1}{(a+b)\sqrt{a+b \tanh^2(x)}}$$

[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)-1/(a+b)/(a+b*tanh(x)^2)^(1/2)

Rubi [A] time = 0.09, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3670, 444, 51, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{1}{(a+b)\sqrt{a+b \tanh^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]/(a + b*Tanh[x]^2)^(3/2), x]

[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) - 1/((a + b)*Sqrt[a + b*Tanh[x]^2])

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x]] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 444

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^(m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned} \int \frac{\tanh(x)}{(a + b \tanh^2(x))^{3/2}} dx &= \text{Subst} \left(\int \frac{x}{(1-x^2)(a+bx^2)^{3/2}} dx, x, \tanh(x) \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1-x)(a+bx)^{3/2}} dx, x, \tanh^2(x) \right) \\ &= -\frac{1}{(a+b)\sqrt{a+b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{1}{(1-x)\sqrt{a+bx}} dx, x, \tanh^2(x) \right)}{2(a+b)} \\ &= -\frac{1}{(a+b)\sqrt{a+b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}} dx, x, \sqrt{a+b \tanh^2(x)} \right)}{b(a+b)} \\ &= \frac{\tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}} \right)}{(a+b)^{3/2}} - \frac{1}{(a+b)\sqrt{a+b \tanh^2(x)}} \end{aligned}$$

Mathematica [C] time = 0.04, size = 41, normalized size = 0.84

$$\frac{{}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{b \tanh^2(x) + a}{a+b}\right)}{(a+b)\sqrt{a+b \tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]/(a + b*Tanh[x]^2)^(3/2), x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tanh[x]^2)/(a + b)]/((a + b)*Sqrt[a + b*Tanh[x]^2]))
```

fricas [B] time = 0.67, size = 2277, normalized size = 46.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(3/2), x, algorithm="fricas")
```

```
[Out] [1/4*(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4
+ 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a +
b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(((a^3 + a
^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)
^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh
(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x)
))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)
)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)
```

```

*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 +
(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b
^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)
)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a
^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*c
osh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a
^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*
a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b
- b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^
3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(
x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^
2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 +
4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x)
)/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3
*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + (
(a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a
- b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cos
h(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(-((a + b)*cosh(x)
)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(
3*(a + b)*cosh(x)^2 - b)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
+ sinh(x)^2 - 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*((a + b)*cosh(x)^3
- b*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))
- 4*sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*sinh(x) + (a + b)*sinh(
x)^2 + a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)
^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh
(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sinh(x)^3 + (a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2
*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3 + 3*(a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))*sinh(x)), -1/2*((a + b)
)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*c
osh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3
+ (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(-a - b)*arctan(sqrt(2)*(a*cosh(x)
^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a + b)*sqrt(-a - b)*sqrt(((a + b)*
cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin
h(x)^2)))/((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 +
a*b)*sinh(x)^4 + (2*a^2 + a*b - b^2)*cosh(x)^2 + (6*(a^2 + a*b)*cosh(x)^2 +
2*a^2 + a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*cosh(x)
)^3 + (2*a^2 + a*b - b^2)*cosh(x))*sinh(x)) + ((a + b)*cosh(x)^4 + 4*(a +
b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a +
b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*s
inh(x) + a + b)*sqrt(-a - b)*arctan(sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x)
+ sinh(x)^2 - 1)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a + b)*cosh(x)^4 + 4
*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3
*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh
(x))*sinh(x) + a + b)) + 2*sqrt(2)*((a + b)*cosh(x)^2 + 2*(a + b)*cosh(x)*s
inh(x) + (a + b)*sinh(x)^2 + a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(
x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/((a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*cosh(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)*sin
h(x)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 3*a*b^
2 + b^3 + 2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2
- b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 +
3*a^2*b + 3*a*b^2 + b^3)*cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*cosh(x))*s
inh(x))]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep+1)]Warning, replacing 0 by `u`, a substitution variable should perha
ps be purged.Warning, replacing 0 by `u`, a substitution variable should p
erhaps be purged.Warning, replacing 0 by `u`, a substitution variable shou
ld perhaps be purged.Error: Bad Argument Type
```

maple [B] time = 0.08, size = 273, normalized size = 5.57

$$\frac{1}{2(a+b)\sqrt{(\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a + b}} + \frac{b(2(\tanh(x)-1)b + 2b)}{(a+b)(4b(a+b) - 4b^2)\sqrt{(\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a + b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*tanh(x)^2)^(3/2),x)
```

```
[Out] -1/2/(a+b)/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+b/(a+b)*(2*(tanh(x)-
1)*b+2*b)/(4*b*(a+b)-4*b^2)/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+1/2
/(a+b)^(3/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(
tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))-1/2/(a+b)/((1+tanh(x))^2*b-2*(1+tanh(
x))*b+a+b)^(1/2)-b/(a+b)*(2*(1+tanh(x))*b-2*b)/(4*b*(a+b)-4*b^2)/((1+tanh(x)
))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b-2*(1+tanh(x)
)*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{(b \tanh(x)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(x)/(b*tanh(x)^2 + a)^(3/2), x)
```

mupad [B] time = 1.94, size = 41, normalized size = 0.84

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b \tanh(x)^2 + a}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{1}{(a+b)\sqrt{b \tanh(x)^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a + b*tanh(x)^2)^(3/2),x)
```

```
[Out] atanh((a + b*tanh(x)^2)^(1/2)/(a + b)^(1/2))/(a + b)^(3/2) - 1/((a + b)*(a
+ b*tanh(x)^2)^(1/2))
```

sympy [A] time = 23.26, size = 51, normalized size = 1.04

$$\frac{1}{(a+b)\sqrt{a + b \tanh^2(x)}} - \frac{\operatorname{atan}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] -1/((a + b)*sqrt(a + b*tanh(x)**2)) - atan(sqrt(a + b*tanh(x)**2)/sqrt(-a - b))/(sqrt(-a - b)*(a + b))
```

$$3.243 \quad \int \frac{1}{(a+b \tanh^2(x))^{3/2}} dx$$

Optimal. Leaf size=56

$$\frac{b \tanh(x)}{a(a+b)\sqrt{a+b \tanh^2(x)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{(a+b)^{3/2}}$$

[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(3/2)+b*tanh(x)/a/(a+b)/(a+b*tanh(x)^2)^(1/2)

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3661, 382, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{(a+b)^{3/2}} + \frac{b \tanh(x)}{a(a+b)\sqrt{a+b \tanh^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tanh[x]^2)^(-3/2), x]

[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(3/2) + (b*Tanh[x])/(a*(a + b)*Sqrt[a + b*Tanh[x]^2])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 382

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[(b*c + n*(p+1)*(b*c - a*d))/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p+q+2)+1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]

Rule 3661

Int[((a_) + (b_.)*((c_.)*tan[e_.] + (f_.)*(x_)))^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(ff*x)^n]^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b \tanh^2(x))^{3/2}} dx &= \text{Subst} \left(\int \frac{1}{(1-x^2)(a+bx^2)^{3/2}} dx, x, \tanh(x) \right) \\
&= \frac{b \tanh(x)}{a(a+b)\sqrt{a+b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}} dx, x, \tanh(x) \right)}{a+b} \\
&= \frac{b \tanh(x)}{a(a+b)\sqrt{a+b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{1}{1-(a+b)x^2} dx, x, \frac{\tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right)}{a+b} \\
&= \frac{\tanh^{-1} \left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right)}{(a+b)^{3/2}} + \frac{b \tanh(x)}{a(a+b)\sqrt{a+b \tanh^2(x)}}
\end{aligned}$$

Mathematica [C] time = 4.60, size = 223, normalized size = 3.98

$$\sinh^2(x) \left(\sqrt{2} a^2 (a+b) \tanh(x) {}_2F_1 \left(2, 2; \frac{7}{2}; -\frac{(a+b) \sinh^2(x)}{a} \right) \left(-\frac{(a+b) \sinh^2(x) ((a+b) \cosh(2x) + a - b)}{a^2} \right)^{3/2} + \frac{15}{4} \operatorname{acsch}(x) \operatorname{sech}(x) \right)$$

$$15a^4 \left(-\frac{(a+b)}{4} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Tanh[x]^2)^(-3/2), x]

[Out] -1/15*(Sinh[x]^2*((15*a*(3*a - 2*b + (3*a + 2*b)*Cosh[2*x])*Csch[x]*Sech[x]*((a - b)*ArcSin[Sqrt[-((a + b)*Sinh[x]^2)/a]]) + (a + b)*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a]])*Cosh[2*x] - 2*a*Sqrt[-((a + b)*(b + a*Coth[x]^2)*Sinh[x]^4)/a^2]]))/4 + Sqrt[2]*a^2*(a + b)*Hypergeometric2F1[2, 2, 7/2, -((a + b)*Sinh[x]^2)/a])*(-((a + b)*(a - b + (a + b)*Cosh[2*x])*Sinh[x]^2)/a^2)^(3/2)*Tanh[x]))/(a^4*(-((a + b)*Sinh[x]^2)/a)^(3/2)*Sqrt[Cosh[x]^2 + (b*Sinh[x]^2)/a]*Sqrt[a + b*Tanh[x]^2])

fricas [B] time = 0.72, size = 2509, normalized size = 44.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tanh(x)^2)^(3/2), x, algorithm="fricas")

[Out] [1/4*((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*b)*sinh(x)^4 + 2*(a^2 - a*b)*cosh(x)^2 + 2*(3*(a^2 + a*b)*cosh(x)^2 + a^2 - a*b)*sinh(x)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(x)^3 + (a^2 - a*b)*cosh(x)*sinh(x))*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a

$$\begin{aligned}
&^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*\cosh(x)^2*\sinh(x) \\
&^2 + \sqrt{2}*(b^2*\cosh(x)^6 + 6*b^2*\cosh(x)*\sinh(x)^5 + b^2*\sinh(x)^6 - 3*b \\
&^2*\cosh(x)^4 + 3*(5*b^2*\cosh(x)^2 - b^2)*\sinh(x)^4 + 4*(5*b^2*\cosh(x)^3 - 3 \\
&*b^2*\cosh(x))*\sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*\cosh(x)^2 + (15*b^2*\cosh(x) \\
&^4 - 18*b^2*\cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*\sinh(x)^2 - a^2 - 2*a*b - b^2 \\
&+ 2*(3*b^2*\cosh(x)^5 - 6*b^2*\cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*\cosh(x))*\sin \\
&h(x))*\sqrt{a + b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cos \\
&h(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))} + 4*(2*(a*b^2 + b^3)*\cosh(x)^7 - 3 \\
&*(a*b^2 + 2*b^3)*\cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*\cosh(x)^3 + (a \\
&^3 - 3*a*b^2 - 2*b^3)*\cosh(x))*\sinh(x))/(\cosh(x)^6 + 6*\cosh(x)^5*\sinh(x) + \\
&15*\cosh(x)^4*\sinh(x)^2 + 20*\cosh(x)^3*\sinh(x)^3 + 15*\cosh(x)^2*\sinh(x)^4 + \\
&6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6)) + ((a^2 + a*b)*\cosh(x)^4 + 4*(a^2 + a*b)* \\
&\cosh(x)*\sinh(x)^3 + (a^2 + a*b)*\sinh(x)^4 + 2*(a^2 - a*b)*\cosh(x)^2 + 2*(3* \\
&(a^2 + a*b)*\cosh(x)^2 + a^2 - a*b)*\sinh(x)^2 + a^2 + a*b + 4*((a^2 + a*b)*\c \\
&osh(x)^3 + (a^2 - a*b)*\cosh(x))*\sinh(x))*\sqrt{a + b}*\log(((a + b)*\cosh(x)^4 \\
&+ 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a + b)*\sinh(x)^4 + 2*a*\cosh(x)^2 + 2*(3*(\\
&a + b)*\cosh(x)^2 + a)*\sinh(x)^2 + \sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \\
&\sinh(x)^2 + 1))*\sqrt{a + b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a \\
&- b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))} + 4*((a + b)*\cosh(x)^3 + \\
&a*\cosh(x))*\sinh(x) + a + b)/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) + \\
&4*\sqrt{2}*((a*b + b^2)*\cosh(x)^2 + 2*(a*b + b^2)*\cosh(x)*\sinh(x) + (a*b + b \\
&^2)*\sinh(x)^2 - a*b - b^2)*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a \\
&- b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))}/((a^4 + 3*a^3*b + 3*a^2* \\
&b^2 + a*b^3)*\cosh(x)^4 + 4*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*\cosh(x)*\sinh \\
&(x)^3 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*\sinh(x)^4 + a^4 + 3*a^3*b + 3*a \\
&^2*b^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*\cosh(x)^2 + 2*(a^4 + a^3* \\
&b - a^2*b^2 - a*b^3 + 3*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*\cosh(x)^2)*\sin \\
&h(x)^2 + 4*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*\cosh(x)^3 + (a^4 + a^3*b - \\
&a^2*b^2 - a*b^3)*\cosh(x))*\sinh(x)), -1/2*((a^2 + a*b)*\cosh(x)^4 + 4*(a^2 + \\
&a*b)*\cosh(x)*\sinh(x)^3 + (a^2 + a*b)*\sinh(x)^4 + 2*(a^2 - a*b)*\cosh(x)^2 + \\
&2*(3*(a^2 + a*b)*\cosh(x)^2 + a^2 - a*b)*\sinh(x)^2 + a^2 + a*b + 4*((a^2 + \\
&a*b)*\cosh(x)^3 + (a^2 - a*b)*\cosh(x))*\sinh(x))*\sqrt{-a - b}*\arctan(\sqrt{2})* \\
&(b*\cosh(x)^2 + 2*b*\cosh(x)*\sinh(x) + b*\sinh(x)^2 - a - b)*\sqrt{-a - b}*\sqrt \\
&(((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sin \\
&h(x) + \sinh(x)^2)))/((a*b + b^2)*\cosh(x)^4 + 4*(a*b + b^2)*\cosh(x)*\sinh(x)^3 \\
&+ (a*b + b^2)*\sinh(x)^4 + (a^2 - a*b - 2*b^2)*\cosh(x)^2 + (6*(a*b + b^2)*\c \\
&osh(x)^2 + a^2 - a*b - 2*b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b \\
&^2)*\cosh(x)^3 + (a^2 - a*b - 2*b^2)*\cosh(x))*\sinh(x))) + ((a^2 + a*b)*\cosh(\\
&x)^4 + 4*(a^2 + a*b)*\cosh(x)*\sinh(x)^3 + (a^2 + a*b)*\sinh(x)^4 + 2*(a^2 - a \\
&*b)*\cosh(x)^2 + 2*(3*(a^2 + a*b)*\cosh(x)^2 + a^2 - a*b)*\sinh(x)^2 + a^2 + a \\
&*b + 4*((a^2 + a*b)*\cosh(x)^3 + (a^2 - a*b)*\cosh(x))*\sinh(x))*\sqrt{-a - b})* \\
&\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1))*\sqrt{-a - b} \\
&*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x) \\
&)*\sinh(x) + \sinh(x)^2))}/((a + b)*\cosh(x)^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + \\
&(a + b)*\sinh(x)^4 + 2*(a - b)*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 + a - b)*\sin \\
&h(x)^2 + 4*((a + b)*\cosh(x)^3 + (a - b)*\cosh(x))*\sinh(x) + a + b) - 2*\sqrt{2} \\
&*((a*b + b^2)*\cosh(x)^2 + 2*(a*b + b^2)*\cosh(x)*\sinh(x) + (a*b + b^2)* \\
&\sinh(x)^2 - a*b - b^2)*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b) \\
&/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))}/((a^4 + 3*a^3*b + 3*a^2*b^2 \\
&+ a*b^3)*\cosh(x)^4 + 4*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*\cosh(x)*\sinh(x)^ \\
&3 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*\sinh(x)^4 + a^4 + 3*a^3*b + 3*a^2*b \\
&^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*\cosh(x)^2 + 2*(a^4 + a^3*b - \\
&a^2*b^2 - a*b^3 + 3*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*\cosh(x)^2)*\sinh(x) \\
&^2 + 4*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*\cosh(x)^3 + (a^4 + a^3*b - a^2* \\
&b^2 - a*b^3)*\cosh(x))*\sinh(x))]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
 constant sign by intervals (correct if the argument is real):Check [abs(t_n
 ostep+1)]Warning, replacing 0 by `u`, a substitution variable should perha
 ps be purged.Warning, replacing 0 by `u`, a substitution variable should p
 erhaps be purged.Warning, replacing 0 by `u`, a substitution variable shou
 ld perhaps be purged.Error: Bad Argument Type

maple [B] time = 0.09, size = 272, normalized size = 4.86

$$\frac{1}{2(a+b)\sqrt{(\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a + b}} + \frac{b(2(\tanh(x)-1)b + 2b)}{(a+b)(4b(a+b) - 4b^2)\sqrt{(\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a + b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*tanh(x)^2)^(3/2),x)

[Out]
$$-1/2/(a+b)/((\tanh(x)-1)^2*b+2*(\tanh(x)-1)*b+a+b)^{(1/2)}+b/(a+b)*(2*(\tanh(x)-1)*b+2*b)/(4*b*(a+b)-4*b^2)/((\tanh(x)-1)^2*b+2*(\tanh(x)-1)*b+a+b)^{(1/2)}+1/2/(a+b)^{(3/2)}*\ln((2*a+2*b+2*(\tanh(x)-1)*b+2*(a+b)^{(1/2)}*((\tanh(x)-1)^2*b+2*(\tanh(x)-1)*b+a+b)^{(1/2)})/(\tanh(x)-1))+1/2/(a+b)/((1+\tanh(x))^2*b-2*(1+\tanh(x))*b+a+b)^{(1/2)}+b/(a+b)*(2*(1+\tanh(x))*b-2*b)/(4*b*(a+b)-4*b^2)/((1+\tanh(x))^2*b-2*(1+\tanh(x))*b+a+b)^{(1/2)}-1/2/(a+b)^{(3/2)}*\ln((2*a+2*b-2*(1+\tanh(x))*b+2*(a+b)^{(1/2)}*((1+\tanh(x))^2*b-2*(1+\tanh(x))*b+a+b)^{(1/2)})/(1+\tanh(x)))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \tanh(x)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*tanh(x)^2 + a)^(-3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(b \tanh(x)^2 + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*tanh(x)^2)^(3/2),x)

[Out] int(1/(a + b*tanh(x)^2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \tanh^2(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tanh(x)**2)**(3/2),x)

[Out] Integral((a + b*tanh(x)**2)**(-3/2), x)

$$3.244 \quad \int \frac{\coth(x)}{(a+b \tanh^2(x))^{3/2}} dx$$

Optimal. Leaf size=78

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{b}{a(a+b)\sqrt{a+b \tanh^2(x)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}$$

[Out] $-\operatorname{arctanh}\left(\frac{(a+b \tanh(x)^2)^{1/2}}{a^{1/2}}\right)/a^{3/2} + \operatorname{arctanh}\left(\frac{(a+b \tanh(x)^2)^{1/2}}{(a+b)^{1/2}}\right)/(a+b)^{3/2} + b/a/(a+b)/(a+b \tanh(x)^2)^{1/2}$

Rubi [A] time = 0.15, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3670, 446, 85, 156, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{b}{a(a+b)\sqrt{a+b \tanh^2(x)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}$$

Antiderivative was successfully verified.

[In] `Int[Coth[x]/(a + b*Tanh[x]^2)^(3/2), x]`

[Out] $-(\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b \operatorname{Tanh}[x]^2]/\operatorname{Sqrt}[a]]/a^{3/2}) + \operatorname{ArcTanh}[\operatorname{Sqrt}[a + b \operatorname{Tanh}[x]^2]/\operatorname{Sqrt}[a + b]]/(a + b)^{3/2} + b/(a*(a + b)*\operatorname{Sqrt}[a + b \operatorname{Tanh}[x]^2])$

Rule 63

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 85

`Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)), x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[((b*d*e - b*c*f - a*d*f - b*d*f*x)*(e + f*x)^(p + 1))/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[p, -1]`

Rule 156

`Int[((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]`

Rule 208

`Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*((c_)*tan[(e_) +
(f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x],
x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(x)}{(a + b \tanh^2(x))^{3/2}} dx &= \text{Subst} \left(\int \frac{1}{x(1-x^2)(a+bx^2)^{3/2}} dx, x, \tanh(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1-x)x(a+bx)^{3/2}} dx, x, \tanh^2(x) \right) \\
&= \frac{b}{a(a+b)\sqrt{a+b \tanh^2(x)}} - \frac{\text{Subst} \left(\int \frac{-a-b+bx}{(1-x)x\sqrt{a+bx}} dx, x, \tanh^2(x) \right)}{2a(a+b)} \\
&= \frac{b}{a(a+b)\sqrt{a+b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \tanh^2(x) \right)}{2a} + \frac{\text{Subst} \left(\int \frac{1}{(1-x)\sqrt{a+bx}} dx, x, \tanh^2(x) \right)}{2a} \\
&= \frac{b}{a(a+b)\sqrt{a+b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a+b \tanh^2(x)} \right)}{ab} + \frac{\text{Subst} \left(\int \frac{1}{1-x} dx, x, \sqrt{a+b \tanh^2(x)} \right)}{2a} \\
&= -\frac{\tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a}} \right)}{a^{3/2}} + \frac{\tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}} \right)}{(a+b)^{3/2}} + \frac{b}{a(a+b)\sqrt{a+b \tanh^2(x)}}
\end{aligned}$$

Mathematica [C] time = 0.08, size = 70, normalized size = 0.90

$$\frac{(a+b) {}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{b \tanh^2(x)}{a} + 1 \right) - a {}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{b \tanh^2(x)+a}{a+b} \right)}{a(a+b)\sqrt{a+b \tanh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(a + b*Tanh[x]^2)^(3/2), x]
```

```
[Out] (- (a*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tanh[x]^2)/(a + b)]) + (a + b)*
Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Tanh[x]^2)/a]) / (a*(a + b)*Sqrt[a + b
*Tanh[x]^2])
```

fricas [B] time = 1.01, size = 6955, normalized size = 89.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/4*((a^3 + a^2*b)*\cosh(x)^4 + 4*(a^3 + a^2*b)*\cosh(x)*\sinh(x)^3 + (a^3 + \\ & a^2*b)*\sinh(x)^4 + a^3 + a^2*b + 2*(a^3 - a^2*b)*\cosh(x)^2 + 2*(a^3 - a^2*b \\ & b + 3*(a^3 + a^2*b)*\cosh(x)^2)*\sinh(x)^2 + 4*((a^3 + a^2*b)*\cosh(x)^3 + (a^3 \\ & - a^2*b)*\cosh(x))*\sinh(x))*\sqrt{a + b}*\log(((a^3 + a^2*b)*\cosh(x)^8 + 8*(\\ & a^3 + a^2*b)*\cosh(x)*\sinh(x)^7 + (a^3 + a^2*b)*\sinh(x)^8 + 2*(2*a^3 + a^2*b \\ &)*\cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*\cosh(x)^2)*\sinh(x)^6 + 4* \\ & (14*(a^3 + a^2*b)*\cosh(x)^3 + 3*(2*a^3 + a^2*b)*\cosh(x))*\sinh(x)^5 + (6*a^3 \\ & + 4*a^2*b - a*b^2 + b^3)*\cosh(x)^4 + (70*(a^3 + a^2*b)*\cosh(x)^4 + 6*a^3 + \\ & 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*\cosh(x)^2)*\sinh(x)^4 + 4*(14*(a \\ & ^3 + a^2*b)*\cosh(x)^5 + 10*(2*a^3 + a^2*b)*\cosh(x)^3 + (6*a^3 + 4*a^2*b - a \\ & *b^2 + b^3)*\cosh(x))*\sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + \\ & 3*a^2*b - b^3)*\cosh(x)^2 + 2*(14*(a^3 + a^2*b)*\cosh(x)^6 + 15*(2*a^3 + a^2 \\ & *b)*\cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*\c \\ & osh(x)^2)*\sinh(x)^2 + \sqrt{2}*(a^2*\cosh(x)^6 + 6*a^2*\cosh(x)*\sinh(x)^5 + a^2 \\ & *2*\sinh(x)^6 + 3*a^2*\cosh(x)^4 + 3*(5*a^2*\cosh(x)^2 + a^2)*\sinh(x)^4 + 4*(5* \\ & a^2*\cosh(x)^3 + 3*a^2*\cosh(x))*\sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*\cosh(x)^2 \\ & + (15*a^2*\cosh(x)^4 + 18*a^2*\cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*\sinh(x)^2 + a \\ & ^2 + 2*a*b + b^2 + 2*(3*a^2*\cosh(x)^5 + 6*a^2*\cosh(x)^3 + (3*a^2 + 2*a*b - \\ & b^2)*\cosh(x))*\sinh(x))*\sqrt{a + b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x) \\ &)^2 + a - b}/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) + 4*(2*(a^3 + a^2 \\ & *b)*\cosh(x)^7 + 3*(2*a^3 + a^2*b)*\cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3 \\ &)*\cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*\cosh(x))*\sinh(x))/(\cosh(x)^6 + 6*\cosh \\ & (x)^5*\sinh(x) + 15*\cosh(x)^4*\sinh(x)^2 + 20*\cosh(x)^3*\sinh(x)^3 + 15*\cosh(x) \\ & ^2*\sinh(x)^4 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6)) + 2*((a^3 + 3*a^2*b + 3* \\ & a*b^2 + b^3)*\cosh(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(x)*\sinh(x)^3 \\ & + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\sinh(x)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 \\ & + 2*(a^3 + a^2*b - a*b^2 - b^3)*\cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3 \\ & + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(x)^2)*\sinh(x)^2 + 4*((a^3 + 3*a^2 \\ & *b + 3*a*b^2 + b^3)*\cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*\cosh(x))*\sinh(x) \\ &)*\sqrt{a}*\log(-((2*a + b)*\cosh(x)^4 + 4*(2*a + b)*\cosh(x)*\sinh(x)^3 + (2*a \\ & + b)*\sinh(x)^4 + 2*(2*a - b)*\cosh(x)^2 + 2*(3*(2*a + b)*\cosh(x)^2 + 2*a - \\ & b)*\sinh(x)^2 - 2*\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1))*\sqrt{ \\ & a}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh \\ & (x)*\sinh(x) + \sinh(x)^2)) + 4*((2*a + b)*\cosh(x)^3 + (2*a - b)*\cosh(x))* \\ & \sinh(x) + 2*a + b)/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh \\ & (x)^2 - 1)*\sinh(x)^2 - 2*\cosh(x)^2 + 4*(\cosh(x)^3 - \cosh(x))*\sinh(x) + 1)) \\ & + ((a^3 + a^2*b)*\cosh(x)^4 + 4*(a^3 + a^2*b)*\cosh(x)*\sinh(x)^3 + (a^3 + a^2 \\ & *b)*\sinh(x)^4 + a^3 + a^2*b + 2*(a^3 - a^2*b)*\cosh(x)^2 + 2*(a^3 - a^2*b + \\ & 3*(a^3 + a^2*b)*\cosh(x)^2)*\sinh(x)^2 + 4*((a^3 + a^2*b)*\cosh(x)^3 + (a^3 - \\ & a^2*b)*\cosh(x))*\sinh(x))*\sqrt{a + b}*\log(-((a + b)*\cosh(x)^4 + 4*(a + b)*\cosh \\ & (x)*\sinh(x)^3 + (a + b)*\sinh(x)^4 - 2*b*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 \\ & - b)*\sinh(x)^2 + \sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1))* \\ & \sqrt{a + b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 \\ & - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) + 4*((a + b)*\cosh(x)^3 - b*\cosh(x))*\sinh \\ & (x) + a + b)/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) + 4*\sqrt{2}*(a^2*b \\ & + a*b^2 + (a^2*b + a*b^2)*\cosh(x)^2 + 2*(a^2*b + a*b^2)*\cosh(x)*\sinh(x) + \\ & (a^2*b + a*b^2)*\sinh(x)^2)*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a \\ & - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/(a^5 + 3*a^4*b + 3*a^3*b^2 \\ & + a^2*b^3 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*\cosh(x)^4 + 4*(a^5 + \\ & 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*\cosh(x)*\sinh(x)^3 + (a^5 + 3*a^4*b + 3*a^3*b^2 \\ & + a^2*b^3)*\sinh(x)^4 + 2*(a^5 + a^4*b - a^3*b^2 - a^2*b^3)*\cosh(x)^2 + 2* \\ & (a^5 + a^4*b - a^3*b^2 - a^2*b^3 + 3*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3) \\ &)*\cosh(x)^2)*\sinh(x)^2 + 4*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*\cosh(x)^3 \\ & + (a^5 + a^4*b - a^3*b^2 - a^2*b^3)*\cosh(x))*\sinh(x)), 1/4*(4*((a^3 + 3*a^2 \\ & *b + 3*a*b^2 + b^3)*\cosh(x)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(x)*\sinh \\ & (x)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\sinh(x)^4 + a^3 + 3*a^2*b + 3*a \end{aligned}$$

$$\begin{aligned}
& b^2 + b^3 + 2*(a^3 + a^2*b - a*b^2 - b^3)*\cosh(x)^2 + 2*(a^3 + a^2*b - a*b^2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(x)^2)*\sinh(x)^2 + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*\cosh(x)^3 + (a^3 + a^2*b - a*b^2 - b^3)*\cosh(x)*\sinh(x))*\sqrt{-a}*\arctan(\sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1)*\sqrt{-a}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)})/((a + b)*\cosh(x)^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a + b)*\sinh(x)^4 + 2*(a - b)*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 + a - b)*\sinh(x)^2 + 4*((a + b)*\cosh(x)^3 + (a - b)*\cosh(x))*\sinh(x) + a + b)) + ((a^3 + a^2*b)*\cosh(x)^4 + 4*(a^3 + a^2*b)*\cosh(x)*\sinh(x)^3 + (a^3 + a^2*b)*\sinh(x)^4 + a^3 + a^2*b + 2*(a^3 - a^2*b)*\cosh(x)^2 + 2*(a^3 - a^2*b + 3*(a^3 + a^2*b)*\cosh(x)^2)*\sinh(x)^2 + 4*((a^3 + a^2*b)*\cosh(x)^3 + (a^3 - a^2*b)*\cosh(x))*\sinh(x))*\sqrt{a + b}*\log(((a^3 + a^2*b)*\cosh(x)^8 + 8*(a^3 + a^2*b)*\cosh(x)*\sinh(x)^7 + (a^3 + a^2*b)*\sinh(x)^8 + 2*(2*a^3 + a^2*b)*\cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*\cosh(x)^2)*\sinh(x)^6 + 4*(14*(a^3 + a^2*b)*\cosh(x)^3 + 3*(2*a^3 + a^2*b)*\cosh(x))*\sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x)^4 + (70*(a^3 + a^2*b)*\cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*\cosh(x)^2)*\sinh(x)^4 + 4*(14*(a^3 + a^2*b)*\cosh(x)^5 + 10*(2*a^3 + a^2*b)*\cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x))*\sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*\cosh(x)^2 + 2*(14*(a^3 + a^2*b)*\cosh(x)^6 + 15*(2*a^3 + a^2*b)*\cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x)^2)*\sinh(x)^2 + \sqrt{2}*(a^2*\cosh(x)^6 + 6*a^2*\cosh(x)*\sinh(x)^5 + a^2*\sinh(x)^6 + 3*a^2*\cosh(x)^4 + 3*(5*a^2*\cosh(x)^2 + a^2)*\sinh(x)^4 + 4*(5*a^2*\cosh(x)^3 + 3*a^2*\cosh(x))*\sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*\cosh(x)^2 + (15*a^2*\cosh(x)^4 + 18*a^2*\cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*\cosh(x)^5 + 6*a^2*\cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*\cosh(x))*\sinh(x))*\sqrt{a + b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))} + 4*(2*(a^3 + a^2*b)*\cosh(x)^7 + 3*(2*a^3 + a^2*b)*\cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*\cosh(x))*\sinh(x))/(\cosh(x)^6 + 6*\cosh(x)^5*\sinh(x) + 15*\cosh(x)^4*\sinh(x)^2 + 20*\cosh(x)^3*\sinh(x)^3 + 15*\cosh(x)^2*\sinh(x)^4 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6)) + ((a^3 + a^2*b)*\cosh(x)^4 + 4*(a^3 + a^2*b)*\cosh(x)*\sinh(x)^3 + (a^3 + a^2*b)*\sinh(x)^4 + a^3 + a^2*b + 2*(a^3 - a^2*b)*\cosh(x)^2 + 2*(a^3 - a^2*b + 3*(a^3 + a^2*b)*\cosh(x)^2)*\sinh(x)^2 + 4*((a^3 + a^2*b)*\cosh(x)^3 + (a^3 - a^2*b)*\cosh(x))*\sinh(x))*\sqrt{a + b}*\log(-((a + b)*\cosh(x)^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a + b)*\sinh(x)^4 - 2*b*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 - b)*\sinh(x)^2 + \sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 - 1)*\sqrt{a + b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)} + 4*((a + b)*\cosh(x)^3 - b*\cosh(x))*\sinh(x) + a + b)/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) + 4*\sqrt{2}*(a^2*b + a*b^2 + (a^2*b + a*b^2)*\cosh(x)^2 + 2*(a^2*b + a*b^2)*\cosh(x)*\sinh(x) + (a^2*b + a*b^2)*\sinh(x)^2)*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))}/(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*\cosh(x)^4 + 4*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*\cosh(x)*\sinh(x)^3 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*\sinh(x)^4 + 2*(a^5 + a^4*b - a^3*b^2 - a^2*b^3)*\cosh(x)^2 + 2*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + 3*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*\cosh(x)^2)*\sinh(x)^2 + 4*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*\cosh(x)^3 + (a^5 + a^4*b - a^3*b^2 - a^2*b^3)*\cosh(x))*\sinh(x)), -1/2*((a^3 + a^2*b)*\cosh(x)^4 + 4*(a^3 + a^2*b)*\cosh(x)*\sinh(x)^3 + (a^3 + a^2*b)*\sinh(x)^4 + a^3 + a^2*b + 2*(a^3 - a^2*b)*\cosh(x)^2 + 2*(a^3 - a^2*b + 3*(a^3 + a^2*b)*\cosh(x)^2)*\sinh(x)^2 + 4*((a^3 + a^2*b)*\cosh(x)^3 + (a^3 - a^2*b)*\cosh(x))*\sinh(x))*\sqrt{-a - b}*\arctan(\sqrt{2}*(a*\cosh(x)^2 + 2*a*\cosh(x)*\sinh(x) + a*\sinh(x)^2 + a + b)*\sqrt{-a - b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)})/((a^2 + a*b)*\cosh(x)^4 + 4*(a^2 + a*b)*\cosh(x)*\sinh(x)^3 + (a^2 + a*b)*\sinh(x)^4 + (2*a^2 + a*b - b^2)*\cosh(x)^2 + (6*(a^2 + a*b)*\cosh(x)^2 + 2*a^2 + a*b - b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*\cosh(x)^3 + (2*a^2 + a*b - b^2)*\cosh(x))*\sinh(x))) +
\end{aligned}$$

$$\begin{aligned}
& ((a^3 + a^2b) \cosh(x)^4 + 4(a^3 + a^2b) \cosh(x) \sinh(x)^3 + (a^3 + a^2b) \sinh(x)^4 + a^3 + a^2b + 2(a^3 - a^2b) \cosh(x)^2 + 2(a^3 - a^2b + 3(a^3 + a^2b) \cosh(x)^2) \sinh(x)^2 + 4((a^3 + a^2b) \cosh(x)^3 + (a^3 - a^2b) \cosh(x)) \sinh(x)) \sqrt{-a - b} \arctan(\sqrt{2}(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) \sqrt{-a - b} \sqrt{((a + b) \cosh(x)^2 + (a + b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / ((a + b) \cosh(x)^4 + 4(a + b) \cosh(x) \sinh(x)^3 + (a + b) \sinh(x)^4 + 2(a - b) \cosh(x)^2 + 2(3(a + b) \cosh(x)^2 + a - b) \sinh(x)^2 + 4((a + b) \cosh(x)^3 + (a - b) \cosh(x)) \sinh(x) + a + b)) - ((a^3 + 3a^2b + 3ab^2 + b^3) \cosh(x)^4 + 4(a^3 + 3a^2b + 3ab^2 + b^3) \cosh(x) \sinh(x)^3 + (a^3 + 3a^2b + 3ab^2 + b^3) \sinh(x)^4 + a^3 + 3a^2b + 3ab^2 + b^3 + 2(a^3 + a^2b - ab^2 - b^3) \cosh(x)^2 + 2(a^3 + a^2b - ab^2 - b^3 + 3(a^3 + 3a^2b + 3ab^2 + b^3) \cosh(x)^2) \sinh(x)^2 + 4((a^3 + 3a^2b + 3ab^2 + b^3) \cosh(x)^3 + (a^3 + a^2b - ab^2 - b^3) \cosh(x)) \sinh(x)) \sqrt{a} \log(-((2a + b) \cosh(x)^4 + 4(2a + b) \cosh(x) \sinh(x)^3 + (2a + b) \sinh(x)^4 + 2(2a - b) \cosh(x)^2 + 2(3(2a + b) \cosh(x)^2 + 2a - b) \sinh(x)^2 - 2\sqrt{2}(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1) \sqrt{a} \sqrt{((a + b) \cosh(x)^2 + (a + b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) + 4((2a + b) \cosh(x)^3 + (2a - b) \cosh(x)) \sinh(x) + 2a + b) / (\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2(3 \cosh(x)^2 - 1) \sinh(x)^2 - 2 \cosh(x)^2 + 4(\cosh(x)^3 - \cosh(x)) \sinh(x) + 1)) - 2\sqrt{2}(a^2b + ab^2 + (a^2b + ab^2) \cosh(x)^2 + 2(a^2b + ab^2) \cosh(x) \sinh(x) + (a^2b + ab^2) \sinh(x)^2) \sqrt{((a + b) \cosh(x)^2 + (a + b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / (a^5 + 3a^4b + 3a^3b^2 + a^2b^3 + (a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x)^4 + 4(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x) \sinh(x)^3 + (a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \sinh(x)^4 + 2(a^5 + a^4b - a^3b^2 - a^2b^3) \cosh(x)^2 + 2(a^5 + a^4b - a^3b^2 - a^2b^3 + 3(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x)^2) \sinh(x)^2 + 4((a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x)^3 + (a^5 + a^4b - a^3b^2 - a^2b^3) \cosh(x)) \sinh(x)), 1/2(2((a^3 + 3a^2b + 3ab^2 + b^3) \cosh(x)^4 + 4(a^3 + 3a^2b + 3ab^2 + b^3) \cosh(x) \sinh(x)^3 + (a^3 + 3a^2b + 3ab^2 + b^3) \sinh(x)^4 + a^3 + 3a^2b + 3ab^2 + b^3 + 2(a^3 + a^2b - ab^2 - b^3) \cosh(x)^2 + 2(a^3 + a^2b - ab^2 - b^3 + 3(a^3 + 3a^2b + 3ab^2 + b^3) \cosh(x)^2) \sinh(x)^2 + 4((a^3 + 3a^2b + 3ab^2 + b^3) \cosh(x)^3 + (a^3 + a^2b - ab^2 - b^3) \cosh(x)) \sinh(x)) \sqrt{-a} \arctan(\sqrt{2}(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1) \sqrt{-a} \sqrt{((a + b) \cosh(x)^2 + (a + b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / ((a + b) \cosh(x)^4 + 4(a + b) \cosh(x) \sinh(x)^3 + (a + b) \sinh(x)^4 + 2(a - b) \cosh(x)^2 + 2(3(a + b) \cosh(x)^2 + a - b) \sinh(x)^2 + 4((a + b) \cosh(x)^3 + (a - b) \cosh(x)) \sinh(x) + a + b)) - ((a^3 + a^2b) \cosh(x)^4 + 4(a^3 + a^2b) \cosh(x) \sinh(x)^3 + (a^3 + a^2b) \sinh(x)^4 + a^3 + a^2b + 2(a^3 - a^2b) \cosh(x)^2 + 2(a^3 - a^2b + 3(a^3 + a^2b) \cosh(x)^2) \sinh(x)^2 + 4((a^3 + a^2b) \cosh(x)^3 + (a^3 - a^2b) \cosh(x)) \sinh(x)) \sqrt{-a - b} \arctan(\sqrt{2}((a \cosh(x)^2 + 2a \cosh(x) \sinh(x) + a \sinh(x)^2 + a + b) \sqrt{-a - b} \sqrt{((a + b) \cosh(x)^2 + (a + b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / ((a^2 + ab) \cosh(x)^4 + 4(a^2 + ab) \cosh(x) \sinh(x)^3 + (a^2 + ab) \sinh(x)^4 + (2a^2 + ab - b^2) \cosh(x)^2 + (6(a^2 + ab) \cosh(x)^2 + 2a^2 + ab - b^2) \sinh(x)^2 + a^2 + 2ab + b^2 + 2(2(a^2 + ab) \cosh(x)^3 + (2a^2 + ab - b^2) \cosh(x)) \sinh(x))) - ((a^3 + a^2b) \cosh(x)^4 + 4(a^3 + a^2b) \cosh(x) \sinh(x)^3 + (a^3 + a^2b) \sinh(x)^4 + a^3 + a^2b + 2(a^3 - a^2b) \cosh(x)^2 + 2(a^3 - a^2b + 3(a^3 + a^2b) \cosh(x)^2) \sinh(x)^2 + 4((a^3 + a^2b) \cosh(x)^3 + (a^3 - a^2b) \cosh(x)) \sinh(x)) \sqrt{-a - b} \arctan(\sqrt{2}(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) \sqrt{-a - b} \sqrt{((a + b) \cosh(x)^2 + (a + b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / ((a + b) \cosh(x)^4 + 4(a + b) \cosh(x) \sinh(x)^3 + (a + b) \sinh(x)^4 + 2(a - b) \cosh(x)^2 + 2(3(a + b) \cosh(x)^2 + a - b) \sinh(x)^2 + 4((a + b) \cosh(x)^3 + (a - b) \cosh(x)) \sinh(x) + a + b)) + 2\sqrt{2}(a^2b + ab^2 + (a^2b + ab^2) \cosh(x)^2 + 2(a^2b + ab^2) \cosh(x) \sinh(x)
\end{aligned}$$

```
*sinh(x) + (a^2*b + a*b^2)*sinh(x)^2)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^4 + 4*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)*sinh(x)^3 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sinh(x)^4 + 2*(a^5 + a^4*b - a^3*b^2 - a^2*b^3)*cosh(x)^2 + 2*(a^5 + a^4*b - a^3*b^2 - a^2*b^3 + 3*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*cosh(x)^3 + (a^5 + a^4*b - a^3*b^2 - a^2*b^3)*cosh(x))*sinh(x))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep+1)]Error: Bad Argument Type
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{(a + b(\tanh^2(x)))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*tanh(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)/(a+b*tanh(x)^2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{(b \tanh(x)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/(b*tanh(x)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)}{(b \tanh(x)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a + b*tanh(x)^2)^(3/2),x)
```

```
[Out] int(coth(x)/(a + b*tanh(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{(a + b \tanh^2(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*tanh(x)**2)**(3/2),x)
```

```
[Out] Integral(coth(x)/(a + b*tanh(x)**2)**(3/2), x)
```

$$3.245 \quad \int \frac{\coth^2(x)}{(a+b \tanh^2(x))^{3/2}} dx$$

Optimal. Leaf size=85

$$-\frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a^2(a+b)} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} + \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(3/2)+b*coth(x)/a/(a+b)/(a+b*tanh(x)^2)^(1/2)-(a+2*b)*coth(x)*(a+b*tanh(x)^2)^(1/2)/a^2/(a+b)

Rubi [A] time = 0.16, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3670, 472, 583, 12, 377, 206}

$$-\frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a^2(a+b)} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} + \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^2/(a + b*Tanh[x]^2)^(3/2), x]

[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(3/2) + (b*Cot h[x])/(a*(a + b)*Sqrt[a + b*Tanh[x]^2]) - ((a + 2*b)*Coth[x]*Sqrt[a + b*Tan h[x]^2])/(a^2*(a + b))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match Q[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt Q[a, 0] || LtQ[b, 0])

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 472

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 583


```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f*f^2*x^2), x], x, (c*Tan[e + f*x])/ff, x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))
```

Rubi steps

$$\begin{aligned} \int \frac{\coth^2(x)}{(a + b \tanh^2(x))^{3/2}} dx &= \text{Subst} \left(\int \frac{1}{x^2 (1-x^2) (a + bx^2)^{3/2}} dx, x, \tanh(x) \right) \\ &= \frac{b \coth(x)}{a(a+b)\sqrt{a + b \tanh^2(x)}} - \frac{\text{Subst} \left(\int \frac{-a-2b+2bx^2}{x^2(1-x^2)\sqrt{a+bx^2}} dx, x, \tanh(x) \right)}{a(a+b)} \\ &= \frac{b \coth(x)}{a(a+b)\sqrt{a + b \tanh^2(x)}} - \frac{(a+2b) \coth(x) \sqrt{a + b \tanh^2(x)}}{a^2(a+b)} + \frac{\text{Subst} \left(\int \frac{a^2}{(1-x^2)\sqrt{a+bx^2}} dx, x, \tanh(x) \right)}{a^2(a+b)} \\ &= \frac{b \coth(x)}{a(a+b)\sqrt{a + b \tanh^2(x)}} - \frac{(a+2b) \coth(x) \sqrt{a + b \tanh^2(x)}}{a^2(a+b)} + \frac{\text{Subst} \left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}} dx, x, \tanh(x) \right)}{a} \\ &= \frac{b \coth(x)}{a(a+b)\sqrt{a + b \tanh^2(x)}} - \frac{(a+2b) \coth(x) \sqrt{a + b \tanh^2(x)}}{a^2(a+b)} + \frac{\text{Subst} \left(\int \frac{1}{1-(a+b)x^2} dx, x, \tanh(x) \right)}{a} \\ &= \frac{\tanh^{-1} \left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right)}{(a+b)^{3/2}} + \frac{b \coth(x)}{a(a+b)\sqrt{a + b \tanh^2(x)}} - \frac{(a+2b) \coth(x) \sqrt{a + b \tanh^2(x)}}{a^2(a+b)} \end{aligned}$$

Mathematica [C] time = 7.99, size = 230, normalized size = 2.71

$$\frac{\sinh(2x)\text{sech}^2(x) \left(\sqrt{2} a^3 \sqrt{\frac{\text{csch}^2(x)((a+b)\cosh(2x)+a-b)}{b}} \Pi \left(\frac{b}{a+b}; \sin^{-1} \left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}}{\sqrt{2}} \right) \right) \right)}{2\sqrt{2} a^2 (a+b)^2 \sqrt{\text{sech}^2(x)}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]^2/(a + b*Tanh[x]^2)^(3/2), x]

```
[Out] -1/2*(((a + b)*(a^2 - 2*b^2 + (a^2 + 2*a*b + 2*b^2)*Cosh[2*x])*Csch[x]^2 -
Sqrt[2]*a^2*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*Elliptic
cF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] + Sq
rt[2]*a^3*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticPi[b/(a +
b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1])*S
ech[x]^2*Sinh[2*x])/(Sqrt[2]*a^2*(a + b)^2*Sqrt[(a - b + (a + b)*Cosh[2*x])
*Sech[x]^2])
```

fricas [B] time = 0.93, size = 3929, normalized size = 46.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^3 + a^2*b)*cosh(x)^6 + 6*(a^3 + a^2*b)*cosh(x)*sinh(x)^5 + (a^3 +
a^2*b)*sinh(x)^6 + (a^3 - 3*a^2*b)*cosh(x)^4 + (a^3 - 3*a^2*b + 15*(a^3 +
a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^3 + a^2*b)*cosh(x)^3 + (a^3 - 3*a^2*b
)*cosh(x))*sinh(x)^3 - a^3 - a^2*b - (a^3 - 3*a^2*b)*cosh(x)^2 + (15*(a^3 +
a^2*b)*cosh(x)^4 - a^3 + 3*a^2*b + 6*(a^3 - 3*a^2*b)*cosh(x)^2)*sinh(x)^2
+ 2*(3*(a^3 + a^2*b)*cosh(x)^5 + 2*(a^3 - 3*a^2*b)*cosh(x)^3 - (a^3 - 3*a^2
*b)*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2
+ b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh
(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a
*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b
+ 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b +
4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 +
b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6
*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2
- 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*co
sh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)
^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh
(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*co
sh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*
b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2
*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*c
osh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*co
sh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cos
h(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5
*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*s
inh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + ((a^3 + a^2*b)*cosh(x)^6 + 6
*(a^3 + a^2*b)*cosh(x)*sinh(x)^5 + (a^3 + a^2*b)*sinh(x)^6 + (a^3 - 3*a^2*b
)*cosh(x)^4 + (a^3 - 3*a^2*b + 15*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(5
*(a^3 + a^2*b)*cosh(x)^3 + (a^3 - 3*a^2*b)*cosh(x))*sinh(x)^3 - a^3 - a^2*b
- (a^3 - 3*a^2*b)*cosh(x)^2 + (15*(a^3 + a^2*b)*cosh(x)^4 - a^3 + 3*a^2*b
+ 6*(a^3 - 3*a^2*b)*cosh(x)^2)*sinh(x)^2 + 2*(3*(a^3 + a^2*b)*cosh(x)^5 + 2
*(a^3 - 3*a^2*b)*cosh(x)^3 - (a^3 - 3*a^2*b)*cosh(x))*sinh(x))*sqrt(a + b)*
log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 +
2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 +
(a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4
*((a + b)*cosh(x)^3 + a*cosh(x))*sinh(x) + a + b)/(cosh(x)^2 + 2*cosh(x)*si
nh(x) + sinh(x)^2)) - 4*sqrt(2)*((a^3 + 3*a^2*b + 4*a*b^2 + 2*b^3)*cosh(x)^
4 + 4*(a^3 + 3*a^2*b + 4*a*b^2 + 2*b^3)*cosh(x)*sinh(x)^3 + (a^3 + 3*a^2*b
+ 4*a*b^2 + 2*b^3)*sinh(x)^4 + a^3 + 3*a^2*b + 4*a*b^2 + 2*b^3 + 2*(a^3 + a
^2*b - 2*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(a^3 + a^2*b - 2*a*b^2 - 2*b^3 + 3*(a
^3 + 3*a^2*b + 4*a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 + 3*a^2*b +
4*a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 + a^2*b - 2*a*b^2 - 2*b^3)*cosh(x))*sinh(
x))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cos
```

$$\frac{h(x) \sinh(x) + \sinh(x)^2}{((a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x)^6 + 6(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x) \sinh(x)^5 + (a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \sinh(x)^6 - a^5 - 3a^4b - 3a^3b^2 - a^2b^3 + (a^5 - a^4b - 5a^3b^2 - 3a^2b^3) \cosh(x)^4 + (a^5 - a^4b - 5a^3b^2 - 3a^2b^3 + 15(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x)^2) \sinh(x)^4 + 4(5(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x)^3 + (a^5 - a^4b - 5a^3b^2 - 3a^2b^3) \cosh(x)) \sinh(x)^3 - (a^5 - a^4b - 5a^3b^2 - 3a^2b^3) \cosh(x)^2 - (a^5 - a^4b - 5a^3b^2 - 3a^2b^3 - 15(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x)^4 - 6(a^5 - a^4b - 5a^3b^2 - 3a^2b^3) \cosh(x)^2) \sinh(x)^2 + 2(3(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x)^5 + 2(a^5 - a^4b - 5a^3b^2 - 3a^2b^3) \cosh(x)^3 - (a^5 - a^4b - 5a^3b^2 - 3a^2b^3) \cosh(x)) \sinh(x)}, -1/2(((a^3 + a^2b) \cosh(x)^6 + 6(a^3 + a^2b) \cosh(x) \sinh(x)^5 + (a^3 + a^2b) \sinh(x)^6 + (a^3 - 3a^2b) \cosh(x)^4 + (a^3 - 3a^2b + 15(a^3 + a^2b) \cosh(x)^2) \sinh(x)^4 + 4(5(a^3 + a^2b) \cosh(x)^3 + (a^3 - 3a^2b) \cosh(x)) \sinh(x)^3 - a^3 - a^2b - (a^3 - 3a^2b) \cosh(x)^2 + (15(a^3 + a^2b) \cosh(x)^4 - a^3 + 3a^2b + 6(a^3 - 3a^2b) \cosh(x)^2) \sinh(x)^2 + 2(3(a^3 + a^2b) \cosh(x)^5 + 2(a^3 - 3a^2b) \cosh(x)^3 - (a^3 - 3a^2b) \cosh(x)) \sinh(x)) \sqrt{-a - b} \arctan(\sqrt{2} * (b \cosh(x)^2 + 2b \cosh(x) \sinh(x) + b \sinh(x)^2 - a - b) \sqrt{-a - b} \sqrt{((a + b) \cosh(x)^2 + (a + b) \sinh(x)^2 + a - b) / ((\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)) / ((a * b + b^2) \cosh(x)^4 + 4 * (a * b + b^2) \cosh(x) \sinh(x)^3 + (a * b + b^2) \sinh(x)^4 + (a^2 - a * b - 2 * b^2) \cosh(x)^2 + (6 * (a * b + b^2) \cosh(x)^2 + a^2 - a * b - 2 * b^2) \sinh(x)^2 + a^2 + 2 * a * b + b^2 + 2 * (2 * (a * b + b^2) \cosh(x)^3 + (a^2 - a * b - 2 * b^2) \cosh(x)) \sinh(x))) + ((a^3 + a^2b) \cosh(x)^6 + 6(a^3 + a^2b) \cosh(x) \sinh(x)^5 + (a^3 + a^2b) \sinh(x)^6 + (a^3 - 3a^2b) \cosh(x)^4 + (a^3 - 3a^2b + 15(a^3 + a^2b) \cosh(x)^2) \sinh(x)^4 + 4(5(a^3 + a^2b) \cosh(x)^3 + (a^3 - 3a^2b) \cosh(x)) \sinh(x)^3 - a^3 - a^2b - (a^3 - 3a^2b) \cosh(x)^2 + (15(a^3 + a^2b) \cosh(x)^4 - a^3 + 3a^2b + 6(a^3 - 3a^2b) \cosh(x)^2) \sinh(x)^2 + 2(3(a^3 + a^2b) \cosh(x)^5 + 2(a^3 - 3a^2b) \cosh(x)^3 - (a^3 - 3a^2b) \cosh(x)) \sinh(x)) \sqrt{-a - b} \arctan(\sqrt{2} * (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1) \sqrt{-a - b} \sqrt{((a + b) \cosh(x)^2 + (a + b) \sinh(x)^2 + a - b) / ((\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)) / ((a + b) \cosh(x)^4 + 4 * (a + b) \cosh(x) \sinh(x)^3 + (a + b) \sinh(x)^4 + 2 * (a - b) \cosh(x)^2 + 2 * (3 * (a + b) \cosh(x)^2 + a - b) \sinh(x)^2 + 4 * ((a + b) \cosh(x)^3 + (a - b) \cosh(x)) \sinh(x) + a + b)) + 2 * \sqrt{2} * ((a^3 + 3a^2b + 4a * b^2 + 2b^3) \cosh(x)^4 + 4 * (a^3 + 3a^2b + 4a * b^2 + 2b^3) \cosh(x) \sinh(x)^3 + (a^3 + 3a^2b + 4a * b^2 + 2b^3) \sinh(x)^4 + a^3 + 3a^2b + 4a * b^2 + 2b^3 + 2 * (a^3 + a^2b * b - 2a * b^2 - 2b^3) \cosh(x)^2 + 2 * (a^3 + a^2b - 2a * b^2 - 2b^3 + 3 * (a^3 + 3a^2b + 4a * b^2 + 2b^3) \cosh(x)^2) \sinh(x)^2 + 4 * ((a^3 + 3a^2b + 4a * b^2 + 2b^3) \cosh(x)^3 + (a^3 + a^2b - 2a * b^2 - 2b^3) \cosh(x)) \sinh(x)) \sqrt{((a + b) \cosh(x)^2 + (a + b) \sinh(x)^2 + a - b) / ((\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)) / ((a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x)^6 + 6(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x) \sinh(x)^5 + (a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \sinh(x)^6 - a^5 - 3a^4b - 3a^3b^2 - a^2b^3 + (a^5 - a^4b - 5a^3b^2 - 3a^2b^3) \cosh(x)^4 + (a^5 - a^4b - 5a^3b^2 - 3a^2b^3 + 15(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x)^2) \sinh(x)^4 + 4(5(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x)^3 + (a^5 - a^4b - 5a^3b^2 - 3a^2b^3) \cosh(x)) \sinh(x)^3 - (a^5 - a^4b - 5a^3b^2 - 3a^2b^3) \cosh(x)^2 - (a^5 - a^4b - 5a^3b^2 - 3a^2b^3 - 15(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x)^4 - 6(a^5 - a^4b - 5a^3b^2 - 3a^2b^3) \cosh(x)^2) \sinh(x)^2 + 2(3(a^5 + 3a^4b + 3a^3b^2 + a^2b^3) \cosh(x)^5 + 2(a^5 - a^4b - 5a^3b^2 - 3a^2b^3) \cosh(x)^3 - (a^5 - a^4b - 5a^3b^2 - 3a^2b^3) \cosh(x)) \sinh(x)]$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
 constant sign by intervals (correct if the argument is real):Check [abs(t_n
 ostep+1)]Evaluation time: 1.12Error: Bad Argument Type

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{(a + b(\tanh^2(x)))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x)

[Out] int(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)^2}{(b \tanh(x)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(coth(x)^2/(b*tanh(x)^2 + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)^2}{(b \tanh(x)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^2/(a + b*tanh(x)^2)^(3/2),x)

[Out] int(coth(x)^2/(a + b*tanh(x)^2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{(a + b \tanh^2(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**2/(a+b*tanh(x)**2)**(3/2),x)

[Out] Integral(coth(x)**2/(a + b*tanh(x)**2)**(3/2), x)

$$3.246 \quad \int \frac{\tanh^6(x)}{(a+b \tanh^2(x))^{5/2}} dx$$

Optimal. Leaf size=118

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{b^{5/2}} + \frac{a(a+2b) \tanh(x)}{b^2(a+b)^2 \sqrt{a+b \tanh^2(x)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{a \tanh^3(x)}{3b(a+b)(a+b \tanh^2(x))^{3/2}}$$

[Out] $-\operatorname{arctanh}(b^{1/2} \tanh(x) / (a+b \tanh(x)^2)^{1/2}) / b^{5/2} + \operatorname{arctanh}((a+b)^{1/2} \tanh(x) / (a+b \tanh(x)^2)^{1/2}) / (a+b)^{5/2} + a(a+2b) \tanh(x) / b^2 (a+b)^2 (a+b \tanh(x)^2)^{1/2} + 1/3 a \tanh(x)^3 / b (a+b) (a+b \tanh(x)^2)^{3/2}$

Rubi [A] time = 0.22, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3670, 470, 578, 523, 217, 206, 377}

$$\frac{a(a+2b) \tanh(x)}{b^2(a+b)^2 \sqrt{a+b \tanh^2(x)}} - \frac{\tanh^{-1}\left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{b^{5/2}} + \frac{a \tanh^3(x)}{3b(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}}\right)}{(a+b)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^6/(a + b*Tanh[x]^2)^(5/2), x]

[Out] $-(\operatorname{ArcTanh}[(\operatorname{Sqrt}[b] \operatorname{Tanh}[x]) / \operatorname{Sqrt}[a + b \operatorname{Tanh}[x]^2]]) / b^{5/2} + \operatorname{ArcTanh}[(\operatorname{Sqrt}[a + b] \operatorname{Tanh}[x]) / \operatorname{Sqrt}[a + b \operatorname{Tanh}[x]^2]] / (a + b)^{5/2} + (a \operatorname{Tanh}[x]^3) / (3 * b * (a + b) * (a + b \operatorname{Tanh}[x]^2)^{3/2}) + (a * (a + 2 * b) \operatorname{Tanh}[x]) / (b^2 * (a + b)^2 * \operatorname{Sqrt}[a + b \operatorname{Tanh}[x]^2])$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 523

Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*Sqrt[(c_) + (d_)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 578

Int[((g_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]

Rule 3670

Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*((c_)*tan[(e_) + (f_)*(x_)])^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
 \int \frac{\tanh^6(x)}{(a + b \tanh^2(x))^{5/2}} dx &= \text{Subst} \left(\int \frac{x^6}{(1-x^2)(a+bx^2)^{5/2}} dx, x, \tanh(x) \right) \\
 &= \frac{a \tanh^3(x)}{3b(a+b)(a+b \tanh^2(x))^{3/2}} - \frac{\text{Subst} \left(\int \frac{x^2(3a-3(a+b)x^2)}{(1-x^2)(a+bx^2)^{3/2}} dx, x, \tanh(x) \right)}{3b(a+b)} \\
 &= \frac{a \tanh^3(x)}{3b(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{a(a+2b) \tanh(x)}{b^2(a+b)^2 \sqrt{a+b \tanh^2(x)}} - \frac{\text{Subst} \left(\int \frac{3a(a+2b)-3(a+b)}{(1-x^2)\sqrt{a+bx^2}} dx, x, \tanh(x) \right)}{3b^2(a+b)} \\
 &= \frac{a \tanh^3(x)}{3b(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{a(a+2b) \tanh(x)}{b^2(a+b)^2 \sqrt{a+b \tanh^2(x)}} - \frac{\text{Subst} \left(\int \frac{1}{\sqrt{a+bx^2}} dx, x, \tanh(x) \right)}{b^2} \\
 &= \frac{a \tanh^3(x)}{3b(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{a(a+2b) \tanh(x)}{b^2(a+b)^2 \sqrt{a+b \tanh^2(x)}} - \frac{\text{Subst} \left(\int \frac{1}{1-bx^2} dx, x, \tanh(x) \right)}{b^2} \\
 &= -\frac{\tanh^{-1} \left(\frac{\sqrt{b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right)}{b^{5/2}} + \frac{\tanh^{-1} \left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right)}{(a+b)^{5/2}} + \frac{a \tanh^3(x)}{3b(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{a(a+2b) \tanh(x)}{b^2(a+b)^2 \sqrt{a+b \tanh^2(x)}} - \frac{\text{Subst} \left(\int \frac{1}{1-bx^2} dx, x, \tanh(x) \right)}{b^2}
 \end{aligned}$$

Mathematica [C] time = 2.03, size = 231, normalized size = 1.96

$$\frac{\sqrt{\operatorname{sech}^2(x)((a+b)\cosh(2x)+a-b)} \left(\frac{a(a+b)\sinh(2x)((3a^2+10ab+7b^2)\cosh(2x)+3a^2+2ab-7b^2)}{((a+b)\cosh(2x)+a-b)^2} - \frac{3\sqrt{2}a\coth(x)\left(a^2+3ab+2b^2\right)F\left(\frac{\operatorname{arcsin}\left(\frac{a-b+(a+b)\cosh(2x)}{b}\right)}{\sqrt{2}}\right)}{b\sqrt{2}} \right)}{3\sqrt{2}b^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^6/(a + b*Tanh[x]^2)^(5/2), x]
```

```
[Out] (Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2]*((-3*Sqrt[2]*a*Coth[x]*((a^2 + 3*a*b + 2*b^2)*EllipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] + b^2*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]))/(b*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]) + (a*(a + b)*(3*a^2 + 2*a*b - 7*b^2 + (3*a^2 + 10*a*b + 7*b^2)*Cosh[2*x])*Sinh[2*x])/(a - b + (a + b)*Cosh[2*x]^2))/(3*Sqrt[2]*b^2*(a + b)^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*tanh(x)^2)^(5/2), x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^6/(a+b*tanh(x)^2)^(5/2), x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep+1)]Evaluation time: 0.73Error: Bad Argument Type
```

maple [B] time = 0.10, size = 549, normalized size = 4.65

$$\frac{\tanh^3(x)}{3b(a+b(\tanh^2(x)))^{\frac{3}{2}}} + \frac{\tanh(x)}{b^2\sqrt{a+b(\tanh^2(x))}} - \frac{\ln\left(\sqrt{b}\tanh(x) + \sqrt{a+b(\tanh^2(x))}\right)}{b^{\frac{5}{2}}} + \frac{\tanh(x)}{3b(a+b(\tanh^2(x)))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)^6/(a+b*tanh(x)^2)^(5/2), x)
```

```
[Out] 1/3*tanh(x)^3/b/(a+b*tanh(x)^2)^(3/2)+1/b^2*tanh(x)/(a+b*tanh(x)^2)^(1/2)-1/b^(5/2)*ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2))+1/3*tanh(x)/b/(a+b*tanh(x)^2)^(3/2)-1/3/a/b*tanh(x)/(a+b*tanh(x)^2)^(1/2)-1/3*tanh(x)/a/(a+b*tanh(x)^2)^(3/2)-2/3/a^2*tanh(x)/(a+b*tanh(x)^2)^(1/2)-1/6/(a+b)/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(3/2)+1/6*b/(a+b)/a/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+
```

$$\begin{aligned}
 & (a+b)^{3/2} \tanh(x) + 1/3 * b / (a+b) / a^{1/2} / ((\tanh(x)-1)^{2*b+2} * (\tanh(x)-1) * b + a + b)^{(1/2)} * \tanh(x) - 1/2 / (a+b)^{1/2} / ((\tanh(x)-1)^{2*b+2} * (\tanh(x)-1) * b + a + b)^{(1/2)} + 1/2 / (a+b)^{1/2} / a / ((\tanh(x)-1)^{2*b+2} * (\tanh(x)-1) * b + a + b)^{(1/2)} * b * \tanh(x) + 1/2 / (a+b)^{(5/2)} * \ln((2*a+2*b+2 * (\tanh(x)-1) * b + 2 * (a+b)^{(1/2)} * ((\tanh(x)-1)^{2*b+2} * (\tanh(x)-1) * b + a + b)^{(1/2))} / (\tanh(x)-1)) + 1/6 / (a+b) / ((1+\tanh(x))^{2*b-2} * (1+\tanh(x)) * b + a + b)^{(3/2)} + 1/6 * b / (a+b) / a / ((1+\tanh(x))^{2*b-2} * (1+\tanh(x)) * b + a + b)^{(3/2)} * \tanh(x) + 1/3 * b / (a+b) / a^{1/2} / ((1+\tanh(x))^{2*b-2} * (1+\tanh(x)) * b + a + b)^{(1/2)} * \tanh(x) + 1/2 / (a+b)^{1/2} / ((1+\tanh(x))^{2*b-2} * (1+\tanh(x)) * b + a + b)^{(1/2)} + 1/2 / (a+b)^{1/2} / a / ((1+\tanh(x))^{2*b-2} * (1+\tanh(x)) * b + a + b)^{(1/2)} * b * \tanh(x) - 1/2 / (a+b)^{(5/2)} * \ln((2*a+2*b-2 * (1+\tanh(x)) * b + 2 * (a+b)^{(1/2)} * ((1+\tanh(x))^{2*b-2} * (1+\tanh(x)) * b + a + b)^{(1/2))} / (1+\tanh(x)))
 \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^6}{(b \tanh(x)^2 + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^6/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(tanh(x)^6/(b*tanh(x)^2 + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^6}{(b \tanh(x)^2 + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^6/(a + b*tanh(x)^2)^(5/2),x)

[Out] int(tanh(x)^6/(a + b*tanh(x)^2)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^6(x)}{(a + b \tanh^2(x))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)**6/(a+b*tanh(x)**2)**(5/2),x)

[Out] Integral(tanh(x)**6/(a + b*tanh(x)**2)**(5/2), x)

$$3.247 \quad \int \frac{\tanh^5(x)}{(a+b \tanh^2(x))^{5/2}} dx$$

Optimal. Leaf size=84

$$-\frac{a^2}{3b^2(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{a(a+2b)}{b^2(a+b)^2 \sqrt{a+b \tanh^2(x)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}$$

[Out] $\operatorname{arctanh}\left(\frac{(a+b \tanh(x)^2)^{1/2}}{(a+b)^{1/2}}\right) / (a+b)^{5/2} + a(a+2b) / b^2 (a+b)^{3/2} - 1/3 a^2 / b^2 (a+b) / (a+b \tanh(x)^2)^{3/2}$

Rubi [A] time = 0.18, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {3670, 446, 87, 63, 208}

$$-\frac{a^2}{3b^2(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{a(a+2b)}{b^2(a+b)^2 \sqrt{a+b \tanh^2(x)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Tanh}[x]^5 / (a + b \cdot \text{Tanh}[x]^2)^{5/2}, x]$

[Out] $\text{ArcTanh}[\text{Sqrt}[a + b \cdot \text{Tanh}[x]^2] / \text{Sqrt}[a + b]] / (a + b)^{5/2} - a^2 / (3 \cdot b^2 \cdot (a + b) \cdot (a + b \cdot \text{Tanh}[x]^2)^{3/2}) + (a \cdot (a + 2 \cdot b)) / (b^2 \cdot (a + b)^2 \cdot \text{Sqrt}[a + b \cdot \text{Tanh}[x]^2])$

Rule 63

$\text{Int}[(a_. + (b_.)(x_.))^{(m_.)}((c_.) + (d_.)(x_.))^{(n_.)}, x_Symbol] \rightarrow \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p(m+1)-1)}(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 87

$\text{Int}[(c_. + (d_.)(x_.))^{(n_.)}((e_.) + (f_.)(x_.))^{(p_.)} / ((a_.) + (b_.)(x_.)), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(e + f*x)^{\text{FractionalPart}[p]}, ((c + d*x)^n * (e + f*x)^{\text{IntegerPart}[p]}) / (a + b*x), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1] \&\& \text{FractionQ}[p]$

Rule 208

$\text{Int}[(a_. + (b_.)(x_.)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2] * \text{ArcTanh}[x / \text{Rt}[-(a/b), 2]]) / a, x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b]$

Rule 446

$\text{Int}[(x_.)^{(m_.)}((a_.) + (b_.)(x_.)^{(n_.)})^{(p_.)}((c_.) + (d_.)(x_.)^{(n_.)})^{(q_.)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n] - 1)}(a + b*x)^p * (c + d*x)^q, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, m, n, p, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IntegerQ}[\text{Simplify}[(m+1)/n]]$

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_.)])^(m_.)*((a_.) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_.)])^(n_.))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^(m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^5(x)}{(a + b \tanh^2(x))^{5/2}} dx &= \text{Subst} \left(\int \frac{x^5}{(1-x^2)(a+bx^2)^{5/2}} dx, x, \tanh(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{(1-x)(a+bx)^{5/2}} dx, x, \tanh^2(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \left(\frac{a^2}{b(a+b)(a+bx)^{5/2}} - \frac{a(a+2b)}{b(a+b)^2(a+bx)^{3/2}} - \frac{1}{(a+b)^2(-1+x)\sqrt{a+bx}} \right) dx, x, \tanh^2(x) \right) \\
&= -\frac{a^2}{3b^2(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{a(a+2b)}{b^2(a+b)^2\sqrt{a+b \tanh^2(x)}} - \frac{\text{Subst} \left(\int \frac{1}{(-1+x)\sqrt{a+bx}} dx, x, \tanh^2(x) \right)}{2(a+b)} \\
&= -\frac{a^2}{3b^2(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{a(a+2b)}{b^2(a+b)^2\sqrt{a+b \tanh^2(x)}} - \frac{\text{Subst} \left(\int \frac{1}{-1-\frac{a}{b}+\frac{x^2}{b}} dx, x, \tanh^2(x) \right)}{b(a+b)} \\
&= \frac{\tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}} \right)}{(a+b)^{5/2}} - \frac{a^2}{3b^2(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{a(a+2b)}{b^2(a+b)^2\sqrt{a+b \tanh^2(x)}}
\end{aligned}$$

Mathematica [C] time = 0.12, size = 68, normalized size = 0.81

$$\frac{(a+b)(2a+3b \tanh^2(x)+b) - b^2 {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; \frac{b \tanh^2(x)+a}{a+b}\right)}{3b^2(a+b)(a+b \tanh^2(x))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^5/(a + b*Tanh[x]^2)^(5/2), x]

[Out] $(-b^2 \text{Hypergeometric2F1}[-3/2, 1, -1/2, (a + b \text{Tanh}[x]^2)/(a + b)]) + (a + b)(2a + b + 3b \text{Tanh}[x]^2)/(3b^2(a + b)(a + b \text{Tanh}[x]^2)^{3/2})$

fricas [B] time = 1.58, size = 7033, normalized size = 83.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(5/2), x, algorithm="fricas")

[Out] $[1/12*(3*((a^2*b^2 + 2*a*b^3 + b^4)*\cosh(x)^8 + 8*(a^2*b^2 + 2*a*b^3 + b^4)*\cosh(x)*\sinh(x)^7 + (a^2*b^2 + 2*a*b^3 + b^4)*\sinh(x)^8 + 4*(a^2*b^2 - b^4)*\cosh(x)^6 + 4*(a^2*b^2 - b^4 + 7*(a^2*b^2 + 2*a*b^3 + b^4)*\cosh(x)^2)*\sinh(x)^6 + 8*(7*(a^2*b^2 + 2*a*b^3 + b^4)*\cosh(x)^3 + 3*(a^2*b^2 - b^4)*\cosh(x))*\sinh(x)^5 + 2*(3*a^2*b^2 - 2*a*b^3 + 3*b^4)*\cosh(x)^4 + 2*(35*(a^2*b^2$

$$\begin{aligned}
& + 2*a*b^3 + b^4)*\cosh(x)^4 + 3*a^2*b^2 - 2*a*b^3 + 3*b^4 + 30*(a^2*b^2 - b^4) \\
& * \cosh(x)^2 * \sinh(x)^4 + a^2*b^2 + 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 + 2*a*b^3 \\
& + b^4)*\cosh(x)^5 + 10*(a^2*b^2 - b^4)*\cosh(x)^3 + (3*a^2*b^2 - 2*a*b^3 + 3 \\
& * b^4)*\cosh(x))*\sinh(x)^3 + 4*(a^2*b^2 - b^4)*\cosh(x)^2 + 4*(7*(a^2*b^2 + 2* \\
& a*b^3 + b^4)*\cosh(x)^6 + 15*(a^2*b^2 - b^4)*\cosh(x)^4 + a^2*b^2 - b^4 + 3*(\\
& 3*a^2*b^2 - 2*a*b^3 + 3*b^4)*\cosh(x)^2)*\sinh(x)^2 + 8*((a^2*b^2 + 2*a*b^3 + \\
& b^4)*\cosh(x)^7 + 3*(a^2*b^2 - b^4)*\cosh(x)^5 + (3*a^2*b^2 - 2*a*b^3 + 3*b^4) \\
& *\cosh(x)^3 + (a^2*b^2 - b^4)*\cosh(x))*\sinh(x))*\sqrt{a+b}*\log(((a^3 + a^2*b) \\
& *\cosh(x)^8 + 8*(a^3 + a^2*b)*\cosh(x))*\sinh(x)^7 + (a^3 + a^2*b)*\sinh(x)^8 \\
& + 2*(2*a^3 + a^2*b)*\cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*\cosh(x) \\
& ^2)*\sinh(x)^6 + 4*(14*(a^3 + a^2*b)*\cosh(x)^3 + 3*(2*a^3 + a^2*b)*\cosh(x) \\
&)*\sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x)^4 + (70*(a^3 + a^2*b) \\
& *\cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*\cosh(x)^2)* \\
& \sinh(x)^4 + 4*(14*(a^3 + a^2*b)*\cosh(x)^5 + 10*(2*a^3 + a^2*b)*\cosh(x)^3 + \\
& (6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x))*\sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 \\
& + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*\cosh(x)^2 + 2*(14*(a^3 + a^2*b)*\cosh(x) \\
& ^6 + 15*(2*a^3 + a^2*b)*\cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2 \\
& * b - a*b^2 + b^3)*\cosh(x)^2)*\sinh(x)^2 + \sqrt{2}*(a^2*\cosh(x)^6 + 6*a^2*\cosh(x) \\
& *\sinh(x)^5 + a^2*\sinh(x)^6 + 3*a^2*\cosh(x)^4 + 3*(5*a^2*\cosh(x)^2 + a^2) \\
& *\sinh(x)^4 + 4*(5*a^2*\cosh(x)^3 + 3*a^2*\cosh(x))*\sinh(x)^3 + (3*a^2 + 2*a \\
& * b - b^2)*\cosh(x)^2 + (15*a^2*\cosh(x)^4 + 18*a^2*\cosh(x)^2 + 3*a^2 + 2*a*b \\
& - b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*\cosh(x)^5 + 6*a^2*\cosh(x)^3 \\
& + (3*a^2 + 2*a*b - b^2)*\cosh(x))*\sinh(x))*\sqrt{a+b}*\sqrt{((a+b)*\cosh(x) \\
&)^2 + (a+b)*\sinh(x)^2 + a-b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2) \\
&)} + 4*(2*(a^3 + a^2*b)*\cosh(x)^7 + 3*(2*a^3 + a^2*b)*\cosh(x)^5 + (6*a^3 + \\
& 4*a^2*b - a*b^2 + b^3)*\cosh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*\cosh(x))*\sinh(x) \\
&)/(\cosh(x)^6 + 6*\cosh(x)^5*\sinh(x) + 15*\cosh(x)^4*\sinh(x)^2 + 20*\cosh(x)^3* \\
& \sinh(x)^3 + 15*\cosh(x)^2*\sinh(x)^4 + 6*\cosh(x)*\sinh(x)^5 + \sinh(x)^6)) + 3* \\
& ((a^2*b^2 + 2*a*b^3 + b^4)*\cosh(x)^8 + 8*(a^2*b^2 + 2*a*b^3 + b^4)*\cosh(x)* \\
& \sinh(x)^7 + (a^2*b^2 + 2*a*b^3 + b^4)*\sinh(x)^8 + 4*(a^2*b^2 - b^4)*\cosh(x) \\
& ^6 + 4*(a^2*b^2 - b^4 + 7*(a^2*b^2 + 2*a*b^3 + b^4)*\cosh(x)^2)*\sinh(x)^6 + \\
& 8*(7*(a^2*b^2 + 2*a*b^3 + b^4)*\cosh(x)^3 + 3*(a^2*b^2 - b^4)*\cosh(x))*\sinh(x) \\
& ^5 + 2*(3*a^2*b^2 - 2*a*b^3 + 3*b^4)*\cosh(x)^4 + 2*(35*(a^2*b^2 + 2*a*b^3 \\
& + b^4)*\cosh(x)^4 + 3*a^2*b^2 - 2*a*b^3 + 3*b^4 + 30*(a^2*b^2 - b^4)*\cosh(x) \\
& ^2)*\sinh(x)^4 + a^2*b^2 + 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 + 2*a*b^3 + b^4)*\cosh(x) \\
& ^5 + 10*(a^2*b^2 - b^4)*\cosh(x)^3 + (3*a^2*b^2 - 2*a*b^3 + 3*b^4)*\cosh(x) \\
& *\sinh(x)^3 + 4*(a^2*b^2 - b^4)*\cosh(x)^2 + 4*(7*(a^2*b^2 + 2*a*b^3 + b^4) \\
& *\cosh(x)^6 + 15*(a^2*b^2 - b^4)*\cosh(x)^4 + a^2*b^2 - b^4 + 3*(3*a^2*b^2 \\
& - 2*a*b^3 + 3*b^4)*\cosh(x)^2)*\sinh(x)^2 + 8*((a^2*b^2 + 2*a*b^3 + b^4)*\cosh(x) \\
& ^7 + 3*(a^2*b^2 - b^4)*\cosh(x)^5 + (3*a^2*b^2 - 2*a*b^3 + 3*b^4)*\cosh(x) \\
& ^3 + (a^2*b^2 - b^4)*\cosh(x))*\sinh(x))*\sqrt{a+b}*\log(-((a+b)*\cosh(x)^4 \\
& + 4*(a+b)*\cosh(x)*\sinh(x)^3 + (a+b)*\sinh(x)^4 - 2*b*\cosh(x)^2 + 2*(3*(\\
& a+b)*\cosh(x)^2 - b)*\sinh(x)^2 + \sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \\
& \sinh(x)^2 - 1)*\sqrt{a+b}*\sqrt{((a+b)*\cosh(x)^2 + (a+b)*\sinh(x)^2 + a \\
& - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)} + 4*((a+b)*\cosh(x)^3 - \\
& b*\cosh(x))*\sinh(x) + a+b)/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) + \\
& 8*\sqrt{2}*((a^4 + 5*a^3*b + 7*a^2*b^2 + 3*a*b^3)*\cosh(x)^6 + 6*(a^4 + 5*a^3 \\
& * b + 7*a^2*b^2 + 3*a*b^3)*\cosh(x))*\sinh(x)^5 + (a^4 + 5*a^3*b + 7*a^2*b^2 + \\
& 3*a*b^3)*\sinh(x)^6 + 3*(a^4 + 3*a^3*b + a^2*b^2 - a*b^3)*\cosh(x)^4 + 3*(a^4 \\
& + 3*a^3*b + a^2*b^2 - a*b^3 + 5*(a^4 + 5*a^3*b + 7*a^2*b^2 + 3*a*b^3)*\cosh(x) \\
& ^2)*\sinh(x)^4 + a^4 + 5*a^3*b + 7*a^2*b^2 + 3*a*b^3 + 4*(5*(a^4 + 5*a^3* \\
& b + 7*a^2*b^2 + 3*a*b^3)*\cosh(x)^3 + 3*(a^4 + 3*a^3*b + a^2*b^2 - a*b^3)*\cosh(x) \\
& *\sinh(x)^3 + 3*(a^4 + 3*a^3*b + a^2*b^2 - a*b^3)*\cosh(x)^2 + 3*(5*(a^4 \\
& + 5*a^3*b + 7*a^2*b^2 + 3*a*b^3)*\cosh(x)^4 + a^4 + 3*a^3*b + a^2*b^2 - a \\
& * b^3 + 6*(a^4 + 3*a^3*b + a^2*b^2 - a*b^3)*\cosh(x)^2)*\sinh(x)^2 + 6*((a^4 + \\
& 5*a^3*b + 7*a^2*b^2 + 3*a*b^3)*\cosh(x)^5 + 2*(a^4 + 3*a^3*b + a^2*b^2 - a*b \\
& ^3)*\cosh(x)^3 + (a^4 + 3*a^3*b + a^2*b^2 - a*b^3)*\cosh(x))*\sinh(x))*\sqrt{((\\
& a+b)*\cosh(x)^2 + (a+b)*\sinh(x)^2 + a-b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) \\
&) + \sinh(x)^2)}}/((a^5*b^2 + 5*a^4*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6
\end{aligned}$$

$$\begin{aligned}
& + b^7) * \cosh(x)^8 + 8 * (a^5 * b^2 + 5 * a^4 * b^3 + 10 * a^3 * b^4 + 10 * a^2 * b^5 + 5 * a * b^6 + b^7) * \cosh(x) * \sinh(x)^7 + (a^5 * b^2 + 5 * a^4 * b^3 + 10 * a^3 * b^4 + 10 * a^2 * b^5 + 5 * a * b^6 + b^7) * \sinh(x)^8 + a^5 * b^2 + 5 * a^4 * b^3 + 10 * a^3 * b^4 + 10 * a^2 * b^5 + 5 * a * b^6 + b^7 + 4 * (a^5 * b^2 + 3 * a^4 * b^3 + 2 * a^3 * b^4 - 2 * a^2 * b^5 - 3 * a * b^6 - b^7) * \cosh(x)^6 + 4 * (a^5 * b^2 + 3 * a^4 * b^3 + 2 * a^3 * b^4 - 2 * a^2 * b^5 - 3 * a * b^6 - b^7) * \cosh(x)^2 * \sinh(x)^6 + 8 * (7 * (a^5 * b^2 + 5 * a^4 * b^3 + 10 * a^3 * b^4 + 10 * a^2 * b^5 + 5 * a * b^6 + b^7) * \cosh(x)^3 + 3 * (a^5 * b^2 + 3 * a^4 * b^3 + 2 * a^3 * b^4 - 2 * a^2 * b^5 - 3 * a * b^6 - b^7) * \cosh(x)) * \sinh(x)^5 + 2 * (3 * a^5 * b^2 + 7 * a^4 * b^3 + 6 * a^3 * b^4 + 6 * a^2 * b^5 + 7 * a * b^6 + 3 * b^7) * \cosh(x)^4 + 2 * (3 * a^5 * b^2 + 7 * a^4 * b^3 + 6 * a^3 * b^4 + 6 * a^2 * b^5 + 7 * a * b^6 + 3 * b^7 + 35 * (a^5 * b^2 + 5 * a^4 * b^3 + 10 * a^3 * b^4 + 10 * a^2 * b^5 + 5 * a * b^6 + b^7) * \cosh(x)^4 + 30 * (a^5 * b^2 + 3 * a^4 * b^3 + 2 * a^3 * b^4 - 2 * a^2 * b^5 - 3 * a * b^6 - b^7) * \cosh(x)^2) * \sinh(x)^4 + 8 * (7 * (a^5 * b^2 + 5 * a^4 * b^3 + 10 * a^3 * b^4 + 10 * a^2 * b^5 + 5 * a * b^6 + b^7) * \cosh(x)^5 + 10 * (a^5 * b^2 + 3 * a^4 * b^3 + 2 * a^3 * b^4 - 2 * a^2 * b^5 - 3 * a * b^6 - b^7) * \cosh(x)^3 + (3 * a^5 * b^2 + 7 * a^4 * b^3 + 6 * a^3 * b^4 + 6 * a^2 * b^5 + 7 * a * b^6 + 3 * b^7) * \cosh(x)) * \sinh(x)^3 + 4 * (a^5 * b^2 + 3 * a^4 * b^3 + 2 * a^3 * b^4 - 2 * a^2 * b^5 - 3 * a * b^6 - b^7) * \cosh(x)^2 + 4 * (a^5 * b^2 + 3 * a^4 * b^3 + 2 * a^3 * b^4 - 2 * a^2 * b^5 - 3 * a * b^6 - b^7 + 7 * (a^5 * b^2 + 5 * a^4 * b^3 + 10 * a^3 * b^4 + 10 * a^2 * b^5 + 5 * a * b^6 + b^7) * \cosh(x)^6 + 15 * (a^5 * b^2 + 3 * a^4 * b^3 + 2 * a^3 * b^4 - 2 * a^2 * b^5 - 3 * a * b^6 - b^7) * \cosh(x)^4 + 3 * (3 * a^5 * b^2 + 7 * a^4 * b^3 + 6 * a^3 * b^4 + 6 * a^2 * b^5 + 7 * a * b^6 + 3 * b^7) * \cosh(x)^2) * \sinh(x)^2 + 8 * ((a^5 * b^2 + 5 * a^4 * b^3 + 10 * a^3 * b^4 + 10 * a^2 * b^5 + 5 * a * b^6 + b^7) * \cosh(x)^7 + 3 * (a^5 * b^2 + 3 * a^4 * b^3 + 2 * a^3 * b^4 - 2 * a^2 * b^5 - 3 * a * b^6 - b^7) * \cosh(x)^5 + (3 * a^5 * b^2 + 7 * a^4 * b^3 + 6 * a^3 * b^4 + 6 * a^2 * b^5 + 7 * a * b^6 + 3 * b^7) * \cosh(x)^3 + (a^5 * b^2 + 3 * a^4 * b^3 + 2 * a^3 * b^4 - 2 * a^2 * b^5 - 3 * a * b^6 - b^7) * \cosh(x)) * \sinh(x)), -1/6 * (3 * ((a^2 * b^2 + 2 * a * b^3 + b^4) * \cosh(x)^8 + 8 * (a^2 * b^2 + 2 * a * b^3 + b^4) * \cosh(x) * \sinh(x)^7 + (a^2 * b^2 + 2 * a * b^3 + b^4) * \sinh(x)^8 + 4 * (a^2 * b^2 - b^4) * \cosh(x)^6 + 4 * (a^2 * b^2 - b^4 + 7 * (a^2 * b^2 + 2 * a * b^3 + b^4) * \cosh(x)^2) * \sinh(x)^6 + 8 * (7 * (a^2 * b^2 + 2 * a * b^3 + b^4) * \cosh(x)^3 + 3 * (a^2 * b^2 - b^4) * \cosh(x)) * \sinh(x)^5 + 2 * (3 * a^2 * b^2 - 2 * a * b^3 + 3 * b^4) * \cosh(x)^4 + 2 * (35 * (a^2 * b^2 + 2 * a * b^3 + b^4) * \cosh(x)^4 + 3 * a^2 * b^2 - 2 * a * b^3 + 3 * b^4 + 30 * (a^2 * b^2 - b^4) * \cosh(x)^2) * \sinh(x)^4 + a^2 * b^2 + 2 * a * b^3 + b^4 + 8 * (7 * (a^2 * b^2 + 2 * a * b^3 + b^4) * \cosh(x)^5 + 10 * (a^2 * b^2 - b^4) * \cosh(x)^3 + (3 * a^2 * b^2 - 2 * a * b^3 + 3 * b^4) * \cosh(x)) * \sinh(x)^3 + 4 * (a^2 * b^2 - b^4) * \cosh(x)^2 + 4 * (7 * (a^2 * b^2 + 2 * a * b^3 + b^4) * \cosh(x)^6 + 15 * (a^2 * b^2 - b^4) * \cosh(x)^4 + a^2 * b^2 - b^4 + 3 * (3 * a^2 * b^2 - 2 * a * b^3 + 3 * b^4) * \cosh(x)^2) * \sinh(x)^2 + 8 * ((a^2 * b^2 + 2 * a * b^3 + b^4) * \cosh(x)^7 + 3 * (a^2 * b^2 - b^4) * \cosh(x)^5 + (3 * a^2 * b^2 - 2 * a * b^3 + 3 * b^4) * \cosh(x)^3 + (a^2 * b^2 - b^4) * \cosh(x)) * \sinh(x)) * \sqrt{-a - b} * \arctan(\sqrt{2} * (a * \cosh(x)^2 + 2 * a * \cosh(x) * \sinh(x) + a * \sinh(x)^2 + a + b) * \sqrt{-a - b}) * \sqrt{((a + b) * \cosh(x)^2 + (a + b) * \sinh(x)^2 + a - b) / ((\cosh(x)^2 - 2 * \cosh(x) * \sinh(x) + \sinh(x)^2)) / ((a^2 + a * b) * \cosh(x)^4 + 4 * (a^2 + a * b) * \cosh(x) * \sinh(x)^3 + (a^2 + a * b) * \sinh(x)^4 + (2 * a^2 + a * b - b^2) * \cosh(x)^2 + (6 * (a^2 + a * b) * \cosh(x)^2 + 2 * a^2 + a * b - b^2) * \sinh(x)^2 + a^2 + 2 * a * b + b^2 + 2 * (2 * (a^2 + a * b) * \cosh(x)^3 + (2 * a^2 + a * b - b^2) * \cosh(x)) * \sinh(x))) + 3 * ((a^2 * b^2 + 2 * a * b^3 + b^4) * \cosh(x)^8 + 8 * (a^2 * b^2 + 2 * a * b^3 + b^4) * \cosh(x) * \sinh(x)^7 + (a^2 * b^2 + 2 * a * b^3 + b^4) * \sinh(x)^8 + 4 * (a^2 * b^2 - b^4) * \cosh(x)^6 + 4 * (a^2 * b^2 - b^4 + 7 * (a^2 * b^2 + 2 * a * b^3 + b^4) * \cosh(x)^2) * \sinh(x)^6 + 8 * (7 * (a^2 * b^2 + 2 * a * b^3 + b^4) * \cosh(x)^3 + 3 * (a^2 * b^2 - b^4) * \cosh(x)) * \sinh(x)^5 + 2 * (3 * a^2 * b^2 - 2 * a * b^3 + 3 * b^4) * \cosh(x)^4 + 2 * (35 * (a^2 * b^2 + 2 * a * b^3 + b^4) * \cosh(x)^4 + 3 * a^2 * b^2 - 2 * a * b^3 + 3 * b^4 + 30 * (a^2 * b^2 - b^4) * \cosh(x)^2) * \sinh(x)^4 + a^2 * b^2 + 2 * a * b^3 + b^4 + 8 * (7 * (a^2 * b^2 + 2 * a * b^3 + b^4) * \cosh(x)^5 + 10 * (a^2 * b^2 - b^4) * \cosh(x)^3 + (3 * a^2 * b^2 - 2 * a * b^3 + 3 * b^4) * \cosh(x)) * \sinh(x)^3 + 4 * (a^2 * b^2 - b^4) * \cosh(x)^2 + 4 * (7 * (a^2 * b^2 + 2 * a * b^3 + b^4) * \cosh(x)^6 + 15 * (a^2 * b^2 - b^4) * \cosh(x)^4 + a^2 * b^2 - b^4 + 3 * (3 * a^2 * b^2 - 2 * a * b^3 + 3 * b^4) * \cosh(x)^2) * \sinh(x)^2 + 8 * ((a^2 * b^2 + 2 * a * b^3 + b^4) * \cosh(x)^7 + 3 * (a^2 * b^2 - b^4) * \cosh(x)^5 + (3 * a^2 * b^2 - 2 * a * b^3 + 3 * b^4) * \cosh(x)^3 + (a^2 * b^2 - b^4) * \cosh(x)) * \sinh(x)) * \sqrt{-a - b} * \arctan(\sqrt{2} * (\cosh(x)^2 + 2 * \cosh(x) * \sinh(x) + \sinh(x)^2 - 1) * \sqrt{-a - b}) * \sqrt{((a + b) * \cosh(x)^2 + (a + b) * \sinh(x)^2 + a - b) / ((\cosh(x)^2 - 2 * \cosh(x) * \sinh(x)
\end{aligned}$$

$$\begin{aligned}
& + \sinh(x)^2) / ((a + b) \cosh(x)^4 + 4(a + b) \cosh(x) \sinh(x)^3 + (a + b) \sinh(x)^4 + 2(a - b) \cosh(x)^2 + 2(3(a + b) \cosh(x)^2 + a - b) \sinh(x)^2 \\
& + 4((a + b) \cosh(x)^3 + (a - b) \cosh(x)) \sinh(x) + a + b) - 4\sqrt{2}((a^4 + 5a^3b + 7a^2b^2 + 3ab^3) \cosh(x)^6 + 6(a^4 + 5a^3b + 7a^2b^2 + 3ab^3) \cosh(x) \sinh(x)^5 + (a^4 + 5a^3b + 7a^2b^2 + 3ab^3) \sinh(x)^6 + 3(a^4 + 3a^3b + a^2b^2 - ab^3) \cosh(x)^4 + 3(a^4 + 3a^3b + a^2b^2 - ab^3 + 5(a^4 + 5a^3b + 7a^2b^2 + 3ab^3) \cosh(x)^2) \sinh(x)^4 + a^4 + 5a^3b + 7a^2b^2 + 3ab^3 + 4(5(a^4 + 5a^3b + 7a^2b^2 + 3ab^3) \cosh(x)^3 + 3(a^4 + 3a^3b + a^2b^2 - ab^3) \cosh(x)) \sinh(x)^3 + 3(a^4 + 3a^3b + a^2b^2 - ab^3) \cosh(x)^2 + 3(5(a^4 + 5a^3b + 7a^2b^2 + 3ab^3) \cosh(x)^4 + a^4 + 3a^3b + a^2b^2 - ab^3 + 6(a^4 + 3a^3b + a^2b^2 - ab^3) \cosh(x)^2) \sinh(x)^2 + 6((a^4 + 5a^3b + 7a^2b^2 + 3ab^3) \cosh(x)^5 + 2(a^4 + 3a^3b + a^2b^2 - ab^3) \cosh(x)^3 + (a^4 + 3a^3b + a^2b^2 - ab^3) \cosh(x)) \sinh(x)) \sqrt{((a + b) \cosh(x)^2 + (a + b) \sinh(x)^2 + a - b) / (\cosh(x)^2 - 2\cosh(x) \sinh(x) + \sinh(x)^2)}} / ((a^5b^2 + 5a^4b^3 + 10a^3b^4 + 10a^2b^5 + 5ab^6 + b^7) \cosh(x)^8 + 8(a^5b^2 + 5a^4b^3 + 10a^3b^4 + 10a^2b^5 + 5ab^6 + b^7) \cosh(x) \sinh(x)^7 + (a^5b^2 + 5a^4b^3 + 10a^3b^4 + 10a^2b^5 + 5ab^6 + b^7) \sinh(x)^8 + a^5b^2 + 5a^4b^3 + 10a^3b^4 + 10a^2b^5 + 5ab^6 + b^7 + 4(a^5b^2 + 3a^4b^3 + 2a^3b^4 - 2a^2b^5 - 3ab^6 - b^7) \cosh(x)^6 + 4(a^5b^2 + 3a^4b^3 + 2a^3b^4 - 2a^2b^5 - 3ab^6 - b^7 + 7(a^5b^2 + 5a^4b^3 + 10a^3b^4 + 10a^2b^5 + 5ab^6 + b^7) \cosh(x)^2) \sinh(x)^6 + 8(7(a^5b^2 + 5a^4b^3 + 10a^3b^4 + 10a^2b^5 + 5ab^6 + b^7) \cosh(x)^3 + 3(a^5b^2 + 3a^4b^3 + 2a^3b^4 - 2a^2b^5 - 3ab^6 - b^7) \cosh(x)) \sinh(x)^5 + 2(3a^5b^2 + 7a^4b^3 + 6a^3b^4 + 6a^2b^5 + 7ab^6 + 3b^7) \cosh(x)^4 + 2(3a^5b^2 + 7a^4b^3 + 6a^3b^4 + 6a^2b^5 + 7ab^6 + 3b^7 + 35(a^5b^2 + 5a^4b^3 + 10a^3b^4 + 10a^2b^5 + 5ab^6 + b^7) \cosh(x)^4 + 30(a^5b^2 + 3a^4b^3 + 2a^3b^4 - 2a^2b^5 - 3ab^6 - b^7) \cosh(x)^2) \sinh(x)^4 + 8(7(a^5b^2 + 5a^4b^3 + 10a^3b^4 + 10a^2b^5 + 5ab^6 + b^7) \cosh(x)^5 + 10(a^5b^2 + 3a^4b^3 + 2a^3b^4 - 2a^2b^5 - 3ab^6 - b^7) \cosh(x)^3 + (3a^5b^2 + 7a^4b^3 + 6a^3b^4 + 6a^2b^5 + 7ab^6 + 3b^7) \cosh(x)) \sinh(x)^3 + 4(a^5b^2 + 3a^4b^3 + 2a^3b^4 - 2a^2b^5 - 3ab^6 - b^7) \cosh(x)^2 + 4(a^5b^2 + 3a^4b^3 + 2a^3b^4 - 2a^2b^5 - 3ab^6 - b^7 + 7(a^5b^2 + 5a^4b^3 + 10a^3b^4 + 10a^2b^5 + 5ab^6 + b^7) \cosh(x)^6 + 15(a^5b^2 + 3a^4b^3 + 2a^3b^4 - 2a^2b^5 - 3ab^6 - b^7) \cosh(x)^4 + 3(3a^5b^2 + 7a^4b^3 + 6a^3b^4 + 6a^2b^5 + 7ab^6 + 3b^7) \cosh(x)^2) \sinh(x)^2 + 8((a^5b^2 + 5a^4b^3 + 10a^3b^4 + 10a^2b^5 + 5ab^6 + b^7) \cosh(x)^7 + 3(a^5b^2 + 3a^4b^3 + 2a^3b^4 - 2a^2b^5 - 3ab^6 - b^7) \cosh(x)^5 + (3a^5b^2 + 7a^4b^3 + 6a^3b^4 + 6a^2b^5 + 7ab^6 + 3b^7) \cosh(x)^3 + (a^5b^2 + 3a^4b^3 + 2a^3b^4 - 2a^2b^5 - 3ab^6 - b^7) \cosh(x)) \sinh(x)]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep+1)]Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Evaluation time: 0.79Error: Bad Argument Type

maple [B] time = 0.10, size = 469, normalized size = 5.58

$$\frac{\frac{\tanh^2(x)}{b(a+b(\tanh^2(x)))^{\frac{3}{2}}} + \frac{2a}{3b^2(a+b(\tanh^2(x)))^{\frac{3}{2}}} + \frac{1}{3b(a+b(\tanh^2(x)))^{\frac{3}{2}}} - \frac{1}{6(a+b)((\tanh(x)-1)^2 b + 2(\tanh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^5/(a+b*tanh(x)^2)^(5/2), x)

[Out] tanh(x)^2/b/(a+b*tanh(x)^2)^(3/2)+2/3*a/b^2/(a+b*tanh(x)^2)^(3/2)+1/3/b/(a+b*tanh(x)^2)^(3/2)-1/6/(a+b)/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(3/2)+1/6*b/(a+b)/a/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+1/2/(a+b)^2/a/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))-1/6/(a+b)/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(3/2)-1/6*b/(a+b)/a/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(3/2)*tanh(x)-1/3*b/(a+b)/a^2/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)-1/2/(a+b)^2/a/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^5}{(b \tanh(x)^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^5/(a+b*tanh(x)^2)^(5/2), x, algorithm="maxima")

[Out] integrate(tanh(x)^5/(b*tanh(x)^2 + a)^(5/2), x)

mupad [B] time = 4.01, size = 92, normalized size = 1.10

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b \tanh(x)^2 + a} (2a^2 + 4ab + 2b^2)}{2(a+b)^{5/2}}\right)}{(a+b)^{5/2}} - \frac{\frac{a^2}{3(a+b)} - \frac{(a^2 + 2ba)(b \tanh(x)^2 + a)}{(a+b)^2}}{b^2 (b \tanh(x)^2 + a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^5/(a + b*tanh(x)^2)^(5/2), x)

[Out] atanh(((a + b*tanh(x)^2)^(1/2)*(4*a*b + 2*a^2 + 2*b^2))/(2*(a + b)^(5/2)))/(a + b)^(5/2) - (a^2/(3*(a + b)) - ((2*a*b + a^2)*(a + b*tanh(x)^2))/(a + b)^2)/(b^2*(a + b*tanh(x)^2)^(3/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^5(x)}{(a + b \tanh^2(x))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)**5/(a+b*tanh(x)**2)**(5/2), x)

[Out] Integral(tanh(x)**5/(a + b*tanh(x)**2)**(5/2), x)

$$3.248 \quad \int \frac{\tanh^4(x)}{(a+b \tanh^2(x))^{5/2}} dx$$

Optimal. Leaf size=90

$$-\frac{(a+4b)\tanh(x)}{3b(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{a\tanh(x)}{3b(a+b)(a+b\tanh^2(x))^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}$$

[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(5/2)-1/3*(a+4*b)*tanh(x)/b/(a+b)^2/(a+b*tanh(x)^2)^(1/2)+1/3*a*tanh(x)/b/(a+b)/(a+b*tanh(x)^2)^(3/2)

Rubi [A] time = 0.14, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3670, 470, 527, 12, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} - \frac{(a+4b)\tanh(x)}{3b(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{a\tanh(x)}{3b(a+b)(a+b\tanh^2(x))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^4/(a + b*Tanh[x]^2)^(5/2), x]

[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(5/2) + (a*Tanh[x])/(3*b*(a + b)*(a + b*Tanh[x]^2)^(3/2)) - ((a + 4*b)*Tanh[x])/(3*b*(a + b)^2*Sqrt[a + b*Tanh[x]^2])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 470

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 527

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3670

```
Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*((c_)*tan[(e_) + (f_)*(x_)])^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))
```

Rubi steps

$$\int \frac{\tanh^4(x)}{(a + b \tanh^2(x))^{5/2}} dx = \text{Subst} \left(\int \frac{x^4}{(1-x^2)(a+bx^2)^{5/2}} dx, x, \tanh(x) \right)$$

$$= \frac{a \tanh(x)}{3b(a+b)(a+b \tanh^2(x))^{3/2}} - \frac{\text{Subst} \left(\int \frac{a+(-a-3b)x^2}{(1-x^2)(a+bx^2)^{3/2}} dx, x, \tanh(x) \right)}{3b(a+b)}$$

$$= \frac{a \tanh(x)}{3b(a+b)(a+b \tanh^2(x))^{3/2}} - \frac{(a+4b) \tanh(x)}{3b(a+b)^2 \sqrt{a+b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{3ab}{(1-x^2)\sqrt{a+bx^2}} dx, x, \tanh(x) \right)}{3ab(a+b)}$$

$$= \frac{a \tanh(x)}{3b(a+b)(a+b \tanh^2(x))^{3/2}} - \frac{(a+4b) \tanh(x)}{3b(a+b)^2 \sqrt{a+b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}} dx, x, \tanh(x) \right)}{(a+b)^2}$$

$$= \frac{a \tanh(x)}{3b(a+b)(a+b \tanh^2(x))^{3/2}} - \frac{(a+4b) \tanh(x)}{3b(a+b)^2 \sqrt{a+b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{1}{1-(a+b)x^2} dx, x, \tanh(x) \right)}{(a+b)^2}$$

$$= \frac{\tanh^{-1} \left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right)}{(a+b)^{5/2}} + \frac{a \tanh(x)}{3b(a+b)(a+b \tanh^2(x))^{3/2}} - \frac{(a+4b) \tanh(x)}{3b(a+b)^2 \sqrt{a+b \tanh^2(x)}}$$

Mathematica [A] time = 3.13, size = 132, normalized size = 1.47

$$\frac{\tanh^3(x) \left(3 \tanh^{-1} \left(\frac{\sqrt{\frac{(a+b) \tanh^2(x)}{a}}}{\sqrt{\frac{b \tanh^2(x)}{a} + 1}} \right) \sqrt{\frac{(a+b) \tanh^2(x)}{a}} (a \coth^2(x) + b)^2 - (a+b) \sqrt{\frac{b \tanh^2(x)}{a}} + 1 (3a \coth^2(x) + a + 4b) \right)}{3(a+b)^3 (a+b \tanh^2(x))^{3/2} \sqrt{\frac{b \tanh^2(x)}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]^4/(a + b*Tanh[x]^2)^(5/2),x]

[Out] (Tanh[x]^3*(3*ArcTanh[Sqrt[((a + b)*Tanh[x]^2)/a]/Sqrt[1 + (b*Tanh[x]^2)/a]]*(b + a*Coth[x]^2)^2*Sqrt[((a + b)*Tanh[x]^2)/a] - (a + b)*(a + 4*b + 3*a*Coth[x]^2)*Sqrt[1 + (b*Tanh[x]^2)/a]))/(3*(a + b)^3*(a + b*Tanh[x]^2)^(3/2)*Sqrt[1 + (b*Tanh[x]^2)/a])

fricas [B] time = 1.80, size = 5719, normalized size = 63.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")

[Out] [1/12*(3*((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(x)^2 + 3*a^2 - 2*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + (3*a^2 - 2*a*b + 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2 - b^2)*cosh(x)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)^5 + (3*a^2 - 2*a*b + 3*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 3*((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(x)^2 + 3*a^2 - 2*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + (3*a^2 - 2*a*b + 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2 - b^2)*cosh(x)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)^5 + (3*a^2 - 2*a*b + 3*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*log(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*a*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a)*sinh(x)^2 + sqrt(2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b))*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x)

$$\begin{aligned}
& + \sinh(x)^2)) + 4*((a + b)*\cosh(x)^3 + a*\cosh(x))*\sinh(x) + a + b)/(\cosh(x) \\
&)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2)) - 16*\sqrt{2}*((a^2 + 2*a*b + b^2)*\cos \\
& h(x)^6 + 6*(a^2 + 2*a*b + b^2)*\cosh(x)*\sinh(x)^5 + (a^2 + 2*a*b + b^2)*\sinh \\
& (x)^6 - 3*(a*b + b^2)*\cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*\cosh(x)^2 - a*b \\
& - b^2)*\sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*\cosh(x)^3 - 3*(a*b + b^2)*\cosh(x) \\
&)*\sinh(x)^3 + 3*(a*b + b^2)*\cosh(x)^2 + 3*(5*(a^2 + 2*a*b + b^2)*\cosh(x)^4 \\
& - 6*(a*b + b^2)*\cosh(x)^2 + a*b + b^2)*\sinh(x)^2 - a^2 - 2*a*b - b^2 + 6* \\
& ((a^2 + 2*a*b + b^2)*\cosh(x)^5 - 2*(a*b + b^2)*\cosh(x)^3 + (a*b + b^2)*\cosh \\
& (x))*\sinh(x))*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x) \\
&)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2 \\
& *b^3 + 5*a*b^4 + b^5)*\cosh(x)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 \\
& + 5*a*b^4 + b^5)*\cosh(x)*\sinh(x)^7 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2 \\
& *b^3 + 5*a*b^4 + b^5)*\sinh(x)^8 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 \\
& - 3*a*b^4 - b^5)*\cosh(x)^6 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a \\
& *b^4 - b^5 + 7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*\co \\
& sh(x)^2)*\sinh(x)^6 + 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 \\
& + b^5)*\cosh(x)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b \\
& ^5)*\cosh(x))*\sinh(x)^5 + a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 \\
& + b^5 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*\cosh(x) \\
& ^4 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5 + 35*(a \\
& ^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*\cosh(x)^4 + 30*(a^5 \\
& + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*\cosh(x)^2)*\sinh(x)^4 + \\
& 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*\cosh(x)^5 + \\
& 10*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*\cosh(x)^3 + (3*a \\
& ^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*\cosh(x))*\sinh(x)^3 \\
& + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*\cosh(x)^2 + 4*(\\
& 7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*\cosh(x)^6 + a^5 \\
& + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 15*(a^5 + 3*a^4*b + 2* \\
& a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*\cosh(x)^4 + 3*(3*a^5 + 7*a^4*b + 6*a^3 \\
& *b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*\cosh(x)^2)*\sinh(x)^2 + 8*((a^5 + 5*a^4* \\
& b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*\cosh(x)^7 + 3*(a^5 + 3*a^4*b + \\
& 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*\cosh(x)^5 + (3*a^5 + 7*a^4*b + 6*a^ \\
& 3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*\cosh(x)^3 + (a^5 + 3*a^4*b + 2*a^3*b^2 \\
& - 2*a^2*b^3 - 3*a*b^4 - b^5)*\cosh(x))*\sinh(x)), -1/6*(3*((a^2 + 2*a*b + b^ \\
& 2)*\cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*\cosh(x)*\sinh(x)^7 + (a^2 + 2*a*b + b^2) \\
&)*\sinh(x)^8 + 4*(a^2 - b^2)*\cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2)*\cosh(x)^2 \\
& + a^2 - b^2)*\sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2)*\cosh(x)^3 + 3*(a^2 - b^2) \\
&)*\cosh(x))*\sinh(x)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*\cosh(x)^4 + 2*(35*(a^2 + 2* \\
& a*b + b^2)*\cosh(x)^4 + 30*(a^2 - b^2)*\cosh(x)^2 + 3*a^2 - 2*a*b + 3*b^2)*\si \\
& nh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*\cosh(x)^5 + 10*(a^2 - b^2)*\cosh(x)^3 + (\\
& 3*a^2 - 2*a*b + 3*b^2)*\cosh(x))*\sinh(x)^3 + 4*(a^2 - b^2)*\cosh(x)^2 + 4*(7* \\
& (a^2 + 2*a*b + b^2)*\cosh(x)^6 + 15*(a^2 - b^2)*\cosh(x)^4 + 3*(3*a^2 - 2*a*b \\
& + 3*b^2)*\cosh(x)^2 + a^2 - b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + \\
& 2*a*b + b^2)*\cosh(x)^7 + 3*(a^2 - b^2)*\cosh(x)^5 + (3*a^2 - 2*a*b + 3*b^2)* \\
& \cosh(x)^3 + (a^2 - b^2)*\cosh(x))*\sinh(x))*\sqrt{-a - b}*\arctan(\sqrt{2}*(b*\co \\
& sh(x)^2 + 2*b*\cosh(x)*\sinh(x) + b*\sinh(x)^2 - a - b)*\sqrt{-a - b}*\sqrt{((a \\
& + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) \\
& + \sinh(x)^2)))/((a*b + b^2)*\cosh(x)^4 + 4*(a*b + b^2)*\cosh(x)*\sinh(x)^3 + (a \\
& *b + b^2)*\sinh(x)^4 + (a^2 - a*b - 2*b^2)*\cosh(x)^2 + (6*(a*b + b^2)*\cosh(x) \\
&)^2 + a^2 - a*b - 2*b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(2*(a*b + b^2)*\c \\
& osh(x)^3 + (a^2 - a*b - 2*b^2)*\cosh(x))*\sinh(x))) + 3*((a^2 + 2*a*b + b^2)* \\
& \cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*\cosh(x)*\sinh(x)^7 + (a^2 + 2*a*b + b^2)*\s \\
& inh(x)^8 + 4*(a^2 - b^2)*\cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2)*\cosh(x)^2 + a \\
& ^2 - b^2)*\sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2)*\cosh(x)^3 + 3*(a^2 - b^2)*\co \\
& sh(x))*\sinh(x)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*\cosh(x)^4 + 2*(35*(a^2 + 2*a*b \\
& + b^2)*\cosh(x)^4 + 30*(a^2 - b^2)*\cosh(x)^2 + 3*a^2 - 2*a*b + 3*b^2)*\sinh(\\
& x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*\cosh(x)^5 + 10*(a^2 - b^2)*\cosh(x)^3 + (3*a \\
& ^2 - 2*a*b + 3*b^2)*\cosh(x))*\sinh(x)^3 + 4*(a^2 - b^2)*\cosh(x)^2 + 4*(7*(a^ \\
& 2 + 2*a*b + b^2)*\cosh(x)^6 + 15*(a^2 - b^2)*\cosh(x)^4 + 3*(3*a^2 - 2*a*b +
\end{aligned}$$

$$\begin{aligned}
& 3b^2 \cosh(x)^2 + a^2 - b^2) \sinh(x)^2 + a^2 + 2ab + b^2 + 8((a^2 + 2ab + b^2) \cosh(x)^7 + 3(a^2 - b^2) \cosh(x)^5 + (3a^2 - 2ab + 3b^2) \cosh(x)^3 + (a^2 - b^2) \cosh(x) \sinh(x)) \sqrt{-a-b} \arctan(\sqrt{2} \sqrt{-a-b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a-b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / ((a+b) \cosh(x)^2 + 2(a+b) \cosh(x) \sinh(x) + (a+b) \sinh(x)^2 + a+b)) + 8\sqrt{2}((a^2 + 2ab + b^2) \cosh(x)^6 + 6(a^2 + 2ab + b^2) \cosh(x) \sinh(x)^5 + (a^2 + 2ab + b^2) \sinh(x)^6 - 3(ab + b^2) \cosh(x)^4 + 3(5(a^2 + 2ab + b^2) \cosh(x)^2 - ab - b^2) \sinh(x)^4 + 4(5(a^2 + 2ab + b^2) \cosh(x)^3 - 3(ab + b^2) \cosh(x)) \sinh(x)^3 + 3(ab + b^2) \cosh(x)^2 + 3(5(a^2 + 2ab + b^2) \cosh(x)^4 - 6(ab + b^2) \cosh(x)^2 + ab + b^2) \sinh(x)^2 - a^2 - 2ab - b^2 + 6((a^2 + 2ab + b^2) \cosh(x)^5 - 2(ab + b^2) \cosh(x)^3 + (ab + b^2) \cosh(x)) \sinh(x)) \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a-b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / ((a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5) \cosh(x)^8 + 8(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5) \cosh(x) \sinh(x)^7 + (a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5) \sinh(x)^8 + 4(a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5) \cosh(x)^6 + 4(a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5) + 7(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5) \cosh(x)^2) \sinh(x)^6 + 8(7(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5) \cosh(x)^3 + 3(a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5) \cosh(x)) \sinh(x)^5 + a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5 + 2(3a^5 + 7a^4b + 6a^3b^2 + 6a^2b^3 + 7ab^4 + 3b^5) \cosh(x)^4 + 2(3a^5 + 7a^4b + 6a^3b^2 + 6a^2b^3 + 7ab^4 + 3b^5 + 35(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5) \cosh(x)^4 + 30(a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5) \cosh(x)^2) \sinh(x)^4 + 8(7(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5) \cosh(x)^5 + 10(a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5) \cosh(x)^3 + (3a^5 + 7a^4b + 6a^3b^2 + 6a^2b^3 + 7ab^4 + 3b^5) \cosh(x)) \sinh(x)^3 + 4(a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5) \cosh(x)^2 + 4(7(a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5) \cosh(x)^6 + a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5 + 15(a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5) \cosh(x)^4 + 3(3a^5 + 7a^4b + 6a^3b^2 + 6a^2b^3 + 7ab^4 + 3b^5) \cosh(x)^2) \sinh(x)^2 + 8((a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5) \cosh(x)^7 + 3(a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5) \cosh(x)^5 + (3a^5 + 7a^4b + 6a^3b^2 + 6a^2b^3 + 7ab^4 + 3b^5) \cosh(x)^3 + (a^5 + 3a^4b + 2a^3b^2 - 2a^2b^3 - 3ab^4 - b^5) \cosh(x)) \sinh(x)]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep+1)]Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Evaluation time: 0.54Error: Bad Argument Type

maple [B] time = 0.10, size = 491, normalized size = 5.46

$$\frac{\tanh(x)}{3b(a+b(\tanh^2(x)))^{\frac{3}{2}}} - \frac{\tanh(x)}{3ab\sqrt{a+b(\tanh^2(x))}} - \frac{\tanh(x)}{3a(a+b(\tanh^2(x)))^{\frac{3}{2}}} - \frac{2\tanh(x)}{3a^2\sqrt{a+b(\tanh^2(x))}} - \frac{2\tanh(x)}{6(a+b)(\tanh^2(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x)`

[Out] $\frac{1}{3} \frac{\tanh(x)}{b} (a+b \tanh(x)^2)^{-3/2} - \frac{1}{3} \frac{a}{b} \frac{\tanh(x)}{(a+b \tanh(x)^2)^{1/2}} - \frac{1}{3} \frac{\tanh(x)}{a} (a+b \tanh(x)^2)^{-3/2} - \frac{2}{3} \frac{1}{a^2} \frac{\tanh(x)}{(a+b \tanh(x)^2)^{1/2}} - \frac{1}{6} \frac{1}{(a+b)} \frac{1}{((\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a+b)^{3/2}} + \frac{1}{6} \frac{b}{(a+b)} \frac{1}{a} \frac{1}{((\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a+b)^{3/2}} * \tanh(x) + \frac{1}{3} \frac{b}{(a+b)} \frac{1}{a^2} \frac{1}{((\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a+b)^{1/2}} * \tanh(x) - \frac{1}{2} \frac{1}{(a+b)^2} \frac{1}{((\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a+b)^{1/2}} + \frac{1}{2} \frac{1}{(a+b)^2} \frac{1}{a} \frac{1}{((\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a+b)^{1/2}} * b \tanh(x) + \frac{1}{2} \frac{1}{(a+b)^{5/2}} * \ln\left(\frac{(2a+2b+2(\tanh(x)-1)b+2(a+b)^{1/2}) * ((\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a+b)^{1/2})}{(\tanh(x)-1)} + \frac{1}{6} \frac{1}{(a+b)} \frac{1}{((1+\tanh(x))^2 b - 2(1+\tanh(x))b + a+b)^{3/2}} + \frac{1}{6} \frac{b}{(a+b)} \frac{1}{a} \frac{1}{((1+\tanh(x))^2 b - 2(1+\tanh(x))b + a+b)^{3/2}} * \tanh(x) + \frac{1}{3} \frac{b}{(a+b)} \frac{1}{a^2} \frac{1}{((1+\tanh(x))^2 b - 2(1+\tanh(x))b + a+b)^{1/2}} * \tanh(x) + \frac{1}{2} \frac{1}{(a+b)^2} \frac{1}{((1+\tanh(x))^2 b - 2(1+\tanh(x))b + a+b)^{1/2}} + \frac{1}{2} \frac{1}{(a+b)^2} \frac{1}{a} \frac{1}{((1+\tanh(x))^2 b - 2(1+\tanh(x))b + a+b)^{1/2}} * b \tanh(x) - \frac{1}{2} \frac{1}{(a+b)^{5/2}} * \ln\left(\frac{(2a+2b-2(1+\tanh(x))b+2(a+b)^{1/2}) * ((1+\tanh(x))^2 b - 2(1+\tanh(x))b + a+b)^{1/2})}{(1+\tanh(x))}\right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^4}{(b \tanh(x)^2 + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")`

[Out] `integrate(tanh(x)^4/(b*tanh(x)^2 + a)^(5/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^4}{(b \tanh(x)^2 + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)^4/(a + b*tanh(x)^2)^(5/2),x)`

[Out] `int(tanh(x)^4/(a + b*tanh(x)^2)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^4(x)}{(a + b \tanh^2(x))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)**4/(a+b*tanh(x)**2)**(5/2),x)`

[Out] `Integral(tanh(x)**4/(a + b*tanh(x)**2)**(5/2), x)`

$$3.249 \quad \int \frac{\tanh^3(x)}{(a+b \tanh^2(x))^{5/2}} dx$$

Optimal. Leaf size=74

$$\frac{a}{3b(a+b)(a+b \tanh^2(x))^{3/2}} - \frac{1}{(a+b)^2 \sqrt{a+b \tanh^2(x)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}$$

[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)-1/(a+b)^2/(a+b*tanh(x)^2)^(1/2)+1/3*a/b/(a+b)/(a+b*tanh(x)^2)^(3/2)

Rubi [A] time = 0.14, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3670, 446, 78, 51, 63, 208}

$$\frac{a}{3b(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{1}{(a+b)^2 \sqrt{a+b \tanh^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^3/(a + b*Tanh[x]^2)^(5/2), x]

[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(5/2) + a/(3*b*(a + b)*(a + b*Tanh[x]^2)^(3/2)) - 1/((a + b)^2*Sqrt[a + b*Tanh[x]^2])

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*((c_)*tan[(e_) +
(f_)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x],
x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(x)}{(a + b \tanh^2(x))^{5/2}} dx &= \text{Subst} \left(\int \frac{x^3}{(1-x^2)(a+bx^2)^{5/2}} dx, x, \tanh(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{x}{(1-x)(a+bx)^{5/2}} dx, x, \tanh^2(x) \right) \\
&= \frac{a}{3b(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{\text{Subst} \left(\int \frac{1}{(1-x)(a+bx)^{3/2}} dx, x, \tanh^2(x) \right)}{2(a+b)} \\
&= \frac{a}{3b(a+b)(a+b \tanh^2(x))^{3/2}} - \frac{1}{(a+b)^2 \sqrt{a+b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{1}{(1-x)\sqrt{a+bx}} dx, x, \tanh^2(x) \right)}{2(a+b)^2} \\
&= \frac{a}{3b(a+b)(a+b \tanh^2(x))^{3/2}} - \frac{1}{(a+b)^2 \sqrt{a+b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}} dx, x, \sqrt{a+b \tanh^2(x)} \right)}{b(a+b)^2} \\
&= \frac{\tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}} \right)}{(a+b)^{5/2}} + \frac{a}{3b(a+b)(a+b \tanh^2(x))^{3/2}} - \frac{1}{(a+b)^2 \sqrt{a+b \tanh^2(x)}}
\end{aligned}$$

Mathematica [C] time = 0.09, size = 63, normalized size = 0.85

$$\frac{a(a+b) - 3b(a+b \tanh^2(x)) {}_2F_1 \left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{b \tanh^2(x)+a}{a+b} \right)}{3b(a+b)^2 (a+b \tanh^2(x))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[x]^3/(a + b*Tanh[x]^2)^(5/2), x]
```

```
[Out] (a*(a + b) - 3*b*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tanh[x]^2)/(a + b)]
*(a + b*Tanh[x]^2))/(3*b*(a + b)^2*(a + b*Tanh[x]^2)^(3/2))
```

fricas [B] time = 1.43, size = 6621, normalized size = 89.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")

[Out]
$$\frac{1}{12} \cdot (3 \cdot (a^2 b + 2 a b^2 + b^3) \cosh(x)^8 + 8 \cdot (a^2 b + 2 a b^2 + b^3) \cosh(x) \sinh(x)^7 + (a^2 b + 2 a b^2 + b^3) \sinh(x)^8 + 4 \cdot (a^2 b - b^3) \cosh(x)^6 + 4 \cdot (a^2 b - b^3 + 7 \cdot (a^2 b + 2 a b^2 + b^3) \cosh(x)^2) \sinh(x)^6 + 8 \cdot (7 \cdot (a^2 b + 2 a b^2 + b^3) \cosh(x)^3 + 3 \cdot (a^2 b - b^3) \cosh(x)) \sinh(x)^5 + 2 \cdot (3 a^2 b - 2 a b^2 + 3 b^3) \cosh(x)^4 + 2 \cdot (35 \cdot (a^2 b + 2 a b^2 + b^3) \cosh(x)^4 + 3 a^2 b - 2 a b^2 + 3 b^3 + 30 \cdot (a^2 b - b^3) \cosh(x)^2) \sinh(x)^4 + 8 \cdot (7 \cdot (a^2 b + 2 a b^2 + b^3) \cosh(x)^5 + 10 \cdot (a^2 b - b^3) \cosh(x)^3 + (3 a^2 b - 2 a b^2 + 3 b^3) \cosh(x)) \sinh(x)^3 + a^2 b + 2 a b^2 + b^3 + 4 \cdot (a^2 b - b^3) \cosh(x)^2 + 4 \cdot (7 \cdot (a^2 b + 2 a b^2 + b^3) \cosh(x)^6 + 15 \cdot (a^2 b - b^3) \cosh(x)^4 + a^2 b - b^3 + 3 \cdot (3 a^2 b - 2 a b^2 + 3 b^3) \cosh(x)^2) \sinh(x)^2 + 8 \cdot ((a^2 b + 2 a b^2 + b^3) \cosh(x)^7 + 3 \cdot (a^2 b - b^3) \cosh(x)^5 + (3 a^2 b - 2 a b^2 + 3 b^3) \cosh(x)^3 + (a^2 b - b^3) \cosh(x)) \sinh(x) \cdot \sqrt{a + b} \cdot \log\left(\frac{(a^3 + a^2 b) \cosh(x)^8 + 8 \cdot (a^3 + a^2 b) \cosh(x) \sinh(x)^7 + (a^3 + a^2 b) \sinh(x)^8 + 2 \cdot (2 a^3 + a^2 b) \cosh(x)^6 + 2 \cdot (2 a^3 + a^2 b + 14 \cdot (a^3 + a^2 b) \cosh(x)^2) \sinh(x)^6 + 4 \cdot (14 \cdot (a^3 + a^2 b) \cosh(x)^3 + 3 \cdot (2 a^3 + a^2 b) \cosh(x)) \sinh(x)^5 + (6 a^3 + 4 a^2 b - a b^2 + b^3) \cosh(x)^4 + (70 \cdot (a^3 + a^2 b) \cosh(x)^4 + 6 a^3 + 4 a^2 b - a b^2 + b^3 + 30 \cdot (2 a^3 + a^2 b) \cosh(x)^2) \sinh(x)^4 + 4 \cdot (14 \cdot (a^3 + a^2 b) \cosh(x)^5 + 10 \cdot (2 a^3 + a^2 b) \cosh(x)^3 + (6 a^3 + 4 a^2 b - a b^2 + b^3) \cosh(x)) \sinh(x)^3 + a^3 + 3 a^2 b + 3 a b^2 + b^3 + 2 \cdot (2 a^3 + 3 a^2 b - b^3) \cosh(x)^2 + 2 \cdot (14 \cdot (a^3 + a^2 b) \cosh(x)^6 + 15 \cdot (2 a^3 + a^2 b) \cosh(x)^4 + 2 a^3 + 3 a^2 b - b^3 + 3 \cdot (6 a^3 + 4 a^2 b - a b^2 + b^3) \cosh(x)^2) \sinh(x)^2 + \sqrt{2} \cdot (a^2 \cosh(x)^6 + 6 a^2 \cosh(x) \sinh(x)^5 + a^2 \sinh(x)^6 + 3 a^2 \cosh(x)^4 + 3 \cdot (5 a^2 \cosh(x)^2 + a^2) \sinh(x)^4 + 4 \cdot (5 a^2 \cosh(x)^3 + 3 a^2 \cosh(x)) \sinh(x)^3 + (3 a^2 + 2 a b - b^2) \cosh(x)^2 + (15 a^2 \cosh(x)^4 + 18 a^2 \cosh(x)^2 + 3 a^2 + 2 a b - b^2) \sinh(x)^2 + a^2 + 2 a b + b^2 + 2 \cdot (3 a^2 \cosh(x)^5 + 6 a^2 \cosh(x)^3 + (3 a^2 + 2 a b - b^2) \cosh(x)) \sinh(x)\right) \cdot \sqrt{a + b} \cdot \sqrt{\frac{(a + b) \cosh(x)^2 + (a + b) \sinh(x)^2 + a - b}{(\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}} + 4 \cdot (2 \cdot (a^3 + a^2 b) \cosh(x)^7 + 3 \cdot (2 a^3 + a^2 b) \cosh(x)^5 + (6 a^3 + 4 a^2 b - a b^2 + b^3) \cosh(x)^3 + (2 a^3 + 3 a^2 b - b^3) \cosh(x)) \sinh(x) / (\cosh(x)^6 + 6 \cosh(x)^5 \sinh(x) + 15 \cosh(x)^4 \sinh(x)^2 + 20 \cosh(x)^3 \sinh(x)^3 + 15 \cosh(x)^2 \sinh(x)^4 + 6 \cosh(x) \sinh(x)^5 + \sinh(x)^6) + 3 \cdot ((a^2 b + 2 a b^2 + b^3) \cosh(x)^8 + 8 \cdot (a^2 b + 2 a b^2 + b^3) \cosh(x) \sinh(x)^7 + (a^2 b + 2 a b^2 + b^3) \sinh(x)^8 + 4 \cdot (a^2 b - b^3) \cosh(x)^6 + 4 \cdot (a^2 b - b^3 + 7 \cdot (a^2 b + 2 a b^2 + b^3) \cosh(x)^2) \sinh(x)^6 + 8 \cdot (7 \cdot (a^2 b + 2 a b^2 + b^3) \cosh(x)^3 + 3 \cdot (a^2 b - b^3) \cosh(x)) \sinh(x)^5 + 2 \cdot (3 a^2 b - 2 a b^2 + 3 b^3) \cosh(x)^4 + 2 \cdot (35 \cdot (a^2 b + 2 a b^2 + b^3) \cosh(x)^4 + 3 a^2 b - 2 a b^2 + 3 b^3 + 30 \cdot (a^2 b - b^3) \cosh(x)^2) \sinh(x)^4 + 8 \cdot (7 \cdot (a^2 b + 2 a b^2 + b^3) \cosh(x)^5 + 10 \cdot (a^2 b - b^3) \cosh(x)^3 + (3 a^2 b - 2 a b^2 + 3 b^3) \cosh(x)) \sinh(x)^3 + a^2 b + 2 a b^2 + b^3 + 4 \cdot (a^2 b - b^3) \cosh(x)^2 + 4 \cdot (7 \cdot (a^2 b + 2 a b^2 + b^3) \cosh(x)^6 + 15 \cdot (a^2 b - b^3) \cosh(x)^4 + a^2 b - b^3 + 3 \cdot (3 a^2 b - 2 a b^2 + 3 b^3) \cosh(x)^2) \sinh(x)^2 + 8 \cdot ((a^2 b + 2 a b^2 + b^3) \cosh(x)^7 + 3 \cdot (a^2 b - b^3) \cosh(x)^5 + (3 a^2 b - 2 a b^2 + 3 b^3) \cosh(x)^3 + (a^2 b - b^3) \cosh(x)) \sinh(x) \cdot \sqrt{a + b} \cdot \log\left(-\frac{(a + b) \cosh(x)^4 + 4 \cdot (a + b) \cosh(x) \sinh(x)^3 + (a + b) \sinh(x)^4 - 2 b \cosh(x)^2 + 2 \cdot (3 \cdot (a + b) \cosh(x)^2 - b) \sinh(x)^2 + \sqrt{2} \cdot (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1) \sqrt{a + b}}{(\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)} + 4 \cdot ((a + b) \cosh(x)^3 - b \cosh(x)) \sinh(x) + a + b / (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2)} + 4 \cdot \sqrt{2} \cdot ((a^3 - a^2 b - 5 a b^2 - 3 b^3) \cosh(x)^6 + 6 \cdot (a^3 - a^2 b - 5 a b^2 - 3 b^3) \cosh(x) \sinh(x)^5 + (a^3 - a^2 b - 5 a b^2 - 3 b^3) \sinh(x)^6 + 3 \cdot (a^3 - a^2 b - a b^2 + b^3) \cosh(x)^4 + 3 \cdot (a^3 - a^2 b - a b^2 + b^3 + 5 \cdot (a^3 - a^2 b - 5 a b^2 - 3 b^3) \cosh(x)^2) \sinh(x)^4 + 4 \cdot (5 \cdot (a^3 - a^2 b - 5 a b^2 - 3 b^3) \cosh(x)^3 + 3 \cdot (a^3 - a^2 b - a b^2 + b^3) \cosh(x)) \sinh(x)^3 + a^3 - a^2 b - 5 a b^2 - 3 b^3 + 3 \cdot (a^3 - a^2 b - a b^2 + b^3) \cosh(x)^2 + 3 \cdot (5 \cdot (a^3 - a^2 b$$

$$\begin{aligned}
& - 5*a*b^2 - 3*b^3)*\cosh(x)^4 + a^3 - a^2*b - a*b^2 + b^3 + 6*(a^3 - a^2*b - \\
& a*b^2 + b^3)*\cosh(x)^2*\sinh(x)^2 + 6*((a^3 - a^2*b - 5*a*b^2 - 3*b^3)*\cos \\
& h(x)^5 + 2*(a^3 - a^2*b - a*b^2 + b^3)*\cosh(x)^3 + (a^3 - a^2*b - a*b^2 + b \\
& ^3)*\cosh(x))*\sinh(x))*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/ \\
& (\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))}/((a^5*b + 5*a^4*b^2 + 10*a^3* \\
& b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*\cosh(x)^8 + 8*(a^5*b + 5*a^4*b^2 + 10*a^3 \\
& *b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*\cosh(x)*\sinh(x)^7 + (a^5*b + 5*a^4*b^2 + \\
& 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*\sinh(x)^8 + 4*(a^5*b + 3*a^4*b^2 \\
& + 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6)*\cosh(x)^6 + 4*(a^5*b + 3*a^4*b^2 + \\
& 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6 + 7*(a^5*b + 5*a^4*b^2 + 10*a^3*b^3 \\
& + 10*a^2*b^4 + 5*a*b^5 + b^6)*\cosh(x)^2)*\sinh(x)^6 + a^5*b + 5*a^4*b^2 + 10 \\
& *a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6 + 8*(7*(a^5*b + 5*a^4*b^2 + 10*a^3*b^ \\
& 3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*\cosh(x)^3 + 3*(a^5*b + 3*a^4*b^2 + 2*a^3*b^ \\
& 3 - 2*a^2*b^4 - 3*a*b^5 - b^6)*\cosh(x))*\sinh(x)^5 + 2*(3*a^5*b + 7*a^4*b^2 \\
& + 6*a^3*b^3 + 6*a^2*b^4 + 7*a*b^5 + 3*b^6)*\cosh(x)^4 + 2*(3*a^5*b + 7*a^4*b \\
& ^2 + 6*a^3*b^3 + 6*a^2*b^4 + 7*a*b^5 + 3*b^6 + 35*(a^5*b + 5*a^4*b^2 + 10*a \\
& ^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*\cosh(x)^4 + 30*(a^5*b + 3*a^4*b^2 + 2* \\
& a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6)*\cosh(x)^2)*\sinh(x)^4 + 8*(7*(a^5*b + 5 \\
& *a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*\cosh(x)^5 + 10*(a^5*b + \\
& 3*a^4*b^2 + 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6)*\cosh(x)^3 + (3*a^5*b + \\
& 7*a^4*b^2 + 6*a^3*b^3 + 6*a^2*b^4 + 7*a*b^5 + 3*b^6)*\cosh(x))*\sinh(x)^3 + 4 \\
& *(a^5*b + 3*a^4*b^2 + 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6)*\cosh(x)^2 + 4* \\
& (7*(a^5*b + 5*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*\cosh(x)^6 \\
& + a^5*b + 3*a^4*b^2 + 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6 + 15*(a^5*b + 3 \\
& *a^4*b^2 + 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6)*\cosh(x)^4 + 3*(3*a^5*b + \\
& 7*a^4*b^2 + 6*a^3*b^3 + 6*a^2*b^4 + 7*a*b^5 + 3*b^6)*\cosh(x)^2)*\sinh(x)^2 + \\
& 8*((a^5*b + 5*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*\cosh(x)^7 \\
& + 3*(a^5*b + 3*a^4*b^2 + 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6)*\cosh(x)^5 \\
& + (3*a^5*b + 7*a^4*b^2 + 6*a^3*b^3 + 6*a^2*b^4 + 7*a*b^5 + 3*b^6)*\cosh(x)^3 \\
& + (a^5*b + 3*a^4*b^2 + 2*a^3*b^3 - 2*a^2*b^4 - 3*a*b^5 - b^6)*\cosh(x))*\sin \\
& h(x)), -1/6*(3*((a^2*b + 2*a*b^2 + b^3)*\cosh(x)^8 + 8*(a^2*b + 2*a*b^2 + b^ \\
& 3)*\cosh(x))*\sinh(x)^7 + (a^2*b + 2*a*b^2 + b^3)*\sinh(x)^8 + 4*(a^2*b - b^3)* \\
& \cosh(x)^6 + 4*(a^2*b - b^3 + 7*(a^2*b + 2*a*b^2 + b^3)*\cosh(x)^2)*\sinh(x)^6 \\
& + 8*(7*(a^2*b + 2*a*b^2 + b^3)*\cosh(x)^3 + 3*(a^2*b - b^3)*\cosh(x))*\sinh(x \\
&)^5 + 2*(3*a^2*b - 2*a*b^2 + 3*b^3)*\cosh(x)^4 + 2*(35*(a^2*b + 2*a*b^2 + b^ \\
& 3)*\cosh(x)^4 + 3*a^2*b - 2*a*b^2 + 3*b^3 + 30*(a^2*b - b^3)*\cosh(x)^2)*\sinh \\
& (x)^4 + 8*(7*(a^2*b + 2*a*b^2 + b^3)*\cosh(x)^5 + 10*(a^2*b - b^3)*\cosh(x)^3 \\
& + (3*a^2*b - 2*a*b^2 + 3*b^3)*\cosh(x))*\sinh(x)^3 + a^2*b + 2*a*b^2 + b^3 + \\
& 4*(a^2*b - b^3)*\cosh(x)^2 + 4*(7*(a^2*b + 2*a*b^2 + b^3)*\cosh(x)^6 + 15*(a \\
& ^2*b - b^3)*\cosh(x)^4 + a^2*b - b^3 + 3*(3*a^2*b - 2*a*b^2 + 3*b^3)*\cosh(x) \\
& ^2)*\sinh(x)^2 + 8*((a^2*b + 2*a*b^2 + b^3)*\cosh(x)^7 + 3*(a^2*b - b^3)*\cosh \\
& (x)^5 + (3*a^2*b - 2*a*b^2 + 3*b^3)*\cosh(x)^3 + (a^2*b - b^3)*\cosh(x))*\sinh \\
& (x))*\sqrt{-a - b}*\arctan(\sqrt{2}*(a*\cosh(x)^2 + 2*a*\cosh(x)*\sinh(x) + a*\sin \\
& h(x)^2 + a + b))*\sqrt{-a - b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + \\
& a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))}/((a^2 + a*b)*\cosh(x)^4 \\
& + 4*(a^2 + a*b)*\cosh(x)*\sinh(x)^3 + (a^2 + a*b)*\sinh(x)^4 + (2*a^2 + a*b - \\
& b^2)*\cosh(x)^2 + (6*(a^2 + a*b)*\cosh(x)^2 + 2*a^2 + a*b - b^2)*\sinh(x)^2 + \\
& a^2 + 2*a*b + b^2 + 2*(2*(a^2 + a*b)*\cosh(x)^3 + (2*a^2 + a*b - b^2)*\cosh(x) \\
&))*\sinh(x))) + 3*((a^2*b + 2*a*b^2 + b^3)*\cosh(x)^8 + 8*(a^2*b + 2*a*b^2 + \\
& b^3)*\cosh(x))*\sinh(x)^7 + (a^2*b + 2*a*b^2 + b^3)*\sinh(x)^8 + 4*(a^2*b - b^3 \\
&)*\cosh(x)^6 + 4*(a^2*b - b^3 + 7*(a^2*b + 2*a*b^2 + b^3)*\cosh(x)^2)*\sinh(x) \\
& ^6 + 8*(7*(a^2*b + 2*a*b^2 + b^3)*\cosh(x)^3 + 3*(a^2*b - b^3)*\cosh(x))*\sinh \\
& (x)^5 + 2*(3*a^2*b - 2*a*b^2 + 3*b^3)*\cosh(x)^4 + 2*(35*(a^2*b + 2*a*b^2 + \\
& b^3)*\cosh(x)^4 + 3*a^2*b - 2*a*b^2 + 3*b^3 + 30*(a^2*b - b^3)*\cosh(x)^2)*\si \\
& nh(x)^4 + 8*(7*(a^2*b + 2*a*b^2 + b^3)*\cosh(x)^5 + 10*(a^2*b - b^3)*\cosh(x) \\
& ^3 + (3*a^2*b - 2*a*b^2 + 3*b^3)*\cosh(x))*\sinh(x)^3 + a^2*b + 2*a*b^2 + b^3 \\
& + 4*(a^2*b - b^3)*\cosh(x)^2 + 4*(7*(a^2*b + 2*a*b^2 + b^3)*\cosh(x)^6 + 15* \\
& (a^2*b - b^3)*\cosh(x)^4 + a^2*b - b^3 + 3*(3*a^2*b - 2*a*b^2 + 3*b^3)*\cosh(x) \\
& ^2)*\sinh(x)^2 + 8*((a^2*b + 2*a*b^2 + b^3)*\cosh(x)^7 + 3*(a^2*b - b^3)*\co
\end{aligned}$$

$$\begin{aligned} & \text{sh}(x)^5 + (3a^2b - 2ab^2 + 3b^3) \cosh(x)^3 + (a^2b - b^3) \cosh(x) \sinh(x) \sqrt{-a-b} \arctan(\sqrt{2}(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 - 1) \sqrt{-a-b} \sqrt{((a+b)\cosh(x)^2 + (a+b)\sinh(x)^2 + a-b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)}) / ((a+b)\cosh(x)^4 + 4(a+b)\cosh(x)\sinh(x)^3 + (a+b)\sinh(x)^4 + 2(a-b)\cosh(x)^2 + 2(3(a+b)\cosh(x)^2 + a-b)\sinh(x)^2 + 4((a+b)\cosh(x)^3 + (a-b)\cosh(x))\sinh(x) + a+b) - 2\sqrt{2}((a^3 - a^2b - 5ab^2 - 3b^3)\cosh(x)^6 + 6(a^3 - a^2b - 5ab^2 - 3b^3)\cosh(x)\sinh(x)^5 + (a^3 - a^2b - 5ab^2 - 3b^3)\sinh(x)^6 + 3(a^3 - a^2b - ab^2 + b^3)\cosh(x)^4 + 3(a^3 - a^2b - ab^2 + b^3 + 5(a^3 - a^2b - 5ab^2 - 3b^3)\cosh(x)^2)\sinh(x)^4 + 4(5(a^3 - a^2b - 5ab^2 - 3b^3)\cosh(x)^3 + 3(a^3 - a^2b - ab^2 + b^3)\cosh(x))\sinh(x)^3 + a^3 - a^2b - 5ab^2 - 3b^3 + 3(a^3 - a^2b - ab^2 + b^3)\cosh(x)^2 + 3(5(a^3 - a^2b - 5ab^2 - 3b^3)\cosh(x)^4 + a^3 - a^2b - ab^2 + b^3 + 6(a^3 - a^2b - ab^2 + b^3)\cosh(x)^2)\sinh(x)^2 + 6((a^3 - a^2b - 5ab^2 - 3b^3)\cosh(x)^5 + 2(a^3 - a^2b - ab^2 + b^3)\cosh(x)^3 + (a^3 - a^2b - ab^2 + b^3)\cosh(x))\sinh(x) \sqrt{((a+b)\cosh(x)^2 + (a+b)\sinh(x)^2 + a-b)/(\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2)}) / ((a^5b + 5a^4b^2 + 10a^3b^3 + 10a^2b^4 + 5ab^5 + b^6)\cosh(x)^8 + 8(a^5b + 5a^4b^2 + 10a^3b^3 + 10a^2b^4 + 5ab^5 + b^6)\cosh(x)\sinh(x)^7 + (a^5b + 5a^4b^2 + 10a^3b^3 + 10a^2b^4 + 5ab^5 + b^6)\sinh(x)^8 + 4(a^5b + 3a^4b^2 + 2a^3b^3 - 2a^2b^4 - 3ab^5 - b^6)\cosh(x)^6 + 4(a^5b + 3a^4b^2 + 2a^3b^3 - 2a^2b^4 - 3ab^5 - b^6 + 7(a^5b + 5a^4b^2 + 10a^3b^3 + 10a^2b^4 + 5ab^5 + b^6)\cosh(x)^2)\sinh(x)^6 + a^5b + 5a^4b^2 + 10a^3b^3 + 10a^2b^4 + 5ab^5 + b^6 + 8(7(a^5b + 5a^4b^2 + 10a^3b^3 + 10a^2b^4 + 5ab^5 + b^6)\cosh(x)^3 + 3(a^5b + 3a^4b^2 + 2a^3b^3 - 2a^2b^4 - 3ab^5 - b^6)\cosh(x))\sinh(x)^5 + 2(3a^5b + 7a^4b^2 + 6a^3b^3 + 6a^2b^4 + 7ab^5 + 3b^6)\cosh(x)^4 + 2(3a^5b + 7a^4b^2 + 6a^3b^3 + 6a^2b^4 + 7ab^5 + 3b^6 + 35(a^5b + 5a^4b^2 + 10a^3b^3 + 10a^2b^4 + 5ab^5 + b^6)\cosh(x)^4 + 30(a^5b + 3a^4b^2 + 2a^3b^3 - 2a^2b^4 - 3ab^5 - b^6)\cosh(x)^2)\sinh(x)^4 + 8(7(a^5b + 5a^4b^2 + 10a^3b^3 + 10a^2b^4 + 5ab^5 + b^6)\cosh(x)^5 + 10(a^5b + 3a^4b^2 + 2a^3b^3 - 2a^2b^4 - 3ab^5 - b^6)\cosh(x)^3 + (3a^5b + 7a^4b^2 + 6a^3b^3 + 6a^2b^4 + 7ab^5 + 3b^6)\cosh(x))\sinh(x)^3 + 4(a^5b + 3a^4b^2 + 2a^3b^3 - 2a^2b^4 - 3ab^5 - b^6)\cosh(x)^2 + 4(7(a^5b + 5a^4b^2 + 10a^3b^3 + 10a^2b^4 + 5ab^5 + b^6)\cosh(x)^6 + a^5b + 3a^4b^2 + 2a^3b^3 - 2a^2b^4 - 3ab^5 - b^6 + 15(a^5b + 3a^4b^2 + 2a^3b^3 - 2a^2b^4 - 3ab^5 - b^6)\cosh(x)^4 + 3(3a^5b + 7a^4b^2 + 6a^3b^3 + 6a^2b^4 + 7ab^5 + 3b^6)\cosh(x)^2)\sinh(x)^2 + 8((a^5b + 5a^4b^2 + 10a^3b^3 + 10a^2b^4 + 5ab^5 + b^6)\cosh(x)^7 + 3(a^5b + 3a^4b^2 + 2a^3b^3 - 2a^2b^4 - 3ab^5 - b^6)\cosh(x)^5 + (3a^5b + 7a^4b^2 + 6a^3b^3 + 6a^2b^4 + 7ab^5 + 3b^6)\cosh(x)^3 + (a^5b + 3a^4b^2 + 2a^3b^3 - 2a^2b^4 - 3ab^5 - b^6)\cosh(x))\sinh(x)] \end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep+1)]Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Warning, replacing 0 by `u`, a substitution variable should perhaps be purged.Evaluation time: 0.56Error: Bad Argument Type

maple [B] time = 0.08, size = 435, normalized size = 5.88

$$\frac{1}{3b \left(a + b \left(\tanh^2(x) \right) \right)^{\frac{3}{2}}} - \frac{1}{6(a+b) \left((\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a + b \right)^{\frac{3}{2}}} + \frac{1}{6(a+b)a \left((\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a + b \right)^{\frac{3}{2}}} + \frac{1}{b \tanh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^3/(a+b*tanh(x)^2)^(5/2), x)

[Out] 1/3/b/(a+b*tanh(x)^2)^(3/2)-1/6/(a+b)/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(3/2)+1/6*b/(a+b)/a/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)+1/2/(a+b)^2/a/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2)*((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))-1/6/(a+b)/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(3/2)-1/6*b/(a+b)/a/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(3/2)*tanh(x)-1/3*b/(a+b)/a^2/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)-1/2/(a+b)^2/a/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b-2*(1+tanh(x))*b+2*(a+b)^(1/2)*((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2))/(1+tanh(x)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^3}{(b \tanh(x)^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^3/(a+b*tanh(x)^2)^(5/2), x, algorithm="maxima")

[Out] integrate(tanh(x)^3/(b*tanh(x)^2 + a)^(5/2), x)

mupad [B] time = 3.82, size = 82, normalized size = 1.11

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b \tanh(x)^2 + a} (2a^2 + 4ab + 2b^2)}{2(a+b)^{5/2}}\right)}{(a+b)^{5/2}} + \frac{\frac{a}{3(a+b)} - \frac{b(b \tanh(x)^2 + a)}{(a+b)^2}}{b(b \tanh(x)^2 + a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^3/(a + b*tanh(x)^2)^(5/2), x)

[Out] atanh(((a + b*tanh(x)^2)^(1/2)*(4*a*b + 2*a^2 + 2*b^2))/(2*(a + b)^(5/2)))/(a + b)^(5/2) + (a/(3*(a + b)) - (b*(a + b*tanh(x)^2))/(a + b)^2)/(b*(a + b*tanh(x)^2)^(3/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^3(x)}{(a + b \tanh^2(x))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)**3/(a+b*tanh(x)**2)**(5/2), x)

[Out] Integral(tanh(x)**3/(a + b*tanh(x)**2)**(5/2), x)

$$3.250 \quad \int \frac{\tanh^2(x)}{(a+b \tanh^2(x))^{5/2}} dx$$

Optimal. Leaf size=88

$$-\frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{\tanh(x)}{3(a+b)(a+b\tanh^2(x))^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}$$

[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(5/2)-1/3*(2*a-b)*tanh(x)/a/(a+b)^2/(a+b*tanh(x)^2)^(1/2)-1/3*tanh(x)/(a+b)/(a+b*tanh(x)^2)^(3/2)

Rubi [A] time = 0.13, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {3670, 471, 527, 12, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} - \frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{\tanh(x)}{3(a+b)(a+b\tanh^2(x))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]^2/(a + b*Tanh[x]^2)^(5/2), x]

[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(5/2) - Tanh[x]/(3*(a + b)*(a + b*Tanh[x]^2)^(3/2)) - ((2*a - b)*Tanh[x])/(3*a*(a + b)^2*Sqrt[a + b*Tanh[x]^2])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 471

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m - n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n + 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x],
x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^2(x)}{(a + b \tanh^2(x))^{5/2}} dx &= \text{Subst} \left(\int \frac{x^2}{(1-x^2)(a+bx^2)^{5/2}} dx, x, \tanh(x) \right) \\
&= -\frac{\tanh(x)}{3(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{\text{Subst} \left(\int \frac{1+2x^2}{(1-x^2)(a+bx^2)^{3/2}} dx, x, \tanh(x) \right)}{3(a+b)} \\
&= -\frac{\tanh(x)}{3(a+b)(a+b \tanh^2(x))^{3/2}} - \frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b \tanh^2(x)}} - \frac{\text{Subst} \left(\int -\frac{3a}{(1-x^2)\sqrt{a+bx^2}} dx, x, \tanh(x) \right)}{3a(a+b)} \\
&= -\frac{\tanh(x)}{3(a+b)(a+b \tanh^2(x))^{3/2}} - \frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}} dx, x, \tanh(x) \right)}{(a+b)} \\
&= -\frac{\tanh(x)}{3(a+b)(a+b \tanh^2(x))^{3/2}} - \frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{1}{1-(a+b)x^2} dx, x, \tanh(x) \right)}{(a+b)} \\
&= \frac{\tanh^{-1} \left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right)}{(a+b)^{5/2}} - \frac{\tanh(x)}{3(a+b)(a+b \tanh^2(x))^{3/2}} - \frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b \tanh^2(x)}}
\end{aligned}$$

Mathematica [C] time = 7.82, size = 259, normalized size = 2.94

$$\cosh^4(x) \coth(x) \left(-12(a+b)^3 \tanh^6(x) (a+b \tanh^2(x)) {}_2F_1 \left(2, 2; \frac{9}{2}; -\frac{(a+b) \sinh^2(x)}{a} \right) \sqrt{-\frac{\sinh^2(x) \cosh^2(x) (a^2 + a(b \tanh^2(x) + 1))}{a^2}} \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Tanh[x]^2/(a + b*Tanh[x]^2)^(5/2), x]
```

```
[Out] (Cosh[x]^4*Coth[x]*(-12*(a + b)^3*Hypergeometric2F1[2, 2, 9/2, -(((a + b)*Sinh[x]^2)/a)]*Tanh[x]^6*(a + b*Tanh[x]^2)*Sqrt[-((Cosh[x]^2*Sinh[x]^2*(a^2 + b^2*Tanh[x]^2 + a*(b + b*Tanh[x]^2)))/a^2)] - 35*a*Sech[x]^2*(-5*a - 2*b*Tanh[x]^2)*(3*ArcSin[Sqrt[-((a + b)*Sinh[x]^2)/a]])*(a + b*Tanh[x]^2)^2 + a*Sech[x]^2*(-4*b*Tanh[x]^2 + a*(-3 - Tanh[x]^2))*Sqrt[-((Cosh[x]^2*Sinh[x]^2*(a^2 + b^2*Tanh[x]^2 + a*(b + b*Tanh[x]^2)))/a^2])))/(315*a^4*(a + b)^2*Sqrt[a + b*Tanh[x]^2]*Sqrt[-((a + b)*Cosh[x]^2*Sinh[x]^2*(a + b*Tanh[x]^2)/a^2)]*(1 + (b*Tanh[x]^2)/a))
```

fricas [B] time = 1.60, size = 6507, normalized size = 73.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 4*(a^3 - a*b^2)*cosh(x)^6 + 4*(a^3 - a*b^2 + 7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(a^3 - a*b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x)^4 + 2*(35*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 3*a^3 - 2*a^2*b + 3*a*b^2 + 30*(a^3 - a*b^2)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(a^3 - a*b^2)*cosh(x)^3 + (3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*a^2*b + a*b^2 + 4*(a^3 - a*b^2)*cosh(x)^2 + 4*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(a^3 - a*b^2)*cosh(x)^4 + a^3 - a*b^2 + 3*(3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x)^2)*sinh(x)^2 + 8*((a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(a^3 - a*b^2)*cosh(x)^5 + (3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x)^3 + (a^3 - a*b^2)*cosh(x))*sinh(x)*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 3*((a^3 + 2*a^2*b + a*b^2)*cosh(x)^8 + 8*(a^3 + 2*a^2*b + a*b^2)*cosh(x)*sinh(x)^7 + (a^3 + 2*a^2*b + a*b^2)*sinh(x)^8 + 4*(a^3 - a*b^2)*cosh(x)^6 + 4*(a^3 - a*b^2 + 7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^3 + 3*(a^3 - a*b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x)^4 + 2*(35*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^4 + 3*a^3 - 2*a^2*b + 3*a*b^2 + 30*(a^3 - a*b^2)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^5 + 10*(a^3 - a*b^2)*cosh(x)^3 + (3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x))*sinh(x)^3 + a^3 + 2*a^2*b + a*b^2 + 4*(a^3 - a*b^2)*cosh(x)^2 + 4*(7*(a^3 + 2*a^2*b + a*b^2)*cosh(x)^6 + 15*(a^3 - a*b^2)*cosh(x)^4 + a^3 - a*b^2 + 3*(3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x)^2)*sinh(x)^2 + 8*((a^3 + 2*a^2*b + a*b^2)*cosh(x)^7 + 3*(a^3 - a*b^2)*cosh(x)^5 + (3*a^3 - 2*a^2*b + 3*a*b^2)*cosh(x)^3 + (a^3 - a*b^2)*co
```

$$\begin{aligned}
& \text{sh}(x) \cdot \sinh(x) \cdot \sqrt{a+b} \cdot \log\left(\left((a+b) \cdot \cosh(x)^4 + 4(a+b) \cdot \cosh(x) \cdot \sinh(x)^3 + (a+b) \cdot \sinh(x)^4 + 2a \cdot \cosh(x)^2 + 2(3(a+b) \cdot \cosh(x)^2 + a) \cdot \sinh(x)^2 + \sqrt{2} \cdot (\cosh(x)^2 + 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2 + 1) \cdot \sqrt{a+b}\right) \cdot \sqrt{\left(\frac{(a+b) \cdot \cosh(x)^2 + (a+b) \cdot \sinh(x)^2 + a - b}{\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2}\right)} + 4 \cdot \left(\frac{(a+b) \cdot \cosh(x)^3 + a \cdot \cosh(x) \cdot \sinh(x) + a + b}{\cosh(x)^2 + 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2}\right) - 4 \cdot \sqrt{2} \cdot \left(\frac{3a^3 + 5a^2b + ab^2 - b^3}{\cosh(x)^6 + 6(3a^3 + 5a^2b + ab^2 - b^3) \cdot \cosh(x) \cdot \sinh(x)^5 + (3a^3 + 5a^2b + ab^2 - b^3) \cdot \sinh(x)^6 + 3(a^3 - a^2b - ab^2 + b^3) \cdot \cosh(x)^4 + 3(a^3 - a^2b - ab^2 + b^3 + 5(3a^3 + 5a^2b + ab^2 - b^3) \cdot \cosh(x)^2) \cdot \sinh(x)^4 + 4(5(3a^3 + 5a^2b + ab^2 - b^3) \cdot \cosh(x)^3 + 3(a^3 - a^2b - ab^2 + b^3) \cdot \cosh(x)) \cdot \sinh(x)^3 - 3a^3 - 5a^2b - ab^2 + b^3 - 3(a^3 - a^2b - ab^2 + b^3) \cdot \cosh(x)^2 + 3(5(3a^3 + 5a^2b + ab^2 - b^3) \cdot \cosh(x)^4 - a^3 + a^2b + ab^2 - b^3 + 6(a^3 - a^2b - ab^2 + b^3) \cdot \cosh(x)^2) \cdot \sinh(x)^2 + 6 \cdot \left(\frac{3a^3 + 5a^2b + ab^2 - b^3}{\cosh(x)^5 + 2(a^3 - a^2b - ab^2 + b^3) \cdot \cosh(x)^3 - (a^3 - a^2b - ab^2 + b^3) \cdot \cosh(x)) \cdot \sinh(x)\right) \cdot \sqrt{\left(\frac{(a+b) \cdot \cosh(x)^2 + (a+b) \cdot \sinh(x)^2 + a - b}{\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2}\right)}\right) / \left(\frac{a^6 + 5a^5b + 10a^4b^2 + 10a^3b^3 + 5a^2b^4 + ab^5}{\cosh(x)^8 + 8(a^6 + 5a^5b + 10a^4b^2 + 10a^3b^3 + 5a^2b^4 + ab^5) \cdot \cosh(x) \cdot \sinh(x)^7 + (a^6 + 5a^5b + 10a^4b^2 + 10a^3b^3 + 5a^2b^4 + ab^5) \cdot \sinh(x)^8 + 4(a^6 + 3a^5b + 2a^4b^2 - 2a^3b^3 - 3a^2b^4 - ab^5) \cdot \cosh(x)^6 + 4(a^6 + 3a^5b + 2a^4b^2 - 2a^3b^3 - 3a^2b^4 - ab^5 + 7(a^6 + 5a^5b + 10a^4b^2 + 10a^3b^3 + 5a^2b^4 + ab^5) \cdot \cosh(x)^2) \cdot \sinh(x)^6 + a^6 + 5a^5b + 10a^4b^2 + 10a^3b^3 + 5a^2b^4 + ab^5 + 8(7(a^6 + 5a^5b + 10a^4b^2 + 10a^3b^3 + 5a^2b^4 + ab^5) \cdot \cosh(x)^3 + 3(a^6 + 3a^5b + 2a^4b^2 - 2a^3b^3 - 3a^2b^4 - ab^5) \cdot \cosh(x)) \cdot \sinh(x)^5 + 2(3a^6 + 7a^5b + 6a^4b^2 + 6a^3b^3 + 7a^2b^4 + 3ab^5) \cdot \cosh(x)^4 + 2(3a^6 + 7a^5b + 6a^4b^2 + 6a^3b^3 + 7a^2b^4 + 3ab^5 + 35(a^6 + 5a^5b + 10a^4b^2 + 10a^3b^3 + 5a^2b^4 + ab^5) \cdot \cosh(x)^4 + 30(a^6 + 3a^5b + 2a^4b^2 - 2a^3b^3 - 3a^2b^4 - ab^5) \cdot \cosh(x)^2) \cdot \sinh(x)^4 + 8(7(a^6 + 5a^5b + 10a^4b^2 + 10a^3b^3 + 5a^2b^4 + ab^5) \cdot \cosh(x)^5 + 10(a^6 + 3a^5b + 2a^4b^2 - 2a^3b^3 - 3a^2b^4 - ab^5) \cdot \cosh(x)^3 + (3a^6 + 7a^5b + 6a^4b^2 + 6a^3b^3 + 7a^2b^4 + 3ab^5) \cdot \cosh(x)) \cdot \sinh(x)^3 + 4(a^6 + 3a^5b + 2a^4b^2 - 2a^3b^3 - 3a^2b^4 - ab^5) \cdot \cosh(x)^2 + 4(7(a^6 + 5a^5b + 10a^4b^2 + 10a^3b^3 + 5a^2b^4 + ab^5) \cdot \cosh(x)^6 + a^6 + 3a^5b + 2a^4b^2 - 2a^3b^3 - 3a^2b^4 - ab^5 + 15(a^6 + 3a^5b + 2a^4b^2 - 2a^3b^3 - 3a^2b^4 - ab^5) \cdot \cosh(x)^4 + 3(3a^6 + 7a^5b + 6a^4b^2 + 6a^3b^3 + 7a^2b^4 + 3ab^5) \cdot \cosh(x)^2) \cdot \sinh(x)^2 + 8 \cdot \left(\frac{a^6 + 5a^5b + 10a^4b^2 + 10a^3b^3 + 5a^2b^4 + ab^5}{\cosh(x)^7 + 3(a^6 + 3a^5b + 2a^4b^2 - 2a^3b^3 - 3a^2b^4 - ab^5) \cdot \cosh(x)^5 + (3a^6 + 7a^5b + 6a^4b^2 + 6a^3b^3 + 7a^2b^4 + 3ab^5) \cdot \cosh(x)^3 + (a^6 + 3a^5b + 2a^4b^2 - 2a^3b^3 - 3a^2b^4 - ab^5) \cdot \cosh(x)}\right) \cdot \sinh(x)\right), -1/6 \cdot \left(3 \cdot \left(\frac{a^3 + 2a^2b + ab^2}{\cosh(x)^8 + 8(a^3 + 2a^2b + ab^2) \cdot \cosh(x) \cdot \sinh(x)^7 + (a^3 + 2a^2b + ab^2) \cdot \sinh(x)^8 + 4(a^3 - ab^2) \cdot \cosh(x)^6 + 4(a^3 - ab^2 + 7(a^3 + 2a^2b + ab^2) \cdot \cosh(x)^2) \cdot \sinh(x)^6 + 8(7(a^3 + 2a^2b + ab^2) \cdot \cosh(x)^3 + 3(a^3 - ab^2) \cdot \cosh(x)) \cdot \sinh(x)^5 + 2(3a^3 - 2a^2b + 3ab^2) \cdot \cosh(x)^4 + 2(35(a^3 + 2a^2b + ab^2) \cdot \cosh(x)^4 + 3a^3 - 2a^2b + 3ab^2 + 30(a^3 - ab^2) \cdot \cosh(x)^2) \cdot \sinh(x)^4 + 8(7(a^3 + 2a^2b + ab^2) \cdot \cosh(x)^5 + 10(a^3 - ab^2) \cdot \cosh(x)^3 + (3a^3 - 2a^2b + 3ab^2) \cdot \cosh(x)) \cdot \sinh(x)^3 + a^3 + 2a^2b + ab^2 + 4(a^3 - ab^2) \cdot \cosh(x)^2 + 4(7(a^3 + 2a^2b + ab^2) \cdot \cosh(x)^6 + 15(a^3 - ab^2) \cdot \cosh(x)^4 + a^3 - ab^2 + 3(3a^3 - 2a^2b + 3ab^2) \cdot \cosh(x)^2) \cdot \sinh(x)^2 + 8 \cdot \left(\frac{a^3 + 2a^2b + ab^2}{\cosh(x)^7 + 3(a^3 - ab^2) \cdot \cosh(x)^5 + (3a^3 - 2a^2b + 3ab^2) \cdot \cosh(x)^3 + (a^3 - ab^2) \cdot \cosh(x)}\right) \cdot \sinh(x)\right) \cdot \sqrt{-a-b} \cdot \arctan\left(\sqrt{2} \cdot \frac{b \cdot \cosh(x)^2 + 2b \cdot \cosh(x) \cdot \sinh(x) + b \cdot \sinh(x)^2 - a - b}{\sqrt{-a-b} \cdot \sqrt{\left(\frac{(a+b) \cdot \cosh(x)^2 + (a+b) \cdot \sinh(x)^2 + a - b}{\cosh(x)^2 - 2 \cdot \cosh(x) \cdot \sinh(x) + \sinh(x)^2}\right)}}\right) / \left(\frac{(ab + b^2) \cdot \cosh(x)^4 + 4(ab + b^2) \cdot \cosh(x) \cdot \sinh(x)^3 + (ab + b^2) \cdot \sinh(x)^4 + (a^2 - ab - 2b^2) \cdot \cosh(x)^2 + (6(ab + b^2) \cdot \cosh(x)^2 + a^2 - ab - 2b^2) \cdot \sinh(x)^2 + \right.
\end{aligned}$$

$$\begin{aligned}
& a^2 + 2ab + b^2 + 2(2(a+b)^2 \cosh(x)^3 + (a^2 - ab - 2b^2) \cosh(x) \\
&) \sinh(x)) + 3((a^3 + 2a^2b + ab^2) \cosh(x)^8 + 8(a^3 + 2a^2b + ab^2) \cosh(x) \sinh(x)^7 + (a^3 + 2a^2b + ab^2) \sinh(x)^8 + 4(a^3 - ab^2) \\
&) \cosh(x)^6 + 4(a^3 - ab^2 + 7(a^3 + 2a^2b + ab^2) \cosh(x)^2) \sinh(x)^6 + 8(7(a^3 + 2a^2b + ab^2) \cosh(x)^3 + 3(a^3 - ab^2) \cosh(x)) \sinh \\
& (x)^5 + 2(3a^3 - 2a^2b + 3ab^2) \cosh(x)^4 + 2(35(a^3 + 2a^2b + ab^2) \cosh(x)^4 + 3a^3 - 2a^2b + 3ab^2 + 30(a^3 - ab^2) \cosh(x)^2) \sinh \\
& (x)^4 + 8(7(a^3 + 2a^2b + ab^2) \cosh(x)^5 + 10(a^3 - ab^2) \cosh(x)^3 + (3a^3 - 2a^2b + 3ab^2) \cosh(x)) \sinh(x)^3 + a^3 + 2a^2b + ab^2 \\
& + 4(a^3 - ab^2) \cosh(x)^2 + 4(7(a^3 + 2a^2b + ab^2) \cosh(x)^6 + 15(a^3 - ab^2) \cosh(x)^4 + a^3 - ab^2 + 3(3a^3 - 2a^2b + 3ab^2) \cosh(x) \\
&)^2 \sinh(x)^2 + 8((a^3 + 2a^2b + ab^2) \cosh(x)^7 + 3(a^3 - ab^2) \cosh(x)^5 + (3a^3 - 2a^2b + 3ab^2) \cosh(x)^3 + (a^3 - ab^2) \cosh(x)) \sinh(x) \\
&) \sqrt{-a-b} \arctan(\sqrt{2} \sqrt{-a-b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a-b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / ((a+b) \cosh(x)^2 + 2(a+b) \cosh(x) \sinh(x) + (a+b) \sinh(x)^2 + a+b)) + \\
& 2\sqrt{2} * ((3a^3 + 5a^2b + ab^2 - b^3) \cosh(x)^6 + 6(3a^3 + 5a^2b + ab^2 - b^3) \cosh(x) \sinh(x)^5 + (3a^3 + 5a^2b + ab^2 - b^3) \sinh(x)^6 \\
& + 3(a^3 - a^2b - ab^2 + b^3) \cosh(x)^4 + 3(a^3 - a^2b - ab^2 + b^3 + 5(3a^3 + 5a^2b + ab^2 - b^3) \cosh(x)^2) \sinh(x)^4 + 4(5(3a^3 + 5a^2b + ab^2 - b^3) \cosh(x)^3 + 3(a^3 - a^2b - ab^2 + b^3) \cosh(x)) \sinh \\
& (x)^3 - 3a^3 - 5a^2b - ab^2 + b^3 - 3(a^3 - a^2b - ab^2 + b^3) \cosh(x)^2 + 3(5(3a^3 + 5a^2b + ab^2 - b^3) \cosh(x)^4 - a^3 + a^2b + ab^2 \\
& - b^3 + 6(a^3 - a^2b - ab^2 + b^3) \cosh(x)^2) \sinh(x)^2 + 6((3a^3 + 5a^2b + ab^2 - b^3) \cosh(x)^5 + 2(a^3 - a^2b - ab^2 + b^3) \cosh(x)^3 - \\
& (a^3 - a^2b - ab^2 + b^3) \cosh(x)) \sinh(x) \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a-b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)}) / ((a^6 + 5a^5b + 10a^4b^2 + 10a^3b^3 + 5a^2b^4 + ab^5) \cosh(x)^8 + 8(a^6 + 5a^5b + 10a^4b^2 + 10a^3b^3 + 5a^2b^4 + ab^5) \cosh(x) \sinh(x)^7 + (a^6 + 5a^5b + 10a^4b^2 + 10a^3b^3 + 5a^2b^4 + ab^5) \sinh(x)^8 + 4(a^6 + 3a^5b + 2a^4b^2 - 2a^3b^3 - 3a^2b^4 - ab^5) \cosh(x)^6 + 4(a^6 + 3a^5b + 2a^4b^2 - 2a^3b^3 - 3a^2b^4 - ab^5 + 7(a^6 + 5a^5b + 10a^4b^2 + 10a^3b^3 + 5a^2b^4 + ab^5) \cosh(x)^2) \sinh(x)^6 + a^6 + 5a^5b + 10a^4b^2 + 10a^3b^3 + 5a^2b^4 + ab^5 + 8(7(a^6 + 5a^5b + 10a^4b^2 + 10a^3b^3 + 5a^2b^4 + ab^5) \cosh(x)^3 + 3(a^6 + 3a^5b + 2a^4b^2 - 2a^3b^3 - 3a^2b^4 - ab^5) \cosh(x)) \sinh(x)^5 + 2(3a^6 + 7a^5b + 6a^4b^2 + 6a^3b^3 + 7a^2b^4 + 3ab^5) \cosh(x)^4 + 2(3a^6 + 7a^5b + 6a^4b^2 + 6a^3b^3 + 7a^2b^4 + 3ab^5 + 35(a^6 + 5a^5b + 10a^4b^2 + 10a^3b^3 + 5a^2b^4 + ab^5) \cosh(x)^4 + 30(a^6 + 3a^5b + 2a^4b^2 - 2a^3b^3 - 3a^2b^4 - ab^5) \cosh(x)^2) \sinh(x)^4 + 8(7(a^6 + 5a^5b + 10a^4b^2 + 10a^3b^3 + 5a^2b^4 + ab^5) \cosh(x)^5 + 10(a^6 + 3a^5b + 2a^4b^2 - 2a^3b^3 - 3a^2b^4 - ab^5) \cosh(x)^3 + (3a^6 + 7a^5b + 6a^4b^2 + 6a^3b^3 + 7a^2b^4 + 3ab^5) \cosh(x)) \sinh(x)^3 + 4(a^6 + 3a^5b + 2a^4b^2 - 2a^3b^3 - 3a^2b^4 - ab^5) \cosh(x)^2 + 4(7(a^6 + 5a^5b + 10a^4b^2 + 10a^3b^3 + 5a^2b^4 + ab^5) \cosh(x)^6 + a^6 + 3a^5b + 2a^4b^2 - 2a^3b^3 - 3a^2b^4 - ab^5 + 15(a^6 + 3a^5b + 2a^4b^2 - 2a^3b^3 - 3a^2b^4 - ab^5) \cosh(x)^4 + 3(3a^6 + 7a^5b + 6a^4b^2 + 6a^3b^3 + 7a^2b^4 + 3ab^5) \cosh(x)^2) \sinh(x)^2 + 8((a^6 + 5a^5b + 10a^4b^2 + 10a^3b^3 + 5a^2b^4 + ab^5) \cosh(x)^7 + 3(a^6 + 3a^5b + 2a^4b^2 - 2a^3b^3 - 3a^2b^4 - ab^5) \cosh(x)^5 + (3a^6 + 7a^5b + 6a^4b^2 + 6a^3b^3 + 7a^2b^4 + 3ab^5) \cosh(x)^3 + (a^6 + 3a^5b + 2a^4b^2 - 2a^3b^3 - 3a^2b^4 - ab^5) \cosh(x)) \sinh(x)]
\end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
 constant sign by intervals (correct if the argument is real):Check [abs(t_n
 ostep+1)]Warning, replacing 0 by `u`, a substitution variable should perha
 ps be purged.Warning, replacing 0 by `u`, a substitution variable should p
 erhaps be purged.Warning, replacing 0 by `u`, a substitution variable shou
 ld perhaps be purged.Evaluation time: 0.5Error: Bad Argument Type

maple [B] time = 0.09, size = 454, normalized size = 5.16

$$\frac{\tanh(x)}{3a(a+b(\tanh^2(x)))^{\frac{3}{2}}} - \frac{2\tanh(x)}{3a^2\sqrt{a+b(\tanh^2(x))}} - \frac{1}{6(a+b)((\tanh(x)-1)^2b+2(\tanh(x)-1)b+a+b)^{\frac{3}{2}}} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x)

[Out]
$$-1/3*\tanh(x)/a/(a+b*\tanh(x)^2)^{(3/2)}-2/3/a^2*\tanh(x)/(a+b*\tanh(x)^2)^{(1/2)}-1/6/(a+b)/((\tanh(x)-1)^{2*b+2*(\tanh(x)-1)*b+a+b})^{(3/2)}+1/6*b/(a+b)/a/((\tanh(x)-1)^{2*b+2*(\tanh(x)-1)*b+a+b})^{(3/2)}*\tanh(x)+1/3*b/(a+b)/a^2/((\tanh(x)-1)^{2*b+2*(\tanh(x)-1)*b+a+b})^{(1/2)}*\tanh(x)-1/2/(a+b)^2/((\tanh(x)-1)^{2*b+2*(\tanh(x)-1)*b+a+b})^{(1/2)}+1/2/(a+b)^2/a/((\tanh(x)-1)^{2*b+2*(\tanh(x)-1)*b+a+b})^{(1/2)})*b*\tanh(x)+1/2/(a+b)^{(5/2)}*\ln((2*a+2*b+2*(\tanh(x)-1)*b+2*(a+b)^{(1/2)}*((\tanh(x)-1)^{2*b+2*(\tanh(x)-1)*b+a+b})^{(1/2)})/(\tanh(x)-1))+1/6/(a+b)/((1+\tanh(x))^{2*b-2*(1+\tanh(x))*b+a+b})^{(3/2)}+1/6*b/(a+b)/a/((1+\tanh(x))^{2*b-2*(1+\tanh(x))*b+a+b})^{(3/2)}*\tanh(x)+1/3*b/(a+b)/a^2/((1+\tanh(x))^{2*b-2*(1+\tanh(x))*b+a+b})^{(1/2)}*\tanh(x)+1/2/(a+b)^2/((1+\tanh(x))^{2*b-2*(1+\tanh(x))*b+a+b})^{(1/2)}+1/2/(a+b)^2/a/((1+\tanh(x))^{2*b-2*(1+\tanh(x))*b+a+b})^{(1/2)})*b*\tanh(x)-1/2/(a+b)^{(5/2)}*\ln((2*a+2*b-2*(1+\tanh(x))*b+2*(a+b)^{(1/2)}*((1+\tanh(x))^{2*b-2*(1+\tanh(x))*b+a+b})^{(1/2)})/(1+\tanh(x)))$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)^2}{(b \tanh(x)^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(tanh(x)^2/(b*tanh(x)^2 + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)^2}{(b \tanh(x)^2 + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)^2/(a + b*tanh(x)^2)^(5/2),x)

[Out] int(tanh(x)^2/(a + b*tanh(x)^2)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh^2(x)}{(a + b \tanh^2(x))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)**2/(a+b*tanh(x)**2)**(5/2), x)
```

```
[Out] Integral(tanh(x)**2/(a + b*tanh(x)**2)**(5/2), x)
```

$$3.251 \quad \int \frac{\tanh(x)}{(a+b \tanh^2(x))^{5/2}} dx$$

Optimal. Leaf size=70

$$-\frac{1}{(a+b)^2 \sqrt{a+b \tanh^2(x)}} - \frac{1}{3(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}$$

[Out] arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)-1/(a+b)^2/(a+b*tanh(x)^2)^(1/2)-1/3/(a+b)/(a+b*tanh(x)^2)^(3/2)

Rubi [A] time = 0.10, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3670, 444, 51, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{1}{(a+b)^2 \sqrt{a+b \tanh^2(x)}} - \frac{1}{3(a+b)(a+b \tanh^2(x))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]/(a + b*Tanh[x]^2)^(5/2), x]

[Out] ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(5/2) - 1/(3*(a + b)*(a + b*Tanh[x]^2)^(3/2)) - 1/((a + b)^2*Sqrt[a + b*Tanh[x]^2])

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 3670

```
Int[((d_)*tan[(e_) + (f_)*(x_)]^(m_))*((a_) + (b_)*((c_)*tan[(e_) +
(f_)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(x)}{(a + b \tanh^2(x))^{5/2}} dx &= \text{Subst} \left(\int \frac{x}{(1-x^2)(a+bx^2)^{5/2}} dx, x, \tanh(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1-x)(a+bx)^{5/2}} dx, x, \tanh^2(x) \right) \\
&= -\frac{1}{3(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{\text{Subst} \left(\int \frac{1}{(1-x)(a+bx)^{3/2}} dx, x, \tanh^2(x) \right)}{2(a+b)} \\
&= -\frac{1}{3(a+b)(a+b \tanh^2(x))^{3/2}} - \frac{1}{(a+b)^2 \sqrt{a+b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{1}{(1-x)\sqrt{a+bx}} dx, x, \tanh^2(x) \right)}{2(a+b)} \\
&= -\frac{1}{3(a+b)(a+b \tanh^2(x))^{3/2}} - \frac{1}{(a+b)^2 \sqrt{a+b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}} dx, x, \tanh^2(x) \right)}{b(a+b)} \\
&= \frac{\tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}} \right)}{(a+b)^{5/2}} - \frac{1}{3(a+b)(a+b \tanh^2(x))^{3/2}} - \frac{1}{(a+b)^2 \sqrt{a+b \tanh^2(x)}}
\end{aligned}$$

Mathematica [C] time = 0.05, size = 43, normalized size = 0.61

$$-\frac{{}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; \frac{b \tanh^2(x)+a}{a+b}\right)}{3(a+b)(a+b \tanh^2(x))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]/(a + b*Tanh[x]^2)^(5/2), x]

[Out] -1/3*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tanh[x]^2)/(a + b)]/((a + b)*(a + b*Tanh[x]^2)^(3/2))

fricas [B] time = 1.40, size = 5779, normalized size = 82.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(5/2), x, algorithm="fricas")

[Out] [1/12*(3*((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*

$$\begin{aligned}
& \cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*\cosh(x)^4 + 30*(a^2 - b^2)*\cosh(x)^2 \\
& + 3*a^2 - 2*a*b + 3*b^2)*\sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*\cosh(x)^5 + 1 \\
& 0*(a^2 - b^2)*\cosh(x)^3 + (3*a^2 - 2*a*b + 3*b^2)*\cosh(x))*\sinh(x)^3 + 4*(a \\
& ^2 - b^2)*\cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*\cosh(x)^6 + 15*(a^2 - b^2)*\c \\
& \cosh(x)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*\cosh(x)^2 + a^2 - b^2)*\sinh(x)^2 + a^2 \\
& + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*\cosh(x)^7 + 3*(a^2 - b^2)*\cosh(x)^5 \\
& + (3*a^2 - 2*a*b + 3*b^2)*\cosh(x)^3 + (a^2 - b^2)*\cosh(x))*\sinh(x))*\sqrt{a \\
& + b)*\log(((a^3 + a^2*b)*\cosh(x)^8 + 8*(a^3 + a^2*b)*\cosh(x))*\sinh(x)^7 + (a \\
& ^3 + a^2*b)*\sinh(x)^8 + 2*(2*a^3 + a^2*b)*\cosh(x)^6 + 2*(2*a^3 + a^2*b + 14 \\
& *(a^3 + a^2*b)*\cosh(x)^2)*\sinh(x)^6 + 4*(14*(a^3 + a^2*b)*\cosh(x)^3 + 3*(2* \\
& a^3 + a^2*b)*\cosh(x))*\sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x)^4 \\
& + (70*(a^3 + a^2*b)*\cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 \\
& + a^2*b)*\cosh(x)^2)*\sinh(x)^4 + 4*(14*(a^3 + a^2*b)*\cosh(x)^5 + 10*(2*a^3 + \\
& a^2*b)*\cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x))*\sinh(x)^3 + a^ \\
& 3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*\cosh(x)^2 + 2*(14*(\\
& a^3 + a^2*b)*\cosh(x)^6 + 15*(2*a^3 + a^2*b)*\cosh(x)^4 + 2*a^3 + 3*a^2*b - b \\
& ^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x)^2)*\sinh(x)^2 + \sqrt{2}*(a^2* \\
& \cosh(x)^6 + 6*a^2*\cosh(x))*\sinh(x)^5 + a^2*\sinh(x)^6 + 3*a^2*\cosh(x)^4 + 3*(\\
& 5*a^2*\cosh(x)^2 + a^2)*\sinh(x)^4 + 4*(5*a^2*\cosh(x)^3 + 3*a^2*\cosh(x))*\sinh \\
& (x)^3 + (3*a^2 + 2*a*b - b^2)*\cosh(x)^2 + (15*a^2*\cosh(x)^4 + 18*a^2*\cosh(x) \\
&)^2 + 3*a^2 + 2*a*b - b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*\cosh(x) \\
& ^5 + 6*a^2*\cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*\cosh(x))*\sinh(x))*\sqrt{a + b)* \\
& \sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x) \\
& *\sinh(x) + \sinh(x)^2))} + 4*(2*(a^3 + a^2*b)*\cosh(x)^7 + 3*(2*a^3 + a^2*b)*\c \\
& \cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*\cosh(x)^3 + (2*a^3 + 3*a^2*b - b \\
& ^3)*\cosh(x))*\sinh(x))/(\cosh(x)^6 + 6*\cosh(x)^5*\sinh(x) + 15*\cosh(x)^4*\sinh(x) \\
& ^2 + 20*\cosh(x)^3*\sinh(x)^3 + 15*\cosh(x)^2*\sinh(x)^4 + 6*\cosh(x)*\sinh(x)^ \\
& 5 + \sinh(x)^6)) + 3*((a^2 + 2*a*b + b^2)*\cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)* \\
& \cosh(x))*\sinh(x)^7 + (a^2 + 2*a*b + b^2)*\sinh(x)^8 + 4*(a^2 - b^2)*\cosh(x)^6 \\
& + 4*(7*(a^2 + 2*a*b + b^2)*\cosh(x)^2 + a^2 - b^2)*\sinh(x)^6 + 8*(7*(a^2 + \\
& 2*a*b + b^2)*\cosh(x)^3 + 3*(a^2 - b^2)*\cosh(x))*\sinh(x)^5 + 2*(3*a^2 - 2*a* \\
& b + 3*b^2)*\cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*\cosh(x)^4 + 30*(a^2 - b^2) \\
& *\cosh(x)^2 + 3*a^2 - 2*a*b + 3*b^2)*\sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*\c \\
& \cosh(x)^5 + 10*(a^2 - b^2)*\cosh(x)^3 + (3*a^2 - 2*a*b + 3*b^2)*\cosh(x))*\sinh(\\
& x)^3 + 4*(a^2 - b^2)*\cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*\cosh(x)^6 + 15*(a \\
& ^2 - b^2)*\cosh(x)^4 + 3*(3*a^2 - 2*a*b + 3*b^2)*\cosh(x)^2 + a^2 - b^2)*\sinh \\
& (x)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*\cosh(x)^7 + 3*(a^2 - b^2) \\
&)*\cosh(x)^5 + (3*a^2 - 2*a*b + 3*b^2)*\cosh(x)^3 + (a^2 - b^2)*\cosh(x))*\sinh \\
& (x))*\sqrt{a + b)*\log(-((a + b)*\cosh(x)^4 + 4*(a + b)*\cosh(x))*\sinh(x)^3 + (a \\
& + b)*\sinh(x)^4 - 2*b*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 - b)*\sinh(x)^2 + s \\
& \sqrt{2}*(\cosh(x)^2 + 2*\cosh(x))*\sinh(x) + \sinh(x)^2 - 1)*\sqrt{a + b)*\sqrt{((a \\
& + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x))*\sinh(x) \\
& + \sinh(x)^2))} + 4*((a + b)*\cosh(x)^3 - b*\cosh(x))*\sinh(x) + a + b)/(\cosh(x) \\
& ^2 + 2*\cosh(x))*\sinh(x) + \sinh(x)^2)) - 16*\sqrt{2}*((a^2 + 2*a*b + b^2)*\cos \\
& h(x)^6 + 6*(a^2 + 2*a*b + b^2)*\cosh(x))*\sinh(x)^5 + (a^2 + 2*a*b + b^2)*\sinh \\
& (x)^6 + 3*(a^2 + a*b)*\cosh(x)^4 + 3*(5*(a^2 + 2*a*b + b^2)*\cosh(x)^2 + a^2 \\
& + a*b)*\sinh(x)^4 + 4*(5*(a^2 + 2*a*b + b^2)*\cosh(x)^3 + 3*(a^2 + a*b)*\cosh(\\
& x))*\sinh(x)^3 + 3*(a^2 + a*b)*\cosh(x)^2 + 3*(5*(a^2 + 2*a*b + b^2)*\cosh(x)^ \\
& 4 + 6*(a^2 + a*b)*\cosh(x)^2 + a^2 + a*b)*\sinh(x)^2 + a^2 + 2*a*b + b^2 + 6* \\
& ((a^2 + 2*a*b + b^2)*\cosh(x)^5 + 2*(a^2 + a*b)*\cosh(x)^3 + (a^2 + a*b)*\cosh \\
& (x))*\sinh(x))*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x) \\
& ^2 - 2*\cosh(x))*\sinh(x) + \sinh(x)^2)))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2 \\
& *b^3 + 5*a*b^4 + b^5)*\cosh(x)^8 + 8*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^ \\
& 3 + 5*a*b^4 + b^5)*\cosh(x))*\sinh(x)^7 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2 \\
& *b^3 + 5*a*b^4 + b^5)*\sinh(x)^8 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 \\
& - 3*a*b^4 - b^5)*\cosh(x)^6 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a \\
& *b^4 - b^5 + 7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*\co \\
& sh(x)^2)*\sinh(x)^6 + 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^ \\
& 4 + b^5)*\cosh(x)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b
\end{aligned}$$


```
*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)*sinh(x)^7 + (a^5 + 5*a^4*b + 10*a^3*b^2 +
  10*a^2*b^3 + 5*a*b^4 + b^5)*sinh(x)^8 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a
^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^6 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^
3 - 3*a*b^4 - b^5 + 7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 +
b^5)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 +
5*a*b^4 + b^5)*cosh(x)^3 + 3*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*
b^4 - b^5)*cosh(x))*sinh(x)^5 + a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5
*a*b^4 + b^5 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5
)*cosh(x)^4 + 2*(3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5
+ 35*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^4 +
30*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^2)*sinh(
x)^4 + 8*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(
x)^5 + 10*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^3
+ (3*a^5 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(x))*sin
h(x)^3 + 4*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^
2 + 4*(7*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^
6 + a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5 + 15*(a^5 + 3*a^4
*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^4 + 3*(3*a^5 + 7*a^4*b
+ 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(x)^2)*sinh(x)^2 + 8*((a^5 +
5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*cosh(x)^7 + 3*(a^5 + 3*
a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x)^5 + (3*a^5 + 7*a^4*b
+ 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*cosh(x)^3 + (a^5 + 3*a^4*b + 2*
a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*cosh(x))*sinh(x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep+1)]Warning, replacing 0 by `u`, a substitution variable should perha
ps be purged.Warning, replacing 0 by `u`, a substitution variable should p
erhaps be purged.Warning, replacing 0 by `u`, a substitution variable shou
ld perhaps be purged.Evaluation time: 0.56Error: Bad Argument Type
```

maple [B] time = 0.08, size = 420, normalized size = 6.00

$$\frac{1}{6(a+b)\left((\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a + b\right)^{\frac{3}{2}}} + \frac{b \tanh(x)}{6(a+b)a\left((\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a + b\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(x)/(a+b*tanh(x)^2)^(5/2),x)
```

```
[Out] -1/6/(a+b)/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(3/2)+1/6*b/(a+b)/a/((tanh
(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/((tanh(x)-1)^
2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/((tanh(x)-1)^2*b+2*(tanh
(x)-1)*b+a+b)^(1/2)+1/2/(a+b)^2/a/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/
2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2))*((ta
nh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))-1/6/(a+b)/((1+tanh(x)
)^2*b-2*(1+tanh(x))*b+a+b)^(3/2)-1/6*b/(a+b)/a/((1+tanh(x))^2*b-2*(1+tanh(x)
)*b+a+b)^(3/2)*tanh(x)-1/3*b/(a+b)/a^2/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+
b)^(1/2)*tanh(x)-1/2/(a+b)^2/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)-1/
2/(a+b)^2/a/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)
```

$$\frac{\ln((2a+2b-2(1+\tanh(x))^b+2(a+b)^{1/2}((1+\tanh(x))^b-2(1+\tanh(x))^b+a+b)^{1/2})/(1+\tanh(x)))^{5/2}}{(b \tanh(x)^2 + a)^{5/2}}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{(b \tanh(x)^2 + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(tanh(x)/(b*tanh(x)^2 + a)^(5/2), x)

mupad [B] time = 3.56, size = 76, normalized size = 1.09

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{b \tanh(x)^2 + a} (2a^2 + 4ab + 2b^2)}{2(a+b)^{5/2}}\right)}{(a+b)^{5/2}} - \frac{\frac{1}{3(a+b)} + \frac{b \tanh(x)^2 + a}{(a+b)^2}}{(b \tanh(x)^2 + a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)/(a + b*tanh(x)^2)^(5/2),x)

[Out] $\operatorname{atanh}\left(\frac{(a + b \tanh(x)^2)^{1/2} (4ab + 2a^2 + 2b^2)}{2(a + b)^{5/2}}\right) / (a + b)^{5/2} - \frac{1}{3(a + b)} + \frac{a + b \tanh(x)^2}{(a + b)^2} / (a + b \tanh(x)^2)^{3/2}$

sympy [A] time = 30.76, size = 73, normalized size = 1.04

$$-\frac{1}{3(a+b)(a+b \tanh^2(x))^{3/2}} - \frac{1}{(a+b)^2 \sqrt{a+b \tanh^2(x)}} - \frac{\operatorname{atan}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b} (a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*tanh(x)**2)**(5/2),x)

[Out] $-1/(3(a+b)(a+b \tanh(x)**2)**(3/2)) - 1/((a+b)**2 \sqrt{a+b \tanh(x)**2}) - \operatorname{atan}(\sqrt{a+b \tanh(x)**2}/\sqrt{-a-b})/(\sqrt{-a-b} (a+b)**2)$

$$3.252 \quad \int \frac{1}{(a+b \tanh^2(x))^{5/2}} dx$$

Optimal. Leaf size=93

$$\frac{b(5a+2b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{b\tanh(x)}{3a(a+b)(a+b\tanh^2(x))^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}$$

[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(5/2)+1/3*b*(5*a+2*b)*tanh(x)/a^2/(a+b)^2/(a+b*tanh(x)^2)^(1/2)+1/3*b*tanh(x)/a/(a+b)/(a+b*tanh(x)^2)^(3/2)

Rubi [A] time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3661, 414, 527, 12, 377, 206}

$$\frac{b(5a+2b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{b\tanh(x)}{3a(a+b)(a+b\tanh^2(x))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tanh[x]^2)^(-5/2), x]

[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(5/2) + (b*Tanh[x])/(3*a*(a + b)*(a + b*Tanh[x]^2)^(3/2)) + (b*(5*a + 2*b)*Tanh[x])/(3*a^2*(a + b)^2*Sqrt[a + b*Tanh[x]^2])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1)/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 527


```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3661

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :=
With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(
ff*x)^n]^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a,
b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || E
qQ[n^2, 16])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + b \tanh^2(x))^{5/2}} dx &= \text{Subst} \left(\int \frac{1}{(1-x^2)(a+bx^2)^{5/2}} dx, x, \tanh(x) \right) \\ &= \frac{b \tanh(x)}{3a(a+b)(a+b \tanh^2(x))^{3/2}} - \frac{\text{Subst} \left(\int \frac{b-3(a+b)+2bx^2}{(1-x^2)(a+bx^2)^{3/2}} dx, x, \tanh(x) \right)}{3a(a+b)} \\ &= \frac{b \tanh(x)}{3a(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{b(5a+2b) \tanh(x)}{3a^2(a+b)^2 \sqrt{a+b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{3a^2}{(1-x^2)\sqrt{a+bx^2}} dx, x, \tanh(x) \right)}{3a^2(a+b)} \\ &= \frac{b \tanh(x)}{3a(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{b(5a+2b) \tanh(x)}{3a^2(a+b)^2 \sqrt{a+b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}} dx, x, \tanh(x) \right)}{(a+b)} \\ &= \frac{b \tanh(x)}{3a(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{b(5a+2b) \tanh(x)}{3a^2(a+b)^2 \sqrt{a+b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{1}{1-(a+b)x^2} dx, x, \tanh(x) \right)}{(a+b)} \\ &= \frac{\tanh^{-1} \left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right)}{(a+b)^{5/2}} + \frac{b \tanh(x)}{3a(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{b(5a+2b) \tanh(x)}{3a^2(a+b)^2 \sqrt{a+b \tanh^2(x)}} \end{aligned}$$

Mathematica [C] time = 7.71, size = 976, normalized size = 10.49

$$\cosh(x) \sinh(x) \left(\frac{840b^2(a+b)^2 \sin^{-1} \left(\sqrt{\frac{(a+b) \sinh^2(x)}{a}} \right) \tanh^4(x) \sinh^4(x)}{a^4} + \frac{2100b(a+b)^2 \sin^{-1} \left(\sqrt{\frac{(a+b) \sinh^2(x)}{a}} \right) \tanh^2(x) \sinh^4(x)}{a^3} + \dots \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Tanh[x]^2)^(-5/2), x]
```

```
[Out] (Cosh[x]*Sinh[x]*(1575*ArcSin[Sqrt[-((a + b)*Sinh[x]^2)/a]]) + (3150*(a +
b)*ArcSin[Sqrt[-((a + b)*Sinh[x]^2)/a]])*Sinh[x]^2/a + (1575*(a + b)^2*Ar
cSin[Sqrt[-((a + b)*Sinh[x]^2)/a]])*Sinh[x]^4/a^2 + (2100*b*ArcSin[Sqrt[-
((a + b)*Sinh[x]^2)/a]])*Tanh[x]^2/a + (4200*b*(a + b)*ArcSin[Sqrt[-((a
+ b)*Sinh[x]^2)/a]])*Sinh[x]^2*Tanh[x]^2/a^2 + (2100*b*(a + b)^2*ArcSin[Sq
rt[-((a + b)*Sinh[x]^2)/a]])*Sinh[x]^4*Tanh[x]^2/a^3 + (840*b^2*ArcSin[Sq
rt[-((a + b)*Sinh[x]^2)/a]])*Tanh[x]^4/a^2 + (1680*b^2*(a + b)*ArcSin[Sqr
t[-((a + b)*Sinh[x]^2)/a]])*Sinh[x]^2*Tanh[x]^4/a^3 + (840*b^2*(a + b)^2*
ArcSin[Sqrt[-((a + b)*Sinh[x]^2)/a]])*Sinh[x]^4*Tanh[x]^4/a^4 + 2100*(-((
(a + b)*Sinh[x]^2)/a))^(3/2)*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a] + 96*Hyp
ergeometric2F1[2, 2, 9/2, -((a + b)*Sinh[x]^2)/a]*(-((a + b)*Sinh[x]^2)/
a))^(7/2)*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a] + 24*HypergeometricPFQ[{2,
2, 2}, {1, 9/2}, -((a + b)*Sinh[x]^2)/a]*(-((a + b)*Sinh[x]^2)/a))^(7/2)
*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a] + (2800*b*(-((a + b)*Sinh[x]^2)/a)
^(3/2)*Tanh[x]^2*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a])/a + (168*b*Hypergeo
metric2F1[2, 2, 9/2, -((a + b)*Sinh[x]^2)/a])*(-((a + b)*Sinh[x]^2)/a)^(
7/2)*Tanh[x]^2*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a])/a + (48*b*Hypergeomet
ricPFQ[{2, 2, 2}, {1, 9/2}, -((a + b)*Sinh[x]^2)/a])*(-((a + b)*Sinh[x]^2
)/a))^(7/2)*Tanh[x]^2*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a])/a + (1120*b^2*
(-((a + b)*Sinh[x]^2)/a))^(3/2)*Tanh[x]^4*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2
))/a])/a^2 + (72*b^2*Hypergeometric2F1[2, 2, 9/2, -((a + b)*Sinh[x]^2)/a]
*(-((a + b)*Sinh[x]^2)/a))^(7/2)*Tanh[x]^4*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^
2))/a])/a^2 + (24*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, -((a + b)*Sin
h[x]^2)/a])*(-((a + b)*Sinh[x]^2)/a))^(7/2)*Tanh[x]^4*Sqrt[(Cosh[x]^2*(a +
b*Tanh[x]^2))/a])/a^2 - 1575*Sqrt[-((a + b)*Cosh[x]^2*Sinh[x]^2*(a + b*Ta
nh[x]^2))/a^2]] - (2100*b*Tanh[x]^2*Sqrt[-((a + b)*Cosh[x]^2*Sinh[x]^2*(a
+ b*Tanh[x]^2))/a^2])/a - (840*b^2*Tanh[x]^4*Sqrt[-((a + b)*Cosh[x]^2*Sin
h[x]^2*(a + b*Tanh[x]^2))/a^2])/a^2)/(315*a^2*(-((a + b)*Sinh[x]^2)/a))^(
5/2)*Sqrt[a + b*Tanh[x]^2]*Sqrt[(Cosh[x]^2*(a + b*Tanh[x]^2))/a]*(1 + (b*T
anh[x]^2)/a))
```

fricas [B] time = 1.48, size = 6933, normalized size = 74.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^8 + 8*(a^4 + 2*a^3*b + a^2*b^2)
*cosh(x)*sinh(x)^7 + (a^4 + 2*a^3*b + a^2*b^2)*sinh(x)^8 + 4*(a^4 - a^2*b^2
)*cosh(x)^6 + 4*(a^4 - a^2*b^2 + 7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^2)*sin
h(x)^6 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*cosh(x)^3 + 3*(a^4 - a^2*b^2)*cosh(
x))*sinh(x)^5 + 2*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*cosh(x)^4 + 2*(35*(a^4 + 2*
a^3*b + a^2*b^2)*cosh(x)^4 + 3*a^4 - 2*a^3*b + 3*a^2*b^2 + 30*(a^4 - a^2*b^
2)*cosh(x)^2)*sinh(x)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*(a^4 + 2*a^3*b + a
^2*b^2)*cosh(x)^5 + 10*(a^4 - a^2*b^2)*cosh(x)^3 + (3*a^4 - 2*a^3*b + 3*a^2
*b^2)*cosh(x))*sinh(x)^3 + 4*(a^4 - a^2*b^2)*cosh(x)^2 + 4*(7*(a^4 + 2*a^3*
b + a^2*b^2)*cosh(x)^6 + 15*(a^4 - a^2*b^2)*cosh(x)^4 + a^4 - a^2*b^2 + 3*(
3*a^4 - 2*a^3*b + 3*a^2*b^2)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 + 2*a^3*b + a^2
*b^2)*cosh(x)^7 + 3*(a^4 - a^2*b^2)*cosh(x)^5 + (3*a^4 - 2*a^3*b + 3*a^2*b^
2)*cosh(x)^3 + (a^4 - a^2*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*log(-((a*b^2 +
b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)
^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh
(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x
))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3
)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)
*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 +
(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b
^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)
)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b
```

$$\begin{aligned}
& + 4ab^2 + 6b^3) \cosh(x)^2 \sinh(x)^2 + \sqrt{2} (b^2 \cosh(x)^6 + 6b^2 \cosh(x) \sinh(x)^5 + b^2 \sinh(x)^6 - 3b^2 \cosh(x)^4 + 3(5b^2 \cosh(x)^2 - b^2) \sinh(x)^4 + 4(5b^2 \cosh(x)^3 - 3b^2 \cosh(x)) \sinh(x)^3 - (a^2 - 2ab - 3b^2) \cosh(x)^2 + (15b^2 \cosh(x)^4 - 18b^2 \cosh(x)^2 - a^2 + 2ab + 3b^2) \sinh(x)^2 - a^2 - 2ab - b^2 + 2(3b^2 \cosh(x)^5 - 6b^2 \cosh(x)^3 - (a^2 - 2ab - 3b^2) \cosh(x)) \sinh(x)) \sqrt{a+b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a-b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)} + 4(2(ab^2 + b^3) \cosh(x)^7 - 3(ab^2 + 2b^3) \cosh(x)^5 + (a^3 - a^2b + 4ab^2 + 6b^3) \cosh(x)^3 + (a^3 - 3a^2b - 2b^3) \cosh(x)) \sinh(x) / (\cosh(x)^6 + 6 \cosh(x)^5 \sinh(x) + 15 \cosh(x)^4 \sinh(x)^2 + 20 \cosh(x)^3 \sinh(x)^3 + 15 \cosh(x)^2 \sinh(x)^4 + 6 \cosh(x) \sinh(x)^5 + \sinh(x)^6) + 3((a^4 + 2a^3b + a^2b^2) \cosh(x)^8 + 8(a^4 + 2a^3b + a^2b^2) \cosh(x) \sinh(x)^7 + (a^4 + 2a^3b + a^2b^2) \sinh(x)^8 + 4(a^4 - a^2b^2) \cosh(x)^6 + 4(a^4 - a^2b^2 + 7(a^4 + 2a^3b + a^2b^2) \cosh(x)^2) \sinh(x)^6 + 8(7(a^4 + 2a^3b + a^2b^2) \cosh(x)^3 + 3(a^4 - a^2b^2) \cosh(x)) \sinh(x)^5 + 2(3a^4 - 2a^3b + 3a^2b^2) \cosh(x)^4 + 2(35(a^4 + 2a^3b + a^2b^2) \cosh(x)^4 + 3a^4 - 2a^3b + 3a^2b^2 + 30(a^4 - a^2b^2) \cosh(x)^2) \sinh(x)^4 + a^4 + 2a^3b + a^2b^2 + 8(7(a^4 + 2a^3b + a^2b^2) \cosh(x)^5 + 10(a^4 - a^2b^2) \cosh(x)^3 + (3a^4 - 2a^3b + 3a^2b^2) \cosh(x)) \sinh(x)^3 + 4(a^4 - a^2b^2) \cosh(x)^2 + 4(7(a^4 + 2a^3b + a^2b^2) \cosh(x)^6 + 15(a^4 - a^2b^2) \cosh(x)^4 + a^4 - a^2b^2 + 3(3a^4 - 2a^3b + 3a^2b^2) \cosh(x)^2) \sinh(x)^2 + 8((a^4 + 2a^3b + a^2b^2) \cosh(x)^7 + 3(a^4 - a^2b^2) \cosh(x)^5 + (3a^4 - 2a^3b + 3a^2b^2) \cosh(x)^3 + (a^4 - a^2b^2) \cosh(x)) \sinh(x)) \sqrt{a+b} \log(((a+b) \cosh(x)^4 + 4(a+b) \cosh(x) \sinh(x)^3 + (a+b) \sinh(x)^4 + 2a \cosh(x)^2 + 2(3(a+b) \cosh(x)^2 + a) \sinh(x)^2 + \sqrt{2} (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1) \sqrt{a+b} \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a-b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)} + 4((a+b) \cosh(x)^3 + a \cosh(x)) \sinh(x) + a+b) / (\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2)) + 8\sqrt{2} ((3a^3b + 7a^2b^2 + 5ab^3 + b^4) \cosh(x)^6 + 6(3a^3b + 7a^2b^2 + 5ab^3 + b^4) \cosh(x) \sinh(x)^5 + (3a^3b + 7a^2b^2 + 5ab^3 + b^4) \sinh(x)^6 + 3(a^3b - a^2b^2 - 3ab^3 - b^4) \cosh(x)^4 + 3(a^3b - a^2b^2 - 3ab^3 - b^4 + 5(3a^3b + 7a^2b^2 + 5ab^3 + b^4) \cosh(x)^2) \sinh(x)^4 - 3a^3b - 7a^2b^2 - 5ab^3 - b^4 + 4(5(3a^3b + 7a^2b^2 + 5ab^3 + b^4) \cosh(x)^3 + 3(a^3b - a^2b^2 - 3ab^3 - b^4) \cosh(x)) \sinh(x)^3 - 3(a^3b - a^2b^2 - 3ab^3 - b^4) \cosh(x)^2 + 3(5(3a^3b + 7a^2b^2 + 5ab^3 + b^4) \cosh(x)^4 - a^3b + a^2b^2 + 3ab^3 + b^4 + 6(a^3b - a^2b^2 - 3ab^3 - b^4) \cosh(x)^2) \sinh(x)^2 + 6((3a^3b + 7a^2b^2 + 5ab^3 + b^4) \cosh(x)^5 + 2(a^3b - a^2b^2 - 3ab^3 - b^4) \cosh(x)^3 - (a^3b - a^2b^2 - 3ab^3 - b^4) \cosh(x)) \sinh(x)) \sqrt{((a+b) \cosh(x)^2 + (a+b) \sinh(x)^2 + a-b) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)} / ((a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 + 5a^3b^4 + a^2b^5) \cosh(x)^8 + 8(a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 + 5a^3b^4 + a^2b^5) \cosh(x) \sinh(x)^7 + (a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 + 5a^3b^4 + a^2b^5) \sinh(x)^8 + a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 + 5a^3b^4 + a^2b^5 + 4(a^7 + 3a^6b + 2a^5b^2 - 2a^4b^3 - 3a^3b^4 - a^2b^5) \cosh(x)^6 + 4(a^7 + 3a^6b + 2a^5b^2 - 2a^4b^3 - 3a^3b^4 - a^2b^5 + 7(a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 + 5a^3b^4 + a^2b^5) \cosh(x)^2) \sinh(x)^6 + 8(7(a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 + 5a^3b^4 + a^2b^5) \cosh(x)^3 + 3(a^7 + 3a^6b + 2a^5b^2 - 2a^4b^3 - 3a^3b^4 - a^2b^5) \cosh(x)) \sinh(x)^5 + 2(3a^7 + 7a^6b + 6a^5b^2 + 6a^4b^3 + 7a^3b^4 + 3a^2b^5) \cosh(x)^4 + 2(3a^7 + 7a^6b + 6a^5b^2 + 6a^4b^3 + 7a^3b^4 + 3a^2b^5) \cosh(x)^2) \sinh(x)^4 + 8(7(a^7 + 5a^6b + 10a^5b^2 + 10a^4b^3 + 5a^3b^4 + a^2b^5) \cosh(x)^5 + 10(a^7 + 3a^6b + 2a^5b^2 - 2a^4b^3 - 3a^3b^4 - a^2b^5) \cosh(x)^3 + (3a^7 + 7a^6b + 6a^5b^2 + 6a^4b^3 + 7a^3b^4 + 3a^2b^5) \cosh(x)) \sinh(x)^3 + 4(a^7 + 3a^6b + 2a^5b^2 - 2a^4b^3 - 3a^3b^4 - a^2b^5) \cosh(x)^2
\end{aligned}$$

$$\begin{aligned}
& + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5 + 7*(a^7 \\
& + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*\cosh(x)^6 + 15*(\\
& a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*\cosh(x)^4 + 3* \\
& (3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*\cosh(x)^2 \\
&)*\sinh(x)^2 + 8*((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2 \\
& *b^5)*\cosh(x)^7 + 3*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^ \\
& 2*b^5)*\cosh(x)^5 + (3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3 \\
& *a^2*b^5)*\cosh(x)^3 + (a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - \\
& a^2*b^5)*\cosh(x))*\sinh(x)), -1/6*(3*((a^4 + 2*a^3*b + a^2*b^2)*\cosh(x)^8 + \\
& 8*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(x)*\sinh(x)^7 + (a^4 + 2*a^3*b + a^2*b^2)*\sinh(x)^8 \\
& + 4*(a^4 - a^2*b^2)*\cosh(x)^6 + 4*(a^4 - a^2*b^2 + 7*(a^4 + 2*a^3*b \\
& b + a^2*b^2)*\cosh(x)^2)*\sinh(x)^6 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(x)^ \\
& 3 + 3*(a^4 - a^2*b^2)*\cosh(x))*\sinh(x)^5 + 2*(3*a^4 - 2*a^3*b + 3*a^2*b^2)* \\
& \cosh(x)^4 + 2*(35*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(x)^4 + 3*a^4 - 2*a^3*b + 3 \\
& *a^2*b^2 + 30*(a^4 - a^2*b^2)*\cosh(x)^2)*\sinh(x)^4 + a^4 + 2*a^3*b + a^2*b^ \\
& 2 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(x)^5 + 10*(a^4 - a^2*b^2)*\cosh(x)^3 \\
& + (3*a^4 - 2*a^3*b + 3*a^2*b^2)*\cosh(x))*\sinh(x)^3 + 4*(a^4 - a^2*b^2)*\cosh(x)^2 \\
& + 4*(7*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(x)^6 + 15*(a^4 - a^2*b^2)*\cosh(x)^4 \\
& + a^4 - a^2*b^2 + 3*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*\cosh(x)^2)*\sinh(x)^2 \\
& + 8*((a^4 + 2*a^3*b + a^2*b^2)*\cosh(x)^7 + 3*(a^4 - a^2*b^2)*\cosh(x)^5 + \\
& (3*a^4 - 2*a^3*b + 3*a^2*b^2)*\cosh(x)^3 + (a^4 - a^2*b^2)*\cosh(x))*\sinh(x) \\
&)*\sqrt{-a - b}*\arctan(\sqrt{2}*(b*\cosh(x)^2 + 2*b*\cosh(x)*\sinh(x) + b*\sinh(x) \\
& ^2 - a - b)*\sqrt{-a - b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - \\
& b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)})/((a*b + b^2)*\cosh(x)^4 + 4* \\
& (a*b + b^2)*\cosh(x)*\sinh(x)^3 + (a*b + b^2)*\sinh(x)^4 + (a^2 - a*b - 2*b^2) \\
& *\cosh(x)^2 + (6*(a*b + b^2)*\cosh(x)^2 + a^2 - a*b - 2*b^2)*\sinh(x)^2 + a^2 \\
& + 2*a*b + b^2 + 2*(2*(a*b + b^2)*\cosh(x)^3 + (a^2 - a*b - 2*b^2)*\cosh(x))*\sinh(x) \\
&)) + 3*((a^4 + 2*a^3*b + a^2*b^2)*\cosh(x)^8 + 8*(a^4 + 2*a^3*b + a^2*b^2) \\
& *b^2)*\cosh(x)*\sinh(x)^7 + (a^4 + 2*a^3*b + a^2*b^2)*\sinh(x)^8 + 4*(a^4 - a^2 \\
& *b^2)*\cosh(x)^6 + 4*(a^4 - a^2*b^2 + 7*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(x)^2) \\
& *\sinh(x)^6 + 8*(7*(a^4 + 2*a^3*b + a^2*b^2)*\cosh(x)^3 + 3*(a^4 - a^2*b^2)*\cosh(x) \\
&)*\sinh(x)^5 + 2*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*\cosh(x)^4 + 2*(35*(a^4 \\
& + 2*a^3*b + a^2*b^2)*\cosh(x)^4 + 3*a^4 - 2*a^3*b + 3*a^2*b^2 + 30*(a^4 - a^ \\
& 2*b^2)*\cosh(x)^2)*\sinh(x)^4 + a^4 + 2*a^3*b + a^2*b^2 + 8*(7*(a^4 + 2*a^3*b \\
& + a^2*b^2)*\cosh(x)^5 + 10*(a^4 - a^2*b^2)*\cosh(x)^3 + (3*a^4 - 2*a^3*b + 3 \\
& *a^2*b^2)*\cosh(x))*\sinh(x)^3 + 4*(a^4 - a^2*b^2)*\cosh(x)^2 + 4*(7*(a^4 + 2* \\
& a^3*b + a^2*b^2)*\cosh(x)^6 + 15*(a^4 - a^2*b^2)*\cosh(x)^4 + a^4 - a^2*b^2 + \\
& 3*(3*a^4 - 2*a^3*b + 3*a^2*b^2)*\cosh(x)^2)*\sinh(x)^2 + 8*((a^4 + 2*a^3*b + \\
& a^2*b^2)*\cosh(x)^7 + 3*(a^4 - a^2*b^2)*\cosh(x)^5 + (3*a^4 - 2*a^3*b + 3*a^ \\
& 2*b^2)*\cosh(x)^3 + (a^4 - a^2*b^2)*\cosh(x))*\sinh(x))*\sqrt{-a - b}*\arctan(\sqrt{2} \\
&)*\sqrt{-a - b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x) \\
& *\sinh(x) + \sinh(x)^2)})/((a + b)*\cosh(x)^2 + 2*(a + b)*\cosh(x)*\sinh(x) + (a + b) \\
& *\sinh(x)^2 + a + b)) - 4*\sqrt{2}*((3*a^3*b + 7*a^2*b^2 + 5*a*b^3 + b^4)*\cosh(x)^6 + 6*(3*a^3*b \\
& + 7*a^2*b^2 + 5*a*b^3 + b^4)*\cosh(x)*\sinh(x)^5 + (3*a^3*b + 7*a^2*b^2 + 5*a*b^3 + b^4) \\
& *\sinh(x)^6 + 3*(a^3*b - a^2*b^2 - 3*a*b^3 - b^4)*\cosh(x)^4 + 3*(a^3*b - a^2*b^2 - 3*a*b^3 - b^4 \\
& + 5*(3*a^3*b + 7*a^2*b^2 + 5*a*b^3 + b^4)*\cosh(x)^2)*\sinh(x)^4 - 3*a^3*b - 7*a^2*b^2 - 5*a*b^3 - b^4 \\
& + 4*(5*(3*a^3*b + 7*a^2*b^2 + 5*a*b^3 + b^4)*\cosh(x)^3 + 3*(a^3*b - a^2*b^2 - 3*a*b^3 - b^4) \\
& *\cosh(x))*\sinh(x)^3 - 3*(a^3*b - a^2*b^2 - 3*a*b^3 - b^4)*\cosh(x)^2 + 3*(5*(3*a^3*b + 7*a^2*b^2 \\
& + 5*a*b^3 + b^4)*\cosh(x)^4 - a^3*b + a^2*b^2 + 3*a*b^3 + b^4 + 6*(a^3*b - a^2*b^2 - 3*a*b^3 - b^4) \\
& *\cosh(x)^2)*\sinh(x)^2 + 6*((3*a^3*b + 7*a^2*b^2 + 5*a*b^3 + b^4)*\cosh(x)^5 + 2*(a^3*b - a^2*b^2 - 3*a*b^3 - b^4) \\
& *\cosh(x)^3 - (a^3*b - a^2*b^2 - 3*a*b^3 - b^4)*\cosh(x))*\sinh(x))*\sqrt{((a + b)*\cosh(x)^2 + (a + b) \\
& *\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)})/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5) \\
& *\cosh(x)^8 + 8*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*\cosh(x)*\sinh(x)^7 + \\
& (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*\sinh(x)^8 + a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5 + 4*(a^7 +
\end{aligned}$$

```

3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*cosh(x)^6 + 4*(a^7 +
  3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5 + 7*(a^7 + 5*a^6*b +
  10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cosh(x)^2)*sinh(x)^6 + 8*(7
*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cosh(x)^3
+ 3*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*cosh(x))*
sinh(x)^5 + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*
b^5)*cosh(x)^4 + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3
*a^2*b^5 + 35*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^
5)*cosh(x)^4 + 30*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*
b^5)*cosh(x)^2)*sinh(x)^4 + 8*(7*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 +
  5*a^3*b^4 + a^2*b^5)*cosh(x)^5 + 10*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3
  - 3*a^3*b^4 - a^2*b^5)*cosh(x)^3 + (3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^
  3 + 7*a^3*b^4 + 3*a^2*b^5)*cosh(x))*sinh(x)^3 + 4*(a^7 + 3*a^6*b + 2*a^5*b^
  2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*cosh(x)^2 + 4*(a^7 + 3*a^6*b + 2*a^5*b^
  ^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5 + 7*(a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a
  ^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cosh(x)^6 + 15*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2
  *a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*cosh(x)^4 + 3*(3*a^7 + 7*a^6*b + 6*a^5*b^2
  + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*cosh(x)^2)*sinh(x)^2 + 8*((a^7 + 5*a^6
  *b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cosh(x)^7 + 3*(a^7 + 3*
  a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*cosh(x)^5 + (3*a^7 + 7
  *a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*cosh(x)^3 + (a^7 +
  3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*cosh(x))*sinh(x))]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
 constant sign by intervals (correct if the argument is real):Check [abs(t_n
 ostep+1)]Warning, replacing 0 by `u`, a substitution variable should perha
 ps be purged.Warning, replacing 0 by `u`, a substitution variable should p
 erhaps be purged.Warning, replacing 0 by `u`, a substitution variable shou
 ld perhaps be purged.Evaluation time: 0.55Error: Bad Argument Type

maple [B] time = 0.10, size = 420, normalized size = 4.52

$$\frac{1}{6(a+b)\left((\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a + b\right)^{\frac{3}{2}}} + \frac{b \tanh(x)}{6(a+b)a\left((\tanh(x)-1)^2 b + 2(\tanh(x)-1)b + a + b\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*tanh(x)^2)^(5/2),x)

[Out] -1/6/(a+b)/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(3/2)+1/6*b/(a+b)/a/((tanh
 (x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/((tanh(x)-1)^
 2*b+2*(tanh(x)-1)*b+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/((tanh(x)-1)^2*b+2*(tanh
 (x)-1)*b+a+b)^(1/2)+1/2/(a+b)^2/a/((tanh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/
 2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b+2*(tanh(x)-1)*b+2*(a+b)^(1/2))*((ta
 nh(x)-1)^2*b+2*(tanh(x)-1)*b+a+b)^(1/2))/(tanh(x)-1))+1/6/(a+b)/((1+tanh(x)
)^2*b-2*(1+tanh(x))*b+a+b)^(3/2)+1/6*b/(a+b)/a/((1+tanh(x))^2*b-2*(1+tanh(x)
))*b+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+
 b)^(1/2)*tanh(x)+1/2/(a+b)^2/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)+1/
 2/(a+b)^2/a/((1+tanh(x))^2*b-2*(1+tanh(x))*b+a+b)^(1/2)*b*tanh(x)-1/2/(a+b)

$^{(5/2)} \cdot \ln\left(\frac{(2a+2b-2(1+\tanh(x))b+2(a+b)^{1/2}) \cdot ((1+\tanh(x))^{2b-2(1+\tanh(x))b+a+b)^{1/2}}{(1+\tanh(x))}\right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b \tanh(x)^2 + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")

[Out] integrate((b*tanh(x)^2 + a)^(-5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(b \tanh(x)^2 + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*tanh(x)^2)^(5/2),x)

[Out] int(1/(a + b*tanh(x)^2)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \tanh^2(x))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*tanh(x)**2)**(5/2),x)

[Out] Integral((a + b*tanh(x)**2)**(-5/2), x)

$$3.253 \quad \int \frac{\coth(x)}{(a+b \tanh^2(x))^{5/2}} dx$$

Optimal. Leaf size=108

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b \tanh^2(x)}} + \frac{b}{3a(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}$$

[Out] $-\operatorname{arctanh}\left(\frac{(a+b \tanh(x)^2)^{1/2}}{a^{1/2}}\right)/a^{5/2} + \operatorname{arctanh}\left(\frac{(a+b \tanh(x)^2)^{1/2}}{(a+b)^{1/2}}\right)/(a+b)^{5/2} + b(2a+b)/a^2(a+b)^2(a+b \tanh(x)^2)^{1/2} + 1/3 * b/a/(a+b)/(a+b \tanh(x)^2)^{3/2}$

Rubi [A] time = 0.21, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3670, 446, 85, 152, 156, 63, 208}

$$\frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b \tanh^2(x)}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{b}{3a(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]/(a + b*Tanh[x]^2)^(5/2), x]

[Out] $-(\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b \operatorname{Tanh}[x]^2]/\operatorname{Sqrt}[a]]/a^{5/2}) + \operatorname{ArcTanh}[\operatorname{Sqrt}[a + b \operatorname{Tanh}[x]^2]/\operatorname{Sqrt}[a + b]]/(a + b)^{5/2} + b/(3a(a + b)(a + b \operatorname{Tanh}[x]^2)^{3/2}) + (b(2a + b))/(a^2(a + b)^2 \operatorname{Sqrt}[a + b \operatorname{Tanh}[x]^2])$

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 85

Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Simp[(f*(e + f*x)^(p + 1))/((p + 1)*(b*e - a*f)*(d*e - c*f)), x] + Dist[1/((b*e - a*f)*(d*e - c*f)), Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*(e + f*x)^(p + 1)/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[p, -1]

Rule 152

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h]*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[2*m, 2*n, 2*p]

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x],
x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(x)}{(a + b \tanh^2(x))^{5/2}} dx &= \text{Subst} \left(\int \frac{1}{x(1-x^2)(a+bx^2)^{5/2}} dx, x, \tanh(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1-x)x(a+bx)^{5/2}} dx, x, \tanh^2(x) \right) \\
&= \frac{b}{3a(a+b)(a+b \tanh^2(x))^{3/2}} - \frac{\text{Subst} \left(\int \frac{-a-b+bx}{(1-x)x(a+bx)^{3/2}} dx, x, \tanh^2(x) \right)}{2a(a+b)} \\
&= \frac{b}{3a(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{b(2a+b)}{a^2(a+b)^2 \sqrt{a+b \tanh^2(x)}} - \frac{\text{Subst} \left(\int \frac{-\frac{1}{2}(a+b)^2 + \frac{1}{2}b(2a+b)}{(1-x)x \sqrt{a+bx}} dx, x, \tanh^2(x) \right)}{a^2(a+b)} \\
&= \frac{b}{3a(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{b(2a+b)}{a^2(a+b)^2 \sqrt{a+b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{1}{x \sqrt{a+bx}} dx, x, \tanh^2(x) \right)}{2a^2} \\
&= \frac{b}{3a(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{b(2a+b)}{a^2(a+b)^2 \sqrt{a+b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \tanh^2(x) \right)}{a^2 b} \\
&= -\frac{\tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a}} \right)}{a^{5/2}} + \frac{\tanh^{-1} \left(\frac{\sqrt{a+b \tanh^2(x)}}{\sqrt{a+b}} \right)}{(a+b)^{5/2}} + \frac{b}{3a(a+b)(a+b \tanh^2(x))^{3/2}} + \frac{b}{a^2(a+b)^2 \sqrt{a+b \tanh^2(x)}}
\end{aligned}$$

Mathematica [C] time = 0.07, size = 73, normalized size = 0.68

$$\frac{(a+b) {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; \frac{b \tanh^2(x)}{a} + 1\right) - a {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; \frac{b \tanh^2(x)+a}{a+b}\right)}{3a(a+b)(a+b \tanh^2(x))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]/(a + b*Tanh[x]^2)^(5/2), x]

[Out] $(- (a \text{Hypergeometric2F1}[-3/2, 1, -1/2, (a + b \text{Tanh}[x]^2)/(a + b)]) + (a + b) \text{Hypergeometric2F1}[-3/2, 1, -1/2, 1 + (b \text{Tanh}[x]^2)/a]) / (3 * a * (a + b) * (a + b \text{Tanh}[x]^2)^{(3/2)})$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*tanh(x)^2)^(5/2), x, algorithm="fricas")

[Out] Timed out

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*tanh(x)^2)^(5/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep+1)]Evaluation time: 0.74Error: Bad Argument Type

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{(a+b(\tanh^2(x)))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)/(a+b*tanh(x)^2)^(5/2), x)

[Out] int(coth(x)/(a+b*tanh(x)^2)^(5/2), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{(b \tanh(x)^2 + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*tanh(x)^2)^(5/2), x, algorithm="maxima")

[Out] integrate(coth(x)/(b*tanh(x)^2 + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)}{(b \tanh(x)^2 + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(x)/(a + b*tanh(x)^2)^(5/2), x)`

[Out] `int(coth(x)/(a + b*tanh(x)^2)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)}{(a + b \tanh^2(x))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)/(a+b*tanh(x)**2)**(5/2), x)`

[Out] `Integral(coth(x)/(a + b*tanh(x)**2)**(5/2), x)`

$$3.254 \quad \int \frac{\coth^2(x)}{(a+b \tanh^2(x))^{5/2}} dx$$

Optimal. Leaf size=131

$$-\frac{(3a+2b)(a+4b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a^3(a+b)^2} + \frac{b(7a+4b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{1}{3a(a+b)}$$

[Out] arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(5/2)+1/3*b*(7*a+4*b)*coth(x)/a^2/(a+b)^2/(a+b*tanh(x)^2)^(1/2)-1/3*(3*a+2*b)*(a+4*b)*coth(x)*(a+b*tanh(x)^2)^(1/2)/a^3/(a+b)^2+1/3*b*coth(x)/a/(a+b)/(a+b*tanh(x)^2)^(3/2)

Rubi [A] time = 0.24, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3670, 472, 579, 583, 12, 377, 206}

$$-\frac{(3a+2b)(a+4b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a^3(a+b)^2} + \frac{b(7a+4b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{1}{3a(a+b)}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^2/(a + b*Tanh[x]^2)^(5/2), x]

[Out] ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(5/2) + (b*Coth[x])/(3*a*(a + b)*(a + b*Tanh[x]^2)^(3/2)) + (b*(7*a + 4*b)*Coth[x])/(3*a^2*(a + b)^2*Sqrt[a + b*Tanh[x]^2]) - ((3*a + 2*b)*(a + 4*b)*Coth[x]*Sqrt[a + b*Tanh[x]^2])/(3*a^3*(a + b)^2)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 472

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*(e*x)^(m+1)*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*e*n*(b*c - a*d)*(p+1)), x] + Dist[1/(a*n*(b*c - a*d)*(p+1)), Int[(e*x)^m*(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[c*b*(m+1) + n*(b*c - a*d)*(p+1) + d*b*(m+n*(p+q+2)+1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(x)}{(a + b \tanh^2(x))^{5/2}} dx &= \text{Subst} \left(\int \frac{1}{x^2 (1-x^2) (a + bx^2)^{5/2}} dx, x, \tanh(x) \right) \\
&= \frac{b \coth(x)}{3a(a+b) (a + b \tanh^2(x))^{3/2}} - \frac{\text{Subst} \left(\int \frac{-3a-4b+4bx^2}{x^2(1-x^2)(a+bx^2)^{3/2}} dx, x, \tanh(x) \right)}{3a(a+b)} \\
&= \frac{b \coth(x)}{3a(a+b) (a + b \tanh^2(x))^{3/2}} + \frac{b(7a+4b) \coth(x)}{3a^2(a+b)^2 \sqrt{a + b \tanh^2(x)}} + \frac{\text{Subst} \left(\int \frac{(3a+2b)(a+b)}{x^2(1-x^2)} dx, x, \tanh(x) \right)}{3a^2(a+b)^2} \\
&= \frac{b \coth(x)}{3a(a+b) (a + b \tanh^2(x))^{3/2}} + \frac{b(7a+4b) \coth(x)}{3a^2(a+b)^2 \sqrt{a + b \tanh^2(x)}} - \frac{(3a+2b)(a+4b)}{3a^2(a+b)^2} \\
&= \frac{b \coth(x)}{3a(a+b) (a + b \tanh^2(x))^{3/2}} + \frac{b(7a+4b) \coth(x)}{3a^2(a+b)^2 \sqrt{a + b \tanh^2(x)}} - \frac{(3a+2b)(a+4b)}{3a^2(a+b)^2} \\
&= \frac{b \coth(x)}{3a(a+b) (a + b \tanh^2(x))^{3/2}} + \frac{b(7a+4b) \coth(x)}{3a^2(a+b)^2 \sqrt{a + b \tanh^2(x)}} - \frac{(3a+2b)(a+4b)}{3a^2(a+b)^2} \\
&= \frac{b \coth(x)}{3a(a+b) (a + b \tanh^2(x))^{3/2}} + \frac{b(7a+4b) \coth(x)}{3a^2(a+b)^2 \sqrt{a + b \tanh^2(x)}} - \frac{(3a+2b)(a+4b)}{3a^2(a+b)^2} \\
&= \frac{\tanh^{-1} \left(\frac{\sqrt{a+b} \tanh(x)}{\sqrt{a+b \tanh^2(x)}} \right)}{(a+b)^{5/2}} + \frac{b \coth(x)}{3a(a+b) (a + b \tanh^2(x))^{3/2}} + \frac{b(7a+4b) \coth(x)}{3a^2(a+b)^2 \sqrt{a + b \tanh^2(x)}}
\end{aligned}$$

Mathematica [C] time = 7.77, size = 246, normalized size = 1.88

$$\frac{\sqrt{\text{sech}^2(x)((a+b) \cosh(2x) + a - b)}}{b \sqrt{\frac{\text{csch}^2(x)((a+b) \cosh(2x) + a - b)}{b}}} \left(\frac{3\sqrt{2} a^3 \coth(x) \left((a+b) F \left[\sin^{-1} \left(\frac{\sqrt{\frac{(a-b+(a+b) \cosh(2x) \text{csch}^2(x))}{b}}}{\sqrt{2}} \right) \right] \right) - a \Pi \left[\frac{b}{a+b}; \sin^{-1} \left(\frac{\sqrt{\frac{(a-b+(a+b) \cosh(2x) \text{csch}^2(x))}{b}}}{\sqrt{2}} \right) \right]}{b \sqrt{\frac{\text{csch}^2(x)((a+b) \cosh(2x) + a - b)}{b}}} \right)$$

$3\sqrt{2} a^3 (a + b)$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]^2/(a + b*Tanh[x]^2)^(5/2), x]

[Out] (Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2]*((3*Sqrt[2]*a^3*Coth[x]*((a + b)*EllipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] - a*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1])))/(b*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]) - ((a + b)*(3*(a + b)^2*(a - b + (a + b)*Cosh[2*x])^2*Coth[x] + 2*a*b^3*Sinh[2*x] + b^2*(9*a + 5*b)*(a - b + (a + b)*Cosh[2*x])*Sinh[2*x]))/(a - b + (a + b)*Cosh[2*x])^2)/(3*Sqrt[2]*a^3*(a + b)^3)

fricas [B] time = 2.49, size = 10671, normalized size = 81.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

$$\begin{aligned}
& 3*b^2)*\cosh(x)^4 - 12*(a^5 - 2*a^4*b + 5*a^3*b^2)*\cosh(x)^2*\sinh(x)^2 + 2* \\
& (5*(a^5 + 2*a^4*b + a^3*b^2)*\cosh(x)^9 + 4*(3*a^5 - 2*a^4*b - 5*a^3*b^2)*\cosh(x)^7 + 6*(a^5 - 2*a^4*b + 5*a^3*b^2)*\cosh(x)^5 - 4*(a^5 - 2*a^4*b + 5*a^3*b^2)*\cosh(x)^3 - (3*a^5 - 2*a^4*b - 5*a^3*b^2)*\cosh(x))*\sinh(x))*\sqrt{a + b}*\log(((a + b)*\cosh(x)^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a + b)*\sinh(x)^4 + 2*a*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 + a)*\sinh(x)^2 + \sqrt{2}*(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1))*\sqrt{a + b}*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)}) + 4*((a + b)*\cosh(x)^3 + a*\cosh(x))*\sinh(x) + a + b)/(\cosh(x)^2 + 2*\cosh(x))*\sinh(x) + \sinh(x)^2)) - 4*\sqrt{2}*((3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5)*\cosh(x)^8 + 8*(3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5)*\cosh(x)*\sinh(x)^7 + (3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5)*\sinh(x)^8 + 4*(3*a^5 + 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5)*\cosh(x)^6 + 4*(3*a^5 + 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5 + 7*(3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5))*\cosh(x)^2)*\sinh(x)^6 + 8*(7*(3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5)*\cosh(x)^3 + 3*(3*a^5 + 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5)*\cosh(x))*\sinh(x)^5 + 3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5 + 6*(3*a^5 + 7*a^4*b + 3*a^3*b^2 + 9*a^2*b^3 + 18*a*b^4 + 8*b^5)*\cosh(x)^4 + 2*(9*a^5 + 21*a^4*b + 9*a^3*b^2 + 27*a^2*b^3 + 54*a*b^4 + 24*b^5 + 35*(3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5))*\cosh(x)^4 + 30*(3*a^5 + 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5)*\cosh(x)^2)*\sinh(x)^4 + 8*(7*(3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5))*\cosh(x)^5 + 10*(3*a^5 + 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5)*\cosh(x)^3 + 3*(3*a^5 + 7*a^4*b + 3*a^3*b^2 + 9*a^2*b^3 + 18*a*b^4 + 8*b^5)*\cosh(x))*\sinh(x)^3 + 4*(3*a^5 + 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5)*\cosh(x)^2 + 4*(7*(3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5))*\cosh(x)^6 + 3*a^5 + 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5 + 15*(3*a^5 + 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5)*\cosh(x)^4 + 9*(3*a^5 + 7*a^4*b + 3*a^3*b^2 + 9*a^2*b^3 + 18*a*b^4 + 8*b^5)*\cosh(x)^2)*\sinh(x)^2 + 8*((3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5))*\cosh(x)^7 + 3*(3*a^5 + 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5)*\cosh(x)^5 + 3*(3*a^5 + 7*a^4*b + 3*a^3*b^2 + 9*a^2*b^3 + 18*a*b^4 + 8*b^5)*\cosh(x)^4 + 8*b^5)*\cosh(x)^3 + (3*a^5 + 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5)*\cosh(x))*\sinh(x))*\sqrt{((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2)))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*\cosh(x)^10 + 10*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*\cosh(x)*\sinh(x)^9 + (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*\sinh(x)^10 + (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*\cosh(x)^8 + (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5 + 45*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5))*\cosh(x)^2)*\sinh(x)^8 - a^8 - 5*a^7*b - 10*a^6*b^2 - 10*a^5*b^3 - 5*a^4*b^4 - a^3*b^5 + 8*(15*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5))*\cosh(x)^3 + (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*\cosh(x))*\sinh(x)^7 + 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*\cosh(x)^6 + 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5 + 105*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5))*\cosh(x)^4 + 14*(3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*\cosh(x)^2)*\sinh(x)^6 + 4*(63*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5))*\cosh(x)^5 + 14*(3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*\cosh(x)^3 + 3*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*\cosh(x))*\sinh(x)^5 - 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*\cosh(x)^4 - 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5 - 105*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5))*\cosh(x)^6 - 35*(3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*\cosh(x)^4 - 15*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*\cosh(x)^2)*
\end{aligned}$$

$$\begin{aligned}
& ^4*b - 5*a^3*b^2)*\cosh(x)^6 - 3*a^5 + 2*a^4*b + 5*a^3*b^2 + 30*(a^5 - 2*a^4 \\
& *b + 5*a^3*b^2)*\cosh(x)^4 - 12*(a^5 - 2*a^4*b + 5*a^3*b^2)*\cosh(x)^2)*\sinh(x) \\
& ^2 + 2*(5*(a^5 + 2*a^4*b + a^3*b^2)*\cosh(x)^9 + 4*(3*a^5 - 2*a^4*b - 5*a^3 \\
& *b^2)*\cosh(x)^7 + 6*(a^5 - 2*a^4*b + 5*a^3*b^2)*\cosh(x)^5 - 4*(a^5 - 2*a^4 \\
& *b + 5*a^3*b^2)*\cosh(x)^3 - (3*a^5 - 2*a^4*b - 5*a^3*b^2)*\cosh(x))*\sinh(x)) \\
& *sqrt(-a - b)*\arctan(sqrt(2)*sqrt(-a - b)*sqrt(((a + b)*\cosh(x)^2 + (a + b) \\
& *\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sinh(x) + \sinh(x)^2))/((a + b)*c \\
& osh(x)^2 + 2*(a + b)*\cosh(x)*\sinh(x) + (a + b)*\sinh(x)^2 + a + b)) + 2*sqrt \\
& (2)*((3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5)*\cosh(x) \\
&)^8 + 8*(3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5)*cos \\
& h(x)*\sinh(x)^7 + (3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8 \\
& *b^5)*\sinh(x)^8 + 4*(3*a^5 + 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - \\
& 8*b^5)*\cosh(x)^6 + 4*(3*a^5 + 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - \\
& 8*b^5 + 7*(3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5))* \\
& \cosh(x)^2)*\sinh(x)^6 + 8*(7*(3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 3 \\
& 4*a*b^4 + 8*b^5)*\cosh(x)^3 + 3*(3*a^5 + 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - \\
& 22*a*b^4 - 8*b^5)*\cosh(x))*\sinh(x)^5 + 3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a \\
& ^2*b^3 + 34*a*b^4 + 8*b^5 + 6*(3*a^5 + 7*a^4*b + 3*a^3*b^2 + 9*a^2*b^3 + 18 \\
& *a*b^4 + 8*b^5)*\cosh(x)^4 + 2*(9*a^5 + 21*a^4*b + 9*a^3*b^2 + 27*a^2*b^3 + \\
& 54*a*b^4 + 24*b^5 + 35*(3*a^5 + 15*a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b \\
& ^4 + 8*b^5)*\cosh(x)^4 + 30*(3*a^5 + 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a \\
& *b^4 - 8*b^5)*\cosh(x)^2)*\sinh(x)^4 + 8*(7*(3*a^5 + 15*a^4*b + 39*a^3*b^2 + \\
& 53*a^2*b^3 + 34*a*b^4 + 8*b^5)*\cosh(x)^5 + 10*(3*a^5 + 9*a^4*b + 6*a^3*b^2 \\
& - 14*a^2*b^3 - 22*a*b^4 - 8*b^5)*\cosh(x)^3 + 3*(3*a^5 + 7*a^4*b + 3*a^3*b^2 \\
& + 9*a^2*b^3 + 18*a*b^4 + 8*b^5)*\cosh(x))*\sinh(x)^3 + 4*(3*a^5 + 9*a^4*b + \\
& 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5)*\cosh(x)^2 + 4*(7*(3*a^5 + 15*a^4 \\
& *b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5)*\cosh(x)^6 + 3*a^5 + 9*a^4* \\
& b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5 + 15*(3*a^5 + 9*a^4*b + 6*a^3 \\
& *b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5)*\cosh(x)^4 + 9*(3*a^5 + 7*a^4*b + 3*a^ \\
& 3*b^2 + 9*a^2*b^3 + 18*a*b^4 + 8*b^5)*\cosh(x)^2)*\sinh(x)^2 + 8*((3*a^5 + 15 \\
& *a^4*b + 39*a^3*b^2 + 53*a^2*b^3 + 34*a*b^4 + 8*b^5)*\cosh(x)^7 + 3*(3*a^5 + \\
& 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5)*\cosh(x)^5 + 3*(3*a^5 \\
& + 7*a^4*b + 3*a^3*b^2 + 9*a^2*b^3 + 18*a*b^4 + 8*b^5)*\cosh(x)^3 + (3*a^5 + \\
& 9*a^4*b + 6*a^3*b^2 - 14*a^2*b^3 - 22*a*b^4 - 8*b^5)*\cosh(x))*\sinh(x))*sqrt \\
& (((a + b)*\cosh(x)^2 + (a + b)*\sinh(x)^2 + a - b)/(\cosh(x)^2 - 2*\cosh(x)*\sin \\
& h(x) + \sinh(x)^2))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + \\
& a^3*b^5)*\cosh(x)^10 + 10*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4* \\
& b^4 + a^3*b^5)*\cosh(x)*\sinh(x)^9 + (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 \\
& + 5*a^4*b^4 + a^3*b^5)*\sinh(x)^10 + (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5* \\
& b^3 - 17*a^4*b^4 - 5*a^3*b^5)*\cosh(x)^8 + (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18 \\
& *a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5 + 45*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5 \\
& *b^3 + 5*a^4*b^4 + a^3*b^5)*\cosh(x)^2)*\sinh(x)^8 - a^8 - 5*a^7*b - 10*a^6*b \\
& ^2 - 10*a^5*b^3 - 5*a^4*b^4 - a^3*b^5 + 8*(15*(a^8 + 5*a^7*b + 10*a^6*b^2 + \\
& 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*\cosh(x)^3 + (3*a^8 + 7*a^7*b - 2*a^6*b^2 \\
& - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*\cosh(x))*\sinh(x)^7 + 2*(a^8 + a^7*b \\
& + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*\cosh(x)^6 + 2*(a^8 + a^ \\
& 7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5 + 105*(a^8 + 5*a^7*b \\
& + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*\cosh(x)^4 + 14*(3*a^8 + 7* \\
& a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*\cosh(x)^2)*\sinh(x) \\
& ^6 + 4*(63*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)* \\
& \cosh(x)^5 + 14*(3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a \\
& ^3*b^5)*\cosh(x)^3 + 3*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + \\
& 5*a^3*b^5)*\cosh(x))*\sinh(x)^5 - 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 1 \\
& 3*a^4*b^4 + 5*a^3*b^5)*\cosh(x)^4 - 2*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 \\
& + 13*a^4*b^4 + 5*a^3*b^5 - 105*(a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5 \\
& *a^4*b^4 + a^3*b^5)*\cosh(x)^6 - 35*(3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^ \\
& 3 - 17*a^4*b^4 - 5*a^3*b^5)*\cosh(x)^4 - 15*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^ \\
& 5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*\cosh(x)^2)*\sinh(x)^4 + 8*(15*(a^8 + 5*a^7*b \\
& + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*\cosh(x)^7 + 7*(3*a^8 + 7*
\end{aligned}$$

$$a^7b - 2a^6b^2 - 18a^5b^3 - 17a^4b^4 - 5a^3b^5) \cosh(x)^5 + 5(a^8 + a^7b + 2a^6b^2 + 10a^5b^3 + 13a^4b^4 + 5a^3b^5) \cosh(x)^3 - (a^8 + a^7b + 2a^6b^2 + 10a^5b^3 + 13a^4b^4 + 5a^3b^5) \cosh(x) \sinh(x)^3 - (3a^8 + 7a^7b - 2a^6b^2 - 18a^5b^3 - 17a^4b^4 - 5a^3b^5) \cosh(x)^2 + (45(a^8 + 5a^7b + 10a^6b^2 + 10a^5b^3 + 5a^4b^4 + a^3b^5) \cosh(x)^8 - 3a^8 - 7a^7b + 2a^6b^2 + 18a^5b^3 + 17a^4b^4 + 5a^3b^5 + 28(3a^8 + 7a^7b - 2a^6b^2 - 18a^5b^3 - 17a^4b^4 - 5a^3b^5) \cosh(x)^6 + 30(a^8 + a^7b + 2a^6b^2 + 10a^5b^3 + 13a^4b^4 + 5a^3b^5) \cosh(x)^4 - 12(a^8 + a^7b + 2a^6b^2 + 10a^5b^3 + 13a^4b^4 + 5a^3b^5) \cosh(x)^2) \sinh(x)^2 + 2(5(a^8 + 5a^7b + 10a^6b^2 + 10a^5b^3 + 5a^4b^4 + a^3b^5) \cosh(x)^9 + 4(3a^8 + 7a^7b - 2a^6b^2 - 18a^5b^3 - 17a^4b^4 - 5a^3b^5) \cosh(x)^7 + 6(a^8 + a^7b + 2a^6b^2 + 10a^5b^3 + 13a^4b^4 + 5a^3b^5) \cosh(x)^5 - 4(a^8 + a^7b + 2a^6b^2 + 10a^5b^3 + 13a^4b^4 + 5a^3b^5) \cosh(x)^3 - (3a^8 + 7a^7b - 2a^6b^2 - 18a^5b^3 - 17a^4b^4 - 5a^3b^5) \cosh(x)) \sinh(x))]$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep+1)]Evaluation time: 2.08Error: Bad Argument Type

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{(a + b(\tanh^2(x)))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x)

[Out] int(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth(x)^2}{(b \tanh(x)^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(coth(x)^2/(b*tanh(x)^2 + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\coth(x)^2}{(b \tanh(x)^2 + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)^2/(a + b*tanh(x)^2)^(5/2),x)

[Out] `int(coth(x)^2/(a + b*tanh(x)^2)^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\coth^2(x)}{(a + b \tanh^2(x))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)**2/(a+b*tanh(x)**2)**(5/2), x)`

[Out] `Integral(coth(x)**2/(a + b*tanh(x)**2)**(5/2), x)`

$$3.255 \quad \int \frac{1}{\sqrt{1+\tanh^2(x)}} dx$$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{\tanh^2(x)+1}}\right)}{\sqrt{2}}$$

[Out] 1/2*arctanh(2^(1/2)*tanh(x)/(1+tanh(x)^2)^(1/2))*2^(1/2)

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3661, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{\tanh^2(x)+1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 + Tanh[x]^2], x]

[Out] ArcTanh[(Sqrt[2]*Tanh[x])/Sqrt[1 + Tanh[x]^2]]/Sqrt[2]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 3661

Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(ff*x)^n]^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{1+\tanh^2(x)}} dx &= \text{Subst} \left(\int \frac{1}{(1-x^2)\sqrt{1+x^2}} dx, x, \tanh(x) \right) \\ &= \text{Subst} \left(\int \frac{1}{1-2x^2} dx, x, \frac{\tanh(x)}{\sqrt{1+\tanh^2(x)}} \right) \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 1.40

$$\frac{\sinh^{-1}\left(\sqrt{2}\sinh(x)\right)\sqrt{\cosh(2x)}\operatorname{sech}(x)}{\sqrt{2}\sqrt{\tanh^2(x)+1}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[1 + Tanh[x]^2], x]

[Out] (ArcSinh[Sqrt[2]*Sinh[x]]*Sqrt[Cosh[2*x]]*Sech[x])/(Sqrt[2]*Sqrt[1 + Tanh[x]^2])

fricas [B] time = 0.45, size = 543, normalized size = 21.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+tanh(x)^2)^(1/2), x, algorithm="fricas")

[Out] 1/8*sqrt(2)*log(-2*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(x)^2 - 3)*sinh(x)^6 - 3*cosh(x)^6 + 2*(28*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 + 5*(14*cosh(x)^4 - 9*cosh(x)^2 + 1)*sinh(x)^4 + 5*cosh(x)^4 + 4*(14*cosh(x)^5 - 15*cosh(x)^3 + 5*cosh(x))*sinh(x)^3 + (28*cosh(x)^6 - 45*cosh(x)^4 + 30*cosh(x)^2 - 4)*sinh(x)^2 - 4*cosh(x)^2 + 2*(4*cosh(x)^7 - 9*cosh(x)^5 + 10*cosh(x)^3 - 4*cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^4 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x)^3 + (15*sqrt(2)*cosh(x)^4 - 18*sqrt(2)*cosh(x)^2 + 4*sqrt(2))*sinh(x)^2 + 4*sqrt(2)*cosh(x)^2 + 2*(3*sqrt(2)*cosh(x)^5 - 6*sqrt(2)*cosh(x)^3 + 4*sqrt(2)*cosh(x))*sinh(x) - 4*sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4)/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 1/8*sqrt(2)*log(2*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(x)^2 + 2*(2*cosh(x)^3 + cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))

giac [B] time = 0.12, size = 58, normalized size = 2.32

$$-\frac{1}{4}\sqrt{2}\left(\log\left(\sqrt{e^{4x}+1}-e^{2x}+1\right)+\log\left(\sqrt{e^{4x}+1}-e^{2x}\right)-\log\left(-\sqrt{e^{4x}+1}+e^{2x}+1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+tanh(x)^2)^(1/2), x, algorithm="giac")

[Out] -1/4*sqrt(2)*(log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) + log(sqrt(e^(4*x) + 1) - e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))

maple [B] time = 0.13, size = 62, normalized size = 2.48

$$\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{(2\tanh(x)+2)\sqrt{2}}{4\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}\right)}{4}-\frac{\sqrt{2}\operatorname{arctanh}\left(\frac{(2-2\tanh(x))\sqrt{2}}{4\sqrt{(1+\tanh(x))^2-2\tanh(x)}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+tanh(x)^2)^(1/2), x)

[Out] $\frac{1}{4} \cdot 2^{1/2} \cdot \operatorname{arctanh}\left(\frac{1}{4} \cdot (2 \cdot \tanh(x) + 2)\right) \cdot 2^{1/2} / ((\tanh(x) - 1)^2 + 2 \cdot \tanh(x))^{1/2} - \frac{1}{4} \cdot 2^{1/2} \cdot \operatorname{arctanh}\left(\frac{1}{4} \cdot (2 - 2 \cdot \tanh(x))\right) \cdot 2^{1/2} / ((1 + \tanh(x))^2 - 2 \cdot \tanh(x))^{1/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\tanh(x)^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+tanh(x)^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/sqrt(tanh(x)^2 + 1), x)`

mupad [B] time = 0.17, size = 63, normalized size = 2.52

$$\frac{\sqrt{2} \left(\ln(\tanh(x) + 1) - \ln\left(\sqrt{2} \sqrt{\tanh(x)^2 + 1} - \tanh(x) + 1\right) \right)}{4} + \frac{\sqrt{2} \left(\ln\left(\tanh(x) + \sqrt{2} \sqrt{\tanh(x)^2 + 1} + 1\right) - \ln(\tanh(x) - 1) \right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(tanh(x)^2 + 1)^(1/2),x)`

[Out] $\frac{2^{1/2} \cdot (\log(\tanh(x) + 1) - \log(2^{1/2} \cdot (\tanh(x)^2 + 1)^{1/2} - \tanh(x) + 1))}{4} + \frac{2^{1/2} \cdot (\log(\tanh(x) + 2^{1/2} \cdot (\tanh(x)^2 + 1)^{1/2} + 1) - \log(\tanh(x) - 1))}{4}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\tanh^2(x) + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+tanh(x)**2)**(1/2),x)`

[Out] `Integral(1/sqrt(tanh(x)**2 + 1), x)`

$$3.256 \quad \int \frac{1}{\sqrt{-1-\tanh^2(x)}} dx$$

Optimal. Leaf size=27

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)}{\sqrt{2}}$$

[Out] 1/2*arctan(2^(1/2)*tanh(x)/(-1-tanh(x)^2)^(1/2))*2^(1/2)

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3661, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-1 - Tanh[x]^2], x]

[Out] ArcTan[(Sqrt[2]*Tanh[x])/Sqrt[-1 - Tanh[x]^2]]/Sqrt[2]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 3661

Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(ff*x)^n]^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-1-\tanh^2(x)}} dx &= \text{Subst} \left(\int \frac{1}{\sqrt{-1-x^2} (1-x^2)} dx, x, \tanh(x) \right) \\ &= \text{Subst} \left(\int \frac{1}{1+2x^2} dx, x, \frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}} \right) \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)}{\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 1.37

$$\frac{\sinh^{-1}\left(\sqrt{2} \sinh(x)\right) \sqrt{\cosh(2x)} \operatorname{sech}(x)}{\sqrt{2} \sqrt{-\tanh^2(x) - 1}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-1 - Tanh[x]^2], x]

[Out] (ArcSinh[Sqrt[2]*Sinh[x]]*Sqrt[Cosh[2*x]]*Sech[x])/(Sqrt[2]*Sqrt[-1 - Tanh[x]^2])

fricas [C] time = 0.41, size = 175, normalized size = 6.48

$$\frac{1}{8}i\sqrt{2} \log\left(\frac{1}{2}\left(i\sqrt{2}\sqrt{-2e^{4x}-2} + 2e^{2x} + 2\right)e^{(-2x)}\right) - \frac{1}{8}i\sqrt{2} \log\left(\frac{1}{2}\left(-i\sqrt{2}\sqrt{-2e^{4x}-2} + 2e^{2x} + 2\right)e^{(-2x)}\right) -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1-tanh(x)^2)^(1/2), x, algorithm="fricas")

[Out] 1/8*I*sqrt(2)*log(1/2*(I*sqrt(2)*sqrt(-2*e^(4*x) - 2) + 2*e^(2*x) + 2)*e^(-2*x)) - 1/8*I*sqrt(2)*log(1/2*(-I*sqrt(2)*sqrt(-2*e^(4*x) - 2) + 2*e^(2*x) + 2)*e^(-2*x)) - 1/8*I*sqrt(2)*log((sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 2) + I*sqrt(2)*e^(4*x) - I*sqrt(2)*e^(2*x) + 2*I*sqrt(2))*e^(-4*x)) + 1/8*I*sqrt(2)*log((sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 2) - I*sqrt(2)*e^(4*x) + I*sqrt(2)*e^(2*x) - 2*I*sqrt(2))*e^(-4*x))

giac [C] time = 0.14, size = 58, normalized size = 2.15

$$\frac{1}{4}i\sqrt{2} \left(\log\left(\sqrt{e^{4x}+1} - e^{2x} + 1\right) + \log\left(\sqrt{e^{4x}+1} - e^{2x}\right) - \log\left(-\sqrt{e^{4x}+1} + e^{2x} + 1\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1-tanh(x)^2)^(1/2), x, algorithm="giac")

[Out] 1/4*I*sqrt(2)*(log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) + log(sqrt(e^(4*x) + 1) - e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))

maple [B] time = 0.12, size = 66, normalized size = 2.44

$$-\frac{\sqrt{2} \arctan\left(\frac{(-2-2 \tanh(x))\sqrt{2}}{4\sqrt{-(\tanh(x)-1)^2-2 \tanh(x)}}\right)}{4} + \frac{\sqrt{2} \arctan\left(\frac{(-2+2 \tanh(x))\sqrt{2}}{4\sqrt{-(1+\tanh(x))^2+2 \tanh(x)}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-1-tanh(x)^2)^(1/2), x)

[Out] -1/4*2^(1/2)*arctan(1/4*(-2-2*tanh(x))*2^(1/2)/(-(tanh(x)-1)^2-2*tanh(x))^(1/2))+1/4*2^(1/2)*arctan(1/4*(-2+2*tanh(x))*2^(1/2)/(-(1+tanh(x))^2+2*tanh(x))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\tanh(x)^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1-tanh(x)^2)^(1/2), x, algorithm="maxima")

[Out] integrate(1/sqrt(-tanh(x)^2 - 1), x)

mupad [B] time = 1.20, size = 22, normalized size = 0.81

$$\frac{\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2} \tanh(x)}{\sqrt{-\tanh(x)^2-1}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-tanh(x)^2 - 1)^(1/2), x)

[Out] (2^(1/2)*atan((2^(1/2)*tanh(x))/(-tanh(x)^2 - 1)^(1/2)))/2

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\tanh^2(x) - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1-tanh(x)**2)**(1/2), x)

[Out] Integral(1/sqrt(-tanh(x)**2 - 1), x)

3.257 $\int (a + b \tanh^3(c + dx))^2 dx$

Optimal. Leaf size=89

$$x(a^2 + b^2) - \frac{ab \tanh^2(c + dx)}{d} + \frac{2ab \log(\cosh(c + dx))}{d} - \frac{b^2 \tanh^5(c + dx)}{5d} - \frac{b^2 \tanh^3(c + dx)}{3d} - \frac{b^2 \tanh(c + dx)}{d}$$

[Out] (a^2+b^2)*x+2*a*b*ln(cosh(d*x+c))/d-b^2*tanh(d*x+c)/d-a*b*tanh(d*x+c)^2/d-1/3*b^2*tanh(d*x+c)^3/d-1/5*b^2*tanh(d*x+c)^5/d

Rubi [A] time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3661, 1810, 633, 31}

$$\frac{ab \tanh^2(c + dx)}{d} - \frac{(a + b)^2 \log(1 - \tanh(c + dx))}{2d} + \frac{(a - b)^2 \log(\tanh(c + dx) + 1)}{2d} - \frac{b^2 \tanh^5(c + dx)}{5d} - \frac{b^2 \tanh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Tanh[c + d*x]^3)^2, x]

[Out] -((a + b)^2*Log[1 - Tanh[c + d*x]])/(2*d) + ((a - b)^2*Log[1 + Tanh[c + d*x]])/(2*d) - (b^2*Tanh[c + d*x])/d - (a*b*Tanh[c + d*x]^2)/d - (b^2*Tanh[c + d*x]^3)/(3*d) - (b^2*Tanh[c + d*x]^5)/(5*d)

Rule 31

Int[((a_) + (b_.)*(x_))^-1, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 633

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[-(a*c)]

Rule 1810

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 3661

Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(ff*x)^n]^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned}
\int (a + b \tanh^3(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \frac{(a+bx^3)^2}{1-x^2} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(-b^2 - 2abx - b^2x^2 - b^2x^4 + \frac{a^2+b^2+2abx}{1-x^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\
&= -\frac{b^2 \tanh(c + dx)}{d} - \frac{ab \tanh^2(c + dx)}{d} - \frac{b^2 \tanh^3(c + dx)}{3d} - \frac{b^2 \tanh^5(c + dx)}{5d} + \dots \\
&= -\frac{b^2 \tanh(c + dx)}{d} - \frac{ab \tanh^2(c + dx)}{d} - \frac{b^2 \tanh^3(c + dx)}{3d} - \frac{b^2 \tanh^5(c + dx)}{5d} + \dots \\
&= -\frac{(a + b)^2 \log(1 - \tanh(c + dx))}{2d} + \frac{(a - b)^2 \log(1 + \tanh(c + dx))}{2d} - \frac{b^2 \tanh(c + dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.76, size = 95, normalized size = 1.07

$$\frac{30ab \tanh^2(c + dx) + 15\left((a + b)^2 \log(1 - \tanh(c + dx)) - (a - b)^2 \log(\tanh(c + dx) + 1)\right) + 6b^2 \tanh^5(c + dx)}{30d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Tanh[c + d*x]^3)^2, x]

[Out] -1/30*(15*((a + b)^2*Log[1 - Tanh[c + d*x]] - (a - b)^2*Log[1 + Tanh[c + d*x]]) + 30*b^2*Tanh[c + d*x] + 30*a*b*Tanh[c + d*x]^2 + 10*b^2*Tanh[c + d*x]^3 + 6*b^2*Tanh[c + d*x]^5)/d

fricas [B] time = 0.43, size = 2074, normalized size = 23.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")

[Out] 1/15*(15*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^10 + 150*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)*sinh(d*x + c)^9 + 15*(a^2 - 2*a*b + b^2)*d*x*sinh(d*x + c)^10 + 15*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^8 + 15*(45*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + 5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*sinh(d*x + c)^8 + 120*(15*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + (5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 30*(5*(a^2 - 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x + c)^6 + 30*(105*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 5*(a^2 - 2*a*b + b^2)*d*x + 14*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^2 + 6*a*b + 6*b^2)*sinh(d*x + c)^6 + 60*(63*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^5 + 14*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^3 + 3*(5*(a^2 - 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(15*(a^2 - 2*a*b + b^2)*d*x + 18*a*b + 28*b^2)*cosh(d*x + c)^4 + 10*(315*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 105*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^4 + 15*(a^2 - 2*a*b + b^2)*d*x + 45*(5*(a^2 - 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x + c)^2 + 18*a*b + 28*b^2)*sinh(d*x + c)^4 + 40*(45*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^7 + 21*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^5 + 15*(5*(a^2 - 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x + c)^3 + (15*(a^2 - 2*a*b + b^2)*d*x + 18*a*b + 28*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 15*(a^2 - 2*a*b + b^2)*d*x + 5*(15*(a^2 - 2*a*b + b^2)*d*x + 12*a*b + 28*b^2)*cosh(d*x + c)^2 + 5*(135*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 84*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b +

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6*b^2)*cosh(d*x + c)^6 + 90*(5*(a^2 - 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*co
sh(d*x + c)^4 + 15*(a^2 - 2*a*b + b^2)*d*x + 12*(15*(a^2 - 2*a*b + b^2)*d*x
+ 18*a*b + 28*b^2)*cosh(d*x + c)^2 + 12*a*b + 28*b^2)*sinh(d*x + c)^2 + 46
*b^2 + 30*(a*b*cosh(d*x + c)^10 + 10*a*b*cosh(d*x + c)*sinh(d*x + c)^9 + a*
b*sinh(d*x + c)^10 + 5*a*b*cosh(d*x + c)^8 + 5*(9*a*b*cosh(d*x + c)^2 + a*b
)*sinh(d*x + c)^8 + 10*a*b*cosh(d*x + c)^6 + 40*(3*a*b*cosh(d*x + c)^3 + a*
b*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*a*b*cosh(d*x + c)^4 + 14*a*b*cosh
(d*x + c)^2 + a*b)*sinh(d*x + c)^6 + 10*a*b*cosh(d*x + c)^4 + 4*(63*a*b*cos
h(d*x + c)^5 + 70*a*b*cosh(d*x + c)^3 + 15*a*b*cosh(d*x + c))*sinh(d*x + c)
^5 + 10*(21*a*b*cosh(d*x + c)^6 + 35*a*b*cosh(d*x + c)^4 + 15*a*b*cosh(d*x
+ c)^2 + a*b)*sinh(d*x + c)^4 + 5*a*b*cosh(d*x + c)^2 + 40*(3*a*b*cosh(d*x
+ c)^7 + 7*a*b*cosh(d*x + c)^5 + 5*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))
*sinh(d*x + c)^3 + 5*(9*a*b*cosh(d*x + c)^8 + 28*a*b*cosh(d*x + c)^6 + 30*a
*b*cosh(d*x + c)^4 + 12*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^2 + a*b +
10*(a*b*cosh(d*x + c)^9 + 4*a*b*cosh(d*x + c)^7 + 6*a*b*cosh(d*x + c)^5 + 4
*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c
)/(cosh(d*x + c) - sinh(d*x + c))) + 10*(15*(a^2 - 2*a*b + b^2)*d*x*cosh(d*
x + c)^9 + 12*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^7 +
18*(5*(a^2 - 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x + c)^5 + 4*(15*(a^
2 - 2*a*b + b^2)*d*x + 18*a*b + 28*b^2)*cosh(d*x + c)^3 + (15*(a^2 - 2*a*b
+ b^2)*d*x + 12*a*b + 28*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c
)^10 + 10*d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + 5*d*cosh(d
*x + c)^8 + 5*(9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 40*(3*d*cosh(d*x
+ c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^7 + 10*d*cosh(d*x + c)^6 + 10*(21*d
*cosh(d*x + c)^4 + 14*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 4*(63*d*cosh
(d*x + c)^5 + 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^5 +
10*d*cosh(d*x + c)^4 + 10*(21*d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 + 15
*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 40*(3*d*cosh(d*x + c)^7 + 7*d*cos
h(d*x + c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^3 + 5*d
*cosh(d*x + c)^2 + 5*(9*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6 + 30*d*cos
h(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 10*(d*cosh(d*x +
c)^9 + 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x + c)^5 + 4*d*cosh(d*x + c)^3 + d
*cosh(d*x + c))*sinh(d*x + c) + d)

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giac [A] time = 0.16, size = 142, normalized size = 1.60

$$\frac{30ab \log(e^{2dx+2c} + 1) + 15(a^2 - 2ab + b^2)(dx + c) + \frac{2(23b^2 + 15(2ab + 3b^2)e^{8dx+8c} + 90(ab + b^2)e^{6dx+6c} + 10(9ab + 14b^2)e^{4dx+4c} + 10(3ab + 7b^2)e^{2dx+2c})}{(e^{2dx+2c} + 1)^5}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")

[Out] 1/15*(30*a*b*log(e^(2*d*x + 2*c) + 1) + 15*(a^2 - 2*a*b + b^2)*(d*x + c) + 2*(23*b^2 + 15*(2*a*b + 3*b^2)*e^(8*d*x + 8*c) + 90*(a*b + b^2)*e^(6*d*x + 6*c) + 10*(9*a*b + 14*b^2)*e^(4*d*x + 4*c) + 10*(3*a*b + 7*b^2)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c) + 1)^5)/d

maple [A] time = 0.02, size = 163, normalized size = 1.83

$$\frac{b^2(\tanh^5(dx + c))}{5d} - \frac{b^2(\tanh^3(dx + c))}{3d} - \frac{ab(\tanh^2(dx + c))}{d} - \frac{b^2 \tanh(dx + c)}{d} - \frac{\ln(\tanh(dx + c) - 1)a^2}{2d} - \frac{\ln(\tanh(dx + c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*tanh(d*x+c)^3)^2,x)

[Out] -1/5*b^2*tanh(d*x+c)^5/d-1/3*b^2*tanh(d*x+c)^3/d-a*b*tanh(d*x+c)^2/d-b^2*tanh(d*x+c)/d-1/2/d*ln(tanh(d*x+c)-1)*a^2-1/d*ln(tanh(d*x+c)-1)*a*b-1/2/d*ln(tanh(dx + c))

$\tanh(dx+c)-1) \cdot b^2 + 1/2/d \cdot \ln(1+\tanh(dx+c)) \cdot a^2 - 1/d \cdot \ln(1+\tanh(dx+c)) \cdot a \cdot b + 1/2/d \cdot \ln(1+\tanh(dx+c)) \cdot b^2$

maxima [B] time = 0.41, size = 194, normalized size = 2.18

$$\frac{1}{15} b^2 \left(15x + \frac{15c}{d} - \frac{2(70e^{(-2dx-2c)} + 140e^{(-4dx-4c)} + 90e^{(-6dx-6c)} + 45e^{(-8dx-8c)} + 23)}{d(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} \right) + 2ab \left(x + \frac{c}{d} + \log(e^{(-2dx-2c)} + 1)/d + 2e^{(-2dx-2c)}/(d(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1)) \right) + a^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(dx+c))^3)^2,x, algorithm="maxima")

[Out] 1/15*b^2*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 2*a*b*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a^2*x

mupad [B] time = 1.16, size = 91, normalized size = 1.02

$$x(a^2 + 2ab + b^2) - \frac{b^2 \tanh(c + dx)}{d} - \frac{b^2 \tanh(c + dx)^3}{3d} - \frac{b^2 \tanh(c + dx)^5}{5d} - \frac{2ab \ln(\tanh(c + dx) + 1)}{d} - \frac{ab \tanh(c + dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*tanh(c + dx))^3)^2,x

[Out] x*(2*a*b + a^2 + b^2) - (b^2*tanh(c + dx))/d - (b^2*tanh(c + dx)^3)/(3*d) - (b^2*tanh(c + dx)^5)/(5*d) - (2*a*b*log(tanh(c + dx) + 1))/d - (a*b*tanh(c + dx)^2)/d

sympy [A] time = 0.62, size = 100, normalized size = 1.12

$$\begin{cases} a^2 x + 2abx - \frac{2ab \log(\tanh(c+dx)+1)}{d} - \frac{ab \tanh^2(c+dx)}{d} + b^2 x - \frac{b^2 \tanh^5(c+dx)}{5d} - \frac{b^2 \tanh^3(c+dx)}{3d} - \frac{b^2 \tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x(a + b \tanh^3(c))^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*tanh(dx+c)**3)**2,x)

[Out] Piecewise((a**2*x + 2*a*b*x - 2*a*b*log(tanh(c + dx) + 1)/d - a*b*tanh(c + dx)**2/d + b**2*x - b**2*tanh(c + dx)**5/(5*d) - b**2*tanh(c + dx)**3/(3*d) - b**2*tanh(c + dx)/d, Ne(d, 0)), (x*(a + b*tanh(c)**3)**2, True))

$$3.258 \quad \int \frac{1}{1+\tanh^3(x)} dx$$

Optimal. Leaf size=38

$$\frac{x}{2} - \frac{1}{6(\tanh(x) + 1)} - \frac{2 \tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}$$

[Out] 1/2*x-2/9*arctan(1/3*(1-2*tanh(x))*3^(1/2))*3^(1/2)-1/6/(1+tanh(x))

Rubi [A] time = 0.07, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3661, 2074, 207, 618, 204}

$$\frac{x}{2} - \frac{1}{6(\tanh(x) + 1)} - \frac{2 \tan^{-1}\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + Tanh[x]^3)^(-1), x]

[Out] x/2 - (2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) - 1/(6*(1 + Tanh[x]))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2074

Int[(P_)^(p_)*(Q_)^(q_), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]

Rule 3661

Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(a + b*(ff*x)^n]^p/(c^2 + ff^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && (IntegerQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] || EqQ[n^2, 16])

Rubi steps

$$\begin{aligned}
\int \frac{1}{1 + \tanh^3(x)} dx &= \text{Subst} \left(\int \frac{1}{(1-x^2)(1+x^3)} dx, x, \tanh(x) \right) \\
&= \text{Subst} \left(\int \left(\frac{1}{6(1+x)^2} - \frac{1}{2(-1+x^2)} + \frac{1}{3(1-x+x^2)} \right) dx, x, \tanh(x) \right) \\
&= -\frac{1}{6(1+\tanh(x))} + \frac{1}{3} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, \tanh(x) \right) - \frac{1}{2} \text{Subst} \left(\int \frac{1}{-1+x^2} dx, x, \tanh(x) \right) \\
&= \frac{x}{2} - \frac{1}{6(1+\tanh(x))} - \frac{2}{3} \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, -1+2\tanh(x) \right) \\
&= \frac{x}{2} - \frac{2 \tan^{-1} \left(\frac{1-2\tanh(x)}{\sqrt{3}} \right)}{3\sqrt{3}} - \frac{1}{6(1+\tanh(x))}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 40, normalized size = 1.05

$$\frac{1}{2} \tanh^{-1}(\tanh(x)) - \frac{1}{6(\tanh(x)+1)} - \frac{2 \tan^{-1} \left(\frac{1-2 \tanh(x)}{\sqrt{3}} \right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Tanh[x]^3)^(-1), x]

[Out] (-2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) + ArcTanh[Tanh[x]]/2 - 1/(6*(1 + Tanh[x]))

fricas [B] time = 0.53, size = 95, normalized size = 2.50

$$\frac{18x \cosh(x)^2 + 36x \cosh(x) \sinh(x) + 18x \sinh(x)^2 - 8(\sqrt{3} \cosh(x)^2 + 2\sqrt{3} \cosh(x) \sinh(x) + \sqrt{3} \sinh(x)^2)}{36(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+tanh(x)^3), x, algorithm="fricas")

[Out] 1/36*(18*x*cosh(x)^2 + 36*x*cosh(x)*sinh(x) + 18*x*sinh(x)^2 - 8*(sqrt(3)*cosh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2)*arctan(-1/3*(sqrt(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - sinh(x))) - 3)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)

giac [A] time = 0.11, size = 25, normalized size = 0.66

$$\frac{2}{9} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} e^{(2x)} \right) + \frac{1}{2} x - \frac{1}{12} e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+tanh(x)^3), x, algorithm="giac")

[Out] 2/9*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) + 1/2*x - 1/12*e^(-2*x)

maple [A] time = 0.06, size = 41, normalized size = 1.08

$$-\frac{\ln(\tanh(x)-1)}{4} - \frac{1}{6(1+\tanh(x))} + \frac{\ln(1+\tanh(x))}{4} + \frac{2\sqrt{3} \arctan \left(\frac{(2 \tanh(x)-1)\sqrt{3}}{3} \right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(1+tanh(x)^3),x)`

[Out] $-1/4*\ln(\tanh(x)-1)-1/6/(1+\tanh(x))+1/4*\ln(1+\tanh(x))+2/9*3^{(1/2)}*\arctan(1/3*(2*\tanh(x)-1)*3^{(1/2)})$

maxima [B] time = 0.41, size = 73, normalized size = 1.92

$$\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}+3^{\frac{1}{4}}\sqrt{2}\right)\right)-\frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}-3^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{2}x-\frac{1}{12}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+tanh(x)^3),x, algorithm="maxima")`

[Out] $2/9*\sqrt{3}*\arctan(1/6*3^{(3/4)}*\sqrt{2}*(2*\sqrt{3}*e^{(-x)}+3^{(1/4)}*\sqrt{2})) - 2/9*\sqrt{3}*\arctan(1/6*3^{(3/4)}*\sqrt{2}*(2*\sqrt{3}*e^{(-x)}-3^{(1/4)}*\sqrt{2})) + 1/2*x - 1/12*e^{(-2*x)}$

mupad [B] time = 0.10, size = 38, normalized size = 1.00

$$\frac{\frac{x}{2} + \frac{\tanh(x)}{6} + \frac{x \tanh(x)}{2}}{\tanh(x) + 1} + \frac{2\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}(2 \tanh(x) - 1)}{3}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(tanh(x)^3 + 1),x)`

[Out] $(x/2 + \tanh(x)/6 + (x*\tanh(x))/2)/(tanh(x) + 1) + (2*3^{(1/2)}*\operatorname{atan}((3^{(1/2)}*(2*\tanh(x) - 1))/3))/9$

sympy [B] time = 0.56, size = 102, normalized size = 2.68

$$\frac{9x \tanh(x)}{18 \tanh(x) + 18} + \frac{9x}{18 \tanh(x) + 18} + \frac{4\sqrt{3} \tanh(x) \operatorname{atan}\left(\frac{2\sqrt{3} \tanh(x)}{3} - \frac{\sqrt{3}}{3}\right)}{18 \tanh(x) + 18} + \frac{4\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3} \tanh(x)}{3} - \frac{\sqrt{3}}{3}\right)}{18 \tanh(x) + 18} - \frac{3}{18 \tanh(x) + 18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+tanh(x)**3),x)`

[Out] $9*x*\tanh(x)/(18*\tanh(x) + 18) + 9*x/(18*\tanh(x) + 18) + 4*\sqrt{3}*\tanh(x)*\operatorname{atan}(2*\sqrt{3}*\tanh(x)/3 - \sqrt{3}/3)/(18*\tanh(x) + 18) + 4*\sqrt{3}*\operatorname{atan}(2*\sqrt{3}*\tanh(x)/3 - \sqrt{3}/3)/(18*\tanh(x) + 18) - 3/(18*\tanh(x) + 18)$

3.259 $\int \tanh(x) (a + b \tanh^4(x))^{3/2} dx$

Optimal. Leaf size=124

$$-\frac{1}{6} (a + b \tanh^4(x))^{3/2} - \frac{1}{4} (2(a + b) + b \tanh^2(x)) \sqrt{a + b \tanh^4(x)} + \frac{1}{2} (a+b)^{3/2} \tanh^{-1} \left(\frac{a + b \tanh^2(x)}{\sqrt{a+b} \sqrt{a + b \tanh^4(x)}} \right)$$

[Out] $1/2*(a+b)^{(3/2)}*\operatorname{arctanh}((a+b*\tanh(x)^2)/(a+b)^{(1/2)}/(a+b*\tanh(x)^4)^{(1/2)}) - 1/4*(3*a+2*b)*\operatorname{arctanh}(b^{(1/2)}*\tanh(x)^2/(a+b*\tanh(x)^4)^{(1/2)})*b^{(1/2)} - 1/4*(a+b*\tanh(x)^4)^{(1/2)}*(2*a+2*b+b*\tanh(x)^2) - 1/6*(a+b*\tanh(x)^4)^{(3/2)}$

Rubi [A] time = 0.24, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3670, 1248, 735, 815, 844, 217, 206, 725}

$$\frac{1}{2} (a+b)^{3/2} \tanh^{-1} \left(\frac{a + b \tanh^2(x)}{\sqrt{a+b} \sqrt{a + b \tanh^4(x)}} \right) - \frac{1}{6} (a + b \tanh^4(x))^{3/2} - \frac{1}{4} \sqrt{b} (3a+2b) \tanh^{-1} \left(\frac{\sqrt{b} \tanh^2(x)}{\sqrt{a + b \tanh^4(x)}} \right)$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]*(a + b*Tanh[x]^4)^(3/2), x]

[Out] $-(\operatorname{Sqrt}[b]*(3*a + 2*b)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Tanh}[x]^2)/\operatorname{Sqrt}[a + b*\operatorname{Tanh}[x]^4]])/4 + ((a + b)^{(3/2)}*\operatorname{ArcTanh}[(a + b*\operatorname{Tanh}[x]^2)/(\operatorname{Sqrt}[a + b]*\operatorname{Sqrt}[a + b*\operatorname{Tanh}[x]^4])])/2 - ((2*(a + b) + b*\operatorname{Tanh}[x]^2)*\operatorname{Sqrt}[a + b*\operatorname{Tanh}[x]^4])/4 - (a + b*\operatorname{Tanh}[x]^4)^{(3/2)}/6$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 735

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

Rule 815

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p

```

+ 1) + g*c*e*(m + 2*p + 1)*x*(a + c*x^2)^p)/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2)), x] + Dist[(2*p)/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] || !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) && !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

```

Rule 844

```

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

```

Rule 1248

```

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
:= Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]

```

Rule 3670

```

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_.))^p, x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x],
x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))

```

Rubi steps

$$\begin{aligned}
\int \tanh(x) (a + b \tanh^4(x))^{3/2} dx &= \text{Subst} \left(\int \frac{x (a + bx^4)^{3/2}}{1 - x^2} dx, x, \tanh(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{(a + bx^2)^{3/2}}{1 - x} dx, x, \tanh^2(x) \right) \\
&= -\frac{1}{6} (a + b \tanh^4(x))^{3/2} - \frac{1}{2} \text{Subst} \left(\int \frac{(-a - bx)\sqrt{a + bx^2}}{1 - x} dx, x, \tanh^2(x) \right) \\
&= -\frac{1}{4} (2(a + b) + b \tanh^2(x)) \sqrt{a + b \tanh^4(x)} - \frac{1}{6} (a + b \tanh^4(x))^{3/2} - \frac{\text{Subst}}{2} \\
&= -\frac{1}{4} (2(a + b) + b \tanh^2(x)) \sqrt{a + b \tanh^4(x)} - \frac{1}{6} (a + b \tanh^4(x))^{3/2} + \frac{1}{2} (a + b \tanh^4(x))^{3/2} \\
&= -\frac{1}{4} (2(a + b) + b \tanh^2(x)) \sqrt{a + b \tanh^4(x)} - \frac{1}{6} (a + b \tanh^4(x))^{3/2} - \frac{1}{2} (a + b \tanh^4(x))^{3/2} \\
&= -\frac{1}{4} \sqrt{b} (3a + 2b) \tanh^{-1} \left(\frac{\sqrt{b} \tanh^2(x)}{\sqrt{a + b \tanh^4(x)}} \right) + \frac{1}{2} (a + b)^{3/2} \tanh^{-1} \left(\frac{a + b \tanh^4(x)}{\sqrt{a + b \tanh^4(x)}} \right)
\end{aligned}$$

Mathematica [A] time = 4.81, size = 166, normalized size = 1.34

$$\frac{1}{12} \left(-\sqrt{a + b \tanh^4(x)} (8a + 2b \tanh^4(x) + 3b \tanh^2(x) + 6b) + 6(a + b)^{3/2} \tanh^{-1} \left(\frac{a + b \tanh^2(x)}{\sqrt{a + b} \sqrt{a + b \tanh^4(x)}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]*(a + b*Tanh[x]^4)^(3/2), x]

[Out] (-6*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a + b*Tanh[x]^4]] + 6*(a + b)^(3/2)*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])] - Sqrt[a + b*Tanh[x]^4]*(8*a + 6*b + 3*b*Tanh[x]^2 + 2*b*Tanh[x]^4) - (3*Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a]]*Sqrt[a + b*Tanh[x]^4])/Sqrt[1 + (b*Tanh[x]^4)/a])/12

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)*(a+b*tanh(x)^4)^(3/2), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \tanh(x)^4 + a)^{\frac{3}{2}} \tanh(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)*(a+b*tanh(x)^4)^(3/2), x, algorithm="giac")

[Out] integrate((b*tanh(x)^4 + a)^(3/2)*tanh(x), x)

maple [C] time = 0.24, size = 620, normalized size = 5.00

$$\frac{b(\tanh^4(x))\sqrt{a+b(\tanh^4(x))}}{6} - \frac{b(\tanh^2(x))\sqrt{a+b(\tanh^4(x))}}{4} - \frac{2\sqrt{a+b(\tanh^4(x))}a}{3} - \frac{b\sqrt{a+b(\tanh^4(x))}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)*(a+b*tanh(x)^4)^(3/2), x)

[Out] -1/6*b*tanh(x)^4*(a+b*tanh(x)^4)^(1/2)-1/4*b*tanh(x)^2*(a+b*tanh(x)^4)^(1/2)-2/3*(a+b*tanh(x)^4)^(1/2)*a-1/2*b*(a+b*tanh(x)^4)^(1/2)-1/2*(5/3*a*b+b^2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticF(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2), I)-3/4*ln(2*b^(1/2)*tanh(x)^2+2*(a+b*tanh(x)^4)^(1/2))*b^(1/2)*a-1/2*ln(2*b^(1/2)*tanh(x)^2+2*(a+b*tanh(x)^4)^(1/2))*b^(3/2)-1/2*I*(-7/5*a*b-b^2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)/b^(1/2)*(EllipticF(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2), I)-EllipticE(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2), I))+1/2*a^2/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))+a*b/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))+1/2*b^2/(a+b)^(1/2)*arctanh(1

$$\frac{1}{2} \frac{(2b \tanh(x)^2 + 2a)}{(a+b)^{1/2} (a+b \tanh(x)^4)^{1/2}} - \frac{1}{2} \frac{(-5/3 a b - b^2)}{(I/a^{1/2} b^{1/2})^{1/2} (1 - I/a^{1/2} b^{1/2} \tanh(x)^2)^{1/2} (1 + I/a^{1/2} b^{1/2} \tanh(x)^2)^{1/2}} (a+b \tanh(x)^4)^{1/2} \text{EllipticF}(\tanh(x) * (I/a^{1/2} b^{1/2}))^{1/2}, I) - \frac{1}{2} I * (7/5 a b + b^2) a^{1/2} / (I/a^{1/2} b^{1/2})^{1/2} (1 - I/a^{1/2} b^{1/2} \tanh(x)^2)^{1/2} (1 + I/a^{1/2} b^{1/2} \tanh(x)^2)^{1/2} / (a+b \tanh(x)^4)^{1/2} / b^{1/2} * (\text{EllipticF}(\tanh(x) * (I/a^{1/2} b^{1/2}))^{1/2}, I) - \text{EllipticE}(\tanh(x) * (I/a^{1/2} b^{1/2}))^{1/2}, I)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \tanh(x)^4 + a)^{\frac{3}{2}} \tanh(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)*(a+b*tanh(x)^4)^(3/2),x, algorithm="maxima")

[Out] integrate((b*tanh(x)^4 + a)^(3/2)*tanh(x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x) (b \tanh(x)^4 + a)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)*(a + b*tanh(x)^4)^(3/2),x)

[Out] int(tanh(x)*(a + b*tanh(x)^4)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \tanh^4(x))^{\frac{3}{2}} \tanh(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)*(a+b*tanh(x)**4)**(3/2),x)

[Out] Integral((a + b*tanh(x)**4)**(3/2)*tanh(x), x)

3.260 $\int \tanh(x) \sqrt{a + b \tanh^4(x)} dx$

Optimal. Leaf size=89

$$-\frac{1}{2}\sqrt{a + b \tanh^4(x)} - \frac{1}{2}\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \tanh^2(x)}{\sqrt{a + b \tanh^4(x)}}\right) + \frac{1}{2}\sqrt{a + b} \tanh^{-1}\left(\frac{a + b \tanh^2(x)}{\sqrt{a + b} \sqrt{a + b \tanh^4(x)}}\right)$$

[Out] $-1/2*\operatorname{arctanh}(b^{(1/2)}*\tanh(x)^2/(a+b*\tanh(x)^4)^{(1/2)})*b^{(1/2)}+1/2*\operatorname{arctanh}((a+b*\tanh(x)^2)/(a+b)^{(1/2)}/(a+b*\tanh(x)^4)^{(1/2)})*(a+b)^{(1/2)}-1/2*(a+b*\tanh(x)^4)^{(1/2)}$

Rubi [A] time = 0.13, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3670, 1248, 735, 844, 217, 206, 725}

$$-\frac{1}{2}\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \tanh^2(x)}{\sqrt{a + b \tanh^4(x)}}\right) + \frac{1}{2}\sqrt{a + b} \tanh^{-1}\left(\frac{a + b \tanh^2(x)}{\sqrt{a + b} \sqrt{a + b \tanh^4(x)}}\right) - \frac{1}{2}\sqrt{a + b \tanh^4(x)}$$

Antiderivative was successfully verified.

[In] `Int[Tanh[x]*Sqrt[a + b*Tanh[x]^4], x]`

[Out] $-(\operatorname{Sqrt}[b]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[b]*\operatorname{Tanh}[x]^2)/\operatorname{Sqrt}[a + b*\operatorname{Tanh}[x]^4]])/2 + (\operatorname{Sqrt}[a + b]*\operatorname{ArcTanh}[(a + b*\operatorname{Tanh}[x]^2)/(\operatorname{Sqrt}[a + b]*\operatorname{Sqrt}[a + b*\operatorname{Tanh}[x]^4])])/2 - \operatorname{Sqrt}[a + b*\operatorname{Tanh}[x]^4]/2$

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 217

`Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]`

Rule 725

`Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]`

Rule 735

`Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]`

Rule 844

`Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,`

e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

Rule 1248

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rule 3670

Int[((d_)*tan[(e_) + (f_)*(x_)])^(m_)*((a_) + (b_)*((c_)*tan[(e_) + (f_)*(x_)])^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
 \int \tanh(x) \sqrt{a + b \tanh^4(x)} dx &= \text{Subst} \left(\int \frac{x \sqrt{a + bx^4}}{1 - x^2} dx, x, \tanh(x) \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{a + bx^2}}{1 - x} dx, x, \tanh^2(x) \right) \\
 &= -\frac{1}{2} \sqrt{a + b \tanh^4(x)} - \frac{1}{2} \text{Subst} \left(\int \frac{-a - bx}{(1 - x) \sqrt{a + bx^2}} dx, x, \tanh^2(x) \right) \\
 &= -\frac{1}{2} \sqrt{a + b \tanh^4(x)} - \frac{1}{2} (-a - b) \text{Subst} \left(\int \frac{1}{(1 - x) \sqrt{a + bx^2}} dx, x, \tanh^2(x) \right) - \\
 &= -\frac{1}{2} \sqrt{a + b \tanh^4(x)} - \frac{1}{2} b \text{Subst} \left(\int \frac{1}{1 - bx^2} dx, x, \frac{\tanh^2(x)}{\sqrt{a + b \tanh^4(x)}} \right) - \frac{1}{2} (a + b) \\
 &= -\frac{1}{2} \sqrt{b} \tanh^{-1} \left(\frac{\sqrt{b} \tanh^2(x)}{\sqrt{a + b \tanh^4(x)}} \right) + \frac{1}{2} \sqrt{a + b} \tanh^{-1} \left(\frac{a + b \tanh^2(x)}{\sqrt{a + b} \sqrt{a + b \tanh^4(x)}} \right)
 \end{aligned}$$

Mathematica [A] time = 0.06, size = 86, normalized size = 0.97

$$\frac{1}{2} \left(-\sqrt{a + b \tanh^4(x)} - \sqrt{b} \tanh^{-1} \left(\frac{\sqrt{b} \tanh^2(x)}{\sqrt{a + b \tanh^4(x)}} \right) + \sqrt{a + b} \tanh^{-1} \left(\frac{a + b \tanh^2(x)}{\sqrt{a + b} \sqrt{a + b \tanh^4(x)}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]*Sqrt[a + b*Tanh[x]^4], x]

[Out] -(Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a + b*Tanh[x]^4]]) + Sqrt[a + b]*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])] - Sqrt[a + b*Tanh[x]^4])/2

fricas [B] time = 0.65, size = 5136, normalized size = 57.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)*(a+b*tanh(x)^4)^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{4} \left((\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2(3 \cosh(x)^2 + 1) \sinh(x)^2 + 2 \cosh(x)^2 + 4(\cosh(x)^3 + \cosh(x)) \sinh(x) + 1) \sqrt{b} \log(-(\right.$
 $(a + 2b) \cosh(x)^8 + 8(a + 2b) \cosh(x) \sinh(x)^7 + (a + 2b) \sinh(x)^8 +$
 $4(a - 2b) \cosh(x)^6 + 4(7(a + 2b) \cosh(x)^2 + a - 2b) \sinh(x)^6 + 8$
 $(7(a + 2b) \cosh(x)^3 + 3(a - 2b) \cosh(x)) \sinh(x)^5 + 6(a + 2b) \cosh(x)^4 + 2(35(a + 2b) \cosh(x)^4 + 30(a - 2b) \cosh(x)^2 + 3a + 6b) \sinh(x)^4 + 8(7(a + 2b) \cosh(x)^5 + 10(a - 2b) \cosh(x)^3 + 3(a + 2b) \cosh(x)) \sinh(x)^3 + 4(a - 2b) \cosh(x)^2 + 4(7(a + 2b) \cosh(x)^6 + 15(a - 2b) \cosh(x)^4 + 9(a + 2b) \cosh(x)^2 + a - 2b) \sinh(x)^2 - 2\sqrt{2}(\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2(3 \cosh(x)^2 - 1) \sinh(x)^2 - 2 \cosh(x)^2 + 4(\cosh(x)^3 - \cosh(x)) \sinh(x) + 1) \sqrt{b} \sqrt{((a + b) \cosh(x)^4 + (a + b) \sinh(x)^4 + 4(a - b) \cosh(x)^2 + 2(3(a + b) \cosh(x)^2 + 2a - 2b) \sinh(x)^2 + 3a + 3b) / (\cosh(x)^4 - 4 \cosh(x)^3 \sinh(x) + 6 \cosh(x)^2 \sinh(x)^2 - 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4)) + 8((a + 2b) \cosh(x)^7 + 3(a - 2b) \cosh(x)^5 + 3(a + 2b) \cosh(x)^3 + (a - 2b) \cosh(x)) \sinh(x) + a + 2b) / (\cosh(x)^8 + 8 \cosh(x) \sinh(x)^7 + \sinh(x)^8 + 4(7 \cosh(x)^2 + 1) \sinh(x)^6 + 4 \cosh(x)^6 + 8(7 \cosh(x)^3 + 3 \cosh(x)) \sinh(x)^5 + 2(35 \cosh(x)^4 + 30 \cosh(x)^2 + 3) \sinh(x)^4 + 6 \cosh(x)^4 + 8(7 \cosh(x)^5 + 10 \cosh(x)^3 + 3 \cosh(x)) \sinh(x)^3 + 4(7 \cosh(x)^6 + 15 \cosh(x)^4 + 9 \cosh(x)^2 + 1) \sinh(x)^2 + 4 \cosh(x)^2 + 8(\cosh(x)^7 + 3 \cosh(x)^5 + 3 \cosh(x)^3 + \cosh(x)) \sinh(x) + 1) + (\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2(3 \cosh(x)^2 + 1) \sinh(x)^2 + 2 \cosh(x)^2 + 4(\cosh(x)^3 + \cosh(x)) \sinh(x) + 1) \sqrt{a + b} \log(((a^2 + 2ab + b^2) \cosh(x)^8 + 8(a^2 + 2ab + b^2) \cosh(x) \sinh(x)^7 + (a^2 + 2ab + b^2) \sinh(x)^8 + 4(a^2 - b^2) \cosh(x)^6 + 4(7(a^2 + 2ab + b^2) \cosh(x)^2 + a^2 - b^2) \sinh(x)^6 + 8(7(a^2 + 2ab + b^2) \cosh(x)^3 + 3(a^2 - b^2) \cosh(x)) \sinh(x)^5 + 2(3a^2 + 2ab + 3b^2) \cosh(x)^4 + 2(35(a^2 + 2ab + b^2) \cosh(x)^4 + 30(a^2 - b^2) \cosh(x)^2 + 3a^2 + 2ab + 3b^2) \sinh(x)^4 + 8(7(a^2 + 2ab + b^2) \cosh(x)^5 + 10(a^2 - b^2) \cosh(x)^3 + (3a^2 + 2ab + 3b^2) \cosh(x)) \sinh(x)^3 + 4(a^2 - b^2) \cosh(x)^2 + 4(7(a^2 + 2ab + b^2) \cosh(x)^6 + 15(a^2 - b^2) \cosh(x)^4 + 3(3a^2 + 2ab + 3b^2) \cosh(x)^2 + a^2 - b^2) \sinh(x)^2 + \sqrt{2}((a + b) \cosh(x)^4 + 4(a + b) \cosh(x) \sinh(x)^3 + (a + b) \sinh(x)^4 + 2(a - b) \cosh(x)^2 + 2(3(a + b) \cosh(x)^2 + a - b) \sinh(x)^2 + 4((a + b) \cosh(x)^3 + (a - b) \cosh(x)) \sinh(x) + a + b) \sqrt{a + b} \sqrt{((a + b) \cosh(x)^4 + (a + b) \sinh(x)^4 + 4(a - b) \cosh(x)^2 + 2(3(a + b) \cosh(x)^2 + 2a - 2b) \sinh(x)^2 + 3a + 3b) / (\cosh(x)^4 - 4 \cosh(x)^3 \sinh(x) + 6 \cosh(x)^2 \sinh(x)^2 - 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4)) + a^2 + 2ab + b^2 + 8((a^2 + 2ab + b^2) \cosh(x)^7 + 3(a^2 - b^2) \cosh(x)^5 + (3a^2 + 2ab + 3b^2) \cosh(x)^3 + (a^2 - b^2) \cosh(x)) \sinh(x) / (\cosh(x)^4 + 4 \cosh(x)^3 \sinh(x) + 6 \cosh(x)^2 \sinh(x)^2 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4) - 2\sqrt{2} \sqrt{((a + b) \cosh(x)^4 + (a + b) \sinh(x)^4 + 4(a - b) \cosh(x)^2 + 2(3(a + b) \cosh(x)^2 + 2a - 2b) \sinh(x)^2 + 3a + 3b) / (\cosh(x)^4 - 4 \cosh(x)^3 \sinh(x) + 6 \cosh(x)^2 \sinh(x)^2 - 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4)) / (\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2(3 \cosh(x)^2 + 1) \sinh(x)^2 + 2 \cosh(x)^2 + 4(\cosh(x)^3 + \cosh(x)) \sinh(x) + 1) \sqrt{-a - b} \arctan(\sqrt{2}((a + b) \cosh(x)^4 + 4(a + b) \cosh(x) \sinh(x)^3 + (a + b) \sinh(x)^4 + 2(a - b) \cosh(x)^2 + 2(3(a + b) \cosh(x)^2 + a - b) \sinh(x)^2 + 4((a + b) \cosh(x)^3 + (a - b) \cosh(x)) \sinh(x) + a + b) \sqrt{a + b} \sqrt{((a + b) \cosh(x)^4 + (a + b) \sinh(x)^4 + 4(a - b) \cosh(x)^2 + 2(3(a + b) \cosh(x)^2 + 2a - 2b) \sinh(x)^2 + 3a + 3b) / (\cosh(x)^4 - 4 \cosh(x)^3 \sinh(x) + 6 \cosh(x)^2 \sinh(x)^2 - 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4)) / ((a^2 + 2ab + b^2) \cosh(x)^8 + 8(a^2 + 2ab + b^2) \cosh(x) \sinh(x)^7 + (a^2 + 2ab + b^2) \sinh(x)^8 + 4(a^2 - b^2) \cosh(x)^6 + 4(7(a^2 + 2ab + b^2) \cosh(x)^2 + a^2 - b^2) \sinh(x)^6 + 8(7(a^2 + 2ab + b^2) \cosh(x)^3 + 3(a^2 - b^2) \cosh(x)) \sinh(x)^5 + 6(a^2 + 2ab + b^2) \cosh(x)^4 + 2(35(a^2 + 2ab + b^2) \cosh(x)^4 + 30(a^2 - b^2) \cosh(x)^2 + 3a^2$

$$\begin{aligned}
& + 6*a*b + 3*b^2)*\sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*\cosh(x)^5 + 10*(a^2 - \\
& b^2)*\cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*\cosh(x))*\sinh(x)^3 + 4*(a^2 - b^2)* \\
& \cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*\cosh(x)^6 + 15*(a^2 - b^2)*\cosh(x)^4 + \\
& 9*(a^2 + 2*a*b + b^2)*\cosh(x)^2 + a^2 - b^2)*\sinh(x)^2 + a^2 + 2*a*b + b^2 \\
& + 8*((a^2 + 2*a*b + b^2)*\cosh(x)^7 + 3*(a^2 - b^2)*\cosh(x)^5 + 3*(a^2 + 2* \\
& a*b + b^2)*\cosh(x)^3 + (a^2 - b^2)*\cosh(x))*\sinh(x))) - (\cosh(x)^4 + 4*\cosh \\
& (x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 + 1)*\sinh(x)^2 + 2*\cosh(x)^2 + 4 \\
& *(cosh(x)^3 + cosh(x))*\sinh(x) + 1)*\sqrt{b}*\log(-((a + 2*b)*\cosh(x)^8 + 8*(\\
& a + 2*b)*\cosh(x)*\sinh(x)^7 + (a + 2*b)*\sinh(x)^8 + 4*(a - 2*b)*\cosh(x)^6 + \\
& 4*(7*(a + 2*b)*\cosh(x)^2 + a - 2*b)*\sinh(x)^6 + 8*(7*(a + 2*b)*\cosh(x)^3 + \\
& 3*(a - 2*b)*\cosh(x))*\sinh(x)^5 + 6*(a + 2*b)*\cosh(x)^4 + 2*(35*(a + 2*b)*\cosh \\
& (x)^4 + 30*(a - 2*b)*\cosh(x)^2 + 3*a + 6*b)*\sinh(x)^4 + 8*(7*(a + 2*b)*\cosh \\
& (x)^5 + 10*(a - 2*b)*\cosh(x)^3 + 3*(a + 2*b)*\cosh(x))*\sinh(x)^3 + 4*(a - \\
& 2*b)*\cosh(x)^2 + 4*(7*(a + 2*b)*\cosh(x)^6 + 15*(a - 2*b)*\cosh(x)^4 + 9*(a + \\
& 2*b)*\cosh(x)^2 + a - 2*b)*\sinh(x)^2 - 2*\sqrt{2}*(\cosh(x)^4 + 4*\cosh(x)*\sinh \\
& (x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 - 1)*\sinh(x)^2 - 2*\cosh(x)^2 + 4*(\cosh(x) \\
& ^3 - \cosh(x))*\sinh(x) + 1)*\sqrt{b}*\sqrt{((a + b)*\cosh(x)^4 + (a + b)*\sinh \\
& (x)^4 + 4*(a - b)*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 + 2*a - 2*b)*\sinh(x)^2 \\
& + 3*a + 3*b)/(\cosh(x)^4 - 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 - 4* \\
& \cosh(x)*\sinh(x)^3 + \sinh(x)^4)) + 8*((a + 2*b)*\cosh(x)^7 + 3*(a - 2*b)*\cosh \\
& (x)^5 + 3*(a + 2*b)*\cosh(x)^3 + (a - 2*b)*\cosh(x))*\sinh(x) + a + 2*b)/(\cosh \\
& (x)^8 + 8*\cosh(x)*\sinh(x)^7 + \sinh(x)^8 + 4*(7*\cosh(x)^2 + 1)*\sinh(x)^6 + 4 \\
& *\cosh(x)^6 + 8*(7*\cosh(x)^3 + 3*\cosh(x))*\sinh(x)^5 + 2*(35*\cosh(x)^4 + 30*\cosh \\
& (x)^2 + 3)*\sinh(x)^4 + 6*\cosh(x)^4 + 8*(7*\cosh(x)^5 + 10*\cosh(x)^3 + 3*\cosh \\
& (x))*\sinh(x)^3 + 4*(7*\cosh(x)^6 + 15*\cosh(x)^4 + 9*\cosh(x)^2 + 1)*\sinh(x) \\
& ^2 + 4*\cosh(x)^2 + 8*(\cosh(x)^7 + 3*\cosh(x)^5 + 3*\cosh(x)^3 + \cosh(x))*\sinh \\
& (x) + 1)) + 2*\sqrt{2}*\sqrt{((a + b)*\cosh(x)^4 + (a + b)*\sinh(x)^4 + 4*(a - \\
& b)*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 + 2*a - 2*b)*\sinh(x)^2 + 3*a + 3*b)/ \\
& (\cosh(x)^4 - 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 - 4*\cosh(x)*\sinh(x) \\
&)^3 + \sinh(x)^4))/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh \\
& (x)^2 + 1)*\sinh(x)^2 + 2*\cosh(x)^2 + 4*(\cosh(x)^3 + \cosh(x))*\sinh(x) + 1), \\
& 1/4*(2*(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 + 1)*\sinh \\
& (x)^2 + 2*\cosh(x)^2 + 4*(\cosh(x)^3 + \cosh(x))*\sinh(x) + 1)*\sqrt{-b}*\arctan \\
& (\sqrt{2}*\sqrt{-b}*\sqrt{((a + b)*\cosh(x)^4 + (a + b)*\sinh(x)^4 + 4*(a - b) \\
& *\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 + 2*a - 2*b)*\sinh(x)^2 + 3*a + 3*b)/(\cosh \\
& (x)^4 - 4*\cosh(x)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 - 4*\cosh(x)*\sinh(x)^3 \\
& + \sinh(x)^4)))/(b*\cosh(x)^4 + 4*b*\cosh(x)*\sinh(x)^3 + b*\sinh(x)^4 - 2*b*\cosh \\
& (x)^2 + 2*(3*b*\cosh(x)^2 - b)*\sinh(x)^2 + 4*(b*\cosh(x)^3 - b*\cosh(x))*\sinh \\
& (x) + b)) + (\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 2*(3*\cosh(x)^2 + \\
& 1)*\sinh(x)^2 + 2*\cosh(x)^2 + 4*(\cosh(x)^3 + \cosh(x))*\sinh(x) + 1)*\sqrt{a + \\
& b}*\log(((a^2 + 2*a*b + b^2)*\cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*\cosh(x)*\sinh \\
& (x)^7 + (a^2 + 2*a*b + b^2)*\sinh(x)^8 + 4*(a^2 - b^2)*\cosh(x)^6 + 4*(7*(a^2 \\
& + 2*a*b + b^2)*\cosh(x)^2 + a^2 - b^2)*\sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2) \\
& *\cosh(x)^3 + 3*(a^2 - b^2)*\cosh(x))*\sinh(x)^5 + 2*(3*a^2 + 2*a*b + 3*b^2)*\cosh \\
& (x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*\cosh(x)^4 + 30*(a^2 - b^2)*\cosh(x)^2 + \\
& 3*a^2 + 2*a*b + 3*b^2)*\sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*\cosh(x)^5 + 10 \\
& *(a^2 - b^2)*\cosh(x)^3 + (3*a^2 + 2*a*b + 3*b^2)*\cosh(x))*\sinh(x)^3 + 4*(a^2 \\
& - b^2)*\cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*\cosh(x)^6 + 15*(a^2 - b^2)*\cosh \\
& (x)^4 + 3*(3*a^2 + 2*a*b + 3*b^2)*\cosh(x)^2 + a^2 - b^2)*\sinh(x)^2 + \sqrt{2} \\
& (2)*((a + b)*\cosh(x)^4 + 4*(a + b)*\cosh(x)*\sinh(x)^3 + (a + b)*\sinh(x)^4 + \\
& 2*(a - b)*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 + a - b)*\sinh(x)^2 + 4*((a + b) \\
&)*\cosh(x)^3 + (a - b)*\cosh(x))*\sinh(x) + a + b)*\sqrt{a + b}*\sqrt{((a + b)*\cosh \\
& (x)^4 + (a + b)*\sinh(x)^4 + 4*(a - b)*\cosh(x)^2 + 2*(3*(a + b)*\cosh(x)^2 \\
& + 2*a - 2*b)*\sinh(x)^2 + 3*a + 3*b)/(\cosh(x)^4 - 4*\cosh(x)^3*\sinh(x) + 6*\cosh \\
& (x)^2*\sinh(x)^2 - 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4)) + a^2 + 2*a*b + b^2 \\
& + 8*((a^2 + 2*a*b + b^2)*\cosh(x)^7 + 3*(a^2 - b^2)*\cosh(x)^5 + (3*a^2 + 2*a \\
& *b + 3*b^2)*\cosh(x)^3 + (a^2 - b^2)*\cosh(x))*\sinh(x))/(\cosh(x)^4 + 4*\cosh(x) \\
&)^3*\sinh(x) + 6*\cosh(x)^2*\sinh(x)^2 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4)) - 2 \\
& *\sqrt{2}*\sqrt{((a + b)*\cosh(x)^4 + (a + b)*\sinh(x)^4 + 4*(a - b)*\cosh(x)^2
\end{aligned}$$

+ 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1), -1/2*((cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*sqrt(-a - b)*arctan(sqrt(2)*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 6*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(x)^2 + 3*a^2 + 6*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2 - b^2)*cosh(x)^4 + 9*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)^5 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))) - (cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*sqrt(-b)*arctan(sqrt(2)*sqrt(-b)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/(b*cosh(x)^4 + 4*b*cosh(x)*sinh(x)^3 + b*sinh(x)^4 - 2*b*cosh(x)^2 + 2*(3*b*cosh(x)^2 - b)*sinh(x)^2 + 4*(b*cosh(x))^3 - b*cosh(x))*sinh(x) + b)) + sqrt(2)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tanh(x)^4 + a} \tanh(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)*(a+b*tanh(x)^4)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*tanh(x)^4 + a)*tanh(x), x)

maple [A] time = 0.12, size = 116, normalized size = 1.30

$$\frac{\sqrt{a + b(\tanh^4(x))}}{2} - \frac{\sqrt{b} \ln\left(2\sqrt{b}(\tanh^2(x)) + 2\sqrt{a + b(\tanh^4(x))}\right)}{2} + \frac{a \operatorname{arctanh}\left(\frac{2b(\tanh^2(x)) + 2a}{2\sqrt{a+b}\sqrt{a+b(\tanh^4(x))}}\right)}{2\sqrt{a+b}} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)*(a+b*tanh(x)^4)^(1/2),x)

[Out] -1/2*(a+b*tanh(x)^4)^(1/2)-1/2*b^(1/2)*ln(2*b^(1/2)*tanh(x)^2+2*(a+b*tanh(x)^4)^(1/2))+1/2*a/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))+1/2*b/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \tanh(x)^4 + a} \tanh(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)*(a+b*tanh(x)^4)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*tanh(x)^4 + a)*tanh(x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tanh(x) \sqrt{b \tanh(x)^4 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)*(a + b*tanh(x)^4)^(1/2),x)

[Out] int(tanh(x)*(a + b*tanh(x)^4)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \tanh^4(x)} \tanh(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)*(a+b*tanh(x)**4)**(1/2),x)

[Out] Integral(sqrt(a + b*tanh(x)**4)*tanh(x), x)

$$3.261 \quad \int \frac{\tanh(x)}{\sqrt{a+b \tanh^4(x)}} dx$$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{a+b \tanh^2(x)}{\sqrt{a+b} \sqrt{a+b \tanh^4(x)}}\right)}{2\sqrt{a+b}}$$

[Out] 1/2*arctanh((a+b*tanh(x)^2)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))/(a+b)^(1/2)

Rubi [A] time = 0.08, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3670, 1248, 725, 206}

$$\frac{\tanh^{-1}\left(\frac{a+b \tanh^2(x)}{\sqrt{a+b} \sqrt{a+b \tanh^4(x)}}\right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]/Sqrt[a + b*Tanh[x]^4], x]

[Out] ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(2*Sqrt[a + b])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 1248

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_.))^p, x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(x)}{\sqrt{a + b \tanh^4(x)}} dx &= \text{Subst} \left(\int \frac{x}{(1-x^2)\sqrt{a+bx^4}} dx, x, \tanh(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1-x)\sqrt{a+bx^2}} dx, x, \tanh^2(x) \right) \\
&= - \left(\frac{1}{2} \text{Subst} \left(\int \frac{1}{a+b-x^2} dx, x, \frac{-a-b \tanh^2(x)}{\sqrt{a+b \tanh^4(x)}} \right) \right) \\
&= \frac{\tanh^{-1} \left(\frac{a+b \tanh^2(x)}{\sqrt{a+b} \sqrt{a+b \tanh^4(x)}} \right)}{2\sqrt{a+b}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 1.00

$$\frac{\tanh^{-1} \left(\frac{a+b \tanh^2(x)}{\sqrt{a+b} \sqrt{a+b \tanh^4(x)}} \right)}{2\sqrt{a+b}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]/Sqrt[a + b*Tanh[x]^4], x]

[Out] ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(2*Sqrt[a + b])

fricas [B] time = 0.64, size = 1286, normalized size = 32.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(1/2), x, algorithm="fricas")

[Out] [1/4*log(((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 3*b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(x)^2 + 3*a^2 + 2*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 3*b^2)*cosh(x))*sinh(x)^3 + 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2 - b^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b + 3*b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + sqrt(2)*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)^5 + (3*a^2 + 2*a*b + 3*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4))/sqrt(a + b), -1/2*sqrt(-a - b)*arctan(sqrt(2)*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(

$$\frac{(a-b)\cosh(x)^2 + 2(3(a+b)\cosh(x)^2 + 2a - 2b)\sinh(x)^2 + 3a + 3b}{(\cosh(x)^4 - 4\cosh(x)^3\sinh(x) + 6\cosh(x)^2\sinh(x)^2 - 4\cosh(x)\sinh(x)^3 + \sinh(x)^4)} \cdot \frac{1}{((a^2 + 2ab + b^2)\cosh(x)^8 + 8(a^2 + 2ab + b^2)\cosh(x)\sinh(x)^7 + (a^2 + 2ab + b^2)\sinh(x)^8 + 4(a^2 - b^2)\cosh(x)^6 + 4(7(a^2 + 2ab + b^2)\cosh(x)^2 + a^2 - b^2)\sinh(x)^6 + 8(7(a^2 + 2ab + b^2)\cosh(x)^3 + 3(a^2 - b^2)\cosh(x))\sinh(x)^5 + 6(a^2 + 2ab + b^2)\cosh(x)^4 + 2(35(a^2 + 2ab + b^2)\cosh(x)^4 + 30(a^2 - b^2)\cosh(x)^2 + 3a^2 + 6ab + 3b^2)\sinh(x)^4 + 8(7(a^2 + 2ab + b^2)\cosh(x)^5 + 10(a^2 - b^2)\cosh(x)^3 + 3(a^2 + 2ab + b^2)\cosh(x))\sinh(x)^3 + 4(a^2 - b^2)\cosh(x)^2 + 4(7(a^2 + 2ab + b^2)\cosh(x)^6 + 15(a^2 - b^2)\cosh(x)^4 + 9(a^2 + 2ab + b^2)\cosh(x)^2 + a^2 - b^2)\sinh(x)^2 + a^2 + 2ab + b^2 + 8((a^2 + 2ab + b^2)\cosh(x)^7 + 3(a^2 - b^2)\cosh(x)^5 + 3(a^2 + 2ab + b^2)\cosh(x)^3 + (a^2 - b^2)\cosh(x))\sinh(x))} (a+b)}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{b \tanh(x)^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(1/2),x, algorithm="giac")

[Out] integrate(tanh(x)/sqrt(b*tanh(x)^4 + a), x)

maple [A] time = 0.13, size = 37, normalized size = 0.92

$$\frac{\operatorname{arctanh}\left(\frac{2b(\tanh^2(x))+2a}{2\sqrt{a+b}\sqrt{a+b(\tanh^4(x))}}\right)}{2\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)/(a+b*tanh(x)^4)^(1/2),x)

[Out] 1/2/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{b \tanh(x)^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(1/2),x, algorithm="maxima")

[Out] integrate(tanh(x)/sqrt(b*tanh(x)^4 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tanh(x)}{\sqrt{b \tanh(x)^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)/(a + b*tanh(x)^4)^(1/2),x)

[Out] int(tanh(x)/(a + b*tanh(x)^4)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{\sqrt{a + b \tanh^4(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*tanh(x)**4)**(1/2),x)

[Out] Integral(tanh(x)/sqrt(a + b*tanh(x)**4), x)

$$3.262 \quad \int \frac{\tanh(x)}{(a+b \tanh^4(x))^{3/2}} dx$$

Optimal. Leaf size=74

$$\frac{\tanh^{-1}\left(\frac{a+b \tanh^2(x)}{\sqrt{a+b} \sqrt{a+b \tanh^4(x)}}\right)}{2(a+b)^{3/2}} - \frac{a-b \tanh^2(x)}{2a(a+b)\sqrt{a+b \tanh^4(x)}}$$

[Out] 1/2*arctanh((a+b*tanh(x)^2)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))/(a+b)^(3/2)+
1/2*(-a+b*tanh(x)^2)/a/(a+b)/(a+b*tanh(x)^4)^(1/2)

Rubi [A] time = 0.12, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3670, 1248, 741, 12, 725, 206}

$$\frac{\tanh^{-1}\left(\frac{a+b \tanh^2(x)}{\sqrt{a+b} \sqrt{a+b \tanh^4(x)}}\right)}{2(a+b)^{3/2}} - \frac{a-b \tanh^2(x)}{2a(a+b)\sqrt{a+b \tanh^4(x)}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]/(a + b*Tanh[x]^4)^(3/2), x]

[Out] ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(2*(a + b)^(3/2)) - (a - b*Tanh[x]^2)/(2*a*(a + b)*Sqrt[a + b*Tanh[x]^4])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 741

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[(((d + e*x)^(m + 1)*(a + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

Rule 1248

Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ

[{a, c, d, e, p, q}, x]

Rule 3670

Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_.))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(c*ff)/f, Subst[Int[((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p]/(c^2 + f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && RationalQ[n]))

Rubi steps

$$\begin{aligned}
 \int \frac{\tanh(x)}{(a + b \tanh^4(x))^{3/2}} dx &= \text{Subst} \left(\int \frac{x}{(1-x^2)(a+bx^4)^{3/2}} dx, x, \tanh(x) \right) \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1-x)(a+bx^2)^{3/2}} dx, x, \tanh^2(x) \right) \\
 &= -\frac{a-b \tanh^2(x)}{2a(a+b)\sqrt{a+b \tanh^4(x)}} + \frac{\text{Subst} \left(\int \frac{a}{(1-x)\sqrt{a+bx^2}} dx, x, \tanh^2(x) \right)}{2a(a+b)} \\
 &= -\frac{a-b \tanh^2(x)}{2a(a+b)\sqrt{a+b \tanh^4(x)}} + \frac{\text{Subst} \left(\int \frac{1}{(1-x)\sqrt{a+bx^2}} dx, x, \tanh^2(x) \right)}{2(a+b)} \\
 &= -\frac{a-b \tanh^2(x)}{2a(a+b)\sqrt{a+b \tanh^4(x)}} - \frac{\text{Subst} \left(\int \frac{1}{a+b-x^2} dx, x, \frac{-a-b \tanh^2(x)}{\sqrt{a+b \tanh^4(x)}} \right)}{2(a+b)} \\
 &= \frac{\tanh^{-1} \left(\frac{a+b \tanh^2(x)}{\sqrt{a+b} \sqrt{a+b \tanh^4(x)}} \right)}{2(a+b)^{3/2}} - \frac{a-b \tanh^2(x)}{2a(a+b)\sqrt{a+b \tanh^4(x)}}
 \end{aligned}$$

Mathematica [A] time = 0.54, size = 73, normalized size = 0.99

$$\frac{1}{2} \left(\frac{\tanh^{-1} \left(\frac{a+b \tanh^2(x)}{\sqrt{a+b} \sqrt{a+b \tanh^4(x)}} \right)}{(a+b)^{3/2}} - \frac{a-b \tanh^2(x)}{a(a+b)\sqrt{a+b \tanh^4(x)}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]/(a + b*Tanh[x]^4)^(3/2), x]

[Out] (ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(a + b)^(3/2) - (a - b*Tanh[x]^2)/(a*(a + b)*Sqrt[a + b*Tanh[x]^4]))/2

fricas [B] time = 0.81, size = 3914, normalized size = 52.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(3/2),x, algorithm="fricas")

[Out]
$$\frac{1}{4} \left((a^2 + a*b) \cosh(x)^8 + 8*(a^2 + a*b) \cosh(x) \sinh(x)^7 + (a^2 + a*b) \sinh(x)^8 + 4*(a^2 - a*b) \cosh(x)^6 + 4*(7*(a^2 + a*b) \cosh(x)^2 + a^2 - a*b) \sinh(x)^6 + 8*(7*(a^2 + a*b) \cosh(x)^3 + 3*(a^2 - a*b) \cosh(x)) \sinh(x)^5 + 6*(a^2 + a*b) \cosh(x)^4 + 2*(35*(a^2 + a*b) \cosh(x)^4 + 30*(a^2 - a*b) \cosh(x)^2 + 3*a^2 + 3*a*b) \sinh(x)^4 + 8*(7*(a^2 + a*b) \cosh(x)^5 + 10*(a^2 - a*b) \cosh(x)^3 + 3*(a^2 + a*b) \cosh(x)) \sinh(x)^3 + 4*(a^2 - a*b) \cosh(x)^2 + 4*(7*(a^2 + a*b) \cosh(x)^6 + 15*(a^2 - a*b) \cosh(x)^4 + 9*(a^2 + a*b) \cosh(x)^2 + a^2 - a*b) \sinh(x)^2 + a^2 + a*b + 8*((a^2 + a*b) \cosh(x)^7 + 3*(a^2 - a*b) \cosh(x)^5 + 3*(a^2 + a*b) \cosh(x)^3 + (a^2 - a*b) \cosh(x)) \sinh(x) \sqrt{a+b} \log\left((a^2 + 2*a*b + b^2) \cosh(x)^8 + 8*(a^2 + 2*a*b + b^2) \cosh(x) \sinh(x)^7 + (a^2 + 2*a*b + b^2) \sinh(x)^8 + 4*(a^2 - b^2) \cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2) \cosh(x)^2 + a^2 - b^2) \sinh(x)^6 + 8*(7*(a^2 + 2*a*b + b^2) \cosh(x)^3 + 3*(a^2 - b^2) \cosh(x)) \sinh(x)^5 + 2*(3*a^2 + 2*a*b + 3*b^2) \cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2) \cosh(x)^4 + 30*(a^2 - b^2) \cosh(x)^2 + 3*a^2 + 2*a*b + 3*b^2) \sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2) \cosh(x)^5 + 10*(a^2 - b^2) \cosh(x)^3 + (3*a^2 + 2*a*b + 3*b^2) \cosh(x)) \sinh(x)^3 + 4*(a^2 - b^2) \cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2) \cosh(x)^6 + 15*(a^2 - b^2) \cosh(x)^4 + 3*(3*a^2 + 2*a*b + 3*b^2) \cosh(x)^2 + a^2 - b^2) \sinh(x)^2 + \sqrt{2} * ((a+b) \cosh(x)^4 + 4*(a+b) \cosh(x) \sinh(x)^3 + (a+b) \sinh(x)^4 + 2*(a-b) \cosh(x)^2 + 2*(3*(a+b) \cosh(x)^2 + a-b) \sinh(x)^2 + 4*((a+b) \cosh(x)^3 + (a-b) \cosh(x)) \sinh(x) + a+b) \sqrt{a+b} \sqrt{((a+b) \cosh(x)^4 + (a+b) \sinh(x)^4 + 4*(a-b) \cosh(x)^2 + 2*(3*(a+b) \cosh(x)^2 + 2*a - 2*b) \sinh(x)^2 + 3*a + 3*b) / (\cosh(x)^4 - 4*\cosh(x)^3 \sinh(x) + 6*\cosh(x)^2 \sinh(x)^2 - 4*\cosh(x) \sinh(x)^3 + \sinh(x)^4))} + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2) \cosh(x)^7 + 3*(a^2 - b^2) \cosh(x)^5 + (3*a^2 + 2*a*b + 3*b^2) \cosh(x)^3 + (a^2 - b^2) \cosh(x)) \sinh(x) / (\cosh(x)^4 + 4*\cosh(x)^3 \sinh(x) + 6*\cosh(x)^2 \sinh(x)^2 + 4*\cosh(x) \sinh(x)^3 + \sinh(x)^4) - 2*\sqrt{2} * ((a^2 - b^2) \cosh(x)^4 + 4*(a^2 - b^2) \cosh(x) \sinh(x)^3 + (a^2 - b^2) \sinh(x)^4 + 2*(a^2 + 2*a*b + b^2) \cosh(x)^2 + 2*(3*(a^2 - b^2) \cosh(x)^2 + a^2 + 2*a*b + b^2) \sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2) \cosh(x)^3 + (a^2 + 2*a*b + b^2) \cosh(x)) \sinh(x) \sqrt{((a+b) \cosh(x)^4 + (a+b) \sinh(x)^4 + 4*(a-b) \cosh(x)^2 + 2*(3*(a+b) \cosh(x)^2 + 2*a - 2*b) \sinh(x)^2 + 3*a + 3*b) / (\cosh(x)^4 - 4*\cosh(x)^3 \sinh(x) + 6*\cosh(x)^2 \sinh(x)^2 - 4*\cosh(x) \sinh(x)^3 + \sinh(x)^4)} \right) / ((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3) \cosh(x)^8 + 8*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3) \cosh(x) \sinh(x)^7 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3) \sinh(x)^8 + 4*(a^4 + a^3*b - a^2*b^2 - a*b^3) \cosh(x)^6 + 4*(a^4 + a^3*b - a^2*b^2 - a*b^3 + 7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3) \cosh(x)^2) \sinh(x)^6 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3) \cosh(x)^3 + 3*(a^4 + a^3*b - a^2*b^2 - a*b^3) \cosh(x)) \sinh(x)^5 + 6*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3) \cosh(x)^4 + 2*(35*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3) \cosh(x)^4 + 3*a^4 + 9*a^3*b + 9*a^2*b^2 + 3*a*b^3 + 30*(a^4 + a^3*b - a^2*b^2 - a*b^3) \cosh(x)^2) \sinh(x)^4 + a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3) \cosh(x)^5 + 10*(a^4 + a^3*b - a^2*b^2 - a*b^3) \cosh(x)^3 + 3*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3) \cosh(x)) \sinh(x)^3 + 4*(a^4 + a^3*b - a^2*b^2 - a*b^3) \cosh(x)^2 + 4*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3) \cosh(x)^6 + 15*(a^4 + a^3*b - a^2*b^2 - a*b^3) \cosh(x)^4 + a^4 + a^3*b - a^2*b^2 - a*b^3 + 9*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3) \cosh(x)^2) \sinh(x)^2 + 8*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3) \cosh(x)^7 + 3*(a^4 + a^3*b - a^2*b^2 - a*b^3) \cosh(x)^5 + 3*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3) \cosh(x)^3 + (a^4 + a^3*b - a^2*b^2 - a*b^3) \cosh(x)) \sinh(x) \right), -1/2 * ((a^2 + a*b) \cosh(x)^8 + 8*(a^2 + a*b) \cosh(x) \sinh(x)^7 + (a^2 + a*b) \sinh(x)^8 + 4*(a^2 - a*b) \cosh(x)^6 + 4*(7*(a^2 + a*b) \cosh(x)^2 + a^2 - a*b) \sinh(x)^6 + 8*(7*(a^2 + a*b) \cosh(x)^3 + 3*(a^2 - a*b) \cosh(x)) \sinh(x)^5 + 6*(a^2 + a*b) \cosh(x)^4 + 2*(35*(a^2 + a*b) \cosh(x)^4 + 30*(a^2 - a*b) \cosh(x)^2 + 3*a^2 + 3*a*b) \sinh(x)^4 + 8*(7*(a^2 + a*b) \cosh(x)^5 + 10*(a^2 - a*b) \cosh(x)^3 + 3*(a^2 + a*b) \cosh(x)) \sinh(x)^3 + 4*(a^2 - a*b) \cosh(x)^2 + 4*(7*(a^2 + a*b) \cosh(x)^6 + 15*(a^2 - a*b) \cosh(x)^4 + 9*(a^2 + a*b) \cosh(x)^2 + a^2 - a*b) \sinh(x)^2 + a^2 + a*b + 8*((a^2 +$$

```

a*b)*cosh(x)^7 + 3*(a^2 - a*b)*cosh(x)^5 + 3*(a^2 + a*b)*cosh(x)^3 + (a^2 -
a*b)*cosh(x))*sinh(x))*sqrt(-a - b)*arctan(sqrt(2)*((a + b)*cosh(x)^4 + 4*
(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*
(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(
x))*sinh(x) + a + b)*sqrt(-a - b)*sqrt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)
^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 +
3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cos
h(x)*sinh(x)^3 + sinh(x)^4))/((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*
b + b^2)*cosh(x)*sinh(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*
cosh(x)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(
7*(a^2 + 2*a*b + b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 6*(a^2
+ 2*a*b + b^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 -
b^2)*cosh(x)^2 + 3*a^2 + 6*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^
2)*cosh(x)^5 + 10*(a^2 - b^2)*cosh(x)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(x))*si
nh(x)^3 + 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15
*(a^2 - b^2)*cosh(x)^4 + 9*(a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(
x)^2 + a^2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)
*cosh(x)^5 + 3*(a^2 + 2*a*b + b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x)
)) + sqrt(2)*((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a^
2 - b^2)*sinh(x)^4 + 2*(a^2 + 2*a*b + b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cos
h(x)^2 + a^2 + 2*a*b + b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^
3 + (a^2 + 2*a*b + b^2)*cosh(x))*sinh(x))*sqrt(((a + b)*cosh(x)^4 + (a + b)
*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + 2*a - 2*b)*sinh
(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2
- 4*cosh(x)*sinh(x)^3 + sinh(x)^4)))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*
cosh(x)^8 + 8*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)*sinh(x)^7 + (a^4
+ 3*a^3*b + 3*a^2*b^2 + a*b^3)*sinh(x)^8 + 4*(a^4 + a^3*b - a^2*b^2 - a*b^3)
)*cosh(x)^6 + 4*(a^4 + a^3*b - a^2*b^2 - a*b^3 + 7*(a^4 + 3*a^3*b + 3*a^2*b
^2 + a*b^3)*cosh(x)^2)*sinh(x)^6 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)
*cosh(x)^3 + 3*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x))*sinh(x)^5 + 6*(a^4
+ 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^4 + 2*(35*(a^4 + 3*a^3*b + 3*a^2*b^2
+ a*b^3)*cosh(x)^4 + 3*a^4 + 9*a^3*b + 9*a^2*b^2 + 3*a*b^3 + 30*(a^4 + a^3
*b - a^2*b^2 - a*b^3)*cosh(x)^2)*sinh(x)^4 + a^4 + 3*a^3*b + 3*a^2*b^2 + a*
b^3 + 8*(7*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^5 + 10*(a^4 + a^3*b
- a^2*b^2 - a*b^3)*cosh(x)^3 + 3*(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(
x))*sinh(x)^3 + 4*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^2 + 4*(7*(a^4 + 3*
a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)^6 + 15*(a^4 + a^3*b - a^2*b^2 - a*b^3)*c
osh(x)^4 + a^4 + a^3*b - a^2*b^2 - a*b^3 + 9*(a^4 + 3*a^3*b + 3*a^2*b^2 + a
*b^3)*cosh(x)^2)*sinh(x)^2 + 8*((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cosh(x)
^7 + 3*(a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x)^5 + 3*(a^4 + 3*a^3*b + 3*a^2
*b^2 + a*b^3)*cosh(x)^3 + (a^4 + a^3*b - a^2*b^2 - a*b^3)*cosh(x))*sinh(x)
]

```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{(b \tanh(x)^4 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(3/2),x, algorithm="giac")

[Out] integrate(tanh(x)/(b*tanh(x)^4 + a)^(3/2), x)

maple [C] time = 0.12, size = 431, normalized size = 5.82

$$\frac{b \left(\frac{\tanh^3(x)}{4a(a+b)} + \frac{\tanh^2(x)}{4a(a+b)} + \frac{\tanh(x)}{4a(a+b)} - \frac{1}{4(a+b)b} \right)}{\sqrt{\left(\tanh^4(x) + \frac{a}{b}\right)b}} - \frac{\operatorname{arctanh}\left(\frac{2b(\tanh^2(x)+2a)}{2\sqrt{a+b}\sqrt{a+b(\tanh^4(x))}}\right)}{2\sqrt{a+b}} - \frac{\sqrt{1-\frac{i\sqrt{b}(\tanh^2(x))}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}(\tanh^2(x))}{\sqrt{a}}}\operatorname{EllipticPi}\left(\frac{\tanh(x)}{\sqrt{a+b(\tanh^4(x))}}\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{a+b(\tanh^4(x))}}}{2(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)/(a+b*tanh(x)^4)^(3/2), x)`

[Out] $b*(1/4/a/(a+b)*\tanh(x)^3+1/4/a/(a+b)*\tanh(x)^2+1/4/a/(a+b)*\tanh(x)-1/4/(a+b)/b)/((\tanh(x)^4+a/b)*b)^(1/2)-1/2/(a+b)*(-1/2/(a+b)^(1/2)*\operatorname{arctanh}(1/2*(2*b*\tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*\tanh(x)^4)^(1/2))-1/(I/a^(1/2)*b^(1/2))^(1/2))*(1-I/a^(1/2)*b^(1/2)*\tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*\tanh(x)^2)^(1/2)/(a+b*\tanh(x)^4)^(1/2)*\operatorname{EllipticPi}(\tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),-I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)))+b*(-1/4/a/(a+b)*\tanh(x)^3+1/4/a/(a+b)*\tanh(x)^2-1/4/a/(a+b)*\tanh(x)-1/4/(a+b)/b)/((\tanh(x)^4+a/b)*b)^(1/2)-1/2/(a+b)*(-1/2/(a+b)^(1/2)*\operatorname{arctanh}(1/2*(2*b*\tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*\tanh(x)^4)^(1/2))+1/(I/a^(1/2)*b^(1/2))^(1/2))*(1-I/a^(1/2)*b^(1/2)*\tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*\tanh(x)^2)^(1/2)/(a+b*\tanh(x)^4)^(1/2)*\operatorname{EllipticPi}(\tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),-I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{(b \tanh(x)^4 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)/(a+b*tanh(x)^4)^(3/2), x, algorithm="maxima")`

[Out] `integrate(tanh(x)/(b*tanh(x)^4 + a)^(3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)}{(b \tanh(x)^4 + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(x)/(a + b*tanh(x)^4)^(3/2), x)`

[Out] `int(tanh(x)/(a + b*tanh(x)^4)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{(a + b \tanh^4(x))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(x)/(a+b*tanh(x)**4)**(3/2), x)`

[Out] `Integral(tanh(x)/(a + b*tanh(x)**4)**(3/2), x)`

$$3.263 \quad \int \frac{\tanh(x)}{(a+b \tanh^4(x))^{5/2}} dx$$

Optimal. Leaf size=118

$$\frac{3a^2 - b(5a + 2b) \tanh^2(x)}{6a^2(a+b)^2 \sqrt{a+b \tanh^4(x)}} - \frac{a - b \tanh^2(x)}{6a(a+b) (a+b \tanh^4(x))^{3/2}} + \frac{\tanh^{-1}\left(\frac{a+b \tanh^2(x)}{\sqrt{a+b} \sqrt{a+b \tanh^4(x)}}\right)}{2(a+b)^{5/2}}$$

[Out] 1/2*arctanh((a+b*tanh(x)^2)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))/(a+b)^(5/2)+1/6*(-3*a^2+b*(5*a+2*b)*tanh(x)^2)/a^2/(a+b)^2/(a+b*tanh(x)^4)^(1/2)+1/6*(-a+b*tanh(x)^2)/a/(a+b)/(a+b*tanh(x)^4)^(3/2)

Rubi [A] time = 0.20, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3670, 1248, 741, 823, 12, 725, 206}

$$-\frac{3a^2 - b(5a + 2b) \tanh^2(x)}{6a^2(a+b)^2 \sqrt{a+b \tanh^4(x)}} + \frac{\tanh^{-1}\left(\frac{a+b \tanh^2(x)}{\sqrt{a+b} \sqrt{a+b \tanh^4(x)}}\right)}{2(a+b)^{5/2}} - \frac{a - b \tanh^2(x)}{6a(a+b) (a+b \tanh^4(x))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[x]/(a + b*Tanh[x]^4)^(5/2), x]

[Out] ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(2*(a + b)^(5/2)) - (a - b*Tanh[x]^2)/(6*a*(a + b)*(a + b*Tanh[x]^4)^(3/2)) - (3*a^2 - b*(5*a + 2*b)*Tanh[x]^2)/(6*a^2*(a + b)^2*Sqrt[a + b*Tanh[x]^4])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 741

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol]
:= Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 3670

```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x],
x]}, Dist[(c*ff)/f, Subst[Int[(((d*ff*x)/c)^m*(a + b*(ff*x)^n)^p)/(c^2 + f
f^2*x^2), x], x, (c*Tan[e + f*x])/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ration
alQ[n]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(x)}{(a + b \tanh^4(x))^{5/2}} dx &= \text{Subst} \left(\int \frac{x}{(1-x^2)(a+bx^4)^{5/2}} dx, x, \tanh(x) \right) \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1-x)(a+bx^2)^{5/2}} dx, x, \tanh^2(x) \right) \\
&= -\frac{a-b \tanh^2(x)}{6a(a+b)(a+b \tanh^4(x))^{3/2}} - \frac{\text{Subst} \left(\int \frac{-3a-2b+2bx}{(1-x)(a+bx^2)^{3/2}} dx, x, \tanh^2(x) \right)}{6a(a+b)} \\
&= -\frac{a-b \tanh^2(x)}{6a(a+b)(a+b \tanh^4(x))^{3/2}} - \frac{3a^2-b(5a+2b) \tanh^2(x)}{6a^2(a+b)^2 \sqrt{a+b \tanh^4(x)}} + \frac{\text{Subst} \left(\int \frac{3a^2b}{(1-x)\sqrt{a+bx^2}} dx, x, \tanh^2(x) \right)}{6a^2b} \\
&= -\frac{a-b \tanh^2(x)}{6a(a+b)(a+b \tanh^4(x))^{3/2}} - \frac{3a^2-b(5a+2b) \tanh^2(x)}{6a^2(a+b)^2 \sqrt{a+b \tanh^4(x)}} + \frac{\text{Subst} \left(\int \frac{1}{(1-x)\sqrt{a+bx^2}} dx, x, \tanh^2(x) \right)}{2(a+b)} \\
&= -\frac{a-b \tanh^2(x)}{6a(a+b)(a+b \tanh^4(x))^{3/2}} - \frac{3a^2-b(5a+2b) \tanh^2(x)}{6a^2(a+b)^2 \sqrt{a+b \tanh^4(x)}} - \frac{\text{Subst} \left(\int \frac{1}{a+b-x^2} dx, x, \tanh^2(x) \right)}{2(a+b)} \\
&= \frac{\tanh^{-1} \left(\frac{a+b \tanh^2(x)}{\sqrt{a+b} \sqrt{a+b \tanh^4(x)}} \right)}{2(a+b)^{5/2}} - \frac{a-b \tanh^2(x)}{6a(a+b)(a+b \tanh^4(x))^{3/2}} - \frac{3a^2-b(5a+2b) \tanh^2(x)}{6a^2(a+b)^2 \sqrt{a+b \tanh^4(x)}}
\end{aligned}$$

Mathematica [A] time = 0.85, size = 113, normalized size = 0.96

$$\frac{1}{6} \left(\frac{-3a^2b \tanh^4(x) - a^2(4a + b) + b^2(5a + 2b) \tanh^6(x) + 3ab(2a + b) \tanh^2(x)}{a^2(a + b)^2 (a + b \tanh^4(x))^{3/2}} + \frac{3 \tanh^{-1} \left(\frac{a + b \tanh^2(x)}{\sqrt{a+b} \sqrt{a+b \tanh^4(x)}} \right)}{(a + b)^{5/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[x]/(a + b*Tanh[x]^4)^(5/2), x]

[Out] ((3*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4]])/(a + b)^(5/2) + (-a^2*(4*a + b)) + 3*a*b*(2*a + b)*Tanh[x]^2 - 3*a^2*b*Tanh[x]^4 + b^2*(5*a + 2*b)*Tanh[x]^6)/(a^2*(a + b)^2*(a + b*Tanh[x]^4)^(3/2)))/6

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(5/2), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{(b \tanh(x)^4 + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(5/2), x, algorithm="giac")

[Out] integrate(tanh(x)/(b*tanh(x)^4 + a)^(5/2), x)

maple [C] time = 0.14, size = 637, normalized size = 5.40

$$\frac{\left(-\frac{\tanh^3(x)}{6a(a+b)b} - \frac{\tanh^2(x)}{6a(a+b)b} - \frac{\tanh(x)}{6a(a+b)b} + \frac{1}{6(a+b)b^2} \right) \sqrt{a + b (\tanh^4(x))} + b \left(\frac{(3a+b)(\tanh^3(x))}{8a^2(a+b)^2} + \frac{(5a+2b)(\tanh^2(x))}{12a^2(a+b)^2} + \frac{(11a+5b) \tanh(x)}{24a^2(a+b)^2} \right)}{2 \left(\tanh^4(x) + \frac{a}{b} \right)^2 \sqrt{\left(\tanh^4(x) + \frac{a}{b} \right) b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)/(a+b*tanh(x)^4)^(5/2), x)

[Out] -1/2*(-1/6/a/(a+b)/b*tanh(x)^3-1/6/a/(a+b)/b*tanh(x)^2-1/6/a/(a+b)/b*tanh(x)+1/6/(a+b)/b^2)*(a+b*tanh(x)^4)^(1/2)/(tanh(x)^4+a/b)^2+b*(1/8*(3*a+b)/a^2/(a+b)^2*tanh(x)^3+1/12*(5*a+2*b)/a^2/(a+b)^2*tanh(x)^2+1/24/a^2*(11*a+5*b)/(a+b)^2*tanh(x)-1/4/(a+b)^2/b)/((tanh(x)^4+a/b)*b)^(1/2)-1/2/(a+b)^2*(-1/2/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))-1/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticPi(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2), -I*a^(1/2)/b^(1/2), (-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))-1/2*(1/6/a/(a+b)/b*tanh(x)^3-1/6/a/(a+b)/b*tanh(x)^2+1/6/a/(a+b)/b*tanh(x)+1/6/(a+b)/b^2)*(a+b*tanh(x)^4)^(1/2)/(tanh(x)^4+a/

$b)^2 + b * (-1/8 * (3 * a + b) / a^2 / (a + b)^2 * \tanh(x)^3 + 1/12 * (5 * a + 2 * b) / a^2 / (a + b)^2 * \tanh(x)^2 - 1/24 / a^2 * (11 * a + 5 * b) / (a + b)^2 * \tanh(x) - 1/4 / (a + b)^2 / b) / ((\tanh(x)^4 + a/b) * b)^{(1/2)} - 1/2 / (a + b)^2 * (-1/2 / (a + b)^{(1/2)} * \operatorname{arctanh}(1/2 * (2 * b * \tanh(x)^2 + 2 * a) / (a + b)^{(1/2)} / (a + b * \tanh(x)^4)^{(1/2)}) + 1 / (I/a^{(1/2)} * b^{(1/2)})^{(1/2)} * (1 - I/a^{(1/2)} * b^{(1/2)} * \tanh(x)^2)^{(1/2)} * (1 + I/a^{(1/2)} * b^{(1/2)} * \tanh(x)^2)^{(1/2)} / (a + b * \tanh(x)^4)^{(1/2)} * \operatorname{EllipticPi}(\tanh(x) * (I/a^{(1/2)} * b^{(1/2)})^{(1/2)}, -I * a^{(1/2)} / b^{(1/2)}, (-I/a^{(1/2)} * b^{(1/2)})^{(1/2)} / (I/a^{(1/2)} * b^{(1/2)})^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{(b \tanh(x)^4 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*tanh(x)^4)^(5/2),x, algorithm="maxima")

[Out] integrate(tanh(x)/(b*tanh(x)^4 + a)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tanh(x)}{(b \tanh(x)^4 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(x)/(a + b*tanh(x)^4)^(5/2),x)

[Out] int(tanh(x)/(a + b*tanh(x)^4)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tanh(x)}{(a + b \tanh^4(x))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(x)/(a+b*tanh(x)**4)**(5/2),x)

[Out] Integral(tanh(x)/(a + b*tanh(x)**4)**(5/2), x)

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*   is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*   antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
        "B"],
      "C"],
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      "C",
      "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```

(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
    If[ListQ[expn],
      Max[Map[ExpnType, expn]],
      If[Head[expn]===Power,
        If[IntegerQ[expn[[2]]],
          ExpnType[expn[[1]]],
          If[Head[expn[[2]]]===Rational,
            If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
              1,
              Max[ExpnType[expn[[1]], 2]],
            Max[ExpnType[expn[[1]], ExpnType[expn[[2]], 3]],
          If[Head[expn]===Plus || Head[expn]===Times,
            Max[ExpnType[First[expn]], ExpnType[Rest[expn]]],
          If[ElementaryFunctionQ[Head[expn]],
            Max[3, ExpnType[expn[[1]]],
          If[SpecialFunctionQ[Head[expn]],
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 4]],
          If[HypergeometricFunctionQ[Head[expn]],
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 5]],
          If[AppellFunctionQ[Head[expn]],
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
          If[Head[expn]===RootSum,
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
          If[Head[expn]===Integrate || Head[expn]===Int,
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
          9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]

```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
  debug:=false;

  leaf_count_result:=leafcount(result);
  #do NOT call ExpnType() if leaf size is too large. Recursion problem
  if leaf_count_result > 500000 then
    return "B";
  fi;

  leaf_count_optimal:=leafcount(optimal);

  ExpnType_result:=ExpnType(result);
  ExpnType_optimal:=ExpnType(optimal);

  if debug then
    print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
  ExpnType_optimal);
  fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
  return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
  if debug then
    print("ExpnType_result<=ExpnType_optimal");
  fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do
not as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function

```

```

# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+' or type(expn,'*') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

```

```

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

4.0.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
    asinh,acosh,atanh,acoth,asech,acsch
  ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
    gamma,loggamma,digamma,zeta,polylog,LambertW,
    elliptic_f,elliptic_e,elliptic_pi,exp_polar
  ]

def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]

```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,``^``)
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,``+``) or
    type(expn,``*``)
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

```

```

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True

```



```

        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M',
hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print (">>>>Enter expnType, expn=", expn)
        print (">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer
)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #instance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #instance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands(
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
instance(expn,Add) or instance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
    elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(4,m1) #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(5,m1) #max(5,m1)
    elif is_appell_function(expn.operator()):

```

```

        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
expn.args)))
        return max(6,m1)      #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
expn.args)))
        return max(8,m1)      #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```